Augmentation of AUSGeoid98 with GRACE satellite gravity data

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ABSTRACT

The AUSGeoid98 gravimetric quasigeoid model of Australia is augmented in the medium- and long-wavelength bands by removing its EGM96 basis and replacing this with GGM02C and EIGEN-GL04C, whose long wavelengths are derived from Gravity Recovery And Climate Experiment (GRACE) satellite gravimetry. No significant improvement over AUSGeoid98 is seen: agreements with GPS-levelling change from ±28 cm to ±27 cm (acknowledging distortions in the levelling); agreements with astrogeodetic vertical deflections do not change, remaining at about ±1 arc-second. While this remove-replace approach is not theoretically exact, it is likely that errors in the terrestrial gravity data are contaminating these combined GRACE solutions in the medium wavelengths over Australia.

Keywords: Geodesy, quasigeoid modelling, GRACE, GPS-levelling, vertical deflections, Australia

INTRODUCTION

The majority of the long- and medium-wavelength components of the AUSGeoid98 gravimetric quasigeoid model of Australia (Featherstone *et al*., 2001) were provided by the EGM96 global geopotential model (GGM), which itself was derived from a combination of satellite tracking data and terrestrial gravity and terrain data available in 1996 (Lemoine *et al*., 1998). Since then, the CHAMP (Challenging Mini-satellite Payload; Reigber *et al*., 1999) and GRACE (Gravity Recovery and Climate Experiment; Tapley *et al*., 2004a; 2004b) dedicated satellite gravimetry missions have been launched (on 15

July 2000 and 17 March 2002, respectively). Overviews of the CHAMP, GRACE and the imminent GOCE (Gravity Field and Steady-State Ocean Circulation Explorer) mission concepts and aims are given by, e.g., Nerem *et al*. (1995), Balmino *et al*. (1999), Reigber *et al*. (1999), Rummel *et al*., (2002), Featherstone (2002) , Chao (2003) and Tapley *et al*. (2004a; 2004b).

Since the launches of CHAMP and GRACE, over 20 new GGMs have been released. These are distributed freely by the International Centre for Global Earth Models (ICGEM) (http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html) as ASCII files of fully normalised spherical harmonic coefficients of the Earth's external gravitational potential. Various other gravity-field-related quantities can be computed from these coefficients, such as gravity anomalies, quasigeoid heights and vertical deflections. [The subtle differences between the geoid and quasigeoid are outlined in Sjöberg (1995), Rapp (1997), Featherstone and Kirby (1997), Featherstone and Kuhn (2006) and Tenzer *et al.* (2006)]. Importantly, CHAMP/GRACE-based gravity field models are much better in the long wavelengths than previous GGMs (see Figure 3 later), which relied on tracking satellites with a limited range of orbital inclinations (e.g., Featherstone, 2002).

Work is currently underway to produce a new Australian gravimetric quasigeoid model that will incorporate new CHAMP/GRACE-based GGMs, new satellite altimetry, new gravity and new terrain data, as well as using improved mathematical models and computational techniques (see Featherstone *et al*. (2007) for a status report). The release of this new Australian quasigeoid model is deliberately being delayed until mid-2008, after the EGM07 GGM and DNSC07 satellite-altimeter-derived gravity anomalies are released, probably in early-2008. EGM07 is currently being computed by the US National Geospatial Imagery Agency (NGA) in conjunction with SGT Inc. as a replacement for EGM96, and will be the highest-ever spherical harmonic expansion of the Earth's gravity field, being complete to degree and order 2160 (~10 km spatial resolution). DNSC07 marine gravity anomalies will be based on EGM07 and use retracked multi-mission satellite radar altimetry, which will improve the gravity data in the notoriously problematic coastal zone (cf. Deng and Featherstone, 2006).

This deliberate delay will allow EGM07, DNSC07 and other datasets to be tested in the Australian context so as to ensure better longevity of the new gravimetric quasigeoid model. The new model will also be fitted to GPS-levelling data at key Australian Height Datum (AHD) benchmarks across Australia using cross-validated least-squares collocation (cf. Featherstone and Sproule, 2006). This is so as to give a more directly useable product, specifically for GPS heighting (cf. Featherstone, 1998). Meanwhile, it is informative to gauge what improvements the current GRACE-derived combined GGMs may make upon AUSGeoid98.

THE AUGMENTATION METHOD AND ITS LIMITATIONS

Hipkin (1996) computed the EDIN891 gravimetric geoid model of Britain from an earlier regional gravimetric geoid model simply by subtracting the degree-360 geoid expansion of the OSU86E GGM (Rapp and Cruz, 1986), then adding back the degree-360 geoid expansion of the OSU91A GGM (Rapp *et al.*, 1991). In this paper, this procedure is termed *augmentation* of the regional gravimetric quasigeoid model by the GGM. As pointed out by Featherstone and Olliver (2001), this approach is theoretically incorrect because it violates the remove-compute-restore approach (cf. Sjöberg, 2006), as follows.

In the combined solution for the geoid or quasigeoid, a GGM is merged with terrestrial gravity and terrain data via some adaptation of Stokes's integral (see, e.g., Featherstone *et al*. (2007) for a conceptual overview). However, this is not a perfect shift-filtering process when the Stokes integration is over a limited region, and spectral leakage occurs among high and low frequencies (Vaníček and Featherstone, 1998). Therefore, and strictly, the same spherical harmonic degree and order of the same GGM must be subtracted (remove step) from the terrestrial gravity data before Stokes integration (compute step) as is added back later (restore step). If not, spurious long-wavelength errors result from the inconsistent use of different GGMs due to the spectral leakage problem. A more rigorous combination procedure is proposed by Sjöberg and Featherstone (2004), but this is not used here for reasons of convenience.

It is extremely difficult to quantify the error that the augmentation used here may generate, because numerous factors can contribute, such as the relative quality of the GGM and terrestrial gravity data added, the integration radius, etc. (Sjöberg and Featherstone, 2004). However, a crude *guestimate* is of the order of 20 cm, which is deduced from the standard deviation (STD) of the difference between two GGMs (see Table 1, later).

Nevertheless, it is still informative to see if such augmentation of an existing regional gravimetric quasigeoid model can make any improvement in the Australian context. It must be stressed that this is only an experiment designed to give some approximate idea of likely future improvements from GRACE (cf. Featherstone, 2002). As such, these augmented AUSGeoid models are not official product releases, and users should continue to use AUSGeoid98 until the official release of the new model that uses all new datasets in a rigorous combined solution (in mid-2008). AUSGeoid98 and associated software can be downloaded free-of-charge from: http://www.ga.gov.au/geodesy/ausgeoid/.

Two recently released GRACE-based combined (i.e., with global terrestrial data; see Featherstone, 2002) GGMs will be used in these experiments: GGM02C (Tapley *et al*., 2005) and EIGEN-GL04C (Förste *et al*., 2007). GGM02C was produced at the Centre for Space Research (CSR) at the University of Texas at Austin, USA, and is available complete to spherical harmonic degree and order 200 (~100 km spatial resolution). EIGEN-GL04C was produced at the GeoForschungZentrum (GFZ) Potsdam, Germany, and is available complete to spherical harmonic degree and order 360 (~55 km spatial resolution). Both models were downloaded free of charge from the ICGEM website (http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html).

An in-house computer program, `harmonics.f`, which is a modified version of Rapp's (1982) FORTRAN77 code, was used to evaluate EGM96, GGM02C and EIGEN-GL04C quasigeoid undulations from spherical harmonic synthesis on exactly the same 2-arc-minute by 2-arc-minute grid as used for AUSGeoid98 (cf. Featherstone *et al*., 2001). The main modification to `harmonics.f` is the addition of Holmes and Featherstone's (2002) recursion routines, which significantly accelerate the computation speed. Note that GGM coefficients directly give quasigeoid undulations; additional computations are needed if the geoid is desired (Rapp, 1997; also see Sjöberg, 1995; Featherstone and Kirby, 1997; Featherstone and Kuhn, 2006; Tenzer et al., 2007). EGM96 quasigeoid heights were evaluated complete to degrees 200 and 360, GGM02C to 200 and EIGEN-GL04C to 360.

The augmentation (i.e., remove-replace) methodology is then rather straightforward. To augment AUSGeoid98 by GGM02C, the degree-200 EGM96 quasigeoid grid was subtracted (remove step) from the AUSGeoid98 grid, and then the degree-200 GGM02C quasigeoid grid subsequently added (replace step). Likewise, to augment by EIGEN-GL04C, degree-360 EGM96 was removed from AUSGeoid98, then replaced with degree-360 EIGEN-GL04C. The resulting augmented regional gravimetric quasigeoid models are respectively called AG98+GGM02C and AG98+GL04C. They are both on the same two-arc-minute grid as AUSGeoid98 and likewise refer to the GRS80 ellipsoid (Moritz, 1980).

RESULTS, ASSESSMENTS AND DISCUSSION

Differences among GGMs

First, it is informative to see how GGM02C and EIGEN-GL04C differ from EGM96 over the AUSGeoid98 computation area. In Figures 1 and 2, the main differences occur in the long and medium wavelengths, which is expected because of the improved low-frequency gravity field information available from GRACE (Figure 3). However, the treatment of the terrestrial gravity data used in the GGM02C and EIGEN-GL04C combined GGMs also comes into play, also affecting the medium wavelengths. The more detail in Figure 2 is because of the different spatial resolution of the GGMs.

The largest differences (up to 2 m) in Figures 1 and 2 occur near Indonesia, where the gravity field is very variable due to the subduction of the Australian plate beneath the Asian plate (e.g., Hillis and Müller, 2003). There is also a large difference (~1 m) in the Gulf of Carpentaria (centred at ~12°S, ~140°E), but the exact cause of this remains unknown at present. It is possible that the satellite altimeter data in this shallow sea, where tides are poorly modelled, have contaminated the GGMs. Over mainland Australia, the differences in Figures 1 and 2 are around 25 cm.

The statistics of these differences, as well as differences from AUSGeoid98, are summarised in Table 1. While these values are biased by the large differences discussed above, it is important to show the whole picture because there are AUSGeoid98 users in marine areas, and there may even be some users in Indonesia. From the STDs in Table 1, the GRACE-based GGMs contribute around 20 cm differences from EGM96. This reflects the use of GRACE data, as well as the different treatment of the terrestrial gravity data (cf. Lemoine *et al.*, 1998; Tapley *et al.*, 2005; Förste *et al.*, 2007). The STD of the differences between AUSGeoid98 and the various GGMs are around 40-50 cm, which shows the contribution of Australian gravity and terrain data to the GGMs.

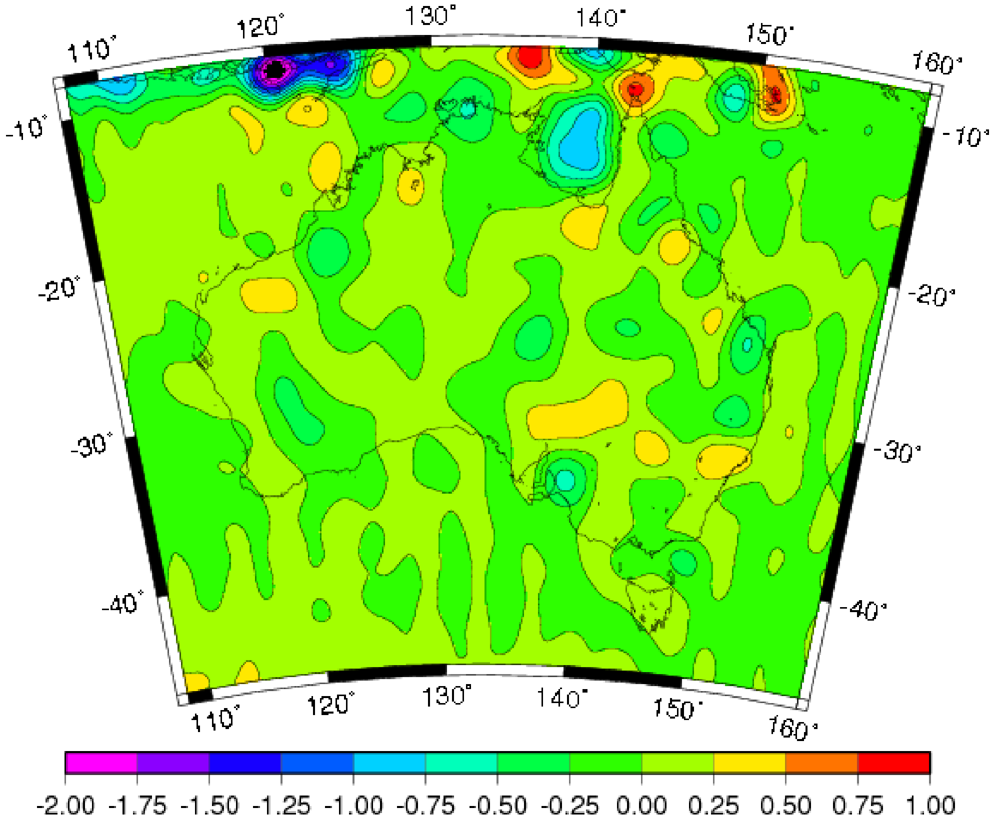

Figure 1 *Differences between EGM96 and GGM02C quasigeoid heights to degree 200 (~100 km resolution) over the AUSGeoid98 area (Units in metres; Lambert projection). This is identical to the differences between AUSGeoid98 and AG98+GGM02C.*

GGM difference	Degree	Max	Min	Mean	STD
EGM96 minus GGM02C	200	+0.944	–2.292	–0.001	±0.207
EGM96 minus EIGEN-GL04C	360	+1.804	–2.395	–0.002	±0.231
AUSGeoid98 minus EGM96	200	+4.743	–11.000	–0.005	±0.505
AUSGeoid98 minus EGM96	360	+3.484	–11.444	–0.005	±0.411
AUSGeoid98 minus GGM02C	200	+4.158	–10.921	–0.006	±0.552
AUSGeoid98 minus EIGEN-GL04C	360	+2.597	–10.968	–0.005	±0.468

Table 1 *Descriptive statistics of the differences among EGM96, GGM02C, EIGEN-GL04C and AUSGeoid98 quasigeoid heights (Units in metres; 1,781,101 points)*

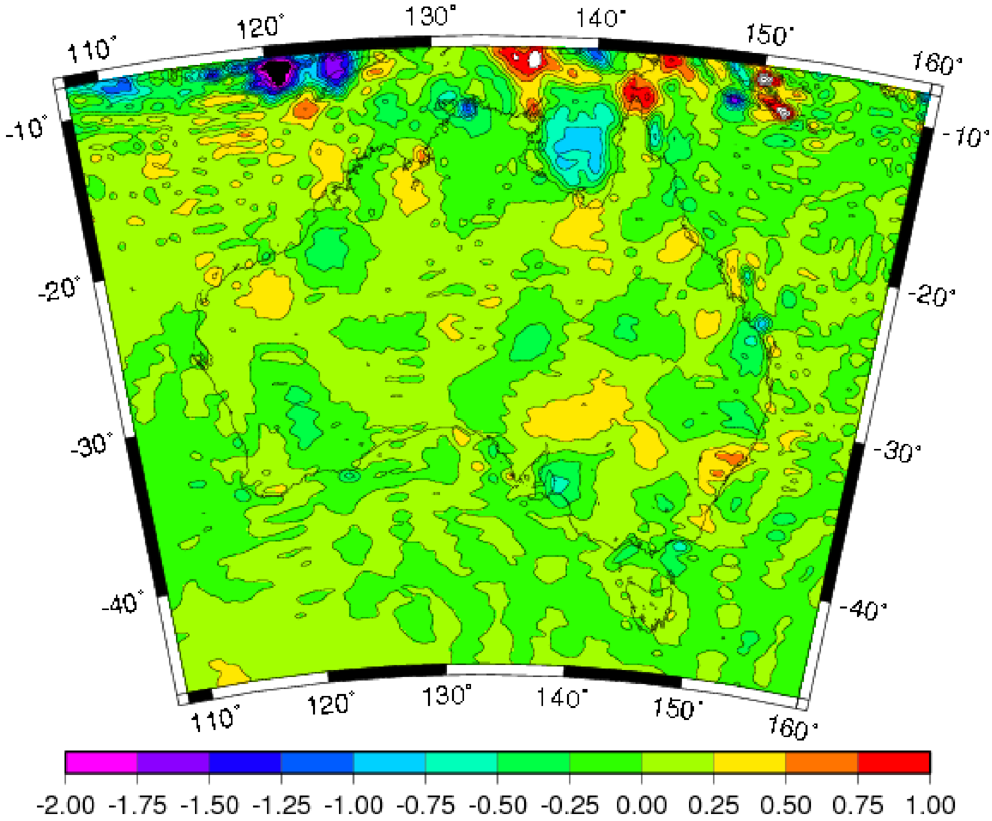

Figure 2 *Differences between EGM96 and EIGEN-GL04C quasigeoid heights to degree 360 (~55 km resolution) over the AUSGeoid98 area (Units in metres; Lambert projection). This is identical to the differences between AUSGeoid98 and AG98+GL04C.*

Comparisons with GPS-levelling

Following, e.g., Featherstone and Guo (2001) and Amos and Featherstone (2003), the AG98+GGM02C and AG98+GL04C augmented quasigeoid models were compared with GPS-levelling data on the AHD. Comparing a quasigeoid with GPS-levelling on the AHD is consistent because the AHD uses a normal-orthometric height system (Featherstone and Kuhn, 2006). This comparison was extended to include the AUSGeoid98, EGM96, GGM02C and EIGEN-GL04C quasigeoid models. The `geoid_abs_tester.f` and `geoid_rel_tester.f` FORTRAN77 software (Featherstone, 2001) was used in both the absolute (single-point) and relative (baseline-by-baseline) modes (Tables 2 and 3, respectively).

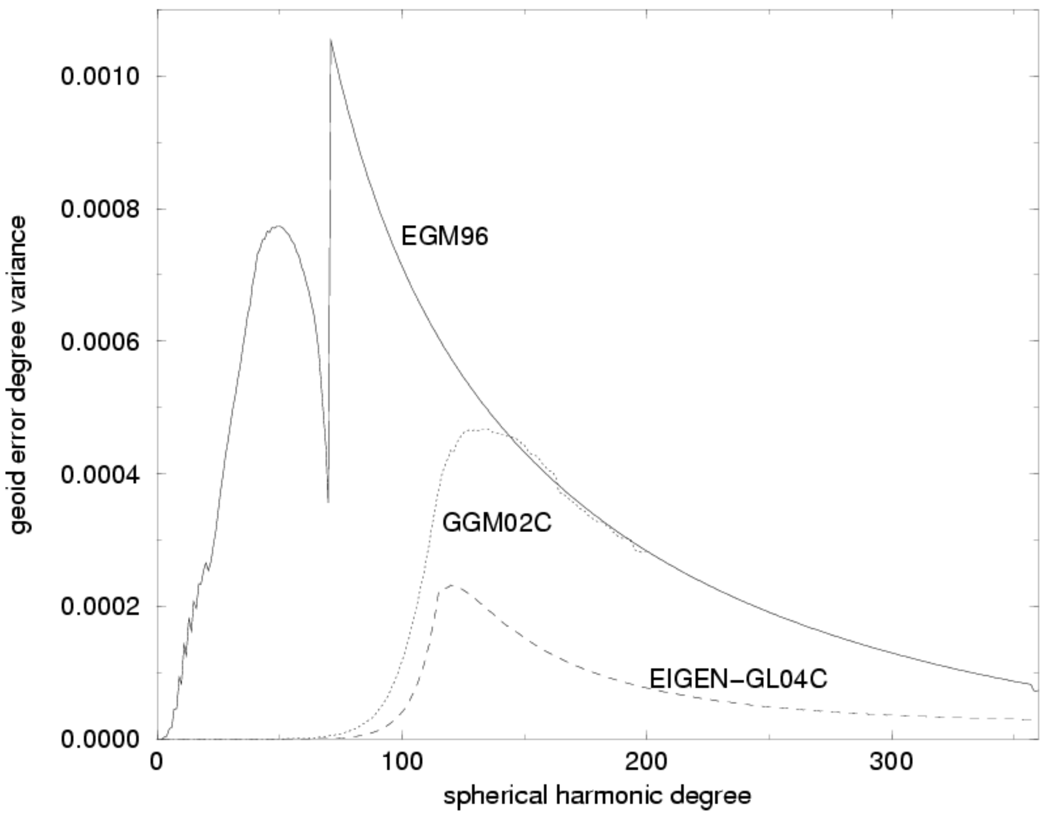

Figure 3 *Error degree variance (spectral power of the error) of quasigeoid heights for EGM96, GGM02C and EIGEN-GL04C, showing the reduced low-frequency errors in the GRACE-based GGMs (Units in metres-squared).*

This comparison uses the newer 254-point co-located GPS-AHD dataset provided by Geoscience Australia (cf. Featherstone and Sproule, 2006; Soltanpour *et al*., 2006; see Figure 4). It is acknowledged that this comparison is not conclusive because of distortions in the AHD (Featherstone, 2004; 2006; 2007; Featherstone and Kuhn, 2006), but the STDs in Table 2 give some *indication* that the GGM02C GRACE-based GGM can make some small (~1 cm) improvements to AUSGeoid98 for single-point GPS heighting (i.e., in the absolute sense). This will be improved much further, by ~15 cm or more in STD, when cross-validated least-squares collocation is used to fit the quasigeoid model to the AHD (cf. Featherstone and Sproule, 2006).

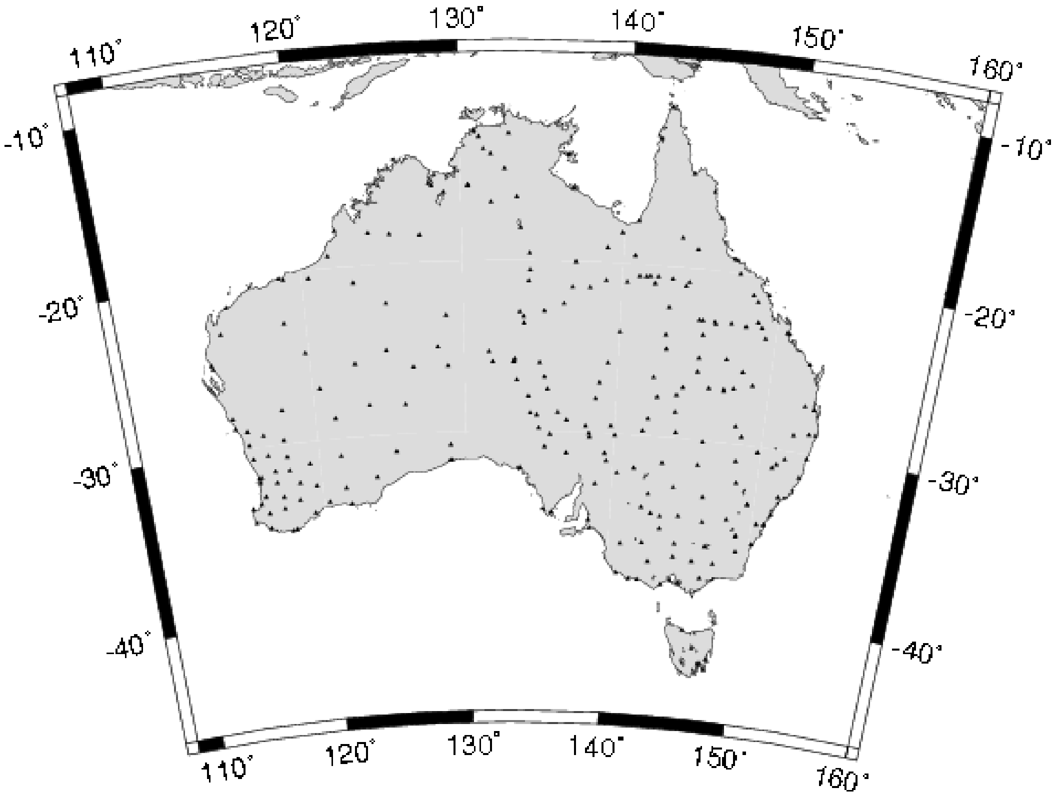

Figure 4 *Coverage of the 254 GPS-levelling points (Lambert projection).*

Quasigeoid	Degree	Max	Min	Mean	STD
EGM96	200	+1.093	–1.299	–0.026	±0.446
EGM96	360	+0.894	–0.961	+0.009	±0.334
GGM02C	200	+0.950	–1.318	+0.007	±0.415
EIGEN-GL04C	360	+0.789	–0.653	+0.059	±0.293
AUSGeoid98	N/A	+0.865	–0.721	+0.077	±0.284
AG98+GGM02C	N/A	+1.066	–0.671	+0.110	±0.268
AG98+GL04C	N/A	+1.079	–0.667	+0.127	±0.286

Table 2 *Descriptive statistics of the absolute differences between quasigeoid models and 254 co-located GPS-AHD points (Units in metres)*

Quasigeoid	Degree	Max	Min	Mean	STD	ppm
EGM96	200	+2.382	–2.391	+0.091	±0.625	2.45
EGM96	360	+1.855	–1.665	+0.076	±0.467	2.31
GGM02C	200	+2.213	–2.268	+0.090	±0.581	2.41
EIGEN-GL04C	360	+1.442	–1.390	+0.066	±0.410	2.27
AUSGeoid98	N/A	+1.409	–1.587	+0.008	±0.402	2.24
AG98+GGM02C	N/A	+1.737	–1.589	+0.007	±0.380	2.23
AG98+GL04C	N/A	+1.747	–1.578	–0.001	±0.406	2.25

Table 3 *Descriptive statistics of the relative differences between quasigeoid models over the 32,131 possible baselines between and 254 GPS-AHD points (Units in metres)*

Another observation from Tables 2 and 3 is that AUSGeoid98 provides a better fit to the AHD in absolute and relative senses than all the GGMs tested. This is expected because of the limited spatial resolution of the GGMs (omission error) versus the higher resolution of AUSGeoid98. The effect of the omission error can be seen when comparing the degree-200 and degree-360 expansions of EGM96, where the higher the degree of expansion gives a better fit to the GPS-levelling. However, the commission errors (errors in the coefficients) of GGM02C and EIGEN-GL04C are smaller than EGM96 (for the same degrees), showing the improvement coming from GRACE (cf. Gunter *et al*., 2006; also see Figure 3).

Comparisons with vertical deflections

An arguably better validation of a regional gravimetric quasigeoid model and GGM is through comparison with astrogeodetically observed vertical deflections (Jekeli, 1999; Featherstone, 2006; 2007; Featherstone and Morgan, 2007), which are more independent and better at sensing the high-frequency components of the gravity field. The validation presented here uses a dataset of 1080 vertical deflections (Figure 5), extending on that used in Featherstone (2006) by including additional points in Western Australia provided by Landgate (Featherstone and Morgan, 2007) and SGT Inc. (see Featherstone, 2007).

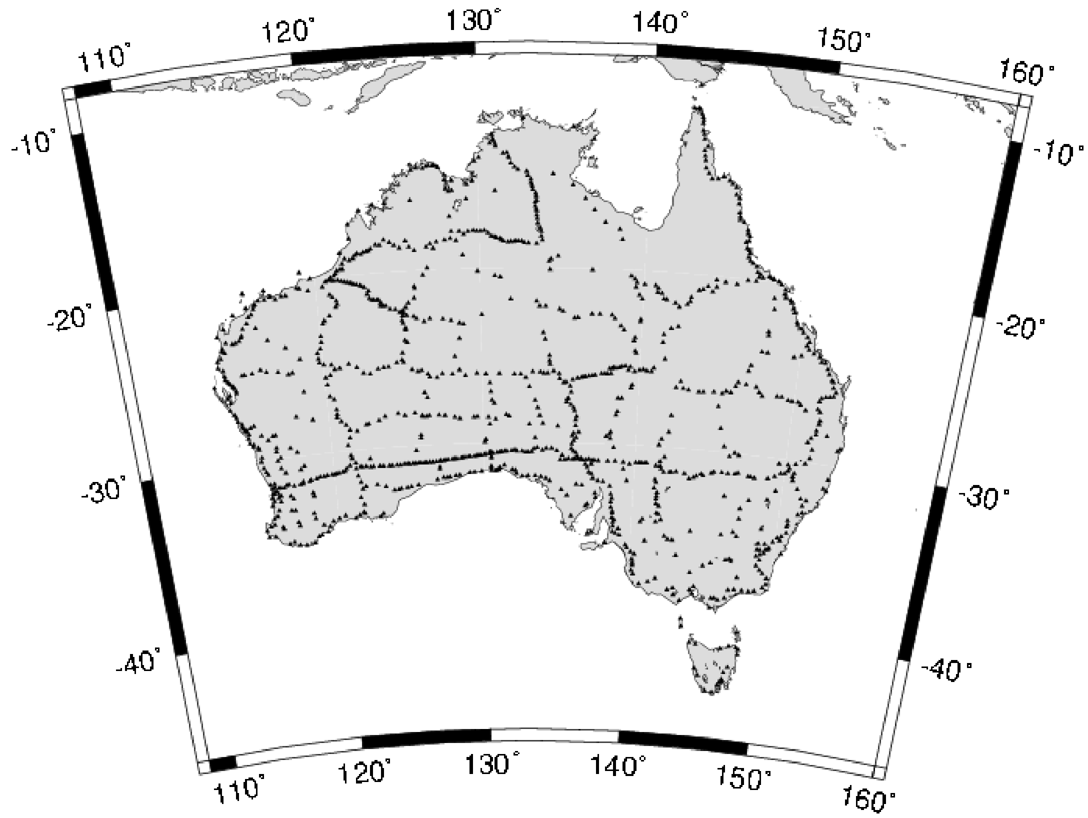

Figure 5. *Coverage of the 1080 astrogeodetically observed vertical deflections [Lambert projection]*

This evaluation required the development of some new software. Whilst `harmonics.f` can determine vertical deflections directly from the fully normalised spherical harmonic coefficients of any GGM, new code was needed to test the AG98+GGM02C and AG98+GL04C augmented models. This code determined the east-west and north-south vertical deflections from the horizontal quasigeoid gradients using the equations in Featherstone and Rüeger (2000); also see Featherstone and Morgan (2007).

The resulting grids of vertical deflections were then compared to the astrogeodetically observed vertical deflections using `geoid_abs_tester.f` (Featherstone, 2001). Tables 4 and 5 show the descriptive statistics of the differences between the astrogeodetically observed vertical deflections and the vertical deflections derived from the various quasigeoid models, after outlier rejection in the east-west and north-south components based on the three-sigma test, which is permitted because the differences are near-normally distributed (cf. Featherstone and Morgan, 2007).

In the outlier rejection, if one component was an outlier, then it was rejected for the other component. This proved to be rather robust in that most of the outliers were common among models and common among deflection components. It should be noted that a limitation of this analysis is that curvature and torsion of the plumbline have been ignored, as well as the quasigeoid not describing an equipotential surface (cf. Featherstone and Morgan, 2007). However, these effects are probably small in Australia, especially when taking into account the quality of the astrogeodetic deflections that were observed over 40 years ago (Featherstone and Rüeger, 2000; Featherstone, 2006; 2007; Featherstone and Morgan, 2007).

Quasigeoid	Degree	Max	Min	Mean	STD	Rejected outliers
EGM96	200	7.73	–8.15	–0.18	±1.79	15
EGM96	360	6.21	–7.31	–0.19	±1.54	19
GGM02C	200	6.99	–6.90	–0.18	±1.64	14
EIGEN-GL04C	360	6.15	–6.28	–0.18	±1.46	21
AUSGeoid98	N/A	3.28	–3.91	–0.25	±0.84	26
AG98+GGM02C	N/A	3.26	–3.87	–0.24	±0.82	28
AG98+GL04C	N/A	3.28	–4.02	–0.26	±0.83	25

Table 4 *Descriptive statistics of the differences between quasigeoid-derived east-west vertical deflections and astrogeodetic east-west vertical deflections (Units in arc-seconds)*

From the results in Tables 4 and 5, the augmented models (AG98+GGM02C and AG98+GL04C) make virtually no difference to the vertical deflections. This is to be expected because vertical deflections have most of their power in the high frequencies, so are relatively insensitive to the change in GGM. [Another factor is the quality of the astrogeodetic vertical deflections.] Instead, the deflections are mainly influenced by the local gravity and terrain data, so reflect the residual quasigeoid computed for AUSGeoid98. This is why the statistics in Tables 4 and 5 are virtually identical for all of AUSGeoid98, AG98+GGM02C and AG98+GL04C.

Quasigeoid	Degree	Max	Min	Mean	STD	Rejected outliers
EGM96	200	9.61	–10.69	–0.15	±2.71	15
EGM96	360	9.45	–9.33	–0.08	±2.19	19
GGM02C	200	9.45	–9.99	–0.12	±2.66	14
EIGEN-GL04C	360	9.76	–9.47	–0.10	±2.23	21
AUSGeoid98	N/A	3.76	–3.62	–0.14	±1.09	26
AG98+GGM02C	N/A	3.77	–3.59	–0.14	±1.08	28
AG98+GL04C	N/A	4.00	–3.72	–0.15	±1.12	25

Table 5 *Descriptive statistics of the absolute differences between quasigeoid-derived north-south vertical deflections and astrogeodetic north-south vertical deflections (Units in arc-seconds)*

CONCLUSION AND DISCUSSION

The experiments reported here on augmenting AUSGeoid98 with two recently released GRACE-based combined GGMs have proven somewhat inconclusive in that they do not provide any significant improvement over AUSGeoid98 alone when compared to GPS-levelling and vertical deflections.

From Figure 3, however, the GRACE-based models do make significant improvements on EGM96 in the low frequencies in a global sense, but this is not manifested in comparisons with Australian GPS-levelling and astrogeodetic vertical deflection data. A limitation is the distortions in the AHD and the vintage of the astrogeodetic vertical deflections. On the other hand, the spatial differences between AUSGeoid98 and the two augmented models (Figures 1 and 2) show ~25 cm differences in most places, which indicates that the spatial sampling of GPS-levelling and astrogeodetic vertical deflections (Figures 4 and 5) does not capture these changes properly.

Another limiting factor is that the GGM02C and EIGEN-GL04C combined GGMs use largely the same terrestrial gravity data as EGM96, which is also largely the same terrestrial gravity data that was used in AUSGeoid98. Therefore, they are highly correlated in the medium frequencies, so the results merely reflect the effect of the same data. If these data are in error, then the improvements from GRACE will be obscured. As such, it will be important to carefully look at the use of filters in the production of the next AUSGeoid (cf. Kern *et al.*, 2003; Featherstone 2003).

Finally, these experimental results should not be seen as a definitive statement that the inclusion of GRACE data will not make an improvement over AUSGeoid98. This is because the remove-replace augmentation procedure used here is theoretically inexact, and thus might not give a complete picture.

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