Exploring The Detailed Structure Of The Local Earth's Gravity Field Using Fractal And Fourier Power Spectrum Techniques

K F Zhang[1] and W E Featherstone[2]

1. *Dept of Land Information, the RMIT University, Melbourne, Victoria 3001, Australia.*
2. *School of Spatial Sciences, Curtin University of Technology, Perth, Western Australia*

Abstract

As one of the world's oldest continents, Australia has experienced a complicated geological history and thus has a distinctive landscape and an extensively weathered topography. As such, the gravity field of Australia behaves differently to that in other countries. This paper investigates the features of the Australian gravity field. The methods used are a simple statistical comparison, Fourier power spectrum analysis and the Hurst fractal technique. Three test gravity and height profiles, which represent extreme features of the Australian gravity field (ie the Hamersley Ranges, Central Australia and The Snowy Mountains), have been selected. It is shown that the gravity field of Australia is not always correlated with the terrain and the topography often contains longer wavelength features than the gravity anomalies. It is demonstrated that the simple statistical analysis and Fourier power spectral methods are the most informative tools for measuring the smoothness of the gravity field. It is revealed that none of the free-air, Bouguer or topographic-isostatic gravity anomalies is consistently the smoothest type in Australia. Sometimes, the Bouguer anomaly is more variable than the free-air anomaly and thus should not necessarily be used for gravity field gridding in Australia.

1. Introduction

With the advent of the Navstar Global Positioning System (GPS), there is a pressing demand for a precise geoid model with which to transform GPS-derived ellipsoidal heights to orthometric heights. This has triggered a large amount of geoid-related research over the past decade. Generally, large amounts of data are involved in the evaluation of the Earth's gravity field, particularly in geoid determination and terrain reduction. For the interpretation of geophysical data, it is often convenient if the quantity to be analysed is decomposed as a sum of components, a process known as spectral analysis. The Fast Fourier Transformation (FFT) and Fast Hartely Transform (FHT) techniques (Brigham, 1988; Bracewell, 1986a, 1986b) have proven to be a very powerful tool for the efficient evaluation of gravity field convolution integrals (eg. Tziavos, 1996; Zhang *et al*, 2000) and present a very attractive alternative to the classical, time consuming numerical methods.

However, the FFT/FHT approaches normally require that gridded gravity data are utilised (Li *et al.*, 1995; Schwarz *et al*, 1990). Therefore, the irregularly spaced gravity observations must be re-sampled onto a regular grid over the area of interest. The conventional approach to such gravity gridding is to smooth the gravity field using reductions - the remove stage, interpolate these smoothed quantities onto the desired grid, then add back the appropriate gravity reduction - the restore stage (Forsberg, 1984). In many instances, the Bouguer anomalies are used for data smooth purposes prior to gravimetrical geoid computation (Bian and Zhang, 1991). However, if the reduced gravity data are not smooth, interpolation errors will result, which will then propagate into the gravimetric geoid (Hipkin, 1988).

This research investigates the special features of the Australian gravity field in terms of relative roughness of various gravity anomalies and its spectral relationships with corresponding topography. The data used are various gravity anomalies and spot heights. The methods used are fractal geometry and Fourier spectrum analysis, in addition to conventional statistics and graphical visualisation. Fourier spectral analysis and the fractal geometry methods provide a new paradigm for understanding detailed structure and features of the Earth's gravity field.

Studies show that the gravity field in Australia is relatively complicated and does not adhere to the traditional axioms. Not one of the free-air, Bouguer or (Airy-Heiskannen) topographic-isostatic gravity anomalies is consistently the smoothest in continental Australia. This suggests that in order to grid the gravity field prior to geoid determination, some hybrid approach should be used, where the smoothest gravity anomaly type is used in different areas. Obviously, this has implications for interpolation, gridding and subsequent gravimetric geoid determination in this part of the world.

2. Data sets and investigation procedures

Gravity data over the Australian continent were supplied by the Australian Geological Survey Organisation (AGSO). This includes 526,091 land and 111,396 marine gravity observations. These data have been reformatted and validated using the procedures described in Featherstone *et al.* (1996) and Zhang (1997). A digital file of approximately six million spot heights was supplied by the Australian Surveying and Land Information Group. These spot heights have been combined with the gravity observation elevations, then gridded at 1km by 1km to produce the digital terrain model (DTM). Then, various gravity anomalies are calculated using the AGSO gravity databank and the 1km by 1km DTM.

Due to the variable features of the gravity field (Zhang, 1997), three profiles were selected (cf Table 1), which exhibit different topographic and gravity features. Profile 1 (P_1) is located in the Hammersley Ranges of Western Australia, Profile 2 (P_2) in central Australia and Profile 3 (P_3) in the Snowy Mountains. Statistics over the whole continent and three specific test profiles which exhibit different topographic and gravity features are used for comparisons. The power spectral and fractal techniques are used to analyse the terrain, free-air anomalies, Bouguer and topographic-isostatic gravity anomalies across the three profiles.

Table 1 Location of the three test profiles

Profiles	P_1	P_2	P_3
Latitude	25°S	26°S	36°S
Longitude	116°~120°E	126°~132°E	145°~149°E

Four different techniques are used to determine which type of gravity anomaly is the smoothest in Australia. These are:

1. Statistics of the various gravity anomalies across different profiles
2. Direct visual comparisons of the profiles
3. Power spectrum comparison at the three profiles
4. Hurst fractal dimension comparisons

Both the power spectral and the fractal analyses are well documented in the reference (cf: Barton and Pointe, 1995; Farge et al, 1993; Brace well, 1986a). However, the fractal analysis method is a relatively new technique and offers a new paradigm for understanding detailed structure and features of the Earth's gravity field. The fractal characterisation technique was initially applied to time-based phenomena, but can also be used for elevation-like profile analysis (Russ, 1994; Mandelbrot and Benoit, 1982; Barnsley, 1988). The best known applications of fractals are to surfaces and profiles (Barton and Pointe, 1995). The fractal dimension of a surface or a profile can be used to quantify its smoothness through its fractal dimension (*ibid.*). The larger the fractal dimension, the rougher the surface or profile. There are many different approaches to estimate fractal dimensions (Russ, 1994; Barnsley, 1988). The underlying characteristic of a fractal set is the self-similarity of the scales in the sense that there are large and small scales which maintain some relation between them (Farge *et al.*, 1993). Therefore, the fractal dimension can be a useful tool to measure the roughness of a profile or a surface in terms of classifying the texture of various gravity anomaly surfaces.

Hurst, or rescaled range analysis of fractals, was initially performed on time-based historical data (Hurst *et al.,* 1965). The basic idea to construct the Hurst fractal is to use a log-log plot of the maximum differences in a given window versus the size of the window (range). The size of this window is progressively increased while the maximum difference is determined accordingly. The slope fitted to the log-log plot of the maximum differences and corresponding window size using linear regression is the slope value ($\hat{S}$). The slope value thus defined is used to determined the Hurst dimension (D).

Assuming the gravity anomalies (Δg) in a limited area to follow self-similarity, the log maximum differences of the gravity anomalies ($\log \Delta g$) and log differences ($\log L$) for a given window size (L) is related by:

$$S_i = \frac{\log_{10}(\Delta g_i)}{\log_{10}(L_i)} \tag{1}$$

and for a profile, the Hurst fractal dimension is:

$$D = 2 - \hat{S} \tag{2}$$

where S_i is the slope of the log-log graph corresponding to the i^{th} window; $\hat{S}$ is the regression slope of the log-log plot with an RMS regression error σ; D is the Hurst fractal dimension for a profile (or a surface) and Δg_i is the maximum differences of the gravity anomalies within the i^{th} window size (L_i).

3. Test results and analysis

3.1 Statistics

The first indication of the relative roughness of the Bouguer anomalies in Australia can be seen by simply comparing the statistical properties of each gravity anomaly type across the whole Australian continent.

Table 2. The statistics of land gravity anomalies over Australia (units in mGal)

	MAX	MIN	MEAN	RMS	STD
Free-air	172.89	-121.10	0.40	25.31	25.10
Bouguer	82.56	-164.00	-24.79	41.27	33.00

Generally, the refined Bouguer anomaly surface should be smoother than the free-air anomaly surface. However, the STD of the Bouguer anomalies is higher than that of free-air anomalies over the whole continent. This indicates that the Bouguer anomalies are more variable, and thus more rough, so would be expected to introduce larger interpolation errors than if free-air anomalies alone are used during the gridding process. This implies that the application of a constant topographical density model can not effectively remove the irregularities of the gravity field of the topography in Australia, and thus it is concluded that a complicated geological structure exists.

The Australian free-air anomaly shows a preponderance of negative features in the southwest and more positive features in the north and east flanks. It is found that a largely negative Bouguer anomaly field is a prominent feature of the Australian gravity (cf. Zhang, 1997). This finding is of importance for subsequent gravity field gridding and precise geoid determination.

Topographic-isostatic (TI) gravity anomalies were computed along these profiles and were also compared statistically to determine whether these are less variable than the Bouguer and free-air anomalies. Table 3 shows the statistics of the free-air, Bouguer and topographic-isostatic gravity anomalies for each test profile.

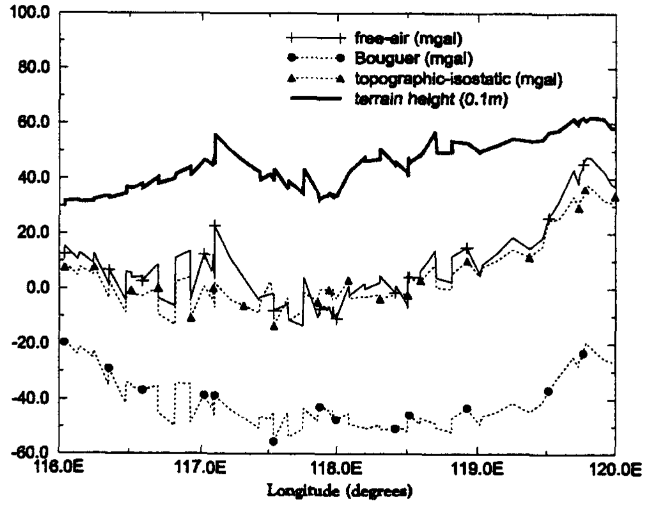
Table 3. The statistics of gravity anomalies over the three test profiles

Statistics	Profile P_1			Profile P_2			Profile P_3		
(mGal)	FA	BG	TI	FA	BG	TI	FA	BG	TI
Max-min	61.3	74.6	51.2	128.0	219.5	137.4	161.0	71.7	53.6
Mean	8.8	-40.2	4.8	-31.4	-48.4	-37.0	2.7	-20.7	1.8
Rms	27.3	64.6	25.7	99.0	48.2	52.1	24.8	25.6	12.7
Std	15.8	9.8	12.9	15.4	71.5	15.2	23.5	18.6	10.2

By comparing the standard deviations values in each of the three study areas, there is no indication that any gravity anomaly type is consistently the smoothest. For example, in the Hammersley Ranges (P_1), the Bouguer anomalies are the least variable, whereas in the Snowy Mountains (P_3), the topographic-isostatic gravity anomalies are the least variable. However, this does not provide conclusive evidence in its own right as to the roughness of these gravity anomaly types. Therefore, spectral and fractal analysis are also used to support this hypothesis.

3.2 Profiles

The profiles of the free-air, Bouguer and topographic-isostatic gravity anomalies and terrain height are plotted for profiles P_1, P_2 and P_3 in Figures 1~3 respectively. Through a simple visual inspection of the profiles in Figure 2, the Bouguer anomalies are not necessarily smoother than the free-air anomalies. This is because the topography is of longer wavelength than the gravity anomalies. This observation suggests that, in some regions of Australia, the Bouguer anomalies are not necessarily the optimum gravity anomalies to use for interpolation.



Fgiure 1 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P_1

In Figure 1, the profiles of free-air, and terrain height are very similar in shape and they thus have a strong correlation. The free-air anomaly profile is rougher than other two kinds of anomaly profiles in this area. The Bouguer anomaly profile is the smoothest.

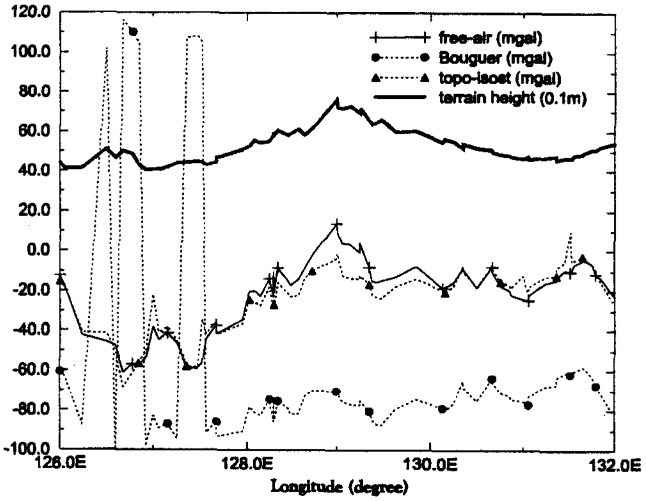


Figure 2 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P_2

For the profile P_2, it is revealed that Bouguer anomalies are relatively rough and some steep gradients exist at the intervals [126.0°E~128.0°E]. The free-air and topographic-isostatic gravity anomalies have a slightly negative correlation with terrain heights. This implies that a complicated geological structure exists in this region and ρ=2,670kgm^{-3} is not representative of the true topographic density. In the intervals [128.0°E~130.0°E] topographic-isostatic anomaly has a slight correlation with terrain height. The topographic-isostatic gravity anomaly profile is the smoothest profile in this area, whereas the free-air gravity anomaly profile is slightly rougher than the topographic-isostatic profile.

The interesting fact is that the terrain profile, in contrast to the gravity anomaly profiles, is relatively smooth which means that the topography is of a longer wavelength in nature than the gravity. Therefore, it is concluded that terrain heights do not have strong correlation with free-air gravity anomalies along this profile at high (or even medium) frequencies, suggesting that there exists a large density anomaly below this region. From this point of view, the topographical density information is very important for the terrain reduction in the application of the remove-restore techniques, and free-air gravity anomaly reduction, where the gradient of the gravity cannot be represented by the normal gravity gradient, can cause errors in the free-air reduction. In addition, the free-air anomalies are correlated with terrain height, but not as strongly as in profiles P_1 and P_2.

This finding can be confirmed by Anfiloff (1982), who used eighteen elevation and gravity profiles across Australia to study crust and tectonic processes using sparse gravity observations (~260,000). Anfiloff (*ibid.*) concluded that the gradient of the Bouguer anomalies along profile 133°E is very steep. Unfortunately, he was not able to give more analysis on several other profiles where similar features can be observed (ie. profiles 22°S, 24°S, 26°S, 29°S, 133°E and 144°E).

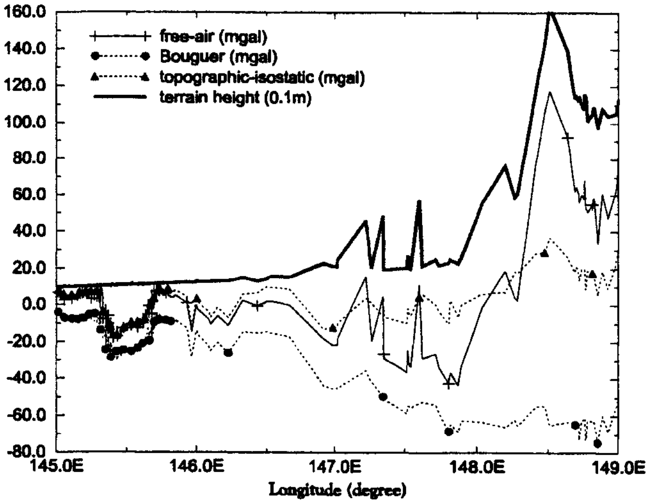


Figure 3 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P_3

As seen from Figure 3, the free-air anomalies are highly correlated with the terrain heights in the interval [146.5°E~149°E]. However, all the three types of gravity anomalies are not correlated with the terrain heights in the interval [145°E~146.5°E]. This is most probably due to the relatively flat terrain and complicated geological structure along this interval. If a usual terrain reduction is applied to the gravity observations in this region using terrain heights only, the residual gravity anomaly will become rougher rather than smoother. Terrain and free-air anomalies are very rough in this area. The Bouguer anomaly is smoother than free-air anomaly. The topographic-isostatic anomaly profile is the smoothest.

This study shows none of the gravity anomaly types are consistently smoothest in the three test profiles.

3.3 Power spectrum analysis

To analyse the spectral characteristics of the gravity anomaly profiles, and thus the gravity field, the Fourier power spectral analysis method (Brigham, 1988; Bracewell, 1986a) has been implemented. This method is very useful to analyse the detailed spectral structure of the gravity field in terms of power distribution versus wavelength. The power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies on profiles P_1, P_2 and P_3 are shown on Figures 4~6 respectively.

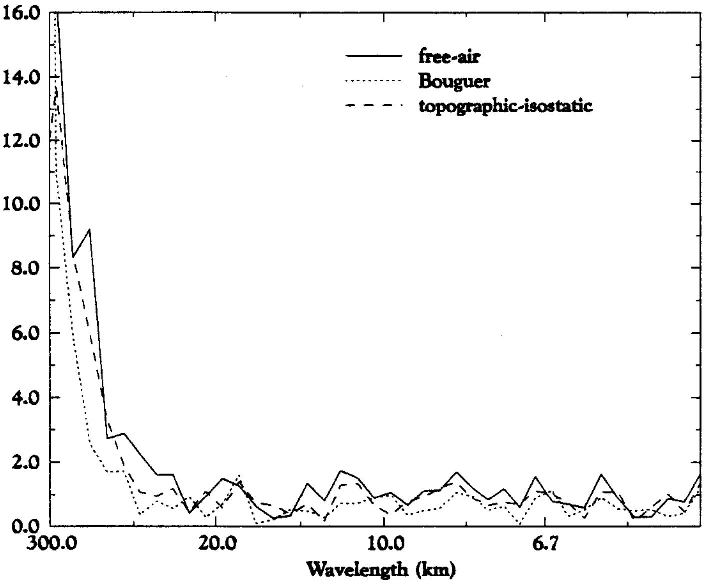


Figure 4 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies on profiles P_1

In Figure 4, all the three gravity profiles (ie. Bouguer, free-air and topographic-isostatic gravity anomalies) contain very similar power spectra in the short wavelength components (5km~20km). However, the free-air profile is the roughest and Bouguer anomaly profile is slightly smoother than that of topographic-isostatic gravity anomaly profile.

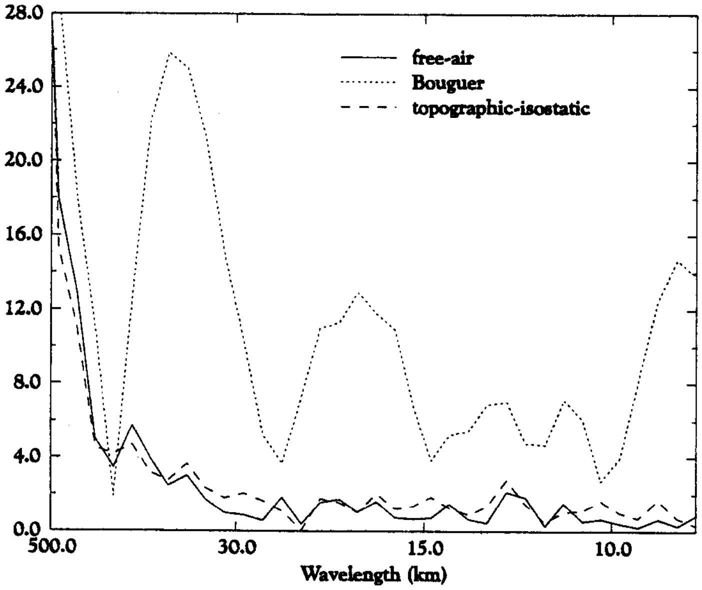


Figure 5 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies at profile P_2

In Figure 5, it is clearly seen that Bouguer anomaly is much rougher than those of free-air and topographic-isostatic anomaly profiles for the short and medium frequency parts (10km~300km). The roughness of the free-air profile is very similar to that of the topographic-isostatic anomaly. It is hard to tell which is smoother directly from Figure 5. The topographic-isostatic anomaly, however, is slightly smoother than free-air anomaly can be observed if Figure 5 is rescaled. This conclusion complies with the previous statistical analysis.

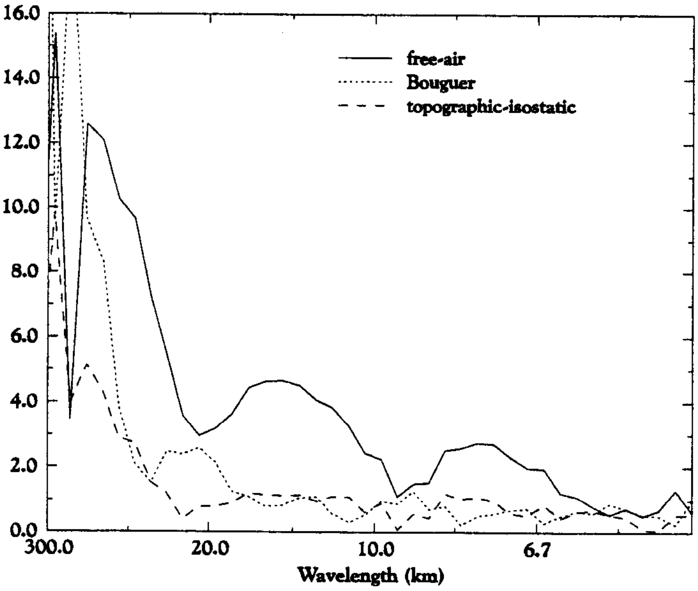


Figure 6 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P_3

From profile P_3, it is concluded that the free-air anomaly is very rough in the short wavelength range (<~50km). The Bouguer and topographic-isostatic profiles are relatively smooth in this wavelength band. However, the topographic- isostatic anomaly profile is slightly smoother than that of Bouguer anomaly.

3.4 Hurst Fractal Dimension

The fractal dimension of a surface can be used to quantify its relative smoothness. The Hurst fractal dimension (Russ, 1994) uses the logarithm of the difference in quantities versus the logarithm of their direct separation. The gradient of this line can give an estimate of the relative smoothness of quantities. This approach has been applied to gravity and terrain profiles across the Australian continent and the results from two typical profiles at P_1, P_2 and P_3 are summarised in Table 4. One advantage using the Hurst fractal dimension method is that the gravity anomaly and terrain height profiles can be inter-comparable, otherwise not possible for other techniques.

Table 4. The Hurst fractal dimension at P_1,P_2 and P_3

Profiles	P_1	P_2	P_3
free-air	1.9907±0.0026	1.9985±0.0001	1.9517±0.0071
Bouguer	1.9920±0.0020	1.9999±0.0000	1.9616±0.0047
TI	1.9923±0.0022	1.9985±0.0001	1.9517±0.0071
terrain	1.9773±0.0028	1.9527±0.0071	1.9165±0.0075

In Table 6, the Hurst fractal dimension indicates that the free-air gravity anomalies are the smoothest at both P_1 and P_2. For these, and most other profiles in Australia, the free-air anomalies appear smoother (low fractal dimension) than the Bouguer anomalies (higher fractal dimension). This implies that interpolation errors could be decreased by using free-air anomalies alone, which also reduce the computational requirements when gridding gravity data prior to geoid determination.

Note that the terrain is smoother than the free-air anomalies, when they are expected be closely correlated, and thus equally rough. This feature suggests that the rougher terrain information is effectively contaminating the Bouguer anomalies and making them rougher instead of smoother. The gravity anomaly is more variable than the terrain which indicates that the terrain in Australia is of long wavelength in nature.

4. Reasons for the Complicated Australian Gravity Field

Australia is one of the world's oldest continents, which has a complicated geological history and a relatively smooth landscape. The highest mountain in Australia is 2228m (Mt Kosciusko) and the lowest point is -15m (Lake Eyre). Correspondingly, the gravity field of Australia behaves quite differently to that in other countries. The gravity field is variable and not always correlated with the terrain. This explains why the Bouguer and terrain corrections will not necessarily smooth the gravity field.

According to this study, it is evident that none of the three gravity anomaly types tested are necessarily the smoothest over the Australian continent. It is most likely that the highly variable gravity field in Australia is due to the complicated geological structure of the continent in conjunction with the topography, which has been highly weathered and is thus relatively smooth. Despite this, the use of terrain data does not necessarily smooth the gravity field and appears to make it more rough. Therefore, the refinement of the Australian gravity field, especially for gravity field gridding prior to geoid determination is a delicate task.

5. Concluding Remarks

The gravity field of Australia has been found to be unusual. The gravity is not strongly correlated with the terrain but more so with the geology in some regions. It is also revealed that the Australian gravity field is highly variable, which is likely due to the complicated geological structure of the continent in conjunction with the relatively smooth topography. None of the gravity anomalies are consistently the smoothest over Australia. The Bouguer and topographical-isostatic anomalies are not necessary smoother than free-air gravity anomalies in Australia. The roughness of the three kinds of gravity anomalies behaves quite differently in the three test areas. More importantly, this finding may also be pertinent to other parts of the world. Therefore the relative roughness of different gravity anomalies should be assessed before these data are gridded.

The power spectral analysis gives results consistent with the statistical comparisons. However, the power spectral analysis can give more detailed information for specific frequencies in the gravity field. The Hurst fractal dimension method can only approximately indicate the relative roughness of the surfaces. Due to regression errors in the estimation of the fractal dimension, the simple 2-D Hurst analysis is not always informative.

Of the three methods used for quantifying the smoothness of the various gravity anomalies, direct statistical comparison and Fourier power spectral method are the most informative. The statistical analysis method is simple, intuitive and easily implemented. The power spectral method also performs very well to scrutinise the detailed spectral distribution. The Hurst fractal analysis can also give relative roughness information approximately. However, when the smoothness of the profiles/surfaces is close, due to regression errors, the Hurst method does not perform very well. Therefore, the simple statistical comparison and Fourier power spectral analysis methods are recommended for the analysis of the gravity anomalies.

Given the complex nature of the Australian gravity field, 3-D density model is required for further refinement of the Australian gravity field where possible.

Acknowledgments

The authors wish to thank all individuals and agencies for kindly providing the data, particularly AGSO and AUSLIG. This research is financially supported by the Australian Research Council through large grant A49331318.

References

Anfiloff, V. (1982) Elevation and Gravity Profiles Across Australia: Some Implications for Tectonism, *BMR Journal of Australian Geology and Geophysics*, Vol. 7, pp. 47-54.

Barnsley, M. (1988) *Fractals Everywhere*, Boston Academic Press, USA, 394 pp.

Barton, C. and Pointe, R.P.L. (1995) *Fractals in the Earth Sciences*, Plenum Press, New York, 265pp.

Bian, SF and Zhang, KF (1991) Remarks on the function of terrain reduction in the accurate determination of the Chinese geoid, *Journal of New Progress in Geophysics*, Vol. 6, No. 4, pp. 64-73.

Bracewell, R.N. (1986a) *The Fourier Transform and its Applications*, Second Edition, McGraw-Hill, New York, 474pp.

Bracewell, R.N. (1986b) *The Hartley Transform*, Oxford University Press, Oxford, 160 pp.

Brigham, E.O. (1988) *The Fast Fourier Transform*, Prentice-Hall, London, 252pp.

Farge, M., Hunt, J.C.R. and Vassilicos, J.C. (eds.) (1993) *Wavelets, Fractals and Fourier Transforms*, Clarendon Press, Oxford, 403pp.

Featherstone, W.E., Kearsley, A.H.W. and Gilliland, J.R. (1996) Data preparations for a new Australian gravimetric geoid. *The Australian Surveyor*, Vol. 41, No. 2.

Forsberg, R. (1984) A study of terrain reduction, density anomalies and geophysical inversion methods in gravity field approximation. *Report No. 355, Department of Geodetic Science and Surveying*, Ohio State University, USA, 129p.

Hipkin, R.G. (1988) Bouguer anomalies and the geoid: a re-assessment of Stokes's method. *Geophysical Journal International,* No. 92, pp. 53-66.

Hurst, H. E., Black, R.P. and Simaika, Y.M. (1965) *Long Term Storage: An Experimental Study*, London, Constable, 145 pp.

Li, Y.C., Sideris, M.G. and Schwarz, K-P. (1995) A numerical investigation on height anomaly prediction in mountainous areas. *Bulletin Geodesique*, Vol. 69, pp.143-156.

Mandelbrot, B. and Benoit, B. (1982) *The Fractal Geometry of Nature*, W.H. Freeman, San Francisco, 468 pp.

Russ, J.C. (1994) *Fractal Surfaces*, Plenum Publishing Corporation, New York, USA, 309p.

Schwarz, K.P., Sideris, M.G. and Forsberg, R. (1990) The use of FFT techniques in physical geodesy, *Geophysical Journal International*, No. 100, pp. 485-514.

Tziavos, I.N. (1996) Comparisons of Spectral Techniques for Geoid Computations Over Large Regions, *Journal of Geodesy*, Vol. 70, pp. 357-373.

Zhang, KF (1997) An Evaluation of geoid determination techniques and their application to Australian GPS heighting, *Ph.D. Thesis*, School of Surveying and Land Information, Curtin University of Technology, Australia.

Zhang KF, Dodson AH and Chen W (2000) Precision evaluation of the geoid determination through FFT technique, *Physics and Chemistry of the Earth*, Vol. 25, No.1 (to appear).