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Filtering for discrete-time nonhomogeneous Markov jump systems with uncertainties

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ABSTRACT

This paper studies the problem of robust H_∞ filtering for a class of uncertain discrete-time nonhomogeneous Markov jump systems. The time-varying jump transition probability matrix is described by a polytope. By Lyapunov function approach, mode-dependent and variation-dependent H_∞ filter is designed such that the resulting error dynamic system is stochastically stable and has a prescribed H_∞ performance index. A numerical example is given to illustrate the effectiveness of the developed techniques.

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1. Introduction

Owing to the pioneering work on Markov jump systems (MJSs) [9] in the mid 1960s, this kind of stochastic system has attracted much attention due to its wide range of applications in many practical dynamical systems, such as economic systems [3], solar thermal receiver systems [17] and communication systems [2]. MJSs can describe realistically many systems, with abrupt structures and parameters variations, which are caused by sudden environmental changes, switching of the system to different working points, unexpected disturbances to the system, failures and repairs of components or interconnections, etc. In the past decades, the stochastic stability issue on MJSs has been widely investigated, leading to the systematic formulations of many stochastic stability, and filtering problems on discrete time MJSs [1,6,7,10–13, 24–27,36]. They are under the assumption that the transition jump probabilities of the MJSs are time-invariant. Intensive research has been carried out for issues related to control [4,8,14,15,19,28,35] and fault detection [30]. Some results are also obtained for cases involving MJSs with completely known transition jump probabilities or partially known transition jump probabilities. See for example, [22,29,37] and the references cited therein.

However, this assumption is not realistic in many situations, and the transition probability of Markov jump system is a time-dependent and time-varying matrix. One typical example is networked systems, in which packet dropouts and network delays evolve in Markov chains or Markov processes, however, internet delay or packet dropouts are different in different period, this will bring in time-varying transition probabilities as the transition rates vary though the whole working region. Another example is the failures and repairs of subsystems on discrete-time Markov systems, which depends deeply on system age and working time, this leads to time-varying transition matrix. A real system is DC Motor system, it is known

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that the dynamics of DC Motors are reasonable to be described by second-order linear systems, and then, angular velocity of the motor shaft and the electrical current are two variables of the motor. In such case, abrupt changes on the power transmitted to the shaft will totally change the speed of motors, and this fact motivates us to model these systems by Markov jump chain, hence, the probabilities of the transition of these multiple circuits are not fixed. One feasible and reasonable assumption is to use a polytope set to describe this characteristics of uncertainties caused by time-varying transition probabilities. The main reason is that although the transition probability of the Markov process is not exactly known, but one can evaluate some values in some points, and assume that the time-varying transition probabilities evolve in this polytope, which belongs to a convex set. This time-varying transition probabilities matrix give rise to the nonhomogeneous Markov jump systems.

On the other hand, it is well known that filtering is an important research topic in control [31–34], and received much attention. In recent years, under the assumption that the transition probabilities are time invariant, many results on filtering and estimation have been reported for stochastic systems, such as Kalman filtering [16], robust filtering [18], and H_∞ filtering and adaptive estimation [5,20,23,38]. Since H_∞ filtering has an advantage in dealing with external unknown noises, it is more appropriate to discuss the H_∞ filtering problem for nonhomogeneous systems.

The rest of the paper is organized as follows: Section 2 contains problem statement and preliminaries results. In Section 3, H_∞ performance for the resulting error dynamic system is analyzed. A robust H_∞ filter is designed such that error dynamic system is stochastically stable and satisfies a prescribed H_∞ performance index in Section 4. A numerical example is given to illustrate the effectiveness of our approach in Section 5. Finally, some concluding remarks are given in Section 6.

In the sequel, the notation \mathbb{R}^n stands for an n -dimensional Euclidean space, the transpose of a matrix A is denoted by A^T ; $E\{\cdot\}$ denotes the mathematical statistical expectation; $L_2^n[0, \infty)$ stands for the space of n -dimensional square integrable functions over $[0, \infty)$; a positive-definite matrix is denoted by $P > 0$; I is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix.

2. Problem statement and preliminaries

Consider a probability space (M, F, P) , where M , F and P represent, respectively, the sample space, the algebra of events and the probability measure defined on F . The uncertain discrete-time Markov jump systems (MJSSs) considered in this paper are given below:

$$\begin{cases} x_{k+1} = A(r_k)x_k + B(r_k)w_k + g(x_k, r_k) \\ y_k = C(r_k)x_k + D(r_k)w_k \\ z_k = H(r_k)x_k + L(r_k)w_k \end{cases} \quad (2.1)$$

where $\{r_k, k \geq 0\}$ is the concerned discrete time Markov stochastic process, which takes values in a finite state set $\mathcal{A} = \{1, 2, 3, \dots, N\}$, and r_0 represents the initial mode; the transition probability matrix is defined as $\Pi(k) = \{\pi_{ij}(k)\}$, $i, j \in \mathcal{A}$ and $\pi_{ij}(k) = P(r_{k+1} = j | r_k = i)$ is the transition probability from mode i at time k to mode j at time $k + 1$, such that $\pi_{ij}(k) \geq 0$ and $\sum_{j=1}^N \pi_{ij}(k) = 1$. $A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$, $H(r_k)$ and $L(r_k)$ are mode-dependent constant matrices with appropriate dimensions at the working instant k ; $g(\cdot)$ is time-dependent and norm-bounded uncertainties; $x_k \in \mathbb{R}^n$ is the state vector of the system; $u_k \in \mathbb{R}^m$ is the input vector of the system; $y_k \in \mathbb{R}^p$ is the output vector of the system; $z_k \in \mathbb{R}^p$ is the controlled output vector of the system; and $w_k \in L_2^q[0, \infty]$ is the external disturbance vector of the system.

Assumption 2.1. The norm-bounded uncertainty $g(\cdot)$ in system (2.1) is assumed to satisfy

$$g(x_k, r_k) = \Delta A(r_k)x_k$$

and

$$\Delta A(r_k) = M(r_k) \cdot Y(r_k) \cdot N(r_k)$$

where $M(r_k)$ and $N(r_k)$ are constant matrices with appropriate dimensions, $Y(r_k)$ is an unknown matrix with Lebesgue measurable elements satisfying $Y^T(r_k)Y(r_k) \leq 1$.

System (2.1) can be written as:

$$\begin{cases} x_{k+1} = (A(r_k) + \Delta A(r_k))x_k + B(r_k)w_k \\ y_k = C(r_k)x_k + D(r_k)w_k \\ z_k = H(r_k)x_k + L(r_k)w_k \end{cases} \quad (2.2)$$

For simplicity, when $r_k = i$, $i \in \mathcal{A}$, the matrices $A(r_k)$, $\Delta A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$, $H(r_k)$ and $L(r_k)$ are, respectively, denoted as $A(i)$, $\Delta A(i)$, $B(i)$, $C(i)$, $D(i)$, $H(i)$ and $L(i)$.

To estimate the signal z_k in system (2.2), a general filter is constructed as follows:

$$\begin{cases} \hat{x}_{k+1} = A_f(i)\hat{x}_k + B_f(i)y_k \\ \hat{z}_k = C_f(i)\hat{x}_k + D_f(i)y_k \end{cases} \quad (2.3)$$

where \hat{x}_k is the filter state vector, y_k is the input of the filter, $A_f(i)$, $B_f(i)$, $C_f(i)$ and $D_f(i)$ are filter gains to be determined. Clearly, system (2.3) is mode-dependent. Augmenting system (2.2) to include the states of the filter, we obtain the following error dynamical system:

$$\begin{cases} \bar{x}_{k+1} = \bar{A}(i)\bar{x}_k + \bar{B}(i)w_k \\ \bar{z}_k = \bar{C}(i)\bar{x}_k + \bar{D}(i)w_k \end{cases} \quad (2.4)$$

where $\bar{z}_k = z_k - \hat{z}_k$, $\bar{x}_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix}$, $\bar{A}(i) = \begin{bmatrix} A(i) + \Delta A(i) & 0 \\ B_f(i)C(i) & A_f(i) \end{bmatrix}$, $\bar{B}(i) = \begin{bmatrix} B(i) \\ B_f(i)D(i) \end{bmatrix}$, $\bar{C}(i) = [H(i) - D_f(i)C(i) \quad -C_f(i)]$, $\bar{D}(i) = [L(i) - D_f(i)D(i)]$.

Note that if $\Pi(k)$ is a constant matrix, the Markov jump system follows a homogeneous Markov chain or Markov process, and on the other hand, if the transition probability matrix is time-varying, then the Markov chain or Markov process is a nonhomogeneous one. The Markov jump system we considered in this paper evolves as a nonhomogeneous Markov process. The variation of the transition probabilities is described as a polytope, and the form is given below:

$$\Pi(k) = \sum_{s=1}^w \alpha_s(k) \Pi^s$$

where $\Pi^s = \{\pi_{ij}^s\}$, $s = 1, \dots, w$, are given matrices representing the vertices of the polytope,

$$0 \leq \alpha_s(k) \leq 1 \text{ and } \sum_{s=1}^w \alpha_s(k) = 1.$$

To proceed further, some definitions and lemmas for system (2.4) are given below:

Definition 2.1. For any initial mode r_0 , and a given initial state \bar{x}_0 , system (2.4) (with $w_k = 0$) is said to be robustly stochastically stable if the following condition holds:

$$\lim_{m \rightarrow \infty} E \left\{ \sum_{k=0}^m \bar{x}_k^T \bar{x}_k | \bar{x}_0, r_0 \right\} < \infty \quad (2.5)$$

Lemma 2.1 [21]. Let Q , W , S and V be real matrices with appropriate dimensions. Suppose that S is chosen that $S^T S \leq I$. Then, for a positive scalar $\alpha > 0$, it holds that

$$Q + WSV + V^T S^T W^T \leq Q + \alpha^{-1} WW^T + \alpha V^T V$$

Definition 2.2. For a given constant $\gamma > 0$, system (2.4) is said to be stochastically stable and satisfies an H_∞ performance index γ , if it is robustly stochastically stable and the following condition is satisfied.

$$E \left\{ \sum_{k=0}^{\infty} \bar{z}_k^T \bar{z}_k \right\} \leq \gamma^2 E \left\{ \sum_{k=0}^{\infty} w_k^T w_k \right\} \quad (2.6)$$

We may now state formally the purpose of the paper as follows. Consider system (2.1) with time-varying jump transition probabilities. Design a mode-dependent and parameter-dependent filter (2.3), such that the resulting filtering error system (2.4) is stochastically stable and satisfies a prescribed H_∞ performance index.

3. H_∞ Error performance analysis

In order to minimize the influences of the disturbances, H_∞ performance index is analyzed for system (2.4) subject to all admissible disturbances. In this way, system (2.4) is stochastically stable and has a prescribed H_∞ index γ .

Theorem 3.1. Consider system (2.4) (with $w_k \neq 0$) and let $\gamma > 0$ be a given constant. Suppose that there exist a set of positive definite symmetric matrices $\bar{P}_s(i)$ and $\bar{P}_q(j)$ such that

$$\Theta_{sq}(i) = \begin{bmatrix} -\bar{P}_{sq}(i) & 0 & \bar{P}_{sq}(i)\bar{A}(i) & \bar{P}_{sq}(i)\bar{B}(i) \\ * & -I & \bar{C}(i) & \bar{D}(i) \\ * & * & -\bar{P}_s(i) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad \forall i \in \mathcal{A} \quad (3.1)$$

where

$$\bar{P}_{sq}(i) = \sum_{j=1}^N \sum_{s=1}^w \sum_{q=1}^w \alpha_s(k) \beta_q(k) \pi_{ij}^s \bar{P}_q(j)$$

$$\tilde{P}_s(i) = \sum_{s=1}^w \alpha_s(k) \bar{P}_s(i)$$

Then, system (2.4) is stochastically stable with $w_k = 0$, and satisfies a prescribed H_∞ performance index γ .

Proof. State equations of system (2.4) (with $w_k = 0$) can be written as:

$$\bar{x}_{k+1} = \bar{A}(i) \bar{x}_k \quad (3.2)$$

Construct a parameter-dependent and mode-dependent Lyapunov function given below:

$$V(\bar{x}_k, i) = \sum_{s=1}^w \alpha_s(k) \bar{x}_k^T \bar{P}_s(i) \bar{x}_k \quad (i \in \mathcal{A}) \quad (3.3)$$

where

$$0 \leq \alpha_s(k) \leq 1, \quad \sum_{s=1}^w \alpha_s(k) = 1, \quad \bar{P}_s(i) > 0$$

We obtain

$$\Delta V(\bar{x}_k, i) = E\{V(\bar{x}_{k+1}, i)\} - V(\bar{x}_k, i) = \bar{x}_k^T \left[\bar{A}^T(i) \sum_{j=1}^N \sum_{s=1}^w \sum_{q=1}^w \alpha_s(k) \alpha_s(k+1) \pi_{ij}^s \bar{P}_s(j) \bar{A}(i) \right] \bar{x}_k - \bar{x}_k^T \sum_{s=1}^w \alpha_s(k) \bar{P}_s(i) \bar{x}_k$$

Denote

$$\sum_{s=1}^w \alpha_s(k+1) \bar{P}_s(j) = \sum_{q=1}^w \beta_q(k) \bar{P}_q(j)$$

Then, we have

$$\Delta V(\bar{x}_k, i) = \bar{x}_k^T \bar{A}^T(i) \left(\sum_{j=1}^N \sum_{s=1}^w \sum_{q=1}^w \alpha_s(k) \beta_q(k) \pi_{ij}^s \bar{P}_q(j) \right) \bar{A}(i) \bar{x}_k - \bar{x}_k^T \sum_{s=1}^w \alpha_s(k) \bar{P}_s(i) \bar{x}_k = \bar{x}_k^T \Xi(i, k) \bar{x}_k$$

Denote

$$\Xi(i, k) = - \sum_{s=1}^w \alpha_s(k) \bar{P}_s(i) + \bar{A}^T(i) \left(\sum_{j=1}^N \sum_{s=1}^w \sum_{q=1}^w \alpha_s(k) \beta_q(k) \pi_{ij}^s \bar{P}_q(j) \right) \bar{A}(i) < 0 \quad \forall i \in \mathcal{A} \quad (3.4)$$

where

$$0 \leq \alpha_s(k) \leq 1, \quad \sum_{s=1}^w \alpha_s(k) = 1$$

$$0 \leq \beta_q(k) \leq 1, \quad \sum_{q=1}^w \beta_q(k) = 1$$

For system (3.2), condition (3.4) implies that

$$\Delta V(\bar{x}_k, i) < 0 \quad (i \in \mathcal{A})$$

Let

$$\eta = \min_k \{\lambda_{\min}(-\Xi(i, k))\} \quad \forall i \in \mathcal{A}$$

where $\lambda_{\min}(-\Xi(i, k))$ is the minimal eigenvalue of $-\Xi(i, k)$. Then,

$$\Delta V(\bar{x}_k, i) \leq -\eta \bar{x}_k^T \bar{x}_k$$

Thus,

$$E \left\{ \sum_{k=0}^T \Delta V(\bar{x}_k, i) \right\} = E \{ V(\bar{x}_{T+1}, i) \} - V(\bar{x}_0, i) \leq -\eta E \left\{ \sum_{k=0}^T \|\bar{x}_k\|^2 \right\}$$

This, in turn, implies that

$$E\left\{\sum_{k=0}^T \|\bar{x}_k\|^2\right\} \leq \frac{1}{\eta} \{V(\bar{x}_0, i) - E\{V(\bar{x}_{T+1}, i)\}\} \leq \frac{1}{\eta} V(\bar{x}_0, i)$$

$$\lim_{T \rightarrow \infty} E\left\{\sum_{k=0}^T \|\bar{x}_k\|^2\right\} \leq \frac{1}{\eta} V(\bar{x}_0, i)$$

By **Definition 2.1**, system (2.4) (with $w_k = 0$) is stochastically stable.

Then, consider the Lyapunov function (3.3) for system (2.4). We can show that

$$\begin{aligned} \Delta V(\bar{x}_k, i) &= E\{V(\bar{x}_{k+1}, i)\} - V(\bar{x}_k, i) = (\bar{A}(i)\bar{x}_k + \bar{B}(i)w_k)^T \tilde{P}_{sq}(i) (\bar{A}(i)\bar{x}_k + \bar{B}(i)w_k) - \bar{x}_k^T \tilde{P}_s(i) \bar{x}_k \\ &= \bar{x}_k^T [\bar{A}^T(i) \tilde{P}_{sq}(i) \bar{A}(i) - \tilde{P}_s(i)] \bar{x}_k + 2\bar{x}_k^T \bar{A}^T(i) \tilde{P}_{sq}(i) \bar{B}(i) w_k + w_k^T \bar{B}^T(i) \tilde{P}_{sq}(i) \bar{B}(i) w_k \end{aligned}$$

To establish the H_∞ performance for the system, the following cost function is introduced for system (2.4):

$$J(T) = E\left\{\sum_{k=0}^T \bar{z}_k^T \bar{z}_k\right\} - \gamma^2 E\left\{\sum_{k=0}^T w_k^T w_k\right\} \quad (3.5)$$

Under zero initial condition, $J(T)$ can be written as:

$$J(T) \leq E\left\{\sum_{k=0}^T [\bar{z}_k^T \bar{z}_k - \gamma^2 w_k^T w_k + \Delta V(\bar{x}_k, i)]\right\} \quad (3.6)$$

Thus, we have

$$\begin{aligned} J(T) &\leq E\left\{\sum_{k=0}^T [\bar{z}_k^T \bar{z}_k - \gamma^2 w_k^T w_k + \Delta V(\bar{x}_k, i)]\right\} = E\left\{\sum_{k=0}^T \{[\bar{C}(i)\bar{x}_k + \bar{D}(i)w_k]^T [\bar{C}(i)\bar{x}_k + \bar{D}(i)w_k] - \gamma^2 w_k^T w_k + \Delta V(\bar{x}_k, i)\}\right\} \\ &= E\left\{\sum_{k=0}^T \{[\bar{C}(i)\bar{x}_k + \bar{D}(i)w_k]^T [\bar{C}(i)\bar{x}_k + \bar{D}(i)w_k] - \gamma^2 w_k^T w_k\}\right\} \\ &\quad + E\left\{\sum_{k=0}^T [\bar{x}_k^T [\bar{A}^T(i) \tilde{P}_{sq}(i) \bar{A}(i) - \tilde{P}_s(i)] \bar{x}_k + 2\bar{x}_k^T \bar{A}^T(i) \tilde{P}_{sq}(i) \bar{B}(i) w_k]\right\} + E\left\{\sum_{k=0}^T w_k^T \bar{B}^T(i) \tilde{P}_{sq}(i) \bar{B}(i) w_k\right\} \end{aligned}$$

By Schur complement, it follows that

$$J(T) \leq \bar{x}_k^T \Theta_{sq}(i) \bar{x}_k$$

where

$$\bar{x}_k = \begin{bmatrix} \bar{x}_k^T & w_k^T \end{bmatrix}$$

Under the assumption that $w_k = 0$, $\Theta_{sq}(i) < 0$ implies inequality 3.4. Following a similar argument given above, we can show that system (2.4) is stochastically stable. On the other hand, as $T \rightarrow \infty$, $\Theta_{sq}(i) < 0$ results in $J(\infty) < -V(x_\infty, i) < 0$, that is

$$E\left\{\sum_{k=0}^{\infty} \bar{z}_k^T \bar{z}_k\right\} \leq \gamma^2 E\left\{\sum_{k=0}^{\infty} w_k^T w_k\right\} \quad (3.7)$$

By **Definition 2.2**, it follows that system (2.4) is stochastically stable and satisfies a prescribed H_∞ performance if 3.1 holds, which completes the proof. \square

Remark 3.1. If we select $[A_f(i) \ B_f(i) \ C_f(i) \ D_f(i)] = [A_f \ B_f \ C_f \ D_f]$ in system (2.4), one can obtain a mode-independent filter which is more conservative. Note that if we set $\sum_{s=1}^w \alpha_s(k) \bar{P}_s(i) = \bar{P}(i)$, then, the results we obtained can be applied to general homogeneous stochastic systems. Clearly, homogeneous Markov jump system is a special case of the system under consideration in this paper.

4. Robust H_∞ filter design

Sufficient conditions for the existence of an admissible mode-dependent H_∞ filter in the form of (2.3) for system (2.1) are presented in the following theorems.

Theorem 4.1. Consider system (2.4) with time-varying jump transition probabilities, and let $\gamma > 0$ be a given constant. Suppose that there exists a set of positive definite symmetric matrices $\bar{P}_s(i)$, $\bar{P}_q(j)$ and mode-dependent matrices $X(i)$ such that

$$\Omega_{sq}(i) = \begin{bmatrix} -X(i) - X^T(i) + \hat{P}_{sq}(i) & 0 & X(i)\bar{A}(i) & X(i)\bar{B}(i) \\ * & -I & \bar{C}(i) & \bar{D}(i) \\ * & * & -\bar{P}_s(i) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad \forall i \in \mathcal{A} \quad (4.1)$$

where

$$\hat{P}_{sq}(i) = \sum_{j=1}^N \pi_{ij}^s \bar{P}_q(j), \quad s, q = 1, \dots, w$$

Then, system (2.4) is stochastically stable and satisfies a prescribed H_∞ performance index γ .

Proof. Note that for system (2.4) to be stochastically stable and has a prescribed H_∞ performance index, it is required that all the vertices of the polytope are to satisfy the stability requirements as shown in Theorem 3.1.

Now, by Theorem 3.1, we have

$$\Omega_{1sq}(i) = \begin{bmatrix} -\check{P}_{sq}(i) & 0 & \check{P}_{sq}(i)\bar{A}(i) & \check{P}_{sq}(i)\bar{B}(i) \\ * & -I & \bar{C}(i) & \bar{D}(i) \\ * & * & -\bar{P}_s(i) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad \forall i \in \mathcal{A} \quad (4.2)$$

where

$$\check{P}_{sq}(i) = \sum_{j=1}^N \sum_{q=1}^w \beta_q(k) \pi_{ij}^s \bar{P}_q(j)$$

which, in turn, implies that

$$\Omega_{2sq}(i) = \begin{bmatrix} -\hat{P}_{sq}(i) & 0 & \hat{P}_{sq}(i)\bar{A}(i) & \hat{P}_{sq}(i)\bar{B}(i) \\ * & -I & \bar{C}(i) & \bar{D}(i) \\ * & * & -\bar{P}_s(i) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad \forall i \in \mathcal{A} \quad (4.3)$$

In order to avoid the cross-coupling of matrix product terms in condition (4.3), a slack matrix $X(i)$ is considered here. Then, after standard matrix manipulation, condition (4.1) is obtained. Therefore, system (2.4) is stochastically stable and has a prescribed H_∞ performance index. This concludes the proof of Theorem 4.1. Next, by Theorem 4.1, we will design the robust H_∞ filter for system (2.1), so as to ensure that the resulting error dynamic system (2.4) is stochastically stable and has a prescribed H_∞ performance index. \square

Theorem 4.2. Consider system (2.4) with time-varying jump transition probabilities, and let $\gamma > 0$ be a given constant. Suppose that there exist matrices $P_{1s}(i) > 0$, $P_{3s}(i) > 0$ and mode-dependent matrices $P_{2s}(i)$, $R(i)$, $Y(i)$, $Z(i)$, $A_F(i)$, $B_F(i)$, $C_F(i)$, $D_F(i)$ and $\alpha(i) > 0$ such that the following condition has a feasible solution

$$\Gamma_{sq}(i) = \begin{bmatrix} a_1 & a_2 & 0 & R(i)A(i) + B_F(i)C(i) & A_F(i) & R(i)B(i) + B_F(i)D(i) & R(i)M(i) \\ * & a_3 & 0 & Z(i)A(i) + B_F(i)C(i) & A_F(i) & Z(i)B(i) + B_F(i)D(i) & Z(i)M(i) \\ * & * & -I & H(i) - D_F(i)C(i) & -C_F(i) & L(i) - D_F(i)D(i) & 0 \\ * & * & * & -P_{1s}(i) + \alpha(i)N^T(i)N(i) & -P_{2s}(i) & 0 & 0 \\ * & * & * & * & -P_{3s}(i) & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -\alpha(i) \end{bmatrix} < 0 \quad (4.4)$$

where $i \in \mathcal{A}$, $s, q = 1, \dots, w$, $a_1 = -R(i) - R^T(i) + P_{1q}(j)$, $a_2 = -Y(i) - Z^T(i) + P_{2q}(j)$, $a_3 = -Y(i) - Y^T(i) + P_{3q}(j)$. Then, a mode-dependent filter (2.3) with the gain matrices shown below is obtained, such that the resulting filtering error system (2.4) is stochastically stable and satisfies a prescribed H_∞ performance index γ , and the gain matrices of the filter are given by $A_F(i) = A_F(i)Y^{-1}(i)$, $B_F(i) = B_F(i)Y^{-1}(i)$, $C_F(i) = C_F(i)$, $D_F(i) = D_F(i)$.

Proof. Consider the filtering error system (2.4) and denote

$$\bar{P}_s(i) = \begin{bmatrix} P_{1s}(i) & P_{2s}(i) \\ * & P_{3s}(i) \end{bmatrix} > 0, \quad X(i) = \begin{bmatrix} R(i) & Y(i) \\ Z(i) & Y(i) \end{bmatrix}$$

Then, by Theorem 4.1, $\Omega_{sq}(i) < 0$ implies

$$\Gamma_{1sq}(i) = \begin{bmatrix} a_1 & a_2 & 0 & a_4 & Y(i)A_f(i) & R(i)B(i) + Y(i)B_f(i)D(i) \\ * & a_3 & 0 & a_5 & Y(i)A_f(i) & Z(i)B(i) + Y(i)B_f(i)D(i) \\ * & * & -I & H(i) - D_f(i)C(i) & -C_f(i) & L(i) - D_f(i)D(i) \\ * & * & * & -P_{1s}(i) & -P_{2s}(i) & 0 \\ * & * & * & * & -P_{3s}(i) & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (4.5)$$

where

$$a_4 = R(i)(A(i) + \Delta A(i)) + Y(i)B_f(i)C(i)$$

$$a_5 = Z(i)(A(i) + \Delta A(i)) + Y(i)B_f(i)C(i)$$

Clearly, $\Gamma_{1sq}(i) < 0$ gives rise to

$$\Gamma_{2sq}(i) + T_1(i)Y(i)T_2(i) + T_2^T(i)Y^T(i)T_1^T(i) < 0$$

where

$$\Gamma_{2sq}(i) = \begin{bmatrix} a_1 & a_2 & 0 & a_6 & Y(i)A_f(i) & R(i)B(i) + Y(i)B_f(i)D(i) \\ * & a_3 & 0 & a_7 & Y(i)A_f(i) & Z(i)B(i) + Y(i)B_f(i)D(i) \\ * & * & -I & H(i) - D_f(i)C(i) & -C_f(i) & L(i) - D_f(i)D(i) \\ * & * & * & -P_{1s}(i) & -P_{2s}(i) & 0 \\ * & * & * & * & -P_{3s}(i) & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (4.6)$$

$$a_6 = R(i)A(i) + Y(i)B_f(i)C(i)$$

$$a_7 = Z(i)A(i) + Y(i)B_f(i)C(i)$$

$$T_1^T(i) = [M^T(i)X^T(i) \quad M^T(i)Z^T(i) \quad 0 \quad 0 \quad 0 \quad 0]$$

$$T_2^T(i) = \begin{bmatrix} 0 & 0 \\ 0 & N(i) \quad 0 \quad 0 \end{bmatrix}$$

Denote

$$A_f(i) = A_f(i)Y(i), \quad B_f(i) = B_f(i)Y(i), \quad C_f(i) = C_f(i), \quad D_f(i) = D_f(i)$$

Then, by Lemma 2.1 and Schur complement, $\Gamma_{2sq}(i) < 0$ holds if $\Gamma_{sq}(i) < 0$.

Therefore, if (4.4) holds, the filtering error system (2.4) is stochastically stable and satisfies a prescribed H_∞ performance index γ . Moreover, the parameters of the filter are given by $A_f(i) = A_f(i)Y^{-1}(i)$, $B_f(i) = B_f(i)Y^{-1}(i)$, $C_f(i) = C_f(i)$, $D_f(i) = D_f(i)$. This completes the proof. \square

Remark 4.1. Note that in order to get the optimal H_∞ performance index γ for system (2.4), we set $\gamma^2 = \varepsilon$. Then, Theorem 4.2 can be cast as an optimization problem as follows:

$$\min \quad \varepsilon$$

$$\text{s.t. LMI (4.7)}$$

$$\Gamma_{3sq}(i) = \begin{bmatrix} a_1 & a_2 & 0 & R(i)A(i) + B_f(i)C(i) & A_f(i) & R(i)B(i) + B_f(i)D(i) & R(i)M(i) \\ * & a_3 & 0 & Z(i)A(i) + B_f(i)C(i) & A_f(i) & Z(i)B(i) + B_f(i)D(i) & Z(i)M(i) \\ * & * & -I & H(i) - D_f(i)C(i) & -C_f(i) & L(i) - D_f(i)D(i) & 0 \\ * & * & * & -P_{1s}(i) + \alpha(i)N^T(i)N(i) & -P_{2s}(i) & 0 & 0 \\ * & * & * & * & -P_{3s}(i) & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -\alpha(i) \end{bmatrix} < 0 \quad (4.7)$$

Remark 4.2. By solving (4.7), one obtain the filter corresponding to the optimal H_∞ performance index. It is worth mentioning that time-invariant jump probability matrix is a special case of time-varying ones, as the transition probabilities in the filter are time-variant, so the filter (2.3) is not only mode-dependent but also variant-dependent. A convex Lyapunov function is addressed in this paper which bring in less conservatism.

5. Simulation results

Consider nonhomogeneous discrete-time MJSSs, which are aggregated into 2 modes, where

$$A(1) = \begin{bmatrix} 0 & -0.45 \\ 0.9 & 0.9 \end{bmatrix}, \quad A(2) = \begin{bmatrix} 0 & -0.29 \\ 0.9 & 1.26 \end{bmatrix}$$

$$B(1) = \begin{bmatrix} 0.5 \\ 1.1 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}$$

$$C(1) = [0.5 \quad 0.4], \quad C(2) = [0.3 \quad 0.1]$$

$$D(1) = [0.9], \quad D(2) = [-0.6]$$

$$H(1) = [0.8 \quad -0.2], \quad H(2) = [0.1 \quad 0.5]$$

$$L(1) = 1.88, \quad L(2) = 1.98$$

$$M(1) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad M(2) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$N(1) = [0.1 \quad 0.1], \quad N(2) = [0.1 \quad 0.1]$$

The vertices of the time-varying transition probability matrix are given by

$$\Pi^1 = \begin{bmatrix} 0.2 & 0.8 \\ 0.35 & 0.65 \end{bmatrix}, \quad \Pi^2 = \begin{bmatrix} 0.55 & 0.45 \\ 0.48 & 0.52 \end{bmatrix}$$

$$\Pi^3 = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}, \quad \Pi^4 = \begin{bmatrix} 0.4 & 0.6 \\ 0.9 & 0.1 \end{bmatrix}$$

Our purpose is to design an H_∞ filter for system (2.1) such that the resulting filtering error system (2.4) is stochastically stable with an H_∞ noise attenuation performance index.

Apply the obtained parameters to filter (2.3), set initial condition as $x_0 = [-0.5 \quad 0.4]^T$, $\gamma = 0.8$, initial condition for the filter as $[0 \quad 0]^T$ and the noise signal as $w_k = 0.5 \exp(-0.1k) \sin(0.01\pi k)$. Then, we obtain the state trajectories of system (2.1), jumping modes and filtering error response of the resulting filtering error system (2.4) as shown in Figs. 1–3. It is

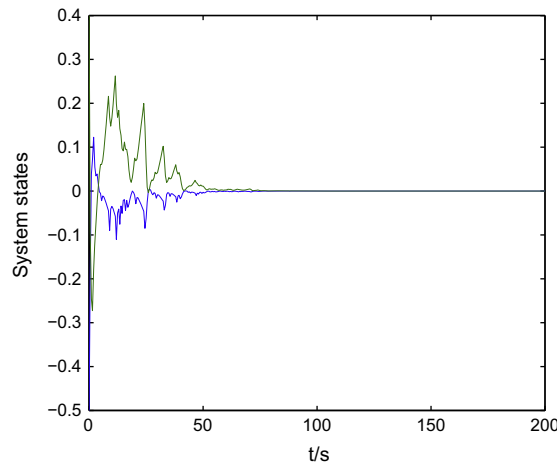


Fig. 1. Trajectories of system states.

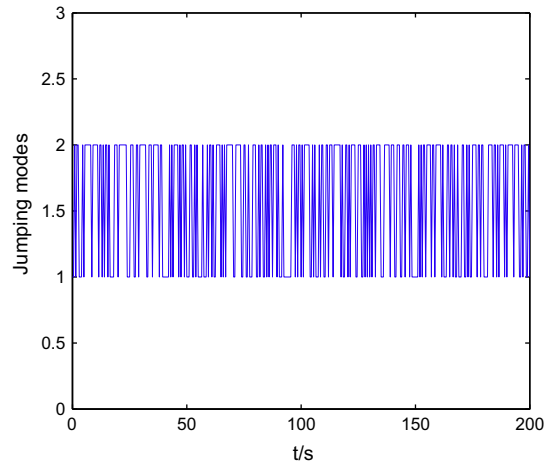


Fig. 2. Jumping modes.

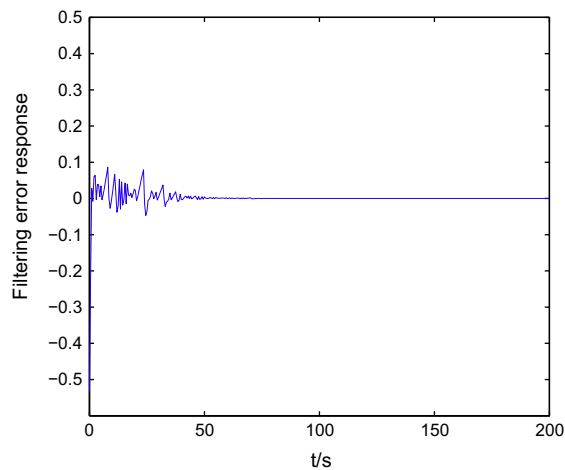


Fig. 3. Filtering error response.

clearly observed from the simulation result that under the designed filter, system (2.1) is stochastically stable and has a prescribed H_∞ performance index.

Remark 5.1. By solving LMI (4.7), one can obtain the optimal value of the H_∞ performance index, the mode-independent H_∞ performance index γ is also given in the following form, and it is obvious that mode-independent filter bring in some conservativeness.

Case	Mode-dependent	Mode-independent
γ_{min}	0.68	1.54

Fig. 1 shows one case of the jumping modes expressed by time-varying transition probabilities.

6. Conclusions

In this paper, the issue on robust H_∞ filtering for a class of uncertain discrete-time nonhomogeneous Markov jump systems is addressed, and the transition probabilities is expressed as a polytope, in which vertices are given a priori, and the filter designed ensures that the resulting error dynamic system is stochastically stable and satisfies a prescribed H_∞ performance index. The simulation result shows the potential of the proposed techniques.

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