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A theoretical study on the bottlenecks of GPS phase ambiguity resolution in a CORS RTK Network

Research Article

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Abstract:

Crucial to the performance of GPS Network RTK positioning is that a user receives and applies correction information from a CORS Network. These corrections are necessary for the user to account for the atmospheric (ionospheric and tropospheric) delays and possibly orbit errors between his approximate location and the locations of the CORS Network stations. In order to provide the most precise corrections to users, the CORS Network processing should be based on integer resolution of the carrier phase ambiguities between the network's CORS stations. One of the main challenges is to reduce the convergence time, thus being able to quickly resolve the integer carrier phase ambiguities between the network's reference stations. Ideally, the network ambiguity resolution should be conducted within one single observation epoch, thus truly in real time.

Unfortunately, single-epoch CORS Network RTK ambiguity resolution is currently not feasible and in the present contribution we study the bottlenecks preventing this. For current dual-frequency GPS the primary cause of these CORS Network integer ambiguity initialization times isthe lack of a sufficiently large number of visible satellites. Although an increase in satellite number shortens the ambiguity convergence times, instantaneous CORS Network RTK ambiguity resolution is not feasible even with 14 satellites. It is further shown that increasing the number of stations within the CORS Network itself does not help ambiguity resolution much, since every new station introduces new ambiguities. The problem with CORS Network RTK ambiguity resolution is the presence of the atmospheric (mainly ionospheric) delays themselves and the fact that there are no external corrections that are sufficiently precise. We also show that external satellite clock corrections hardly contribute to CORS Network RTK ambiguity resolution, despite their quality, since the network satellite clock parameters and the ambiguities are almost completely uncorrelated. One positive is that the foreseen modernized GPS will have a very beneficial effect on CORS ambiguity resolution, because of an additional frequency with improved code precision.

Keywords:
GPS • CORS Network RTK • Ambiguity Resolution • ADOP

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1. Introduction

The basis of high-precision (cm-level) GNSS Network RTK positioning is the presence of a network of Continuously Operating Reference Stations (CORS). The CORS stations permanently collect GNSS data which are sent to a computing center. This computing center then computes a network solution, combining the data of the CORS stations in a least-squares adjustment in order to produce the parameters of interest, which are basically atmospheric (ionospheric and tropospheric) delay parameters. These atmospheric delay estimates from the network are consequently used to predict the (differential) atmospheric errors at the approximate location of the Network RTK rover, by means of sophisticated modeling or interpolation. The rover's position is then estimated with high pre-

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cision using the atmospheric corrections and the data of one of the CORS stations as received from the computing center. Examples of commercial Network RTK systems are described in e.g. Euler et al. (2001) and Vollath et al. (2000).

To obtain the most precise atmospheric error prediction for the users, the processing of the data of the CORS stations should be based on fixing of the integer carrier phase ambiguities. Once these ambiguities are resolved, the high precision of the phase data will be reflected in the network atmospheric estimates. See Figure 1 in which the standard deviation of the estimated double-differenced ionospheric delay is plotted for a high and low elevation satellite in two cases: the first where the ambiguities are float, and the secondwhere they are fixed. A reference satellite is also involved in the double difference and this satellite has the highest elevation. The tremendous improvement due to ambiguity fixing is clearly visible: while the float ionospheric precision is at dm-m level, with the ambiguities fixed this precision improves to mm-cm level. Note that for $k = 1$ the gain in precision due to ambiguity fixing is largest: about a factor 100.

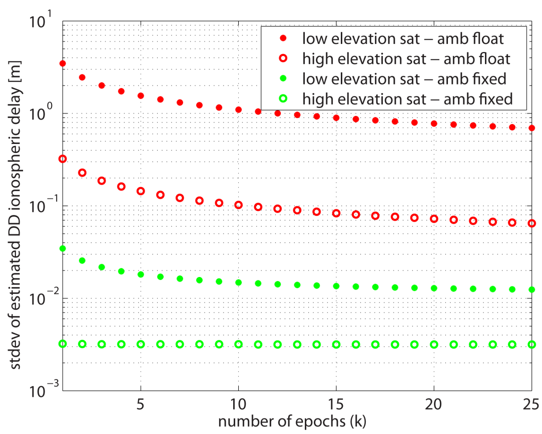

Figure 1. Double-differenced ionospheric precision for a CORS network baseline as function of number of epochs.

Although the CORS station positions are not estimated but held fixed in the network processing, full network ambiguity fixing is not a trivial issue, because of the presence of the unknown ionospheric and tropospheric delays. An efficient practical approach for real-time CORS network processing is to make use of a Kalman filter implementation, based on un-differenced dual-frequency GPS phase and code observations and with the double-differenced (DD) ambiguities, (zenith) tropospheric delays, ionospheric delays, receiver and satellite clocks in the state vector. In the Kalman time update use is made of the time-constant property of the ambiguities (as far as no cycle slips occur). Although the float state vector is solved in real-time, the resolution of the integer ambiguities, carried out by means of the LAMBDA method as invented by Teunissen (1993), requires some initialization time, since the float ambiguity solution has to converge. This network convergence time may be up to a few minutes, depending on the data sampling interval of the CORS stations, their separation, and the actual ionospheric conditions. Ambiguity convergence time is also required for a new satellite that has risen and after a (power) failure of (some of) the CORS stations.

Despite the fact that after the float ambiguity convergence the integer network ambiguities can be estimated in real time, it is obvious that a truly real-time CORS network RTK processing should be conducted on basis of instantaneous ambiguity resolution, thus using just one single epoch of GNSS data. In this paper we will study the limiting factors for instantaneous CORS Network RTK ambiguity resolution, based on dual-frequency GPS data. For this we will use the concept of Ambiguity Dilution of Precision (ADOP), a scalar diagnostic measure for the precision of the float ambiguities. An advantage of using the ADOP is that it is possible to derive analytical closed-form expressions for it, from which the various factors impacting on ambiguity resolution can be easily identified. These factors comprise the number of satellites, stations, frequencies and observation time span. In addition, the ADOP is related to the success rate of integer ambiguity resolution. Besides, the required quality of internal and external data for successful CORS Network RTK ambiguity resolution can be easily assessed. In Section 2 the ADOP concept is reviewed, while in Section 3 the CORS Network model is presented for which a closed-form ADOP expression is derived in Section 4. In relation to this, we will elaborate upon the use of precise external global ionospheric maps (GIM) and satellite clock corrections to lower the CORS Network RTK ADOP. Recently precise real-time satellite clocks have been successfully applied to improve Precise Point Positioning (PPP), see Bree et al. (2009). In Section 5 it is investigated whether such precise satellite clocks may contribute to speed up CORS Network RTK ambiguity resolution. It is emphasized that this paper is restricted to full ambiguity resolution, i.e. resolving all integers in the network. We do not consider partial ambiguity resolution, such as only fixing the wide-lane combination.

2. Ambiguity Dilution Of Precision & Ambiguity Success Rate

In this section we will briefly review the ADOP measure. First, recall that there are three steps to precise carrier-phase based CORS Network parameter estimation: i) float solution, ii) integer ambiguity resolution and iii) fixed solution. The success of the second step -ambiguity resolution- depends on the quality of the float ambiguity estimates: the more precise the float ambiguities, the higher the probability of estimating the correct integer ambiguities. For practical applications it would be helpful if, instead of having to evaluate all the entries of the float ambiguity variance-covariance matrix, one could work with an easy-to-evaluate scalar precision measure. Teunissen (1997) introduced the Ambiguity Dilution Of

Precision (ADOP) as such a measure. It is defined as:

$$ADOP = |Q_{\hat{a}}|^{1/(2n)} \quad (1)$$

where $|\cdot|$ denotes the determinant and $Q_{\hat{a}}$ the variance-covariance matrix of the float ambiguities and n the dimension of this matrix (the number of ambiguities). By taking the determinant of the float ambiguity variance-covariance matrix a simple scalar is obtained, which not only depends on the variances of the ambiguities, but also on their covariances. This is an advantageous property, since the ambiguities can be highly correlated, especially for short time spans, and by taking the determinant the full information in the variance-covariance matrix is taken into account. By raising the determinant to the power of $1/(2n)$, the scalar is, like the ambiguities, expressed in $cycles$. It is emphasized that the ADOP is invariant for the class of admissible ambiguity transformations, amongst others the decorrelating Z-transformation of the LAMBDA method. ADOP is also invariant to a change in the choice of reference satellite, in contrast to a measure that is only based on the diagonal elements (variances) of the ambiguity variance-covariance matrix. A detailed elaboration of the properties of ADOP is given by Teunissen and Odijk (1997). Since the ADOP gives a good approximation to the average precision of the ambiguities, it also provides for a good approximation to the integer least-squares ambiguity success rate, as shown by Verhagen (2005):

$$P(\check{a} = a) \approx \left[2\Phi\left(\frac{1}{2ADOP}\right) - 1\right]^n \quad (2)$$

where $P(\check{a} = a)$ denotes the integer least-squares success rate, i.e. the probability of estimating the correct integer vector (correct integer vector denoted as a, while the estimated integer vector is denoted as $\check{a}$). Furthermore, $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, i.e. $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}v^2\right\} dv$. From the formula it follows that the smaller ADOP, the higher the success rate. In Odijk and Teunissen (2008) it was demonstrated that if ADOP is smaller than about 0.12 cyc, the success rate becomes larger than 0.999, while for ADOP smaller than 0.14 cyc, the success rate is always better than 0.99. A further elaboration on the relation between success rate and ambiguity precision is given by Teunissen (2000).

3. The ionosphere-weighted CORS Network Model

In order to derive a closed-form expression for the ADOP, in this section we will set up the CORS Network RTK model of GPS phase and data observation equations. This will be done for the most general multi-frequency case, since then it is straightforward to obtain the ADOP expression for the modernized triple-frequency GPS case. Although it is possible to use equivalent model formulations based on un-differenced, single-differenced or double-differenced observation equations, we will formulate our CORS Network RTK model based on between-satellite single differences, since this enables an easy incorporation of external satellite clock corrections, which will be described in Section 5.

3.1. The GPS phase, code and ionosphere observation equations

Starting point for the CORS Network RTK model are the un-differenced carrier phase and code or pseudo-range observation equations. They read as follows for a receiver-satellite combination $r-s$ at an observation epoch i and frequency j, in units of metres, e.g. Hofmann-Wellenhof et al. (2001):

$$\begin{aligned} E(\phi^s_{r,j}(i)) =& \rho^s_r(i) + \tau^s_r(i) + c\left[dt_r(i) + \delta_{r,j}(i)\right] - c[dt^s(i) - \\ & \delta^s_{,j}(i)] + \lambda_j[\varphi_{r,j}(0) - \varphi^s_{,j}(0) + N^s_{r,j}] - \mu_j \iota^s_{r,1}(i) \\ E(p^s_{r,j}(i)) =& \rho^s_r(i) + \tau^s_r(i) + c\left[dt_r(i) + d_{r,j}(i)\right] - c[dt^s(i) - \\ & d^s_{,j}(i)] + \mu_j \iota^s_{r,1}(i) \end{aligned} \quad (3)$$

In these observation equations $E(\cdot)$ denotes the mathematical expectation operator, $\phi^s_{r,j}(i)$ and $p^s_{r,j}(i)$ the phase and code observable respectively, $\rho^s_r(i)$ the receiver-satellite range, $\tau^s_r(i)$the slant tropospheric delay, $dt_r(i)$ the receiver clock error, $dt^s(i)$ the satellite clock error, $\delta_{r,j}(i)$ the frequency-dependent receiver phase hardware bias, $d_{r,j}(i)$ the frequency-dependent receiver code hardware bias, $\delta^s_{,j}(i)$ the frequency-dependent satellite phase hardware bias, $d^s_{,j}(i)$ the frequency-dependent satellite code hardware bias, $\varphi_{r,j}(0)$ and $\varphi^s_{,j}(0)$ the initial phases at receiver and satellite, $N^s_{r,j}$ the integer phase ambiguity, $\iota^s_{r,1}(i)$ the slant ionospheric delay on the first frequency, $\mu_j = \lambda_j^2/\lambda_1^2$ the frequency-dependent ionospheric coefficient, and $\lambda_j$the wavelength corresponding to frequency j. For reasonably short time spans Sardon et al. (1994) concluded that the satellite hardware biases may be assumed constant, i.e. $\delta^s_{,j}(i) = \delta^s_{,j}$ and $d^s_{,j}(i) = d^s_{,j}$. If we further lump the receiver clock error with the receiver hardware biases to form observable-dependent receiver clocks, i.e. a receiver clock parameter different for each phase and code observable on each frequency, the observation equations can be simplified as:

$$\begin{aligned} E(\phi^s_{r,j}(i)) =& \rho^s_r(i) + \tau^s_r(i) + c\delta t_{r,j}(i) - cdt^s(i) + \lambda_j M^s_{r,j} - \mu_j \iota^s_{r,1}(i) \\ E(p^s_{r,j}(i)) =& \rho^s_r(i) + \tau^s_r(i) + cdt_{r,j}(i) - cdt^s(i) + cd^s_{,j} + \mu_j \iota^s_{r,1}(i) \end{aligned} \quad (4)$$

with (lumped) receiver clock errors $\delta t_{r,j}(i) = dt_r(i) + \delta_{r,j}(i)$ and $dt_{r,j}(i) = dt_r(i) + d_{r,j}(i)$ and non-integer ambiguity $M^s_{r,j} = \varphi_{r,j}(0) - [\varphi^s_{,j}(0) - f_j\delta^s_{,j}] + N^s_{r,j}$.

In addition to the phase and code observation equations, we assume an $ionosphere-weighted$ model formulation as done by Odijk (1999), incorporating observation equations for the ionospheric delays:

$$E(\iota^s_{r,p}(i)) = \iota^s_{r,1}(i) \quad (5)$$

where $\iota^s_{r,p}(i)$ denotes the ionospheric pseudo observable. Even in absence of external ionospheric corrections an ionosphere-weighted processing is more advantageous than treating the ionospheric delays as completely unknown parameters, since the model is stronger. For example, for ambiguity resolution of CORS Network baselines up to 100 km it was demonstrated by Odijk (1999) that under (mid-latitude) ionospheric conditions it is often more beneficial to include zero ionospheric observations weighted with an (un-differenced) ionospheric standard deviation of 10 cm, than using an ionosphere-float approach. A simple procedure showing how to set the ionospheric standard deviation as a function of baseline length and the time within the solar cycle (since the ionospheric activity is correlated with that) is described in Schaffrin and Bock (1988).

3.2. The CORS Network RTK model formulated

Within the CORS Network RTK model we distinguish between a functional model, relating the observables to the parameters, and a stochastic model, reflecting the noise assumptions of the observables. We will first set up the functional model and identify the estimable parameters.

Despite that we formulate the model based on single differenced observations, the carrier phase ambiguities need to be parameterized in terms of double-differences, to make benefit of the integer property of double-differenced ambiguities: $M^{1s}_{1r,j} = N^{1s}_{1r,j} \in \mathbb{Z}$ for $r = 1, \ldots, n$ and $s = 1, \ldots, m$. Furthermore, we assume for a CORS network processing all receiver positions and satellite positions (computed in real-time from the predicted part of the ultra-rapid IGS orbits) known and not parameterized at all (such that $\rho^s_r(i)$ is subtracted from the observations). Residual tropospheric delays are mapped to local zenith for each CORS station, after a priori tropospheric corrections have been subtracted from the observations, i.e. $\tau_r(i) = G_r(i) g_r$, with $\tau_r(i) = \left[\tau^1_r(i), \ldots, \tau^m_r(i)\right]^T$ the residual (or wet) tropospheric delays, and where $G_r(i)$ denotes a vector with tropospheric mapping function coefficients and g_r a zenith tropospheric delay for each CORS station. For time spans shorter than say 15 minutes, it is often admissible to keep the zenith tropospheric delay constant. In the model we may then replace the time-varying $G_r(i)$ matrices by one time-constant matrix, which is denoted as $\bar{G}_r$. This assumption facilitates the derivation of a closed-form ADOP expression, see Section 4. Furthermore, to avoid an additional (near) rank deficiency, no tropospheric delays are estimated for the master (pivot) station of the CORS network. Thus, $n-1$ relative zenith tropospheric delays are parameterized. According to Rocken et al. (1993) only for networks with spacing larger than ~500 km one can reliably estimate absolute tropospheric delays for all stations.

The full-rank ionosphere-weighted multi-frequency, multi-epoch, multi-receiver, multi-satellite $between-satellite$ single-difference (SD) functional model reads (assuming all n receivers track the same m satellites during the same k observation epochs):

$$E\left(\begin{bmatrix} \begin{pmatrix} \phi^{sd} \\ p^{sd} \end{pmatrix} \\ \iota^{sd}_p \end{bmatrix}\right) =$$

$$\begin{bmatrix} \begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes e_k \otimes (C_n \otimes I_{m-1}) & \begin{pmatrix} e_j \\ e_j \end{pmatrix} \otimes e_k \otimes B & \begin{pmatrix} I_j & 0 \\ 0 & I_j \end{pmatrix} \otimes I_k \otimes (-e_n \otimes I_{m-1}) & \begin{pmatrix} -\mu \\ \mu \end{pmatrix} \otimes I_k \otimes (I_n \otimes I_{m-1}) \\ 0 & 0 & 0 & I_k \otimes (I_n \otimes I_{m-1}) \end{bmatrix} \begin{bmatrix} a^{dd} \\ g \\ \begin{pmatrix} s^{sd}_\phi \\ s^{sd}_p \end{pmatrix} \\ \iota^{sd} \end{bmatrix} \quad (6)$$

where $\phi^{sd} = \left[\phi^{sd^T}_1, \ldots, \phi^{sd^T}_j\right]^T$, with $\phi^{sd}_j = \left[\phi^{sd}_j(1)^T, \ldots, \phi^{sd}_j(k)^T\right]^T$, and likewise vectors for the code data. Furthermore, for $i = 1, \ldots, k$ we have the SD observables:

$$\begin{aligned} \phi^{sd}_j(i) &= \left[\left(\phi^{12}_{1,j}(i), \ldots, \phi^{1m}_{1,j}(i)\right), \ldots, \left(\phi^{12}_{n,j}(i), \ldots, \phi^{1m}_{n,j}(i)\right)\right]^T \\ p^{sd}_j(i) &= \left[\left(p^{12}_{1,j}(i), \ldots, p^{1m}_{1,j}(i)\right), \ldots, \left(p^{12}_{n,j}(i), \ldots, p^{1m}_{n,j}(i)\right)\right]^T \\ \iota^{sd}_p(i) &= \left[\left(\iota^{12}_{1,p}(i), \ldots, \iota^{1m}_{1,p}(i)\right), \ldots, \left(\iota^{12}_{n,p}(i), \ldots, \iota^{1m}_{n,p}(i)\right)\right]^T \end{aligned}$$

For notational convenience and compactness, we have used the matrix Kronecker product $\otimes$ as given by Rao (1973). Unknown parameters of the model are the multi-frequency DD ambiguities, $a^{dd} = \left[a^{dd^T}_1, \ldots, a^{dd^T}_j\right]^T$, zenith tropospheric delay parameters g, satellite clock parameters for phase $s^{sd}_\phi = \left[s^{sd^T}_{\phi_1}, \ldots, s^{sd^T}_{\phi_j}\right]^T$, with $s^{sd}_{\phi_j} = \left[s^{sd}_{\phi_j}(1)^T, \ldots, s^{sd}_{\phi_j}(k)^T\right]^T$, satellite clock parameters for code $s^{sd}_p = \left[s^{sd^T}_{p_1}, \ldots, s^{sd^T}_{p_j}\right]^T$, with $s^{sd}_{p_j} = \left[s^{sd}_{p_j}(1)^T, \ldots, s^{sd}_{p_j}(k)^T\right]^T$, and the SD ionospheric delay parameters, where:

$$
\begin{aligned}
a_j^{dd} &= \left[\left(N_{12,j}^{12},\ldots,N_{12,j}^{1m}\right),\ldots,\left(N_{1n,j}^{12},\ldots,N_{1n,j}^{1m}\right)\right]^T\\
g &= [g_2,\ldots,g_n]^T\\
s_{\phi_j}^{sd}(i) &= \left[\left(cdt^{12}(i)-\lambda_j M_{1,j}^{12}\right),\ldots,\left(cdt^{1m}(i)-\lambda_j M_{1,j}^{1m}\right)\right]^T\\
s_{p_j}^{sd}(i) &= \left[c(dt^{12}(i)-d_{,j}^{12}),\ldots,c(dt^{1m}(i)-d_{,j}^{1m})\right]^T
\end{aligned}
$$

The ionospheric delay parameter vector has the same structure as its observable counterpart. Note that a *biased* satellite clock parameter for each observable is estimable; in case of the phase observables the clocks are lumped with the between satellite SD phase ambiguities, and in case of the code observables with the SD satellite hardware biases.

The stochastic model corresponding to the phase and code and ionosphere observations reads:

$$
D\left(\left[\begin{pmatrix}\phi^{sd}\\ p^{sd}\end{pmatrix}\\ \iota_p^{sd}\right]\right)=\left[\begin{pmatrix}C_\phi & \\ & C_p\end{pmatrix} \\ & c_\iota^2\right]\otimes R_k\otimes\left(I_n\otimes D_m^T W_m^{-1} D_m\right) \quad (7)
$$

with $D(\cdot)$ being the mathematical dispersion operator.

In the functional and stochastic models the following matrices and vectors are used:

$$
\begin{aligned}
\Lambda &= \mathrm{diag}(\lambda_1,\ldots,\lambda_j) && \text{diagonal matrix with wavelengths}\\
\mu &= (\mu_1,\ldots,\mu_j)^T && \text{vector with ionospheric coefficients; } \mu_j=\lambda_j^2/\lambda_1^2\\
e_x &= (1,\ldots,1)^T && \text{vector with ones (dimension is clear from subscript)}\\
I_x &= \mathrm{diag}(1,\ldots,1) && \text{identity matrix (dimension is clear from subscript)}
\end{aligned}
$$

$$
C_n = \begin{pmatrix} 0 & & \\ 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix} \qquad n\times(n-1) \text{ matrix to model DD ambiguities using SD observations}
$$

$$
B = \begin{pmatrix} 0 & & \\ D_m^T\bar{G}_2 & & \\ & \ddots & \\ & & D_m^T\bar{G}_n \end{pmatrix} \qquad n(m-1)\times(n-1) \text{ matrix with SD mapping coefficients per receiver}
$$

$$
D_m^T = \begin{pmatrix} -1 & 1 & \\ \vdots & & \ddots & \\ -1 & & & 1 \end{pmatrix} \qquad (m-1)\times m \text{ between} - \text{satellite difference matrix}
$$

$$
\bar{G}_r = \begin{pmatrix} \bar{\psi}_r^1 \\ \vdots \\ \bar{\psi}_r^m \end{pmatrix} \qquad m\times 1 \text{ time} - \text{constant tropospheric mapping coefficients}
$$

$$
C_\phi = \begin{pmatrix} c_{\phi_1}^2 & \cdots & c_{\phi_1\phi_j} \\ \vdots & \ddots & \vdots \\ c_{\phi_j\phi_1} & \cdots & c_{\phi_j}^2 \end{pmatrix} \qquad j\times j \text{ cofactor matrix of undifferenced phase observations}
$$

$$
C_p = \begin{pmatrix} c_{p_1}^2 & \cdots & c_{p_1p_j} \\ \vdots & \ddots & \vdots \\ c_{p_jp_1} & \cdots & c_{p_j}^2 \end{pmatrix} \qquad j\times j \text{ cofactor matrix of undifferenced code observations}
$$

$$
\begin{aligned}
&c_\iota^2 && \text{undifferenced ionospheric variance factor}\\
&R_k && k\times k \text{ temporal correlation matrix}\\
&W_m = \mathrm{diag}(w_1,\ldots,w_m) && \text{diagonal matrix with satellite weights}
\end{aligned}
$$

The redundancy of the multi-frequency CORS network RTK model reads $(2j+1)n(m-1)k - [j(n-1)(m-1) + (n-1) + 2j(m-1)k + n(m-1)k] = (n-1)[j(m-1)(2k-1)-1]$ and thus $j \geq 2,\ k \geq 1,\ n \geq 2,\ \ m \geq 2$, which means that the model can be solved using one observation epoch, which is a condition for instantaneous ambiguity resolution. In addition, at least two receivers and two satellites are required. The model is solvable already based on two satellites, since no receiver positions are parameterized. We emphasize that the presented model formulation is a *batch* one, but this is only done for the purpose of the ADOP derivation. In practice a real-time implementation should be carried out by means of a Kalman filter; including a dynamic model in which the ambiguities and zenith troposheric delays are assumed to be constant in time (provided that no cycle slips occur).

4. A closed-form expression for the CORS Network RTK ADOP

As shown in (Teunissen and Odijk, 1997), the multi-receiver or network ADOP can be easily computed from the single-baseline ADOP by multiplying the latter with factor $n^{1/2(n-1)}$, with n the number of stations in the network. A closed-form expression for the short-time ionosphere-weighted ADOP of a single baseline was derived in Odijk and Teunissen (2008). Based on these results, the closed-form expression for the CORS Network ADOP can be given as:

$$ADOP = \frac{|C_\phi|^{\frac{1}{2j}}}{\prod_{i=1}^{j}\lambda_i^{1/j}} \frac{1}{\sqrt{e_k^T R_k^{-1} e_k}} \left(\frac{\sum_{s=1}^{m} w_s}{\prod_{s=1}^{m} w_s}\right)^{\frac{1}{2(m-1)}} n^{\frac{1}{2(n-1)}} \left(\frac{c_{\hat{\imath}|g}^2}{c_{\check{\imath}|g}^2}\right)^{\frac{1}{2j}} \left(\frac{c_{\hat{g}}^2}{c_{\check{g}}^2}\right)^{\frac{1}{2j(m-1)}} \quad [cyc] \tag{8}$$

In this formula we need to explain two ratios. First, the ratio of the *float* and *fixed* variance factors of the ionospheric delays conditioned on the zenith tropospheric delays, which is computed as:

$$\frac{c_{\hat{\imath}|g}^2}{c_{\check{\imath}|g}^2} = \frac{\left[\mu^T\left(C_\phi^{-1}+C_p^{-1}\right)\mu\right]+c_\iota^{-2}}{\left[\mu^T C_p^{-1}\mu\right]+c_\iota^{-2}} \tag{9}$$

Second, the ratio of the *float* and *fixed* variance factors of the zenith tropospheric delays, and this is computed as:

$$\frac{c_{\hat{g}}^2}{c_{\check{g}}^2} = \frac{\left[\mu^T C_p^{-1}\mu\right]+c_\iota^{-2}}{\left[e_j^T C_p^{-1} e_j\right]\left(\left[\mu^T C_p^{-1}\mu\right]+c_\iota^{-2}\right)-\left[e_j^T C_p^{-1}\mu\right]^2} \cdot \frac{\left[e_j^T\left(C_\phi^{-1}+C_p^{-1}\right)e_j\right]\left(\left[\mu^T\left(C_\phi^{-1}+C_p^{-1}\right)\mu\right]+c_\iota^{-2}\right)-\left[e_j^T\left(C_\phi^{-1}-C_p^{-1}\right)\mu\right]^2}{\left[\mu^T\left(C_\phi^{-1}+C_p^{-1}\right)\mu\right]+c_\iota^{-2}} \tag{10}$$

Although these variance ratio factors look complex, they can be computed rather easily since they are basically a function of the (inverse) $j \times j$ cofactor matrices for phase and code and the (inverse) ionosphere variance factor.

The following remarks can be made on basis of the ADOP expression. In absence of (cross) correlation between the phase observables, i.e. $c_{\phi_1\phi_j} = 0$, $|C_\phi|^{1/(2j)}$ reduces to $\prod_{i=1}^{j} c_{\phi_j}^{1/j}$, which is the geometric mean of the phase standard deviations. With equal phase standard deviations this further reduces to c_ϕ. From this follows that an improvement of the phase data has a lowering effect on the ADOP. In absence of temporal correlations between the observables, i.e.$R_k = I_k$, scalar $e_k^T R_k^{-1} e_k$ reduces to the number of epochs k. Thus, increasing the number of epochs benefits ADOP. In absence of satellite dependent weighting, i.e. $w_s = 1$ for $s = 1, \ldots, m$, the ratio $\sum_{s=1}^{m} w_s / \prod_{s=1}^{m} w_s$ reduces to m, the number of satellites. From the resulting factor $m^{1/2(m-1)}$ it follows that having more satellites will decrease ADOP. The variance ratio factor $c_{\hat{\imath}|g}^2/c_{\check{\imath}|g}^2$ is due to the *ionosphere* parameterization. It ranges between 1 if no ionospheric delays need to be modeled (for short baselines; $c_\iota = 0$) and about the variance ratio of the code and phase data if ionospheric delays are parameterized but no a priori information is modeled ($c_\iota = \infty$). This variance ratio is approximately the variance ratio of the code and phase data, which is a factor 10^4 in GPS practice. This implies that the ADOP worsens by at most a factor 10 due to ionosphere parameterization for current dual-frequency GPS. With modernized triple-frequency GPS this factor is considerably lower: about 4.6. The variance ratio factor $c_{\hat{g}}^2/c_{\check{g}}^2$ is due to the *troposphere* parameterization. In Odijk and Teunissen (2008) it was shown that this factor is rather insensitive to the ionospheric standard deviation c_ι. It was also demonstrated that this factor approximates the code-phase variance ratio which is about 10^4. However, the effect of this factor on ADOP is damped out when the number of satellites rises, due to the power of $1/2j(m-1)$.

It is emphasized that the presented ADOP formula only holds for short time spans, since this assumption was used for the tropospheric mapping coefficients. As a consequence of this the entries of the time-averaged tropospheric mapping matrices do

not have influence on the ADOP.
Having a closed-form ADOP expression available, we are now able to investigate the impact of several relevant factors affecting CORS Network RTK ambiguity resolution, i.e. number of epochs, number of CORS stations, number of satellites, number of frequencies, ionospheric weight and zenith tropospheric delay estimation. This analysis is based on the following standard assumptions:

1. the CORS network consists of 5 stations simultaneously tracking the same 7 satellites;
2. there are dual-frequency ($j = 2$) L1 and L2 phase and code observables that are uncorrelated and of equal precision, with $c_\phi = 3$ mm, $c_p = 30$ cm (un-differenced);
3. the un-differenced ionospheric standard deviation is set to $c_\iota = 10$ cm;
4. there are no temporal correlations between all observations;
5. all observations are weighted using a realistic elevation dependent function.

In Figure 2, Figure 3 and Figure 4 now one of the factors is varied, keeping the others to their 'standard' values. In the figures we have also plotted a horizontal line corresponding to an ADOP of 0.14 cyc, which is more or less a threshold for successful ambiguity resolution (see Section 2). In Figure 2 (left) the ADOP has been plotted as function of the number of epochs for a varying number of satellites (4, 7 and 14), while in Figure 2 (right) for each number of satellites there is an additional satellite rising on epoch $k = 6$, as to investigate the rising of a new satellite on CORS ambiguity resolution. In Figure 3 (left) again the ADOP is plotted as function of the number of epochs, but for a varying number of CORS stations (2, 5 and 10), while in Figure 3 (right) the ADOP is plotted both in presence and in absence of the zenith tropopsheric delays. The impact of the ionospheric uncertainty can be inferred from Figure 4 (left), where the ADOP is plotted for three levels of the ionospheric standard deviation (10 m, 10 cm and 1 cm). Finally, in Figure 4 (right) we anticipate on the ADOP of modernized GPS with triple-frequency phase and code data, for which it can be reasonably assumed that the quality of the L5 code data is expected to be much better than of the present L1 and L2 code observables. Hence we have assumed a code standard deviation of 30 cm for L1 and L2 and a code standard deviation of 10 cm for L5. All phase observables are assumed to have an equal standard deviation of 3 mm.
From the figures the following conclusions can be drawn:

1. Instantaneous CORS Network RTK ambiguity resolution is virtually impossible based on current dual-frequency GPS data, even when there would be a large number of 14 satellites in view. The number of epochs required for ADOP < 0.14 cyc varies between 3 epochs for 14 satellites and more than 10 epochs using 4 satellites. This means that CORS Network RTK ambiguity convergence times may be decreased when the sampling rate of the observations is increased.
2. The lower the number of satellites, the more a satellite that is rising affects on ADOP. In case of 7 satellites, inclusion of an additional satellite rising may not lead to successful ambiguity resolution immediately, but this can last for several epochs after.
3. While increasing the number of satellites is very beneficial to reduce the CORS Network RTK ADOP, the effect of more CORS stations in the network is limited. This is because every additional station introduces extra ambiguities to be solved.
4. The presence of the ionospheric delays hampers instantaneous CORS Network RTK ambiguity resolution. Only when it is allowed to use an ionospheric standard deviation as small as 1 cm, the single-epoch ADOP is below 0.14 cyc. Unfortunately for CORS Networks this requirement cannot be met, since the precision of external (predicted) ionospheric data is much worse than 1 cm.
5. The absence of zenith tropospheric delay parameters does not bring the single-epoch ADOP below 0.14 cyc, thus any external high quality tropospheric corrections will not enable instantaneous CORS Network RTK ambiguity resolution.
6. CORS Network RTK ambiguity resolution will benefit tremendously from a modernized GPS. In a triple-frequency GPS situation the single-epoch ADOP is smaller than 0.15 cyc when more than 7 satellites are tracked. This implies that instantaneous ambiguity resolution becomes feasible, especially when the precision of the L5 code precision will be much better than of the current dual-frequency code data.

5. Contribution of precise satellite clock corrections to CORS Network Ambiguity Resolution

Now let us assume there are precise corrections for the satellite clock errors and ionospheric delays available in real time, e.g. from the International GNSS Service (IGS). Then, according to Schaer (1999), in the case of current dual-frequency GPS, these satellite clock corrections are biased with an ionosphere-free combination of satellite code hardware biases. In addition, corrections for the ionospheric delays from the Global Ionospheric Maps are biased as well:

$$\begin{aligned} E(dt^s_{,12}(i)) &= dt^s(i) + \tfrac{\mu_2}{\mu_1-\mu_2} d^s_{,1} - \tfrac{\mu_1}{\mu_1-\mu_2} d^s_{,2} \\ E(\iota^s_{r,12}(i)) &= \iota^s_{r,1}(i) + \tfrac{\mu_1}{\mu_1-\mu_2} c(d^s_{,1} - d^s_{,2}) \end{aligned} \quad (11)$$

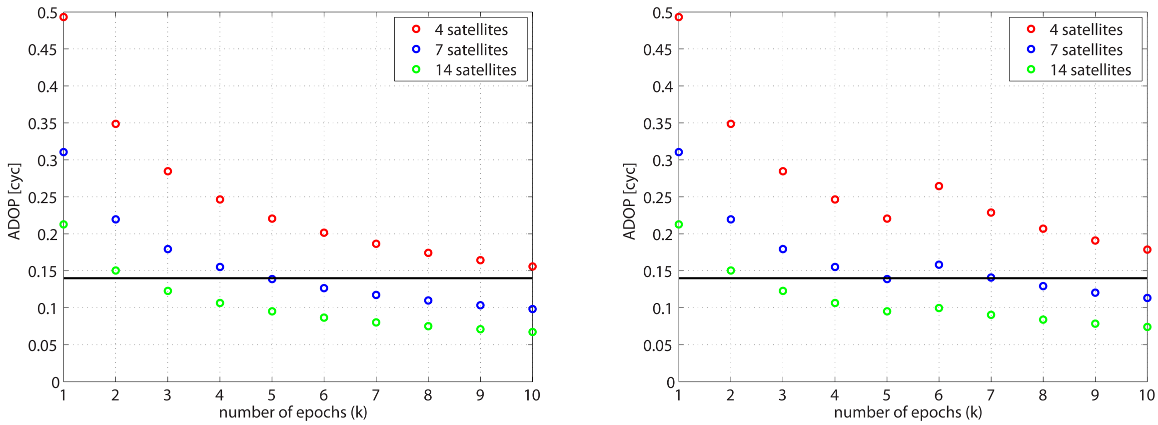

Figure 2. Dual-frequency GPS CORS Network ADOP: impact of increase in satellites (left) and influence of an additional satellite at all stations rising at epoch k = 6 (right).

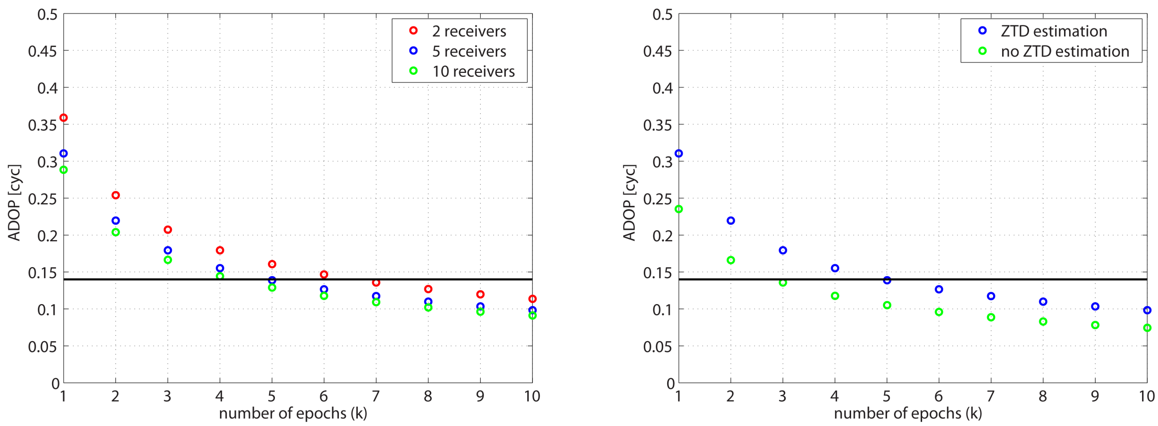

Figure 3. Dual-frequency GPS CORS Network ADOP: impact of increase in stations (left) and impact of Zenith Tropospheric Delay estimation (present vs. absent; right).

To correct for this L1-L2 Differential Code Biases (P1-P2 DCBs) are provided by the Center for Orbit Determination in Europe (CODE, 2010), per day and for all GPS satellites:

$$E(DCB^s) = c(d^s_{,1} - d^s_{,2}) \tag{12}$$

The satellite DCBs may vary within ± 4ns (which corresponds to ± 1.2m). If we assume these DCBs to be deterministic then we may correct the dual-frequency GPS phase and code data for them (applying a scale factor), resulting in the following rewritten observation equations:

$$\begin{aligned}
E(\phi^s_{r,1}(i) - \tfrac{\mu_1}{\mu_1-\mu_2}DCB^s) &= \rho^s_r(i) + \tau^s_r(i) + c\delta t_{r,1}(i) - cdt^s(i) - c[\tfrac{\mu_2}{\mu_1-\mu_2}d^s_{,1} - \tfrac{\mu_1}{\mu_1-\mu_2}d^s_{,2} + d^s_{,1}] + \lambda_1 M^s_{r,1} - \mu_1 \iota^s_{r,1}(i) \\
E(\phi^s_{r,2}(i) - \tfrac{\mu_2}{\mu_1-\mu_2}DCB^s) &= \rho^s_r(i) + \tau^s_r(i) + c\delta t_{r,2}(i) - cdt^s(i) - c[\tfrac{\mu_2}{\mu_1-\mu_2}d^s_{,1} - \tfrac{\mu_1}{\mu_1-\mu_2}d^s_{,2} + d^s_{,2}] + \lambda_2 M^s_{r,2} - \mu_2 \iota^s_{r,1}(i) \\
E(p^s_{r,1}(i) - \tfrac{\mu_1}{\mu_1-\mu_2}DCB^s) &= \rho^s_r(i) + \tau^s_r(i) + cdt_{r,1}(i) - cdt^s(i) - c[\tfrac{\mu_2}{\mu_1-\mu_2}d^s_{,1} - \tfrac{\mu_1}{\mu_1-\mu_2}d^s_{,2}] + \mu_1 \iota^s_{r,1}(i) \\
E(p^s_{r,2}(i) - \tfrac{\mu_2}{\mu_1-\mu_2}DCB^s) &= \rho^s_r(i) + \tau^s_r(i) + cdt_{r,2}(i) - cdt^s(i) - c[\tfrac{\mu_2}{\mu_1-\mu_2}d^s_{,1} - \tfrac{\mu_1}{\mu_1-\mu_2}d^s_{,2}] + \mu_2 \iota^s_{r,1}(i) \\
E(\iota^s_{r,12}(i) - \tfrac{\mu_1}{\mu_1-\mu_2}DCB^s) &= \iota^s_{r,1}(i)
\end{aligned} \tag{13}$$

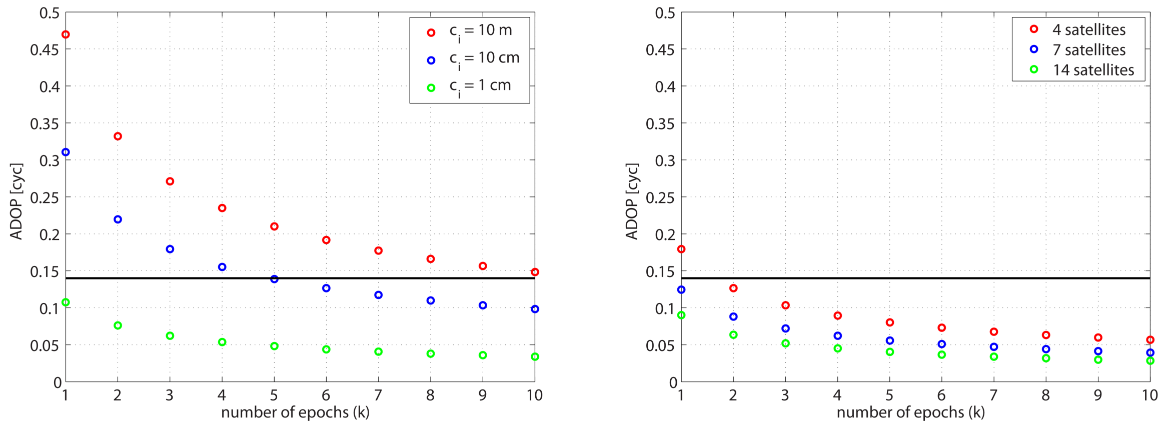

Figure 4. Dual-frequency GPS CORS Network ADOP: impact of ionospheric uncertainty (left) and Modernized triple-frequency GPS CORS Network ADOP for varying number of satellites (right).

It should be mentioned that if C1 code data are used instead of P1, one must account for a P1-C1 correction as well. These P1-C1 DCBs are published on a daily basis by IGS.

If we -- like the ionospheric observations -- include the satellite clock corrections as stochastic observables as well, this results in the following set of DCB-corrected observation equations:

$$\begin{array}{rcl} E(\bar{\phi}^s_{r,1}(i)) & = & \rho^s_r(i) + \tau^s_r(i) + c\delta t_{r,1}(i) - cdt^s_{,12}(i) + \lambda_1 \bar{M}^s_{r,1} - \mu_1 \iota^s_{r,1}(i) \\ E(\bar{\phi}^s_{r,2}(i)) & = & \rho^s_r(i) + \tau^s_r(i) + c\delta t_{r,2}(i) - cdt^s_{,12}(i) + \lambda_2 \bar{M}^s_{r,2} - \mu_2 \iota^s_{r,1}(i) \\ E(\bar{p}^s_{r,1}(i)) & = & \rho^s_r(i) + \tau^s_r(i) + cdt_{r,1}(i) - cdt^s_{,12}(i) + \mu_1 \iota^s_{r,1}(i) \\ E(\bar{p}^s_{r,2}(i)) & = & \rho^s_r(i) + \tau^s_r(i) + cdt_{r,2}(i) - cdt^s_{,12}(i) + \mu_2 \iota^s_{r,1}(i) \\ E(\bar{\iota}^s_{r,1}(i)) & = & \iota^s_{r,1}(i) \\ E(cdt^s_{,12}(i)) & = & cdt^s_{,12}(i) \end{array} \tag{14}$$

with the biased satellite clock error $dt^s_{,12}(i) = dt^s(i) + \frac{\mu_2}{\mu_1-\mu_2} d^s_{,1} - \frac{\mu_1}{\mu_1-\mu_2} d^s_{,2}$ and the extended non-integer ambiguity $\bar{M}^s_{r,j} = \varphi_{r,j}(0) - \varphi^s_{,j}(0) - f_j[d^s_{,j} - \delta^s_{,j}] + N^s_{r,j}$. This un-differenced ambiguity now contains a term for the phase-minus-code satellite hardware bias. In addition, there is now one *common* estimable satellite clock parameter for both phase and code observables.

The full-rank satellite-clock weighted CORS Network RTK model can now be given as:

$$E\left(\begin{bmatrix} \begin{pmatrix} \bar{\phi}^{sd} \\ \bar{p}^{sd} \end{pmatrix} \\ \bar{\iota}^{sd} \\ s^{sd} \end{bmatrix}\right) = \begin{bmatrix} \begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes e_k \otimes (C_n \otimes I_{m-1}) & \begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes e_k \otimes (e_n \otimes I_{m-1}) & \begin{pmatrix} e_j \\ e_j \end{pmatrix} \otimes e_k \otimes B & \begin{pmatrix} e_j \\ e_j \end{pmatrix} \otimes I_k \otimes (-e_n \otimes I_{m-1}) & \begin{pmatrix} -\mu \\ \mu \end{pmatrix} \otimes I_k \otimes (I_n \otimes I_{m-1}) \\ 0 & 0 & 0 & 0 & I_k \otimes (I_n \otimes I_{m-1}) \\ 0 & 0 & 0 & I_k \otimes (1 \otimes I_{m-1}) & 0 \end{bmatrix} \begin{bmatrix} a^{dd} \\ a^{sd} \\ g \\ s^{sd} \\ \iota^{sd} \end{bmatrix} \tag{15}$$

$$D\left(\begin{bmatrix} \begin{pmatrix} \bar{\phi}^{sd} \\ \bar{p}^{sd} \end{pmatrix} \\ \bar{\iota}^{sd} \\ s^{sd} \end{bmatrix}\right) = \begin{bmatrix} \begin{bmatrix} \begin{pmatrix} C_\phi & \\ & C_p \end{pmatrix} & \\ & c_\iota^2 \end{bmatrix} \otimes R_k \otimes \left(I_n \otimes D_m^T W_m^{-1} D_m\right) & \\ & c_s^2 \otimes R_k \otimes \left(1 \otimes D_m^T W_m^{-1} D_m\right) \end{bmatrix} \tag{16}$$

where the following parameters are not defined yet: the (non-integer) between-satellite SD ambiguities $a^{sd} = \left[a_1^{sd^T}, \ldots, a_j^{sd^T}\right]^T$ and the observable-independent satellite clock errors $s^{sd} = \left[s^{sd}(1)^T, \ldots, s^{sd}(k)^T\right]^T$, with $a_j^{sd} = \left[\bar{M}^{12}_{1,j}, \ldots, \bar{M}^{1m}_{1,j}\right]^T$ and $s^{sd}(i) = \left[cdt^{12}_{,12}(i), \ldots, cdt^{1m}_{,12}(i)\right]^T$. The satellite clock standard deviations is denoted as c_s. Note that the phase, code and ionosphere observables are the same as for the model without satellite clock corrections, except that they are now corrected for the (scaled) DCBs.

The drawback of this is that the introduction of the precise satellite clock corrections in the CORS Network RTK model has introduced additional unknowns as well: the between-satellite SD ambiguities of the (master) reference receiver. In the absence of satellite clock corrections, these between-satellite SD ambiguities were lumped with the satellite clocks to form observable-dependent estimable satellite clocks. Unfortunately, it can be shown that the satellite clock corrections do hardly affect the DD ambiguity solution, whether they are of high quality or not! This is because it can be proved that the DD ambiguities and satellite clocks are almost completely *decorrelated*. The only (very!) slight improvement in ADOP is due to an improvement in variance ratio factor $c_{\hat{g}}^2/c_{\check{g}}^2$ when stochastic satellite clock corrections are used. However, the consequence of this is that precise satellite clock corrections will not enable instantaneous CORS Network RTK ambiguity resolution as based on present dual-frequency GPS data.

6. Conclusions

Resolution of the integer carrier phase ambiguities between the stations of a CORS network is a prerequisite for precise ionospheric correction generation for Network RTK users. At present using dual-frequency GPS, CORS Network RTK ambiguity resolution requires convergence time which prevents a true real-time service. By means of ADOP analysis we have identified the bottlenecks for instantaneous or single-epoch full CORS Network RTK ambiguity resolution. These comprise: the absence of sufficiently precise external ionospheric corrections, an insufficient number of satellites and a shortage of frequencies. It was also shown that the availability of precise external tropospheric corrections and satellite clock corrections do hardly affect CORS Network RTK ambiguity resolution. However, the good news is that a future triple-frequency GPS with high-quality L5 code data will ultimately enable instantaneous ambiguity resolution.

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