Effect of Induction Motors Mechanical Load on its Model for Purpose of Static Voltage Stability Analysis

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Abstract: *Load modelling is one of the major issues in voltage stability analysis. It greatly impacts the result of the study and has a significant effect on the stability regions. This paper examines the validity of static models for induction motors, proposed for static voltage stability studies, in their different operating conditions. It has been shown that motor operating condition and mechanical load, largely influences motor model. The effect has been fully studied in this paper and the results have been presented.*

Keywords: Voltage stability, static load modelling, induction motor, mechanical load characteristic.

1. Introduction

Voltage stability had received great attention in the late of 80s and early of 90s [17]. This was mainly because of the incidents occurred around the world and voltage stability has been recognised as the main reason [12],[13]. However, it has not been confined to that period of time. Since then, voltage instability incidents continued to occur [1],[14] . This has been the main reason to motivate researchers and companies to seeking methods in order to study voltage stability and even monitoring it in real time operation [18]. Recent works in this issue can be found in [2],[3].

Voltage stability studies can be categorized in two main sections: static and dynamic. However, the voltage stability is a dynamic phenomenon, influenced by various dynamics in the power system, static studies can be used in order to evaluate voltage stability and the results are valid under certain conditions [4]. Since static analysis is simpler and faster than dynamic ones and gives a glimpse into the mechanisms of voltage instability, it has been widely used as a tool to evaluate voltage stability and also for educational purposes.

It has been shown that load modeling plays an important role in voltage stability analysis [15],[16]. [5] shows that stability region changes with load parameter variation. Voltage dependency of loads can improve voltage stability condition compared to constant power load models. It is because of load reduction in case of reduced system voltages. Load dynamics which recover the load to the pre-disturbance value has an opposite effect. These dynamics mainly includes fast dynamics of induction motors and slow dynamics of load tap changers and thermostatic load recovery. The latter are main dynamics leads to long term voltage instability [16].

Many efforts have been done in order to properly model loads in power system and voltage stability studies. A survey on this issue can be found in [6]. Exponential and polynomial models have been extensively used in static voltage stability analysis. Description of them can be found in [7],[8]. Generic dynamic load models have been presented in [5],[10]. Dynamic model for various types of loads has been presented in [9]. [16] presents more explanation on self-restoring generic models. This model has been widely used in dynamic voltage stability studies.

This paper deals with induction motor representation for static voltage stability studies. Main concept is to examine whether mechanical load magnitude and characteristic can affect motor static model or not. This was investigated in [11], but in this reference influence of mechanical conditions on active power has been only investigated. This paper deals with both active and reactive parts. It has been shown that variation for reactive power is so huge that cannot be neglected in studies.

The paper is organized as follows. In section 2, static load model has been introduced. In section 3 induction motor steady state load characteristic has been investigated. In section 4 effects of different mechanical loads have been presented.

2. Static Load Model Representation

As it has been previously mentioned, loads can be presented either static or dynamic. Static model only considers the dependency of loads to voltage and frequency variation (frequency term is usually omitted in voltage stability studies); and it does not change by time. On the other, dynamic load model considers voltage and frequency dependency of loads as well as its variation by time. This enables to model restoration of loads by time in dynamic studies.

In static model, the steady state characteristic of load is concerned. Exponential load model has been presented as below [7]:

$$P = P_0\left(\frac{V}{V_0}\right)^{pv}$$
$$Q = Q_0\left(\frac{V}{V_0}\right)^{qv} \tag{1}$$

Where P_0, Q_0 and V_0 are the values of active and reactive power and voltage at base load respectively. *pv* and *qv* are voltage dependency coefficients. This model relates the steady state characteristic of load to its voltage. As it is evident from equation (1), where coefficients *pv* and *qv* are positive, load active and reactive power decreases in case of voltage decrease. On the other hand, when they are negative, load increases when voltage decreases.

Another famous static model is polynomial load model as shown below [7]. It consists of three exponential load models: constant impedance (*pv*= *qv*=2), constant current (*pv*= *qv*=1) and constant power (*pv*= *qv*=0). It is also known as ZIP model

$$\frac{P}{P_0} = a_p\left(\frac{V}{V_0}\right)^2 + b_p\frac{V}{V_0} + c_p$$
$$\frac{Q}{Q_0} = a_q\left(\frac{V}{V_0}\right)^2 + b_q\frac{V}{V_0} + c_q \tag{2}$$

Load parameters are calculated considering steady state power variation with respect to voltage. TABLE I represents load parameters from [13]. Two last rows of this table are the most concerned. Small value of *pv* indicates almost complete recovery of active power in case of voltage variation. This is because of load recovery by motor dynamics. Positive value for *qv* showes reactive power reduction in case of voltage decrease. As it has been shown in next section this may not be always true.

TABLE I: Exponential Load Model Parameters

Load Type	*pv*	*qv*
resistance space heater	2	0
fluorescent lighting	1	3
arc furnace	2.3	1.61
small industrial motors	0.1	0.6
large industrial motors	0.05	0.5

3. Induction Motor Modelling

In this section induction motor model has been presented. Electrical equivalent circuit of an induction motor has been presented in Fig. 1 [16]. In this figure R_s and X_s are resistance and reactance of stator, R_r and X_r are the transferred resistance and reactance of rotor and X_m is the magnetizing reactance of machine. s is the machine slip.

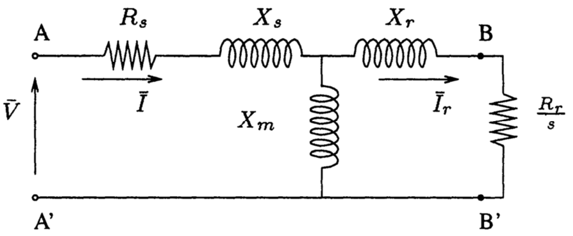

Fig. 1: Equivalent circuit of induction motor

Following equations for electrical torque and absorbed active and reactive power of machine can be obtained[16]:

$$T_e = \frac{\bar{V}^2 . X_m{}^2}{[(R_1 + R_r/s)^2 + (X_1 + X_r{}^2)]} \times \frac{R_r/s}{(R_s{}^2 + (X_m + X_s)^2}$$
$$P_e = \frac{\bar{V}^2(R_s + R_e)}{(R_s + R_e)^2 + (X_s + X_e)^2} \tag{3}$$
$$Q_e = \frac{\bar{V}^2(X_s + X_e)}{(R_s + R_e)^2 + (X_s + X_e)^2}$$

Where R_1, X_1 and R_e, X_e are real and reactive parts of thevenin impedances seen from rotor and stator respectively:

$$R_1 + jX_1 = \frac{jX_m(R_s + jX_s)}{R_s + j(X_m + X_s)}$$
$$R_e + jX_e = \frac{jX_m(R_r/s + jX_r)}{R_r/s + j(X_m + X_r)} \tag{4}$$

In steady state condition, electrical torque and mechanical torque are in balance and motor does not accelerate or decelerate, hence following equation can be written as follow:

$$T_e(\bar{V}, s) = T_m(s) \tag{5}$$

Utilizing above equations, steady state characteristics of an induction motor in any condition can be obtained. In this paper parameters for two types of motors have been considered from[16]. These are small industrial and large industrial motors. Parameters are shown in TABLE II.

Fig. 2 represents T-s characteristic of typical small and large industrial motors ($\bar{V}$=1). A quadratic characteristic for mechanical torque has been considered as below. It represents typical mechanical load characteristics of industrial pumps and compressors.

$$T_m(s) = T_0(1 - s)^2 \tag{6}$$

TABLE II: Typical Induction Motor Parameters

Motor Type	R_s	X_s	X_m	R_r	X_r
small industrial motor	0.031	0.1	3.2	0.018	0.18
large industrial motor	0.013	0.067	3.8	0.009	0.17

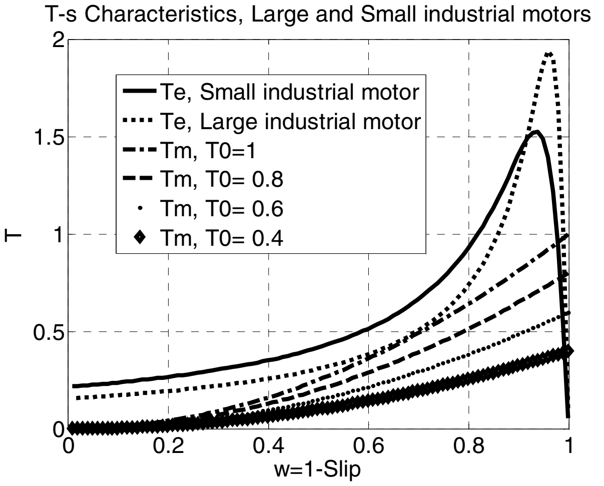

Fig. 2: T_e-s and T_m-s characteristics for large and small industrial motors

It is evident from figure that increasing mechanical load will decrease motor speed and hence increasing motor slip. In case of light load (T_0=0.4, 0.6) motor has slip nearly zero, on the other hand in case of heavy load (T_0=0.8, 1) motor works in higher slips and approaches region near maximum torque. Increasing mechanical torque will lead to a point where mechanical and electrical curves are tangent to each other at one point. After then motor does not have stable operational point and will not start.

Since in voltage stability studies voltage varies extensively, the behavior of motor under variation of terminal voltage is of interest. Fig. 3 and Fig. 4 represent P-V and Q-V characteristics of a large industrial motor under light load condition with quadratic mechanical torque characteristics. In this case, reduction of terminal voltage of motor decreases absorbed active and reactive power. The reduction in active power is so small which motor absorbed power can be considered approximately constant. This is because of load restoration by motor dynamic: speed. When terminal voltage decreases, T_e decreases as it is clear from (3). Now T_m is greater than T_e and motor decelerates until equality in (5) is reached again. Motor power reaches nearly same value as it was before in this dynamic process. So in steady state and after dynamics played their role, it can be treated as constant load.

Fig. 4 shows that story is quite different for variation of steady state absorbed reactive power by motor. Reactive power shows quite large reduction when terminal voltage decreases. This is because of increased motor current in reduced voltage levels. This process reduces voltage on magnetizing reactance (X_m in Fig. 1) and a reduction in motor reactive power. On the other hand increased motor current leads to increase in reactive losses in rotor and stator reactance (X_s and X_r). These two effects are opposite to each other. In certain voltage levels the latter becomes stronger and leads to increase of reactive power of motor by further decreasing of voltage. This leads to a turning point in V-Q characteristic curve and has been indicated by TP in Fig. 4.

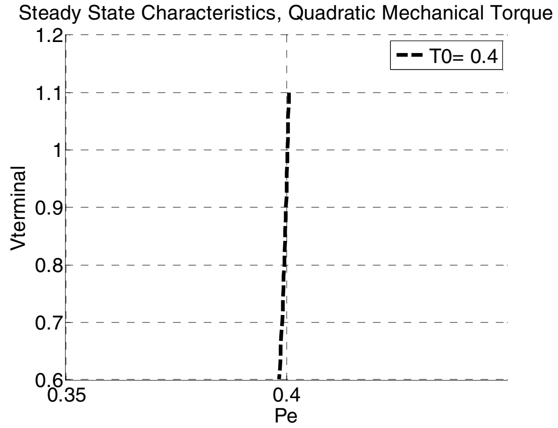

Fig. 3: P-V curve for large industrial motor

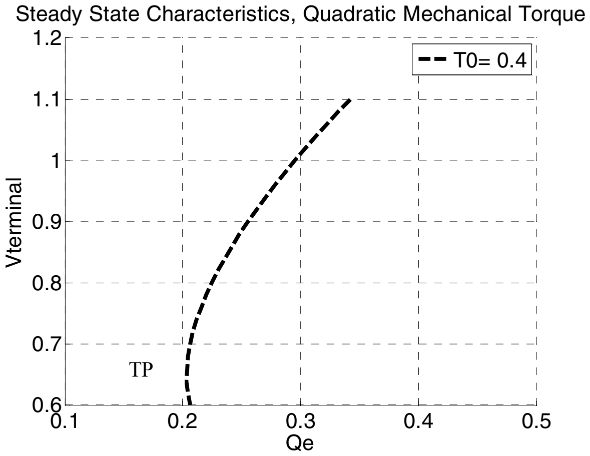

Fig. 4: Q-V curve for large industrial motor

Motor parameters according to (1) can be obtained by a suitable curve fitting process in Fig. 3 and Fig. 4. This has been done by utilizing "fit" command in MatLab software which fits data to specified equation. By doing so, following equations have been obtained for this kind of motor. Small *pv* indicates load restoration as it was expected. Large positive value for *qv*, models reactive power reduction when voltage decreases. As it can be seen from the results, calculated *pv* and *qv* values are quite different from those are in TABLE I last row. This shows that typical values can not be trusted and motor parameters shall be calculated for each case.

$$P = 0.4 \times V^{0.009}$$
$$Q = 0.3 \times V^{0.98} \qquad (7)$$

The validity range of above equation for static motor modeling is the major question. As it is clear from Fig. 4, positive value for *qv* is valid until voltages near 0.7 p.u. After that, negative value for *qv* shall be substituted to model reactive power increase by further voltage reduction. Since reactive power has a significant effect on voltage stability, TPs are in much interest for the application of static voltage stability analysis. In the studied motor, TP occurs in such a low voltage and then the static model in (7) is valid for almost all voltage stability studies (it shall be noted than voltage range for static analysis is almost [1.1~0.7] p.u. static models for lower voltages are not valid, this is because of nonlinearities in loads, i.e. motor stalls in lower voltages).

In next section, it has been shown that motor mechanical load extensively affects both *qv* and TP of the Q-V characteristics of the motor and therefore the static model of the motor. So motor model shall be subject to change, according to its mechanical load situation.

4. Influence of Mechanical Load on Motor Model

In previous section P-V and Q-V characteristic for a motor in light load condition have been fully investigated. It has been showed that exponential load model can be used for this motor in almost for the full voltage range of the study. In this section the effect of mechanical load of the motor, on motor load model has been studied. For this purpose the quadratic characteristics presented in (6) has been considered. Four operating condition of motor has been considered by increasing T_0 as below:

- $T_0 = 0.4, 0.6$: light load condition
- $T_0 = 0.8, 1.0$: heavy load condition

Results for large industrial motor (with parameters indicated in TABLE II) are depicted in Fig. 5 and Fig 6. Active power characteristic of motor does not change significantly as mechanical load increases. The only difference is the turning points in P curve for higher load levels. This point indicates maximum stable steady state operating point of the motor. After that point, motor will stall. So in first step mechanical load level of the motor changes validity range of the model but it seems it does not change the model parameter.

Considering Q curves in Fig. 6, reveals that the effect of motor load on reactive power is so great. For light load condition, TP occurs at relatively low voltages. But for higher load levels, it moves to higher voltage magnitudes. For the extreme case of motor load (T_0=1), it occurs at voltage greater than 1 p.u. in this case reactive power of motor increases extensively as the terminal voltage decreases. This characteristic has a great impact on voltage degraded networks and may lead to serious voltage problems.

By a suitable curve fitting process following equations for reactive power have been obtained:

$$\begin{array}{ll} T_0 = 0.4, & Q = 0.3 \times V^{0.98} \\ T_0 = 0.6, & Q = 0.36 \times V^{0.083} \\ T_0 = 0.8, & Q = 0.38 \times V^{-1.12} \\ T_0 = 1.0, & Q = 0.50 \times V^{-1.35} \end{array} \qquad (8)$$

The negative values for *qv* show the aforementioned increase in Q in the process of voltage reduction. The results clearly show that a single model can not be used for modeling of motor in all conditions. Changing mechanical load of the motor will change validity range of the model as well as model parameters. So usage of a constant model for motor may lead to incorrect results in the study.

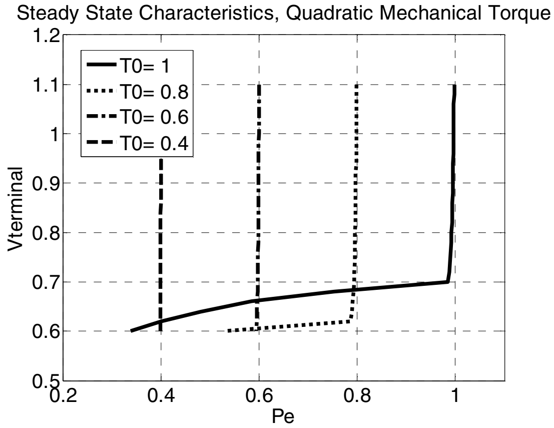

Fig. 5: P-V curve for large industrial motor

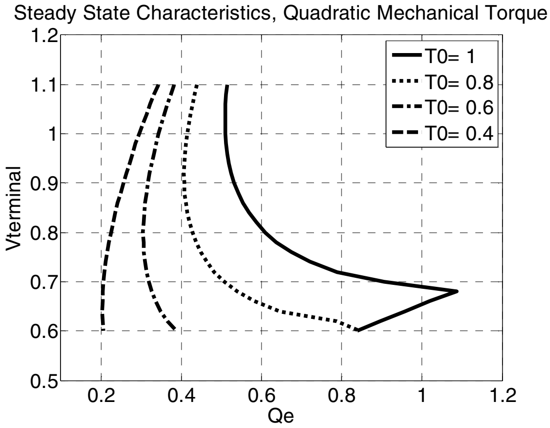

Fig. 6: Q-V curve for large industrial motor

In order to examine the effect of motor parameters on its Q-V Curve, the calculation have been repeated for small industrial motor and for various values of X_m/X_s, in order to consider different motor designs. In addition it has been repeated for constant torque mechanical load model. Results showed that the behavior obtained before, is also true for these cases. TABLE III shows turning points in Q-V curves in these cases. In addition for comparison to previous case, Fig. 7 shows Q-V curves for small industrial motor case and quadratic mechanical torque.

As it can be seen from TABLE III, increasing mechanical load in any cases, increases the voltage which TP occurs in Q-V curve. It means change in *qv* value with mechanical load variation. It is true for both constant and quadratic mechanical torque characteristics. In addition TP has higher values for constant torque cases in comparison to quadratic ones. This indicates that not only magnitude of mechanical load affects the motor model, but also its characteristic influences the model too.

Regarding above results, it can be obtained that for the same motors with different mechanical load magnitudes and/or different mechanical load characteristics, motor model parameters are not equal to each other. Utilizing same model for these motors, especially for voltage

TABLE III: Turning Point Voltage in Q-V Curve

a) Small Industrial Motor

	Constant Torque				Quadratic Torque			
	T0=1	T0=0.8	T0=0.6	T0=0.4	T0=1	T0=0.8	T0=0.6	T0=0.4
Xm/Xs=10	0.96	0.86	0.74	<0.6	0.9	0.8	0.7	<0.6
Xm/Xs=20	1.02	0.9	0.78	0.64	0.98	0.86	0.75	<0.6
Xm/Xs=30*	1.08	0.96	0.84	0.68	1.04	0.92	0.81	0.66
Xm/Xs=40	>1.1	1.01	0.88	0.72	>1.1	0.98	0.84	0.69
Xm/Xs=50	>1.1	1.05	0.91	0.74	>1.1	1.02	0.88	0.72
Xm/Xs=60	>1.1	>1.1	0.95	0.75	>1.1	1.06	0.92	0.75
Xm/Xs=70	>1.1	>1.1	0.97	0.8	>1.1	>1.1	0.95	0.77

*: indicates base case according to TABLE I

b) Large Industrial Motor

	T0=1	T0=0.8	T0=0.6	T0=0.4	T0=1	T0=0.8	T0=0.6	T0=0.4
Xm/Xs=20	0.86	0.8	0.68	<0.6	0.86	0.77	0.67	<0.6
Xm/Xs=30	0.94	0.84	0.72	<0.6	0.92	0.82	0.71	<0.6
Xm/Xs=40	0.98	0.88	0.76	0.62	0.96	0.86	0.74	<0.6
Xm/Xs=50	1.02	0.91	0.79	0.64	1	0.9	0.78	0.64
Xm/Xs=60*	1.06	0.94	0.82	0.66	1.04	0.92	0.8	0.66
Xm/Xs=70	1.08	0.97	0.84	0.69	1.07	0.96	0.83	0.68
Xm/Xs=80	>1.1	1	0.86	0.7	>1.1	0.98	0.85	0.7

*: indicates base case according to TABLE I

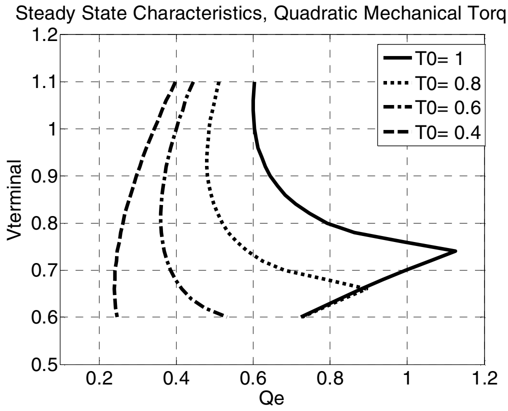

Fig. 7: Q-V curve for small industrial motor with quadratic mechanical load characteristic

stability studies which is so sensitive to reactive power, may leads to misleading results.

Further works are now in progress to analyse the results in a power system and study the effects of motor load conditions on voltage stability boundary regions, and will be published in future.

5. Conclusion

The effect of different motor mechanical load conditions and characteristics, on its steady state representation has been investigated. It has been shown that they have a great impact on motor static model. The effect is so intensive for reactive power part. Active power part is less subject to change. Since the reactive power is a crucial issue in voltage stability analysis, the use of static models for induction motors in static voltage stability studies shall be undertaken with care. It has been shown that static model parameters are subject to change in different motor operation conditions. It is expected that different voltage stability margins being obtained for different motor mechanical load conditions where electrical parameters of the motors are held unchanged.

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