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The effect of 'noise' on estimates of the elastic thickness of the continental lithosphere by the coherence method

C. J. Swain[1] and J. F. Kirby

Dept. of Spatial Sciences, Curtin Univ., Australia

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[1] We model the lithosphere as a uniform elastic plate overlying an inviscid fluid and loaded with both surface and subsurface fractal loads to generate synthetic topography and gravity data. To simulate data having low (topographic) signal to (gravity) noise ratio we use an algebraically larger exponent for the subsurface load in the spectral synthesis fractal algorithm.The gravity power spectrum then decays less rapidly than that of topography, the spectra resembling those for central Australia. We find that the coherence method using multitaper spectral estimation yields significant underestimates of plate thickness for both low and normal signal-to-noise ratio data, unless the data window is larger than several times the true flexural wavelength. We quantify this bias for the parameters used here and apply it as a correction to an effective elastic thickness estimate for central Australia, obtaining a value of 115 ± 25 km. *INDEX TERMS:* 8110 Tectonophysics: Continental tectonics—general (0905); 8159 Tectonophysics: Rheology—crust and lithosphere; 1236 Geodesy and Gravity: Rheology of the lithosphere and mantle (8160). **Citation:** Swain, C. J., and J. F. Kirby, The effect of 'noise' on estimates of the elastic thickness of the continental lithosphere by the coherence method, *Geophys. Res. Lett.*, *30*(11), 1574, doi:10.1029/2003GL017070, 2003.

1. Introduction

[2] A widely used model for the isostatic response of the lithosphere is a thin elastic plate overlying a fluid asthenosphere [*Watts*, 2001]. In this model the plate responds by flexure to loads both at the surface, i. e., topography, and beneath the surface. Evidence for the latter (as well as for the plate flexure itself) can be seen in gravity anomalies. Two statistical methods for estimating the flexural rigidity of the plate D (or its effective elastic thickness T_e) involve modeling the observed admittance and coherence between gravity and topography. These are usually calculated from rectangular grids using the FFT, with the assumption of an isotropic isostatic mechanism so the admittance and coherence are functions of scalar wavenumber k. In the admittance method the model values are calculated from simple formulae for a layered earth involving densities and depths of the layers as well as T_e and an assumed ratio f of initial subsurface-to-surface loading. In *Forsyth*'s [1985] coherence method T_e and $f(\mathbf{k})$ are determined iteratively. A T_e value is first assumed, then the topography and Bouguer gravity used to solve for the initial loads, which are assumed to be statistically independent and used to calculate a predicted coherence. A minimum misfit model is then found. Thus $f(\mathbf{k})$ is estimated directly from the 2-D gravity and topography grids and does not have to be assumed - a distinct advantage. The coherence typically rolls off smoothly from 1, for long wavelength topographic features that are fully compensated, to 0 at short (i.e., uncompensated) wavelengths, over a narrow band whose central wavenumber, where the coherence is 0.5, is characteristic of the elastic thickness. Coherence is much less affected by f than is admittance.

[3] The coherence method has been used extensively for estimating T_e of whole tectonic provinces [e. g., *Zuber et al.*, 1989] and for systematic mapping of T_e [e. g., *Lowry and Smith*, 1994], yielding values between 5 and 134 km [*Watts*, 2001, Table 5. 2], with cratons commonly giving values >50 km. However, *McKenzie and Fairhead* [1997] have stated that for stable continental regions T_e should not be expected to much exceed 25 km on the basis that (a) focal depths for earthquakes under continents are always <30 km and (b) geotherms indicate a <50 km depth to the 450°C isotherm (assumed to correspond to T_e in oceanic regions). They argue, moreover, that use of the coherence method over shields often yields upper bounds, rather than estimates, of T_e, because erosion has reduced the topography, and hence the coherence, at the intermediate wavelengths. If the coherence would otherwise have been appreciable at these wavelengths this will push the roll-off to longer wavelengths. To test if this is the case, they advocate comparing the power spectrum of the free air gravity, which here constitutes the "noise" and is due to loads in the crust, with that of the Bouguer correction ($2\pi G\rho h$). They refer to the latter as the "uncompensated gravity from the topography", which is effectively the "signal". If the signal is substantially smaller than the noise at intermediate wavelengths, they consider that the method will not give a valid estimate of T_e but only an upper bound. *Banks et al.* [2001] interpret the ratio of noise-to-signal here as equivalent to f. For a method where T_e is estimated by fitting an observed coherence curve with a model having a fixed value of f, they argue that underestimating f pushes up the roll-off wavelength, as predicted by *McKenzie and Fairhead* [1997], but only if f < 0.1. If f > 1 it has the opposite effect. However, two points that are not clear from this argument are firstly how this applies to Forsyth's method, in which $f(\mathbf{k})$ is not an assumed constant but is derived directly from the Bouguer gravity and topography, and secondly whether this really answers the argument of *McKenzie and Fairhead* [1997]. Judging by a recent comment by *Lamb*

[1]Permanently at 20 Bedwell Crescent, Booragoon, W. A. 6154, Australia.

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[2002], others have also remained unconvinced by the points made by *Banks et al.* [2001].

[4] In this paper we show, by means of synthetic models which simulate these low S/N ratios, that there is no reason to reject T_e estimates made by Forsyth's method in such cases. Thus it appears that previous T_e estimates for cratons of >100 km made with the coherence method are valid (to the extent that the model of an elastic plate over a fluid is valid).

2. Synthetic Models

[5] We have used a method similar to one described by *Macario et al.* [1995] for generating grids of topography and gravity from a model consisting of uncorrelated random fractal surface and subsurface loads emplaced on a thin elastic plate. The power spectra of such surfaces are very similar to those of real continental topography. We used the method of "spectral synthesis" [*Peitgen and Saupe*, 1988] for the initial loads, and for the subsurface load have followed *Banks et al.* [2001] and assumed a thin, shallow sheet of variable density, rather than Moho undulations as in *Forsyth*'s [1985] model. This seems to us more realistic for simulating "noise" due to density anomalies in the crust. We used equations 16–18 from *Banks et al.* [2001] to calculate plate flexure and the resulting topography and gravity anomaly.

[6] In order to produce models with "normal" S/N ratios we made $f = 1$ and used a spectral exponent of -3 for both loads, implying a fractal dimension of 2.5, as was used by *Macario et al.* [1995]. However, they note that use of any value between 2.3 and 2.7 makes no significant difference to the results. For models that aim to reproduce the behavior of power spectra in regions of low S/N ratio such as western Australia [*McKenzie and Fairhead*, 1997, Figures 2b, 3r] we increased the spectral exponent for the subsurface load only, to -2. This produces a subsurface load that is much "rougher" than the surface load. We also scaled the subsurface load by a factor of 2 after standardizing both loads to unit variance [*Macario et al.*, 1995]. The resulting f is about 1 at the longest wavelengths increasing to >10 at the shortest. Figure 1 compares gravity and topography spectra of a typical synthetic model (with low S/N ratio) to spectra for Central Australia.

3. Spectral Estimation

[7] Many T_e studies based on the coherence method have followed *Bechtel et al.* [1987] and mirrored rectangular data sets in their edges. This removes first order discontinuities that would otherwise cause leakage in spectral estimates principally in the higher wavenumbers (where the signals are smallest), but leaves discontinuities in the derivatives and in trends in the data which can produce anomalies in the longer wavelength spectral estimates. These can be important in the coherence method where one is frequently trying to measure a roll-off at wavelengths close to the longest in the data.

[8] *Macario et al.* [1995], although they do not explicitly state this, appear to have used mirroring for calculating their spectral estimates, since their longest wavelengths are twice the width of their data grids. Their results, based on synthetic data, show that the method does not suffer from bias, unless the initial loads are correlated.

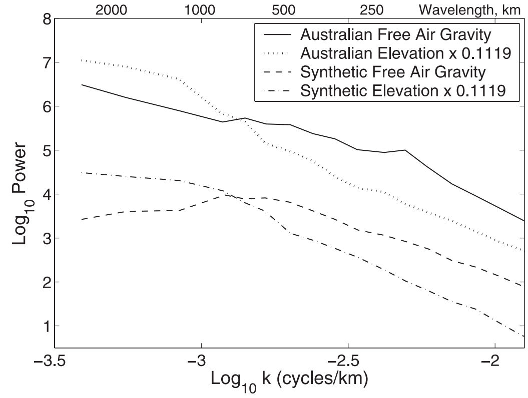

Figure 1. Multitaper ($K = 2$, $NW = 2$) power spectra of free air gravity and topography for Australian and synthetic data. Both data sets are for 2500 km square areas. The Australian area is centred near Alice Springs. The synthetic data are for a model with a T_e of 120 km and a low S/N ratio (see text). The elevations have been multiplied by $2\pi G\rho$.

[9] *McKenzie and Fairhead* [1997] used the method of 2-D multitapers for estimating both admittance and coherence. Multitaper spectral estimation should minimize leakage and moreover, by using a number of windows, it can generate several independent estimates of each spectral quantity, and hence an average with a smaller variance. However, *Simons et al.* [2000] note that their T_e estimates made with multitapers are lower than those made by mirroring. *McNutt* [1983] noticed this problem in the context of admittance estimates using a single Gaussian or cosine taper. She explained it terms of the taper "forcing a perfect correlation between gravity and topography at wavelengths corresponding to the width of the taper" which would lead to a larger estimate of the coherence, a decrease in the transition wavelength and hence to underestimating T_e.

[10] We have quantified this bias by carrying out a similar analysis to that of *Macario et al.* [1995] with synthetic data sets but using multitapers instead of mirroring. We found that, unless the data window is more than several times the true flexural wavelength λ, T_e estimates are always biased downward by an amount that increases with decreasing window size, for a given T_e. [Note that $\lambda = \pi(4D/\Delta\rho g)^{1/4} \sim 29T_e^{3/4}$ for T_e in km, with $\Delta\rho$ = 500 kg/m^3, a Young's modulus of 10^{11} (Pa) and Poisson's ratio of 0.25].

[11] By generating sets of 100 random synthetic data for a range of T_e values, and for a specific loading model, multitaper parameters (K, NW) and window width, and then inverting each pair to get a T_e estimate (see section 5), we have constructed "error curves" of apparent versus true T_e. Figure 2 shows two such curves based on models with normal and low S/N ratios. We have produced a number of such curves for different parameters and also for the admittance method: The latter are quantitatively quite similar to those for coherence.

[12] We have found that the bias increases with both the number of tapers K and the resolution bandwidth NW [*Simons et al.*, 2000]. For the power spectra and coherence results shown below we have minimized the bias by using K = 2, NW = 2, which are probably the smallest useful values. As a rule of thumb, for K = 3, NW = 3, the bias is

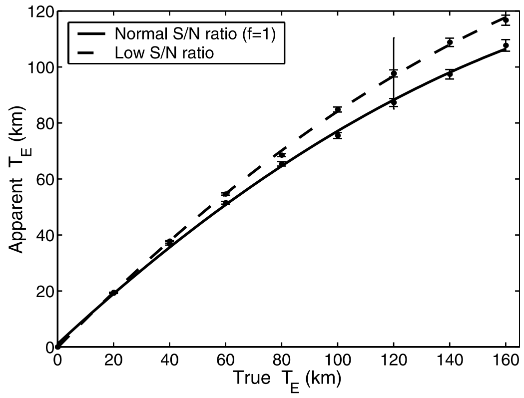

Figure 2. Error curves for multitaper coherence T_e estimates (with $K = 2$, $NW = 2$) for a square window of width 2200 km. Each point is the result of applying the T_e estimation program to a set of 100 pairs of synthetic topography and gravity grids. The curves are second degree polynomials. The error bars are standard errors (s.e.). The larger error bar is 1 standard deviation (10 s.e.) and illustrates the effect of random errors on a single T_e estimate.

typically 50% larger. Halving the (square) window width roughly doubles the bias, provided the width is still no smaller than λ.

4. Real and Synthetic Power Spectra

[13] Power spectra for Australian data are shown in Figure 1, together with those for a typical synthetic model with a T_e value of 120 km and a low S/N ratio. We used load and Moho depths of 3 km and 33 km here, but these are not critical. Although there are some differences between the two pairs of spectra, they do show essentially the same behavior in the wavelength range 200–1000 km, where the S/N ratio is much the same. We have produced such spectra for models with different T_e values and note that all of them show a crossover between the free air and "uncompensated topography" spectra. The wavelength at which this occurs is directly related to the flexural wavelength (and hence to T_e) because at shorter wavelengths the plate supports the loads while at longer wavelengths it deflects under them, compensating both the surface topography and the subsurface load. This decreases the power in the free air gravity but increases the power in the topography because the subsurface loads are much greater.

5. Predicting Coherence

[14] Because our model assumes internal loading by a thin sheet in the upper crust, whereas *Forsyth*'s [1985] assumes initial undulations on the Moho, we use modified "load deconvolution" equations [*Lowry and Smith*, 1994]. Using the formulation of the problem given by *Banks et al.* [2001] we solve simultaneously their equations (7) and (8) to determine the initial loads from the topography and gravity grids. The coefficients in these equations are obtained from their equations (19), but unfortunately these appear to be wrong. They can be corrected by replacing $B(|k|)/(\rho_0 g)$ in each of their equations by $(Dk^4 + \rho_m g)^{-1}$, where ρ_0, ρ_m are crust and mantle density and g is gravitational acceleration. The predicted coherence was calculated assuming uncorrelated initial loads, using equation 25 from *Forsyth* [1985].

[15] The load deconvolution was performed on the same (tapered) data windows as were used for calculating the observed coherence. We followed *Macario et al.* [1995] in using an automatic line-search procedure to find T_e, by minimizing the penalty function given by *McKenzie and Fairhead* [1997, Equation 13].

6. Results From the Coherence Method

[16] Figure 2 summarises the results of applying the coherence method using multitapers (with $K = 2$, $NW = 2$) to synthetic data. We see that multitaper T_e estimates for the low S/N data show slightly less downward bias than those for the data with normal S/N ratio (i. e., $f = 1$).

[17] Figure 3 shows observed and best-fitting predicted coherence curves for central Australia, which are remarkably similar to typical curves for synthetic data for T_e = 120 km (summarised in Figure 2). From this apparent T_e estimate of 93 km we used the low S/N curve in Figure 2 to estimate a true T_e of 115 ± 25 km, estimating the error by reading off the "True T_e" values corresponding to the ends of the 1SD error bar.

7. Admittance for Central Australia

[18] There is other evidence for $T_e > 100$ km in Australia. In Figure 4 we show the observed Bouguer admittance $-Q(k)$ for the same area as Figure 3, together with predicted admittance for a model including both surface and subsurface loading, calculated as described by *Banks et al.* [2001] but using the corrected equations (2). Here we have used multitapers with $K = 4$, $NW = 3$, in order to reduce the variance of the estimates, and accepted the larger bias in the T_e estimate that this produces. The roll-off at 500 km wavelength is very clear and this largely controls the T_e estimate of 70 km: Correcting for the multitaper bias would take this to over 100 km. For the model we have assumed $f = 1$ except for the point at 200 km wavelength

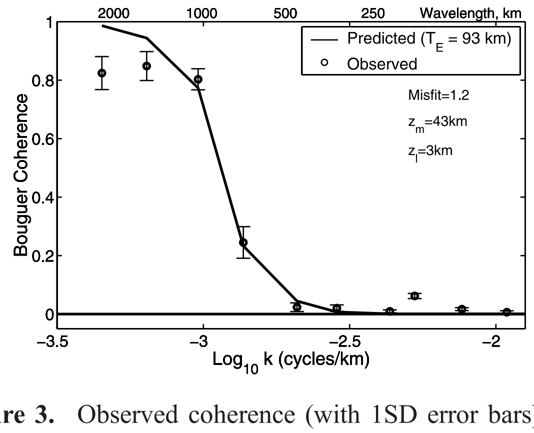

Figure 3. Observed coherence (with 1SD error bars) and best-fitting predicted coherence, using multitapers ($K = 2$, $NW = 2$), for gravity and topography data over a 2200 km square area of central Australia.

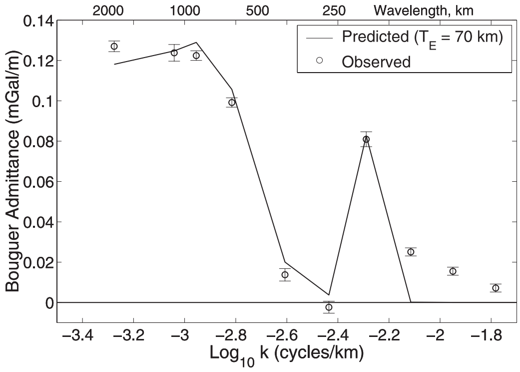

Figure 4. Bouguer admittance, using multitapers (K = 4, NW = 3), for the same data as Figure 4. The observed admittances have been fitted with a model having both surface topography and subsurface loading consisting of a thin layer of variable density at a depth of 5 km. The ratio of subsurface-to-surface loading (f) is 1 at all wavelengths except at 200 km where it is 10.

where we have used f = 10 to illustrate how subsurface loading can affect the admittance.

[19] We have been unable to reproduce the admittance result for western Australia in *McKenzie and Fairhead* [1997], despite using free air admittance, the same area as theirs and a linear wavenumber scale, as in their Figure 7o. Calculating the admittance from free air anomalies produces $Q + 2\pi G\rho_0$ as expected [*Banks et al.*, 2001], though the error estimates are slightly different from those of Q. For other areas of Australia we observed roll-off at similar wavelengths as long as the areas are larger than about 1600 km square, but the points at shorter wavelengths can be somewhat different, which may be due to variations in subsurface loading.

8. Discussion

[20] All the models discussed so far are isotropic. However, *Simons et al.* [2000] have shown by mapping the Bouguer coherence in 2-D that the isostatic response of the central Australian lithosphere is anisotropic. Their Plate 3b shows that the roll-off wavenumber is smaller in the N-S direction than E-W. We have confirmed this by computing the 2-D multitaper coherence (with K = NW = 3) for our 2200 km square window. Our coherence contours are roughly elliptical with a WNW-ESE long axis. Thus our T_e estimate is an average, since it was obtained by azimuthal averaging. The fact that the average T_e given in *Simons et al.* [2000] is so low (35 km) is readily explained by extension of our synthetic model results described in section 3. *Simons et al.* [2000] used K = 7 and NW = 4. They also used a window about half the size that we used, which, as noted above, roughly doubles the bias, but all of these factors bias the estimate downward and cumulatively they reduce it by factor of 3 or more from the true value.

[21] *McKenzie and Fairhead* [1997] state that the free air coherence γ_F^2 should be close to 1 at short wavelengths and since it is not for their Australian data they argue that Bouguer coherence cannot be used to estimate T_e. *Banks et al.* [2001], however, show with their Equation (26) that γ_F^2 can be small at all but the shortest wavelengths when subsurface loading dominates. We have confirmed this by computing γ_F^2 for our synthetic models. At the longest wavelengths these generally show γ_F^2 increasing but not necessarily reaching 1 as expected. For example a typical model for T_e = 120 km gives $\gamma_F^2 \leq 0.3$ except at the longest wavelength where it reaches 0.5. For central Australia our results agree with Figure 1c of *McKenzie and Fairhead* [1997] in showing γ_F^2 to be small at all wavelengths. We agree with *Banks et al.* [2001] that the problem lies in the difficulty of estimating spectral features at wavelengths close to the data window size.

[22] In conclusion, we do not dispute that low topographic signal will tend to reduce coherence. However, for a given wavenumber band, the presence of large free air anomalies implies large loads, so absence of topography must mean that the plate is strong.

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C. J. Swain and J. F. Kirby, Dept. of Spatial Sciences, Curtin Univ., Australia. (c_swain@wt.com.au)