Predicting Stress and Strain of FRP Confined Square/Rectangular Columns Using Artificial Neural Networks

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Abstract

This study proposes the use of artificial neural networks (ANNs) to calculate the compressive strength and strain of fiber reinforced polymer (FRP) confined square/rectangular columns. Modeling results have shown that the two proposed ANN models fit the testing data very well. Specifically, the average absolute errors of the two proposed models are less than 5%. The ANNs were trained, validated, and tested on two databases. The first database contains the experimental compressive strength results of 104 FRP confined rectangular concrete columns. The second database consists of the experimental compressive strain of 69 FRP confined square concrete columns. Furthermore, this study proposes a new potential approach to generate a user-friendly equation from a trained ANN model. The proposed equations estimate the compressive strength/strain with small error. As such the equations could be easily used in engineering design instead of the "invisible" processes inside the ANN.

CE Database subject headings: Fiber Reinforced Polymer; Confinement; Concrete columns; Neural networks; Compressive strength; Computer model.

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Introduction

The use of FRP confined concrete columns has been proven in enhancing the strength and the ductility of columns. Over the last two decades, a large number of experimental and analytical studies have been conducted to understand and simulate the compressive behavior of FRP confined concrete. Experimental studies have confirmed the advantages of FRP confined concrete columns in increasing the compressive strength, strain, and ductility of columns (Hadi and Li 2004; Hadi 2006a; Hadi 2006b; Hadi 2007a, b; Rousakis et al. 2007; Hadi 2009; Wu and Wei 2010; Hadi and Widiarsa 2012; Hadi et al. 2013; Pham et al. 2013). Meanwhile, many stress-strain models were developed to simulate the results from experimental studies. Most of the existing models were based on the mechanism of confinement together with calibration of test results to predict the compressive stress and strain of FRP confined concrete columns (Lam and Teng 2003a; Ilki et al. 2008; Wu and Wang 2009; Wu and Wei 2010; Rousakis et al. 2012; Yazici and Hadi 2012; Pham and Hadi 2013; Pham and Hadi 2014). Models developed by this approach provide a good understanding of stress-strain curve of the confined concrete, but their errors in estimating the compressive strength and strain are still considerable. Bisby et al. (2005) had carried out an overview on confinement models for FRP confined concrete and indicated that the average absolute error of strain estimation ranges from 35% to 250% while the error of strength estimation is about 14% - 27%. In addition, Ozbakkaloglu et al. (2013) had reviewed 88 existing FRP confinement models for circular columns. That study showed that the average absolute errors of the above models in estimating stress and strain are greater than 10% and 23%, respectively. Thus, it is necessary for the research community to improve the accuracy of estimating both the compressive stress and strain of FRP confined concrete. This study introduces the use of artificial neural networks (ANNs) to predict the compressive strength and strain of FRP confined square/rectangular concrete columns given the input parameters including geometry of the section and mechanical properties of the materials.

ANN can be applied to problems where patterns of information represented in one form need to be mapped into patterns of information in another form. As a result, various ANN applications can be categorized as classification or pattern recognition or prediction and modeling. ANN is commonly used in many industrial disciplines, for example, banking, finance, forecasting, process engineering, structural control and monitoring, robotics, and transportation. In civil engineering, ANN has been applied to many areas, including damage detection (Wu et al. 1992; Elkordy et al. 1993), identification and control (Masri et al. 1992; Chen et al. 1995), optimization (Hadi 2003; Kim et al. 2006), structural analysis and design (Hajela and Berke 1991; Adeli and Park 1995), and shear resistance of beams strengthened with FRP (Perera et al. 2010a; Perera et al. 2010b).

In addition, ANN has also been used to predict the compressive strength of FRP confined circular concrete columns (Naderpour et al. 2010; Jalal and Ramezanianpour 2012). This study uses ANN to predict both the compressive strength and strain of FRP confined square/rectangular concrete columns. Furthermore, a new potential approach is introduced to generate predictive user-friendly equations for the compressive strength and strain.

Experimental Databases

The test databases used in this study is adopted from the studies by Pham and Hadi (2013; 2014). Details of the databases could be found elsewhere in these studies, but for convenience the main properties of specimens are summarized. It is noted that when the axial strain of unconfined concrete at the peak stress (ε_{co}) is not specified, it can be estimated using the equation proposed by Tasdemir et al. (1998) as follows:

$$\varepsilon_{co} = (-0.067 f_{co}^{'2} + 29.9 f_{co}^{'} + 1053)10^{-6} \tag{1}$$

In the literature, test results of the compressive strain of FRP confined concrete is relatively less than that of the compressive strength. If a database is used to verify both the strain and strength models, the size of this database will be limited by the number of specimens having results of the strain. Thus, in order to maximize the databases' size, this study uses two different databases for the two proposed models. In addition, studies about FRP confined rectangular specimens focused on confined strength but not strain. Thus data about confined strain of rectangular specimens reported are extremely limited. When the number of rectangular specimens is much fewer than that of square columns, it is not reliable to predict the compressive strain of the rectangular specimens by using a mixed database. Therefore, this paper deals with strain of square specimens only.

All specimens collated in the databases were chosen based on similar testing schemes, ratio of the height and the side length, failure modes, and similar stress-strain curves. The ratio of the height and the side length is 2. The aspect ratio of the rectangular specimens ranged between 1 and 2.7. Test results of the specimens which have a descending type in the stress-strain curves were excluded from the databases. In addition, a few studies concluded that square columns confined with FRP provide a little (Mirmiran et al. 1998) or no strength improvement (Wu and Zhou 2010). Thus, this study deals only with specimens with round corner, as such specimens with sharp corners were excluded from the databases. After excluding all the above, the databases contained the test results of 104 FRP confined rectangular concrete columns and 69 FRP confined square concrete columns for the strength and strain models, respectively.

Artificial Neural Network Modeling

Compressive Strength of FRP Confined Rectangular Columns

The ANN strength model was developed by the ANN toolbox of MATLAB R2012b (MATLAB) to estimate the compressive strength of FRP confined rectangular specimens. The data used to train, validate and test the proposed model were obtained from the paper by Pham and Hadi (2014). The database contained 104 FRP confined rectangular concrete columns having unconfined concrete strength between 18.3 MPa and 55.2 MPa. The database was randomly divided into training (70%), validation (15%), and test (15%) by the function "Dividerand".

Following the data division and preprocessing, the optimum model architecture (the number of hidden layers and the corresponding number of hidden nodes) needs to be investigated. Hornik et al. (1989) provided a proof that multilayer feedforward networks with as few as one hidden layer of neurons are indeed capable of universal approximation in a very precise and satisfactory sense. Thus, one hidden layer was used in this study. The optimal number of hidden nodes was obtained by a trial and error approach in which the network was trained with a set of random initial weights and a fixed learning rate of 0.01.

Since the number of input, hidden, and output neurons is determined, it is possible to estimate an appropriate number of samples in the training data set. Upadhyaya and Eryurek (1992) proposed an equation to calculate the necessary number of training samples as follows:

$$\frac{w}{o} \leq n \leq \frac{w}{o} log_2^{\frac{w}{o}} \tag{2}$$

where w is the number of weights, o is the number of the output parameters, and n is the number of the training samples. Substituting the number of weights and the number of the output parameters into Eq. 2, the following condition is achieved:

$$54 \le n = 73 \le 310 \tag{3}$$

Once the network has been designed and the input/output have been normalized, the network would be trained. The MATLAB neural network toolbox supports a variety of learning algorithms, including gradient descent methods, conjugate gradient methods, the Levenberg-Marquardt (LM) algorithm, and the resilient back-propagation algorithm (Rprop). The LM algorithm was used in this study. In the MATLAB neural network toolbox, the LM method (denoted by function "Trainlm") requires more memory than other methods. However, the LM method is highly recommended because it is often the fastest back-propagation algorithm in the toolbox. In addition, it does not cause any memory problem with the small training dataset though the learning process was performed on a conventional computer.

In brief, the network parameters are: network type is Feed-forward back propagation, number of input layer neurons is 8, number of hidden layer neurons is 6, one neuron of output layer is used, type of back propagation is Levenberg-Marquardt, training function is "Trainlm", adaption learning function is "Learngdm", performance function is MSE, transfer functions in both hidden and output layers are "Tansig". The network architecture of the proposed ANN strength model is illustrated in Fig. 1.

In the development of an artificial neural network to predict the compressive strength of FRP confined rectangular concrete specimens (f_{cc}' in MPa), the selection of the appropriate input parameters is a very important process. The compressive strength of confined concrete should be dependent on the geometric dimensions and the material properties of concrete and FRP. The geometric dimensions are defined as the short side length (b in mm), the long side length (h in mm), and the corner radius (r in mm). Meanwhile, the material properties considered are: the axial compressive strength (f_{co}' in MPa) and strain (ε_{co} in %) of concrete, the nominal thickness of FRP (t_f in mm), the elastic modulus of FRP (E_f in GPa), and the tensile strength of FRP (f_f in MPa).

Compressive Strain of FRP Confined Square Columns

The ANN strain model was developed to estimate the compressive strain of FRP confined square specimens. The data used in this model were adopted from the study by Pham and Hadi (2013). The database contained 69 FRP confined square concrete columns having unconfined concrete strength between 19.5 MPa and 53.9 MPa.

The algorithm and design of the ANN strain model are the same as the proposed ANN strength model with details as follows: network type is Feed-forward back propagation, number of input layer neurons is 7, number of hidden layer neurons is 6, one neuron of output layer, type of back propagation is Levenberg-Marquardt, training function is “Trainlm”, adaption learning function is “Learngdm”, performance function is MSE, transfer functions in both hidden and output layers are “Tansig”. The architecture of the proposed model is similar to Fig. 1 with exclusion of Variable *h*.

Once the network was designed, the necessary number of training samples could be estimated by using Eq. 2 as follows:

$$48 \le n = 48 \le 268 \tag{4}$$

Performance of the Proposed Models

The performance of the proposed ANN strength model was verified by the database of 104 rectangular specimens. Fig. 2 shows the predictions of the ANN strength model as compared to the experimental values. Many existing models for FRP confined concrete were adopted to compare with the proposed model. However, because of space limitations of the paper, five existing models were studied in this verification (Lam and Teng 2003b; Wu and Wang 2009; Toutanji et al. 2010; Wu and Wei 2010; Pham and Hadi 2014). These models were chosen herein because they have had high citations and yielded good agreement with the database. The comparison between the predictions and the test results in Fig. 2 shows improvement of the selected models in predicting the strength of FRP confined rectangular columns over the last decade. The proposed ANN strength model has the highest general correlation factor (R^2 = 96%) for a linear trend between the prediction and the test results while the other models have a correlation factor between approximately 78% and 88%.

In order to examine the accuracy of the proposed strength model, three statistical indicators were used: the mean square error (MSE), the average absolute error (AAE), and the standard deviation (SD). Among the presented models, the proposed ANN strength model depicts a significant improvement in calculation errors as shown in Fig. 3. A low SD of the proposed ANN strength model indicates that the data points tend to be very close to the mean values.

Meanwhile, the performance of the proposed ANN strain model is verified by the database which had 69 square specimens. Fig. 4 shows the compressive strain of the specimens predicted by the ANN strain model versus the experimental values. In order to make a comparison with other models, five existing models were considered in this verification (Shehata et al. 2002; Lam and Teng 2003b; ACI 440.2R-08 2008; Ilki et al. 2008; Pham and Hadi 2013). The proposed ANN strain model outperforms the selected models in estimating the compressive strain of confined square columns as shown in Fig. 4. The highest general correlation factor (R^2 = 98%) was achieved by the proposed model while the correlation factor of the other models was less than 60%. For further evaluation, the values of MSE, AAE, and SD were calculated and presented. Fig. 5 shows that the proposed model significantly reduces the error in estimating the compressive strain of FRP confined square specimens by approximately five times as compared to the other models. The average absolute error (AAE) of the existing models is around 30% while the AAE of the proposed model is approximately 5%.

Proposal of User-Friendly Equations

In the previous section, the "Tansig" transfer function was used in the ANN as it provides better results than "Pureline" transfer function. Although the simulated results from the proposed ANNs have a good agreement with the experimental data, it is inconvenient for engineers to use the networks in engineering design. It is logical and possible that a functional-form equation could be explicitly derived from the trained networks by combining the weight matrix and the bias matrix. Nevertheless, the final equations will become very complicated because the proposed ANN models contain complex transfer functions, which are "Tansig" as shown in Eq. 5 below. Therefore, in order to generate user-friendly equations to calculate stress and strain of FRP confined concrete, the "Tansig" transfer function used in the previous section was replaced by the "Pureline" transfer function (Eq. 6). A method that uses ANNs to generate user-friendly equations for calculating the compressive strength or strain of FRP confined square/rectangular columns is proposed. As a result, the proposed equation could replace the ANN to yield the same results. Once an ANN is trained and yields good results, a user-friendly equation could be derived following the procedure described below.

$$tan\,sig(x) = \frac{2}{1+e^{-2x}} - 1 \tag{5}$$

$$purelin(x) = x \tag{6}$$

Mathematical Derivations

The architecture of the proposed models is modified to create a simpler relationship between the inputs and the output as shown in Fig 6. The following equations illustrate the notation in Fig. 6.

$$\begin{aligned}\mathbf{X} &= \left[b\; h\; r\; f_{co}^{'}\; \varepsilon_{co}\; t_f\; E_f\; f_f\right]^T \\ &= \left[x_1\; x_2\; x_3\; x_4\; x_5\; x_6\; x_7\; x_8\right]^T\end{aligned} \tag{7}$$

where **X** is the input matrix, which contains eight input parameters, and superscript ^{T} denotes a transpose matrix. Functions that illustrate the relationships of neurons inside the network are presented as follows:

$$\mathbf{u} = \mathbf{IW}\;\mathbf{X} + \mathbf{b}_1 = \sum_{j=1}^{6}\sum_{i=1}^{8} IW_{j,i}x_i + b_{1j} \tag{8}$$

$$\mathbf{u_1} = purelin\left(\mathbf{u}\right) = \mathbf{u} \tag{9}$$

$$\mathbf{u_2} = \mathbf{LW}\mathbf{u_1} + \mathbf{b}_2 = \sum_{i=1}^{6} LW_i u_{1i} + b_{2i} \tag{10}$$

$$\mathbf{y} = purelin\left(\mathbf{u_2}\right) = \mathbf{u_2} \tag{11}$$

where **u**, $\mathbf{u_1}$, and $\mathbf{u_2}$ are the intermediary matrices; "Purelin" is the transfer function; **y** is the output parameter which is the compressive strength of FRP confined square/rectangular columns ($f_{cc}^{'}$ in MPa); **IW** is the input weight matrix; $\mathbf{b_1}$ is the bias matrix of Layer 1; **LW** is the layer weight matrix; and $\mathbf{b_2}$ is the bias matrix of Layer 2.

From Eqs. 7-11 and Fig. 6, the output could be calculated from the input parameters by the following equation:

$$\mathbf{y} = \mathbf{LW}\times\mathbf{IW}\times\mathbf{X} + \mathbf{LW}\times\mathbf{b_1} + \mathbf{b_2} \tag{12}$$

Based on Eq. 12, it is obvious that a user-friendly equation could be derived from a trained network. In order to simplify the above equation, another expression could be derived as follows:

$$\mathbf{y} = \mathbf{W} \times \mathbf{X} + \mathbf{a} \tag{13}$$

where **W** is a proportional matrix and **a** is a scalar, which are calculated as follows:

$$\mathbf{W} = \mathbf{LW} \times \mathbf{IW} \tag{14}$$

$$\mathbf{a} = \mathbf{LW} \times \mathbf{b}_1 + \mathbf{b}_2 \tag{15}$$

where the matrix **W** is denoted as follows:

$$\mathbf{W} = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \end{bmatrix} \tag{16}$$

Proposed Equation for Compressive Strength

A modified ANN strength model was proposed to estimate the compressive strength of FRP confined rectangular concrete columns. The modified ANN strength model was trained on the database of 104 FRP confined rectangular concrete columns. All procedures introduced in the previous sections were applied for this model with exception of the transfer function. As described in Fig. 6, the "Purelin" transfer function was used instead of the "Tansig" transfer function. After training, the input weight matrix (**IW**), the layer weight matrix (**LW**), and the bias matrices ($\mathbf{b_1}$ and $\mathbf{b_2}$) were obtained. From Eqs. 14 – 15, the proportional matrix (**W**) and the scalar (**a**) were determined as follows:

$$\begin{gathered} \mathbf{W} = \mathbf{LW} \times \mathbf{IW} \\ \mathbf{W} = \begin{bmatrix} -0.21 & -0.36 & 0.39 & 5.68 & -5.36 & 1.33 & 0.40 & 0.64 \end{bmatrix} \end{gathered} \tag{17}$$

$$\mathbf{a} = \mathbf{LW} \times \mathbf{b}_1 + \mathbf{b}_2 = 0.24 \tag{18}$$

It is to be noted that the inputs and the output in Eq. 13 are normalized. The relationship between the actual inputs and the actual output is presented in the equations below:

$$y = \frac{y_{max} + y_{min}}{2} + \frac{y_{max} - y_{iin}}{2}\left[\sum_{i=1}^{8} w_i \left(\frac{2(x_i - x_{i\,min})}{x_{i\,max} - x_{i\,min}} - 1\right) + \mathbf{a}\right] \tag{19}$$

$$\begin{aligned} y = {} & \sum_{i=1}^{8} \frac{(y_{max} - y_{min})w_i}{x_{i\,max} - x_{i\,min}} x_i + \left(\frac{y_{max} + y_{min}}{2} + \frac{y_{max} - y_{min}}{2}\mathbf{a}\right) \\ & - \sum_{i=1}^{8}\left(\frac{(y_{max} - y_{min})w_i x_{i\,min}}{x_{i\,max} - x_{i\,min}} + \frac{y_{max} + y_{min}}{2} w_i\right) \end{aligned} \tag{20}$$

Based on the equations above, the output could be calculated from the inputs as follows:

$$y = \sum_{i=1}^{8} k_i x_i + c \tag{21}$$

where k_i are proportional factors, and c is a constant.

$$k_i = \sum_{i=1}^{8} \frac{(y_{max} - y_{min})w_i}{x_{i\,max} - x_{i\,min}} \tag{22}$$

$$c = \frac{(y_{max} + y_{min})}{2} + \frac{(y_{max} - y_{min})}{2}\mathbf{a} - \sum_{i=1}^{8}\left(\frac{(y_{max} - y_{min})w_i x_{i\,min}}{x_{i\,max} - x_{i\,min}} + \frac{(y_{max} - y_{min})}{2} w_i\right) \tag{23}$$

Based on the trained ANN and Eqs. 22 – 23, the constant c is 414.61 while the proportional factor k_i is obtained as follows:

$$\mathbf{k} = [-0.1 \quad -0.12 \quad 0.6 \quad 11.07 \quad -4170.85 \quad 67.21 \quad 0.15 \quad 0.01] \tag{24}$$

In brief, the user-friendly equation was successfully derived from the trained ANN. The compressive strength of FRP confined rectangular concrete column now is calculated by using Eqs. 21 and 24.

Proposed Equation for the Compressive Strain

A modified ANN strain model was proposed to estimate the compressive strain of FRP confined square concrete columns. The proposed ANN strain model was verified by the database which contained 69 FRP confined square concrete columns having unconfined concrete strength between 19.5 MPa and 53.9 MPa. All procedures introduced in the sections above were applied for this model with the exception of the transfer function, which was the "Purelin" function. It is to be noted that the total number of input parameters herein is 7 with exclusion of one variable as shown in Fig. 6. The architecture of the proposed ANN strain model and the size of the weight matrices and biases are also similar to Fig. 6 but with 7 inputs. Following the same procedure of the proposed strength model, the proportional matrix (**W**) and the scalar (**a**) are determined as follows:

$$\begin{gathered}\mathbf{W} = \mathbf{LW} \times \mathbf{IW} \\ \mathbf{W} = \begin{bmatrix} 1.49 & 0.05 & -5.99 & 5.08 & 0.66 & 4.32 & -3.30 \end{bmatrix}\end{gathered} \tag{25}$$

$$\mathbf{a} = \mathbf{LW} \times \mathbf{b}_1 + \mathbf{b}_2 = -1.76 \tag{26}$$

The compressive strain now could be calculated by using Eq. 21 in which the proportional factor k_i and the constant c are as follows:

$$\mathbf{k} = \begin{bmatrix} 0.284 & 0.004 & -0.618 & 209.593 & 1.24 & 0.076 & -0.003 \end{bmatrix} \tag{27}$$

$$c = -66.012 \tag{28}$$

In brief, the user-friendly equation was successfully derived from the trained ANN. The compressive strain of FRP confined square concrete columns now is calculated by using Eqs. 21 and 27-28.

Performance of the Proposed User-Friendly Equations

The performance of the proposed strength equation (Eqs. 21 and 24) is shown in Fig. 7. This figure shows that the proposed user-friendly equation for strength estimation provides the compressive strength that fits the experimental results well. In addition, the proposed model's performance was compared with other existing models as shown in Fig. 7. The five existing models mentioned in the section above were studied in this comparison. The performance of these models is comparable in calculating the compressive strength of FRP confined rectangular columns.

In addition, Fig. 8 shows the performance of the proposed strain equation (Eqs. 21, 27 - 28). This figure illustrates the compressive strain of the specimens estimated by the proposed strain equation versus the experimental results. In addition, the proposed strain equation's performance was compared with other existing models as shown in Fig. 8. The five models mentioned in the above sections were adopted. The proposed ANN strain equation outperforms the selected models in estimating the compressive strain of confined concrete as shown in Fig. 8. The highest general correlation factor (R^2 = 90%) was achieved by the proposed model while the corresponding number of other models is less than 60%. This general correlation factor (R^2) is less than that in the above sections when the "Tansig" transfer function was replaced by the "Purelin" transfer function. Although using the "Purelin" transfer function reduces the accuracy of the proposed models, it provides a much simpler derivation of the proposed equations. For further evaluation, the values of AAE were calculated and are presented in Fig. 8. It demonstrates that the proposed equation significantly reduces the error in estimating the compressive strain of FRP confined square specimens by approximately three times as compared to the other models. The average absolute error of the selected models is around 30% while the corresponding number of the proposed model is approximately 12%.

Analysis and Discussion

Effect of corner radius on the compressive strength and strain

Based on the proportional matrix (**W**) as presented in Eq. 12, the contribution of the input parameters to the output could be examined. The magnitude of the elements in the proportional matrix of the proposed ANN strength equation is comparable, which was presented in Eq. 16. Thus all eight input parameters significantly contribute to the compressive strength of the columns. On the other hand, the element w_2 of the proportional matrix in the proposed ANN strain equation is extremely small as compared to the others (Eq. 25). Hence, the contribution of the input r to the compressive strain of the columns could be negligible.

The proposed ANN strain equation was modified by using 6 input parameters, in which the Input r was removed. The input parameters are: the side length, the unconfined concrete strength and its corresponding strain, the tensile strength of FRP, the nominal thickness of FRP, and the elastic modulus of FRP. The performance of the modified strain equation is shown in Fig. 9 which shows that the AAE of the predictions increased slightly from 12% to 13%. Therefore, it is concluded that the contribution of the corner radius to the compressive strain of the columns is negligible. The proportional factor k_i and the constant c are as follows:

$$\mathbf{k} = \begin{bmatrix} 0.26 & 0.038 & -51.314 & 1.329 & 0.059 & -0.002 \end{bmatrix} \tag{29}$$

$$c = -32.119 \tag{30}$$

Scope and Applicability of the Proposed ANN Models

From the performance of the proposed models, it can be seen that artificial neural networks are a powerful regression tool. The proposed ANN models significantly increase the accuracy of predicting the compressive stress and strain of FRP confined concrete. It is to be noted that, the distribution of the training data within the problem domain can have a significant effect on the learning and generation performance of a network (Flood and Kartam 1994). The function "Deviderand" recommended by MATLAB was used to evenly distribute the training data. Artificial neural networks are not usually able to extrapolate, so the straining data should go at most to the edges of the problem domain in all dimensions. In other words, future test data should fall between the maximum and the minimum of the training data in all dimensions. Table 1 presents the maximum and the minimum values of each input parameter. It is recommended that the proposed ANN models are applicable for the range shown in Table 1 only. In order to extend the applicability of the proposed ANN models, a larger database containing a large number of specimens reported should be used to retrain and test the models. When the artificial neural network has been properly trained, verified, and tested with a comprehensive experimental database, it can be used with a high degree of confidence.

Simulating an ANN by MS Excel

The finding in this study indicates that a trained ANN could be used to generate a user-friendly equation if the following conditions are satisfied. Firstly, the problem is well simulated by the ANN, which yields a small error and high value of general correlation factor (R^2). Secondly, the "Purelin" transfer function must be used in that algorithm. A very complicated problem is then simulated by using a user-friendly equation as followed in the proposed procedure.

However, if using the "Purelin" transfer function instead of other transfer functions increases significantly errors of the model, the proposed ANN models that have the "Tansig" transfer function should be used. So, a user-friendly equation cannot be generated in such a case. The following procedure could be used to simulate the trained ANN by using MS Excel:

Step 1: Normalize the inputs to fall in the interval [-1, 1].

Step 2: Calculate the proportional matrix **W** and the scalar **a** by using Eqs. 14 – 15, respectively.

Step 3: Calculate the normalized output *y'* by using Eq. 13.

Step 4: Return the output to the actual values.

By following the four steps above, a MS Excel file was built to confirm that the predicted results from the MS Excel file are identical with those results yielded from the ANN.

Conclusions

Two ANN strength and strain models are proposed to calculate the compressive strength and strain of FRP confined square/rectangular columns. The prediction of the proposed ANN models fits well the experimental results. They yield results with marginal errors, about half of the errors of the other existing models. This study also develops new models coming up with a user-friendly equation rather than the complex computational models. The findings in this paper are summarized as follows:

1. The two proposed ANN models accurately estimate the compressive strength and strain of FRP confined square/rectangular columns with very small errors (AAE < 5%), which outperform the existing models.

2. The proposed ANN strength equation provides a simpler predictive equation as compared to the existing strength models with comparable errors.

3. The proposed ANN strain equation also delivers a simple-form equation with very small errors. The proposed model's error is approximately 12%, which is one third in comparison with the existing strain models.
4. For FRP confined rectangular columns, the corner radius significantly affects the compressive strength but marginally affects the compressive strain.

The ANN has been successfully applied for calculating the compressive strength and strain of FRP confined concrete columns. It is a promising approach to provide better accuracy in estimating the compressive strength and strain of FRP confined concrete than the existing conventional methods.

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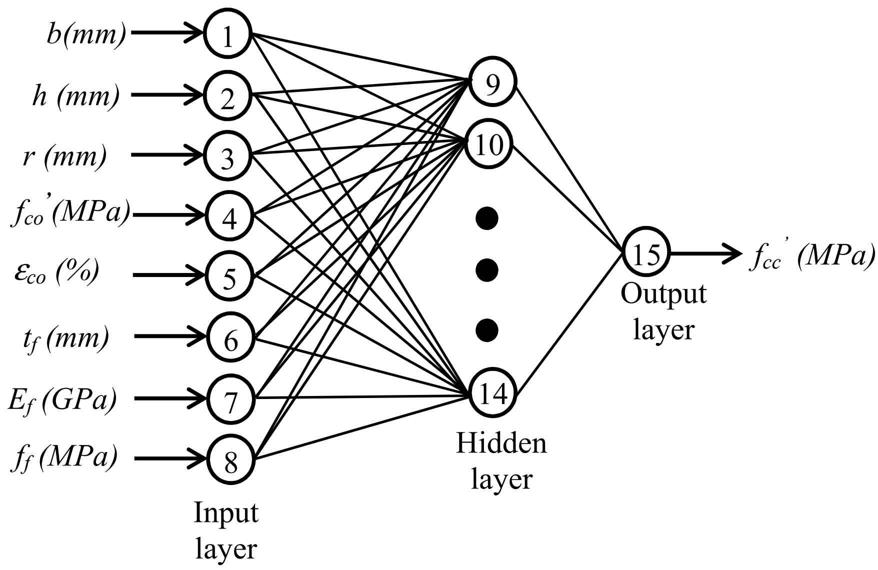
List of Figures

List of Tables

Table 1. Statistics of the input parameters for the proposed models

Input/Output parameters	Strength model		Strain model	
	Maximum	Minimum	Maximum	Minimum
b (mm)	250	100	152	133
h (mm)	305	100	-	-
r (mm)	60	15	60	15
f_{co}' (MPa)	53.9	18.3	53.9	19.5
ε_{co} (%)	0.25	0.16	0.25	0.16
t_f *(mm)*	1.5	0.13	2	0.12
E_f (GPa)	257	75.1	241	38.1
f_f (MPa)	4519	935	4470	580
f_{cc} (MPa)	90.9	21.5	-	-
ε_{cc} (%)	-	-	3.9	0.4

Figure 1



$b(mm)$
$h\ (mm)$
$r\ (mm)$
$f_{co}'(MPa)$
$\varepsilon_{co}\ (\%)$
$t_f\ (mm)$
$E_f\ (GPa)$
$f_f\ (MPa)$
1
2
3
4
5
6
7
8
Input layer
9
10
14
Hidden layer
15
Output layer
$f_{cc}'\ (MPa)$

Figure 2

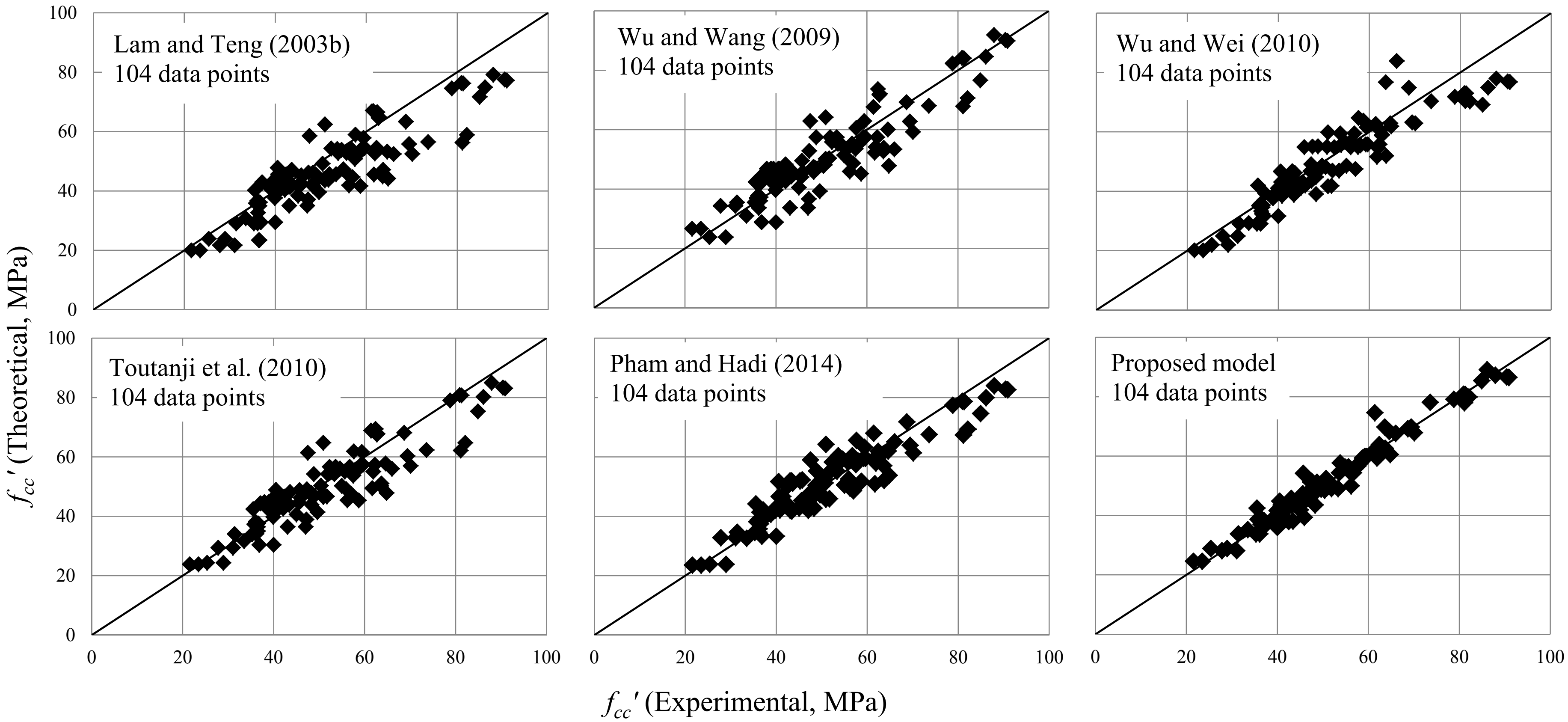

Lam and Teng (2003b)
104 data points
Wu and Wang (2009)
104 data points
Wu and Wei (2010)
104 data points
Toutanji et al. (2010)
104 data points
Pham and Hadi (2014)
104 data points
Proposed model
104 data points
f_{cc}' (Theoretical, MPa)
f_{cc}' (Experimental, MPa)
0
20
40
60
80
100

Figure 3

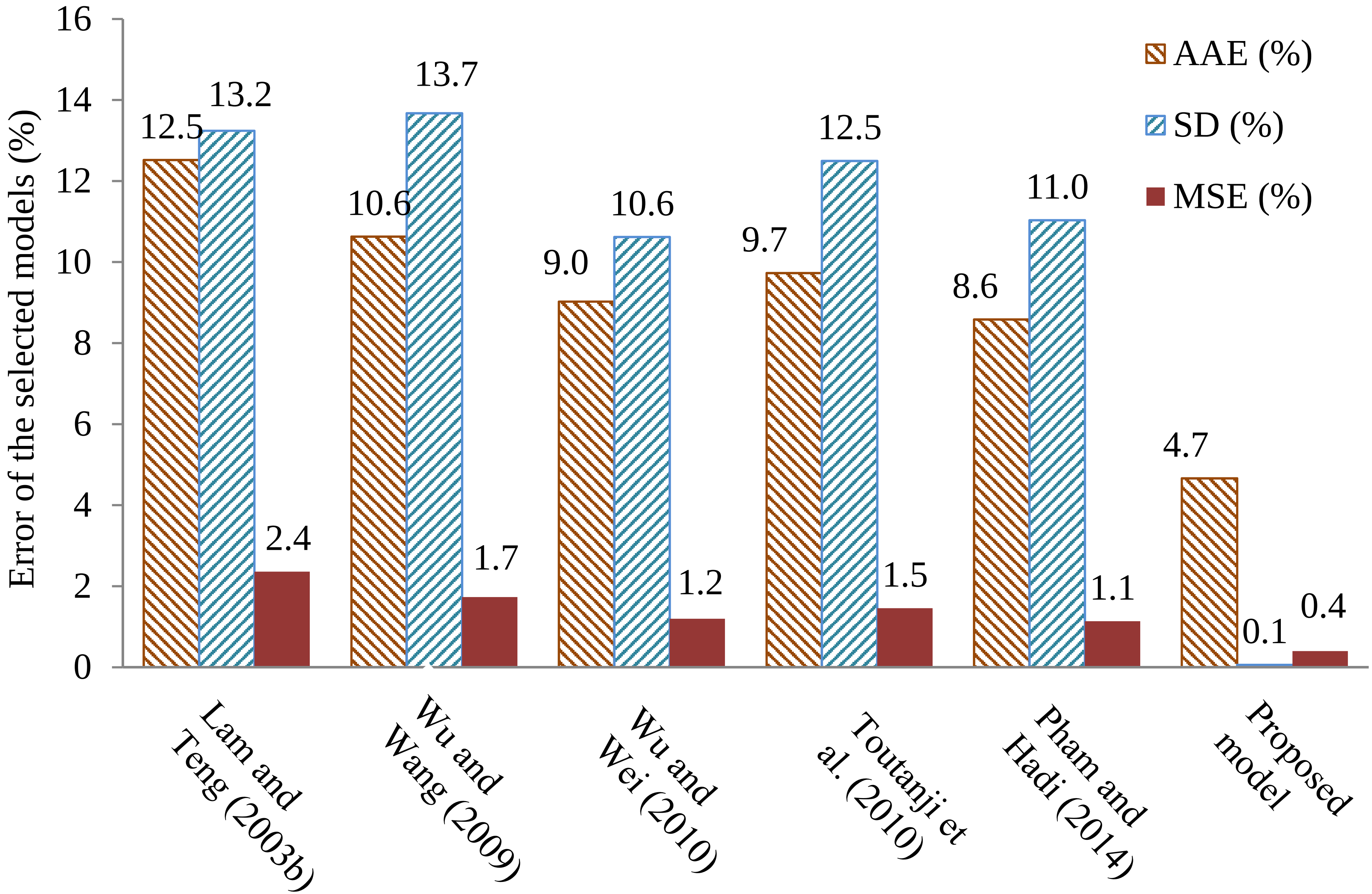

16
14
12
10
8
6
4
2
0
Error of the selected models (%)
AAE (%)
SD (%)
MSE (%)
12.5
13.2
2.4
10.6
13.7
1.7
9.0
10.6
1.2
9.7
12.5
1.5
8.6
11.0
1.1
4.7
0.1
0.4
Lam and Teng (2003b)
Wu and Wang (2009)
Wu and Wei (2010)
Toutanji et al. (2010)
Pham and Hadi (2014)
Proposed model

Figure 4

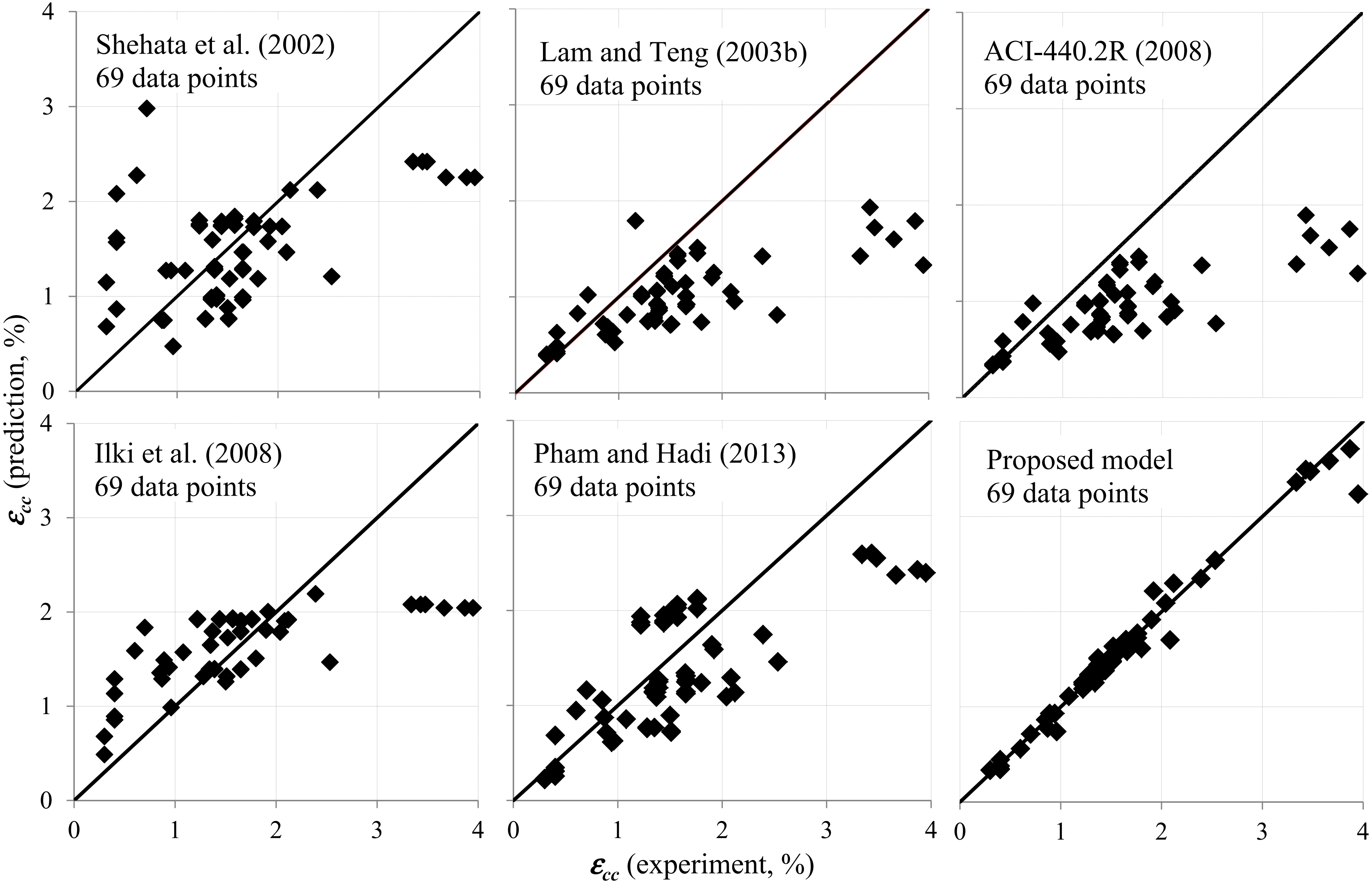
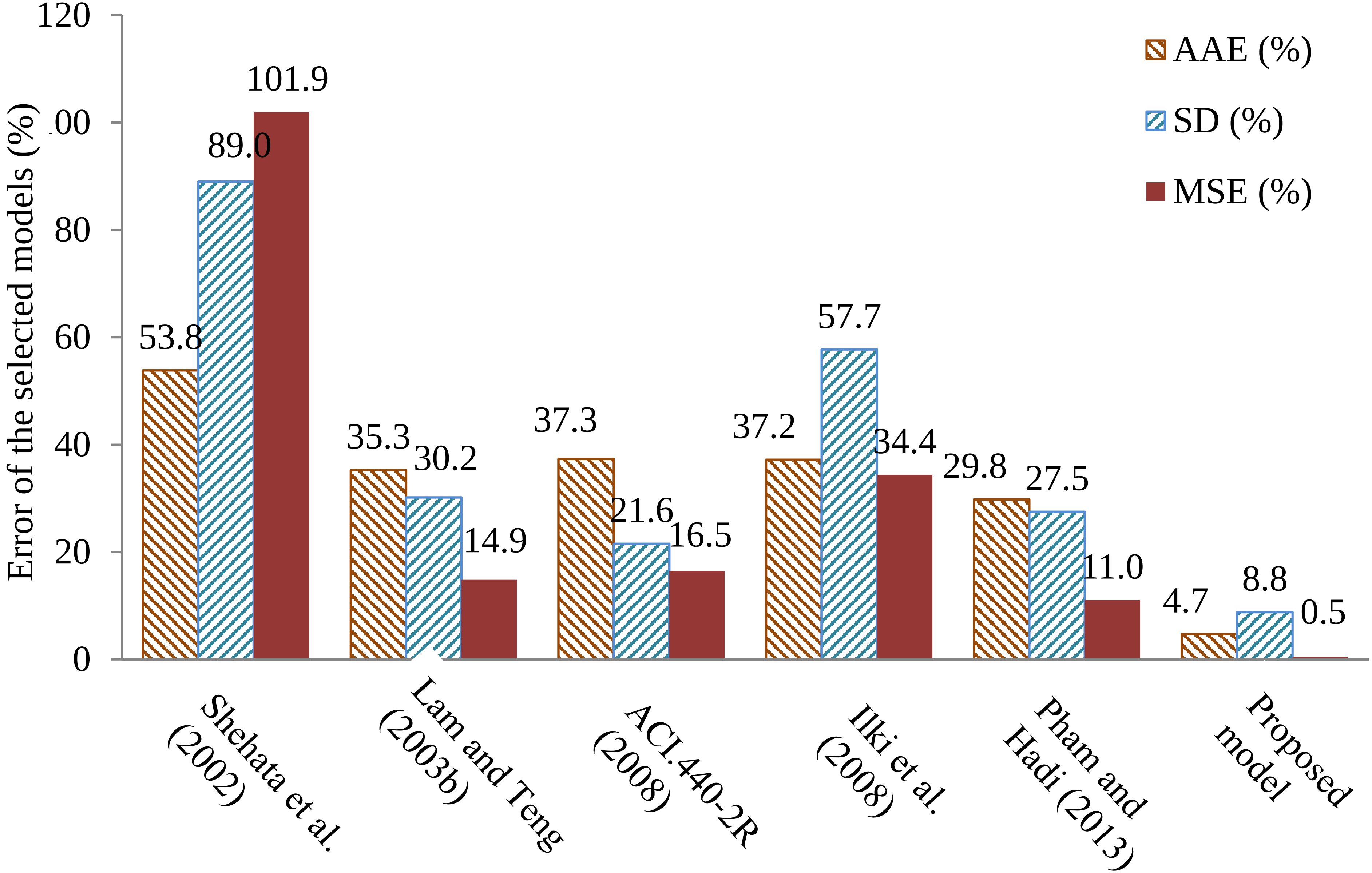
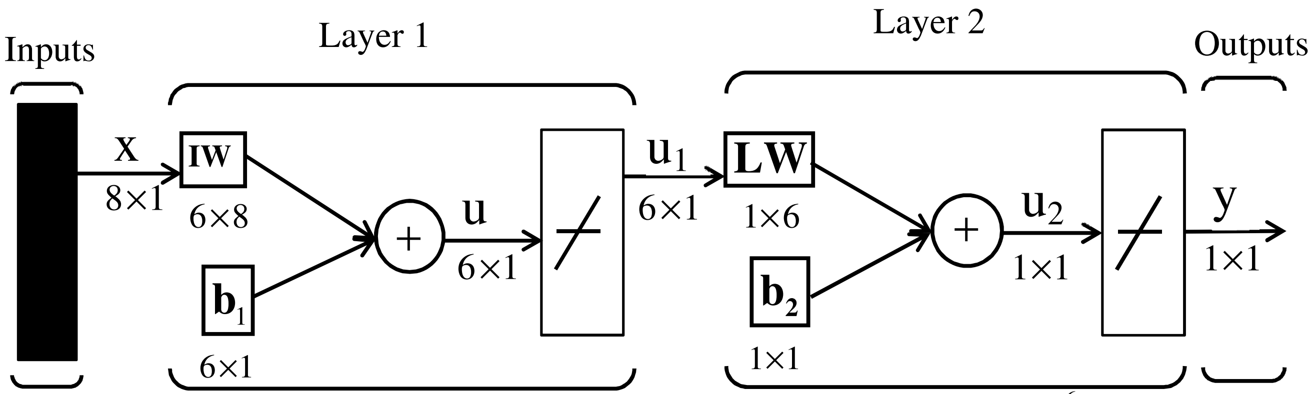

Shehata et al. (2002)
69 data points
Lam and Teng (2003b)
69 data points
ACI-440.2R (2008)
69 data points
Ilki et al. (2008)
69 data points
Pham and Hadi (2013)
69 data points
Proposed model
69 data points
ε_{cc} (prediction, %)
ε_{cc} (experiment, %)
0
1
2
3
4

Figure 5



120
100
80
60
40
20
0
Error of the selected models (%)
AAE (%)
SD (%)
MSE (%)
53.8
89.0
101.9
35.3
30.2
14.9
37.3
21.6
16.5
37.2
57.7
34.4
29.8
27.5
11.0
4.7
8.8
0.5
Shehata et al. (2002)
Lam and Teng (2003b)
ACI.440-2R (2008)
Ilki et al. (2008)
Pham and Hadi (2013)
Proposed model

Figure 6



$$\mathbf{u} = \mathbf{IW}\,\mathbf{X} + \mathbf{b}_1 = \sum_{j=1}^{6}\sum_{i=1}^{8} IW_{j,i}\,x_i + b_{1j}$$

$$\mathbf{u_1} = purelin\,(\mathbf{u}) = \mathbf{u}$$

$$\mathbf{u_2} = \mathbf{LW}\mathbf{u_1} + \mathbf{b}_2 = \sum_{i=1}^{6} LW_i\,u_{1i} + b_{2i}$$

$$\mathbf{y} = purelin(\mathbf{u_2}) = \mathbf{u_2}$$

Figure 7

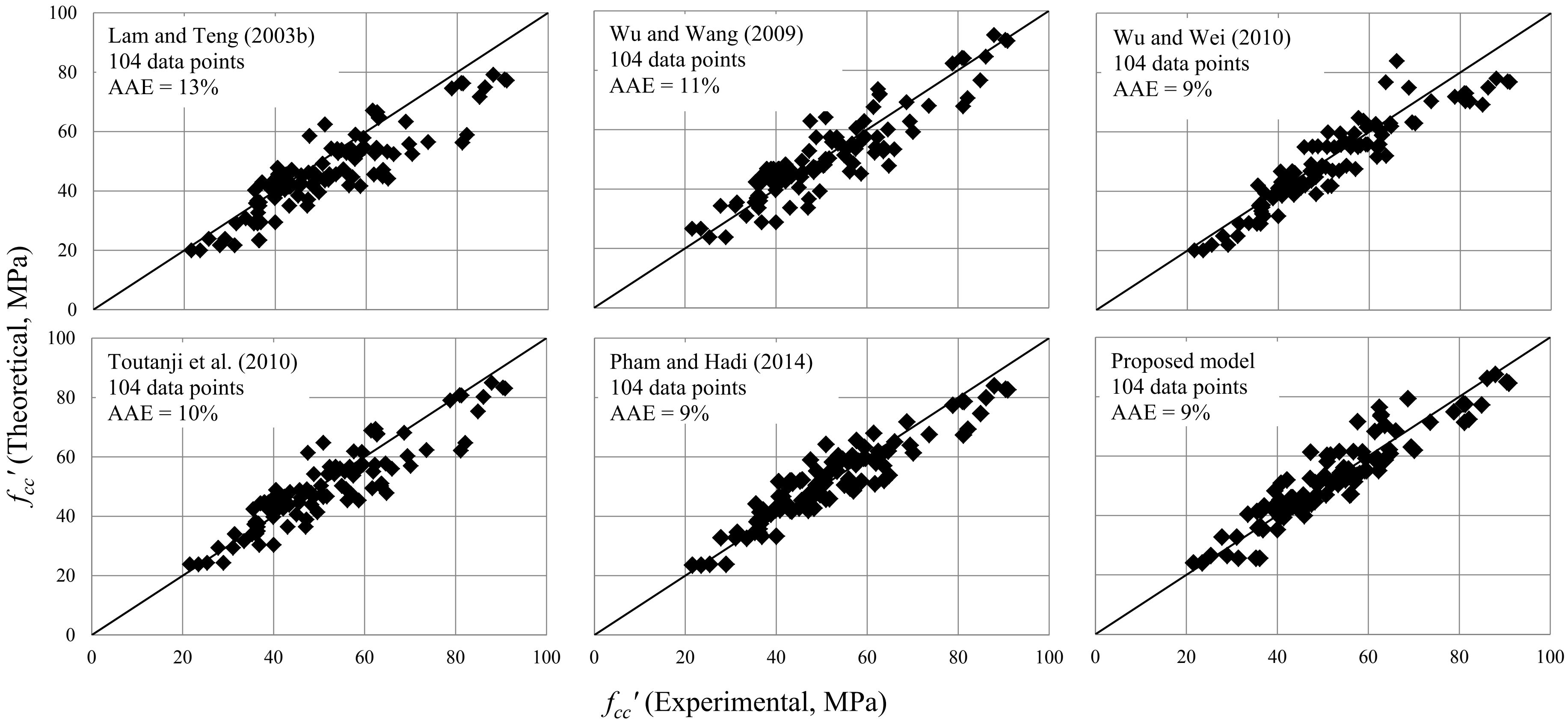
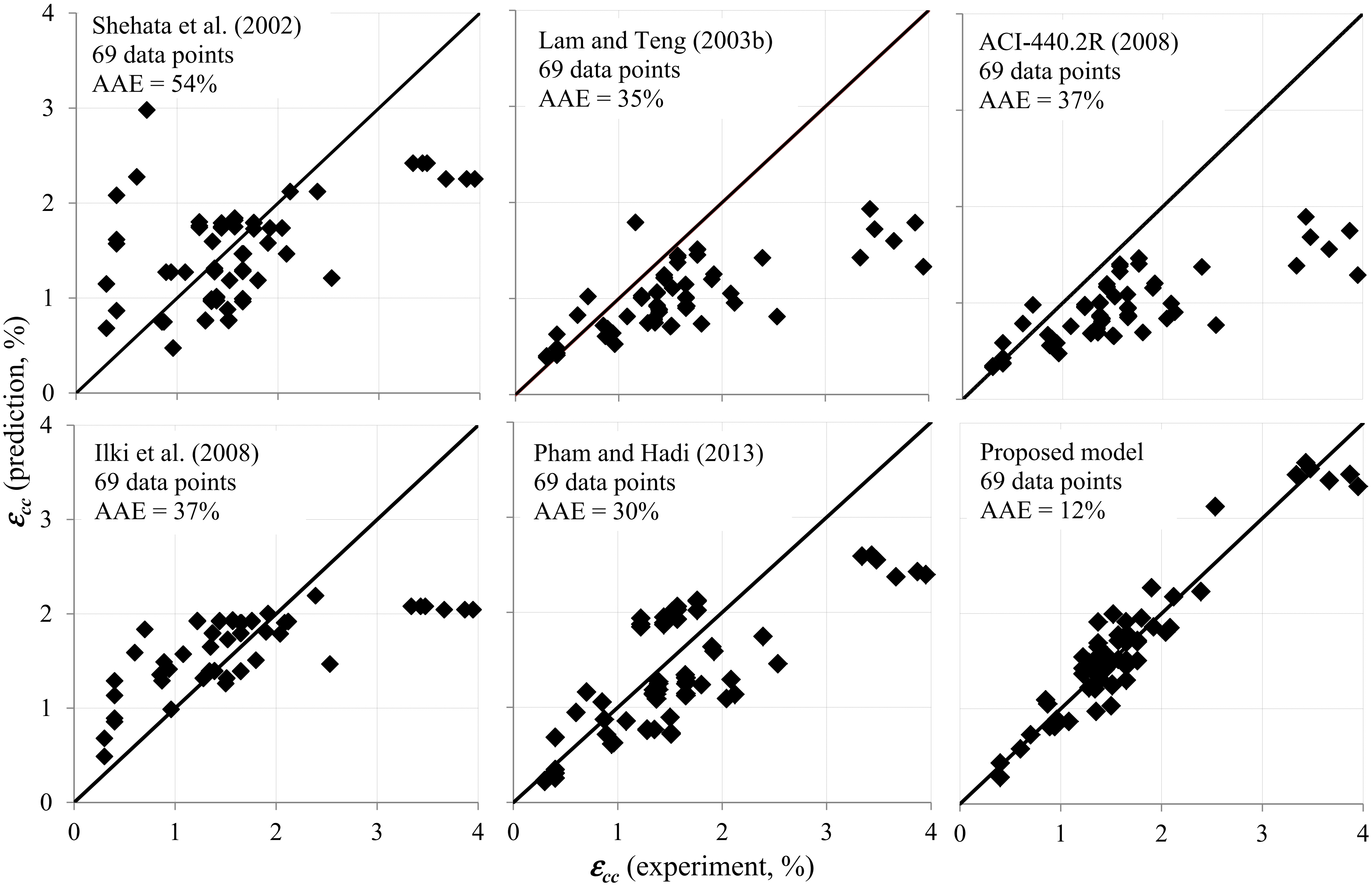
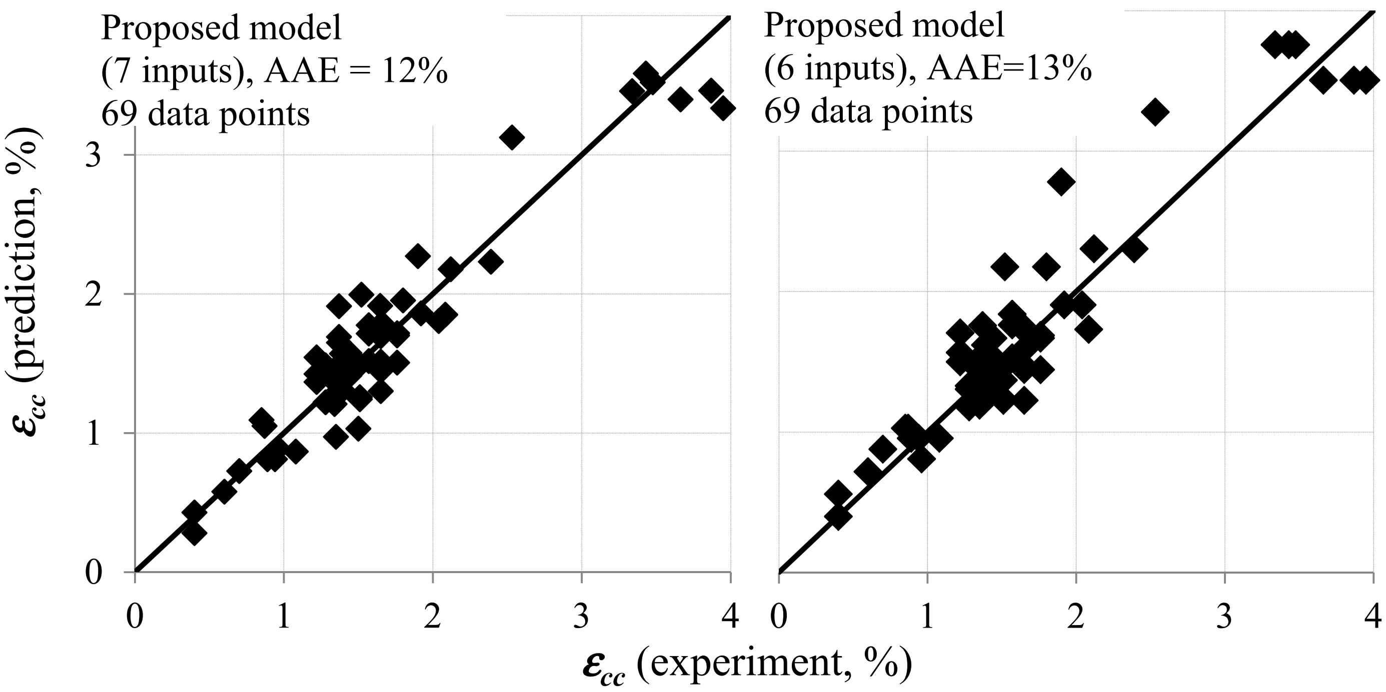

Lam and Teng (2003b)
104 data points
AAE = 13%
Wu and Wang (2009)
104 data points
AAE = 11%
Wu and Wei (2010)
104 data points
AAE = 9%
Toutanji et al. (2010)
104 data points
AAE = 10%
Pham and Hadi (2014)
104 data points
AAE = 9%
Proposed model
104 data points
AAE = 9%
0
20
40
60
80
100
f_{cc}' (Theoretical, MPa)
f_{cc}' (Experimental, MPa)

Figure 8



Shehata et al. (2002)
69 data points
AAE = 54%
Lam and Teng (2003b)
69 data points
AAE = 35%
ACI-440.2R (2008)
69 data points
AAE = 37%
Ilki et al. (2008)
69 data points
AAE = 37%
Pham and Hadi (2013)
69 data points
AAE = 30%
Proposed model
69 data points
AAE = 12%
ε_{cc} (prediction, %)
ε_{cc} (experiment, %)
0
1
2
3
4

Figure 9



(a) The proposed model with 7 inputs; (b) The proposed model with 6 inputs