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Analytical Wavefront Curvature Correction for Spherical Wave AVO

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SUMMARY

For three-parameter AVO inversion it may be important to use large incidence angles close to the critical angle, where spherical wave effects become important. For the amplitude of a spherical wave reflected from a fluid-fluid interface, Brekhovskikh and Godin (1997) developed an analytical approximation, which is accurate at all angles. For the amplitude of a spherical wave reflected from a solid/solid interface, we propose a formula which combines this analytical approximation with a standard linearised plane-wave AVO equation of Thomsen (1990). The proposed approximation shows reasonable agreement with numerical simulations for a range of frequencies. Using this solution, we constructed a two-layer three-parameter least-squares inversion algorithm. Application of this algorithm to synthetic data for a single interface shows promising results.

Introduction

Most AVO analysis and inversion techniques are based on the Zoeppritz equations for plane-wave reflection coefficients or their linearised approximations. Real seismic surveys use localised sources that produce spherical waves, rather than plane waves. AVO response for a spherical wave differs from that for a plane wave, especially for angles close to or beyond the critical angle (Červený, 1961; Krail and Brysk, 1983; Winterstein and Hanten, 1985; Alhussain et al., 2008).

Recently, Ursenbach et al. (2007) developed an approach that accounts for the spherical wave AVO effects. Unlike the Zoeppritz equations for plane waves, the reflection amplitude for spherical waves is represented by a double integral over frequency and wavenumber, which has to be computed numerically. Spherical wave AVO inversion requires these computations to be repeated multiple times in an iterative fashion, making the procedure computationally expensive. Ursenbach et al. (2007) simplified these computations by using an analytical form of the source wavelet, which allows integration over the frequency to be done analytically and numerical integration over the wavenumber to be optimised. In this paper, we propose an alternative approach based on an analytical approximation for the amplitude of the reflected spherical wave.

Analytical expression for fluid/fluid interface

The leading (zero-order) term in the high-frequency asymptotic approximation for the amplitude of a reflected spherical wave can be written as $R(i)/r$, where R is a plane-wave reflection coefficient for an incidence angle i and r is the length of the ray path from the source to the receiver. The next (first-order) approximation can be written as $R(i)(1/r + B(i)/kr^2)$, where k is the characteristic wavenumber and B is a dimensionless quantity of order 1. For typical situations in petroleum seismology, P-wave velocity V_P> 3 km/s, central frequency f>30 Hz and r>2 km, so that $kr=2\pi fr/V_P$ >120. Thus, the zero-order approximation is very accurate and the first-order corrections are only important for near-surface applications.

However, numerical simulations and theoretical analysis show that the ray theory approximations described above break down in the vicinity of the critical angle. Furthermore, Brekhovskikh and Godin (1999) show that in the vicinity of the critical angle, the spherical wave correction is on the order $(kr)^{-1/4}$, and therefore is important for much larger values of kr than the first-order correction $B(kr)^{-1}$. According to Brekhovskikh and Godin (1999) and Godin (2010), the reflection coefficient for a spherical wave reflected from an interface between two fluids with sound velocities V_{P1} and V_{P2} and densities ρ_1 and ρ_2 can be approximated as a sum

$$R(\theta_0) = V_1 + p_2, \tag{1}$$

where the regular part V_1 of the reflection coefficient is given by

$$V_1 = \frac{m^2+n^2-(m^2+1)q^2}{m^2-n^2-(m^2+1)q^2} \tag{2}$$

with $m = \rho_2/\rho_1$, $n = V_{P1}/V_{P2}$ and $q = \sin\theta$. In turn, the 'singular' part p_2 (the part with a singularity at the critical angle in the high-frequency limit) is

$$p_2 = \frac{2^{3/2}\sin\delta\exp\left[ikR_1\left(\frac{\cos(\theta_0-\delta)}{2}\right)^2-\frac{7i\pi}{8}\right]}{m(kR_1)^{\frac{1}{4}}\left[\sin\theta_0\cos\delta\left(\cos\left(\frac{(\theta_0-\delta)}{2}\right)\right)^3\right]^{\frac{1}{2}}} \times \frac{\left[D_{\frac{1}{2}}(u)+\frac{A-1}{u}D_{\frac{3}{2}}(u)\right]}{\exp(ikR_1)}, \tag{3}$$

where θ_0 is the angle of incidence, δ is the critical angle, k is the wave number in the upper layer, R_1 is the distance between the image source in the lower layer and the receiver, and $D_{\frac{1}{2}}$ and $D_{\frac{3}{2}}$ are parabolic cylinder functions. Quantities u and A are given by

$$u = 2\exp(3\pi i/4)(kR_1)^{\frac{1}{2}}\sin\left(\frac{\theta_0-\delta}{2}\right) \tag{4}$$

and

$$A = \frac{m^2[0.5\sin\theta_0\cos\delta\sin(\theta_0+\delta)]^{\frac{1}{2}}\cos\theta_0\left(\cos\frac{(\theta_0-\delta)}{2}\right)^2}{\sin\delta(m^2(\cos\theta_0)^2+(\sin\theta_0)^2-(\sin\delta)^2)}. \tag{5}$$

Figure 1 shows the reflection coefficient versus the incidence angle curves extracted from numerically computed synthetic seismograms for an acoustic wave reflected from a single interface between two fluids with sound velocities $V_{P1} = 1500$ m/s and V_{P2}=2000 m/s and densities ρ_1=1 g/cm^3 and ρ_2=2 g/cm^3. The point source and receiver were 500 m above the reflector and the source wavelet was the Ricker wavelet with central frequency of 50 Hz. Also shown are the real parts of the plane wave reflection coefficient and the spherical wave approximation computed with equations (1)-(5). We see that the spherical correction greatly improves the match with the synthetic AVA curve around the critical angle.

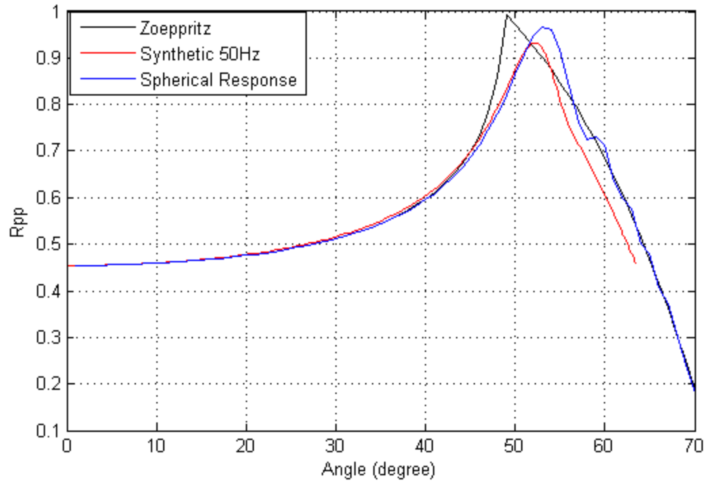

Figure 1 *Comparisons of AVO curves extracted from synthetic data (blue), Zoeppritz equations (black) and spherical wave analytical solution at fluid-fluid interface (red).*

Analytical approximation for solid/solid interface

Analytical expressions (1)-(5) are for fluid/fluid interface, and thus are not useful for practical AVO analysis. It is possible to derive similar expressions for a solid/solid interface, but the expressions will be very cumbersome. Alternatively, we can try to adopt a curvature correction to a linearised approximation widely used for plane-wave AVO.

Thomsen (1990) showed that in case of small contrasts between properties of two solid media, the PP plane-wave reflection coefficient for an interface between these media can be written as

$$R_P(i) = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}\left\{\frac{\Delta V_P}{\bar{V}_P} - \left(\frac{2\bar{V}_S}{\bar{V}_P}\right)^2\frac{\Delta G}{\bar{G}}\right\}\sin^2 i + \frac{1}{2}\frac{\Delta V_P}{\bar{V}_P}\sin^2 i\tan^2 i, \tag{6}$$

where Z denotes acoustic impedance, $G = \rho V_S^2$ is the shear modulus, i is average of the incidence angle and the refraction angle, and $\Delta x/\bar{x}$ denotes relative contrast in the property x between the media 1 and 2. Equation (6) can be rewritten in the form

$$R_P(i) = R_f(i) - \frac{1}{2}\left(\frac{2\bar{V}_S}{\bar{V}_P}\right)^2\frac{\Delta G}{\bar{G}}\sin^2 i \quad , \tag{7}$$

where $R_f(i)$ is the reflection coefficient from an interface between two fluids with the same P-wave velocities and densities as in the two solid layers. To account for the wavefront curvature, we propose to replace the plane wave coefficient for fluid/fluid interface, $R_f(i)$, with the spherical wave reflection coefficient given by equation (1),

$$R_P(i) = V_1 + p_2 - {}^1\!/_2 \left(\frac{2\bar{V}_S}{\bar{V}_P}\right)^2 \frac{\Delta G}{\bar{G}} (\sin i)^2 \qquad (8)$$

with V_1 and p_2 given by equations (2) and (3).

In order to examine the accuracy of this analytical approximation, AVO curves were extracted from synthetic seismograms (computed by the reflectivity method) for the model shown in Table 1 using a Ricker wavelet with frequencies 30Hz, 50 Hz and 80Hz, again for a depth of 500 m. The critical angle is 56.44˚. AVO curves for this model were also computed using plane-wave Zoeppritz equations and Thomsen (1990) approximation, equation (6) and our proposed spherical wave approximation, equation (8). Figure 2 shows these curves along with the AVO curves extracted from synthetic seismograms. It is apparent that AVO curves of our analytical solution are reasonably close to the AVO curves of the synthetic data. Hence, inversion results are expected to be better than those based on the Zoeppritz equations. Also, inversion will not be limited to the data below the critical angle.

Table 1. Properties of two solid media in the synthetic model

	Vp (m/s)	Vs(m/s)	Density (g/cm^3)
Upper Layer	2500	1200	2.00
Lower Layer	3000	1300	2.20

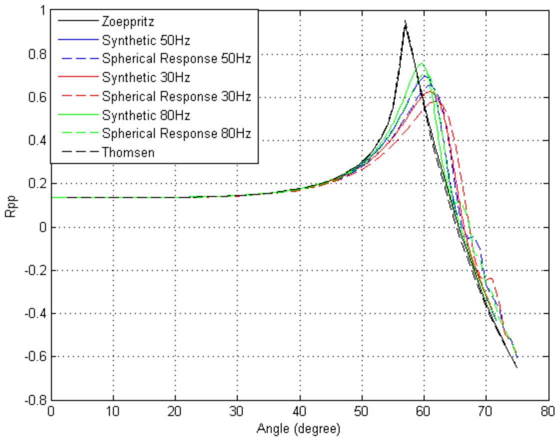

Figure 2 *Comparisons of AVO curves extracted from synthetic data, and given by the Zoeppritz equations, Thomsen's linearised approximation, spherical wave theoretical approximation at solid-solid interface. Model parameters are given in Table 1.*

Inversion

It is clear from earlier studies as well as from our results that the spherical curvature effects are only important at long offsets, close to or beyond the critical angle. These large angles are unnecessary for standard two-parameter AVO analysis, but may be important for three-parameter inversion, when independent recovery of P and S velocities and density is desired. To test how our approximation performs for this purpose, we have developed a simple least-squares three-parameter inversion procedure for a single interface. The algorithm (similar to the one described by Alhussain et al., 2008) assumes that the properties of the upper layer (medium 1) are known, and attempts to find the properties of the bottom layer (medium 2). The algorithm attempts to estimate P and S velocities and density of medium 2 by fitting the AVA curve extracted from our 50 Hz synthetic data using exact

Zoeppritz equations, Thomsen's approximation and the spherical wave approximation, equation (8). The relative error in estimating V_{P2} as a function of the offset range is shown in Figure 3 for data without noise (a) and with added random noise (b). We see that for noise-free data all the algorithms provide accurate estimation of V_{P2}, and moderate angles (below 45 degrees) give best results. However, in the presence of noise, use of long offsets is really important, and our spherical curvature approximation provides the most robust solution for a broad range of angles.

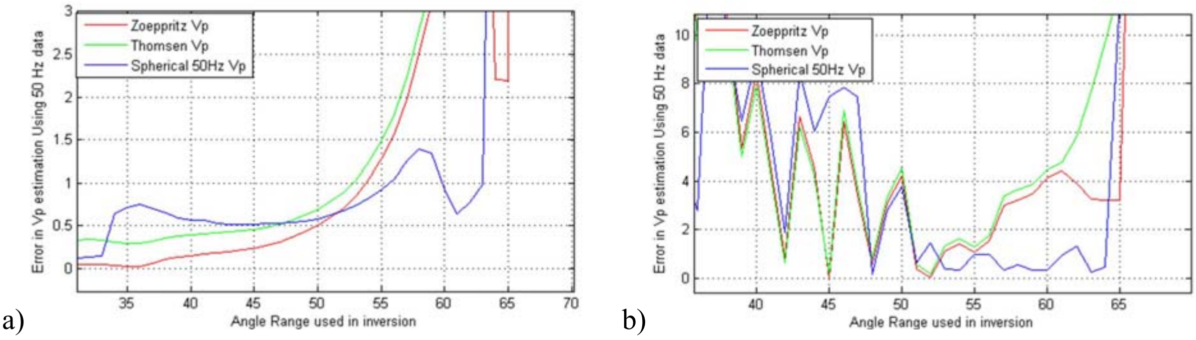

Figure 3 *Error in estimating* $\boldsymbol{V_{P2}}$ *in 3-parameter inversion from synthetic data without (a) and with (b) noise*

Conclusions

For three-parameter AVO inversion it may be important to use angles close to the critical angle, where spherical wave effects become important. By combining a standard linearised plane-wave AVO equation with the known acoustic spherical wave solution, we proposed a new analytical spherical wave approximation. Use of this solution in an iterative two-layer 3-parameter inversion shows promising results.

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