GEOPHYSICS, VOL. 69, NO. 6 (NOVEMBER-DECEMBER 2004); P. 1372–1377, 5 FIGS., 3 TABLES.
10.1190/1.1836811

Short Note

Simple expressions for normal incidence reflection coefficients from an interface between fluid-saturated porous materials

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INTRODUCTION

Hydrocarbon reservoirs as well as many other sedimentary rocks are fluid-saturated porous materials, whose elastic properties can be described by the theory of poroelasticity (Biot, 1962). This theory predicts the effects of movement of the pore fluid relative to the solid skeleton on seismic waves propagating through the rock. This opens potential opportunities to estimate fluid and rock transport properties by measuring seismic waves. However, these opportunities are somewhat limited by the fact that, at low frequencies, relative fluid movement becomes negligible and the rock behaves like an elastic solid with the equivalent elastic moduli given by Gassmann's (1951) equations. In particular, the theory of poroelasticity predicts that elastic wave attenuation and dispersion only become significant at frequencies comparable to the so-called Biot's characteristic frequency $\omega_c = \eta\phi/\kappa\rho_f$, where ϕ and κ are the porosity and permeability of the rock matrix, respectively, and η and ρ_f are steady-state shear viscosity and density of the pore fluid, respectively. (See Table 1 for a glossary of symbols.) For commonly encountered natural rocks such as sandstones or limestones saturated with water, oil or gas, ω_c is usually 0.1 MHz or higher. This is much higher than the frequency of waves of surface seismic exploration (20–70 Hz) and well logging (5–50 kHz). At frequencies lower than ω_c, the fluid-related contribution to (complex) phase velocity of the compressional wave is proportional to frequency. That is, for frequencies much smaller than ω_c, both attenuation and dispersion are very small.

More recently, it was realised that dynamic poroelastic effects may be pronounced at lower frequencies when the porous medium is macroscopically heterogeneous (White, 1983). In particular, it was shown that heterogeneity of porous materials can produce significant additional attenuation and dispersion of the propagating wave. These phenomena are related to the fact that when a compressional or shear wave encounters an interface between two different porous saturated materials, a fluid flow across the interface may occur, which results in the loss of energy from the propagating wave. For a single interface, this phenomenon manifests itself in the fact that the reflection coefficient from an interface between two porous media is proportional to the square root of frequency. In other words, unlike the velocity of the elastic wave in an unbounded porous medium, the reflection coefficient decreases much more gradually with decreasing frequency (Geertsma and Smit, 1961; Dutta and Odé, 1983; Gurevich et al., 1994). Thus, fluid effects on reflection coefficients may be observed at lower frequencies than similar effects in an unbounded medium. This opens the possibility of using reflection sounding at relatively low frequencies to examine the properties of pore fluids. The success of such techniques, however, depends on the numerical values of reflection coefficients for realistic materials. General expressions for reflection coefficients from an interface between two porous materials as a function of material properties, frequency, and angle are very complicated (Geertsma and Smit, 1961; Bourbié et al., 1987; Denneman et al., 2002), and their direct theoretical analysis is still lacking. The former numerical analysis carried out by Gurevich (1996) showed the tiny difference between the poroelastic and elastic solutions for a gas-water contact. However, recent numerical calculations of Denneman et al. (2002) show that, in some cases, the reflection coefficient exhibits strong deviations from its elastic value at frequencies much lower than ω_c. In particular, numerical calculations have shown this behaviour for an interface between a gas-saturated porous solid and a free liquid (Figure 1). In the case of a sealed-pore interface, there is no fluid flow across the interface. Thus, deformation of this system is equivalent to

Manuscript received by the Editor August 15, 2003; revised manuscript received March 29, 2004.
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the deformation of an effective elastic medium. This corresponds to the classical elastic modeling using velocities calculated by the static Gassmann formula. This is the elastic model used in this study for comparison with derived poroelastic reflection coefficients. These indications may form a basis for a significant advance in the use of low-frequency sounding for detecting pore-fluid properties.

In order to understand the nature of the substantial difference between poroelastic and elastic reflection coefficients at an interface between a free fluid and an air-filled porous medium, and to explore implications for reflections from an interface between two porous media, it is desirable to obtain analytical expressions for the reflection coefficients. As mentioned above, they are too complicated for arbitrary incidence angles (Denneman et al., 2002). However, the problem is greatly simplified for the normal incidence. Since the significant difference between poroelastic and elastic reflection coefficients at an interface between a free fluid and an air-filled porous medium is observed already for normal incidence (see Figure 1), closed-form expressions provide a necessary insight into these observations. The primary objective of this short note is to analyze and reveal the physical cause of this phenomenon.

Table 1. Glossary of symbols.

Symbol	Meaning
e, e_f, ξ	Solid frame and fluid dilatation, fluid content increase
k_1, k_2	Wave numbers of fast and slow P-wave
$\mathbf{u}, \mathbf{U}, \mathbf{w}$	Solid, fluid, and relative fluid displacement
v_1	Fast P-wave velocity
H, C, M, L, N, σ	Poroelastic constants
K_f, K_g, K	Fluid, grain, and solid skeleton bulk moduli
$R_{11}, R_{12}, T_{11}, T_{12}$	Reflection and transmission coefficients
$X, \tilde{X}, Y, \tilde{Y}$	Factors expressing difference between poroelastic and elastic R/T coefficients
Z	Proportionality constant: $Y = Z\sqrt{\omega/\omega_c}$ and $X = Z\sqrt{\omega/\omega_c}$
ϕ	Porosity of a solid skeleton
κ	Steady-state permeability of a solid skeleton
μ_g, μ	Grain and bulk shear moduli
η	Steady-state shear viscosity of a pore fluid
ω, ω_c	Frequency, Biot's characteristic frequency
ρ_f, ρ_g, ρ	Fluid, grain, and bulk density
τ, P	Stress tensor and fluid pressure

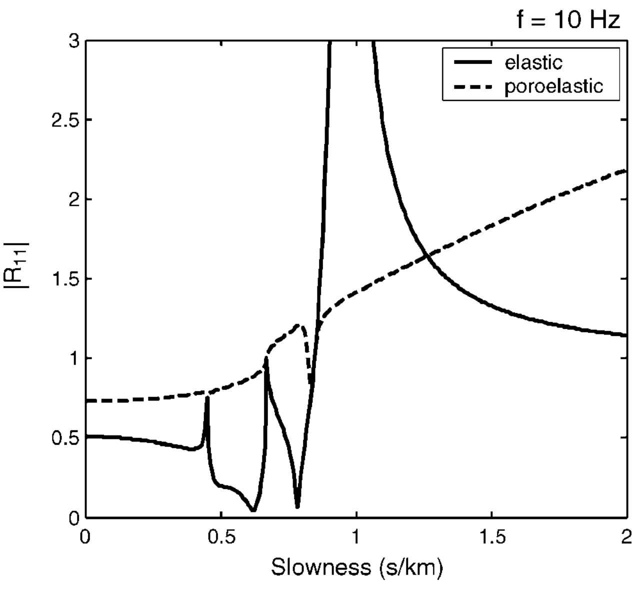

Figure 1. Reflection at the interface between free fluid and air-filled porous medium. The numerical computation shows significant differences between the open-pore poroelastic model (dashed line) and the sealed-pore poroelastic model (solid line), which corresponds to the elastic solution. The magnitude value represents the absolute value of the reflection coefficient for a fast P-wave as a function of slowness, which represents the incidence angle.

NOTATION FOR POROELASTIC ANALYSIS

According to Biot's theory of poroelasticity, the interconnected continuum and its fluid filling are treated in the manner of two interpenetrating elastic continua. The elastic properties of a rock frame saturated by a compressible fluid are described by Gassmann's theory (Gassmann, 1951). We consider a porous medium with the uniformly distributed porosity ϕ whose pores are filled with a viscous fluid. The fluid is characterized by the bulk modulus K_f, density ρ_f, and steady-state shear viscosity η. The grains of the solid are characterized by bulk and shear moduli K_g and μ_g, respectively, and density ρ_g. The drained solid matrix formed from grains and pore spaces are characterized by bulk and shear moduli and density K, μ, and ρ, respectively, and steady-state permeability κ.

For long wavelength acoustic pulses propagating through such a porous medium, we can define average values of local displacement in the solid $\mathbf{u}$ and in the fluid $\mathbf{U}$. A more convenient way of quantifying the fluid displacement introduced by Biot (1962) is to define relative average displacement of fluid $\mathbf{w}$ relative to the solid frame as

$$\mathbf{w} = \phi(\mathbf{U} - \mathbf{u}), \tag{1}$$

and we can define the increased fluid content as

$$\xi = -\phi \operatorname{div}(\mathbf{U} - \mathbf{u}) = \phi(e - e_f), \tag{2}$$

where $e = \operatorname{div}\mathbf{u}$ is the solid frame dilatation and $e_f = \operatorname{div}\mathbf{U}$ is the fluid dilatation. The components of stress tensor τ for the saturated porous medium are

$$\tau_{ij} = [(H - 2\mu)e - C\xi]\delta_{ij} + 2\mu e_{ij}, \tag{3}$$

and fluid pressure P is

$$P = M\xi - Ce = C\left(\frac{\xi}{\sigma} - e\right), \tag{4}$$

where the relations among poroelastic constants are defined in the following way (Biot, 1962):

$$H = K + \frac{4}{3}\mu + \sigma C, \tag{5}$$

$$C = \sigma M, \tag{6}$$

$$M^{-1} = \left[\frac{\sigma - \phi}{K_g} + \frac{\phi}{K_f}\right], \tag{7}$$

$$\sigma = 1 - \frac{K}{K_g}. \tag{8}$$

Biot's theory of poroelasticity gives rise to the three types of waves propagating through a saturated porous medium: the fast compressional wave, the shear wave, and the slow compressional wave often called Biot's slow wave. In a plane wave of any particular type within a homogenous poroelastic medium, $\mathbf{u}$ and $\mathbf{w}$ are coupled. In particular, in the fast P- and S-waves at very low frequencies, the relative displacement is very small and can be neglected, i.e., $\mathbf{w}=0$. The corresponding relation for the slow P-wave is (Geertsma and Smit, 1961)

$$\mathbf{w} = -\left(\frac{H}{C}\right)\mathbf{u}. \tag{9}$$

Within the low-frequency approximation of Biot theory (i.e., for frequencies below the Biot's characteristic frequency $\omega_c = \eta\phi/\kappa\rho_f$), the wavenumbers of fast and slow P-waves are given by the following relations:

$$k_1 = \frac{\omega}{v_1}, \tag{10}$$

$$k_2 = \sqrt{\frac{i\omega\eta}{\kappa N}}, \tag{11}$$

where the index 1 denotes fast P-wave and index 2 denotes slow P-wave. The velocity of the fast wave is $v_1 = \sqrt{H/\rho}$ and the parameter N is

$$N = \frac{C}{\sigma} - \frac{C^2}{H} = \frac{ML}{H}, \tag{12}$$

where $L = K + (4/3)\mu$. The above overview of Biot's theory provides the basis for the derivation of reflection coefficients on the plane surface between two saturated porous media.

REFLECTION COEFFICIENTS FOR THE INTERFACE BETWEEN TWO FLUID-SATURATED POROELASTIC MEDIA

Let us assume that the parameters of the medium depend on the z-coordinate only. Consider the harmonic plane P-wave with unit amplitude and time dependence of the form $e^{-i\omega t}$ normally incident from the media denoted by b. The model is an interface between two elastic porous half-spaces as shown in Figure 2.

The incident P-wave generates two reflected compressional waves (fast and slow) in the medium b and two transmitted P-waves in the poroelastic solid half-space: a fast P-wave and Biot's slow P-wave. The displacement vectors in those porous media, using relations 9–12, may be expressed as

$$u_a = T_{11}e^{-ik_{1a}z} + T_{12}e^{-ik_{2a}z}, \tag{13}$$

$$u_b = e^{-ik_{1b}z} + R_{11}e^{ik_{1b}z} + R_{12}e^{ik_{2b}z}, \tag{14}$$

$$w_a = -T_{12}\frac{H_a}{C_a}e^{-ik_{2a}z}, \tag{15}$$

$$w_b = -R_{12}\frac{H_b}{C_b}e^{ik_{2b}z}, \tag{16}$$

where amplitudes R_{11}, R_{12}, T_{11}, and T_{12} represent the unknown values of the reflection and transmission coefficients. These amplitudes must satisfy the following boundary conditions on the interface for the porous-medium a and porous medium b (Deresiewicz and Skalak, 1963; Gurevich and Schoenberg, 1999):

$$u_a = u_b, \tag{17}$$

$$w_a = w_b, \tag{18}$$

$$\tau_{zz}^a = \tau_{zz}^b, \tag{19}$$

$$P_a = P_b. \tag{20}$$

The constitutive relations for the 1D case are

$$\tau_{zz} = He_{zz} - C\xi, \quad e_{zz} = \frac{\partial u}{\partial z},$$

$$P = C\left(\frac{\xi}{\sigma} - e_{zz}\right), \quad \xi = -\frac{\partial w}{\partial z}. \tag{21}$$

Substituting equations 13–16 into boundary conditions 17–20 and solving the linear system of equations gives the reflection and transmission coefficients R_{11} and T_{11} for the fast P-wave:

$$R_{11} = \frac{H_b k_{1b} - (1-X)H_a k_{1a}}{H_b k_{1b} + (1+X)H_a k_{1a}} = \frac{\rho_b v_{1b} - (1-X)\rho_a v_{1a}}{\rho_b v_{1b} + (1+X)\rho_a v_{1a}}, \tag{22}$$

$$T_{11} = \frac{2H_b k_{1b}}{H_b k_{1b} + (1+X)H_a k_{1a}} = \frac{2\rho_b v_{1b}}{\rho_b v_{1b} + (1+X)\rho_a v_{1a}}, \tag{23}$$

where $\rho v_1 = k_1 H/\omega$, and the term X is given by

$$X = \frac{H_b k_{1b}\left(\dfrac{C_a}{H_a} - \dfrac{C_b}{H_b}\right)^2}{k_{2a}N_a + k_{2b}N_b}. \tag{24}$$

The coefficients R_{12}, T_{12} for reflected and transmitted slow P-waves are expressed as

$$R_{12} = \frac{2\tilde{X}_b \rho_a v_{1a}}{\rho_b v_{1b} + (1+X)\rho_a v_{1a}}, \tag{25}$$

$$T_{12} = \frac{2\tilde{X}_a \rho_a v_{1a}}{\rho_b v_{1b} + (1+X)\rho_a v_{1a}}, \tag{26}$$

where the parameters $\tilde{X}_a$ and $\tilde{X}_b$ are

$$\tilde{X}_a = \frac{X}{\dfrac{H_a}{C_a}\left(\dfrac{C_a}{H_a} - \dfrac{C_b}{H_b}\right)}, \tag{27}$$

$$\tilde{X}_b = \frac{X}{\dfrac{H_b}{C_b}\left(\dfrac{C_a}{H_a} - \dfrac{C_b}{H_b}\right)}. \tag{28}$$

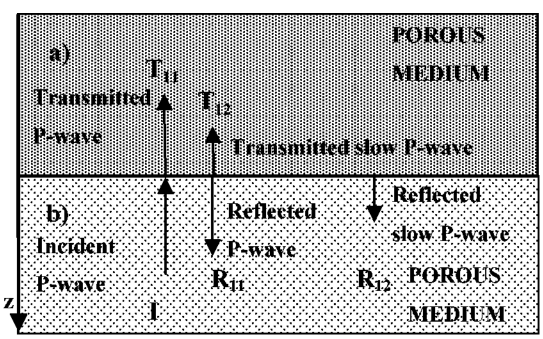

Figure 2. Reflection and transmission coefficients from an interface between two porous media.

REFLECTION COEFFICIENTS FOR THE INTERFACE BETWEEN A FREE FLUID AND A FLUID-SATURATED POROELASTIC MEDIUM

If the poroelastic half-space b in Figure 1 is a free fluid with density ρ_b and bulk modulus K_b, we can derive the expressions for the reflection and transmission coefficients from an interface between a free fluid and a porous medium. Briefly, following the procedure above, we assume the incident compressional wave coming from half-space b filled with fluid. The displacement vectors are

$$U_b = e^{-ik_{1b}z} + R_{11}e^{ik_{1b}z}, \quad (29)$$

$$u_a = T_{11}e^{-ik_{1a}z} + T_{12}e^{-ik_{2a}z}, \quad (30)$$

$$w_a = -T_{12}\frac{H_a}{C_a}e^{-ik_{2a}z}, \quad (31)$$

and amplitudes R_{11}, T_{11}, and T_{12} must satisfy following three boundary conditions, which are valid on the open pore interface:

$$w_a = U_b - u_a, \quad (32)$$

$$\tau_{zz}^a = -P_b, \quad (33)$$

$$P_a = P_b, \quad (34)$$

where the relation for the pressure in the free fluid for the z-component only is expressed as

$$P_b = -K_b\frac{\partial U_b}{\partial z}. \quad (35)$$

Introducing the expressions 29–31 into boundary conditions 32–34 and solving the system, we obtain the following closed form expressions for R/T coefficients from an interface between a free fluid and a porous medium:

$$R_{11} = \frac{\rho_b v_{1b} - (1-Y)\rho_a v_{1a}}{\rho_b v_{1b} + (1+Y)\rho_a v_{1a}}, \quad (36)$$

$$T_{11} = \frac{2\rho_b v_{1b}}{\rho_b v_{1b} + (1+Y)\rho_a v_{1a}}, \quad (37)$$

$$T_{12} = \frac{2\tilde{Y}\rho_a v_{1a}}{\rho_b v_{1b} + (1+Y)\rho_a v_{1a}}, \quad (38)$$

where the parameters Y and $\tilde{Y}$ are

$$Y = \left(\frac{C_a}{H_a} - 1\right)^2\sqrt{-\frac{i\omega\kappa_a}{\eta_a N_a}}\rho_b v_{1b}, \quad (39)$$

$$\tilde{Y} = \frac{Y}{1 - \dfrac{H_a}{C_a}} = \frac{C_a}{H_a}\left(\frac{C_a}{H_a} - 1\right)\sqrt{-\frac{i\omega\kappa_a}{\eta_a N_a}}\rho_b v_{1b}. \quad (40)$$

The R/T coefficients given by expressions (36–38) are a special case of expressions 22, 23, and 26, respectively. The expressions for X and $\tilde{X}_a$ in expressions 24 and 27 tend to the ones for Y and $\tilde{Y}$ in expressions 39 and 40, respectively, if one substitutes the porous half-space b with a free fluid (i.e., $H_b = C_b = K_b$) and at the same time assuming "permeability" of free water to be infinite.

DISCUSSION

The central result of this paper is the expression for the reflection coefficient R_{11} for the fast P-wave. Equations 22 and 36 show that the expression for the reflection coefficient is the same as the familiar expression for two elastic media (e.g., Brekhovskikh and Godin, 1990),

$$R_{11} = \frac{\rho_b v_{1b} - \rho_a v_{1a}}{\rho_b v_{1b} + \rho_a v_{1a}}, \quad (41)$$

except for the factors X and Y, respectively. As previously shown (e.g., Bourbié et al., 1987), these factors are proportional to the square root of frequency. The difference between poroelastic and elastic reflection coefficients is given mainly by the difference in ratios of poroelastic parameters C_a/H_a and C_b/H_b between the two poroelastic materials, given by the factor X in equation 24. The effect of the fluid flow is significant when this difference is high.

Since there is a significant difference between normal incidence poroelastic and elastic reflection coefficients at an interface between a free fluid and an air-filled porous medium (Figure 1), closed-from expressions 36 and 39 would provide a necessary insight into these observations. To understand this effect, in Figure 3 we plot the reflection coefficient as given by equation 36 versus frequency for an interface between

Table 2. Mechanical properties of the porous rock (Denneman et al., 2002).

	K_g (GPa)	K (GPa)	μ (GPa)	ϕ	κ (darcy)	ρ_g ($kg \cdot m^{-3}$)
Sand	40	5.8	3.4	0.26	0.95	2760

Table 3. Mechanical properties of the sample pore fluid (Denneman et al., 2002).

	K_f (GPa)	ρ_f ($kg \cdot m^{-3}$)	η (Pa s)
Water	2.22	1000	0.001
Air	0.0001	1.2	1.82×10^{-5}

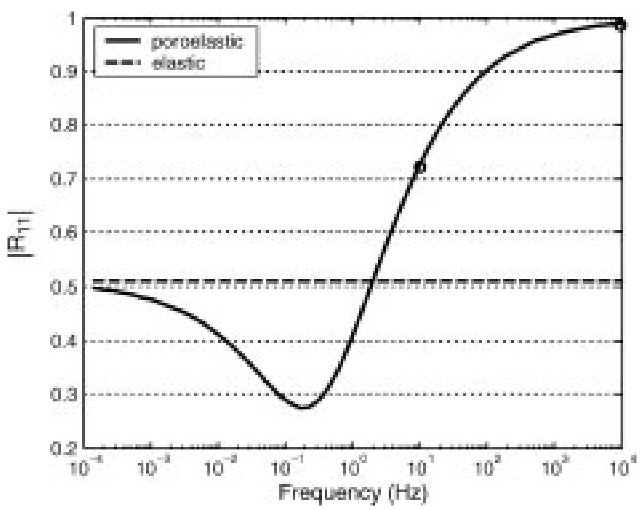

Figure 3. Reflection coefficient at an interface between a free fluid (water) and an air-saturated porous medium. Circles denote the exploration frequency range 10 Hz–10 kHz.

water and an air-saturated porous sandstone. The parameters of both porous rock and two pore fluids are given in the Tables 2 and 3. Several key observations may be drawn from this plot and analytical expressions 36 and 39:

1) The results at 10 Hz and at 10 kHz are in agreement with the results of Denneman et al. (2002, Figure 8, p. 289) for normal incidence, and indeed is very different from the elastic reflection coefficient at frequencies as low as 10 Hz, as indicated by the circles in Figure 3.
2) Nevertheless, this is not the result for the low-frequency limit. Indeed, in the low-frequency limit, the poroelastic reflection coefficient reverts to the elastic value. However, this only occurs at frequencies as low as 10^{-3} Hz.
3) Thus, 10 Hz is not "in the low frequency limit" for equation 36.
4) Within the assumptions of low-frequency Biot's theory (and thus up to at least 10 kHz), factor Y scales linearly with $\sqrt{\omega}$. However, it translates into the same linear dependency of the reflection coefficient R_{11} only if $Y \ll 1$. If Y is not small, then R_{11} as given by equation 36 is still a function of $\sqrt{\omega}$, but it is no longer a linear function of it, but a rational function.
5) Therefore, the large deviation of the poroelastic reflection coefficient from the elastic one for an interface between a free fluid and an air-saturated porous medium at very low frequencies is due to the fact that the proportionality constant between poroelastic parameters and the square root of frequency is very large.

To analyze this problem further, we assume both $K_{fa} \ll K_{ga}$ and $K_a \ll K_{ga}$. Then, the poroelastic parameter N_a can be written as $N_a \approx K_{fa}/\phi_a$ and $C_a = K_{fa}/\phi_a \ll H_a$. Thus, Y can be written as

$$Y = \sqrt{-\frac{i\omega\kappa_a\phi_a\rho_b}{\eta_a}}\sqrt{\frac{K_b}{K_{fa}}} = Z\sqrt{\frac{\omega}{\omega_c}}, \tag{42}$$

where

$$Z = \phi_a\sqrt{\frac{1}{i}\frac{\rho_b}{\rho_{fa}}\frac{K_b}{K_{fa}}}, \tag{43}$$

and ω_c is Biot's characteristic frequency. If the pore fluid is liquid (say, water), then ρ_b/ρ_{fa}, K_b/K_{fa}, and Z are of the order 1 (or smaller), and hence Y is small for frequencies $\omega \ll \omega_c$. However, if the pore fluid is air, Y becomes finite (of order 1) at frequencies much smaller than ω_c. Note that the first square root of equation (42) increases, if one replaces the water in pores by air. More significantly, with air in the pores, the second square root of equation 42 becomes very large due to the small value of bulk modulus K_{fa}. It is this large contrast in fluid bulk moduli, not density contrast as suggested by Denneman et al. (2002), that is the cause of the anomalous difference between elastic and poroelastic reflection coefficients at the low frequencies.

To illustrate this difference between free fluid (water) and an air-saturated medium, we plot in Figure 4 these reflection coefficients for a fixed frequency (10^{-1} Hz) against the ratio of bulk moduli of those two fluids K_b/K_{fa}. The free fluid is water and the pore fluid has bulk modulus K_{fa} gradually decreasing from the value for water to its value for air. Solid and dashed lines correspond to the case where the pore fluid density is kept constant. The dotted and dash-dotted lines represent the reflection coefficients when the density is calculated assuming that half-space a is saturated with an uniform mixture of air and water:

$$\rho_{fa} = S_w\rho_{water} + (1 - S_w)\rho_{air}, \tag{44}$$

where water saturation S_w is derived from the Wood formula

$$\frac{1}{K_{fa}} = \frac{S_w}{K_{water}} + \frac{(1 - S_w)}{K_{air}}. \tag{45}$$

We see (Figure 4) that the contrast in bulk moduli of the free fluid and the pore fluid has a greater effect on R/T coefficients than the density contrast.

Expressions 42 and 43 contain all factors which have influence on the observed effect. So, the poroelastic effect will be slightly larger for high porosity and permeability, but as mentioned above the leading term in this expression is given by the large contrast between the bulk moduli of the free fluid and gas pore fill. The difference between the poroelastic and elastic reflection coefficients is contained in the factor Y. The proportionality constant Z in equation 43 indicates when the fluid flow effects will be important.

Can the same effect occur for an interface between two porous rocks? To answer this question, consider an interface between two rocks with all properties except pore fluid properties being of the same order of magnitude, and with stiff grain material, so that $M \approx N \approx K_f/\phi$. In this case equation 24 can

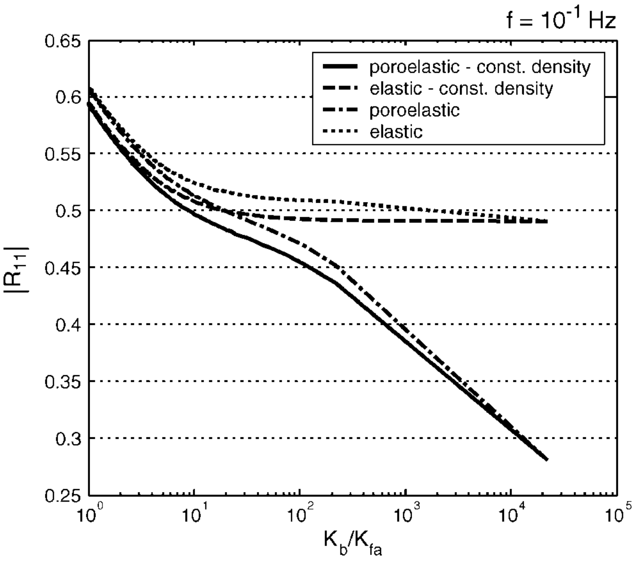

Figure 4. Reflection coefficient at an interface between free fluid (water) and a fluid-saturated porous medium expressed as a function of the ratio of the bulk moduli of the pore fluid in half-space a and the free fluid in half-space b. The solid and dashed lines for poroelastic and elastic coefficients are calculated by keeping the density constant. The pore fluid has bulk modulus K_{fa} gradually decreasing from the value for water to the value for air. The dotted and dash-dotted lines represent the reflection coefficients when the density is calculated assuming that half-space a is saturated with an uniform mixture of air and water, where water saturation is derived from the Wood formula (equation 45).

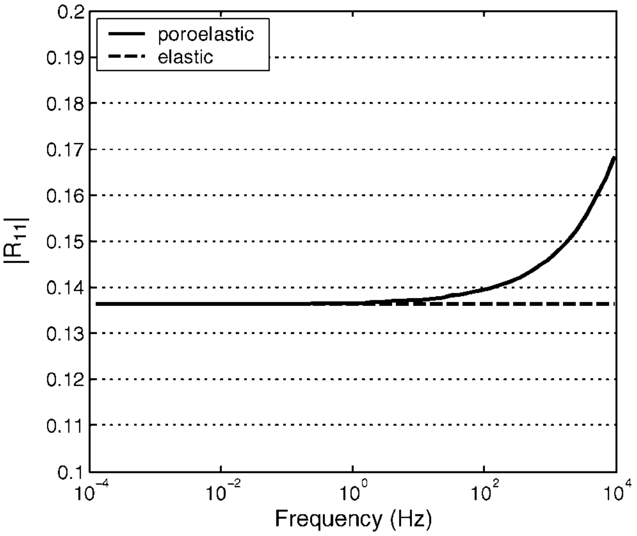

Figure 5. Reflection coefficient at an interface between a water-saturated and an air-saturated porous medium. The incoming wave is from the water-saturated porous medium.

be simplified to give

$$X = \frac{\rho_b v_{1b}\left(\frac{C_b}{H_b}\right)^2}{\sqrt{\frac{i}{\omega\kappa_b\phi_b}}\sqrt{\eta_b K_{fb}}} = \frac{\sigma_b^2}{\phi_b}\sqrt{\frac{\rho_b\omega\kappa_b K_{fb}^3}{i\phi_b\eta_b H_b^3}}$$

$$= \frac{\sigma_b^2}{\phi_b}\sqrt{\frac{1}{i}\frac{\rho_b}{\rho_{fb}}\left(\frac{K_{fb}}{H_b}\right)^3\frac{\omega}{\omega_{cb}}}, \qquad (46)$$

where the bulk modulus of pore fluid in half-space b is much higher then the one in the half-space a. Equation 46 shows that for an interface between two similar rocks saturated with two different fluids, the proportionality constant $Z = X/\sqrt{\omega/\omega_{cb}}$ is of order 1 (or smaller), and hence X is small for frequencies $\omega \ll \omega_{cb}$. This is illustrated in Figure 5, which shows poroelastic and equivalent elastic reflection coefficients for a gas/water contact. We conclude that poroelastic effects on normal-incidence reflection coefficients are unlikely to be observed at low frequencies.

CONCLUSIONS

The derived closed-form expressions for reflection and transmission coefficients for poroelastic interfaces confirm the expected dependence on the square root of frequency. The results we have obtained at 10 Hz and 10 kHz are in agreement with the results of Denneman et al. (2002). In the low-frequency limit, the poroelastic reflection coefficient reverts to the elastic value. However, this only occurs at frequencies as low as 10^{-3} Hz. The large deviation of poroelastic coefficient from the elastic one for an interface between water and an air-saturated porous medium at very low frequencies is due to the fact that the proportionality constant between poroelastic parameters and the square root of frequency is very large due to large contrast in compressibilities of the pore fluid and the free fluid. However, for an interface between two fluid-saturated porous media, the difference between poroelastic and elastic coefficients at seismic frequencies is only several percent, and the coefficients can be approximated by the elastic ones. The possible realistic situation where the derived expression could be applied is the fluid-filled borehole at the borehole wall when the shallow formation is filled with gas, as already suggested by Denneman et al. (2002). The other situation may occur in polar conditions when snow saturated with air is floating on the surface of an unfrozen lake (Johnson, 1982).

ACKNOWLEDGMENTS

This work was funded by the Curtin University of Technology small discovery/linkage grants scheme. The work of Boris Gurevich was partially supported by the Centre of Excellence for Exploration and Production Geophysics, and by Curtin Reservoir Geophysics Consortium. The work of Radim Ciz was supported by CSIRO Postdoctoral Fellowship Program. We thank J. Carcione, H. B. Helle, and two anonymous reviewers for helpful comments and suggestions.

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