An intelligent fuzzy regression approach for affective product design that captures nonlinearity and fuzziness

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Abstract

Affective product design aims at incorporating customers' affective needs into design variables of a new product so as to optimize customers' affective satisfaction. Faced with fierce competition in marketplaces, companies try to determine the settings in order to maximize customers' affective satisfaction with products. To achieve this, a set of customer survey data is required in order to develop a model which relates customers' affective responses to the design variables of a new product. Customer survey data is usually fuzzy since human feeling is usually fuzzy, and the relationship between customers' affective responses and design variables is usually nonlinear. However, previous research on modelling the relationship between affective response and design variables has not addressed the development of explicit models involving either nonlinearity or fuzziness. In this paper, an intelligent fuzzy regression approach is proposed to generate models which represent this nonlinear and fuzzy relationship between affective responses and design variables. In order to do this,

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we extend the existing work on fuzzy regression by first utilizing an evolutionary algorithm to construct branches of a tree representing structures of a model where the nonlinearity of the model can be addressed. The fuzzy regression algorithm is then used to determine the fuzzy coefficients of the model. The models thus developed are explicit, and consist of fuzzy, nonlinear terms which relate affective responses to design variables. A case study of affective product design of mobile phones is used to illustrate the proposed method.

Keywords: Fuzzy regression, evolutionary algorithm, affective product design

Nomenclature

$\tilde{y}$ is the affective response of the product

x_k is the k-th design variable of the product where k=1,2, …N

$\tilde{A}_i$ is the i-th fuzzy coefficient of the fuzzy regression model.

c_i is the center of i-th fuzzy coefficient of the fuzzy regression model.

α_i is the spread of i-th fuzzy coefficient of the fuzzy regression model.

N_{NR} is the number of fuzzy coefficient of the fuzzy regression model.

x_k' is the transformed k-th design variable of the fuzzy regression model where

$\qquad$ k=1,2, …N_{NR}

M is the number of data sets.

h refers to the degree to which the fuzzy linear model fits the data sets in developing

the fuzzy model.

J is the total fuzziness of the regression model.

$y(i)$ is the i-th data set with respect to the affective response of the product w

$x'_j(i)$ is the j-th transformed variable of the fuzzy polynomial model of the i-th data

set

1   Introduction

In today's competitive world, optimization of customer satisfaction is essential in

product design in almost every company (Cross 2000). To achieve this, the basic

functions and operations of the products must perform satisfactorily. Consumers are

ever mindful that the products with which we surround and engage ourselves during

our daily tasks need to satisfy needs beyond the functional and operational aspects.

For example, in mobile phone design, the basic functions and operations of

transmission and receiver must work satisfactorily. After these basic functions and

operations have been achieved, higher quality levels are often required. The mobile

phone should feel comfortable when hand-held by customers. The buttons of a mobile

phone should be able to be punched easily, and voices should be clearly heard by both

the transmitter and receiver. At a more advanced level, products in these sectors often

need to appeal to a user on an affective level in order to be successful. This is now

evident in the mobile phone market where the sector has very successfully

transformed its products from being merely functional items to lifestyle or fashion accessories (Kuang and Jiang 2008).

This forces designers to find new ways to differentiate their products from those of their competitors in affective product design. Consequently, designers are increasingly focusing on affective satisfaction that products can bring about rather than on their functional benefits. Fulfilling affective satisfaction enriches a person's life and can increase one's general experience of well-being (Diener and Lucas 2000, Kouprie and Visser 2009, Mugge et al. 2009). Accordingly, emotional responses to products can be a decisive factor in purchase decisions (Jordan 2000, Desmet 2002, Norman 2004, Artacho et al. 2009, Nurkka et al. 2009). Therefore, it is essential to optimize the affective satisfaction of a new product. To do this, it is essential to develop an explicit model that relates design variables of a new product to its affective satisfaction.

1.1 Previous methods

Kansei engineering (Nagamachi 1995) introduced a concept in product design that emphasized a method for translating affective responses such as 'pretty', 'nice', 'spirited' into design variables. It aims to integrate the customers' feelings and demands into the product design elements. More recent research shows that

relationships between affective responses and design variables were established mainly through artificial intelligence methods or statistical methods.

Artificial neural networks were first proposed for modeling the relationship between affective responses and design variables in automotive design (Nagamachi 1995). Also, a Kansei engineering rule based system was proposed to generate rules which describe associations automatically based on an approach which incorporated neural networks and Principle Component Analysis (PCA) (Tanoue et al 1997). As a result, a better approximation of women's shoes design can be achieved than the ones achieved by the conventional PCA, using less computational time. More recently, Gu et al. (2006) proposed an evolutionary system based on neural networks to analyze people's aesthetic evaluations which is able to approximate their implicit aesthetic intentions. It allows the framework to infer which designs the users may find desirable. Chen et al. (2006) developed a prototype system for affective design in which the Kohonen self-organizing map neural network was employed to consolidate the relationship between affective responses and design variables from consumers. The aforementioned neural network based approaches are able to model a nonlinear system as a simplified black-box structure, but these models lack transparency. Therefore, these methods are not appreciated very much by engineering users in model development.

To address the fuzziness of consumers' responses and the non-linearity of the modeling, artificial intelligence based fuzzy techniques have been used. Hsiao et al. (2005) proposed a method that enables an automatic product form or product image evaluation by means of a neural network-based fuzzy reasoning and genetic algorithm, which was applied to establish relationships between design variables and affective variables. Park and Han (2004), and Lau et al. (2006) adopted the fuzzy rule-based approach for building models which relates affective responses to consumer satisfaction. The above fuzzy modeling based approaches consist of fuzzifiers, fuzzy inference engines, fuzzy rules, and defuzzifiers to determine affective responses based on design variables. Explicit information can be found between affective responses and design variables based on the generated fuzzy rules. However, compared with statistical regression methods that can generate relationships between affective responses and design variables in polynomial form, more explicit information that can express the significance of each design variables and interaction between design variables can be found. Design engineers still prefer to use statistical methods because more explicit information can be found than by using the fuzzy modeling based approaches.

The approach of multiple linear regression (Chuang and Ma 2001, Han et al. 1999, Tanoue et al. 1997, Kuang and Jiang 2008) has been used to model relationships

between design variables and affective responses. This approach is simple to apply but it assumes that the design variables in the regression are linear, and that the effect of an independent design variable is constant throughout the entire range of the affective response. Under this assumption, more and more terms of design variables are included in the model to fit a wide range of affective responses. It increases the number of terms of design variables in the model of the affective response, and the terms of the design variables involved cannot be guaranteed to be significant for the affective response. Thus, the resulting model is more complex and more difficult to interpret when there are many design variables involved (Han and Hong 2003). A decision support system has been proposed to provide guidelines for optimizing affective satisfaction based on principal component analysis and multiple regression (Barnes and Lillford 2009). An explicit modeling method based on a vectors field based approach has been proposed for modeling the relationship between affective satisfaction and design variables (Petiot and Grognet 2006). You et al. (2006) developed affective satisfaction models using quantification I analysis, which was used to identify significant or preferred design variables for the interior parts of a new product. It uses the statistical method, ANOVA, to screen out the insignificant design variables and then it uses the generalized linear model method to model the affective satisfaction in a perceptual space within which the affective satisfaction of a family of

products can be described. However, affective satisfaction involve fuzziness which the above statistical approaches are unable to address.

In contrast, fuzzy regression has a distinct advantage in model development, which has a high degree of fuzziness, and it uses only small or incomplete data sets in model development (Tanaka et al. 1982, Takagi and Sugeno 1985, Tanaka and Watada 1988). Fuzzy regression has been applied by Shimizu and Jindo (1995) to model the relationship between affective responses and design variables in order to account for fuzziness of human feelings towards car interior design. However, the existing fuzzy regression approaches cannot be used to develop models which contain nonlinear terms. Kim et al. (1996) reported that fuzzy regression may not be superior to the traditional statistical regression in modeling nonlinear systems. In fact, nonlinear human feelings commonly exist. If nonlinear terms were to be integrated into the approach of fuzzy regression, more accurate models could be developed.

Genetic programming is an evolutionary computational method which can be used to generate models in polynomial form with nonlinear terms (Koza 1992, Koza 1994). It uses the evolutionary operations in genetic programming to generate the structure of models, and the least squares algorithm is then used to perform the coefficient estimation in the models. However, the relationship between design variables and affective responses involves fuzziness due to the nature of human

feelings. Hence, genetic programming together with the least squares algorithm may not yield the best modeling results, since the methods do not consider the fuzziness due to human feelings.

1.2 Proposed method

To overcome the deficiencies of the above approaches, in this paper, an intelligent fuzzy regression approach which can generate explicit models in fuzzy polynomial forms is proposed by considering the fuzziness of consumers' responses and the non-linearity of the model which captures the relationship between affective responses and design variables. The intelligent fuzzy regression also involves methodological extensions through the use of genetic programming to construct the structure of models in fuzzy polynomial form based on a tree representation where both linear and nonlinear terms can be included. Then, fuzzy regression is used to calculate the fuzzy coefficients of the fuzzy polynomials. Since both linearity and nonlinearity are represented in branches of the tree based on the intelligent fuzzy regression, fuzzy regression models with both fuzzy linear and fuzzy nonlinear terms can be generated. It eliminates the deficiencies of the above approaches which ignore nonlinear terms of relations between design variables and affective satisfactions, as well as the fuzzy nature of data. Moreover, they produce black-box models not usually recommended by engineering users. To evaluate the effectiveness of the proposed intelligent fuzzy

regression for modeling the relationship between affective variables and design variables, we use mobile phone design. Results of the modeling were compared with those based on the existing fuzzy regression methods (Tanaka et al. 1982, Peters 1994) and statistical regression (Seber 2003).

2 Intelligent fuzzy regression

The intelligent fuzzy regression approach generates the fuzzy regression model which relates affective responses and design variables. When defining, the approach needs to:

a) specify the form of the fuzzy model

b) determine the fuzzy coefficients which characterise the model

c) define the algorithm

Specification of the form of the fuzzy model

In the fuzzy regression model, interactions between design variables and nonlinear terms of design variables are represented in a form of a higher order high-dimensional Kolmogorov-Gabor polynomial (Friedman 1991), which can be written as

$$\tilde{y} = \tilde{f}_{NR}(x) = \tilde{A}_0 + \sum_{i_1=1}^{N} \tilde{A}_{i_1} x_{i_1} + \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} \tilde{A}_{i_1 i_2} x_{i_1} x_{i_2} + ... \sum_{i_1=1}^{N} ... \sum_{i_d=1}^{N} \tilde{A}_{i_1 ... i_d} \prod_{j=1}^{d} x_j \tag{1}$$

where $\tilde{y}$ is the affective response; x_k is the k-th design variable with $k=1,2, ...N$; and

$$\tilde{A}_0 = (\alpha_0, c_0), \ \tilde{A}_1 = (\alpha_1, c_1), \ \tilde{A}_2 = (\alpha_2, c_2), \ ...$$

$$\widetilde{A}_N = (\alpha_N, c_N), \widetilde{A}_{11} = (\alpha_{11}, c_{11}), \widetilde{A}_{12} = (\alpha_{12}, c_{12}), \dots \widetilde{A}_{NN} = (\alpha_{NN}, c_{NN}), \dots$$

$$\widetilde{A}_{N\dots N} = (\alpha_{N\dots N}, c_{N\dots N}).$$

Kolmogorov-Gabor polynomials have been used widely to evolve general nonlinear models by evolutionary methods (Nikolaev and Iba 2001), but no fuzzy coefficient has been considered. The purpose of the proposed fuzzy regression model is to overcome the traditional fuzzy regression (Tanaka et al. 1982, Takagi and Sugeno 1985, Tanaka and Watada 1988) which considers only the linear terms; therefore, only the first two terms of the fuzzy regression model (1) are considered and the rest of the terms are all ignored. It is also intended to supplant the evolutionary methods whereby no fuzzy coefficient can be generated and thus no fuzziness can be addressed.

The fuzzy regression model (1) can be rewritten as:

$$\widetilde{y} = \widetilde{A}'_0 \, x'_0 + \widetilde{A}'_1 \, x'_1 + \widetilde{A}'_2 \, x'_2 \dots \widetilde{A}'_{N_{NR}} \, x'_{N_{NR}} \tag{2}$$

or $\widetilde{y} = (c'_0, \alpha'_0) x'_0 + (c'_1, \alpha'_1) x'_1 + (c'_2, \alpha'_2) x'_2 + \dots (c'_{N_{NR}}, \alpha'_{N_{NR}}) x'_{N_{NR}}$ (3)

where $1 + N_{NR}$ is the number of terms of design variables in (1), (2) and (3); $\widetilde{A}'_0 = \widetilde{A}_0$,

$\widetilde{A}'_1 = \widetilde{A}_1$, $\widetilde{A}'_2 = \widetilde{A}_2, \dots \widetilde{A}'_{N_{NR}} = \widetilde{A}_{N\dots N}$; $x'_0 = 1$, $x'_1 = x_1$, $x'_2 = x_2$, $\dots x'_{N_{NR}} = x_1 \cdot x_2 \cdot \dots x_d$;

and $\widetilde{A}'_0 = (c_0, \alpha_0)$, $\widetilde{A}'_1 = (c'_1, \alpha'_1)$, $\dots \widetilde{A}'_{N_{NR}} = (c'_{N_{NR}}, \alpha'_{N_{NR}})$. $\widetilde{A}'_i$ and x'_i are called the fuzzy coefficient and the transformed design variable respectively, where $i = 0,1,2,\dots N_{NR}$.

Determination of fuzzy coefficients

The vectors of the fuzzy coefficients are defined as:

$$\widetilde{A}' = \left(\widetilde{A}'_0, \widetilde{A}'_1, \ldots \widetilde{A}'_{N_{NR}}\right) = \left((c'_0, \alpha'_0), (c'_1, \alpha'_1), \ldots (c'_{N_{NR}}, \alpha'_{N_{NR}})\right), \tag{4}$$

$$c' = \left(c'_0, c'_1, \ldots c'_{N_{NR}}\right), \tag{5}$$

and $\quad \alpha' = \left(\alpha'_0, \alpha'_1, \ldots \alpha'_{N_{NR}}\right).$ \hfill (6)

The vector of the transformed design variables is defined as:

$$x' = \left(x'_0, x'_1, x'_2, \ldots x'_{N_{NR}}\right). \tag{7}$$

Using the vectors of the fuzzy coefficient and the vector of transformed design variables, equation (2) can be rewritten as:

$$\widetilde{y} = \widetilde{A}' \cdot x'^T \tag{8}$$

Since some terms in (1) may be redundant, prudent selection of significant terms or orders is advisable if a more economical and adequate model is desired. In the intelligent fuzzy regression approach, genetic programming is used to generate the fuzzy regression model which consists of significant terms.

Pseudocode of Algorithm

The flowchart of the intelligent fuzzy regression is shown in Figure 1 and the pseudocode of the intelligent fuzzy regression used is shown below:

Intelligent fuzzy regression algorithm pseudocode

Step 1: $t \leftarrow 0$

Step 2: Initialize the population $\Omega(t) = [\theta_1(t), \theta_2(t), \ldots \theta_{POP}(t)]$.

    // $\Omega(t)$ is the population of the fuzzy regression at the t-th generation.

// $\theta_i(t)$ is the i-th individual of $\Omega(t)$ which represents the structure of the fuzzy

// regression model(1).

//where $\theta_k(t)$ is in polynomial form but no fuzzy coefficient is assigned.

Step 3: Assign fuzzy coefficients to all terms in $\theta_i(t)$ by Tanaka et al. (1982)'s

fuzzy regression algorithm.

Step 4: Evaluate all $\theta_i(t)$ based on the fitness function (14).

while (Terminational condition not fulfilled) **do {**

      Step 5: Parent Selection $\Omega(t+1) \leftarrow \Omega(t)$

      Step 6: Crossover $\Omega(t+1)$

      Step 7: Mutation $\Omega(t+1)$

      Step 8: $t \leftarrow t+1$

      Step 9: Step 3

      Step 10: Step 4

}

In Step 1, the generation number t is set to 0. In Step 2, genetic programming

creates a random initial population $\Omega(t)$ with *POP* individuals, where $\theta_i(t)$ is the i-th

individual. Each individual $\theta_i(t)$ is in the form of a tree representation, which can be

used to represent the structure of the fuzzy regression model as defined in equation

(1). In Step 3, the fuzzy coefficients are assigned to each term of the individual $\theta_i(t)$

by applying Tanaka's fuzzy regression (Tanaka et al. 1982, Tanaka and Watada's

1988). In Step 4, all individuals are evaluated based on a defined fitness function

which aims at evaluating the goodness-of-fit of the fuzzy regression model. In Step 5,

the parent selection process uses the goodness-of-fit of each individual to determine

the selection of potential individuals for performing crossover in Step 6 and mutation in Step 7. In Step 8, the new individuals with the determined fuzzy coefficients are evaluated using the fitness function in order to create a new population $\Omega(t+1)$. In Step 8, the generation number t is added by one. The process continues until the pre-defined termination condition is fulfilled. The major aspects of applying the intelligent fuzzy regression to generate the fuzzy regression model are discussed below:

2.1 Model Representation

In intelligent fuzzy regression, hierarchical trees, which are composed of functions in the set F and terminals in the set T (Koza 1992), are used to represent the structure shown in equation (1). F consists of two arithmetic operations, + and *, which exist in the fuzzy regression model (1). $T = \{\boldsymbol{x}, \tilde{\mathbf{a}}\}$ contains the design variable set $\boldsymbol{x} = \{x_1, x_2, \dots x_N\}$ of the fuzzy regression model and the fuzzy coefficient set $\tilde{\mathbf{a}} = \{\tilde{a}_0, \tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_{N_{NS}}\}$ of the fuzzy regression model, where n is the number of design variables and N_{NS} is the number of terms of the fuzzy regression model. The structure of the fuzzy regression model is depicted as a labeled tree with ordered branches, which consists of operations (internal nodes of the tree) F from the function set and arguments (terminal nodes of the tree) from the terminal set T. For example, the i-th

individual $\theta_i(t)$ represents the following structure of the fuzzy regression model:

$$\theta_i(t) = x_1{}^2 - x_2{}^2 + x_1 \cdot x_2 \cdot x_4$$

The fuzzy regression model with fuzzy coefficients can be represented by:

$$\tilde{a}_0 + \tilde{a}_1 \cdot x_1{}^2 - \tilde{a}_2 \cdot x_2{}^2 + \tilde{a}_3 \cdot x_1 x_2 x_4,$$

where $\tilde{a}_0$, $\tilde{a}_1$, $\tilde{a}_2$ and $\tilde{a}_3$ are the fuzzy coefficients. It can also be rewritten as:

$$\tilde{a}_0 + \tilde{a}_1 \cdot x'_1 - \tilde{a}_2 \cdot x'_2 + \tilde{a}_3 \cdot x'_3,$$

where $x'_1 = x_1{}^2$, $x'_2 = x_2{}^2$ and $x'_3 = x_1 \cdot x_2 \cdot x_4$.

For a fuzzy regression model with N_{NR} terms, the fuzzy coefficients, $\tilde{a}_0$, $\tilde{a}_1$, $\ldots$

$\tilde{a}_{NS}$ can be determined by solving the following linear programming problem:

$$Minimize \quad J = \sum_{j=0}^{N_{NR}} \left(c'_j \sum_{i=1}^{M} \left| x'_j(i) \right| \right) \tag{9}$$

where $\tilde{a}_j = \left(a_j^c, a_j^s \right)$, M is the number of data sets, and $x'_j(i)$ is the j-th transformed

variable of the fuzzy polynomial model of the i-th data set, subject to:

$$\sum_{j=0}^{N_{NR}} a_j^s x'_j(i) + (1-h) \sum_{j=0}^{N_{NR}} a_j^c \left| x'_j(i) \right| \geq y(i) \tag{10}$$

$$\sum_{j=0}^{N_{NR}} a_j^s x'_j(i) + (1-h) \sum_{j=0}^{N_{NR}} a_j^c \left| x'_j(i) \right| \leq y(i) \tag{11}$$

$$a_j^c \geq 0, \quad a_j^s \in R, \quad x_0'(i) = 1 \text{ for all } i, \tag{12}$$

$$0 \leq h \leq 1, i = 1, 2, \cdots M, \quad j = 0, 1, 2, \cdots N_{NR}. \tag{13}$$

J in equation (9) is the total fuzziness of the regression model. The value of h in

equation (10) and equation (11) is between 0 and 1. h refers to the degree to which the

fuzzy linear model fits the given data sets, and is subjectively chosen by decision

makers. Constraints (10) and (11) impose the restriction that the observation of the i-th data set $y(i)$ has at least h degrees of belonging to $\tilde{y}(i)$ as $\mu_{\tilde{y}(i)}(y(i)) \geq h \ (i = 1, \ 2, \cdots, M)$. Therefore, the objective of solving the linear programming problem (9-13) is to determine the fuzzy nonlinear parameters $\tilde{a}_j = \left(a_j^c, a_j^s\right)$ such that the total vagueness J is minimized subject to $\mu_{\tilde{y}(i)}(y(i)) \geq h \ (i = 1, \ 2, \cdots, M)$.

2.2 Fitness function

In intelligent fuzzy regression, the fitness function is defined as:

$$fitness_j = \frac{1 - MAE_j}{\left(1 + \exp\left(c_1\left(L_j - c_2\right)\right)\right)} \tag{14}$$

where $fitness_j$ is the fitness value of the j-th individual, L_j is the number of nodes of the j-th individual, c_1 and c_2 are both penalty terms, and the mean absolute error of the j-th individual MAE_j is defined as the following formulation:

$$MAE_j = 100\% \times \sqrt{\frac{1}{M} \sum_{k=1}^{M} \left| \frac{y(k) - F_j(x(k))}{y(k)} \right|} , \tag{15}$$

where F_j is the fuzzy regression model represented by the j-th individual, $(y(k), x(k)) = (y(k), (x_1(k), x_2(k), \ldots x_N(k)))$ is the k-th training data set, N is the number of variables of the training data set, and M is the number of training data sets used for developing the fuzzy regression model.

Equation (15) aims at fitting training data sets to the fuzzy regression model, and it avoids generating fuzzy regression models with too many insignificant terms. It is designed to find a balance between minimizing the number of terms and maximizing model accuracy, since a fuzzy regression model which contains many insignificant terms reduces its interpretation (Madar et al. 2005).

2.3 Evolutionary operators

Like other evolutionary algorithms, the two main evolutionary operators are crossover and mutation. The crossover operation produces a pair of offspring that inherit characteristics from both selected individuals by selecting a random node in each of the hierarchical tree structures of the individuals and exchanging the associated sub-expressions of the hierarchical tree structures. For example, two individuals, θ_i and θ_j, are selected for crossover; they are represented as Figure 2a and are shown as follows:

$$\theta_i = x_1 * x_2 + (-x_3)$$

$$\theta_j = +x_2 - (x_3 * x_4)$$

After the crossover operation, the individuals are represented as Figure 2b and are shown as follows:

$$\theta_i = x_1 * x_2 + (-x_3)$$

$$\theta_j = +x_2 - (x_3 * x_4)$$

Because of the dynamic representation used in GP-FR, typically the parents are different in size, shape and content. The process of mapping the genotype onto the phenotype does not correspond to a one-to-one relationship. Therefore, the resulting offspring can be expressed by more than one different tree structure and some diversification of the population is allowed.

Mutation is performed by randomly selecting a node that is an internal or terminal node and replacing the associated sub-expression with a randomly generated sub-expression. For example, an individual θ_i (as shown in Figure 3) is selected for mutation:

$$\theta_i = (x_1 - x_2) + (x_3 * x_4)$$

As shown in Figure 3, the individual is mutated by replacing a minus in the node with a multiplier. After the mutation has been performed, the individual became:

$$\theta_i = (x_1 - x_2) + (x_3 - x_4)$$

1-st individual: θ_i
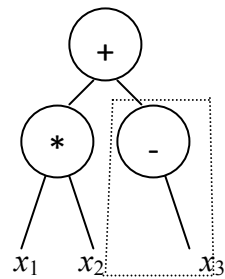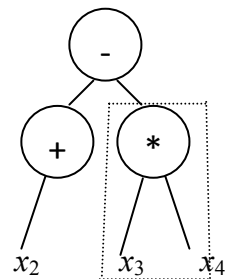
2-nd individual: θ_j

Figure 2 (a) The individuals θ_i and θ_j prior to crossover

1-st individual: θ_i'
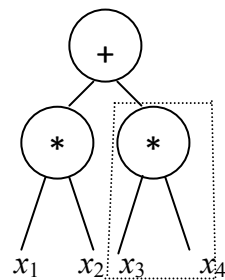
2-nd individual: θ_j'
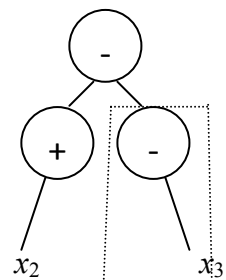
Figure 2 (b) The individuals θ_i' and θ_j' after crossover has been performed

The individual θ_k before the mutation
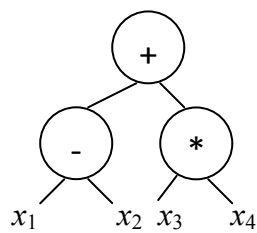
The individual θ_k' before the mutation
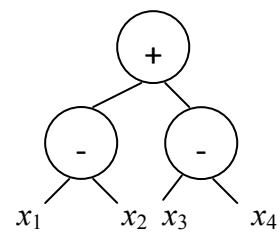
Figure 3 Individual produced by the mutation

2.4 Selection and convergence

After performing crossover and mutation, individuals from the current population

$\Omega(t)$ with relatively better fitness are selected to serve as parents for the next

generation $\Omega(t+1)$. The approach of a roulette-wheel, which is one of the most

common selection methods used for selecting individuals to perform reproduction

operations in evolutionary algorithms (Goldberg 1989), is used in intelligent fuzzy

regression. Regarding the jth individual, its fitness is assigned a value, fitness$_j$, and

the selection probability value, prob$_j$, is defined as:

$$\text{prob}_j = \frac{\text{fitness}_j}{\sum_{j=1}^{POP} \text{fitness}_j} \tag{20}$$

where *POP* is the population size of the intelligent fuzzy regression. Equation (20)

shows that the individual with a larger fitness value has a higher probability of being

selected.

After the selection, the population evolves and improves iteratively until a

stopping condition is met. In the intelligent fuzzy regression, the stopping criterion is

met when the number of generations is equal to a pre-defined number of generations.

Otherwise, the intelligent fuzzy regression goes on to the next iteration.

3 An Illustration Example

3.1 Mobile phone design

The example of mobile phone design is used to illustrate the intelligent fuzzy regression approach to modeling the relationships between affective responses and design variables defined in equation (2) in Section 2. In the highly competitive market of mobile phones, the product designers provide the consumers with various styles for different brands and different product series. To capture the trend of the recent market in mobile phones, 32 recent mobile phones of various brands including Nokia, Sony Ericsson and Motorola were selected and are shown in Figures 1 and 2. Morphological analysis was used to extract representative elements of mobile phones as numerical data sets, in which both the shape profiles and the product components of the mobile phones were used. Table 1 shows the 9 representative elements "top shape", "bottom shape", "side shape", "function button shape", "number button style", "length width ratio", "thickness", "layout" and "Border and frame", which are denoted as $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ and x_9 respectively. They were identified from the 32 mobile phone samples. Each representative element has various types of form variations, which ranged from 3 to 6.

The four most representative affective responses for mobile phone design, "simple - complex" (S-C), "unique - general" (U-G), "high-tech - classic" (H-C),

and "handy - bulky" (H-B) (Lai et al. 2004), were collected from the 14 image word pairs for micro-electronic products (Chuang and Ma 1999), and they were used for evaluating the images values of the mobile phones. A survey was conducted using an online questionnaire to ask about the appearance of mobile phones on the S-C, U-G, H-C, and H-B responses, in which their degrees of image values are denoted as y_1, y_2, y_3 and y_4 respectively. The ranking scale ranges from 1 to 5 in which 1 is the smallest and 5 is the largest. Both front and side views of mobile phones were used for evaluating the product images since both front and side views of mobile phones influence the product image of mobile phones. Figure 3 shows one of the illustrations of the mobile phones in the survey in which 32 illustrations were given. Table 2 shows the morphological matrix of the 32 mobile phones samples based on the 9 representative elements. Also, it shows the means of the affective responses S-C, U-G, H-C, and H-B with respect to 34 interviewers.

3.2 Model development

The objective of modeling is to relate the design variables x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 and x_9 to one of the affective responess y_1, y_2, y_3 and y_4. The modeling results based on intelligent fuzzy regression are compared with those based on statistical regression (Seber 2003), Peters' fuzzy regression (Peters 1994) and Takagi's fuzzy regression (Takagi and Sugeno 1985). Evaluation of the effectiveness of the models is carried out

by investigating the mean of training errors as shown below:

$$\text{Re} = \frac{1}{M} \sum_{i=1}^{M} \frac{\left| y_k(i) - P_k\left(x_1(i), x_2(i), ..., x_9(i)\right) \right|}{y_k(i)} \times 100\% \qquad (21)$$

In equation (21) above, M is the number of data sets; $y_k(i)$ is the k-th affective response of the i-th data set; $x_1(i)$, $x_2(i)$, ... and $x_9(i)$ are the i-th data set for the design variables; and $P_k\left(x_1(i), x_2(i), x_3(i), ..., x_9(i)\right)$ is the prediction of the k-th affective response for the i-th data.

Using the 32 experimental data sets and their results shown in Table 2, the proposed intelligent fuzzy regression was implemented using Matlab to generate models to relate the image values (y_1, y_2, y_3, y_4) and the design values $(x_1, x_2, x_3, ...$ $x_9)$. The parameters used in the intelligent fuzzy regression were set as shown in Table 3 with reference to (Madar et al. 2005). Since intelligent fuzzy regression is a stochastic method, different results will be obtained from different runs. To evaluate its overall performance, 30 runs on intelligent fuzzy regression were carried out, and the mean of the 30 runs was calculated. The models developed for the four affective responses S-C, U-G, H-C, and H-B by using the four methods, and the training errors (Re) of the developed models are summarized in Table 4. It shows that the Re of the proposed intelligent fuzzy regression is smaller than those of the statistical regression, Takagi's fuzzy regression and Peter fuzzy regression. This indicates that the proposed intelligent fuzzy regression can fit the data sets with the smallest mean errors.

To further validate the modeling performance of the intelligent fuzzy regression, two data sets were randomly selected from the 32 data sets, as shown in Table 2, as testing data sets and the remaining 30 data sets were used to develop an intelligent fuzzy regression model, a statistical regression model, a Takagi's fuzzy regression model and Peters' fuzzy regression model. Their prediction errors were calculated. The validations were repeated 32 times. Because intelligent fuzzy regression is a stochastic algorithm, we ran the GP-FR 30 times in each validation test, and the mean of the 30 runs was calculated. Results of the 32 validation tests for models of S-C, U-G, H-C and H-B are shown in Figures 4, 5, 6 and 7 respectively. The results of the statistical regression, Takagi's fuzzy regression, Peters' fuzzy regression and intelligent fuzzy regression are shown as circle dot lines "o-", star dot lines "*-", plus dot lines "+-" and solid lines "--" respectively. It can be seen that the prediction errors of the intelligent fuzzy regression, for all the tests, are usually the smallest. Table 5 summarizes the means and variances of the prediction errors of the S-C, U-G, H-C and H-B for the four methods respectively. From the table, it can be seen that intelligent fuzzy regression yields the smallest means of prediction errors and variances of prediction errors for all S-C, U-G, H-C and H-B.

3.3 Optimization of affective design

In optimization of affective design of the mobile phones, it is necessary to determine a setting of design variables so as to maximize the four affective responses, S-C, U-G,

H-C and H-B. To determine the optimal design variables, the multi-objective function is formulated by maximizing the four affective responses, S-C, U-G, H-C and H-B. Because the proposed intelligent fuzzy regression can achieve the best affective models among the four methods, the models developed by the proposed intelligent fuzzy regression are used and the multi-objective function is formulated by:

$$\max \begin{cases} y_1 = (0.04529, 0.6639) + (2.0400, 0.0453) \cdot (x_9 + x_6 \cdot x_7) \\ y_2 = (2.7266, 0.0025) \cdot x_5 \cdot x_8 + (-0.1769, 1.1952) \cdot x_8 + (1.0102, 0.0023) \\ y_3 = (1.1201, 0) \cdot x_6{}^2 \cdot x_5 + (-0.0206, 0.3143) \cdot x_6 + (1.0444, 0) \cdot x_2 + (0.0366, 0) \\ y_4 = (3.6920, 0) \cdot x_4 + (-0.1531, 0) \cdot x_8 + (0.0165, 0.2423) \cdot x_8{}^2 + \\ \quad (-0.5189, 0) \cdot (x_8 + x_7{}^2 + x_4 \cdot x_7) \end{cases}$$

subject to: $x_1, x_2, x_3, x_4, x_5, x_9 \in \{1, 2, ..., 6\}$;

$$x_6, x_8 \in \{1, 2, 3\};$$

$$x_7 \in \{1, 2, 3, 4\}$$

(21) is a Pareto-based multi-objective problem which can be solved by the multi-objective genetic algorithm richly represented in the literature for solving multi-objective problems (Knowles and Corne, 2000; Zitzler and Thiele, 1999). Therefore, GA was used in this research. The GA optimization model aims at minimizing the objective function (21) from which an optimal design variables (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 and x_9) setting can be obtained to achieve the maximum of y_1, y_2, y_3 and y_4. The detailed description of the multi-objective genetic algorithm, which is not the contribution or scope of this paper, is not presented in this paper. The operations and the parameters used in the genetic algorithm can be referred to (Chan et al. 2009). The optimal design variables are found as:

$x_1 = 3$, $x_2 = 2$, $x_3 = 6$, $x_4 = 2$, $x_5 = 1$, $x_6 = 2$, $x_7 = 4$, $x_8 = 3$ and $x_9 = 5$

The optimal affective responses can be achieved on the mobile phone based on optimal design variables.

4 Conclusion

Existing modeling methods which relate customers' affective responses and the design variables of a new product have not addressed the development of explicit models which represents the nonlinearity or fuzziness inherent in such relationships. In this paper, we have proposed an intelligent fuzzy regression to generate models for relating design variables to affective responses in which both nonlinearity and fuzziness are considered. In order to develop a methodology which is capable of addressing both fuzziness and nonlinearity in fuzzy regression, we employed the evolutionary algorithm to first construct branches of a tree which represents the nonlinear structures of the model. Then we used the fuzzy regression algorithm to calculate the fuzzy coefficients of the model. The resulting model is explicit, and consists of fuzzy nonlinear terms in relating affective responses to design variables.

A case study of the affective product design of mobile phones is used to illustrate the proposed method. To validate this intelligent fuzzy regression approach in modeling the relationship between affective responses to design variables, it has been applied for generating models of affective product design of mobile phones. The four affective responses of customers and nine design variables in mobile phones were considered. It has been compared with the other commonly used explicit modeling methods, statistical regression and two fuzzy regression methods. The results show

that models with the smallest training errors are generated by the intelligent fuzzy regression rather than the ones generated by the other methods. This indicates that the intelligent fuzzy regression approach is more capable of modeling the survey data sets, which are fuzzy and nonlinear. The validation results show that the smallest prediction errors and errors in variance are also achieved by intelligent fuzzy regression rather than by other methods. Better results were achieved and can be explained by the fact that nonlinearity and fuzziness were taken into consideration using the intelligent fuzzy regression approach, which is not represented in commonly used existing methods.

There is no evidence to show that stochastic methods outperform the others. In the future, an evaluation of the performance of the fuzzy regression based polynomial modeling will be implemented using other stochastic methods like particles swarm optimization and immune optimization algorithm. A more appropriate and effective modeling method is expected to be reported in the future.

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Appendix

Step 1 — Set $t \leftarrow 0$

Step 2 — Initialize the population

$\Omega(t)=[\theta_1(t), \theta_2(t),\ldots, \theta_{POP}(t)]$

where the i-th individual $\theta_i(t)$ is in the polynomial form. For example,

$\theta_i(t)=x_1x_2+x_2^2$

Step 3/ Step 9 — Assign fuzzy coefficient to each individual. For example,

$\theta_i(t)=A_{12}x_1x_2+A_{22}x_2^2$

Step 4/ Step 10 — Evaluate all $\theta_i(t)$ based on the fitness function (14)

Termination condition met

Yes → Return the individual with the best fitness

No

Step 5 — Selection the individuals from $\Omega(t)$ to $\Omega(t+1)$.

Step 6 — Performing crossover on some of the individuals on $\Omega(t+1)$.

Step 7 — Performing mutation on some of the individuals on $\Omega(t+1)$.

Step 8 — Performing mutation on some of the individuals on $\Omega(t+1)$.

Figure 1 The flowchart of the intelligent fuzzy regression

Figure 1 The 32 representative mobile phone samples

Figure 2 The 32 representative mobile phone samples (continued)

Does the shape feel?

		1	2	3	4	5
1*	simple or complex? (simple =1, complex =5)	○	○	○	○	○
2*	unique or general? (unique =1, general =5)	○	○	○	○	○
3*	hi-tech or classic? (hi-tech =1, classic =5)	○	○	○	○	○
4*	handy or bulky? (handy =1, bulky =5)	○	○	○	○	○

Figure 3 The format of the questionnaires for each mobile phone sample
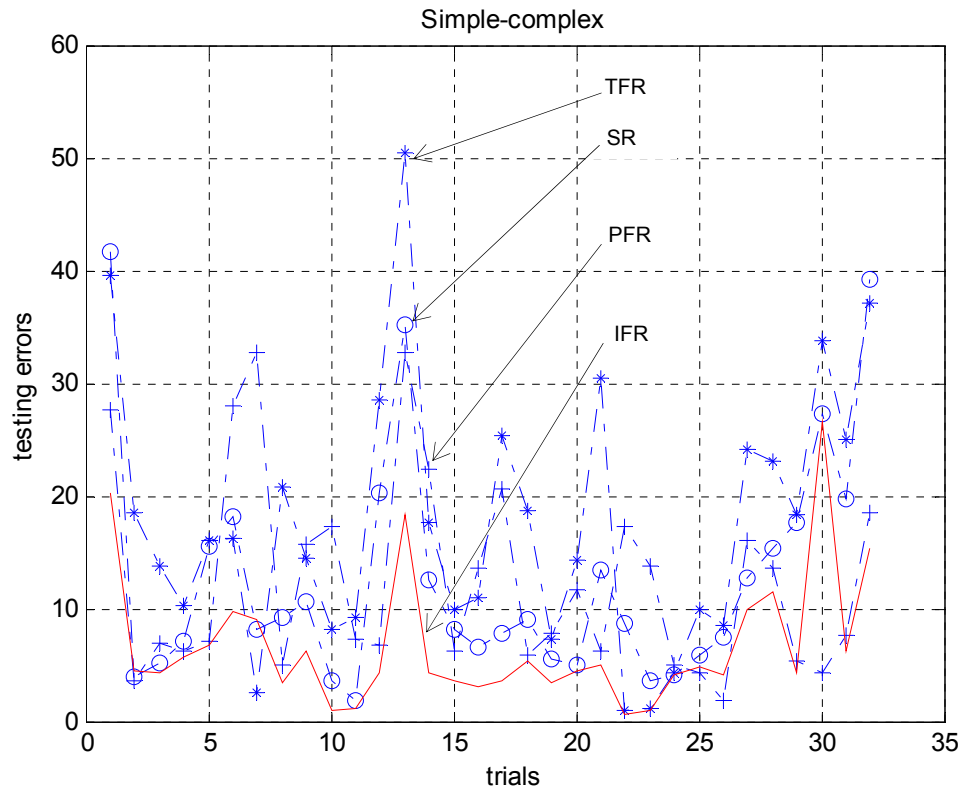
Figure 4 Validation results of the models for S-C
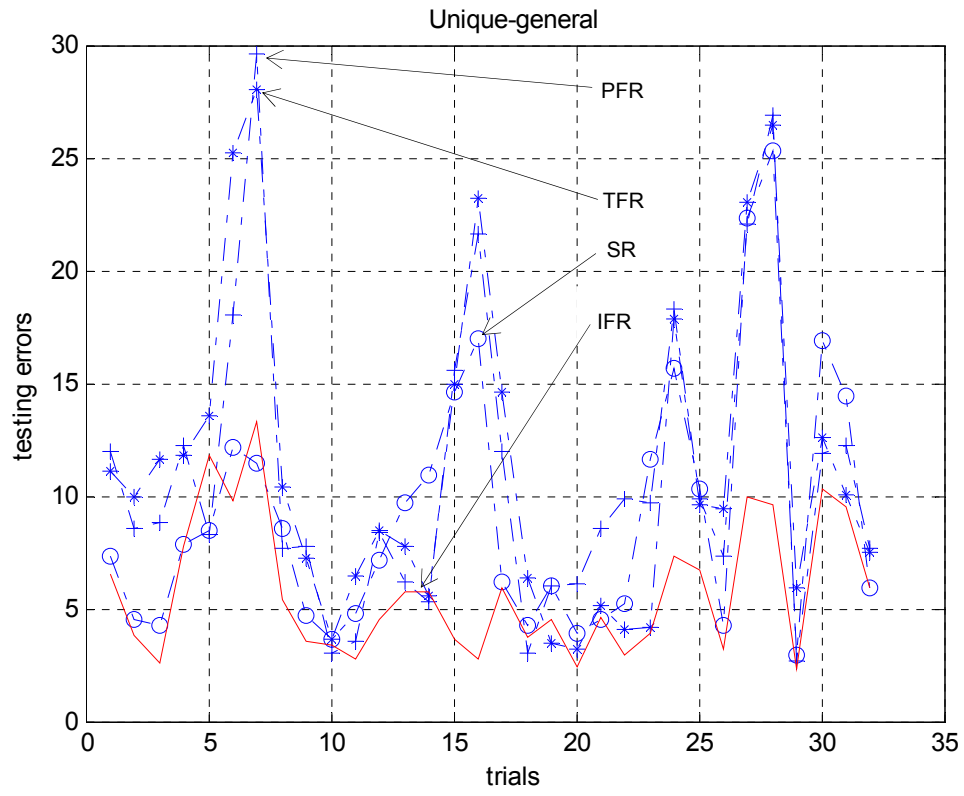
Figure 5 Validation results of the models for U-G
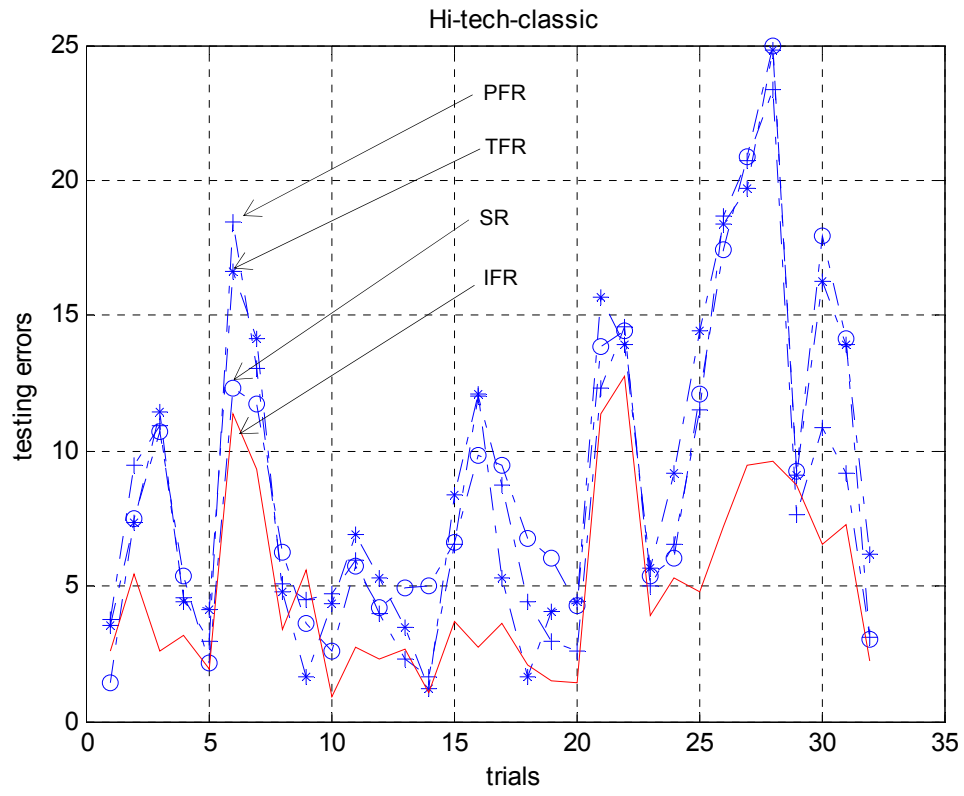
Figure 6 Validation results of the models for H-C
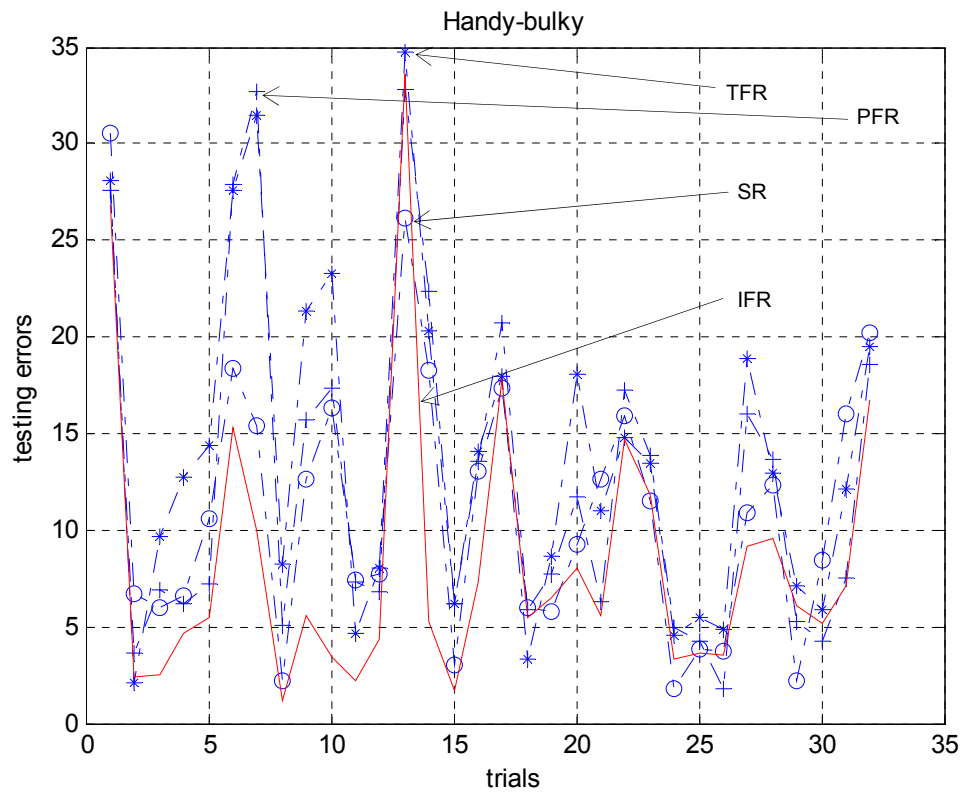
Figure 7 Validation results of the models for H-B

Table 1 Morphological analysis on the 32 representative mobile phone samples

	Elements	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
1	Top Shape (x_1)	Line and no fillet	Arc and no fillet	Line and small fillet	Arc and small fillet	Irregular	Curve
2	Bottom Shape (x_2)	Line and no fillet	Arc and no fillet	Line and small fillet	Arc and small fillet	Irregular	Curve
3	Side Shape (x_3)	Trapezoidal rear	Rounded end	Parallelogram	Bowed	Trapezoid fore	Polygonal
4	Function Button Shape (x_4)	Round	Square and round inner	Small squares	Large square	Wide large	Other Shape
5	Number Buttons Style (x_5)	Regular grid	Shaped grid	Bars	One piece	Other style	No Number buttons
6	Length-width ratio (x_6)	16:9	2:1	5:2			
7	Thickness (x_7)	≦10 mm	11-14 mm	15-18 mm	≧19 mm		
8	Layout (x_8)	Bar	Slide	Large screen	Other Layout		
9	Border or frame (x_9)	Narrow	Wide	3 sided	2 sided	1 Sided	No borders

Table 2 Form design matrix of 32 mobile phone samples

Phone no.	A₁	A₂	A₃	A₄	A₅	A₆	A₇	A₈	A₉	S-C	U-G	H-C	H-B
1	3	3	1	3	2	2	3	1	1	1.85	3.62	2.97	2.56
2	3	3	2	2	1	2	2	1	2	2.59	3.44	3.15	2.79
3	6	6	1	1	5	2	4	1	2	2.88	2.76	3.21	3.32
4	4	4	3	1	6	2	2	2	2	2.41	2.65	2.88	2.59
5	3	4	3	4	6	2	2	2	2	2.06	2.85	2.53	2.47
6	3	3	1	5	6	1	3	2	1	2.71	2.41	2.15	3.18
7	1	1	2	4	6	3	4	2	6	3.26	2.53	2.47	3.18
8	1	1	1	2	6	2	2	2	6	2.79	2.74	2.50	2.71
9	3	4	6	1	6	2	3	2	2	2.91	2.65	2.85	3.12
10	4	4	3	6	4	3	2	1	2	2.65	2.82	3.00	2.15
11	2	2	6	5	6	2	4	2	2	2.76	2.62	2.47	3.18
12	2	2	6	3	6	2	3	2	4	2.71	2.56	2.41	3.38
13	6	6	6	4	6	2	3	2	2	2.09	2.76	2.85	2.71
14	4	4	2	6	6	1	2	3	3	2.21	2.09	2.09	1.94
15	4	3	6	1	6	2	3	2	2	2.44	2.82	2.71	3.09
16	3	3	6	5	6	2	3	2	1	2.62	2.15	2.35	2.94
17	3	3	2	6	6	1	2	3	1	2.12	2.53	2.35	3.03
18	2	4	6	5	2	3	1	1	5	2.50	3.38	2.97	2.59
19	3	3	1	4	5	3	3	1	1	2.41	3.00	3.00	3.03
20	4	4	6	5	1	3	2	1	1	2.68	3.68	3.53	3.06
21	4	4	1	1	2	3	2	1	2	2.88	3.35	3.29	3.12
22	6	4	3	1	4	3	2	1	2	2.88	2.94	2.97	2.97
23	3	3	6	2	3	2	3	1	6	3.12	3.38	3.15	3.56
24	5	5	1	4	3	3	2	1	1	2.50	2.85	3.24	2.62
25	4	4	6	1	6	2	3	2	2	2.44	3.21	3.06	3.09
26	3	6	5	1	6	3	3	2	2	2.68	2.97	2.85	3.32
27	1	1	5	1	6	2	2	2	4	2.65	2.79	2.79	2.91
28	3	3	4	1	6	1	2	3	2	2.00	1.91	1.91	2.53
29	4	4	2	1	6	1	2	2	2	2.41	2.47	2.21	2.56
30	4	4	4	5	2	2	3	1	2	3.26	3.15	2.82	3.03
31	3	3	1	6	6	2	3	4	2	3.38	2.79	2.76	3.18
32	3	3	1	1	6	2	3	2	1	2.32	2.62	2.56	3.50

Table 3 GP parameters implemented in the GP-FR

Population size	50
Maximum number of evaluated individuals	5000
Generation gap	0.9
Probability of crossover	0.5
Probability of mutation	0.5
Probability of changing terminal via non-terminal	0.25
Penalty factors in the fitness function (20)	$c_1=0.5$ and $c_2=30$
Maximum depth of tree	30

Table 4 Training errors (Re in percentage) of the four modeling methods

Affective responses	Training method	Trained models	Training error (%)
S-C	SR	$y = 1.2798 + 0.2506 \cdot x_1 - 0.1986 \cdot x_2 - 0.0272 \cdot x_3 - 0.0010 \cdot x_4 - 0.4208 \cdot x_5 + 0.4547 \cdot x_6 + 0.6401 \cdot x_7 + 0.3634 \cdot x_8 + 0.2580 \cdot x_9$	9.2081
	TFR	$y = (1.2066,0) + (0.4963,0) \cdot x_1 + (-0.5252,3.5191) \cdot x_2 + (0.2057,0) \cdot x_3 + (0.2182,0.9900) \cdot x_4 + (-0.0123,0) \cdot x_5 + (0.5136,0) \cdot x_6 + (0.0680,0) \cdot x_7 + (0.1819,0) \cdot x_8 + (0.2863,0) \cdot x_9$	11.836
	PFR	$y = (0.8671,0.0747) + (0.1073,0.0252) \cdot x_1 + (-0.04815,0.0257) \cdot x_2 + (0.0166,0.1164) \cdot x_3 + (0.0352,0.0409) \cdot x_4 + (-0.0147,0.0035) \cdot x_5 + (0.1358,0.0201) \cdot x_6 + (0.1407,0.2719) \cdot x_7 + (0.1597,0.0144) \cdot x_8 + (0.2863,0.0089) \cdot x_9$	9.2132
	IFR	$y = (0.04529,0.6639) + (2.0400,0.0453) \cdot (x_9 + x_6 \cdot x_7)$	0.091471
U-G	SR	$y = 3.3302 - 0.1971 \cdot x_1 + 0.1514 \cdot x_2 + 0.0480 \cdot x_3 - 0.1195 \cdot x_4 - 0.9145 \cdot x_5 + 0.2959 \cdot x_6 + 0.1916 \cdot x_7 + 0.0522 \cdot x_8 - 0.0106 \cdot x_9$	8.3444
	TFR	$y = (3.4331,0) + (-0.5917,0) \cdot x_1 + (0.4612,0) \cdot x_2 + (0.1273,0.3813) \cdot x_3 + (-0.2291,0) \cdot x_4 + (-1.1061,0) \cdot x_5 + (0.3110,0) \cdot x_6 + (0.4329,0) \cdot x_7 + (-0.0143,2.9794) \cdot x_8 + (0.0042,0) \cdot x_9$	8.5260
	PFR	$y = (3.7118,0) + (-0.1918,0.0091) \cdot x_1 + (0.1403,0.0007) \cdot x_2 + (0.0159,0) \cdot x_3 + (-0.0698,0) \cdot x_4 + (-0.2007,0) \cdot x_5 + (0.0893,0) \cdot x_6 + (0.1284,0) \cdot x_7 + (-0.0773,0.1393) \cdot x_8 + (-0.0038,0) \cdot x_9$	7.6739
	IFR	$y = (2.7266,0.0025) \cdot x_5 \cdot x_8 + (-0.1769,1.1952) \cdot x_8 + (1.0102,0.0023)$	0.07278
H-C	SR	$y = 2.2971 + 0.1752 \cdot x_1 + 0.0767 \cdot x_2 + 0.0321 \cdot x_3 - 0.1197 \cdot x_4 - 0.5569 \cdot x_5 + 0.6110 \cdot x_6 + 0.1731 \cdot x_7 + 0.0673 \cdot x_8 - 0.0042 \cdot x_9$	6.7134
	TFR	$y = (2.4004,0.0072) + (0.0023,0.0031) \cdot x_1 + (0.0452,0.0024) \cdot x_2 + (0.0150,0.0097) \cdot x_3 + (-0.0343,0.0014) \cdot x_4 + (-0.0921,0.0028) \cdot x_5 + (0.2860,0.0057) \cdot x_6 + (0.0833,0.0050) \cdot x_7 + (-0.0514,0.0814) \cdot x_8 + (-0.0228,0.0143) \cdot x_9$	6.3738
	PFR	$y = (2.4584,0) + (-0.0336,0.5852) \cdot x_1 + (0.1332,0) \cdot x_2 + (0.0121,0) \cdot x_3 + (-0.1086,0) \cdot x_4 + (-0.4029,0) \cdot x_5 + (0.6217,0) \cdot x_6 + (0.2961,0.095) \cdot x_7 + (-0.1230,1.3129) \cdot x_8 + (-0.1026,0.4762) \cdot x_9$	7.6739
	IFR	$y = (1.1201,0) \cdot x_6^2 \cdot x_5 + (-0.0206,0.3143) \cdot x_6 + (1.0444,0) \cdot x_2 + (0.0366,0)$	0.0582
H-B	SR	$y = 2.2905 - 0.1365 \cdot x_1 - 0.0710 \cdot x_2 + 0.0776 \cdot x_3 - 0.2116 \cdot x_4 - 0.3376 \cdot x_5 + 0.2183 \cdot x_6 + 0.9195 \cdot x_7 + 0.2235 \cdot x_8 - 0.0471 \cdot x_9$	9.8682
	TFR	$y = (1.7676,1.4951) + (0.0167,0) \cdot x_1 + (-0.0845,0) \cdot x_2 + (0.1098,0) \cdot x_3 + (-0.2344,0.9374) \cdot x_4 + (0.0383,0) \cdot x_5 + (0.2897,0) \cdot x_6 + (0.8183,0) \cdot x_7 + (0.1816,1.3757) \cdot x_8 + (0.0702,0) \cdot x_9$	8.8092
	PFR	$y = (2.3254,0.0412) + (-0.0364,0) \cdot x_1 + (-0.0064,0) \cdot x_2 + (0.0290,0) \cdot x_3 + (-0.0923,0.0260) \cdot x_4 + (-0.0609,0) \cdot x_5 + (0.0299,0) \cdot x_6 + (0.4065,0) \cdot x_7 + (-0.0900,0.0901) \cdot x_8 + (0.0007,0) \cdot x_9$	8.2113
	IFR	$y = (3.6920,0) \cdot x_4 + (-0.1531,0) \cdot x_8 + (0.0165,0.2423) \cdot x_8^2 + (-0.5189,0) \cdot (x_8 + x_7^2 + x_4 \cdot x_7)$	0.0756

Table 5 Means and variances of the prediction errors of the affective responses

		Statistical regression	Takagi's fuzzy regression	Peters' fuzzy regression	Intelligent fuzzy regression
S-C	Mean (%)	12.825	17.794	12.473	6.7548
	Variance (%)	106.31	136.68	76.801	34.693
U-G	Mean (%)	9.279	11.319	10.953	5.8147
	Variance (%)	31.849	50.093	45.178	9.1224
H-C	Mean (%)	8.9346	9.1336	8.5013	4.9788
	Variance (%)	32.941	36.819	33.346	11.505
H-B	Mean (%)	11.216	13.923	12.473	8.3124
	Variance (%)	48.739	73.713	76.801	52.673