Optimal forward contracting in LNG supply capacity investment

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ABSTRACT

This paper constructs a stylized model of an LNG producer's decision on the level of commitment to long-term supply arrangement. The model extends a conventional two-stage model of optimal hedging by accommodating two features commonly observed with LNG trading practice: (1) the forward price of LNG is stochastic at the time of forward contracting as it is linked to the spot price of an alternative fuel, namely crude oil, and (2) the producer has a choice over multiple regional gas markets to which it supplies LNG in short-term trading. The model also allows the producer to hedge its price risk through the futures markets of related energy commodities.

For the second feature, a numerical example is provided to illustrate the distributional properties of the maximum of the regional spot prices and how changes in the stochastic properties of one regional (i.e., US) gas price affect the firm's optimal forward position. For range of sensible parameter values, an increase in mean price in one regional market always increases the expected value of the maximum spot price while it can decrease or increase the variance of the maximum spot price. In contrast, an increase in variance of one regional price increases both mean and variance of the maximum price. These two changes, as observed in the US, affect the firm's forward position in opposite ways. The net effect is indeterminate. These results imply that the transition from a conventional trading model through long-term supply-purchase agreement to more flexible short-term trade will be gradual and decelerated by high volatile regional price differentials—what has been motivating recent discussions on potential returns from spot LNG trading.

INTRODUCTION

Two major changes in the world natural gas markets have been emphasized in the last decade: (1) an increase in the level and volatility of regional natural gas prices, and (2) a continuous reduction in costs for liquefaction, shipping, and regasification of LNG. These observations have followed an extensive discussion on potential gains from short-term LNG trading and resulting integration of geographically sparse regional gas markets (see for example, EIA 2005). Although the amount of spot LNG transaction has been increasing gradually and flexible contractual arrangements have become more common, it is often discussed that a conventional model of LNG trading under long-term supply and purchase agreement will be dominant. Such projection is often reason to a large up-front capital investment. Nonetheless, there has been little attempt to elucidate analytically how observed changes in surrounding natural gas market conditions affect forward contracting decisions of LNG suppliers and buyers.

Forward contracting decision has been long studied in the economics literature (Phlips, 1990). In a simple two-stage model widely considered, a hedger, either producer or buyer of the commodity, decides the level of forward cover—the amount to sell (purchase) under forward contract—so as to maximize its utility, which, in a popular mean-variance utility function, is the weighted sum of the expected value and variance of its total profit. In this framework, the forward price is known at the time of forward contracting, and hence, allows the hedger to reduce the variance of its profit. In essence, a hedger is willing to accept for lower expected profit, a risk premium, in order to reduce the variance of the profit.

As widely discussed, an LNG project requires a large investment for development of natural gas reserve and construction of liquefaction train, specialized LNG cargos, loading and other facilities, and, hence, is usually coupled with a long-term supply arrangement. Nonetheless, there are, at least for the sake research interest here, two important features that make forward contracting of LNG distinctive from a standard model of forward contracting. First, the forward price of LNG is often linked to the spot price of an alternative fuel under long-term supply/purchase arrangement. Since this price is not observable at the time of forward trading, the forward price is stochastic unlike in the case considered in a conventional forward contracting literature.

Second, the LNG producer, if it does not supply under long-term forward contract, can choose from multiple regional gas markets to supply its product through short-term trading. This feature provides two potential benefits to the LNG producer. First, it allows the firm to arbitrage price differentials across geographically sparse regional gas markets. Second,

diversifying supply destinations allow firms to reduce the price risk, as the firm can supply to the regional market where the price is the highest of the all the possible destinations. These two features potentially make general implications of the conventional forward contracting literature not applicable to make inference about how the recently observed changes in the world gas markets affect LNG trading practices.

In this paper, I construct a stylized model of an LNG producer's decision on the level of commitment to a long-term supply arrangement. The model extends a conventional two-stage model of optimal hedging by accommodating two features unique to the LNG industry; (1) stochastic forward price and (2) gains from spatial arbitrage as well as from diversification of its supply destinations. The model also allows the producer to hedge its spot and forward price risk through cross-hedging with the futures markets for an alternative fuel. Such hedging strategy is possible to implement in the presence of actively traded futures markets (e.g. NYMEX) for energy commodities to which LNG forward prices are linked to in many long-term supply-purchase arrangements. The optimal forward position for the LNG producer is obtained under the two scenarios—with or without such cross-hedging, and the comparative statics of these solutions with respect to model parameters are examined. In regards to recent discussions on potential benefits from short-term trading, how changes in the mean and variance of one regional gas price affect on the firm's optimal forward position is examined numerically, using the sensible parameter values obtained from the estimates of price and volatility dynamics of natural gas and crude oil.

2. MODEL OF FORWARD CONTRACTING OF LNG PRODUCER

2.1 Model of forward contracting for an LNG producer - No cross-hedging with futures markets for an alternative fuel

In modeling the optimal forward contracting decision of an LNG producer, I consider a company that has finalized its decision to invest on LNG facilities including a liquefaction train and trading cargo. This assumption allows me to focus on the firm's potential benefit from holding some of its production capacity for spot trading rather than committing all its capacity for supply under long-term arrangement. The model also assumes that the firm is a price taker in both forward and spot market.

In a general setting, the profit to an LNG producer is given as,

(1) $\qquad \pi = F\,x + P\,q - C(q + x)$

where P and F are the spot and forward price of LNG, net of transportation cost, respectively, q and x are the amount of LNG sold in spot and forward market.

The producer decides its production level in two steps. First, in the forward market, it chooses the amount it supplies under a long-term forward contract given the forward price. The LNG forward price, under many long-term supply-purchase arrangements, is linked to the spot price of an alternative fuel. For example, it is well known that, in many of bilateral supply-purchase agreements signed by Japanese utility companies, the LNG price is linked to the quantity weighted average of the crude oil prices, often called Japanese Crude Cocktail (JCC). Therefore, unlike in a conventional model of forward contracting considered in the literature, forward price in (1) is stochastic at the time of forward trading.

Second, in the spot market, the firm chooses its total production level, $q + x$, given its forward position and observing the forward price (as the spot price of alternative fuel is realized) and the spot prices of gas in all regional markets. For its spot sales, the producer has a choice over multiple regional markets to which it supplies LNG. Since the producer supplies to the market where the price is the highest, the spot price in equation (1) is, $P = \max\{P_i\}$ with P_i representing the spot price in the regional gas market, i ($i = 1, \dots n$). For notational ease, I use P to refer to the maximum of n regional prices in the rest of the paper.

Since the firm can choose its production level after observing the maximum spot price, P, the firm's optimal decision is solved in backward. First, in the spot market, the first order condition to the firm's profit maximization problem is,

$$(2) \qquad \frac{\partial \pi}{\partial q} = P - C'(q + x) = 0$$

If the cost function is non-linear, then the marginal cost, $C'(\cdot)$, in (2) varies by the total production level, $Q = q + x$. Thus, (2) defines the firm's optimal spot sales, q^*, as a function of spot price, P and its predetermined forward position, x. Here, I assume that the firm's cost function is linear in the production level, $C(Q) = cQ$, so that the marginal cost is constant, $C'(Q) = c$. Besides, I assume that the firm's total production is constrained by the installed capacity Q_{max} per period of operation. Thus, the optimal solution is,

$$(3) \qquad q^*(x) = Q_{max} - x \quad \text{if } P > c$$
$$q^*(x) = 0 \qquad\qquad \text{if } P > c$$

(3) shows that the firm operates at its maximum production capacity when the realized spot price is above the constant marginal cost and produces just enough to fulfill its forward commitment when the spot price is below the marginal cost.[1]

Substituting (3) into the profit function yields,

(4) $\pi = x\,F + q^*(x)\,P - C(q^*(x) + x) = x\,(F - c) + q^*(x)\,(P - c)$

For the firm's optimization problem in the forward market, suppose that the LNG producer has a mean-variance utility function,

(5) $U(\pi) = E[\pi] - \lambda V[\pi]$

where λ is the coefficient of risk aversion. In (5), $E[\pi]$ and $V[\pi]$ are,

(6) $E[\pi] = x\,(E[F] - c) + E[q^*(x)\,(P - c)]$

$V[\pi] = x^2\,V[F] + V[q^*(x)\,(P - c)] + 2x\,\text{cov}(F, q^*(x)\,(P - c))$

In (6),

(7) $E[q^*(x)\,(P - c)] = (Q_{\max} - x)\int_{c}^{\infty}(p - c)\varphi(p)dp = (Q_{max} - x)\alpha$

$V[q^*(x)\,(P - c)] = \int_{-\infty}^{c}(Q_{\max} - x)^2\alpha^2\varphi(p)dp + \int_{c}^{\infty}\big((p - c)(Q_{\max} - x) - \alpha(Q_{\max} - x)\big)^2\varphi(p)dp = (Q_{max} - x)^2\beta$

$\text{cov}(F, q^*(x)\,(P - c)) = \int_{f}\int_{p}(f - E[F])\big(q^*(x)(p - c) - E[q^*(x)(p - c)]\big)\phi(f, p)dpdf = (Q_{max} - x)\chi$

where $\varphi(p)$ is the density function of P and $\phi(p, f)$ is the joint density function of P and F. In (7), $\alpha = \int_{c}^{\infty}(p - c)\varphi(p)dp$ and $\beta = \int_{-\infty}^{c}\alpha^2\varphi(p)dp + \int_{c}^{\infty}\big((p - c) - \alpha\big)^2\varphi(p)dp$ represent the expected value and variance of the per-unit profit margin from spot LNG sales, respectively. Similarly, $\chi = \int_{\infty}^{-\infty}\left(-\int_{-\infty}^{c}(f - E[F])\alpha\phi(f, p)dp + \int_{c}^{\infty}(f - E[F])(p - c - \alpha)\phi(f, p)dp\right)df$ is the covariance between the

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[1] Here, it is implicit that the firm cannot purchase LNG in the spot market, rather than producing by itself, to fulfill its forward position even when the spot price prevailing in the market is below its marginal cost.

forward price and the per-unit profit margin from spot LNG sales. Substituting these expressions into (6) yields,

(8) $E[\pi] = x\,(E[F] - c) + (Q_{max} - x)\alpha$

   $V[\pi] = x^2\,V[F] + (Q_{max} - x)^2\beta + 2x(Q_{max} - x)\chi$

The first and second order conditions for the maximization of $U(\pi)$ are:

(9) $\partial U(\pi)/\partial x = \partial E[\pi]/\partial x - \lambda\,\partial V[\pi]/\partial x = E[F] - c - \alpha - 2\lambda(xV[F] - (Q_{max} - x)\beta - (Q_{max} - 2x)\chi) = 0$

(10) $\partial^2\,U(\pi)/\partial x^2 = \partial^2 E[\pi]/\partial x^2 - \lambda\,\partial^2 V[\pi]/\partial x^2 = -2\lambda(V[F] + \beta + 2\chi) < 0$

Solving (9) for x yields the firm's optimal forward position,

(11) $x^* = \dfrac{1}{V[F] + \beta + 2\chi}\left(\dfrac{E[F] - c - \alpha}{2\lambda} + Q_{max}(\beta + \chi)\right)$

The optimal forward position in (11) differs from the solution to a usual forward contracting problem due to the uncertainty of the forward price. If the forward price is deterministic, then $E[F] = F$, $V[F] = \chi = 0$ and (11) reduces to,

(12) $x^* = \dfrac{F - c - \alpha}{2\lambda\beta} + Q$

(12) implies that when the forward price is equal to the expected spot price, the firm sells all its capacity under forward contract. If the firm is risk averse ($\lambda > 0$) and forward price is below the expected spot price, the firm's forward cover is only partial ($x^* < Q$). The firm commits to sell more than its production capacity if the forward price is above the expected spot price. In either case, the firm's optimal forward cover depends on the three attributes; the forward premium or the level forward price relative to the expected spot price, the firm's risk aversion coefficient, and volatility of spot price.

In contrast, (11) indicates that the optimal solution, when the forward price is stochastic, depends on, in addition to these three attributes, the expected value and variance of forward price as well as the covariance of spot and forward prices.

Comparative statics

The comparative statics of (11) are,

(13a) $\dfrac{\partial x^*}{\partial \lambda} = -\dfrac{1}{V[F]+\beta+2\chi}\left(\dfrac{E[F]-c-\alpha}{2\lambda^2}\right) > 0$ if $E[F] - c < \alpha$

(13b) $\dfrac{\partial x^*}{\partial Q_{max}} = \dfrac{\beta+\chi}{V[F]+\beta+2\chi} > 0$

(13c) $\dfrac{\partial x^*}{\partial E[F]} = \dfrac{1}{2\lambda(V[F]+\beta+2\chi)} > 0$

(13d) $\dfrac{\partial x^*}{\partial \alpha} = -\dfrac{1}{2\lambda(V[F]+\beta+2\chi)} < 0$

(13e) $\dfrac{\partial x^*}{\partial V[F]} = -\dfrac{1}{(V[F]+\beta+2\chi)^2}\left(\dfrac{E[F]-c-\alpha}{2\lambda} + Q_{max}(\beta+\chi)\right) < 0$ if $x^* > 0$

(13f) $\dfrac{\partial x^*}{\partial \beta} = -\dfrac{1}{(V[F]+\beta+2\chi)^2}\left(\dfrac{E[F]-c-\alpha}{2\lambda} - Q_{max}(V[F]+\chi)\right) > 0$ if $E[F] - c < \alpha$

(13g) $\dfrac{\partial x^*}{\partial \chi} = -\dfrac{1}{(V[F]+\beta+2\chi)^2}\left(\dfrac{E[F]-c-\alpha}{\lambda} - Q_{max}(V[F]-\beta)\right)$

The first results, (13a) through (13e), are rather straightforward. The firm's optimal forward is higher when the firm is more risk averse (13a) and increases with its production capacity but less proportionately so when the forward price is stochastic (13b). The forward position also increases (decreases) with the expected forward price (the expected profit margin in spot sales) while it decreases with the variance of the forward price. In (13f) and (13g), the effects of increase in the variance of per-unit profit margin from spot LNG sales and its covariance with the forward price are somewhat ambiguous. In (13f), an increase in the variance of profit margin from spot sales raises the forward position if the expected profit margin is higher for spot sales than for forward sales. In (13g), the effect of increase in the covariance between the per-unit-profit margin from spot sales and that from forward price depends on the relative magnitude of the mean and variance of the per-unit profit margin from spot sales and those from forward sales.

2.2   Cross hedging with futures markets for an alternative fuel

The model in previous sub-section has accommodated the stochastic nature of the forward price. What has not been discussed in the existing literature is that the producer (and consumer as well) can mitigate the risk associated with the stochastic forward price through

cross-hedging with futures contracts of an alternative fuel to which the LNG forward price is linked.

To illustrate such cross-hedging strategy, suppose that the LNG forward price is linked to a price of an alternative fuel, say crude oil, through the following linear pricing formula,[2]

(14) $F = a + b\,PA$

where PA is the spot price of crude oil. Futures contracts for crude oil have been actively traded in an organized exchange, such as NYMEX, and covers as long as eight years in trading horizon. Naturally, the price of futures contract provides good forecast about the spot price in the delivery period and converges to the spot price as the contract approaches to maturity. The LNG producer can mitigate the price risk by taking a short position in the crude oil futures and clears its position as it approaches to the time for physical delivery of LNG.

With this cross-hedging strategy, the firm's profit becomes,

(15) $\pi = (a + b\,PA)\,x + P\,q - C(q + x) + f_0\,y - f_1\,y$

where P, x, q, and $C(\cdot)$ are as defined before and the forward price of LNG, F, is substituted with the pricing formula (14). f_0 and f_1 are the prices of crude oil futures contracts that mature in the same period as physical delivery of LNG takes place. The subscripts, 0 and 1, denote that the two prices represent the futures prices at the time of forward contracting and at the time of physical delivery of LNG, respectively.

Assuming that the futures market is efficient so that the price converges to the spot price as it approaches to the maturity date, i.e., $f_1 = PA$ where PA is the spot price of crude oil. Substituting this expression into (15) yields,

(16) $\pi = ax + f_0\,y + PA\,(bx - y) + P\,q - C(q + x)$

If the firm takes a short position in the futures market in the amount b per unit of its forward position, it fully covers the forward price risk. This is seen by setting $y = bx$ in (16),

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[2] Exact formulas used in defining LNG forward price under long-term forward contracts are not publicly available. However, it has been often reported that the three types of formulas are commonly used: (i) affine, (ii) linear, and (iii) S-curve (Chabrelie, 2003; Fujime, 2002; Bartsch, 1998).

(17) $\pi = (a + bf_0) x + P q - C(q + x)$

In (17), only stochastic component is the spot price of LNG, P. The forward price of LNG, F, does not appear in (17) as it is replaced with $(a + bf_0)$. This term can be interpreted as the price of LNG futures contract replicated with the crude oil futures contract.

To find the firm's optimal position in forward and futures market, the problem is solved in backward as in previous case. Since the firm's futures position does not impact on the firm's production decision in (16), the optimal production level, $q + x$, is identical with (3). In forward market, the firm chooses its futures and forward position to maximize the value of the mean-variance utility function. The expected value and variance of the firm's revenue, with possibility of cross-hedging with futures market are,

(18) $E[\pi] = (a - c)x + f_0 y + E[PA](bx - y) + E[(P - c) q^*(x)]$

$V[\pi] = (bx - y)^2 V[PA] + V[(P - c) q^*(x)] + 2(bx - y) \text{cov}(PA, (P - c) q^*(x))$

where $E[(P - c) q^*(x)]$ and $V[(P - c) q^*(x)]$ have the expressions as given in (8). The covariance term has the expression,

(19) $\text{cov}(PA, (P - c) q^*(x)) = (Q_{max} - x)\eta$

where $\eta = \int_{-\infty}^{\infty} \left(-\int_{-\infty}^{c} (pa - E[PA])\alpha dp + \int_{c}^{\infty} (pa - E[PA])(p - c - \alpha)dp \right) dpa$.

The first and second order conditions are,

(20) $\partial U(\pi)/ \partial x = (a - c) + bE[PA] - \alpha - 2\lambda\{b(bx - y) V[PA] - (Q_{max} - x) \beta + (bQ_{max} - 2bx + y)\eta\} = 0$

$\partial U(\pi)/ \partial y = f_0 - E[PA] - 2\lambda \{-(bx - y) V[PA] - (Q_{max} - x) \eta\} = 0$

(21) $\partial^2 U(\pi)/\partial x^2 = -2\lambda(b^2 V[PA] + \beta - b\eta) \leq 0$

$\partial^2 U(\pi)/\partial y^2 = -2\lambda V[PA] \leq 0$

$(\partial^2 U(\pi)/\partial x^2) (\partial^2 U(\pi)/\partial y^2) - (\partial^2 U(\pi)/\partial x \partial y)^2 = 4\lambda^2 \{V[PA] (b^2 V[PA] + \beta - 2b\eta) - (\eta - bV[PA])^2\} \geq 0$

Solving the equations (20) for x and y yields,

$$(22) \qquad x^* = Q_{max} + \frac{V[PA]\big((a+bf_0-c)-\alpha\big)-\eta\big(f_0-E[PA]\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)}$$

$$(23) \qquad y^* = bQ_{max} + \frac{\big(bV[PA]-\eta\big)\big((a+bf_0-c)-\alpha\big)+(\beta-b\eta)\big(f_0-E[PA]\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)} \quad \text{or}$$

$$y^* = bx^* + \frac{-\eta\big((a+bf_0-c)-\alpha\big)+\beta\big(f_0-E[PA]\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)}$$

If the futures price is unbiased in forecasting future spot price, that is, $f_0 = E[PA]$, then (22) and (23) reduce to,

$$(24) \qquad x^* = Q_{max} + \frac{V[PA]\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)}$$

$$(25) \qquad y^* = bQ_{max} + \frac{\big(bV[PA]-\eta\big)\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)}$$

In (24) and (25), $a + bf_0 - c$ is the difference between the marginal production cost and the price of LNG futures contract replicated with the crude oil futures contract. In other words, it represents the per-unit profit margin from the forward sales when the firm cross-hedges its price risk with the crude oil futures. (24) and (25) show that if this profit margin is below the expected per-unit profit margin in spot sales, i.e., $a + bf_0 - c < \alpha$, the producer will cover its production capacity only partially under forward contract. Moreover, the firm's futures position is above its forward position if the future spot price of an alternative fuel is positively correlated with the profit per-unit of LNG sold in the spot market (i.e., $\eta > 0$). The firm supplies all its production capacity under the forward contract (i.e., $x^* = Q_{max}$) when the forward price formula is designed as such that the resulting forward price according to the current futures price of crude oil less the marginal cost of production is equal to the expected per-unit profit margin from the spot sales ($a + bf_0 - c = \alpha$). In such case, the firm fully hedges its forward price risk by taking a short position in the futures market for crude oil (i.e., $x^* = Q_{max}$ and $y^* = b\, x^*$).

Comparative statics:

The effects of changes in some of key parameters on the firm's optimal forward position, when the price of the futures contract of an alternative fuel is unbiased in forecasting the futures spot price, are

$$\text{(26a)} \quad \frac{\partial x^*}{\partial Q_{max}} = 1 \qquad\qquad\qquad > 0$$

$$\text{(26b)} \quad \frac{\partial x^*}{\partial \lambda} = -\frac{V[PA]\big((a+bf_0-c)-\alpha\big)}{2\lambda^2\big(V[PA]\beta-\eta^2\big)} \qquad > 0 \text{ if } a + bf_0 - c < \alpha, \text{ and} < 0 \text{ if } a + bf_0 - c > \alpha$$

$$\text{(26c)} \quad \frac{\partial x^*}{\partial (a+bf_0)} = \frac{V[PA]}{2\lambda\big(V[PA]\beta-\eta^2\big)} \qquad > 0$$

$$\text{(26d)} \quad \frac{\partial x^*}{\partial \alpha} = -\frac{V[PA]}{2\lambda\big(V[PA]\beta-\eta^2\big)} \qquad < 0$$

$$\text{(26e)} \quad \frac{\partial x^*}{\partial \beta} = -\frac{V[PA]^2\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2} \qquad > 0 \text{ if } a + bf_0 - c < \alpha, \text{ and} < 0 \text{ if } a + bf_0 - c > \alpha$$

$$\text{(26f)} \quad \frac{\partial x^*}{\partial V[PA]} = -\frac{\eta^2\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2} \qquad > 0 \text{ if } a + bf_0 - c < \alpha, \text{ and} > 0 \text{ if } a + bf_0 - c > \alpha$$

$$\text{(26g)} \quad \frac{\partial x^*}{\partial \eta} = \frac{2\eta V[PA]\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2} \qquad < 0 \text{ if } a + bf_0 - c < \alpha \text{ and } \eta > 0 \text{ or} > 0 \text{ if } a + bf_0 - c > \alpha \text{ and } \eta > 0$$

The effects of changes in some of key parameters on the firm's optimal futures position are,

$$\text{(27a)} \quad \frac{\partial y^*}{\partial Q_{max}} = b \qquad\qquad\qquad > 0$$

$$\text{(27b)} \quad \frac{\partial y^*}{\partial \lambda} = -\frac{\big(bV[PA]-\eta\big)\big((a+bf_0-c)-\alpha\big)}{2\lambda^2\big(V[PA]\beta-\eta^2\big)} > 0 \text{ if } y^* < bQ_{max} \text{ and} < 0 \text{ if } y^* > bQ_{max}$$

$$\text{(27c)} \quad \frac{\partial y^*}{\partial (a+bf_0)} = \frac{bV[PA]-\eta}{2\lambda\big(V[PA]\beta-\eta^2\big)} \qquad > 0 \text{ if } V[PA] > \eta b^{-1}, < 0 \text{ otherwise}$$

$$\text{(27d)} \quad \frac{\partial y^*}{\partial \alpha} = -\frac{bV[PA]-\eta}{2\lambda\big(V[PA]\beta-\eta^2\big)} \qquad < 0 \text{ if } V[PA] > \eta b^{-1}, > 0 \text{ otherwise}$$

$$\text{(27e)} \quad \frac{\partial y^*}{\partial \beta} = -\frac{V[PA]\big(bV[PA]-\eta\big)\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2} = b\frac{\partial x^*}{\partial \beta} + \frac{\eta V[PA]\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2}$$

$$\text{(27f)} \quad \frac{\partial y^*}{\partial V[PA]} = \frac{\big(\eta\beta-b\eta^2\big)\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2} = b\frac{\partial x^*}{\partial V[PA]} + \frac{\eta\beta\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2}$$

$$\text{(27g)} \quad \frac{\partial y^*}{\partial \eta} = \frac{(a+bf_0-c)-\alpha}{2\lambda\big(V[PA]\beta-\eta^2\big)^2}\Big[2\eta\big(bV[PA]-\eta\big)-\big(V[PA]\beta-\eta^2\big)\Big] = b\frac{\partial x^*}{\partial \eta} - \frac{\big(\eta^2+V[PA]\beta\big)\big((a+bf_0-c)-\alpha\big)}{2\lambda\big(V[PA]\beta-\eta^2\big)^2}$$

The comparative static results in (26a) through (26d) indicates that the firm's forward position increases with (a) the firm's production capacity, (b) the firm's risk aversion coefficient, and (c) the price of LNG futures contract as replicated with the futures contract of an alternative fuel, while it decreases with (d) the expected per-unit profit margin from the

spot sales. (26e) shows that, given the expected per-unit profit margin from spot LNG sales is above the profit margin from the forward sales, the firm's forward position increases with the volatility of per-unit profit margin from spot LNG sales.

The last two effects, (26f) and (26g), are not straightforward. (26f) shows that, given that the expected per-unit profit margin from spot LNG sales is above the profit margin from the forward sales and that these two profit margins are correlated (i.e., η = 0), the firm increases its forward position when the spot crude oil price becomes more volatile. Note that if the two profit margins are uncorrelated (i.e., η = 0), the volatility of the spot crude oil price has no impact on the firm's forward position. This is because the risk associated with the stochastic forward LNG price can be completely mitigated through cross-hedging with crude oil futures. The firm takes even greater forward and futures position if the two profit margins are correlated because this correlation allows the firm to reduce not only the forward price risk but also the spot price risk through cross-hedging with crude oil futures. This strategy becomes more costly if the spot crude oil price becomes more volatile. Thus, the firm increases its forward and futures position simultaneously so as to reduce its exposure to the spot price risk. In another case, when the two profit margins are highly correlated, hedging spot price risk by increasing its forward and futures position becomes more effective. Thus, given that the profit margin is higher from spot sales than from forward sales, the firm increases (decreases) its forward and futures position when the two profit margins are highly positively (negatively) correlated.

In (27a) through (27d), the firm's position in crude oil futures increases with (a) the firm's production capacity, (b) the firms risk aversion coefficient, and (c) the price of LNG futures contract as replicated with the crude oil futures contract, while it decreases with (d) the expected per-unit profit margin from spot LNG sales.

In (27e), an increase in the volatility of the per-unit profit margin from spot LNG sales affects the firm's futures position in two ways. First, the firm's futures position increases proportionally to the change in the firm's forward position to mitigate the associated increase in the firm's exposure to the forward price risk (the first term in the right hand side of the equation). Second, an increased volatility of per-unit profit margin from spot LNG sales decreases the firm's futures position under two conditions: (i) the profit margin is higher for the spot LNG sales than for the forward sales and (ii) the profit margin from spot sales is positively correlated with the spot price of crude oil. The firm reduces its futures position because the spot price risk, when it is more volatile, is difficult to mitigate.

Similarly, an increase in the volatility of spot crude oil price also affects the firm's futures position in two ways. The first effect is, again, proportional to the change in the firm's forward position and is positive given the profit margin is higher for spot LNG sales than for forward sales. The second effect is negative if the profit margin from spot LNG sales and spot crude oil are positively correlated. The firm reduces its futures position because higher volatility of spot crude oil price makes cross-hedging with crude oil futures less effective to mitigate the spot price risk.

Finally, (27g) shows an increase in the covariance between the profit margin from the spot LNG sales and spot crude oil price affects both positively and negatively the firm's futures position. From (26g), the firm reduces its forward position when the covariance is high since the spot price risk is more effectively mitigated through cross-hedging with the futures market. The firm changes its futures position in proportion to the change in its forward position. The second term in the right hand side of (27g) is positive because an increase in the expected spot sales requires the firm to increase its futures position to mitigate the spot price risk.

3. EFFECTS OF INCREASE IN THE LEVEL AND VARIANCE OF ONE REGIONAL GAS PRICE—A NUMERICAL ILLUSTRATION

A feature that has not been explicit in the model so far is the fact that the firm can arbitrage spatial price differentials in spot sales, i.e., it can choose from the multiple regional gas markets to sell its LNG in spot trading. This feature is implicit in the model examined in previous section. Thus, the firm's optimal decisions in production, forward and futures position as well as their comparative static results still hold. With the option for spatial arbitrage, the firm's optimal solution in the spot market is to sell $Q_{max} - x$ in the market where the spot price is the highest of all the market and only if this maximum price is above the marginal cost.

Nonetheless, comparative static effects can be investigated further in detail with respect to the change in the distribution of one regional gas price on the firm's optimal forward cover. Such analysis is of particular interest given much debate on the recent increase in the level and volatility of the US natural gas price and how these changes impact on the firm's forward contracting decision. In the model developed in previous section, the change in the distribution of the spot price in one regional market impacts on the firm's optimal decision through three key parameters, the expectation and variance of per-unit profit margin from spot LNG sales (α and β) and its covariance with the forward price of LNG (χ) or the

covariance with the spot price of crude oil (η) if the firm cross-hedges with the futures markets. Unfortunately, it is not easy to examine analytically how the change in the distributional properties of the spot price in a single regional market affects on these three parameters, because that the non-linearity of the maximum operator, $P = \max\{P_i\}$, makes it almost impossible to obtain analytic expressions of the three moments.

In this section, I take an alternative approach and illustrate a numerical example with a simple case where the firm has only two potential markets to which it supplies its LNG, namely Japan and the US. The analysis in the previous section has illustrated that the firm's optimal decisions in production, forward position, and futures position depend on the distributional properties of the regional gas prices as well as the spot price of an alternative fuel. I generated the three price series; the spot natural gas prices in the US and Japanese market and the spot price of crude oil as an alternative fuel, from the multivariate normal distribution. As summarized in **table 1**, the mean and standard deviation of the three prices, in the base case, are set as follows, $60/bbl and $10/bbl for crude oil, $9MBTu and $3/MBTu for the two regional gas prices. For the US LNG price, the range of values is considered—from $3 to $15/MBTu for mean and from $1 to $6/MBTu for standard deviation. For the correlation between the crude oil and the LNG prices, I considered from 0.10 to 0.90 and set the same value for the two markets. The same range of values is considered for the correlation between the two gas prices. These parameter values are selected so as to cover seasonal variations in mean and variance of the US natural gas and Brent crude oil price as depicted by the model of price and volatility dynamics of the two commodities estimated in Suenaga (2007).[3]

In addition to these parameters determining the distributional properties of the three relevant prices, six parameters also determine the firm's optimal decisions: the marginal cost of production (c), production capacity (Q_{max}), the risk aversion coefficient, (λ), the futures price of an alternative fuel (F_0) and the two parameters defining the LNG pricing formula. For the futures price, I maintain the assumption of its unbiasedness and, hence, set equal to the expected spot rude oil price $60/bbl. For the LNG pricing formula, I consider the affine function and set the constant and slope coefficient 0 and 0.15, respectively. These values

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[3] The estimated model shows that the seasonal mean price ranges from $6.810 to $10.943/MBTu within a year for the US Henry Hub (HH) natural gas and from $55.504 and $61.818/bbl for Brent crude oil. The model also indicates substantial variation in price volatility for natural gas, ranging from $1.145 to $5.788/MBTu in standard deviation. In contrast, the spot price volatility does not exhibit much seasonality for crude oil, with corresponding numbers $9.519 and $10.914/bbl. The deviation from the seasonal mean price exhibits reasonably large correlation 0.5238 between the HH natural gas and Brent crude oil.

assure that the LNG forward price is on average equal to the spot price of natural gas in Japanese market. Finally, I considered a wide range of values for the marginal cost of LNG production.

Figure 1 illustrates the sensitivity of the mean of the maximum of the two LNG prices to the values of the three parameters. In panel (a) of figure 1, the mean of the maximum of the two LNG prices increases with the mean of the US gas price. In panels (b) and (c) of figure 1, the mean price increases the volatility of the US gas price while decreases with the covariance of the two gas prices, even when the mean of the US gas price is fixed. These results are consistent with Ker (2002).

Figure 2 shows the sensitivity of the volatility of the maximum of the two LNG prices to the values of the three parameters. In panel (a) of the figure, the standard deviation of the maximum of the two prices is convex—initially decreasing and then increasing with the mean of the US LNG price. In panels (b) and (c) of the figure, the standard deviation increases with the variance of the US price and covariance of the two regional gas prices, respectively.

Figure 3 illustrates the sensitivity of the firm's optimal forward positions under two scenarios: (1) when the firm cross-hedges the spot and forward price risk with futures markets for an alternative fuel, and (2) when the firm does not cross-hedge with the futures markets. **Figure 3** also shows the firm's optimal futures positions, converted into the unit equivalent to the LNG by dividing the number of crude oil futures contracts by the slope coefficient in the LNG pricing formula (b = 0.15). In the scenario with the base parameter values, the firm, if it does not cross-hedge with the futures contracts, supplies slightly more than 60% of its production capacity under the long-term forward contract. The firm supplies substantially larger volume under long-term forward contract if it mitigates the LNG price risk by cross-hedging with the crude oil futures. Besides, the firm's futures position far exceeds its forward position, indicating that the firm utilizes the crude oil futures to mitigate the spot price risk as well given high correlation between the spot price of crude oil and spot gas prices assumed in the base case.

In panel (a), the firm's forward position and futures position decrease with the marginal cost of production but slightly so for the range of the values considered. Surprisingly, even when the marginal cost is very high—only one dollar below the expected spot price, the firm supplies only less than 60% of its capacity under long-term supply. This low sensitivity of the firm's forward position implies that a recent reduction in the liquefaction and shipping cost alone will not stimulate world LNG trade as often discussed in the literature.

In panel (b), the firm's forward position decreases with the expected spot gas price in the US market. The reduction in the forward position is more evident when the firm cross-hedges with the crude oil futures. The two effects seem to determine the net impact. First, as shown in Figure 1, an increase in the mean of the US gas price increases the mean of the maximum of the two spot gas prices, which will make the spot trading more profitable than before the US gas price increase. The second, an increase in the US gas price initially reduces and then increases the volatility of the maximum of the two regional gas prices. An increase in the volatility of the maximum price, when the mean US price is above \$9/MBTu, exposes the firm to a greater spot price risk. However, for the same range of the US gas price, the mean of the maximum of the two prices increases rapidly. These two effects, as shown in panel (b), indicates rather moderate but constant reduction in the firm's forward position. On the other hand, the firm's position in the crude oil futures market is not very sensitive and stays around the level equivalent to 100 units of LNG. This insensitivity is because the crude oil futures can provide a good hedging mechanism for both spot and forward price risk in LNG market.

In panel (c), an increase in the volatility of the US gas price raises the firm's forward positions regardless of whether the firm cross-hedges with the crude oil futures. Again, increased volatility in the US gas price impacts the firm's forward position in two ways—it increases the mean of the maximum of the two regional gas prices while decreases the volatility of the maximum prices. As shown in Figures 1 and 2, the second effect is substantially greater than the first, resulting in the net increase in the firm's forward supply. The firm's futures position, when it cross-hedges, increases accordingly.

In panel (d), the firm's forward position increases with the correlation between the two regional gas prices in both cases. This is consistent with the sensitivity of the maximum of the two prices observed in Figures 1 and 2—the mean of the maximum price decreases while the variance increases with the correlation of the two regional gas prices. These two effects make the spot transaction less profitable while more risky. The firm's futures position is rather insensitive to the correlation of the two regional gas prices, at least in the case considered here with the based parameter values.

Finally, in panel (e), the firm's forward position decreases with the correlation between the two regional gas prices and the spot crude oil price. The story is very straightforward for the case where the firm does not cross-hedge with crude oil futures. High correlation means the LNG forward price is as risky as the spot gas price. Thus, the firm reduces its forward position in favor of slightly higher expected profit margin from the spot trading than from the

forward trading—the difference created by the option to choose from the two regional markets to supply the commodity. In the other case, where the firm cross-hedges with the crude oil futures, a reduction in the forward position originates more in the increased effectiveness of cross-hedging to mitigate the spot price risk. Since the potential arbitrage profit brings the expected profit margin from spot trading above the profit margin from the forward trading, the firm has greater incentive to supply in the spot markets than in the forward market.

CONCLUSION

In this paper, I examined the forward contracting decision for an LNG producer. A stylized model is developed based on a simple two-stage model yet accommodates the two features unique to the situation that LNG industry faces recently—the uncertainty of LNG forward price and the producer's choice over multiple regional gas markets to supply its product through short-term trading. I also considered how hedging the LNG price risk with the futures markets for an alternative fuel affects the producer's optimal forward position.

Without such cross-hedging strategy, the firm takes a large forward position when the expected profit margin is high in forward trading relative to the profit margin from spot sales and when the forward price is less volatile than the spot price. In contrast, cross-hedging allows the LNG producer to mitigate the risk associated with the stochastic forward price. Therefore the producer takes substantially greater forward position than without cross-hedging. The producer also takes a large short position in futures markets because it reduces not only the forward price risk but also the spot price risk in LNG market to the extent that the futures price of an alternative fuel is correlated with the spot LNG price. Adding futures market does not change the effects of two mean prices on the firm's optimal forward position, but changes substantially the effects of the variance of the two prices on the firm's forward position. Among the striking result is that, under general setting, the firm increases its forward position when the spot price of an alternative fuel becomes more volatile. The firm increases its futures position because the risk associated with the volatility of forward LNG price is completely eliminated through cross-hedging. Rather, an increase in the forward position originates in the fact that when the spot price of crude oil is more volatile, cross-hedging the spot LNG price risk with the crude oil futures becomes more costly as the futures position taken for this purpose needs to be cleared when the contract matures.

The effect of an increase in the level and volatility of regional gas price on firms' forward position, much discussed among the observers of world gas industry, is also examined. A numerical example illustrated that an increase in the mean of one regional price increases the

mean of the maximum of the two regional prices while it initially decreases and increases the volatility of the two prices. An increase in the volatility of one regional price increases both the mean and variance of the maximum regional gas price. These changes affect the firm's forward position in opposite way. The net impact is indeterminate.

The model developed in this paper can be extended in several aspects. Among the most interesting exercise is to extend the model into a general equilibrium setting in which the forward and futures positions as well as prices in these markets to be endogenous to the model. The model here is also limited as it presumes that investment on the development of gas reserve and construction of liquefaction and other facilities has been finalized. In reality, these decisions are most likely made simultaneously with the production and contracting decision. Finally, a numerical example in section 3 should be refined with increasing the number of regional markets from two and/or using more detailed production and shipping cost and other data. These extensions are left for a future research.

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**Table 1. List of the parameter values used in simulation**

| Parameters	Description	Min	Base	Max
c	Marginal cost of production	1	3	8
a	Constant in LNG price formula	-	0	-
b	Slope in LNG price formula	-	0.15	-
F_0	Futures price of crude oil	-	60	-
Q_{max}	Production capacity	-	100	-
λ	Risk aversion coefficient	-	0.05	-
μ_{CO}	Mean of spot crude oil price	-	60	-
σ_{CO}	SD of spot crude oil price	-	10	-
μ_{JP}	Mean Japanese LNG price	-	9	-
σ_{JP}	SD Japanese LNG price	-	3	-
μ_{US}	Mean US LNG price	3	9	15
σ_{US}	SD US LNG price	1	3	6
ρ_{12}	Correlation JP-US	0.10	0.50	0.90
ρ_{13}	Correlation JP-Crude oil	0.10	0.50	0.80
ρ_{23}	Correlation US-Crude oil			

Figure 1. The sensitivity of the mean of the maximum of the two LNG prices to the three parameters determining the distributions of the two prices

(a) Sensitivity to the mean US price

US natural gas price ($/MBTu)

(b) Sensitivity to the SD US price

US natural gas price ($/MBTu)
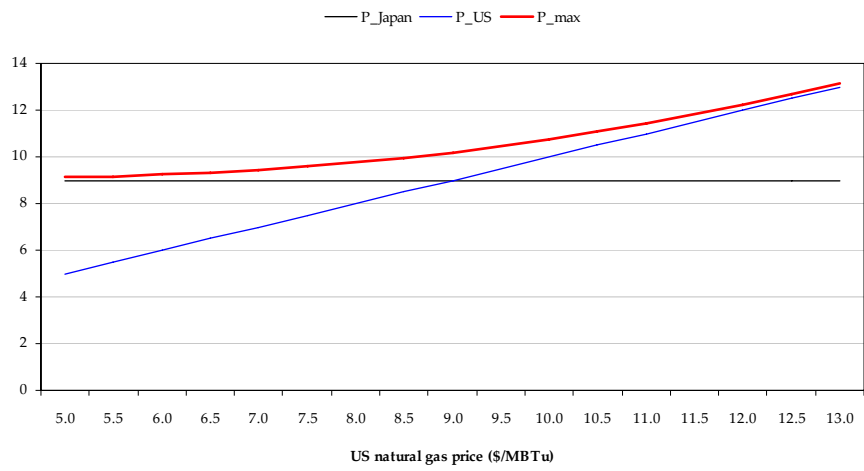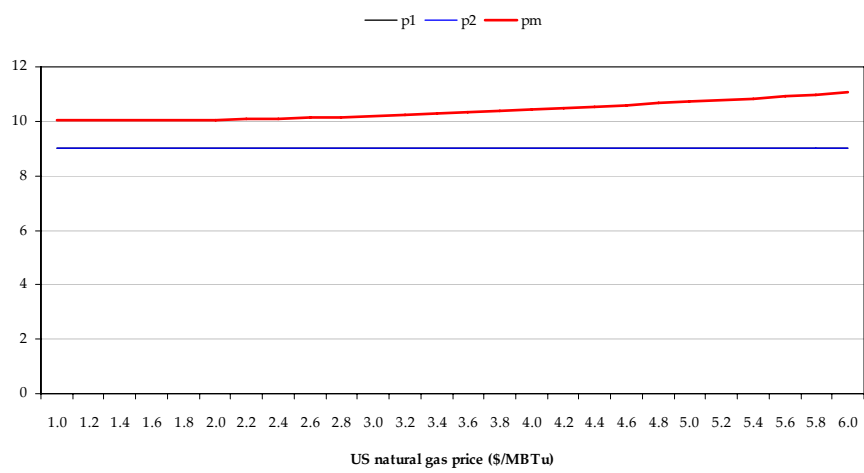
(c) Sensitivity to the correlation between US and Japan price

correlation
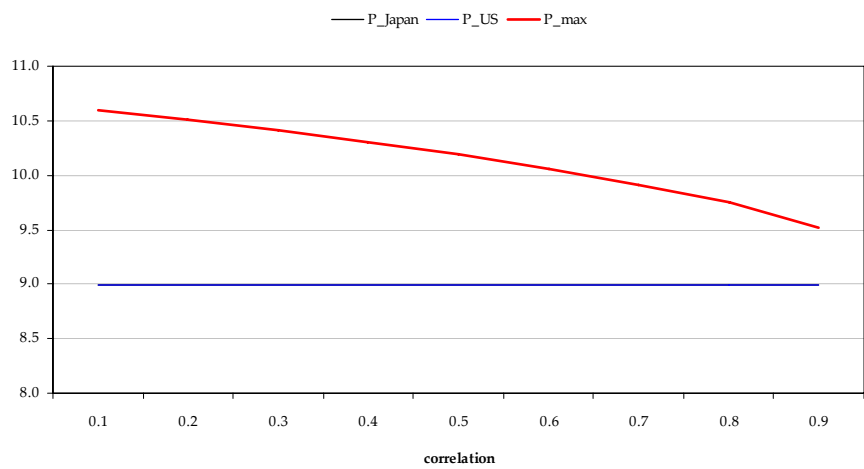
Figure 2. The sensitivity of the standard deviation of the maximum of the two LNG prices to the three parameters determining the distributions of the two prices

(a) Sensitivity to the mean US price

US natural gas price ($/MBTu)

(b) Sensitivity to the SD US price

US natural gas price ($/MBTu)
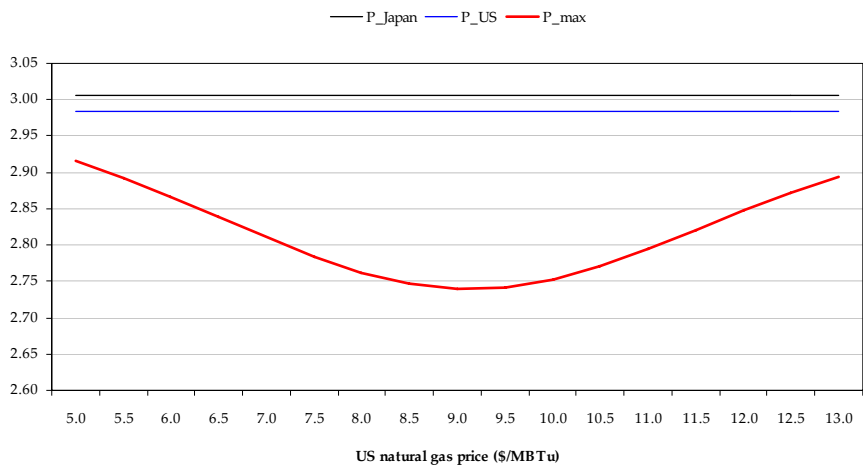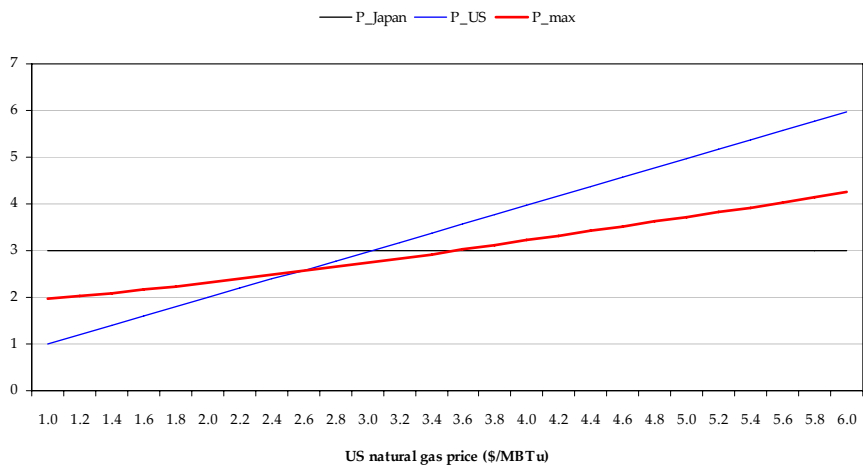
(c) Sensitivity to the correlation between US and Japan price
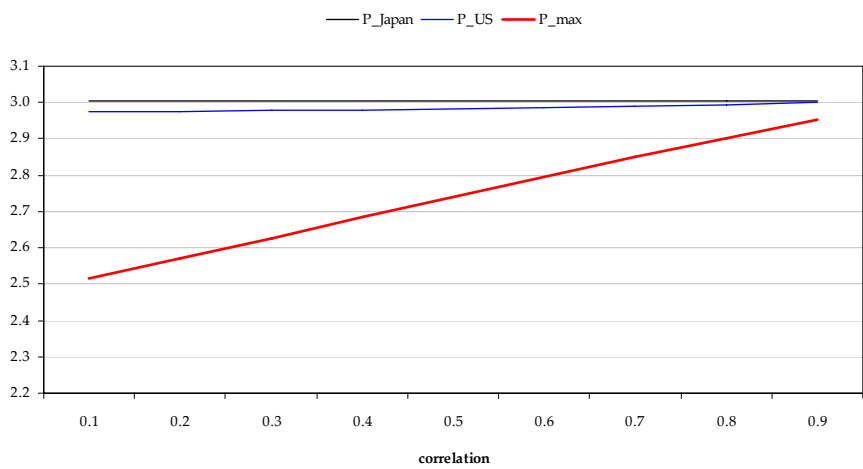
correlation

Figure 3. Optimal forward and futures positions

(a) Sensitivity to the marginal cost

c **($/MBTu)**

(b) Sensitivity to the mean US price

US natural gas price ($/MBTu)

(c) Sensitivity to the SD US price

US natural gas price ($/MBTu)
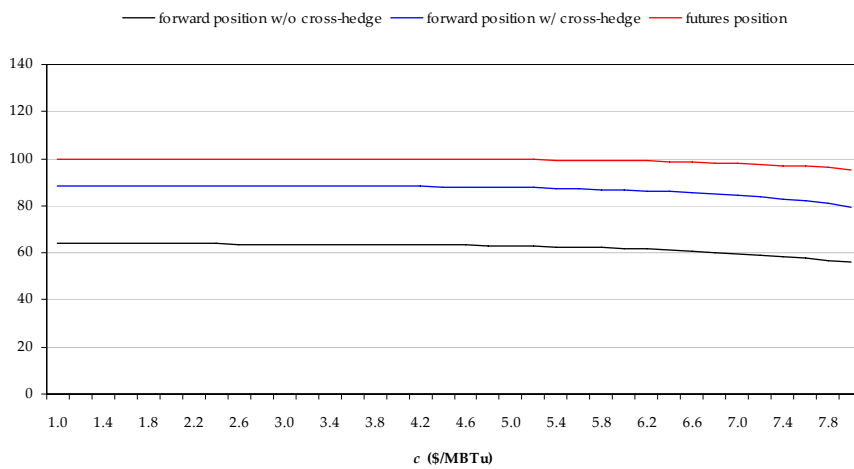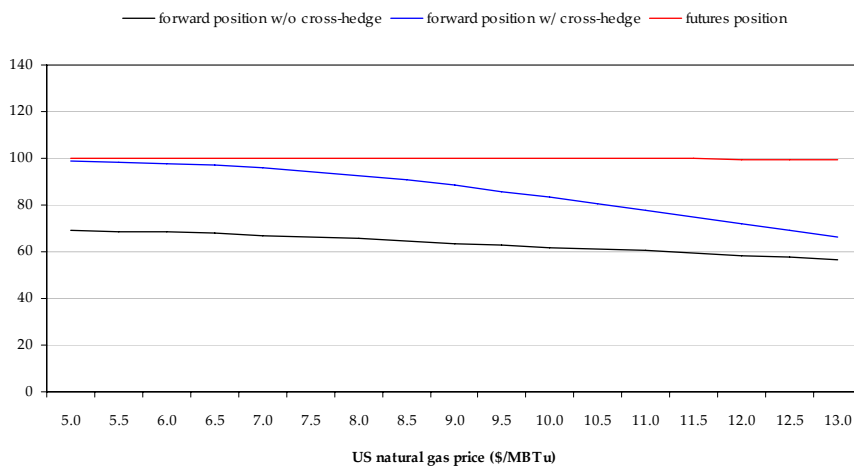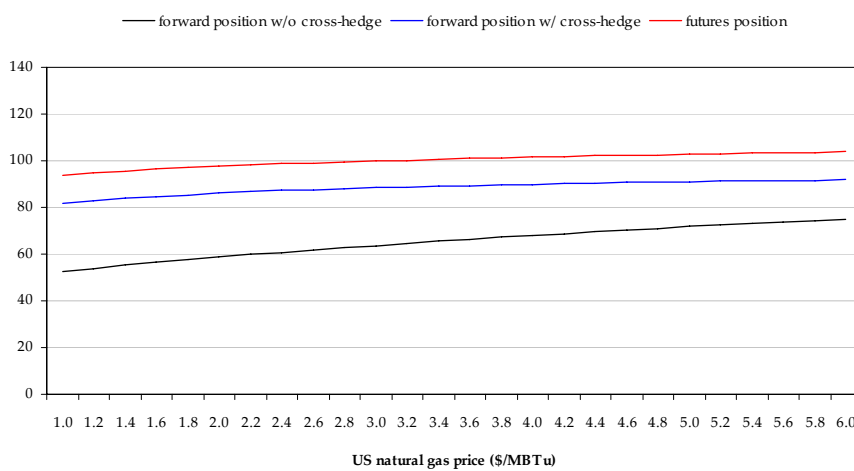
Figure 3. continued.

(d) Sensitivity to the correlation between US and Japan price

—— forward position w/o cross-hedge —— forward position w/ cross-hedge —— futures position
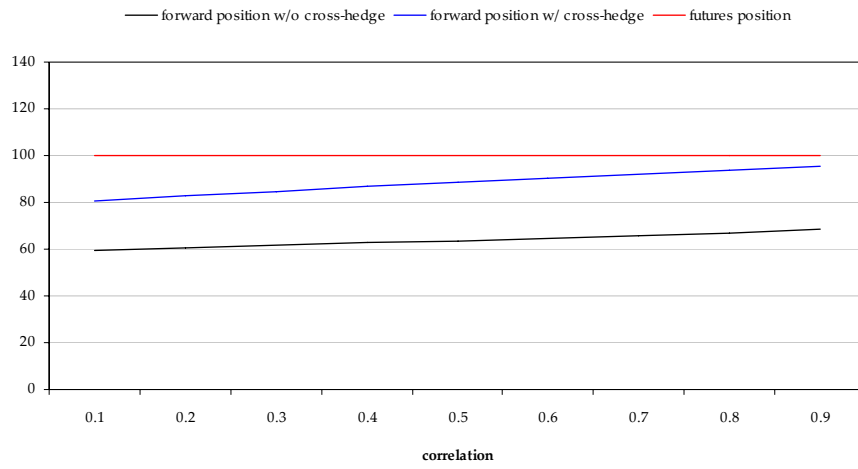
correlation

(e) Sensitivity to the correlation between gas and crude oil price

—— forward position w/o cross-hedge —— forward position w/ cross-hedge —— futures position

correlation