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DAMPENING BULLWHIP EFFECT OF ORDER-UP-TO INVENTORY STRATEGIES VIA AN OPTIMAL CONTROL METHOD

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Abstract. In this paper, we consider the bullwhip effect problem of an Order-Up-To (OUT) inventory strategy for a supply chain system. We firstly establish a new discrete-time dynamical model which is suitable to describe the OUT inventory strategy. Then, we analyze the bullwhip effect for the dynamical model of the supply chain system. We thus transform the bullwhip effect's dampening problem to a discrete-time optimal control problem. By using the Pontryagin's maximum principle, we compute the corresponding optimal control and obtain the optimal manufacturer productivity of goods. Finally, we carry out numerical simulation experiments to show that the devised optimal control strategy is useful to dampen the bullwhip effect which always happens in the supply chain system.

1. Introduction.

With the social division of labor and the grim competition in today's global market, supply chain management plays a very important role to achieve a variety of business goals. The single manufacturer's production is gradually replaced by that of large-scale manufacturers since the raw materials, the sales and other links form a supply chain to enhance the large-scale companies' performance. So, the

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analysis and research of the supply chain become highly demanded, especially for transnational corporations. Since the ways of exchange and sale are not clear-cut, the information's share between the upstream and downstream in the supply chain is imperative. Considering the manufacturers, suppliers, retailers and end consumers as a whole package, it is of importance that we should share the resource and information. Therefore, the problems such as how to optimize the allocation of resources, how to reduce the cost and how to set up a reasonable inventory strategy have become essential issues in the supply chain management recently.

In practice, there exist many uncertain factors between the upstream and downstream of a supply chain, which always cause demand variability's amplification. One of the typical phenomena in supply chain management is the bullwhip effect (since the amplification of the orders' fluctuation looks like a whipping bullwhip, it is called to be the bullwhip effect). Sometimes it is called to be the "whiplash" or the "whipsaw" effect. The bullwhip effect refers to the amplification of demand variability in the supply chain; that's to say, for example, while the customers buy one kind of commodity, the demand orders' variability is amplified as they move to upstream sites of the supply chain. The bullwhip effect from one end of a supply chain to the other is a source of tremendous inefficiencies of a company, which results in the disordered information and the demand variability's amplification between the echelons of the supply chain. It also can lead to a fluctuation of the requirement information. Currently, to dampen the bullwhip effect becomes an important demand for supply chain optimization and control.

It is well known that Procter & Gamble (P & G) found the bullwhip effect when the company examined the order patterns of one best-selling product, Pampers, and noticed that this product's sales at retail stores were always fluctuating. In fact, J. Forrester is the first person to notice the bullwhip effect phenomenon in the supply chain, as early as 1961 [1]. According to the system dynamics, he analyzed a supply chain system composed of three echelons and four nodes, and pointed out that, for seasonal goods, manufacturers always overreact the demand changes comparing with the customers. He believes that the internal structure of a supply chain and their interaction lead to the demand amplification, due to the change of organizational behaviors. Later, J. D. Sterman designed a "beer game" in his class in 1989 and found that the misinterpretation of logistics executives to the feedback information was the main reason to cause the bullwhip effect [2]. D. R. Towill and M. M. Naim discovered that the inventory management was one of main reasons to cause this phenomenon by making a simulation analysis on the bullwhip effect. He found that it would become twice as much as the inventory while the orders deliver from one echelon to others in the supply chain [3]. Thus it is possible to suppress the bullwhip effect by choosing a reasonable inventory strategy. Recently, F. Chen and his collaborators concluded that the more fluctuations and the longer order cycle in the market demand in a short time would make it harder to suppress the bullwhip effect [4]. E. Ricardo and K. Bardia proposed that the members in a supply chain, especially the suppliers, the wholesalers and the retailers, would set up a safe stock for their own economic interests [5]. Thus, the overstock also makes the market demand swing significantly in the entire supply chain. L. L. Hau et al. summarized the main reasons of the bullwhip effect by integrating these issues of the predecessors and identified four sources of the bullwhip effect: demand signal processing, rationing game, order batching, and price variations [6].

Since the bullwhip effect distorts and harms the performance of supply chains fundamentally, it needs to be suppressed. Hence our aim of this paper is to analyze the bullwhip effect of an OUT inventory strategy model and find an appropriate method to dampen the bullwhip effect effectively. In this paper, we use the productivity of manufacturers as the admissible control, establish a discrete-time model and transform the bullwhip effect problem to an optimal control problem. Then, the Pontryagin's maximum principle is applied to solve the obtained optimal control problem. Finally, we conduct numerical simulations to verify the usefulness and effectiveness of the results obtained.

2. **OUT Inventory Strategy.**

The OUT inventory strategy model introduced in this section is a classical supply chain model. This strategy is an optimal one to add the stock level to the desired inventory level and also the simplest inventory strategy for the case of no fixed order cost. This strategy system is widely used to minimize the cost of the stock. For detailed discussions of the inventory theory, see [4, 7, 9, 12, 14] and their references therein. In order to illustrate the OUT inventory strategy clearly, we will first summarize the results in [4, 14, 15].

For convenience, we consider a simple supply chain consisting of one retailer, one manufacturer, and a warehouse. We assume that the members of the supply chain will obey the following rules:

(1) In each pre-determined period t of the cycle, the single retailer receives the order first, and he then observes and satisfies the customer demand of that period, denoted by D_t. The retailer observes his new inventory and sends an order Q_t to the single manufacturer.

(2) All unfilled demands are backlogged and will be postponed to the next delivery cycle. There is a fixed lead time between the time when the retailer receives the product and the time when the order is issued by the retailer. For example, the retailer issues an order at the end of the period t, and he receives the product at the beginning of the period $t + L$. It is should be noted that the delivery lead time L contains the order delay time of a cycle and the logistics delay time T_p. For example, if $L = 1$ and the logistics delay time is 0, then the retailer issues an order at the end of period t and receives the product at the beginning of period $t + 1$.

In this paper, the retailer uses the order decision-making process of an Order-Up-To inventory strategy:

$$O_t = S_t - WIP_t \tag{1}$$

where O_t is the quantity of the order at the period t, S_t is the desired stock level of a certain echelon of the supply chain for the period t, i.e., the manufacturer desired stock level here. WIP_t is the physical inventory for the period t, which consists of the net inventory (the existing inventory) and the work-in-process at the period t. The desired order-up-to point will be updated in each cycle:

$$S_t = L\widehat{D}_t^L + Z\sigma_{D_t} \tag{2}$$

where $\widehat{D}_t^L$ is the total amount of the forecasting demand for all L cycles, σ_{D_t} is the standard deviation of the demand, and Z is a constant which satisfies the expected service level. At the period t, the retailer receives an order and also satisfies the customer demand. Then, he observes a new stock level, and issues a new order at the end of period t, i.e. the retailer sends the order at the start of $t + 1$. In fact, we normally take a longer cycle $L = T_p + 2$, where T_p is the delay time of production and logistics.

In this paper, we only consider the net stock NS_t, i.e., $NS_t = WIP_t$. Hence, we have the classical OUT inventory policy as follows:

$$O_t = S_t - NS_t = L\widehat{D}_t + Z\sigma_{D_t} - NS_t. \tag{3}$$

When a proportional constant $\frac{1}{T_i}$ is introduced according to [15], we have

$$O_t = L\widehat{D}_t + \frac{Z\sigma_{D_t} - NS_t}{T_i} \tag{4}$$

As for the net stock, it will hold that

$$NS_t = NS_{t-1} + O_{t-1} - D_{t-1} \tag{5}$$

Next, let us consider an ARMA process [15] and the forecasting demand will be

$$\widehat{D}_t = d + \rho(\widehat{D}_{t-1} - d) + \varepsilon + \theta(D_{t-1} - \widehat{D}_{t-1}) \tag{6}$$

where ρ and θ are the model parameters, d is the mean of demand, and the initial condition is $\widehat{D}_0 = d+\varepsilon_0$. The forecast error ε is assumed to be a white noise process.

3. **A Discrete-time OUT Model.**

The production order rate of the retailer can be described by the Order-Up-To inventory policy model (4)-(6), i.e.,

$$\begin{cases} O_t = L\widehat{D}_t + \dfrac{Z\sigma_{D_t} - NS_t}{T_i} \\ NS_t = NS_{t-1} + O_{t-1} - D_{t-1} \\ \widehat{D}_t = d + \rho(\widehat{D}_{t-1} - d) + \varepsilon + \theta(D_{t-1} - \widehat{D}_{t-1}) \end{cases}$$

In addition, we can clearly write the above model in the form of the following discrete-time dynamical system,

$$O(t+1) = L\widehat{D}(t+1) + \frac{Z\sigma_D(t) - NS(t+1)}{T_i} \tag{7}$$

$$NS(t+1) = NS(t) + O(t) - D(t) \tag{8}$$

$$\widehat{D}(t+1) = d + \rho(\widehat{D}(t) - d) + \varepsilon + \theta(D(t) - \widehat{D}(t)) \tag{9}$$

Then, we have

$$\begin{aligned} O(t+1) &= L[d+\rho(\widehat{D}(t)-d)+\varepsilon+\theta(D(t)-\widehat{D}(t))] + \frac{Z\sigma_D(t) - NS(t) - O(t) + D(t)}{T_i} \\ &= \frac{-NS(t) - O(t)}{T_i} + L(\rho-\theta)\widehat{D}(t) + \frac{Z\sigma_D(t) + D(t)}{T_i} + L(d - \rho d + \varepsilon + \theta D(t)) \end{aligned}$$

$$NS(t+1) = NS(t) + O(t) - D(t)$$

$$\widehat{D}(t+1) = (\rho-\theta)\widehat{D}(t) + d - \rho d + \varepsilon + \theta D(t)$$

Thus, we can establish the following discrete-time system

$$\begin{bmatrix} O(t+1) \\ NS(t+1) \\ \widehat{D}(t+1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_i} & -\frac{1}{T_i} & L(\rho-\theta) \\ 1 & 1 & 0 \\ 0 & 0 & \rho-\theta \end{bmatrix} \begin{bmatrix} O(t) \\ NS(t) \\ \widehat{D}(t) \end{bmatrix} + \begin{bmatrix} \frac{Z\sigma_D(t)+D(t)}{T_i} + L(d - \rho d + \varepsilon + D(t)) \\ -D(t) \\ d - \rho d + \varepsilon + D(t) \end{bmatrix}$$

Suppose that we do not consider the lead time, then $L = 1$. In fact, many manufacturers manage the demand fluctuations by setting a safety stock in their production inventory system. Such a system has a single manufacturer and a warehouse to store those products, which are manufactured but not sold immediately. Once a product is made and put into stock, it will cause stock holding costs including the cost of physically storing the product, insuring product. The advantages of having products in stock can be listed as follows: 1) it can be used to meet demands for orders immediately; 2) during the low demand period the warehouse can store excess production and during high demand period the warehouse can be available to sell products. Hence, according to [11], it is possible to obtain a smooth production. Our objective is to find a good manufacturer productivity $u(t)$ to minimize the stock holding cost and production cost. Here the manufacturer productivity $u(t)$ can be regarded as the order quantity $O(t)$. We also want to adjust the buffer stock to the target level $\overline{NS(t)}$, so that we can achieve the pre-defined service levels or to lower the shortage cost. Usually, the buffer stock $\overline{NS(t)}$ during a period of time can be taken as a constant. Then, we will include the stock holding cost, production costs and the level of target buffer stock level in the objective function. For the stock holding cost, we can easily minimize the stock level. For the production cost, we can learn from the classical economics theory that the average unit manufacturing cost of a single product is typically convex in its production rate. In fact, for most manufacturing systems, the system is designed to be at operating level where the average unit production cost at this operating level is minimal [14]. With the growth of production rate, the unit production cost usually will be higher along with the increases of wear and tear of equipment, overtime labor, a higher loss or a larger defective rate [10]. In order to model this phenomenon, we use a quadratic function of $f(P) = aP^2 - bP + c$ with respect to production rate as a unit production cost, $f(P)$ is the unit production cost in unit [11]. In fact, as discussed in [8], numerical experimental evidences indicate that there exists a quadratic relationship between the cost of production and marketing. For brevity, we need only consider the following discrete-time systems

$$\begin{bmatrix} NS(t+1) \\ \widehat{D}(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \rho-\theta \end{bmatrix} \begin{bmatrix} NS(t) \\ \widehat{D}(t) \end{bmatrix} + \begin{bmatrix} O(t) \\ 0 \end{bmatrix} + \begin{bmatrix} -D(t) \\ d-\rho d+\varepsilon+D(t) \end{bmatrix} \tag{10}$$

For convenience, let

$$B = \begin{bmatrix} -D(t) \\ d-\rho d+\varepsilon+D(t) \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

then we obtain the objective function

$$J = \sum_{t=0}^{m-1} a[O(t) - \overline{O}(t)]^2 + b[NS(t) - \overline{NS}(t)]^2 \tag{11}$$

where $\overline{O}(t)$ is the desire order by the retailer, m is the cycle number, and a, b are positive coefficients. Without loss of generality, we choose $a = 1.2$ and $b = 1$ in the rest of this paper.

4. **Dampening Bullwhip Effect via Optimal Control Computation.**

In this section, we will solve the optimal control problem (11) by using the Pontryagin's maximum principle [11, 13]. Combining with an adjoint function of equation (10), we rewrite the Hamiltonian function of (11) as

$$\begin{aligned} H(t) =& \lambda_1(t+1)[NS(t)+b_1+O(t)]+\lambda_2(t+1)[(\rho-\theta)\widehat{D}(t)+b_2] \\ &-a[O(t)-\overline{O}(t)]^2+b[NS(t)-\overline{NS}(t)]^2 \end{aligned} \tag{12}$$

where $\lambda_1(t)$ and $\lambda(t)$ are Lagrange multipliers. The adjoint equation and boundary conditions will be

$$\lambda_1(t)=\tfrac{\partial H(t)}{\partial NS(t)}=\lambda_1(t+1)-2b[NS(t)-\overline{NS}(t)] \tag{13}$$

$$\lambda_2(t)=\tfrac{\partial H(t)}{\partial \widehat{D}}(t)=(\rho-\theta)\lambda_2(t+1) \tag{14}$$

$$\lambda_1(12)=\lambda_2(12)=0 \tag{15}$$

Then, it also should be satisfied that

$$\tfrac{\partial H(t)}{\partial O(t)}=\lambda_1(t+1)-2a[O(t)-\overline{O}(t)]=0 \tag{16}$$

Furthermore, we have

$$NS(t)=\tfrac{1}{2b}[\lambda_1(t+1)-\lambda_1(t)]+\overline{NS}(t) \tag{17}$$

$$\lambda_2(t)=(\rho-\theta)\lambda_2(t+1) \tag{18}$$

$$O^*(t)=\tfrac{1}{2a}\lambda_1(t+1)+\overline{O}(t) \tag{19}$$

To solve the optimal control problem, we first need to determine $\lambda_1(t)$. Substituting (16), (17) into (10) yields

$$\frac{1}{2b}[\lambda_1(t+2)-\lambda_1(t+1)]+\overline{NS}(t+1)=\frac{1}{2b}[\lambda_1(t+1)-\lambda_1(t)]+\overline{NS}(t)+O(t)-D(t)$$

then

$$\lambda_1(t+2)-(2+\tfrac{b}{a})\lambda_1(t+1)+\lambda_1(t)=2b[\overline{NS}(t)-\overline{NS}(t+1)+\overline{O}(t)-D(t)] \tag{20}$$

The characteristic equation is

$$\mu^2-(2+\tfrac{b}{a})\mu+1=0 \tag{21}$$

Assume that μ_1, μ_2 are eigenvalues of equation (24) and

$$2b[\overline{NS}(t)-\overline{NS}(t+1)+\overline{O}(t)-D(t)]=d_t, \tag{22}$$

then we have

$$\lambda_1(t)=C_1\mu_1^t+C_2\mu_2^t+c_t \tag{23}$$

It is clear to see that

$$c_{t+2}-(2+\tfrac{b}{a})c_{t+1}+c_t=d_t \tag{24}$$

5. **Numerical simulations.** In this section, we consider a supply chain system whose customer demand and actual net stock are listed below.

t	0	1	2	3	4	5	6	7	8	9	10	11
$D(t)$	3800	3040	6499	1327	3300	3589	4442	2793	4723	4609	2597	3253
NS_0	19442	15728	16103	16099	16450	17559	19263	16942	17954	17932	16050	14546

Let $NS(0) = 2000$, $\overline{NS}(t) = 2000$, $\overline{O}(t) = 4500$. By (22), we have $d_t = 2[\overline{O}(t) - D(t)]$, whose results can be listed as follows.

t	0	1	2	3	4	5	6	7	8	9	10	11
d_t	1400	2920	-3998	6346	2400	1822	116	3414	-446	-218	3806	2494

In order to solve $\lambda_1(t)$, we substitute $a = 1.2$, $b = 1$ into (28) and obtain

$$c_{t+2} - 2.833c_{t+1} + c_t = d_t,$$

Let

$$c_0 = 0$$

$$c_2 - 2.833c_1 = d_0$$

$$c_3 - 2.833c_2 + c_1 = d_1$$

$$\cdots\cdots\cdots\cdots$$

$$c_{12} - 2.833c_{11} + c_{10} = d_{10} \tag{25}$$

Then, it follows from (15), that

$$\lambda_1(12) = C_1\mu_1^{12} + C_2\mu_2^{12} + c_{12} = 0 \tag{26}$$

Solving $\mu^2 - 2.833\mu + 1 = 0$, we obtain $\mu_1 = 2.4201$, $\mu_2 = 0.4132$. By (17), it follows that

$$C_1(\mu_1 - 1) + C_2(\mu_2 - 1) + c_1 = 2[NS(0) - \overline{NS}(t)] = 0 \tag{27}$$

Let $C_2 = -1$, combine (24)-(26) together, then we obtain a linear simultaneous equation with respect to $c_1, c_2, \cdots, c_{12}, C_1$.

$$\begin{bmatrix} -2.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2.8 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.8 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.8 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.8 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.8 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 40365 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.42 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \\ C_1 \end{bmatrix} = \begin{bmatrix} 1400 \\ 2920 \\ -3998 \\ 6346 \\ 2400 \\ 1822 \\ 116 \\ 3414 \\ -446 \\ -218 \\ 3806 \\ 2494 \\ -0.5868 \end{bmatrix}$$

Then, $c_i(i = 1, 2, \cdots, 12)$, and C_1 will be obtained that

c_1	2455.218936
c_2	8355.635246
c_3	24136.29572
c_4	56024.49052
c_5	140927.0859
c_6	345621.9439
c_7	840041.8812
c_8	2034332.705
c_9	4926636.673
c_{10}	11922382.99
c_{11}	28849256.34
c_{12}	69811366.21
C_1	-1729.440659

Clearly, $\lambda_1(t) = -1729.44\mu_1^t - \mu_2^t + c_t$. Thus, $NS(t)$ and $O^*(t)$ will be

$$O^*(t) = \frac{1}{2a}\lambda_1(t+1) + \overline{O}(t) = \frac{1}{2.4}\lambda_1(t+1) + 4000$$

$$NS(t) = [\frac{1}{2b}\lambda_1(t+1) - \lambda_1(t)] + \overline{NS}(t) = \frac{1}{2}[\lambda_1(t+1) - \lambda_1(t)] + 2000$$

Overall, the results of all parameters can be calculated and listed below.

t	$O^*(t)$	$NS(t)$	$NS_0(t)$	$D(t)$
1	3261	1978.5	19442	3800
2	3842.8	2698.2	15728	3040
3	2624.7	538.3030	16103	6499
4	2897.6	2327.5	16099	1327
5	3234	2403.6	16450	3300
6	3647	2495.7	17559	3589
7	3707.2	2072.2	19263	4442
8	4686.3	3175	16942	2793
9	5423.3	2884.4	17954	4723

Numerical simulations of optimal net stock can been seen in Figure 1. It shows that under the optimal strategy the demand of customers will be satisfied very well, the stock shortage will decrease tremendously, and the stock cost will reduce as well.

6. **Conclusion.**

In this paper, we have studied the bullwhip damping problem of a supply chain composed of a retailer, a manufacturer and a warehouse. We have expressed the OUT inventory strategy model by use of a discrete-time dynamical system. Hence, the bullwhip damping problem is transformed to seek an optimal productivity satisfying a set of constraints. By using the Pontryagin's maximum principle, we can obtain a manufacturer's optimal decision. Finally, from numerical simulations, we can see that the optimal productivity strategy can help to suppress the bullwhip effect and satisfy the customer's demand very well.

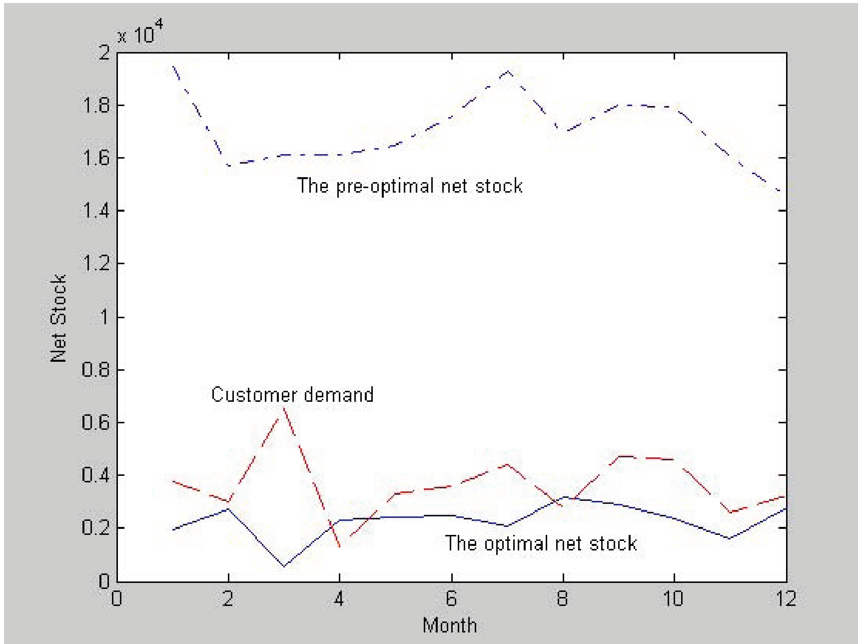

FIGURE 1. Dynamic trajectories of pre-optimal net stock, optimal net stock and customer demand

REFERENCES

[1] J. Forrestor, "Industrial Dynamics", New York: MIT Press and John Wily &Sons, Inc, (1961).

[2] J. D. Sterman, *Modeling managerial behavior: misperceptions of feedback in a dynamic decision making experiment*, Management Science, **35(3)** (1989), 321–339.

[3] D. R. Towill, M. M. Naim, *Industrial dynamics simulation models in the design of supply chains*, International Journal of Physical Distribution and Logistics Management, **22(5)** (1992), 3–12.

[4] F. Chen, Z. Drezner, J. K. Ryan, D. Simchi-Levi, *Quantifying the bullwhip effect in a simple supply chain: the impact of forecasting, lead times, and information*, Management Science, **46(3)** (2000), 436–443.

[5] E. Ricard, K. Baradia, *Evaluation of supply chain structures through modularization and postponement*, European Journal Of Operational Research, **124** (2000), 495–510.

[6] L. L. Hau, V. Padmanabhan, S. Whang, *Information distortion in a supply chain: the bullwhip effect*, Management Science, **43(4)** (1997), 546–558.

[7] S. Karlin, H. Scarf, "One Stage Inventory Models with Uncertainty. In: Studies in the Mathematical Theory of Inventory and Production", Stanford University Press, (1958), 109–134.

[8] S. Makridakis, S. C. Wheelwright, V. E. McGee, "Forecasting: Methods and Applications", John Wiley &Sons, New York, (1978).

[9] J. K. Ryan, "Analysis of inventory models with limited demand information", Ph.D. Dissertation, Department of Industrial Engineering and Management Science, Northwestern University, Evanston, .

[10] R. B. Chase, N. J. Aquilano, "Production and Operations Management", Irwin. 7th ed, (1995).

[11] S. P. Sethi, G. L. Thompson, "Optimal Control Theory-Applications to Management Science", Martinus Nijhoff, Boston, (1981).

[12] S. Eiamkanchanalai, A. Banerjee, *Production lot sizing with variable production rate and explicit idle capacity cost*, International Journal of Production Economics, **59** (1999), 251–259.

[13] K. L. Teo, C. J. Goh and K. H. Wong, "A Unified Computational Approach to Optimal Control Problems", Longman Scientific &Technical, New York, (1991).

[14] K. F. C. Yiu, L. L. Xie, K. L. Mak, *Analysis of bullwhip effect in supply chains with heterogeneous decision models*, Journal of Industrial and Management Optimization, **2** (2009), 81–94.

[15] Y. F. Chen, S. M. Disney, *The myopic order-up-to policy with a proportional feedback controller*, Int J Prod Res, **45** (2007), 351–368.

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