Adaptive OFDM Techniques With One-Bit-Per-Subcarrier Channel-State Feedback

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***Abstract*—In the orthogonal frequency-division multiplexing (OFDM) scheme, some subcarriers may be subject to a deep fading. Adaptive techniques can be applied to mitigate this effect if the channel-state information (CSI) is available at the transmitter. In this paper, we study the performance of an OFDM-based communication system whose transmitter has only one bit of CSI per subcarrier, obtained through a low-rate feedback. Three adaptive approaches are considered to exploit such a CSI feedback: adaptive subcarrier selection; adaptive power allocation (APA); and adaptive modulation selection (AMS). Under the conditions of a constant raw data rate and perfect feedback channel, the performance of these approaches are analyzed and compared in terms of raw bit-error rate. It is shown that one-bit CSI feedback can greatly enhance the system performance. Moreover, imperfections of the feedback channel are considered, and their impact on the performance of these techniques is studied. It is shown that by exploiting the knowledge that the feedback channel is imperfect, the performance of the APA and AMS techniques can be substantially improved.**

***Index Terms*—Channel-state information (CSI), feedback channel, orthogonal frequency-division multiplexing (OFDM).**

I. Introduction

An important advantage of the orthogonal frequency-division multiplexing (OFDM) communication scheme is that, due to the inverse fast Fourier transform (IFFT) at the transmitter and the fast Fourier transform (FFT) at the receiver, the frequency-selective fading channel is converted into parallel flat-fading channels [1], [2]. However, the OFDM approach can suffer from fading that may affect some subcarriers. This makes a reliable detection of the information-bearing symbols at these particular subcarriers very difficult. Therefore, the overall performance of the system may degrade in this case.

One of the recent approaches to mitigate the effect of fading in OFDM systems uses error-correction coding across the subcarriers [1]. Another way to improve the immunity of OFDM systems to fading is based on linear precoding (LP) techniques. Most of the precoding techniques require full channel-state information (CSI), but there are linear precoders that do not require any CSI knowledge, and provide an excellent performance [3]. Unfortunately, all precoding techniques require equalization at the receiver that may dictate quite a complex receiver structure.

If some CSI knowledge is available at the transmitter, adaptive modulation and resource allocation techniques can be applied to allocate bits and transmitted powers to the subcarriers [4]–[6]. However, in cellular communications, it can be difficult to obtain such CSI. For example, if the time-division duplex (TDD) mode is used, the downlink transmit CSI can be obtained by estimating the uplink channel and using the uplink–downlink reciprocity. However, in practical situations, fast channel variability and user mobility may prohibit using the aforementioned reciprocity property. Moreover, this property does not hold if the frequency-division duplex (FDD) mode is used. In the latter case, some feedback has to be exploited to transmit the downlink CSI from the mobile station (MS) to the base station (BS). As the bandwidth consumed by the feedback channel is proportional to the feedback rate, it is interesting to study the performance of systems which enable only a low-rate CSI feedback. For example, the use of one-bit channel-state feedback in Alamouti-type systems has been studied in [7], while an asymptotic lower bound on the minimum feedback rates for multicarrier transmission has been derived in [8]. Adaptive subcarrier selection (ASCS) using channel-dependent thresholds based on waterfilling was studied in [9]. An adaptive power-feedback technique for OFDM systems based on codebooks has been proposed in [10]. Note that the aforementioned adaptive OFDM approaches have a significant advantage, as compared with the LP-OFDM technique. Indeed, these approaches have much easier implementation, because they do not require equalization at the receiver. Moreover, low-rate feedback has already been adopted in many practical communication systems and standards, for example, in the IEEE WiMAX standard [11]. Therefore, the requirement of low-rate CSI feedback does not necessarily complicate the system design.

In this paper (see also [12] and [13]), the performance of OFDM communication systems with one-bit-per-subcarrier CSI feedback is studied. The uncoded transmission case is considered, and the raw bit-error rate (BER) is used as the criterion to evaluate the system performance. Assuming that the feedback channel is perfect, three adaptive approaches, including ASCS, adaptive power allocation (APA), and adaptive modulation selection (AMS) are used to exploit the CSI feedback and compared with each other. For the latter two techniques, closed-form

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expressions for the BER are derived and, based on them, the parameters of these techniques are optimized.

In practical situations, the feedback channel may be erroneous and may suffer from a feedback delay. Therefore, the feedback CSI may be unreliable. Motivated by these facts, the impact of an imperfect CSI feedback on the performance of the ASCS, APA, and AMS techniques is also studied. It is shown that although the ASCS strategy has better performance than the APA and AMS schemes when the feedback channel is perfect, it behaves worse than the latter two schemes in the case of imperfect feedback channel. In particular, for the APA and AMS techniques, it is investigated how to exploit the knowledge of the fact that the feedback is imperfect to optimize the parameters of these techniques.

The remainder of this paper is organized as follows. The OFDM system model is formulated in Section II. In Section III, we consider the perfect feedback channel case and present our analysis of the APA and AMS schemes applied to OFDM systems with one-bit-per-subcarrier feedback. Section IV is devoted to the analysis and optimization of the APA and AMS schemes in the case when the feedback channel suffers from errors or delays. Section V presents simulation results where the performance of the ASCS, APA, and AMS schemes are compared under the conditions of perfect and imperfect feedback channels. Section VI contains our concluding remarks.

II. System Model

Let us consider the point-to-point downlink cellular communication mode, where both the BS and the MS have a single antenna. The frequency-selective multipath Rayleigh fading channel between the BS and the MS is characterized by its gains h_l $(l = 1, \ldots, L)$ and delays τ_l $(l = 1, \ldots, L)$, where L is the total number of paths. We assume that the coefficients h_l $(l = 1, \ldots, L)$ are independent (but not necessarily identically distributed) zero-mean complex Gaussian random variables with the variances σ_l^2 $(l = 1, \ldots, L)$, and that N subcarriers are used. The mth block of information-bearing symbols $\boldsymbol{s}(m) = [s(mN), \cdots, s(mN+N-1)]^T$ is IFFT-modulated, and the cyclic prefix (CP) is inserted to form one OFDM symbol. It is assumed that the length of the CP is longer than the maximum path delay. Finally, the symbol is pulse-shaped and transmitted through the channel. The channel is assumed to be constant during the OFDM symbol transmission time. Hereafter, for notational simplicity, the block index dependence of $\boldsymbol{s}$ is omitted.

After removing the CP, the received $N \times 1$ signal vector $\boldsymbol{y}$ at the MS can be written as [2]

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{F}^H\boldsymbol{P}^{1/2}\boldsymbol{s} + \boldsymbol{v} \tag{1}$$

where $\boldsymbol{P}$ is the $N \times N$ diagonal matrix of the transmitted powers of the symbols corresponding to different subcarriers, $\boldsymbol{F}$ is the $N \times N$ normalized FFT matrix whose (i,l)th entry is given by

$$\boldsymbol{F}_{i,l} = \frac{1}{\sqrt{N}} \exp\left(-\frac{j2\pi(i-1)(l-1)}{N}\right)$$

$j = \sqrt{-1}$, $\boldsymbol{H}$ is the $N \times N$ circulant channel matrix between the MS and BS whose (k,l)th entry is given by $h_{(k-l+1) \bmod N}$, $\boldsymbol{v}$ is the $N \times 1$ vector of the MS additive white Gaussian noise (AWGN) with the covariance matrix $\sigma_v^2\boldsymbol{I}_N$, $(\cdot)^H$ stands for the Hermitian transpose, and $\boldsymbol{I}_N$ denotes an $N \times N$ identity matrix. After the FFT operation, the $N \times 1$ output symbol vector $\boldsymbol{r}$ can be written as $\boldsymbol{r} = \boldsymbol{F}\boldsymbol{y} = \boldsymbol{D}\boldsymbol{P}^{1/2}\boldsymbol{s} + \boldsymbol{n}$, where we used (1) and the fact that $\boldsymbol{F}\boldsymbol{H}\boldsymbol{F}^H = \boldsymbol{D}$ [2]. Here, $\boldsymbol{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ is the diagonal matrix of the subcarrier channel gains, and $\boldsymbol{n} = \boldsymbol{F}\boldsymbol{v}$ with $\mathrm{E}\{\boldsymbol{n}\boldsymbol{n}^H\} = \sigma_v^2\boldsymbol{I}_N$.

The channel gain d_n $(n = 1, \ldots, N)$ of the nth subcarrier is given by [2]

$$d_n = \frac{1}{\sqrt{N}} \sum_{l=1}^{L} h_l e^{-j2\pi n\tau_l/NT}, \quad n = 1, \ldots, N \tag{2}$$

where T is the sampling interval. It is obvious that d_n is a zero-mean complex Gaussian random variable with the variance of $\sum_{l=1}^{L} \sigma_l^2/N$. Without any loss of generality, we normalize the variance of the channel gain at each subcarrier so that $\sum_{l=1}^{L} \sigma_l^2/N = 1$. It can be seen that $d_1, \ldots, d_N$ all are identically distributed. The absolute value of each d_n is Rayleigh-distributed with the probability density function (pdf)

$$p(\alpha) = 2\alpha \exp(-\alpha^2). \tag{3}$$

We assume that the BS transmits at the constant data rate of n_r bits per second (b/s) and that the BS has perfect knowledge of the signal-to-noise ratio (SNR), while the MS has perfect downlink CSI knowledge (which also means that the SNR is known at each MS). The downlink CSI is transmitted back to the BS through a low-rate feedback channel. More specifically, we consider the case when a total number of N bits containing the CSI for all subcarriers (i.e., one bit per subcarrier) is transmitted to the BS in one feedback cycle. The SNR information can be delivered to the BS using another, much lower rate feedback than the one-bit feedback used for the CSI.

In practical applications, not all N subcarriers may be used. For example, the typical choice of the number of used subcarriers in wireless local area networks (WLANs) is 52 out of 64 available subcarriers [1]. However, for the sake of simplicity and with a small abuse of notation, hereafter, we will denote the number of used subcarriers as N.

III. Perfect One-Bit-Per-Subcarrier CSI Feedback

In this section, we assume that the feedback channel is perfect (i.e., there are no feedback errors and/or delays), and study several efficient ways to make use of N feedback bits (one bit per subcarrier) available. Clearly, it is impossible to provide a sufficiently accurate CSI feedback to the BS with only N bits. To illustrate this fact, we note that in wireless communications, the order of the multipath channel can be about $L = 10$ [14], and the typical choice of the number of subcarriers for WLANs is $N = 52$ [1]. Assuming that 16 bits are used to represent a real-valued number, 320 bits are required to feedback the full CSI, and therefore, more than 6 bits of feedback per subcarrier (or, equivalently, more than $6N$ bits in total) are required in this case. Thus, the question how to make use of only one feedback bit per subcarrier in an efficient way is of a great practical interest.

A. Adaptive Subcarrier Selection

The idea of the considered subcarrier-selection strategy is that subcarriers which are affected by a deep fading should be excluded, and only subcarriers with high channel gains should be used.[1]

The feedback in the system with ASCS can be organized in the following way. The MS sorts the channel gains in all N subcarriers, and picks R subcarriers with the highest channel gains. If some particular subcarrier has been selected, "1" is transmitted back to the BS to indicate that this particular subcarrier should be used; otherwise, "0" is transmitted to indicate that this subcarrier should be dropped. The BS equally distributes the available power among the selected subcarriers. In order to keep constant data rate for different numbers of selected subcarriers, different types of signal modulation may be used.

To determine the optimal number of subcarriers, a theoretical analysis of the error probability is required. However, such an analysis appears to be a very difficult task, because it involves order statistics of correlated random variables (channel gains of different subcarriers). Another possibility to determine the optimal number of subcarriers is to resort to offline simulations based on channel profiles. In this case, channel profiles need to be measured for different wave-propagation environments. However, in practice, channel measurements may be prohibitively expensive.

B. Adaptive Power Allocation

As an alternative to the ASCS strategy, the one-bit-per-subcarrier CSI feedback can be used to adaptively allocate transmitted powers according to the channel gain at each subcarrier, under the constraint that the average transmitted power per subcarrier is fixed [4]. In the practical (sufficiently high) SNR range, it is known that more power should be allocated to faded subcarriers than to nonfaded ones to minimize the BER [4]. However, as we will see below, at low SNRs, the situation may be reversed, that is, the BER is minimized when more power is allocated to nonfaded subcarriers with high channel gains.

In what follows, we present a theoretical study of the average BER of the APA strategy, and further optimize this power-allocation scheme.

If the Gray mapping is used to map bits into symbols, the BER can be approximated as [17]

$$P_b \approx \frac{P_s}{\log_2 M} \tag{4}$$

where P_s denotes the symbol-error rate (SER).

Using (4), the BERs in the cases of M-ary phase-shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) can be evaluated as [18]

$$P_b(\text{MPSK}) \approx \frac{1}{\pi \log_2 M} \int_0^{\frac{M-1}{M}\pi} \int_0^{\infty} \exp\left(-\frac{g_{\text{PSK}}\alpha^2 E_s}{\sin^2\phi\, \sigma_v^2}\right) p(\alpha) d\alpha d\phi$$

$$P_b(\text{MQAM}) \approx \frac{1}{\log_2 M} \times \left[\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}}\int_0^{\infty} \exp\left(-\frac{g_{\text{QAM}}\alpha^2 E_s}{\sin^2\phi\, \sigma_v^2}\right) p(\alpha) d\alpha d\phi - \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2 \int_0^{\frac{\pi}{4}}\int_0^{\infty} \exp\left(-\frac{g_{\text{QAM}}\alpha^2 E_s}{\sin^2\phi\, \sigma_v^2}\right) p(\alpha) d\alpha d\phi\right]$$

where E_s is the transmitted signal power, $g_{\text{PSK}} = \sin^2(\pi/M)$, and $g_{\text{QAM}} = 3/(2(M-1))$.

If the channel is quantized to I levels, the BER of the adaptive OFDM system that uses the APA technique can be computed for the cases of MPSK and MQAM as

$$P_b^{\text{APA}}(\text{MPSK}, \boldsymbol{\kappa}, \boldsymbol{\gamma}) \approx \frac{1}{\pi \log_2 M} \times \sum_{i=1}^{I} \int_0^{\frac{M-1}{M}\pi} \int_{\Omega_i} \exp\left(-\frac{g_{\text{PSK}}\alpha^2\gamma_i E_s}{\sin^2\phi\, \sigma_v^2}\right) p(\alpha) d\alpha d\phi \tag{5}$$

$$P_b^{\text{APA}}(\text{MQAM}, \boldsymbol{\kappa}, \boldsymbol{\gamma}) \approx \frac{1}{\log_2 M} \times \left[\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right) \times \sum_{i=1}^{I} \int_0^{\frac{\pi}{2}} \int_{\Omega_i} \exp\left(-\frac{g_{\text{QAM}}\alpha^2\gamma_i E_s}{\sin^2\phi\, \sigma_v^2}\right) p(\alpha) d\alpha d\phi - \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2 \times \sum_{i=1}^{I} \int_0^{\frac{\pi}{4}} \int_{\Omega_i} \exp\left(-\frac{g_{\text{QAM}}\alpha^2\gamma_i E_s}{\sin^2\phi\, \sigma_v^2}\right) p(\alpha) d\alpha d\phi\right] \tag{6}$$

respectively. Here, $\boldsymbol{\kappa} = [\kappa_1, \ldots, \kappa_{I-1}]^T$ is the vector of the thresholds, $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_I]^T$ is the vector of the normalized transmitted powers, and $\Omega_i = [\kappa_{i-1}; \kappa_i)$ $(i = 1, \ldots, I)$ are the channel gain intervals with $\kappa_0 = 0$ and $\kappa_I = \infty$.

Let us now obtain the optimal vectors $\boldsymbol{\kappa}$ and $\boldsymbol{\gamma}$ which minimize (5) or (6), subject to both the average and peak transmit-power constraints. Such optimal vectors $\boldsymbol{\kappa}$ and $\boldsymbol{\gamma}$ can be found as a solution to the following constrained optimization problem:

$$\begin{aligned} \min_{\boldsymbol{\kappa},\boldsymbol{\gamma}} \quad & P_b^{\text{APA}}(\mathcal{M}, \boldsymbol{\kappa}, \boldsymbol{\gamma}) \\ \text{s.t.} \quad & \sum_{i=1}^{I} \int_{\Omega_i} \gamma_i p(\alpha) d\alpha = 1 \\ & 0 < \gamma_i < \gamma_M, \quad i = 1, \ldots, I \\ & 0 < \kappa_l < \infty, \quad l = 1, \ldots, I-1 \end{aligned} \tag{7}$$

where $\mathcal{M}$ denotes either MPSK or MQAM, and γ_M is the normalized maximum transmitted power which is determined by the transmission hardware peak power. The first constraint in

[1]Such a strategy is, however, somewhat different from the techniques used in [9], [15], and [16], where other approaches to subcarrier selection are addressed.

(7) limits the normalized average transmitted power, while the next I constraints in (7) limit the normalized peak transmitted powers.

Inserting (3) into (7), we can see that the objective of the problem (7) is a highly nonlinear and nonconvex function. To solve (7), the method of [19] can be used. Its idea is to quantize the parameters $\boldsymbol{\kappa}$ and $\boldsymbol{\gamma}$ and obtain a suboptimal solution using dynamic programming.

Let us consider three cases for the APA technique with average one-bit-per-subcarrier feedback.

Case A: The feedback for all subcarriers is used, and the channel at each subcarrier is quantized to two levels. Then, the optimization parameters of (7) become $\boldsymbol{\kappa} = \kappa$, $\boldsymbol{\gamma} = [\gamma_1, \gamma_2]^T$ and, with one-bit-per-subcarrier CSI feedback, APA can be implemented in the following way. If the channel gain of some subcarrier is below a certain threshold κ, the feedback bit "0" is transmitted to the BS and, in this case, the BS allocates the transmitted power γ_1 to this particular subcarrier. Otherwise, the feedback bit "1" is transmitted to the BS, and it allocates the transmitted power γ_2 to this subcarrier. We refer to this technique as *conventional APA*. For example, for QPSK modulation, the objective function of (7) becomes

$$P_b^{\mathrm{APA}}(\mathrm{QPSK}, \kappa, \gamma_1, \gamma_2) = \frac{1}{2\pi}\left[\int_0^{\frac{3}{4}\pi}\int_0^{\kappa} \exp\left(-\frac{\alpha^2\gamma_1 E_s}{2\sin^2\phi\,\sigma_v^2}\right) p(\alpha)d\alpha d\phi + \int_0^{\frac{3}{4}\pi}\int_{\kappa}^{\infty} \exp\left(-\frac{\alpha^2\gamma_2 E_s}{2\sin^2\phi\,\sigma_v^2}\right) p(\alpha)d\alpha d\phi\right]. \quad (8)$$

Case B: Let us now consider the effect of correlation of the channel gains between subcarriers. Using (2), this correlation can be computed as

$$\mathrm{E}\{d_i d_k^*\} = \frac{1}{N}\sum_{l=1}^{L}\sigma_l^2 \exp\left(-\frac{j2\pi(i-k)\tau_l}{NT}\right). \quad (9)$$

From (9), it follows that the intersubcarrier correlation reduces when L is increased. However, even for a reasonably large L, the channel gains of adjacent subcarriers remain highly correlated. This fact can be exploited in the following way. The CSI feedback can be provided for every other subcarrier (i.e., for the subcarriers with the indices $2, 4, 6, \ldots$) rather than for each subcarrier. Then, subcarrier 1 uses the CSI feedback of subcarrier 2, and subcarrier 3 uses that of subcarrier 4, and so on. As a result, the CSI feedback is required for $N/2$ subcarriers only. In this case, we can use 2 bits of feedback per subcarrier and still have N bits of feedback in total. If such an approach is adopted, then four normalized transmitted power levels γ_i $(i = 1, 2, 3, 4)$, and correspondingly, three thresholds κ_l $(l = 1, 2, 3)$ can be used in the APA scheme. Hereafter, we refer to this technique as *modified APA*. Using QPSK, the corresponding BER can be computed by substituting $M = 4$ and $I = 4$ in (5).

Case C: Another important question is whether it is beneficial to reduce the total number of subcarriers but to increase the constellation dimension. For example, if the number of subcarriers is reduced twice (to $N/2$), then the same amount of information at the same rate can be transmitted by using the constellation whose dimension is four times higher than in the case of N subcarriers. For example, if the QPSK modulation has been used in the case of N subcarriers, then 16-QAM should be used in the case of $N/2$ subcarriers to maintain the same data transmission rate. Inserting $M = 16$ and $I = 4$ into (6), we obtain the BER of the OFDM scheme with APA that uses N bits of feedback, $N/2$ subcarriers, and 16-QAM. Then, the optimal vectors $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \gamma_3, \gamma_4]^T$ and $\boldsymbol{\kappa} = [\kappa_1, \kappa_2, \kappa_3]^T$ can be found by solving (7) with the objective function $P_b^{\mathrm{APA}}(16\text{-QAM}, \boldsymbol{\kappa}, \boldsymbol{\gamma})$. Hereafter, we refer to this technique as *APA with reduced number of subcarriers*.

TABLE I
OPTIMAL PARAMETERS OF APA FOR CASES A, B, AND C

SNR (dB)		0	5	10	15	20	25
A	κ	0.4724	1.1774	0.7147	0.4724	0.3246	0.2265
	γ_1	0.2	1.1	1.6	2.6	4.6	10.5
	γ_2	1.2	0.7	0.6	0.6	0.6	0.5
B	κ_1	0.3654	0.2792	0.5049	0.2265	0.1591	0.1591
	κ_2	0.6269	0.8936	0.7732	0.4031	0.3246	0.3246
	κ_3	1.5174	1.3774	1.1362	0.7147	0.5972	0.7147
	γ_1	0.1	0.3	2.0	5.8	16.3	28.3
	γ_2	0.7	1.3	1.1	2.2	2.7	1.5
	γ_3	1.3	0.9	0.7	1.0	0.9	0.4
	γ_4	1.0	0.6	0.4	0.4	0.3	0.1
C	κ_1	0.4031	0.2265	0.2792	0.4386	0.2265	0.1591
	κ_2	0.8027	0.5364	0.8326	0.6563	0.4386	0.3246
	κ_3	0.9904	1.5174	1.2213	0.9904	0.8027	0.6856
	γ_1	0.1	0.1	0.3	2.3	6.8	18.7
	γ_2	0.4	0.3	1.4	1.3	2.1	2.5
	γ_3	1.2	1.3	0.9	0.8	0.8	0.8
	γ_4	1.8	0.9	0.6	0.4	0.3	0.2

Table I shows the optimal parameters for the APA scheme for all the aforementioned cases. The optimal values are found by solving the problem (7) in each particular case.

C. Adaptive Modulation Selection

The AMS scheme is based on the following idea. When a certain subcarrier is corrupted by fading, a constellation with smaller dimension and higher transmitted power can be assigned to this particular subcarrier, while constellations of larger dimensions and less transmitted power can be assigned to the subcarriers whose channel gain is high. Similar to the case of ASCS, a low-rate one-bit-per-subcarrier feedback can be used to divide the subcarriers into two groups that use different constellations and transmitted powers.

For example, to achieve the data rate of 2 b/s per subcarrier, we can use the BPSK modulation for faded subcarriers and the 8-PSK modulation for nonfaded subcarriers. In this case, the data rate can be expressed as $\log_2 2\int_0^{\xi} p(\alpha)d\alpha + \log_2 8\int_{\xi}^{\infty} p(\alpha)d\alpha$, where the threshold ξ of the channel gain is used to divide subcarriers into "faded" and "nonfaded" groups. Taking into account that the data rate of 2 b/s per subcarrier is chosen, the value of ξ can be found by solving the following data-rate constraint equation:

$$\int_0^{\xi} p(\alpha)d\alpha + 3\int_{\xi}^{\infty} p(\alpha)d\alpha = 2. \quad (10)$$

TABLE II
OPTIMAL PARAMETERS OF AMS

SNR (dB)	0	5	10	15	20	25
γ_1	1.2925	1.0945	1.0554	1.2629	1.5799	1.8049
γ_2	0.7075	0.9055	0.9446	0.7371	0.4201	0.1951

Using (3), we obtain from (10) that $\xi = \sqrt{\ln 2}$. Then, the BER for this particular AMS scheme can be written as

$$P_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2) = \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{\ln 2}} \exp\left(-\frac{\alpha^2\gamma_1 E_s}{\sin^2\phi\,\sigma_v^2}\right)p(\alpha)d\alpha d\phi + \frac{1}{3}\int_0^{\frac{7\pi}{8}}\int_{\sqrt{\ln 2}}^{\infty} \exp\left(-\frac{\sin^2(\pi/8)\alpha^2\gamma_2 E_s}{\sin^2\phi\,\sigma_v^2}\right)p(\alpha)d\alpha d\phi\right]. \quad (11)$$

The following constrained optimization problem should be solved to obtain the optimum power allocation in this case:

$$\min_{\gamma_1,\gamma_2} \quad P_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2) \quad \text{s.t.} \quad \gamma_1 + \gamma_2 = 2, \quad 0 < \gamma_1, \gamma_2 < 2. \quad (12)$$

This problem is convex with respect to γ_1 and γ_2 and can be easily solved by using, for example, the bisection method. Table II lists the values of optimal power allocation for BPSK and 8-PSK constellations obtained by solving (12).

IV. IMPERFECT ONE-BIT-PER-SUBCARRIER CSI FEEDBACK

In the previous section, we assumed that the feedback channel is perfect. However, in real-world applications, this channel may be erroneous and/or may suffer from a feedback delay. In this section, we extend the results of the previous section to the imperfect-feedback channel case by considering two types of imperfections: feedback errors and delays. Since the theoretical analysis of the BER of the ASCS scheme is intractable, we study the ASCS strategy by simulations presented in Section V. A theoretical analysis of the APA and AMS techniques is given in this section.

A. Erroneous Feedback Channel

We model the erroneous feedback channel as a binary symmetric channel with the error probability p. Note that the performance gain obtained from one-bit CSI feedback decreases with increasing p, and if p is high enough, then such erroneous CSI feedback can sometimes even worsen the system performance. Therefore, it is important to study the impact of erroneous feedback on the performance of the adaptive OFDM techniques.

1) Adaptive Power Allocation: Here, we consider only the conventional APA scheme, because obtaining optimal parameters for modified APA and APA with a reduced number of subcarriers in the case of imperfect feedback seems to be mathematically intractable. Taking feedback errors into account, the BER can be computed as

$$Q_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_1, \gamma_2; p) = (1-p)P_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_1, \gamma_2) + p\,P_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_2, \gamma_1). \quad (13)$$

Inserting (8) into (13), we obtain the BER of the APA scheme. In the case of erroneous CSI feedback, the power constraint should also be modified as follows:

$$p\left(\int_0^{\kappa}\gamma_1 f(\alpha)d\alpha + \int_{\kappa}^{\infty}\gamma_2 f(\alpha)d\alpha\right) + (1-p)\left(\int_0^{\kappa}\gamma_2 f(\alpha)d\alpha + \int_{\kappa}^{\infty}\gamma_1 f(\alpha)d\alpha\right) = 1. \quad (14)$$

Therefore, the optimal parameters from Table I (A) cannot be used in the case of erroneous CSI feedback, because (14) is not satisfied. However, if the error probability p is known at the transmitter, we can find the optimal values of κ, γ_1, and γ_2 that optimize the performance of the conventional APA scheme under erroneous feedback. These values can be found by solving the problem similar to (7), where $Q_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_1, \gamma_2; p)$ of (13) is used as the objective function, and (14) is used as the first constraint, where $I = 2$, $\boldsymbol{\kappa} = \kappa$, and $\boldsymbol{\gamma} = [\gamma_1, \gamma_2]^T$. Table III summarizes the optimal parameters for the conventional APA scheme with erroneous feedback when the probability of error in the feedback channel is equal to 0.15 and 0.4, respectively. It can be seen that when the feedback channel becomes less reliable, it is judicious to equally distribute the power between "faded" and "nonfaded" channel realizations.

2) Adaptive Modulation Selection: For the AMS scheme with erroneous CSI feedback the BER can be calculated as

$$Q_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2; p) = (1-p)P_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2) + pP_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_2, \gamma_1). \quad (15)$$

Inserting (11) into (15), we can obtain the BER for the AMS scheme. In particular, we will find the critical value of the error probability p, above which one-bit-per-subcarrier feedback can only worsen the system performance.[2] Specifically, the feedback remains meaningful only if the following condition is satisfied:

$$Q_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2; p) \le P_b(\text{QPSK}). \quad (16)$$

If (16) holds as equality, we obtain the critical value of p. These values for different SNRs are listed in Table III, in the case when the optimal parameters from Table II are used. We can see from this table that the critical error probability of the feedback

[2]For the APA scheme, the critical p is meaningless, because the power constraint is not satisfied.

TABLE III
CRITICAL PROBABILITIES OF FEEDBACK ERROR AND OPTIMAL PARAMETERS OF CONVENTIONAL APA AND AMS WITH ERRONEOUS FEEDBACK CHANNEL

SNR (dB)			0	5	10	15	20	25
APA	$p = 0.15$	κ	0.61	1.07	0.73	0.49	0.32	0.21
		γ_1	0.71	1.13	1.41	1.84	2.36	2.82
		γ_2	1.17	0.78	0.68	0.64	0.62	0.60
	$p = 0.4$	κ	0.65	1.04	0.72	0.49	0.32	0.20
		γ_1	0.94	1.05	1.12	1.18	1.21	1.23
		γ_2	1.05	0.94	0.89	0.86	0.85	0.84
AMS	p		0	0	0.1975	0.2079	0.1277	0.0603
	$p = 0.15$	γ_1	1.3110	1.0596	0.9812	1.0628	1.0812	1.0653
		γ_2	0.6890	0.9404	1.0188	0.9372	0.9188	0.9347

channel depends on the SNR conditions of the communication channel.

However, if the error probability p is known at the transmitter, we can find the optimal values of γ_1 and γ_2 that optimize the performance of the AMS scheme under the erroneous feedback channel. These optimal values can be found as a solution to the problem similar to (12), where $Q_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2; p)$ of (15) should be used as the objective function. The optimal values of γ_1 and γ_2 for the AMS scheme with erroneous feedback channel with $p = 0.15$ are listed in Table III.

B. Delayed Feedback Channel

The second source of imperfections in the feedback channel is the delay between the actual CSI and the CSI received at the transmitter. Therefore, it is also important to study the impact of outdated CSI on the APA and AMS approaches.

Let α_0 and α_τ be the channel gains at the time slots 0 and τ, respectively. It has been shown in [20] and [21, p. 142] that the joint pdf of α_0^2 and α_τ^2 has the following form:

$$f_{\alpha_0^2,\alpha_\tau^2}(x, y; \rho) = \frac{1}{1-\rho} \exp\left(-\frac{x+y}{1-\rho}\right) I_0\left(\frac{2\sqrt{\rho xy}}{1-\rho}\right) \quad (17)$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind of the order zero, and

$$\rho = \mathrm{cov}(x, y)/\sqrt{\mathrm{var}(x)\mathrm{var}(y)}$$

is the correlation coefficient which characterizes the feedback delay.

1) Adaptive Power Allocation: Using (17), we obtain that in the case of delayed one-bit CSI feedback and QPSK modulation, the BER for the APA scheme can be written as

$$\begin{aligned} &R_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_1, \gamma_2; \rho) \\ &= \frac{1}{2\pi}\left[\int_0^{\kappa^2}\int_0^{\infty}\int_0^{\frac{3\pi}{4}} \exp\left(-\frac{xE_s\gamma_1}{2\sin^2\phi\,\sigma_v^2}\right) f_{\alpha_0^2,\alpha_\tau^2}(x, y; \rho)\, d\phi\, dx\, dy \right. \\ &\quad \left. + \int_{\kappa^2}^{\infty}\int_0^{\infty}\int_0^{\frac{3\pi}{4}} \exp\left(-\frac{xE_s\gamma_2}{2\sin^2\phi\,\sigma_v^2}\right) f_{\alpha_0^2,\alpha_\tau^2}(x, y; \rho)\, d\phi\, dx\, dy\right]. \end{aligned} \quad (18)$$

Using the following property of the first-order Marcum Q-function [21, p. 75]

$$\int_0^{\infty} u \exp\left(-\frac{u^2+v^2}{2}\right) I_0(vu)\, du = 1$$

we can simplify the integral in (18) as

$$\begin{aligned} R_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_1, \gamma_2; \rho) = \frac{1}{2\pi}\int_0^{\frac{3\pi}{4}} \left[\frac{1}{A_1+1}\left(1-\exp(-B_1\kappa^2)\right)\right. \\ \left. + \frac{1}{A_2+1}\exp(-B_2\kappa^2)\right] d\phi \end{aligned} \quad (19)$$

where

$$\begin{aligned} A_i &= \gamma_i E_s/(2\sin^2\phi\,\sigma_v^2), \quad i = 1, 2 \\ B_i &= (\gamma_i E_s + 2\sin^2\phi\,\sigma_v^2)/(\gamma_i E_s(1-\rho) + 2\sin^2\phi\,\sigma_v^2), \\ &\quad i = 1, 2. \end{aligned}$$

It can be seen that if there is no feedback delay (i.e., $\rho = 1$), (19) yields the same BER result as in (8). It is also worth noting that when increasing the delay τ, the coefficient ρ decreases, but the BER increases. Therefore, we can find the critical value of the correlation coefficient ρ under which the CSI feedback becomes meaningless. This can be done by solving the following equation:

$$R_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_1, \gamma_2; \rho) = P_b(\text{QPSK}). \quad (20)$$

The critical values for the coefficient ρ at different SNRs for optimal parameters from Table I (A) can be found using (20), and are listed in Table IV. As we can see, for some values of SNR, the critical value of the coefficient ρ can be quite large and, thus, only very short feedback delays can be tolerated by the communication system. Moreover, similar to the critical value of p in Table III, the critical value of ρ in Table IV depends on SNR nonmonotonically.

If ρ is known at the transmitter, we can find the values of κ, γ_1, and γ_2 that optimize the performance of the APA scheme under delayed CSI feedback. These values can be found by solving the problem similar to (7), where $R_b^{\mathrm{APA}}(\text{QPSK}, \kappa, \gamma_1, \gamma_2; \rho)$ given by (19) should be used as the objective function, and where $I = 2$, $\boldsymbol{\kappa} = \kappa$, and $\boldsymbol{\gamma} = [\gamma_1, \gamma_2]^T$. Table IV summarizes the optimal parameters for the conventional APA scheme when $\rho = 0.7$.

TABLE IV
CRITICAL ρ AND OPTIMAL PARAMETERS OF CONVENTIONAL APA AND AMS WITH FEEDBACK DELAY

SNR (dB)			0	5	10	15	20	25
APA	ρ		0.8121	0.4518	0.6489	0.7556	0.8510	0.9404
	$\rho = 0.7$	κ	0.5364	1.1774	0.9572	0.8326	0.8326	0.9572
		γ_1	0.7	1.1	1.2	1.4	1.5	1.4
		γ_2	1.1	0.7	0.7	0.6	0.5	0.4
AMS	ρ		1	1	0.7515	0.6725	0.7455	0.8172
	$\rho = 0.7$	γ_1	1.3131	1.0426	0.9615	1.0057	1.0834	1.1367
		γ_2	0.6869	0.9574	1.0385	0.9943	0.9166	0.8633

2) Adaptive Modulation Selection: Let us study the performance of the AMS scheme in the case of outdated CSI feedback. Using (11) and (17), we obtain the BER of the AMS scheme in the following form:

$$R_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2; \rho) = \frac{1}{\pi}\left[\int_0^{\ln 2}\int_0^{\infty}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{xE_s\gamma_1}{\sin^2\phi\,\sigma_v^2}\right) f_{\alpha_0^2,\alpha_\tau^2}(x,y;\rho)\,d\phi\,dx\,dy + \frac{1}{3}\int_{\ln 2}^{\infty}\int_0^{\infty}\int_0^{\frac{7\pi}{8}} \exp\left(-\frac{\sin^2(\pi/8)xE_s\gamma_2}{\sin^2\phi\,\sigma_v^2}\right) \times f_{\alpha_0^2,\alpha_\tau^2}(x,y;\rho)\,d\phi\,dx\,dy\right]. \quad (21)$$

Then, the critical value of ρ can be found by solving the following equation:

$$R_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}, \gamma_1, \gamma_2; \rho) = P_b(\text{QPSK}). \quad (22)$$

Table IV summarizes the critical values of the coefficient ρ calculated using (22) for different SNRs and for optimal parameters from Table II. Comparing the critical values of ρ for conventional APA and AMS approaches, we observe that at moderate/high SNRs, the AMS scheme is more robust to CSI feedback delays than the conventional APA approach.

Moreover, the performance of the AMS scheme can be improved if the coefficient ρ is known at the transmitter. In this case, the optimal values of γ_1 and γ_2 can be found by solving the problem similar to (12), where $R_b^{\mathrm{AMS}}(\text{BPSK, 8-PSK}\gamma_1, \gamma_2; \rho)$ of (21) should be used as the objective function. The optimal values of γ_1 and γ_2 for the AMS scheme with delayed CSI feedback when $\rho = 0.7$ are shown in Table IV.

V. SIMULATION EXAMPLES

The channel model used in our simulations is based on the ETSI "Vehicular A" channel environment [14]. In all examples, we assume that the BS transmits at the fixed data rate of $n_r = 128$ b/s, and the number of subcarriers is $N = 64$.

A. Perfect CSI Feedback

1) Adaptive Subcarrier Selection: In the first example, three different system configurations are compared: where no subcarrier selection is used, where 32 "best" subcarriers are selected, and where 16 "best" subcarriers are selected. To keep the same data rate for each system configuration, we use the QPSK modulation for no subcarrier selection, 16-QAM for the selection of 32 subcarriers, and 256-QAM for the selection of 16 subcarriers.

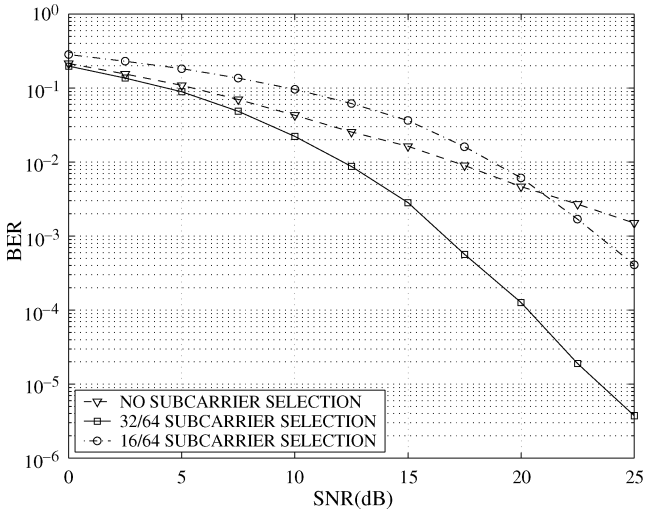

Fig. 1. BER versus SNR; perfect CSI feedback. ASCS with different numbers of selected subcarriers.

Fig. 1 shows the performance of all three system configurations in terms of BER versus SNR. The tradeoff between the number of subcarriers and the modulation used can be seen from this figure. In particular, the adaptive selection of 32 subcarriers has the best performance among the system configurations tested. However, the adaptive selection of 16 subcarriers has much worse performance than that of 32 subcarriers, and at low and moderate SNRs, can even perform worse than the configuration without subcarrier selection.

We stress here that since no theoretical analysis of BER is possible for the ASCS scheme, this may limit its practical application. Moreover, in multiuser OFDM communication scenarios, only a small number of subcarriers may be assigned to each user, and this scheme may not be applicable.

2) Adaptive Power Allocation: Hereafter, in each figure, we display theoretical BER curves that correspond to the derived analytical expressions, and the numerical BER curves obtained via simulations.

BERs of the conventional and modified APA schemes are shown versus SNR in Fig. 2. The QPSK modulation and the optimal parameters from Table I (A) and (B) are used, respectively, for these two schemes. The theoretical BER for the modified APA scheme with the optimal parameters from Table I (B)

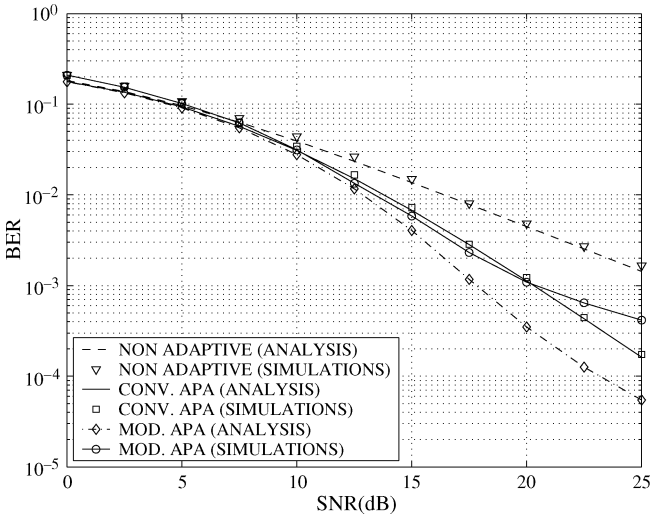

Fig. 2. BER versus SNR; perfect CSI feedback. Conventional and modified APA.

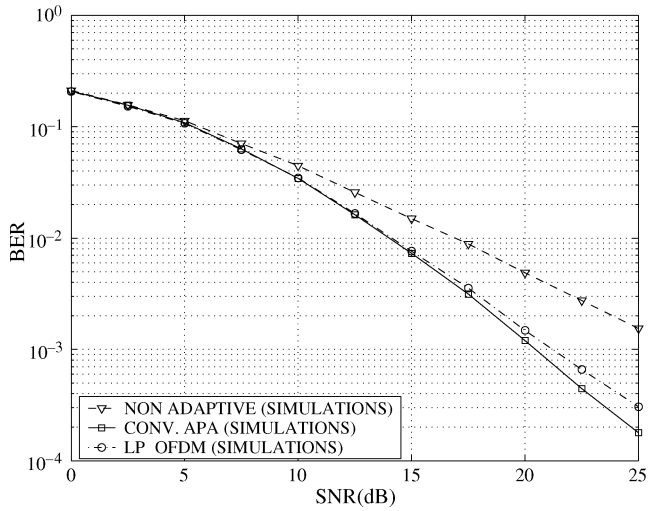

Fig. 3. BER versus SNR; perfect CSI feedback. Conventional APA and LP-OFDM.

assumes that there is a full correlation between each pair of adjacent subcarriers.

From Fig. 2, it can be seen that both tested APA schemes outperform the nonadaptive OFDM scheme. There is a gap between the analysis and simulation results for the BER of the modified APA scheme, because the channel gains of adjacent subcarriers are not fully correlated. In fact, we can not group more than two subcarriers together, even if L is relatively small, since the correlation of channel gains of nonadjacent subcarriers in one subcarrier group is smaller than that of adjacent subcarriers.

Comparing the results of Fig. 2 with Fig. 1, we can see that the APA approach is less efficient than the ASCS scheme with 32 selected subcarriers. This is especially true at high SNRs. However, the APA scheme allows an easier optimization as compared with the ASCS approach.

The BER performances of the conventional APA technique and the LP-OFDM approach of [3] are compared in Fig. 3.

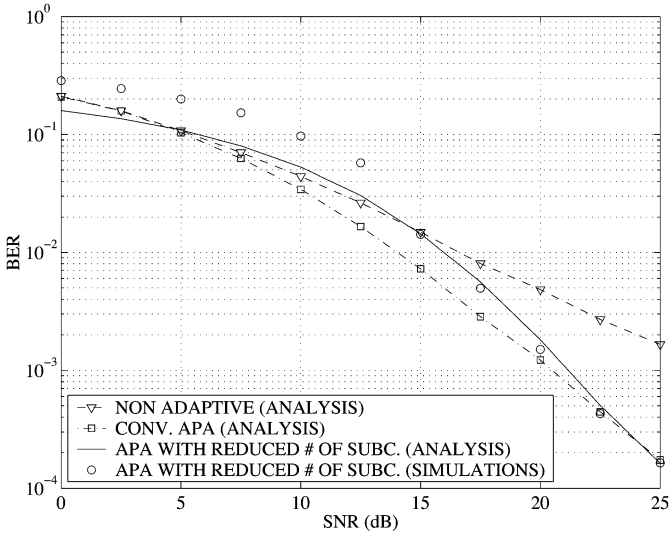

Fig. 4. BER versus SNR; perfect CSI feedback. Conventional APA and APA with reduced number of subcarriers.

The minimal mean-square error (MMSE) equalizer is used in the latter approach, and the optimal subcarrier grouping with $K = 4$ subcarriers[3] in each subcarrier subset is applied. As can be seen from Fig. 3, the conventional APA scheme outperforms the LP-OFDM approach of [3]. It should also be taken into account that the conventional APA scheme enables much simpler receiver structure than the LP-OFDM approach, because for the latter approach, a channel equalizer is needed at each receiver.[4]

Fig. 4 shows BER of the APA scheme with a reduced number of subcarriers versus SNR. The optimal parameters from Table I (C) are used to obtain this figure. It can be seen that this scheme performs better than the nonadaptive OFDM scheme at moderate and high SNRs. However, it has higher BER than the conventional APA scheme and the ASCS scheme with 32 selected subcarriers in the SNR interval of [0; 20] dB. In other words, the APA scheme with a reduced number of subcarriers does not bring any performance improvements, as compared with the conventional APA approach. Note that due to the fact that (4) is an approximation, the theoretical and numerical curves do not coincide at low SNRs in the case when large constellation dimensions are used.

3) Adaptive Modulation Selection: Fig. 5 displays the BER of the AMS scheme with the optimal parameters taken from Table II versus SNR. In this figure, the BPSK and 8-PSK modulations are used at "faded" and "nonfaded" subcarriers, respectively. It can be seen from Fig. 5 that the AMS scheme outperforms the nonadaptive approach. However, the AMS scheme has higher BER than the conventional APA approach. Moreover, comparing Fig. 5 with Figs. 2 and 3, we can notice that the AMS scheme has higher BER than the modified APA approach, but outperforms the APA approach with reduced number of subcarriers at low and moderate SNRs.

[3]Note that this number of subcarriers in each subcarrier subset provides a reasonable complexity-performance tradeoff [3].

[4]Although such an equalizer amounts to diagonal matrix inversion followed by FFT, and therefore, is computationally attractive, its complexity is still much higher than that required for implementation of our adaptive OFDM techniques.

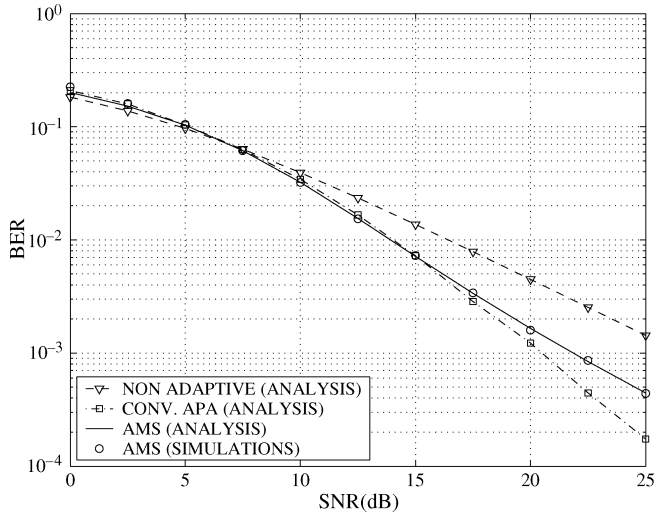

Fig. 5. BER versus SNR; perfect CSI feedback. Conventional APA and AMS.

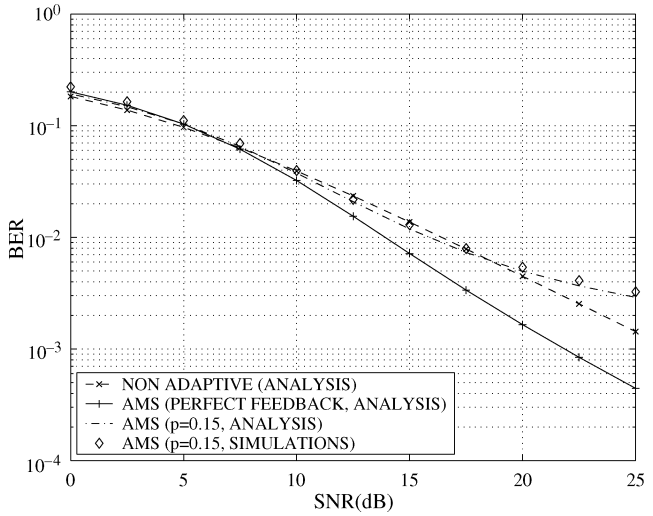

Fig. 6. BER versus SNR; erroneous CSI feedback. AMS.

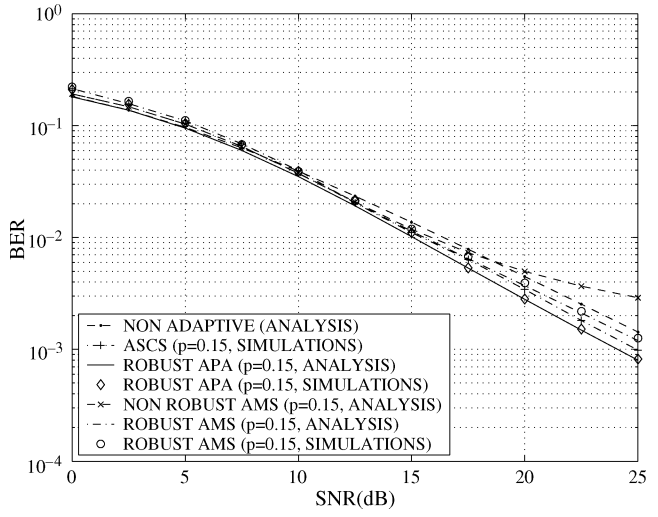

Fig. 7. BER versus SNR; erroneous CSI feedback. ASCS, conventional APA, and AMS (nonrobust and robust).

B. Imperfect CSI Feedback

1) Erroneous Feedback Channel: Fig. 6 displays BER of the AMS scheme with erroneous feedback channel versus SNR. In this figure, $p = 0.15$ is taken. The optimal parameters from Table II are selected, and the BPSK and 8-PSK modulations are used at "faded" and "nonfaded" subcarriers, respectively.

We can see from this figure that the performance of the AMS approach degrades in the case of erroneous feedback, compared with the perfect feedback case. For example, at the BER of $3 \cdot 10^{-3}$, the performance degradation of the AMS approach with erroneous feedback is 7.5 dB, compared with the performance of the AMS approach with perfect feedback. Moreover, at high SNRs, the AMS scheme performs worse than the nonadaptive scheme.

It is worth noting that these simulation results also agree with the results for critical values of p shown in Table III. In particular, from Table III, it can be seen that the critical value of the error probability p at low SNRs of 0 and 5 dB is equal to zero, which means that the AMS scheme cannot tolerate any errors in the feedback channel. Indeed, from Fig. 6, we can observe that the theoretical BER of the AMS scheme is larger than that of the nonadaptive scheme at the SNRs of 0 and 5 dB. Moreover, as we can see from Table III, the critical values of p at the SNRs of 20 and 25 dB are lower than the value of p used in simulations and, indeed, the performance of the AMS scheme is worse than the performance of the nonadaptive scheme in the SNR interval of 20, 25 dB (see Fig. 6).

The BERs of the ASCS, conventional APA, and AMS schemes versus SNR are shown in Fig. 7 for $p = 0.15$. The "best" 32 subcarriers are selected in the ASCS scheme and 16-QAM is applied at the selected subcarriers. The QPSK modulation is used in this figure for the APA technique. The optimal parameters from Table III are used for the APA technique, which we refer to as "robust APA." This terminology reflects the fact that p is known and is used in the APA technique to obtain the optimal parameters κ, γ_1, and γ_2. For the AMS scheme, the BPSK and 8-PSK modulations are used for "faded" and "nonfaded" subcarriers, respectively, and the optimal parameters from Table III are selected. The terms "robust AMS" and "nonrobust AMS" correspond to the cases of known and unknown p, respectively.

As can be seen from Fig. 7, the performance of the AMS schemes can be significantly improved if the error probability p is known at the transmitter. It can also be seen that the robust APA scheme outperforms the ASCS technique and the nonadaptive OFDM system.

2) Delayed Feedback Channel: Jakes' fading model is used in our examples with a delayed feedback channel [17]. The maximal Doppler frequency of 113 Hz is used, which corresponds to the vehicular speed of 72 km/h at the carrier frequency of 2 GHz. We take $\rho = 0.7$, which corresponds to the feedback delay of 24 symbol durations in the IS-136 standard [22].

The BER versus SNR curves for the APA and AMS schemes using the optimal parameters of Tables I and II, respectively, are shown in Fig. 8. The QPSK modulation is used for the conventional APA approach, while the BPSK and 8-PSK modulations are used at "faded" and "nonfaded" subcarriers, respectively, for the AMS scheme.

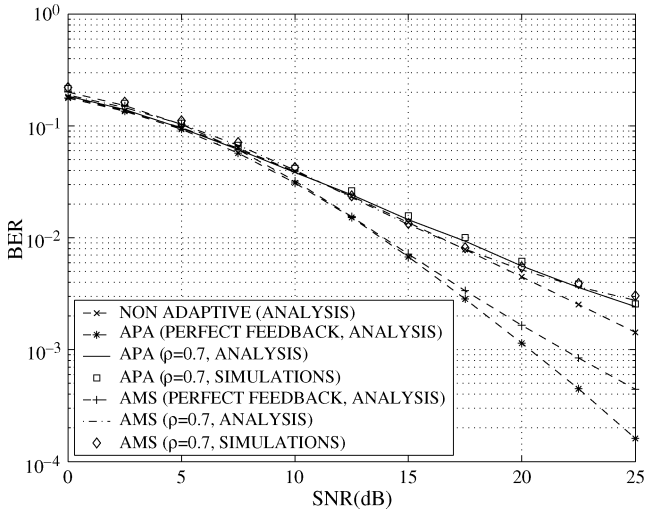

Fig. 8. BER versus SNR; delayed CSI feedback. Conventional APA and AMS.

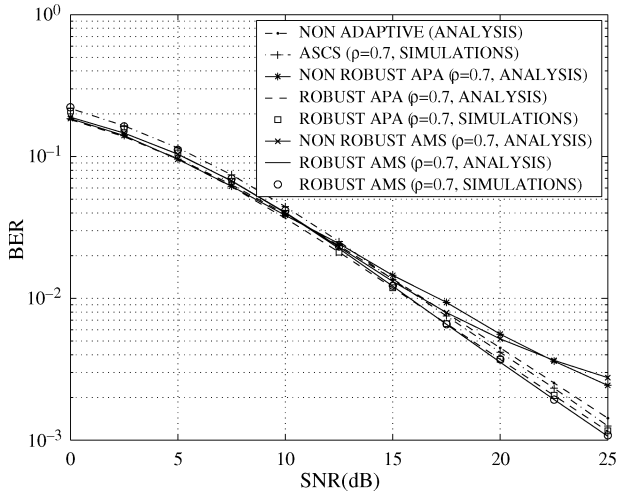

Fig. 9. BER versus SNR; delayed CSI feedback. ASCS, conventional APA, and AMS (nonrobust and robust).

From Fig. 8, we can conclude that at moderate SNRs, the APA approach is more sensitive to the delay in the feedback channel, as compared with the AMS approach, while at high SNRs, the AMS approach has higher sensitivity to the delay in the feedback channel than the APA approach. At the BER of $2 \cdot 10^{-3}$, the performance degradation of the APA and AMS schemes due to the feedback channel delay amounts to 7 dB. Moreover, the APA and AMS schemes show, in this case, worse performance than the nonadaptive OFDM technique.

It is also worth noting that the results of Fig. 8 agree with the results of Table IV. Indeed, the theoretical BER for the AMS technique can be seen to be higher than that for the nonadaptive scheme at SNRs of 0 and 5 dB. For these SNRs, as can be seen from Table IV, the critical value of the correlation coefficient ρ is equal to one, which means that the AMS scheme cannot tolerate any delays in the feedback channel. Moreover, the APA and AMS approaches perform worse than the nonadaptive scheme at the SNRs for which the critical value of ρ given in Table IV is higher than the value of 0.7 that is used in Fig. 8.

Fig. 9 displays the BERs of the ASCS, conventional APA, and AMS schemes with delayed feedback channel versus SNR. In this figure, the best 32 subcarriers are selected in the ASCS scheme and 16-QAM is applied to the selected subcarriers. For the APA and AMS approaches, the coefficient ρ is assumed to be known at the transmitter. Similar to the case of the erroneous feedback channel, we refer to such APA and AMS approaches as robust ones. Vice versa, we call the APA and AMS schemes nonrobust if the optimal parameters are found under the assumption that ρ is unknown. The optimal parameters for the robust APA and AMS techniques are shown in Table IV.

It can be clearly seen from Fig. 9 that the performance of both APA and AMS approaches can be substantially improved if the parameter ρ is known at the transmitter. It can also be seen that the robust APA and AMS schemes outperform the ASCS technique.

Finally, comparing Figs. 7 and 9 with Fig. 3, we can see that although the adaptive APA technique outperforms the LP-OFDM method of [3] in the perfect CSI feedback case, it may perform slightly worse than this method in the imperfect CSI case, if such imperfections are significant.

VI. Conclusions

The performance of OFDM communication systems with one-bit-per-subcarrier CSI feedback has been studied. Three adaptive techniques, including ASCS, APA, and AMS schemes, have been used to exploit such CSI feedback. We found that even one-bit-per-subcarrier CSI feedback can greatly improve the overall system performance if the feedback channel is perfect. Among the three approaches tested, the ASCS approach has the best performance when the feedback channel is perfect. However, the performance of OFDM systems with one-bit-per-subcarrier feedback can be even worse than the performance of the OFDM system without any feedback if the feedback channel is imperfect. It has been demonstrated that the performance of both the APA and AMS approaches can be substantially improved by exploiting the knowledge of how imperfect the feedback channel is. Study of imperfect feedback channel cases, such as the relationship between the critical error probability, SNR, thresholds, and transmit powers can be a proper subject for future study.

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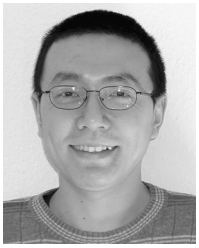

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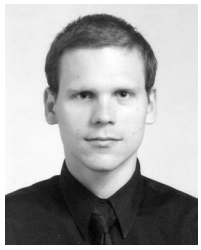

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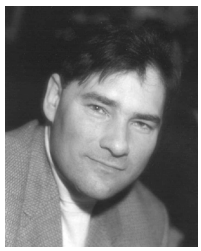

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