Scattering of a longitudinal wave by a circular crack in a fluid-saturated porous medium

R. J. Galvin [a], B. Gurevich [a,b]

[a] *Curtin University of Technology, GPO Box U1987, Perth, Western Australia, 6845, Australia*

[b] *CSIRO Division of Petroleum Resources, ARRC, 26 Dick Perry Avenue, Kensington, Perth, Western Australia, 6151, Australia*

Abstract

Key words: poroelasticity; Biot theory; slow wave; fractured medium

Introduction

Naturally fractured reservoirs are becoming increasingly important for oil and gas exploration in many areas of the World. Because fractures may control the permeability of a reservoir it is important to be able to find and characterise them. In order to characterise a fractured reservoir we need to understand the effect the fractures will have on its overall elastic properties. Fractures are highly compliant compared to the relatively stiff pores, so fluid will flow between pores and fractures during passage of the seismic wave. If the fractures are aligned, the reservoir will exibit long wavelength effective anisotropy. Since the fluid flow and scattering taking place due to the fractures depends upon

seismic frequency, the anisotropy will be frequency-dependent.

In the limit of low frequencies, static models can be used to obtain the effective elastic moduli of the fluid-saturated medium in terms of the properties of the dry skeleton and the saturating fluid (Gassmann (1951), Brown and Korringa (1975), Thomsen (1995), Gurevich 2005, Cardona). For these models to be useful, fluid pressure must have time to fully equilibrate throughout the connected porespace which will only be the case for very low frequencies. To model the attenuation and dispersion that takes place due to fluid flow higher frequencies must be considered, and therefore static effective medium models are no longer appropriate.

A number of schemes tackling this dynamic problem in fractured porous rocks are currently available. Brajanovski et al. model a fractured medium as very thin, highly porous layers in a porous background. Hudson et al. (1996) model fracturing using thin circular cracks, and allow for fluid flow effects by applying the diffusion equation to a single crack and ignoring interaction between cracks. Chapman and Maultzsch also model attenuation and dispersion using a sparse concentration of circular cracks in a porous matrix. Jakobsen? A systematic approach to this problem is to base our model upon Biot's theory of poroelasticity (Biot (1962)). We model fractures using very thin circular cracks in a poroelastic background. We assume that the cracks are mesoscopic (large compared to the pore size, but small compared to the fast wave wavelength). Using the single scattering result obtained previously (Galvin and Gurevich 2006) and the multiple-scattering theory of Waterman and Truell (ref) we estimate the attenuation and dispersion of elastic waves taking place in a porus medium containing a sparse distribution of such cracks (Fig. 1).

1   Scattering from a single crack

1.1   Equations of Poroelasticity

We consider an incident plane longitudinal wave, harmonic in time, propagating in a fluid-saturated porous medium in the positive direction of the z-axis of a cylindrical coordinate system. This wave can be represented as the displacement field $u_z^{(i)} = u_0 e^{ik_1 z}$, where k_1 is the wavenumber. We aim to derive the scattered field $\mathbf{u}(\mathbf{r})$ that results from interaction between the incident wave and the crack, which occupies the circle $0 \leq r \leq a$ in the plane $z = 0$. The total displacement field is therefore $\mathbf{u}^{(t)}(\mathbf{r}) = u_z^{(i)}\mathbf{a}_z + \mathbf{u}(\mathbf{r})$, where $\mathbf{a}_z$ is a unit vector directed along the z-axis.

Since we have geometrical symmetry about the crack plane $z = 0$, both the scattered and total displacement fields satisfy the following equations of dynamic poroelasticity (Biot (1962)) in the semi-infinite poroelastic medium $z \geq 0$:

$$\nabla \cdot \boldsymbol{\sigma} = -\omega^2 \left(\rho\mathbf{u} + \rho_f\mathbf{w} \right) \tag{1}$$

$$\nabla p = \omega^2 \left(\rho_f\mathbf{u} + q\mathbf{w} \right) \tag{2}$$

where $\mathbf{w} = \phi(\mathbf{U} - \mathbf{u})$ is the relative fluid displacement, ϕ is the medium porosity, $\mathbf{U}$ is the average absolute fluid displacement, ω is the angular wave frequency, ρ_f and ρ are the densities of the fluid and of the overall medium. At low frequencies $q(\omega) \approx i\eta/\kappa\omega$ (Biot (1956)) is a frequency-dependent coefficient responsible for viscous and inertial coupling between the solid and fluid displacements where η is the fluid viscosity and κ is the intrinsic permeability of the medium. $\boldsymbol{\sigma}$ and p are the total stress tensor and fluid pressure, which

are related to the displacement vectors via the constitutive relations

$$\boldsymbol{\sigma} = \left[(H - 2\mu) \, \nabla \cdot \mathbf{u} + \alpha M \nabla \cdot \mathbf{w} \right] \mathbf{I} + \mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] \tag{3}$$

$$p = -\alpha M \nabla \cdot \mathbf{u} - M \nabla \cdot \mathbf{w}. \tag{4}$$

In equations (3) and (4) μ is the shear modulus of the solid frame, $\alpha = 1 - K/K_g$ is the Biot-Willis coefficient (Biot and Willis (1957)) ,

$$M = \left[\frac{(\alpha - \phi)}{K_g} + \frac{\phi}{K_f} \right]^{-1} \tag{5}$$

is the so-called pore space modulus,

$$H = K_{sat} + \frac{4}{3}\mu \tag{6}$$

is the P-wave modulus of the saturated poroelastic medium and K_{sat} is the bulk modulus of the saturated medium which is related to the bulk moduli of the fluid K_f, solid K_g, and dry skeleton K by the Gassmann (1951) equation

$$K_{sat} = K + \alpha^2 M. \tag{7}$$

Because of the axial symmetry, there is no dependency upon the transverse angle θ. Using equations (3) and (4) we can therefore decompose equations (1) and (2) into scalar equations in cylindrical coordinates (r, θ, z):

$$\mu \left(\nabla^2 - \frac{1}{r^2} \right) u_r + (\lambda + \mu) \frac{\partial e}{\partial r} + \rho \omega^2 u_r - \alpha \frac{\partial p}{\partial r} = 0 \tag{8}$$

$$\mu \nabla^2 u_z + (\lambda + \mu) \frac{\partial e}{\partial z} + \rho \omega^2 u_z - \alpha \frac{\partial p}{\partial z} = 0 \tag{9}$$

$$\nabla^2 p + \frac{i\omega b}{M} p + i\alpha b \omega e = 0 \tag{10}$$

where $\lambda = K - 4\mu/3$, $b = \eta/\kappa$, and e is the cubical dilatation

$$e = \nabla \cdot \mathbf{u} = \frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{\partial u_z}{\partial z}. \tag{11}$$

Equations (8)-(10) are supplemented by the constitutive relations

$$\sigma_{zz} = 2\mu\frac{\partial u_z}{\partial z} + \lambda e - \alpha p \qquad (12)$$

$$\sigma_{rz} = \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \qquad (13)$$

and using equation (2)

$$w_z = \frac{1}{i\omega b}\left(\frac{\partial p}{\partial z} - \rho_f\omega^2 u_z\right). \qquad (14)$$

1.2   Boundary Conditions

The solution must obey the boundary conditions (Deresiewicz & Skalak, 1963) at a poroelastic interface, i.e. continuity of normal and tangential components of stresses, continuity of solid and relative fluid displacements and continuity of the pore fluid pressure. We assume that the circular crack is in hydraulic communication with the surrounding porous medium.Here and below we assume that the fluid in the crack (but not in the pores!) is incompressible. According to Hudson (1981), the compressibility of the crack-filling fluid can be neglected as long as

$$K_f/\mu \gg c/a, \qquad (15)$$

where as before K_f is the bulk modulus of the fluid in the crack, μ is the shear modulus of the background medium, and c/a is the aspect ratio of the crack. Note that in an elastic medium a crack filled with an incompressible fluid does not cause any scattering of a normally incident P-wave. Analogously, in a porous medium such a crack will not cause any scattering of a normally incident P-wave at frequencies where the size of the crack a is comparable to the wavelength of the incident wave. However, there may still be scattering of incident energy due to fluid flow between the crack and the pore space. As will

be seen, this effect occurs at much lower frequencies where the crack radius is comparable to the wavelength of Biot's slow wave.

For an incompressible fluid in the crack, the volume-fraction-average of the normal displacement $(1 - \phi)u_z + \phi U_z = u_z + w_z$ through the face of the crack must be equal to zero. Continuity of total normal stress tells us that both $\sigma_{zz}^{(t)}$ and $p^{(t)}$ in the *total* field at $z = 0^+$ must be equal to the pressure of the crack-filling fluid, $p^{(c)}$, so that $\sigma_{zz}^{(t)} = p^{(t)}$ or $\sigma_{zz} - p = -(\sigma_{zz}^{(i)} - p^{(i)})$. Also, analogously to the elastic case, σ_{rz} is everywhere zero, and $u_z = w_z = 0$ due to symmetry considerations. Expressing stress and fluid pressure in terms of displacement via constitutive relations (12) and (4) our boundary conditions can be written:

$$\sigma_{rz} = 0 \qquad 0 \leq r < \infty \tag{16}$$

$$u_z = 0 \qquad a < r < \infty \tag{17}$$

$$w_z = 0 \qquad a < r < \infty \tag{18}$$

$$u_z + w_z = 0 \qquad 0 \leq r \leq a \tag{19}$$

$$\sigma_{zz} + p = -ik_1(H - \alpha M)u_0 \qquad 0 \leq r \leq a \tag{20}$$

where it is noted that conditions (**??**)-(**??**) can be combined to give the single condition

$$u_z + w_z = 0 \qquad 0 \leq r \leq \infty \tag{21}$$

1.3   Solution

Galvin and Gurevich (2006) obtained a general solution to equations (8)-(10) using Hankel transform techniques, reducing the scattering problem to finding the three unknown scalar spectral wave amplitude functions $A_1(y)$ (fast

wave), $A_2(y)$ (slow wave) and $A_3(y)$ (shear wave). Application of boundary conditions (16)-(21) allowed the elimination of two of the unknown $A_i(y)$ and resulted in a system of dual integral equations for the single unknown spectral amplitude function $A_1(y)$ which were recast in the form of Noble (1963) using the substitution

$$B(y) = -\frac{2\alpha\mu(1-g)k_3^2 q_1 y}{L\left[2y^2 - k_3^2(1-\frac{\alpha M}{H})\right]} A_1(y).\tag{22}$$

These dual integral equations were shown to be equivalent to a single Fredholm equation of the second kind:

$$B(x) + \frac{1}{\pi}\int_0^\infty R(x,y)T(y)B(y)dy = -p_0 S(x),\tag{23}$$

where

$$R(x,y) = \frac{\sin a(x-y)}{x-y} - \frac{\sin a(x+y)}{x+y},\tag{24}$$

$$S(x) = \frac{2}{\pi}\frac{\sin ax - ax\cos ax}{x^2},\tag{25}$$

$$T(y) = \left[1 + \frac{\alpha M k_3^2}{H(2y^2 - k_3^2)}\right][T_1(y) - T_2(y)] - 1,\tag{26}$$

$$T_1(y) = \frac{M(k_2^2 L - 2\alpha\mu y^2)\left[4\alpha g y^2(y^2 - q_2 q_3) - k_2^2(2y^2 - k_3^2)\right]}{2\mu H(1-g)k_2^2 q_2 y\left[2y^2 - k_3^2(1-\frac{\alpha M}{H})\right]},\tag{27}$$

$$T_2(y) = \frac{\alpha g\left[(2y^2 - k_3^2)^2 - 4y^2 q_1 q_3\right] + (2y^2 - k_3^2)(k_0^2 - k_1^2)}{2\alpha g(1-g)k_3^2 q_1 y},\tag{28}$$

and $p_0 = ik_1(H - \alpha M)u_0$ is the incident pressure. Since an analytical solution to equation (23) exists only when its kernal function $R(x,y)T(y)$ is separable, in general one must obtain $B(y)$ via numerical methods.

$T(y)$ is general with regards to the size of the crack relative to the incident wavelength. However we are mainly interested in situations where the scattering is due to wave-induced fluid flow to and from the cracks (ie. the slow wave). For the scattering to be predominantly due to fluid flow, the crack size must be small relative to the wavelength of the incident compressional wave,

$k_1 a \ll 1$ (mesoscopic cracks).so that there is neglible scattering into a fast wave mode. For significant slow wave scattering to take place y, being a radial wavenumber, should be of the order of $1/a$, or $y \gg k_1$ and we can approximate equations (26-28):

$$T(y) \approx M \frac{(2\alpha g y^2 - k_2^2)^2 - 2yq_2 \alpha g \left[k_2^2(\alpha g - 2) + 2\alpha y^2 g\right]}{2Hg(g-1)yq_2 k_2^2} \tag{29}$$

1.4 Multiple scattering formulae

The multiple scattering theorem of Waterman & Truell (1961) provides the method of computation of attenuation and dispersion of seismic waves in the medium with randomly distributed inhomogeneities. According to Waterman & Truell (1961), effective wave number may be calculated from the amplitudes of the scattered field as

$$\left(\frac{k^*}{k_1}\right)^2 = \left[1 + \frac{2\pi n_0 f(0)}{k_1^2}\right]^2 - \left[\frac{2\pi n_0 f(\pi)}{k_1^2}\right]^2 \tag{30}$$

where k_1 is the real wave number of fast P-wave, n_0 is the density or number of scatterers per unit volume and $f(0)$, $f(\pi)$ are amplitudes of the wave scattered in the forward and backward direction (with respect to incident wave) by a single inclusion. For sufficiently small concentration of inclusions quadratic terms in (30) can be neglected, which yields

$$k^* = k_1 \left[1 + \frac{4\pi n_0 f(0)}{k_1^2}\right]^{1/2} \approx k_1 \left[1 + \frac{2\pi n_0 f(0)}{k_1^2}\right] \tag{31}$$

The real part of (31) gives the effective velocity v^* in media with a low concentration of scatterers

$$\frac{1}{v^*} = \frac{1}{v_1}\left[1 + \frac{2\pi n_0}{k_1^2}\operatorname{Re}\{f(0)\}\right] \tag{32}$$

The imaginary part of (31) gives the dimensionless attenuation (inverse quality factor)

$$Q^{-1} = \frac{4\pi n_0}{k_1^2} \operatorname{Im}\{f(0)\} \tag{33}$$

The above expressions (32) and (33) allow us to model the dispersion and attenuation due to the scattering of a plane elastic wave by poroelastic inclusions randomly distributed throughout a poroelastic medium. In this study we investigate the special case of dispersion and attenuation using the solution for the scattering by a circular crack given in the previous section. Our $f(0)$ is obtained analogously to the elastic case (Robertson 1967) by considering the far-field asymptotics of

$$u_z = \int_0^\infty \left[A_3(y) e^{-q_3 z} - \frac{\alpha}{L} \sum_{i=1}^2 A_i(y) q_i e^{-q_i z} \right] y J_0(yr) \, dy \tag{34}$$

(Galvin and Gurevich 2006) and is given by

$$f(0) = \frac{i\alpha k_1 q_1}{L} A_1(0) = -\frac{ik_1}{u_0} \frac{(H - \alpha M)}{2\mu H(1-g)} \lim \frac{B(y)}{y} \tag{35}$$

2   Results

2.1   Numerical solution

The solution for intermediate frequencies was obtained numerically by the method of quadratures .Figures 1a and 1b show this solution in terms of effective velocity $v(\omega) = \omega / \operatorname{Re} k^*$ (normalized) and dimensionless attenuation $Q^{-1} = 2 \operatorname{Re} k^* / \operatorname{Im} k^*$ as functions of dimensionless frequency Also shown in Figures 1a and 1b are asymptotic solutions of equation (17) in the low- and high-frequency limits (explained in the following sections). This solution exhibits a typical relaxation peak around a normalized frequency ω' of about 10,

or at circular frequency $f = \omega/2\pi \simeq 2\kappa M(K + 4\mu/3)/H\eta a^2$, the frequency where the fluid diffusion length $1/|k_2|$ is of the order of the crack radius a .

2.2 Analytical solution

2.2.1 Simplification of Fredholm equation

For the purposes of using the multiple scattering theory outlined above to estimate effective medium properties, we are only interested in the scattering in the forward direction, in which the radial component of the wavenumber is small. We can exploit this to simplify the function $R(x, y)$. Expanding $R(x, y)$ using the trigonometric identity

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \tag{36}$$

we obtain

$$R(x, y) = \frac{2\left(y \sin ax \cos ay - x \cos ax \sin ay\right)}{x^2 - y^2} \tag{37}$$

If x is small, then $\sin ax \approx ax$, $\cos ax \approx 1$ eq (37) simplifies to

$$R(x, y) \approx \frac{2x\left(\sin ay - ay \cos ay\right)}{y^2} = x\pi S(y). \tag{38}$$

and eq (23) simplifies to

$$B(x) + x \int_0^\infty S(y)T(y)B(y)dy = -p_0 S(x) \tag{39}$$

which can be solved analytically as the kernel has been separated. We now obtain asymptotic solutions in the low and high frequency limits. Note that we are still in the low frequency regime of Biot theory, the low and high frequency limits we evaluate here are with respect to the extent to which fluid is able to diffuse between crack and porespace during passage of the wave field. That

10

is, in the low frequency limit there is plenty of time available for fluid flow to occur, the diffusion length can be taken large compared with the crack radius, or $|k_2| \, a \ll 1$. At increasingly high frequencies there is no time available for appreciable fluid flow to occur and therefore the diffusion length can be taken as small compared with the crack radius, $|k_2| \, a \gg 1$

2.2.2   Low frequency asymptote

For low frequencies $|k_2| \, a \ll 1$ which for y of the order $1/a$ is equivalent to $y \gg |k_2|$. Imposing this on (29) we obtain an asymptotic expression for the transfer function $T(y)$ in the limit $y \gg |k_2|$,

$$T_{low}(y) \approx \frac{-Mk_2^2(2L^2 + 3\alpha^2\mu^2 - 4\alpha\mu L)}{4H\mu(L - \mu)y^2}. \qquad (40)$$

At low frequencies the contribution of the integral in (39) is small and we can solve by iteration. That is, we assume an initial solution $B(x) = -p_0 S(x)$ and then substitute $B(y) = -p_0 S(y)$ and $T(y) \approx T_{low}(y)$ back in to (39) and solve, obtaining

$$B_{low}(x) = \frac{p_0 a^3 \left[M(k_2 a)^2(4\alpha\mu L - 2L^2 - 3\alpha^2\mu^2) - 10H\mu(L - \mu)\right]}{15\pi H\mu(L - \mu)}x \qquad (41)$$

and from (35)

$$f_{low}(0) = \frac{(5 + D_{low})(H - \alpha M)^2 k_1^2 a^3}{15\pi\mu H(1 - g)} \qquad (42)$$

where

$$D_{low} = \frac{M(2 - 4\alpha g + 3\alpha^2 g^2)(k_2 a)^2}{2Hg(1 - g)} \qquad (43)$$

11

By substituting (42) into (32) and taking the real component we can obtain an expression for effective velocity in the low frequency limit

$$v^* = v_1 \left[1 - \frac{2\varepsilon (H - \alpha M)^2}{3\mu H (1 - g)} \right]. \tag{44}$$

In (44) $v_1 = \omega / k_1 = (H/\rho)^{1/2}$ is the velocity of the fast compressional wave in the porous host (crack-free porous medium) and $\varepsilon = n_0 a^3 = (3/4\pi)(a/b)\phi_c$ is the crack density parameter (Hudson) where $\phi_c = (4/3)\pi a^2 b n_0$ is the additional porosity present due to the cracks. For dry open cracks $K_f = M = 0$, $H = K + 4\mu/3$ and equation (44) simplifies to

$$v^* = v_1 \left[1 - \frac{2\varepsilon}{3g(1 - g)} \right] \tag{45}$$

which coincides with the well-known expression for the velocity of compressional waves propagating perpendicular to a system of dry open cracks in an elastic medium in the limit of low crack density (Hudson) Furthermore, equation (44) coincides with the expression for the compressional wave velocity obtained from Gassmann's exact static result for the undrained elastic moduli of an anisotropic fluid-saturated porous medium (Gassmann) with low crack density . This Gassmann consistency is an important feature of the model presented here.

Low-frequency attenuation Q^{-1} is defined by the imaginary part of the function $f_{low}(0)$,

$$Q_{low}^{-1} = \frac{2M(H - \alpha M)^2 (2 - 4\alpha g + 3\alpha^2 g^2) |k_2 a|^2 \varepsilon}{15\mu H^2 g (1 - g)^2} \tag{46}$$

and is proportional to $|k_2 a|^2$, that is, to the first power of frequency.

2.2.3 High frequency asymptote

For high frequencies $|k_2|\, a \gg 1$ which for y of the order $1/a$ is equivalent to $y \ll |k_2|$. Imposing this on equation (29) we obtain an asymptotic expression for the transfer function $T(y)$ in the limit $y \gg |k_2|$,

$$T_{high}(y) \approx \frac{-iML^2 k_2}{2H\mu(L-\mu)y}. \tag{47}$$

In the high frequency limit we cannot solve (39) by iteration because the integral is not small compared with the RHS. However at high frequencies large values of y do not result in a significant contribution to the integral and so we can simplify the RHS in the limit of small x. For small x $\sin ax - ax \cos ax \approx (ax)^3/3$ and therefore

$$S_{high}(x) \approx \frac{2a^3}{3\pi} x \tag{48}$$

We cannot use this approximation for $S(y)$ as the integral will not converge. Substituting (48) into our integral equation we obtain

$$B_{high}(x) = -\left[\frac{2p_0 a^3}{3\pi} + \theta \right] x \tag{49}$$

where

$$\theta = \int_0^\infty S(y) T_{high}(y) B(y) dy. \tag{50}$$

Substituting this back into (39) and solving for θ we get

$$\theta = \frac{-2p_0 \int_0^\infty S(y) T_{high}(y) y dy}{1 + \int_0^\infty S(y) T_{high}(y) y dy} \tag{51}$$

which, when evaluated analytically, gives us

$$B_{high}(x) = \tag{52}$$

and from (35)

$$f_{high}(0) = \frac{i(k_1 a)^2 (H - \alpha M)^2 g}{3\mu M k_2} \tag{53}$$

13

By substituting this expression into the dispersion equation (32) one can see that its relative contribution to the real part of the effective wavenumber vanishes in the high frequency limit, implying that the velocity in that limit tends to the velocity in the porous crack-free medium. This result is logical as at sufficiently high frequencies the fluid has no time to move between pores and cracks, and therefore the cracks behave as if they were isolated In particular, the dry case is excluded, except in the static limit (45).

Attenuation at high frequencies reads

$$Q_{high}^{-1} = \frac{2\sqrt{2}\pi\varepsilon(H-\alpha M)^2 g}{3\mu M \left|k_2 a\right|} \tag{54}$$

and thus scales with $\omega^{-1/2}$. We note that both low and high-frequency asymptotes of attenuation are consistent (but not identical) with the corresponding results for mesoscopic spherical inclusions[12][13] and for a poroelastic medium with smoothly varying material properties[14].

3 Conclusions

4 Acknowledgements

References

Biot, M. A., 1956. Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range. J. Acoust. Soc. Amer. 28, 168–178.

Biot, M. A., 1962. Mechanics of deformation and acoustic propagation in porous media. J. Appl. Phys. 33, 1482–1498.

Biot, M. A., Willis, D. G., 1957. The elastic co-efficients of the theory of consolidation. J. App. Mech. 24, 594–601.

Brown, R. J. S., Korringa, J., 1975. On the dependence of the elastic properties of a porous rock on the compressibility of the pore fluid. Geophysics 40, 608–616.

Gassmann, F., 1951. Über die elastizität poröser medien. Viertel. Naturforsch. Ges. Zürich 96, 1–23.

Hudson, J. A., 1981. Wave speeds and attenuation of elastic waves in material containing cracks. Geophys. J. Roy. Astr. Soc. 64, 133–150.

Hudson, J. A., Liu, E., Crampin, S., 1996. The mechanical properties of materials with interconnected cracks and pores. Geophys. J. Internat. 124, 105–112.

Noble, B., 1963. The solution of bessel function dual integral equations by a multiplying-factor method. Proc. Camb. Phil. Soc. 59, 351–362.

Thomsen, L., 1995. Elastic anisotropy due to aligned cracks in porous rock. Geophysical Prospecting 43, 805–829.

5   Figure Captions

Fig. 1. Scattering cross-section as a function of dimensionless frequency.

Fig. 2. Vertical relative fluid displacement w_z vs normalised horizontal slowness $y/|k_2|$ for a range of dimensionless frequencies.

Fig. 3. Radial relative fluid displacement w_r vs normalised horizontal slowness $y/|k_2|$ for a range of dimensionless frequencies.