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Real-Time Determination of Orthometric Heights Accurate to the Centimeter level Using a Single GPS Receiver: Case Study

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Abstract: Real-time determination of orthometric heights at the cm-level accuracy can be achieved in Dubai, UAE, using a single geodetic-grade GPS receiver. This process requires that the rover receiver uses on-line measurement corrections from multiple reference stations (RTK networks), and geoid heights from a precise gravimetric geoid model. The Dubai RTK network consists of five active reference stations and can provide ellipsoidal heights with external accuracy less than 4 cm and a precision of 3 cm. The Dubai geoid model was recently developed integrating a comprehensive set of gravity, GPS, leveling and digital elevation data to fit GPS/leveling at the 3-5 cm level RMS.

To evaluate this technique, a field test on 41 benchmarks of the Dubai second order leveling network was performed. In this test, orthometric heights derived from the presented method using a single GPS receiver were compared to their precise values determined by spirit leveling. The height differences were analyzed and statistically examined. Results show that orthometric heights determined from the GPS RTK Network and geoid data can be accurate to 2-5 cm with no significant systematic errors. The method can thus be considered a good alternative to traditional leveling, particularly for third order leveling in large areas.

CE Database Keywords: Geoid, Global positioning, Leveling, Topographic surveys, Real-time programming.

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Introduction

Both GPS and traditional leveling have their advantages and disadvantages in height determination with regard to accuracy, cost efficiency, and terrain independence. Traditional leveling is more cost efficient and provides greater accuracy than GPS at the sub 2 cm level in small distance projects (NGS, 1998). In contrast, GPS is more cost efficient in large distance projects and is independent of the terrain surveyed. However, unlike traditional leveling, GPS derived heights are referenced to an ellipsoidal datum (WGS 84), and do not depend on local gravity variations, whereas in most leveling work and mapping, orthometric heights are needed (approximated to leveling heights). These heights reflect changes in topography as well as local variations in gravity. They are referenced to the geoid, which is an equi-potential level surface of the Earth that is closely associated with the mean sea level on a global basis. Thus, the geoid heights are needed to convert ellipsoidal heights from GPS (h_{GPS}) into orthometric heights (H), using the equation:

$$H = h_{GPS} - N \tag{1}$$

where (N) is the geoid height. Most comprehensive GPS software packages have general geopotential models (e.g. EGM96), which can only provide a spatial resolution of the geoid of approximately 0.5 degree $\cong$ 55km. This resolution is usually insufficient for localized GPS surveys. Thus, an accurate geoid is needed, which is usually computed from varying data sources, including gravity data, digital elevation model (DEM), orthometric heights and GPS-observations at leveling benchmarks. The accuracy of the computed geoid varies according to the accuracy, density and type of data involved.

A precise geoid was recently computed in the Emirate of Dubai, UAE. The Emirate consists of a main costal area of approximately 80km x 80km extent, and the 15km x 20km mountainous Hatta enclave. Also, a real-time GPS reference network covering the whole Emirate of Dubai was recently established, such that surveyors can obtain cm-level positioning accuracy with a single receiver without worrying about the limitations of establishing heir own reference stations. The combination of both an accurate geoid and the real-time GPS reference network leads to determining orthometric heights accurate to the cm-level using only a single geodetic-grade GPS receiver. Ignoring the impact of gravity changes between surveyed points in the computation of height differences, the derived heights are considered precise enough to meet at least third order leveling standards.

Development of The Precise Geoid Model for Dubai

Data Used in Geoid Determination

The Dubai Geoid model was developed by integrating a comprehensive set of gravity measurements with GPS, leveling and digital elevation data. Gravity data used in the determination of the Dubai precise geoid consisted of gravity measurements collected on a 1-km square grid covering the entire Dubai Emirate, referenced to three absolute gravity stations. Other available gravity data were also included from marine gravity surveys in the Arabian “Persian” Gulf (provided by BGI, Toulouse) and KMS-01 gravity anomalies derived from satellite altimetry. The heights of the gravity points were measured using a fast static GPS survey. All gravity data were checked for outliers, and the marine gravity data were compared to the satellite altimetry to check for possible datum errors. As part of the gravity processing, gravity values and ellipsoidal heights of the gravity points were converted into conventional free-air and Bouguer gravity anomalies, using the EGM96 geoid model. The Bouguer anomalies were smooth in mainland Dubai, but a very large gradient went through Hatta. One should however note that with the new geoid model, the gravity anomalies could change a fraction of a mgal, but with the GPS leveling fit applied, this will have no practical consequence for the geoid (Forsberg et al. 2001).

For the modeling of terrain effects, the digital elevation models consisted of a number of detailed heights, averaged into a grid of 100 m x 100 m cells, and a basic Digital Elevation Model (DEM) of 30” x 30” from National Geophysical Data Center (NGDC), USA, covering the complete region. In addition, leveling heights of a set of 3750 benchmarks were used, as well as the GPS ellipsoidal heights at these benchmarks. Many of these were determined from repeated RTK surveys, with approximately 1-5 cm positioning accuracy. The GPS data were tied to the base network of Dubai determined in the International Terrestrial Reference Frame (ITRF). Some points, particularly in the built-up areas, were actually determined using terrestrial leveling techniques and tied to nearby GPS points. The leveling elevations were then tied to the local vertical datum by referencing them to a fundamental tide gauge station at Port Rashid, Dubai. Furthermore, in connection with the gravity observations, a leveling line was observed around the perimeter of the Dubai main area and used for constraining the final geoid after an iterative editing of outliers, where the perimeter gravity station heights were determined by GPS.

Geoid Computation

The Dubai precise geoid was computed in cooperation with the National Survey and Cadastre (KMS) and The University of Copenhagen, Denmark. The computations were carried out in two steps:

- A gravimetric geoid model, computed by spherical Fast Fourier Transform (FFT) in a global datum.
- A GPS-tailored local geoid, which fits the GPS observations and the Dubai vertical datum to a few centimeters. This step involved an iterative editing of GPS-leveling outliers, see Forsberg et al. (2001).

The advantage of the two-step method is that the second step, being less complex, can be readily repeated as future GPS-leveling observations detect errors in the original data, and hence the GPS tailored geoid. The method of least-squares collocation has also been used for an alternative preliminary geoid computation, primarily to assess the overall accuracy of the geoid.

The geoid signal (N) is constructed from three parts, such that:

$$N = N_1 + N_2 + N_3 \qquad (2)$$

where the first part (N1) comes from a spherical harmonic expansion complete to degree and order 360 (EGM96), see Lemoine et al. (1996). The second part (N2) comes from the topography, and the third part (N3) from the contributions of "residual" gravity (i.e. gravity anomalies minus the global field contribution and gravimetric terrain effects). The high resolution geoid height model has been invariably computed by a remove-compute-restore technique, see for instance Fotopoulos et al. (1999 & 2003), and Featherstone et al. (2001).

The computed gravimetric geoid was also fitted to local GPS-leveling data for GPS height use in the computation of the orthometric heights, and to eliminate datum shift, residual long-wavelength gravity errors, as well as possible systematic errors in the leveling. The basic principle was to model by least-squares collocation the gravimetric and GPS geoid differences using a smooth function consisting of a trend function **f** (e.g. a polynomial) and a residual (ε’), such that:

$$\varepsilon = N_{grav} - N_{GPS} = f(\phi, \lambda) + \varepsilon' \qquad (3)$$

where (ϕ, λ) are the latitude and longitude. For the final geoid computation, the residual (ε’) was modeled by least-squares collocation, using a 2nd order Markov covariance function, taking:

$$C(s) = C_o (1+ks)e^{-ks} \qquad (4)$$

where (k) is a constant, determined by correlation length, and (s) denotes the distance. The factor (C_o) was determined from the data, whereas (k) and the apriori noise on the GPS leveling may be determined by the user. Thus, the final geoid on average fits the GPS data, but is not affected by individual random errors. More details on the computation of the Dubai geoid are given in Forsberg et al. (2001).

A collocation error estimate of 2 cm RMS was reached, and was considered realistic for the error of the geoid in the Dubai main area. In Hatta, the geoid was difficult to compute due to the lack of gravity data in the neighboring regions (Oman and other UAE Emirates), and due to the apparent larger noise in the GPS leveling. Thus, the geoid might only be accurate to 5-10 cm within this region. **Figure 1** shows the final computed geoid of Dubai. The computed geoid fits GPS/leveling at the 3-4 cm level RMS (Forsberg et al. 2001). This error is, to a large degree, due to uncertainty in GPS heights. The geoidal heights at the desired locations are computed through an interpolation and transformation program, implementing Dubai national coordinate systems (UTM and Dubai Local Transverse Mercator “DLTM”).

The refinement of the Dubai geoid requires building-up of an improved regional gravimetric and highly reliable GPS/leveling database in a uniform reference system, particularly with other Emirates and neighboring countries, e.g. Oman. The European activities in this regard can be taken into consideration, see for instance Kenyeres (2000).

Determination of Ellipsoidal Heights Using a Single GPS Receivers in Dubai

To obtain real-time ellipsoidal heights from GPS at the cm-level accuracy, a versatile approach can be adopted employing a single geodetic-grade GPS receiver using on-line data from a real-time reference network. In this approach, observations from multiple reference stations are gathered and processed in a common network adjustment at a central processing facility. Measurement corrections are computed and sent to the rover online to correct its observations, eliminating in the classical GPS surveying the constraints of using a dedicated reference station, and the need for a short baseline length between the reference receiver and the rover. Additional advantages of this method are: cost-reduction, minimization of number of staff and obtaining consistent coordinates.

In March 2002, the Dubai Municipality started operating such a network system for GPS real-time kinematic (RTK) positioning, whereby users can receive the corrections throughout the Emirate of Dubai. The network, known as Dubai Virtual Reference System (DVRS), consists of five continuously operating reference stations and a control room with a central server. The network baseline lengths range from 23.4 km to 90.8 km. **Figure 2** shows the distribution of the DVRS stations and the network baseline lengths. The five DVRS stations send their GPS measurements online to the control room server by dedicated telephone lines. Three PCs organize, process and archive the data. Communication with the rover is carried out in a duplex mode, whereby the rover sends its approximate position, calculated in a single positioning mode, to the control station via a mobile call in a National Marine Electronics Association (NMEA) format. Then, the corrections are calculated at the user's location and are sent in Radio Technical Commission for Maritime services (RTCM) format, and used to correct the data to reach cm-level positioning. The coordinates of the reference stations are related to the ITRF system and the computed rover position can be represented either in this system or in the local one.

The DVRS network uses the GNSMART software (Wübbena et al. 2001a,b) for data processing, which employs the area correction parameters technique (FKP) to analyze the data and to estimate and represent the state of individual GPS errors in real time. All stations of the network are processed simultaneously using un-differenced observables. Therefore, all error components are estimated including the clock errors. A virtual reference station approach is also employed, which interpolates the state information computed from the FKP to a VRS position near the user using the rover approximate coordinates. The measurement corrections for a user is composed of range corrections and network corrections.

Investigating the Accuracy of the Involved Height Parameters

To assess accuracy of orthometric height determination using a single GPS receiver with the DVRS real-time reference network and the Dubai developed geoid model, each of the involved height parameters, namely the GPS ellipsoidal height and the interpolated geoid height, has to be separately examined and then jointly evaluated. This helps in showing their performance as well as understanding the contribution of each in the total error budget.

Testing Performance of GPS Positioning Using the DVRS Network

A comprehensive testing of height and planimetric coordinates determination from GPS using the DVRS system was carried out in March, 2003. The point coordinates of a set of points, which were previously precisely determined, were compared with their coordinates determined by the DVRS reference network. Results of this comparison showed that the external and relative positioning accuracies obtained were less than 4 cm and 2 cm respectively (El-Mowafy et al. 2003). In addition, different DVRS surveys at different sessions gave a coordinate determination precision for the same test points of less than 2 cm for planimetric coordinates, and less than 3 cm for height determination. These surveys also prove to be consistent in terms of the internal precision of their final output results (i.e. statistically compatible). Also, in the case of failure of one of the reference stations, the system proves to be reliable and robust. Cm-level positioning accuracy was reached for points that were in the area of coverage of the failed reference stations but were more than 25 km away from the nearest working reference station.

Testing the Developed Geoid Model

An external evaluation of the quality of the gravimetric geoid model of Dubai was performed by comparing its interpolated values (N) on a network of benchmarks with the corresponding GPS/leveling-derived geoid heights (N_{GPS}) computed at these benchmarks. Thirty five benchmarks of the Dubai second order leveling network, scattered throughout the Emirate of Dubai within a coverage area of 73.5 km x 84 km were used for this purpose. These benchmarks were established for the purpose of testing the Dubai geoid model, and thus were initially not included in the computation of the model to ensure independence of testing results. The height difference between the highest and lowest points in the test area was approximately 263.2 meters. The orthometric heights were determined by an accurate spirit leveling, while GPS ellipsoidal heights were determined using a fast-static approach. Ellipsoidal heights were obtained in post mission by post processing using a differential carrier-phase fixed solution. Their standard deviations were less than 2 cm.

Figure 3 shows the differences between orthometric heights computed from spirit leveling and those derived from the geoid model + the GPS ellipsoidal heights. The height discrepancies were in the range ± 4 cm. The statistics of these discrepancies are given in **Table 1**, where the mean value was 0.22 cm with random distribution of the differences, and no systematic errors were observed. The average value of the absolute differences was 1.92 cm with 2.33 cm standard deviation. At one benchmark the height difference reached 8.01 cm. However, this single value was considered as an outlier as it was more than 3σ. This large error was attributed to an error in ellipsoidal height determination by GPS. It is worth mentioning in this context that a comprehensive evaluation of the gravimetric geoid of Dubai is currently the subject of another study.

Although results show that there are no systematic errors either in ellipsoidal height determination by GPS or the geoid model computation, the differences in orthometric height computation between using the precise leveling and using the gravimetric model + the GPS ellipsoidal heights can be attributed to several factors. These include: (i) random noise in the values of h, H, N; (ii) datum inconsistencies and other possible systematic distortions in the three height data sets; (iii) various geodynamic effects; and (iv) theoretical approximations in the computation of either H or N. For more discussion on these factors, the interested reader may refer to Fotopoulos et al. (1999).

Testing the Use of RTK GPS Ellipsoidal Heights and the Geoid Model for Determination of Orthometric Heights

A test was conducted in order to assess the accuracy of orthometric height determination combining RTK GPS using the DVRS reference network and the new Dubai geoid model. The heights determined from this method were compared with published orthometric heights for a test network consisting of 41 benchmarks of the Dubai second order leveling network. These benchmarks have previously-determined precise orthometric heights. The test area spanned approximately 22.7 km x 7.8 km in the Easting and Northing directions respectively, representing the area acquiring the most demanding survey works in the Emirate of Dubai. The height difference between the highest and lowest points in this case was approximately 34.5 meters. The GPS ellipsoidal heights were determined using Leica SR530 receivers in an RTK approach employing correctional data from the DVRS reference network. Each test point was occupied for a period of a few seconds, representing an ordinary working environment. The standard deviations of the ellipsoidal height determination for the occupied points of the test network ranged between 1.05 cm and 5.47 cm.

Figure 4 shows the differences in orthometric height computation between the proposed method of using a single GPS receiver employing the DVRS network in addition to the Dubai geoid model, and using the known orthometric heights of the benchmarks determined from the precise spirit leveling. On average, the results agree to within ± 5 cm, with a maximum difference of 7.04 cm. **Figure 5** shows the surface plot of the height discrepancies between the two methods. A frequency histogram of the height discrepancies is also given in **Figure 6**. These figures show that the frequency of occurrence of the errors has a nearly normal distribution, bearing in mind the fact that the number of tested points gives a limited sample size. The results also show that there were no significant systematic errors in the results. The statistical results for the test network are presented in **Table (2)**. The mean value of the differences was 0.44 cm, and the average value of the absolute differences was 2.4 cm with 3.05 cm standard deviation. However, when comparing these results with **Table (1)**, the degradation of accuracy of the orthometric heights in the former case are attributed to using an RTK positioning method versus using the more precise static approach in the latter. However, the benefits of using the RTK approach are substantial, considering that the achieved accuracy is considered precise enough for third order leveling, which represents the majority of leveling works needed.

These results clearly show the feasibility of determining real-time orthometric heights at the cm accuracy level in Dubai using a single geodetic-grade GPS receiver within the proposed framework. In general, this conclusion agrees with findings of another study by the National Geodetic Survey (NGS, 1998), which indicated that GPS can obtain 2-5 centimeter heights at the 95% confidence level when proper field procedures and a good geoid model are utilized. These constraints were met in the case of Dubai by using the online data of the DVRS reference network in addition to the newly developed Dubai gravimetric geoid.

In the future, a refinement of the DVRS reference network (e.g. inclusion of more stations, updating the software and equipment, updating the ITRF-epoch coordinates etc.) is scheduled, which may result in changes in the GPS derived heights. Moreover, the Dubai gravimetric geoid has also the possibility of improvement by adding more data of high accuracy and expanding the working area. Thus, a change in the orthometric heights derived from GPS + geoid is possible compared to the more steady leveling orthometric heights. The height differences can be modeled in this case as a datum shift between the old system and the new one. The model can be formulated for point (i), as follows:

$$(H_{new} - H_{old})_{i\ (\text{GPS RTK Network + Geoid})} = x_1 \cos(\phi_i)\cos(\lambda_i) + x_2 \cos(\phi_i)\sin(\lambda_i) + x_3 \sin(\phi_i) + v_i \quad (5)$$

which is based on equations given in Heiskanen and Moritz (1967), and Fotopoulos et al. (1999). The model of equation (5) is applied to all network points and a least-squares adjustment is performed to estimate the residuals (V_i). The unknown shift parameters (x_1, x_2, x_3) are being solved by minimizing the quantity ($\mathbf{v}^T\mathbf{v}$).The residuals (V_i) contain a combined amount of GPS, leveling, and geoid random errors. An optimal adjustment in a statistical sense would require the proper weighting of the residuals, which is hardly ever applied in reality. From experience, it is also important to include a large number of points of good geometric distribution in the model for a proper estimation of its unknown parameters and residuals. Finally, the expected "new-old" difference in the determined orthometric heights by GPS+geoid at a specific point (ϕ_j, λ_j), which has not been updated by GPS or the geoid, can be anticipated with a small tolerance by means of equation (5) using point coordinates.

Statistical Examination of the Discrepancies of the Orthometric Height Determination between GPS RTK Network + Geoid and the Leveling

The practical differences in height estimation by GPS RTK Network + geoid and leveling techniques were tested by statistical examination of their results for the 41 test benchmarks. Two tests were performed. In these, the height discrepancies between the two techniques were considered as a random sample of small errors. The first test was performed to check the presence of any systematic errors in the sample as a whole by testing the deviation of the sample mean ($\overline{X}$) from the mean of its population, which is assumed to be zero, on the assumption that the sample elements have a normal distribution shape. The test can be formulated as (Anderson and Mikhail, 1998):

$$[\mu - t\frac{\sigma}{\sqrt{n}} < \overline{X} < \mu + t\frac{\sigma}{\sqrt{n}}] \qquad (6)$$

Where (μ) is the population mean value (μ=0), (σ) denotes the sample standard deviation, t is the tabulated students t-distribution value corresponding to the sample size (n) and degrees of freedom equals to (n-1), and taking the 95% confidence level ($\propto$). Using the values given in **Table** (**2**) shows that the sample mean (0.44 cm) lies within the test limits ($\pm$ 0.80 cm), and thus, in general, no significant systematic errors exist in height discrepancies. This indicates that the inherent errors in both GPS+geoid and leveling techniques are of a random nature.

To check that the sample mean was not affected by biases that may be of approximate equal values but with different signs, each individual discrepancy was tested. Thus, undesirable discrepancies could be identified and rejected. The discrepancies are considered as a small sample taken from a population that is assumed to be normally distributed; therefore, they can be individually examined according to the known values of the population mean (μ) and the population variance (σ_p^2). The population variance can be estimated from the sample variance (σ^2) using the following interval (Anderson and Mikhail, 1998):

$$[\frac{(n-1)\times\sigma^2}{\chi^2_{0.975,n-1}} < \sigma^2_p < \frac{(n-1)\times\sigma^2}{\chi^2_{0.025,n-1}}] \qquad (7)$$

where (χ^2) denotes values of Chi-square distribution within [0.975 and 0.025] confidence level and (n-1) degrees of freedom. For the test in hand, the upper and lower limits were 15.25 cm^2 and 6.27 cm^2, respectively. Thus, a population variance with a 3.5 cm standard deviation would be acceptable. This seems realistic considering the expected accuracy obtainable from the RTK-GPS height determination and the geoid model discussed in the previous sections.

The following confidence interval can form a criterion for rejecting the sample element that has a systematic error, which in this case will be one that does not lie within the interval:

$$[\ \mu - t\ \sigma_p < \delta_i < \mu + t\ \sigma_p] \tag{8}$$

where (δ_i) denotes the individual tested height discrepancy. Based on the results of this study, the boundaries of this test were ±5.88 cm. By inspecting height discrepancies, only two points exceeds the boundary value, and can be rejected. All other height discrepancies pass the test and can be considered free from large systematic errors or blunders.

Conclusions

The feasibility of determining real-time orthometric heights at the centimeter accuracy level in Dubai using a single GPS receiver employing online data of the DVRS reference network in addition to the newly developed Dubai gravimetric geoid is shown. It is evident that this approach can be considered an efficient and accurate alternative to the traditional spirit leveling technique for ordinary surveying work in large distance projects. Limited testing shows that there were no systematic errors in computation of the orthometric heights using the RTK-Network GPS ellipsoidal heights + geoid model as compared with precise spirit leveling results. On average, the results agree to within 5 cm. However, further tests of a larger network consisting of a greater number of stations may provide more insight into this method as an alternative to traditional leveling.

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Notations

The following symbols are used in this paper:

$C(s)$ = covariance function;

C_o = correlation factor;

H = orthometric height;

h = ellipsoidal height;

k = constant determined by the correlation length;

N = the geoidal height;

N_1 = the geoidal height part from a spherical harmonic expansion;

N_2 = the geoidal height part from the topography;

N_3 = the geoidal height part from the residual gravity;

N_{GPS} = GPS/leveling-derived geoidal heights;

n = the sample size;

s = the distance;

t = the students t-distribution value;

v_i = residuals in orthometric heights at point i;

$\overline{X}$ = the sample mean;

(x_1, x_2, x_3) = height datum shift parameters in the Cartesian X, Y, and Z directions;

δ_i = the tested height discrepancy;

ε = $N_{grav} - N_{GPS}$;

ε' = residual geoidal height;

(ϕ, λ) = geographic latitude and longitude of a point;

μ = the population mean;

σ = the sample standard deviation;

σ_p^2 = the population variance;

χ^2 = values of Chi-square distribution.

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Table 1. Height Discrepancies between the Model and Precise Leveling (cm)

Average	Average of absolute values	Max. discrepancy	Min. discrepancy	σ
0.2	1.91	4.92	0. 23	2.33

Table 2. Height Differences between the GPS RTK Network + Geoid and Precise Leveling (cm)

Average	Average of absolute values	Max. discrepancy	Min. discrepancy	σ
0.44	2.40	7.04	0.000	3.05

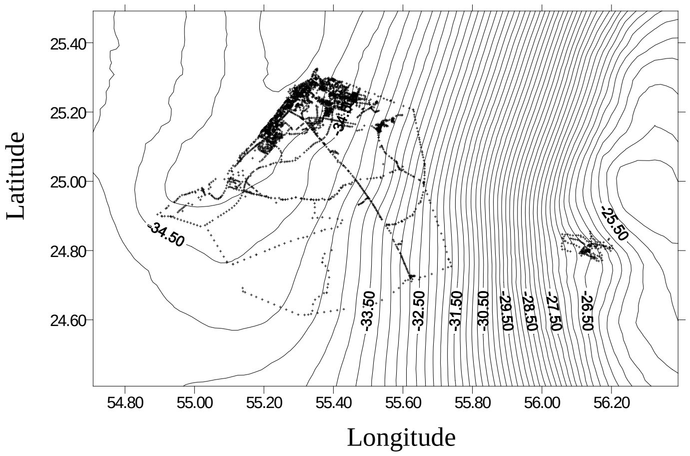

Fig. 1. The Dubai Geoid

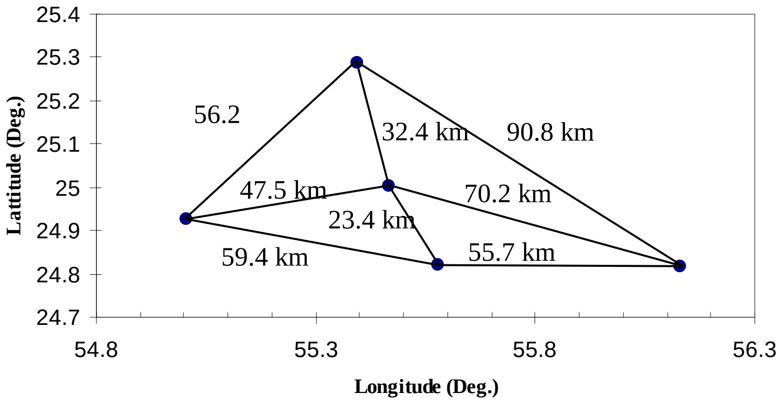

Fig. 2. The DVRS Network

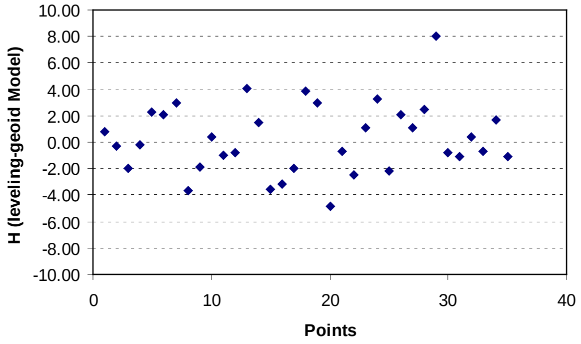

Fig. 3. Height discrepancies between the model and precise leveling (cm)

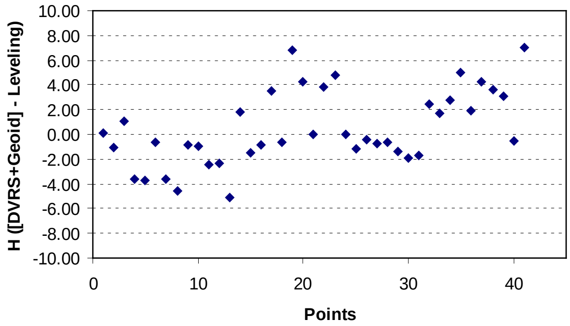

Fig. 4. Height discrepancies between GPS RTK Network +geoid and precise leveling (cm)

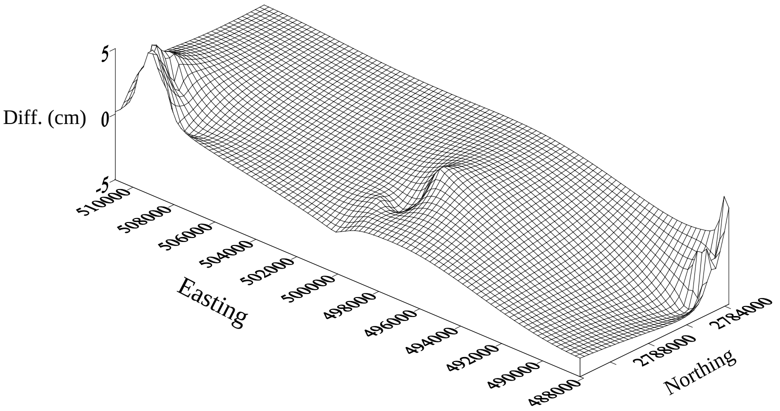

Fig. 5. Surface plot of the height discrepancies

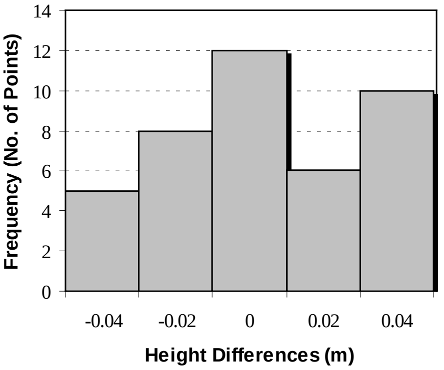

Fig. 6. Frequency histogram of the height discrepancies