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Proposed Constants for Bieniawski's Strength Criterion for Rocks and Coal

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Abstract

Bieniawski's strength criterion is one of the most widely used criteria for strength estimation of intact rocks. This criterion, however, only considers compression loading. In Bieniawski's criterion rocks are distinguished in their properties using two parameters B and α. Selecting these parameters, through lab experiments, as representative as possible for a certain type of rock is significantly important. The quality of lab tests, the number of tests and statistical approaches used to estimate these parameters are some of the important factors, which can influence the accuracy of the estimation. Several attempts have been made by different researchers to propose these parameters for different rock types in different regions. In this paper a similar attempt was made to determine more representative constants for Bieniawski's criterion. This work is different from past studies in that we have based our analysis on a very large number of lab experimental data gathered from the literature and some carried out for the purpose of this study. The studied data includes a wide range of rock types from soft to hard including sandstones, shales and coals. Both linear conversion and nonlinear regression models were applied to the lab data and as a result Bieniawski's constants were proposed for each rock type. For coal, the results are presented as a function of the loading angle with respect to the coal's fractures. The results of nonlinear models were found to be associated with higher correlation coefficients. Also a correlation between parameter B and unconfined compressive strength was proposed. The results of this study were also compared with similar work presented in the past.

Keywords: Bieniawski's strength criterion; UCS; hard rock; soft rock; lab experiments; regression

Introduction

Several failure criteria have been developed in the past by different researchers to estimate rock compression strength at a given stress conditions. Some of these criteria are applicable to intact rocks (e.g. Mohr-Coulomb criterion) whereas some others predict failure of rock masses such as Hoek-Brown criterion [1]. Murrell [2, 3] developed an empirical failure criterion for rock strength under compression. Murrell's criterion is based on Griffith's theoretical failure criterion [4, 5] by considering the hydrostatic pressure in triaxial stress condition. This was modified later by Hoek [6], Bieniawski [7], Yudhbir and Prinzl [8], Das and Sheorey [9] and Sheorey et al. [10] who determined the constants of the criteria for its applications to specific type of rocks.

Murrell's strength criterion [3] proposed for intact rocks is presented as:

$$\sigma_1 = \sigma_c + B\sigma_3^{\alpha} \quad (1)$$

In this equation σ_1 and σ_3 are the major and minor principal stresses applied to the rock; σ_c is the unconfined compressive strength (UCS) and B and α are constants to be determined for each rock type. Bieniawski [7] proposed normalized version of Equation 1 for strength prediction of intact rocks in the form of:

$$\frac{\sigma_1}{\sigma_c} = 1 + B\left(\frac{\sigma_3}{\sigma_c}\right)^{\alpha} \quad (2)$$

In section 4 it is shown how constants B and α can be estimated from lab experiments performed on a certain rock type.

Yudhbir and Prinzl [8] proposed an average value of $\alpha = 0.65$ for all rock types whereas $\alpha = 0.75$ was suggested by Bieniawski [7]. They suggested different values for parameter B depending on rock type. Hossaini [11] suggested an average value of $\alpha = 0.60$ and developed a correlation between B and the UCS. In the subsequent sections constants B and α will be estimated for various rock types using a large number of lab data.

Amongst above and other proposed criteria, Bieniawski's strength criterion [7] has been most widely used for rock strength estimations as it yields closer results to real applications [12]. Bieniawski criterion [7] is a normalized form of Murrell criterion [3], which was developed for some types of rocks like norite, quartzite, sandstone, siltstone and mudstone, but Bieniawski [7] proposed constant values for this criterion to be applicable for all rock types. However, the applications of this criterion are also limited to some assumptions and specific type of rocks, which were tested [13]. The UCS value, which is used as the input to the Bieniawski's strength criterion, is the result of direct UCS tests of samples. This could be different from the regulated UCS (RUCS), which, is estimated from the intercept of the failure envelope with σ_1 axis [13]. This could lead into technical disadvantage in using this criterion for estimating rock strength, as the σ_c derived from Bieniawski's strength criterion is not regulated. Also to fit the Bieniawski criterion to the real triaxial lab data using regression analysis, uniaxial tensile strength (UTS) test data pairs should be eliminated from the data group. Similar procedure should be applied to UCS data pairs when converting the nonlinear form of the criterion to the linear regression. This is not necessary if nonlinear regression is used directly but one should notice that the estimated strength parameters will be different in two cases for a similar data set.

Because of the shortcomings associated with Bieniawski's criterion, attempts have been made to modify its constants or present new criteria for various types of rocks including coals (Hobbs [14]; Bieniawski & Bauer [15]; Yudhbir and Prinzl [8]; Sheorey [16]; and Hossaini [11]). Of course the applications of these criteria are limited to the type of rocks their studies were based upon. The results of a study performed on 12 data groups of different limestone samples indicated that Bieniawski's criterion overestimates the confined ultimate strength in more than 61% of cases when the confining pressure is less than 10 MPa and in 39% of cases when confining pressures is larger than 10 MPa [12].

In this paper constants in Bieniawski's strength criterion were estimated for various types of rocks including coal. The results are based on laboratory triaxial stress testing of more than 1250 samples for more than 150 types of rocks and coals. This data was collected from the authors' previous research works, reported literature and some lab experiments conducted for the purpose of this study. Considering the large number of data used for analysis in this study it is believed that the proposed constants provide a more representative estimation of strength for different types of rocks. The statistical analyses conducted to extract the constants of the criterion are presented and the results are discussed. A comprehensive comparison between the applicability of the considered criterion in this research and other famous applicable criteria has been done before [13]. This research presents practical values for Bieniawski's criterion, which make the strength estimation easy for research and practical purposes, especially in geomechanical projects.

Input Data Sets

For the purpose of this study a total number of 1251 triaxial compression test data was collected from the literature including authors of this paper and Schwartz [17], Horino and Ellikson [18], Ouyang & Elsworth [19], Vutukuri and Hossaini [20], Hossaini [11], Sheorey [16], Mahab Ghodss Engng [21], and Bineshian [22, 23]. The data composes the test results of different intact rock types including igneous, sedimentary and metamorphic rocks. Data belong to coal samples taken at different directions with respect to the direction of major cleat planes to consider transverse isotropic behaviour of coals were also included. Also the results of triaxial tests carried out on 44 limestone specimens with 54 mm diameter and 122 mm height according to the ISRM suggested methods [24] were included in this study. A total of 152 data groups were defined each representing one rock type. In order to ensure that the tests are as representative as possible, below data quality control measure were applied in order to select the final data sets:

- Minimum number of data pairs in each data group should not be less than five [25].
- All data groups should contain both principal stresses in failure and *UCS* result [1].
- All data pairs must satisfy Mogi's transition limit of $\sigma_1 > 4.4\sigma_3$ [26, 11, 16, 22].
- Each data group must contain the results of triaxial tests performed under confined pressure, i.e. $\sigma_3 > 0$ [22].
- Each data group should cover range of tests performed at low to relatively high confining pressures and include at least one test with $\sigma_3 > 0.50\sigma_c$ [27].
- Data groups had to be definable in terms of regressive shaped curve [13]. For example data groups, which their σ_3 vs. σ_1 curve shows an upward concave, should be excepted from the data.
- Data for which the maximum strength shows to be less than the *UCS* (data pairs that their $\sigma_1 < UCS$) should be excluded from analysis [17, 27] because they are recognised as outliers that tend to skew the line away from other data pairs which do not follow the natural trend of strength data.

After eliminating unsuitable data, a total of 1251 data pairs are selected to be used into account in this research for 10 igneous rock types, 7 sedimentary rock types, 3 metamorphic rock types and 4 coal types. For the coal samples, different orientations of main cleats and bedding planes relative to the orientation of the major principal stress were considered to be a coal type. All laboratory tests cited were carried out according to ISRM suggested methods [24].

Statistical Analysis

The statistical analysis in this study includes fitting the best linear or non-linear curve to the lab experimental data, i.e. to failure envelope of σ_1 versus σ_3.

A linear regression model is represented as:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \quad (3)$$

and a non-linear regression model is written in the form of:

$$Y_i = \beta_0 + \beta_1^{-\beta_2 X_i} + \varepsilon_i \quad (4)$$

where, β_0 , β_1 and β_2 are regression model coefficients and ε_i is the error of regression model and $i = 1, 2, \dots, n$.

The coefficient of determination (r^2) is the most known parameter to check the data fitness. This coefficient, which changes between 0 and 1, is represented for linear curves as:

$$r^2 = 1 - \frac{\Sigma[\sigma_{1jexp} - f(\sigma_{3jexp})]^2}{\Sigma\sigma_{1jexp}^2 - (\Sigma\sigma_{1jexp})^2/n} \quad (5)$$

and for non-linear curves as:

$$r^2 = \frac{(\Sigma\sigma_{3jexp}\sigma_{1jexp} - \Sigma\sigma_{3jexp}\Sigma\sigma_{1jexp}/n)^2}{\left[\Sigma\sigma_{3jexp}^2 - (\Sigma\sigma_{3jexp})^2/n\right]\left[\Sigma\sigma_{1jexp}^2 - (\sigma_{1jexp})^2/n\right]} \quad (6)$$

The accordance coefficient (ψ^2) is another statistical parameter, which is defined as [22]:

$$\psi^2 = \frac{\Sigma(\sigma_{1jexp} - \sigma_{1jcal})^2}{\Sigma(\sigma_{1jexp} - \bar{\sigma}_{1jexp})^2} \quad (7)$$

The closer the accordance coefficient to zero the better is the match of the mathematical function fit to the lab data.

In the above equations σ_{1jexp} and σ_{3jexp} are observed values for σ_1 and σ_3 for jth data, n is the number of (σ_3, σ_1) data pairs, $\bar{\sigma}_{1jexp}$ is the average observed value for jth data of σ_1 and σ_{1jcal} is calculated value of σ_1 for jth data.

The parameters of linear conversion model (LCM) for the Bieniawski's criterion are obtained as:

$$X_i = \log\left(\frac{\sigma_3}{\sigma_c}\right) \quad Y_i = \log\left(\frac{\sigma_1}{\sigma_c} - 1\right) \quad B = 10^{\beta_0} \quad \alpha = \beta_1 \quad (8)$$

The constants for non-linear regression model (NRM) are expressed as:

$$X_i = \sigma_3 \quad Y_i = \sigma_1 \quad B \,\&\, \alpha = f(\beta_0, \beta_1 \,\&\, \beta_2) \quad (9)$$

We have used Nelder and Mead method for nonlinear regression [28] in this study.

Proposed Constants For Intact Rocks

Both linear and non-linear regression analysis were performed on experimental data belonging to different rock types in order to obtain the constant parameters in Bieniawski's failure criterion.

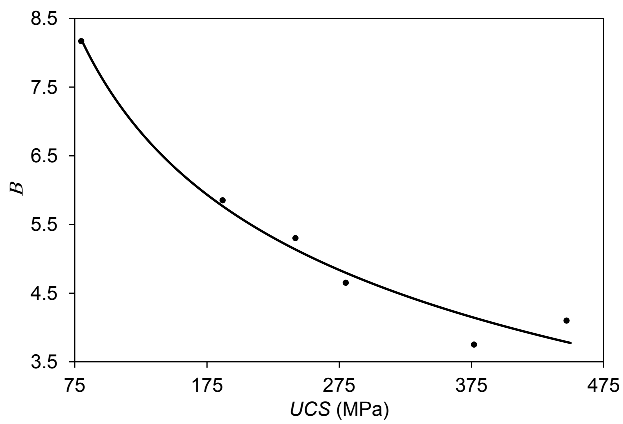
Table 1 (at the end of the paper) shows the average values for B and α obtained from both methods in this study. These values are compared against those proposed by Bieniawski [7] and Yudhbir and Prinzl [8]. While the LCM results are out of range values, the NRM indicated to yield more reliable results. The results of Table 1 show that the mean value for α obtained from the NRM is identical to that of proposed by Bieniawski ($\alpha = 0.75$). However, the value of B obtained from the NRM ($B = 3.85$) is different from the value proposed by Bieniawski ($B = 3.50$).

In calculating the parameters by LCM method, in 17 cases (16 percent of all cases) the value of calculated α was greater than 1.0, which means that the calculated value is out of proposed range of α by Bieniawski and in 2 percent of all cases regression analysis could not be done, however in nonlinear regression method no similar difficulties were experienced. Finally the average values for B and α for each rock type were presented as parameters of Bieniawski's strength criterion. The range of variation of parameters for intact rocks is presented in Table 2 (at the end of the paper) for both analysis methods. As is seen from this Table, there is no limit shown against the LCM for some rocks including andesite, diabase, diorite, granite, quartzdiorite, quartzite, and shale, as the change in values for these rocks showed to be very wide.

An attempt was made to correlate the *UCS* for different rocks with constants in Bieniawski's criterion. No meaningful correlation was observed between *UCS* and constant α but a non-linear correlation was developed between *UCS* and constant B as:

$$B = a + bc^d \quad (10)$$

based on triaxial test data. In this equation $c = \log(UCS)$. Table 3 (at the end of the paper) gives the values for constant parameters a, b, c and d in the above equation. In this Table the correlation coefficient r^2 is shown for each rock type. Figure 1 represents the plot of *UCS* versus B for granite and marble as examples.



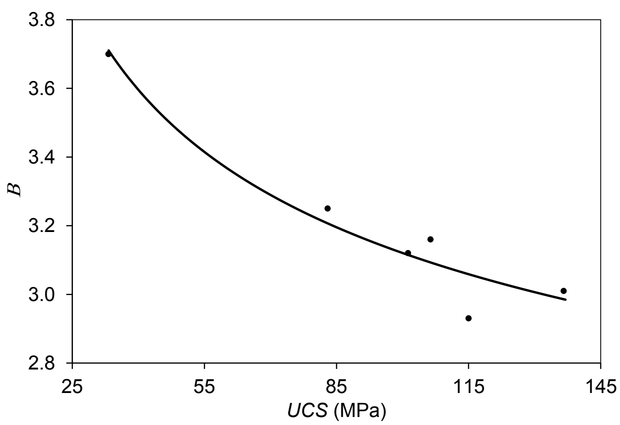


Figure 1. Constant B by the proposed equation for Bieniawski's criterion in this research versus *UCS* for granite (Top, $r = 0.98$) and marble (bottom, $r = 0.96$)

Having obtained the UCS from lab experimental data (preferably extrapolation of triaxial test data), one may use Equation 10 to estimate constant B using parameters proposed in Table 3 and then obtain value α from Table 1.

Variation limit for coefficient of determination for Bieniawski criterion for these 127 rock types is between 0.62 and 1. Table 4 (at the end of the paper) shows a brief record of the coefficient of determination for 127 data groups of intact rocks.

Proposed Constants For Coal

Coal is distinguished from other rocks in that it includes some plane of fractures. As a result of these natural fractures in combination to the bedding plane, coals mechanical behaviour is anisotropic. The triaxial tests performed by Hobbs [14] demonstrated that coals mechanical behaviour is a function of the direction of main fractures with respect to the loading angle. This is why coal mechanical properties are expressed as a function of the loading angle with respect to the bedding plane and the fracture planes.

In this study 25 sets of data belonging to coals composed of 187 triaxial test data were used to determine constants α and B in Bieniawski's failure criterion. The specimens are NX sized with slenderness ratio of 2 in cylindrical shape. Four different cases were considered depending on direction of the sample taken from coal with respect to the bedding plane and fracture planes:

- Loading direction perpendicular to the bedding plane but parallel to the fractures
- Loading direction parallel to both the bedding plane and fractures
- Loading direction is parallel to bedding plane but perpendicular to fractures
- No preferred direction identified

Table 5 shows the Bieniawski's constant values as estimated in this study from both linear and non-linear regression methods and compared to values proposed by Bieniawski [7] and Yudhbir and Prinzl [8]. Table 6 shows the range of variations of these constants. Also, similar correlation proposed in Equation 9 for rocks was fitted to data for coal and Table 7 shows the values of parameters a, b, c and d. Table 5, 6 and 7 are available at the end of the paper.

Figure 2 shows, as an example, the plot of B versus *UCS* for coal when the loading direction is parallel to the bedding plane. No meaningful correlation was observed between UCS and constant α for coal as well as the intact rocks. Similar to intact rocks, one may use Equation 9 to estimate constant B for coals using parameters proposed in Table 7 and then obtain value α from Table 5.

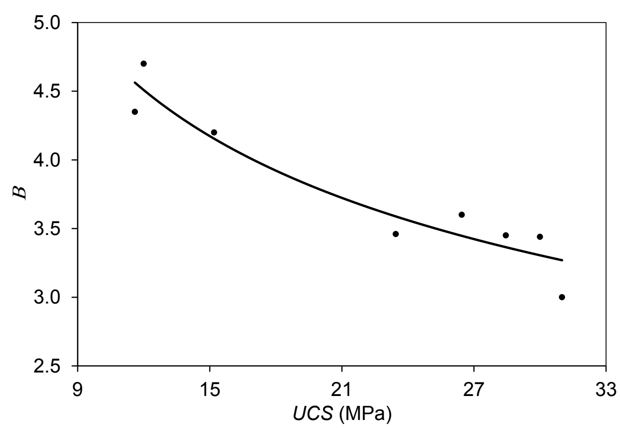
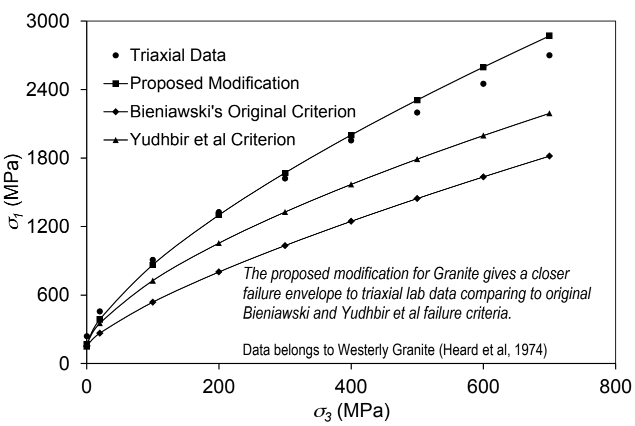


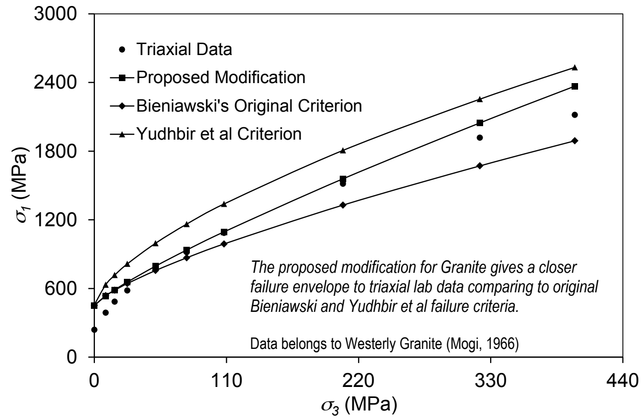
Figure 2. Constant B by the proposed equation for Bieniawski's criterion in this research versus *UCS* for coal ($r = 0.90$)

Authors suggest the results of NRM method because of their precision to estimate the parameters of Bieniawski strength criterion. The variation limit of the coefficient of determination in this research for the data groups of coal is between 0.83 and 1.0. The values of coefficient of determinations for coals are presented in Table 8. Table 9 shows a summary of coefficient of determinations for all data groups of coal and intact rocks. Table 8 and 9 are presented at the end of the paper.

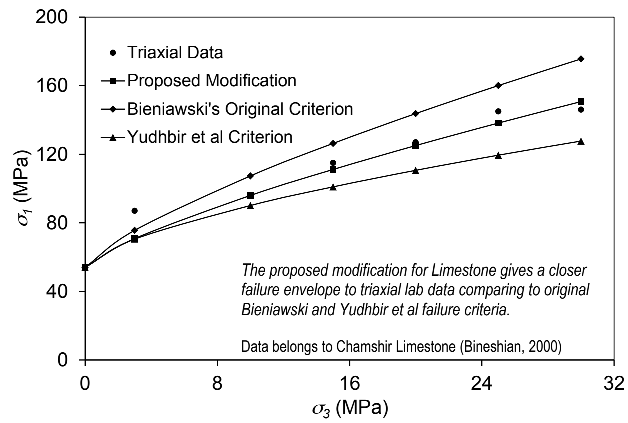
Figures 3 and 4 show a comparison between the proposed modifications to the Bieniawski's strength criterion, original Bieniawski and Yudhbir and Prinzl criteria [7, 8], for granite, limestone, sandstone, and coal. From this figure it is seen how the proposed modification in this research provides a better estimations for major principal stress value at the failure point. The NRM data in Tables 1 and 5 were used to calculate parameters needed to plot curves in Figures 3 and 4. To quantify the accordance of the estimated major principal stress at failure (i.e. how close the estimated values are to real test results) the accordance coefficient was calculated. Table 10 (at the end of the paper) shows the calculated accordance coefficients for the data groups used in Figures 3 and 4 corresponding to different criteria. This Table shows that the proposed modification in this paper yields the least accordance coefficient amongst different strength criteria and therefore is a preferred method to be used.



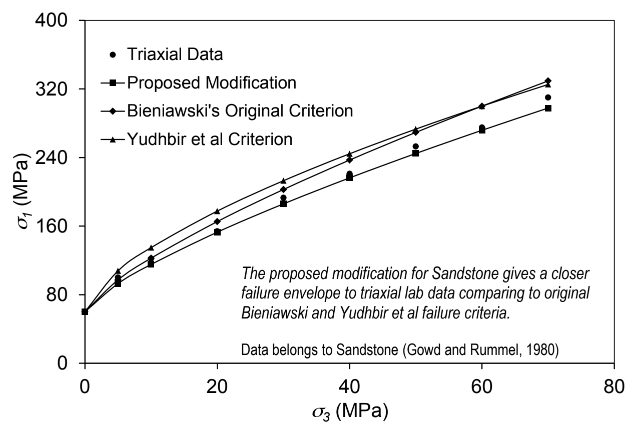
3a- Westerly Granite, Heard et al 1974



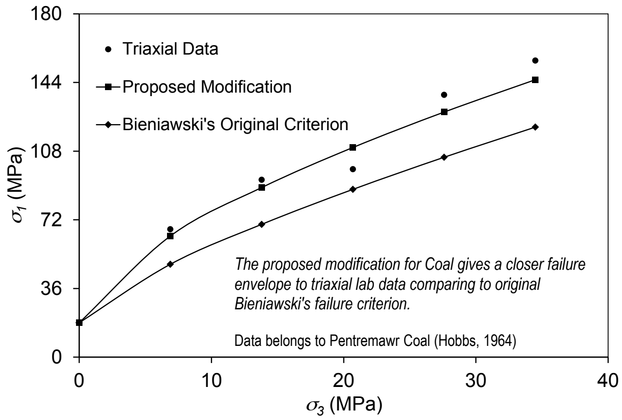
3b- Westerly Granite, Mogi 1966



3c- Chamshir Limestone, Bineshian 2000

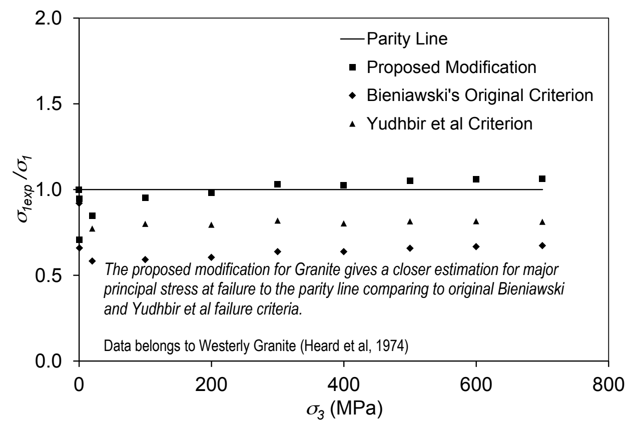


3d- SW Germany Sandstone, Gowd & Rummel 1980

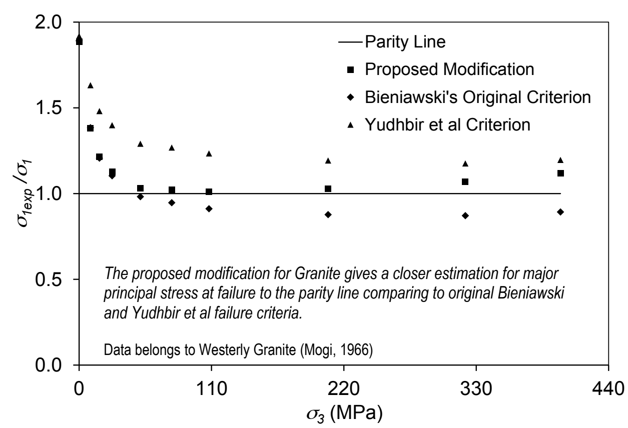


3e- Pentremawr Coal, Hobbs 1964

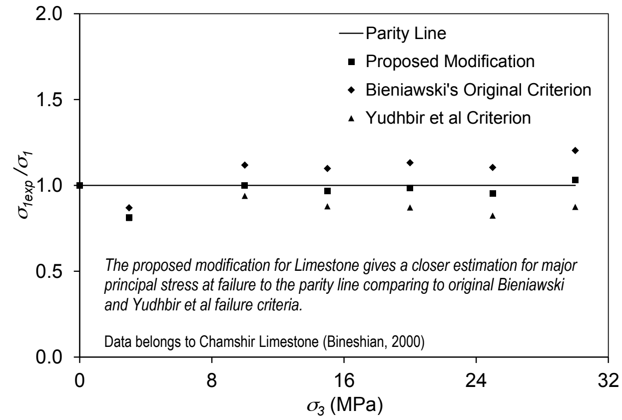
Figure 3. Comparisons between the failure envelopes by proposed modification in this research, original Bieniawski and Yudhbir et al criteria



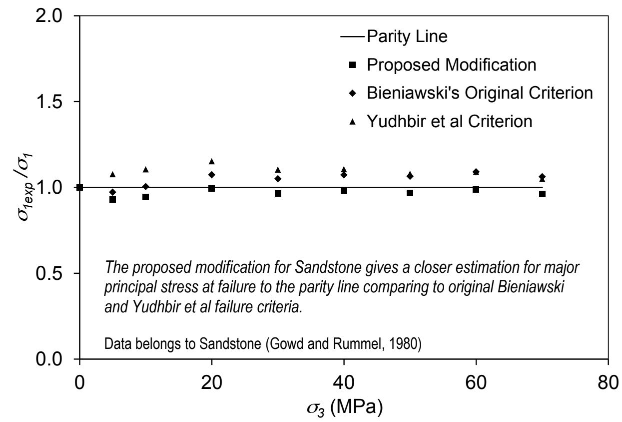
4a- Westerly Granite, Heard et al 1974



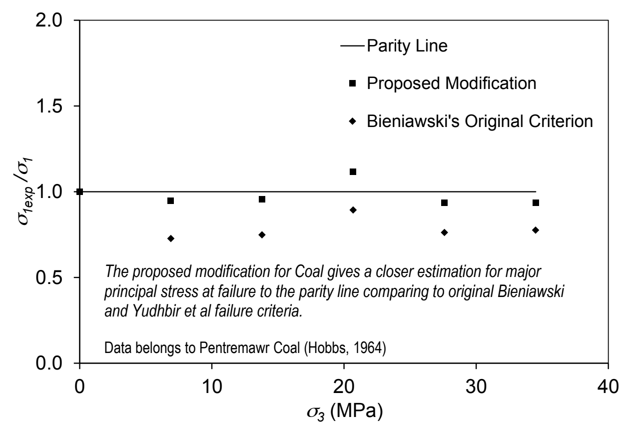
4b- Westerly Granite, Mogi 1966



4c- Chamshir Limestone, Bineshian 2000



4d- SW Germany Sandstone, Gowd & Rummel 1980



4e- Pentremawr Coal, Hobbs 1964

Figure 4. Comparing the estimation for major principal stress at failure as proposed modification in this research, original Bieniawski and Yudhbir et al criteria

Conclusions

In this paper the constants α and B in Bieniawski's failure criterion for intact rocks proposed for a wide range of rocks and coals. A total of 1251 triaxial test data gathered from literature including some triaxial test results performed on limestone were used for this purpose. Total of 152 classes of rocks distinguished for which the constants α and B were estimated through both linear and non-linear regression. The non-linear regression method yielded better correlation results. The constants are presented for different types of rocks. For coals the results are presented as a function of loading direction with respect to coal's bedding plane. Also, correlations developed between UCS and constant B for different type of rocks. One can estimate constant B from its correlation with UCS and then estimate the corresponding value of α from given Tables. Considering that the range of rock types and the input data in this study is very wide it is believed that the developed constant provide a closer estimation of rock strength through the use of Bieniawski's criterion than previously reported values.

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Table 1. Parameters for Bieniawski strength criterion for a wide range of intact rocks

Rock Type	After Bieniawski [7]		After Yudhbir and Prinzl [8]		Suggested in this Research by LCM Method*		Suggested in this Research by NRM Method**	
	B	α	B	α	B	α	B	α
Andesite	-	0.75	-	0.65	5.701	0.733	3.785	0.763
Chert	-	0.75	5	0.65	5.413	0.720	5.358	0.801
Diabase	-	0.75	-	0.65	-	-	4.291	0.662
Diorite	-	0.75	-	0.65	-	-	3.181	0.857
Dolerite	-	0.75	4	0.65	4.584	0.640	3.924	0.685
Dolomite	-	0.75	-	0.65	3.356	0.670	2.912	0.543
Gabbro	-	0.75	-	0.65	4.073	0.598	3.951	0.717
Gneiss	-	0.75	-	0.65	4.030	0.719	2.996	0.737
Granite	-	0.75	5	0.65	6.293	0.687	4.704	0.832
Granodiorite	-	0.75	5	0.65	4.046	0.618	4.084	0.625
Limestone	-	0.75	2	0.65	2.751	0.666	2.804	0.763
Marble	-	0.75	-	0.65	3.315	0.792	3.194	0.861
Mudstone	3	0.75	3	0.65	3.825	0.687	2.859	0.756
Norite	5	0.75	5	0.65	5.518	0.701	4.968	0.793
Quartzdiorite	-	0.75	5	0.65	-	-	4.652	0.784
Quartzite	4.50	0.75	4	0.65	-	-	6.060	0.755
Sandstone	4	0.75	4	0.65	3.847	0.719	3.530	0.748
Shale	-	0.75	2	0.65	-	-	3.791	0.906
Siltstone	3	0.75	3	0.65	2.806	0.682	3.314	0.713
Tuff	-	0.75	2	0.65	2.902	0.778	2.637	0.774
Average	**3.50**	**0.75**	**-**	**0.65**	**4.16**	**0.69**	**3.85**	**0.75**

* Proposed parameters in this research using Linear Conversion Model (LCM)
** Proposed parameters in this research using Nonlinear Regression Model (NRM)

Table 2. Parameters variation for Bieniawski strength criterion suggested in this research for a wide range of rocks

Rock Type	B Suggested in this Research by LCM Method		α Suggested in this Research by LCM Method		B Suggested in this Research by NRM Method		α Suggested in this Research by NRM Method	
	max	min	max	min	max	min	max	min
Andesite	9.162	2.240	0.769	0.697	4.952	2.617	0.953	0.572
Chert	6.252	4.574	0.785	0.665	6.302	4.414	0.921	0.681
Diabase	-	-	-	-	5.022	3.599	0.789	0.534
Diorite	-	-	-	-	3.249	3.113	0.923	0.790
Dolerite	6.103	3.065	0.876	0.404	4.906	2.942	0.829	0.541
Dolomite	5.141	2.455	0.858	0.557	2.935	2.888	0.626	0.460
Gabbro	4.809	3.337	0.627	0.569	4.613	3.289	0.747	0.687
Gneiss	4.824	3.364	0.851	0.596	3.933	2.116	0.771	0.688
Granite	8.770	4.410	0.930	0.523	5.816	3.682	0.941	0.690
Granodiorite	4.906	3.186	0.732	0.504	5.020	3.148	0.681	0.569
Limestone	3.389	2.205	0.973	0.324	3.490	2.051	0.938	0.454
Marble	4.677	2.228	0.917	0.635	3.699	2.920	0.978	0.663
Mudstone	4.371	3.279	0.776	0.598	3.111	2.607	0.790	0.722
Norite	8.148	2.888	0.793	0.609	6.363	3.573	0.913	0.673
Quartzdiorite	-	-	-	-	5.058	4.241	0.913	0.715
Quartzite	-	-	-	-	7.507	2.299	0.925	0.519
Sandstone	5.610	2.549	0.976	0.448	5.598	2.367	0.989	0.536
Shale	-	-	-	-	4.409	3.172	0.991	0.821
Siltstone	3.265	2.347	0.866	0.498	4.058	2.570	0.802	0.624
Tuff	3.525	2.279	0.901	0.655	3.212	2.062	0.798	0.750
All Types	**9.162**	**2.205**	**0.976**	**0.324**	**7.507**	**2.051**	**0.991**	**0.454**

[*] Proposed parameters in this research using Linear Conversion Model (LCM)
[**] Proposed parameters in this research using Nonlinear Regression Model (NRM)

Table 3. Parameter B for Bieniawski strength criterion suggested in this research for wide range of rocks

Rock Type	B			
	a	b	d	r
Diorite	2.2341	0.000	35.423	0.5546
Dolomite	-0.7415	4.4423	$-2.3956*10^{-2}$	0.010
Limestone	$9.5111*10^{-3}$	12.124	-2.0843	0.8043
Gneiss	4.6565	-3.5615	-0.6532	0.169
Granite	-8.7341	30.224	-0.8986	0.9812
Marble	-2.1916	6.9567	-0.3891	0.963
Quartzdiorite	-48.108	54.062	$-3.003*10^{-2}$	0.557
Quartzdiorite & Diorite	-61.574	68.689	$-4.739*10^{-2}$	0.6384
Quartzite	74.775	-66.356	$1.9897*10^{-2}$	0.849
Sandstone	$-4.7774*10^{-2}$	3.7811	$-4.6135*10^{-2}$	0.019
Shale	$3.107*10^{-2}$	3.6008	$9.7642*10^{-2}$	0.066

Table 4. Evaluation of correlation between Bieniawski strength criterion and actual triaxial data for intact rocks

Strength Criterion	Coefficient of Determination (r^2)										
	= 1.00	≥ 0.99	≥ 0.98	≥ 0.97	≥ 0.96	≥ 0.95	≥ 0.94	≥ 0.93	≥ 0.92	≥ 0.91	≥ 0.90
Bieniawski (%)	25	45	55	62	68	72	78	81	87	87	87

Table 5. Parameters for Bieniawski strength criterion for coal with different direction of loading

Coal Type	After Bieniawski [7]		After Yudhbir and Prinzl [8]		Suggested in this Research by LCM Method*		Suggested in this Research by NRM Method**	
	B	α	B	α	B	α	B	α
⊥ bp ‖ mc[1]	-	-	-	-	4.70	0.57	3.71	0.63
‖ bp ‖ mc[2]	-	-	-	-	4.67	0.64	4.02	0.62
‖ bp ⊥ mc[3]	-	-	-	-	4.26	0.58	4.04	0.61
All Types[4]	**3.50**	**0.75**	**-**	**0.65**	**4.54**	**0.60**	**3.92**	**0.62**

[1] Loading direction perpendicular to the bedding plane but parallel to the cleats, [2] Loading direction parallel to both the bedding plane and cleats, [3] Loading direction is parallel to bedding plane but perpendicular to cleats, [4] No preferred direction identified, * Proposed parameters in this research using Linear Conversion Model (LCM), ** Proposed parameters in this research using Nonlinear Regression Model (NRM)

Table 6. Parameters variation for Bieniawski strength criterion suggested in this research for coal with different direction of loading

Coal Type	B Suggested in this Research by LCM Method*		α Suggested in this Research by LCM Method*		B Suggested in this Research by NRM Method**		α Suggested in this Research by NRM Method**	
	max	min	max	min	max	min	max	min
⊥ bp ‖ mc[1]	6.122	3.399	0.850	0.450	4.219	3.243	0.681	0.515
‖ bp ‖ mc[2]	5.812	3.389	0.697	0.580	4.955	3.510	0.690	0.503
‖ bp ⊥ mc[3]	5.048	3.391	0.661	0.531	4.313	3.648	0.661	0.566
All Types[4]	**6.122**	**3.389**	**0.850**	**0.450**	**4.955**	**3.243**	**0.820**	**0.503**

Superscripts 1, 2, 3, 4, * and ** are defined as Table 5.

Table 7. Parameter B for Bieniawski strength criterion suggested in this research for coal

Rock Type	B			
	a	b	d	r
⊥ bp ‖ mc[1]	-536.730	541.230	$-6.065*10^{-3}$	0.84
‖ bp ‖ mc[2]	-536.740	541.620	$-7.3754*10^{-3}$	0.90
‖ bp ⊥ mc[3]	-450.660	455.230	$-4.9785*10^{-3}$	0.59

Superscripts 1, 2, and 3 are defined as Table 5.

Table 8. Evaluation of correlation between Bieniawski strength criterion and actual triaxial data for coal

Strength Criterion	Coefficient of Determination (r^2)										
	= 1.00	≥ 0.99	≥ 0.98	≥ 0.97	≥ 0.96	≥ 0.95	≥ 0.94	≥ 0.93	≥ 0.92	≥ 0.91	≥ 0.90
Bieniawski (%)	06	28	44	50	61	72	89	89	89	89	94

Table 9. Evaluation of correlation between Bieniawski strength criterion and actual triaxial data for coal and intact rocks

Strength Criterion	Coefficient of Determination (r^2)										
	= 1.00	≥ 0.99	≥ 0.98	≥ 0.97	≥ 0.96	≥ 0.95	≥ 0.94	≥ 0.93	≥ 0.92	≥ 0.91	≥ 0.90
Bieniawski (%)	22	43	53	60	67	71	78	81	86	86	87

Table 10. Evaluation of accordance between proposed modification to Bieniawski's strength criterion and the original Bieniawski and Yudhbir and Prinzl criteria [7, 8] for the data groups used in Figures 3 and 4

Data Groups	Accordance Coefficient (ψ^2)		
	Bieniawski Criterion	Yudhbir and Prinzl	Proposed Modification
Westerly Granite, Heard et al [16]	0.3593	0.1071	0.0086
Westerly Granite, Mogi [26]	0.0629	0.1995	0.0433
Chamshir Limestone, Bineshian [22]	0.2645	0.2626	0.0522
SW Germany Sandstone, Gowd & Rummel [16]	0.0306	0.0521	0.0069
Pentremawr Coal, Hobbs [16]	0.2688	NA	0.0279