

Copyright © 2012 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

# Probabilistic Optimisation of Generation Scheduling Considering Wind Power Output and Stochastic Line Capacity

B. Banerjee, *Student Member, IEEE*, D. Jayaweera, *Senior Member, IEEE*, and S.M. Islam, *Senior Member, IEEE*

Department of Electrical and Computer Engineering  
Curtin University  
Perth, Australia

**Abstract**— Optimising the power flow in a system has been a challenge for decades. Due to the complexities that are introduced by new technologies, this problem is evolving. Lately, the effect of integrating wind turbines into the system has been taken into account when solving optimal power flow. However, transmission system constraints are usually modeled as fixed constraints using deterministic methods. Deterministic transmission line ratings have been shown to significantly underestimate the capability of the network. However, probabilistic line ratings are not used in optimization studies. In this paper, stochastic optimisation is used to consider the integration of wind turbines as well as probabilistic real time line capacities. It is shown that optimization considering probabilistic line ratings that lead to dynamic constraints in the OPF problem, represents the operational situation more accurately. This approach further reduces the optimum cost of system operation.

**Keywords** – *optimal generator scheduling; probabilistic line rating; stochastic optimisation; wind power*

## I. INTRODUCTION

Wind integration into the grid is increasing rapidly. Many countries around the world have set aggressive targets to source a part of the generation from renewable energy sources. One of the main challenges of integrating wind energy into the grid is managing the intermittency. Having adequate reserves in the grid are one way of managing this.

As the penetration of wind increases, the reserves required to support the increased integration also increases. Moreover, a lot of these reserves need to be maintained in the form of spinning reserves, i.e. generators that are running at no load to enable rapid support when power from wind turbines is low. The cost of spinning reserves is a prohibitive factor to integrating a large number of wind generators into the grid.

Optimising the amount of reserves in the grid is a continued area of research. Traditional methods of optimizing reserve did not account for the uncertainty in wind power forecasts. However, a number of recent studies use stochastic optimization means to address this. [1-4]

Optimal power flow (OPF) and economic dispatch has been investigated for a long time in the context of power systems. The conventional OPF problem has been to schedule generators appropriately to minimise system operation cost. However, the changing nature of the power system has made it necessary to continuously keep upgrading the considerations in the optimal power flow problem. Integration of renewable energy generators such as wind generators has presented a challenge to the traditional OPF problem. These sources are intermittent and hence cannot be considered in the traditional sense. A lot of research is taking place on how to integrate these into the optimal power flow problem [5-9]. Some methods use an analytical method to assess the expected cost of wind forecast uncertainty [6-8] while others use a Monte Carlo simulation approach to generate a probability distribution of the optimized cost [5, 9].

It has been shown that when the wind power production reaches close to rated capacity (or in some cases even lower), the transmission infrastructure becomes overloaded [10, 11]. This implies that the power transferred in the line exceeds the rated capacity. As a result, wind production has to be curtailed in these instances and shortfalls must be met by having additional reserves. Expanding transmission capacity takes time and is an expensive process.

In an effort to address the line congestion issue, a number of studies [12-15] have been undertaken on dynamic line rating. It has been shown that the amount of power a line can carry is dependent on a number of factors, including temperature, wind speed and other environmental factors [12, 13]. It has also been shown that the rated capacity of lines is usually quite conservative in comparison to the actual capacity at a given time.

Studies on dynamic line rating (DLR) focus on using methods to predict the short term capacity of a transmission line. Power transfer capacity of a line is determined by thermal capacity for short lines and stability limit for long lines. Some studies focus on expressing this as a function of the environmental factors such as wind speed and temperature [12, 13]. Some studies on the other hand use statistical methods to create probability distributions of the short term line ratings [14]. It is important that these models should be incorporated

into optimization studies. However, there is yet to be significant work done on this.

In this paper, a method is proposed to modify the stochastic OPF to incorporate the effect of dynamic line rating. Most studies have focused on modeling stochastic aspects of wind generation but modeled the line capacity as a fixed constraint. Even though it has been shown that deterministic line ratings underestimate the capability of the network, there has not been significant research on using them in optimization studies. The true capacity of a line is not fixed. In this paper, a Monte Carlo simulation method is used to randomly generate different scenarios for the line capacity and wind power availability. For each of these scenarios the system is optimised using quadratic programming. The probability distribution of the optimized variables will allow the system operator to make a judgment on which scenarios are more likely and then schedule generation accordingly.

Section II describes the simulation based stochastic optimization method and the concept of probabilistic line ratings. Section III outlines the detailed simulation parameters and test system. Results are presented in section IV and section V concludes the paper.

## II. STOCHASTIC OPTIMISATION AND LINE RATINGS

### A. Probabilistic Line Ratings

The actual maximum capacity of a line can vary depending on a number of factors, which are usually related to the weather. The probability distribution of power flow in the line and the variation of line capacity are shown in Fig. 1.

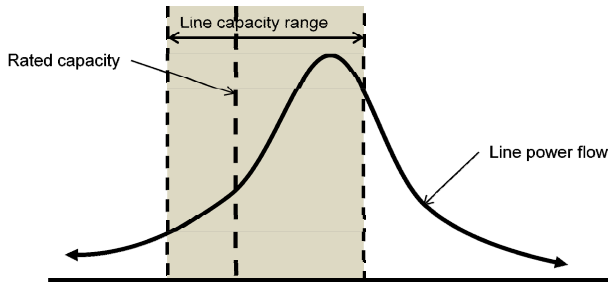


Fig. 1 Probability distribution of power flow in a line compared to the line capacity

In Fig. 1, the shaded region shows the possible range that the actual line capacity can take. The rated line capacity shown by the dotted line appears to be on the lower end of this spectrum, indicating that most line ratings are quite conservative. It has been shown that the probability distribution of the shaded region is given by the generalized extreme value distribution [14]. The equation for this distribution is shown in (1)

$$f(x) = \left(\frac{1}{\sigma}\right) \left(1 + k \frac{(x - \mu)^{-\frac{1}{k}}}{\sigma}\right) e^{\left(-\left(1 + k \frac{(x - \mu)^{-\frac{1}{k}}}{\sigma}\right)\right)} \quad (1)$$

Where  $k$ ,  $\sigma$  and  $\mu$  are parameters defining the shape of the probability distribution. Reference [14] determines the relevant

parameters for the transmission line under consideration over different seasons. These values were scaled and adapted to the system in this paper.

The parameters of the distribution are determined according to the rated maximum limit on the transmission lines. It has been shown that when the probability distribution of DLR is considered, the probability of exceeding the rated capacity ranges from 20 – 30%, depending on the season [12]. The probability is assumed to be 25% in this study. Based on the rated capacity of the lines, and the 25% probability of exceeding this capacity, the parameters for the probability distribution are determined.

The line capacity is determined using a pseudo random number generator. This line capacity determines the constraints of the optimization problem. For each line constraint that is generated, a new optimization problem is solved. Because a D.C. load flow technique is used with quadratic programming approach, it makes this process fast. This is repeated many times and the generated probability distribution can be used to determine the optimum value.

### B. Stochastic Optimisation

The basis of deterministic linear optimisation is that all the variables are known with 100% certainty. Thus, a cost function is minimised subject to constraints. In stochastic programming, not all the variables are known with certainty. However, their probability distributions are known approximately. As a result, a reasonable prediction has to be made for any unknown variables. [16]

In this paper a Monte Carlo simulation procedure is used to determine the probability distribution of the optimised parameters. Scenarios are generated randomly according to the probability distributions of the stochastic variables. For each scenario, the optimisation is carried out using a quadratic programming approach. The cost function of the fuel cost of a thermal power plant is a quadratic function and hence quadratic programming is used.

A number of authors have used a similar approach in their studies [5, 9, 17]. Based on this, the objective function and constraints are proposed as in (2).

Minimise

$$\sum_{i=1}^{N_G} C_i(P_{Gi}) + \sum_{j=1}^{N_W} [e_j(P_{Wj})] \quad (2)$$

Where  $C_i(P_{Gi})$  is the cost of generating power from generator 'i' and  $e_j$  is the feed in tariff for wind power.

Subject to the following constraints

1. Power balance

$$\sum P_i = 0$$

2. Generator power limits

$$P_{j,min} \leq P_j \leq P_{j,max}$$

3. Branch flow limits for branch between nodes m and n

$$P_{mn} \leq P_{mn,max}$$

The branch flow limits can be expressed in terms of generator schedules by using D.C. load flow. The D.C. load flow equations were taken from [18]. The basic D.C. load flow equation is presented in (3)

$$\begin{aligned} P &= B\delta \\ \delta &= XP \end{aligned} \quad (3)$$

Where  $P$  represents the power at the bus,  $B$  is the susceptance matrix for the network,  $X$  is the inverse of  $B$  and  $\delta$  is the vector containing voltage angles.

The branch power flows ( $P_l$ ) can be expressed in terms of branch admittance ( $B_l$ ) and end terminal voltage angles ( $\Phi$ ). This is shown in (4)

$$P_l = B_l\Phi \quad (4)$$

The end terminal voltage angles can be determined from (5).

$$\Phi = A\delta \quad (5)$$

The network incidence matrix is represented by  $A$ .

Substituting (3) into (5) and the resulting equation into (4), the branch power flows can be expressed in terms of the power at each bus as shown in (6).

$$P_l = B_lAXP \quad (6)$$

However, (6) cannot be solved directly since  $B$  is usually not invertible. When solving D.C. load flow problems,  $B$  becomes invertible only after the slack bus is removed. Hence the voltage angle at the slack bus needs to be set to 0 and the appropriate row and column removed from  $B$ . Equation (6) can then be solved.

This optimization represents a single execution of the simulation. It is a simple quadratic optimization problem. This is repeated multiple times to obtain probability distributions of the optimized variables.

A D.C. load flow approach is used to model the system. Due to this, some constraints are not considered, such as reactive power and voltage constraints. D.C. load flow is a fast process allowing a solution to be obtained without the need for multiple iterations. This makes it ideal for use in a simulation approach where multiple runs are required.

### III. SIMULATION PROCEDURE

The test system is shown in Fig. 2

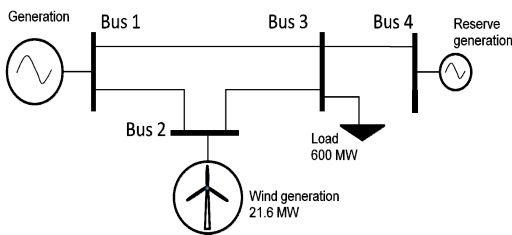


Fig. 2 3 bus test system

Generation and load are on buses 1 and 4 respectively. For this study, the reactive power balance is not considered. A Monte Carlo approach is used which requires fast calculation. Thus, a D.C. load flow technique is used considering only real power balance. The main thermal generation is considered to be the slack bus, so the generation is not constrained. The generation on bus 4 is the reserve generation, which is not constrained. This is because the effect on reserve requirement needs to be observed. The generator cost coefficients and network parameters are shown in Table I.

TABLE I  
PARAMETERS FOR TEST SYSTEM

Generator cost coefficients (cost = a + bx + cx <sup>2</sup> )			
	a	b	c
Generator – Bus 1	100	5	0.0001
Generator – bus 2	200	10	0.0002
Line impedance and limits			
	Impedance	Deterministic Limit	
Line 1-2	0.02 + j0.04 Ω	120 MW	
Line 1-3	0.03 + j0.06 Ω	100 MW	
Line 2-3	0.02 + j0.04 Ω	-	

In Table I, it is assumed that the generation at bus 4 has double the cost as generation at bus 1. This is because, bus 4 contains the spinning reserve which is more expensive. The wind power feed in tariff (e) is set at 2. The absolute costs in this study are chosen arbitrarily and are not expected to reflect actual costs. However, they are meant to give an indication of the relative costs between the proposed method and existing method.

The wind generation is determined based on the Albany Wind Farm capacity. Data from Albany Wind farm was used to approximate the parameters for the Weibull distribution. This was used to simulate the wind speed data. The wind speed data was used with a wind turbine power output curve to simulate the output power. The approximation of Weibull parameters from the histogram of actual data is shown in Fig. 3.

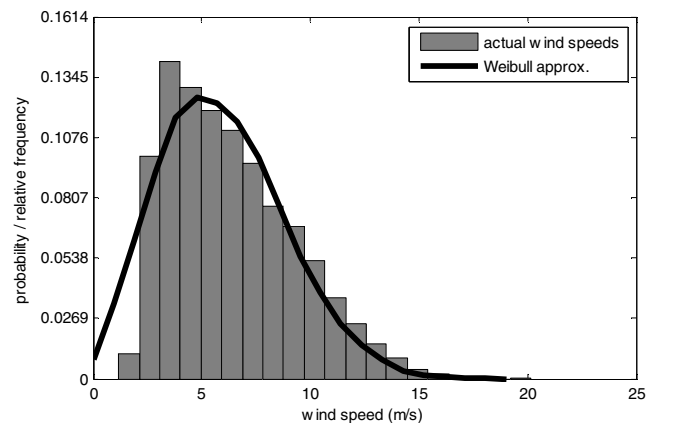


Fig. 3 Weibull approximation of Albany wind farm parameters

The parameters for the Albany Wind Farm, including the Weibull parameters are summarized in Table II. Albany Wind Farm is located approximately 425 km south of Perth, which is the capital of Western Australia. The wind farm is located along the coast.

TABLE II  
PARAMETERS FOR ALBANY WIND FARM

No of Turbines	12
Turbine Type	Enercon E66
Total Rated Power output	21.6 MW
Weibull C Parameter	7.2366
Weibull K parameter	2.3504

Data for the Enercon E66 turbine power curves were obtained from Verve Energy in Western Australia.

The stochastic line capacity was determined using the generalised extreme value distribution as shown in (1). Reference [14] determines the relevant parameters for the transmission line under consideration over different seasons. These values were scaled and adapted to the system in this paper.

For each run the optimisation was carried out using quadratic programming. Initially optimization was carried out by considering the line limits to be fixed and this is later compared to the case when stochastically determined line limits are used.

#### IV. RESULTS AND DISCUSSION

The system was simulated 1000 times on the test system shown. The scheduling of generators was optimised for each scenario. A histogram of the optimum results are shown

Fig. 4 shows the situation when static line ratings are used.

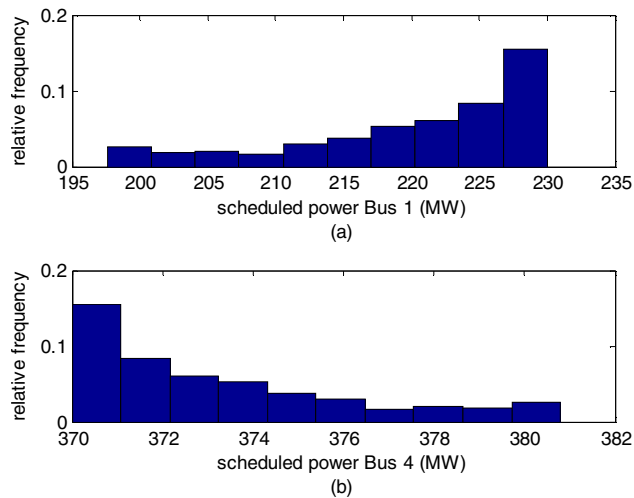


Fig. 4 Probability distribution of optimised generator schedules with deterministic line ratings (a) Generation at bus 1 (b) generation at bus 4

Due to transmission system constraints, not all the power generated by scheduled generators and wind farm can be used by the load. According to Fig. 4(a) it appears as if the generation from the thermal sources has to be capped. This

limit is imposed due to transmission line constraints. Due to capping of generation as bus 1, local spinning reserves (bus 4) have to be used. In this case, the spinning reserve usage is minimum 370 MW (Fig. 4(b)). Therefore, there is 100% probability that the optimally scheduled generation will require more than 370 MW of reserves.

The probability distribution of the objective function (total cost) is simulated and plotted in Fig. 5. The cost figure in Fig. 5 is not indicative of actual costs. The cost figures were chosen arbitrarily. However, they are useful for comparing the effect on cost when the proposed method is used.

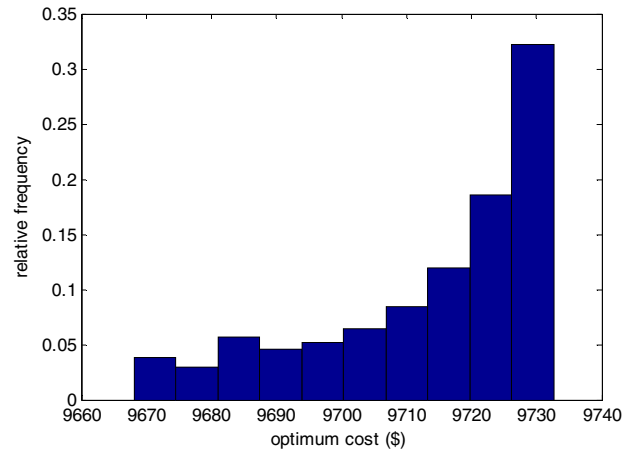


Fig. 5 Simulated probability distribution of optimised total cost using deterministic line ratings

In Fig. 5, the cost ranged from \$9670 to \$9730. The probability of getting a cost on the higher end of this spectrum is quite high. For example, the probability of the optimised cost being less than \$9700 (half way) is 21.1%. Thus the probability distribution is biased towards the high end.

The impact on wind power scheduling is shown in Fig. 6.

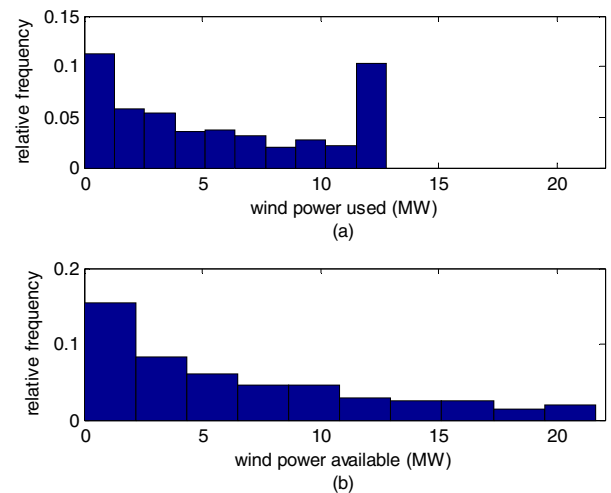


Fig. 6 Wind power scheduling with transmission line constraints (a) Wind power used (b) Available wind power

In Fig. 6(a), it is observed that in some scenarios wind power output has to be capped. The spike on the right hand side of the horizontal axis indicates that any scenarios with wind generation greater than 12.1 MW has to be capped at 12 MW, since the transmission network cannot transfer any more power. The wind generation has to be capped in 18% of the simulated cases. It is not desirable to curtail wind power due to lack of transmission capacity.

Similar probability distributions are simulated considering the transmission line capacity to be a random variable. The probability distribution of optimized generation schedules are shown in Fig. 7

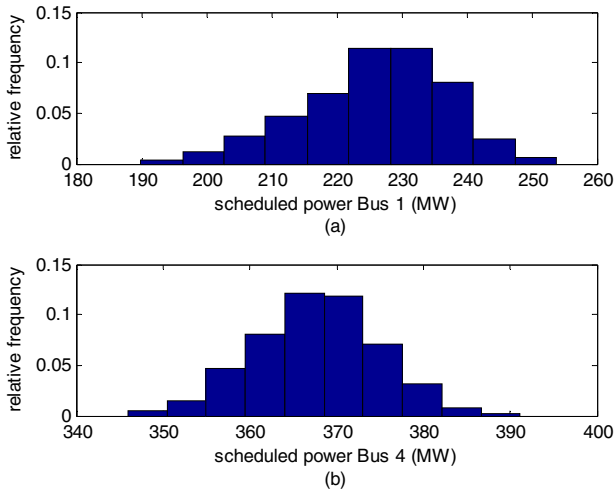


Fig. 7 Probability distribution of optimised generator schedules considering stochastic line ratings (a) Generation at bus 1 (b) generation at bus 4

As seen from Fig. 7(b), the minimum value of scheduled reserves is less than Fig. 4(b) (350 MW compared to 370 MW). The probability of the requiring more than 370 MW of reserve after optimisation is 39.7% as compared to 100% when deterministic line ratings were used. There is no capping of generation in Fig. 7(b). The optimization algorithm prefers to supply the load from generator at bus 1 since the cost of generating is lower. To minimize the cost, the generation from bus 4 has to be kept to a minimum.

The probability distribution of optimum cost with stochastic line ratings is plotted in Fig. 8.

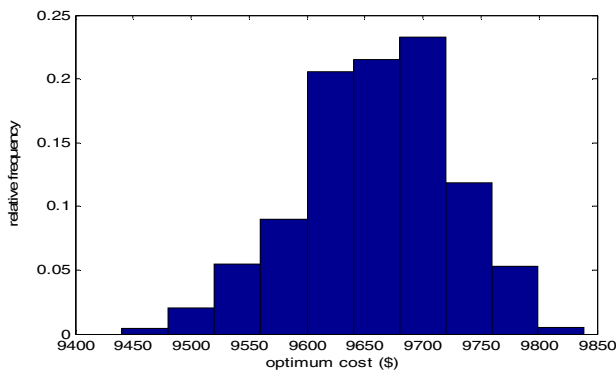


Fig. 8 Simulated probability distribution of optimised total cost using stochastic line ratings

According to Fig. 8, the optimum cost, it is more spread out from \$9450 to \$ 9850. The distribution is no longer biased towards the higher end of the spectrum but is more even on either side of the mean. The high probability scenarios have a lower cost than Fig. 5. As a reference, the probability of the optimised cost being less than \$9700 is 70% as compared to 21% when deterministic line ratings are used. This is because, when the true capacity is considered, the line can be loaded to a greater extent. Power generated by less expensive sources can be used and reliance on spinning reserves can be decreased.

The effect on wind farm output is also investigated. The available wind power and utilised wind power are shown

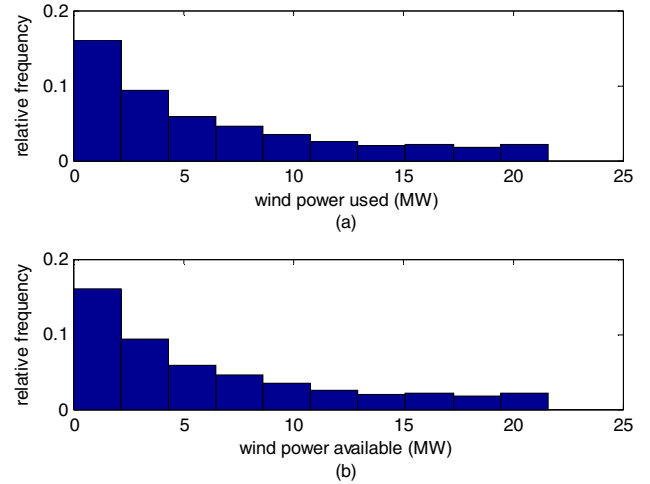


Fig. 9 Wind power scheduling with transmission line constraints (a) Wind power used (b) Available wind power

When the real time capacity of the transmission network is considered, nearly all the available wind power is used. There is no capping.

Probability distributions of the optimized schedule can be used by the system operator to determine the possible variation in system parameters. This can then be used to schedule the generation to cater to the scenarios that have the highest probability. It is observed that when deterministic line ratings are used, these scenarios with higher probability have a high cost and high requirement for reserves. Transmission constraints limit the amount of power that can be utilized from lower cost sources. This is due to the approach being too conservative.

When probabilistic line ratings are used, the high probability scenarios have a much lower cost and lower reserve requirement. Wind power is not curtailed. Thus, by considering the true capacity of the network, more efficient use of generators can be made.

Thus, when traditional deterministic line ratings are used as constraints in optimal power flow problems, the network is underutilized, and the solution is not optimal. When the uncertainty of wind farm output as well as transmission line capacity is considered, scheduling can be optimised to reduce

costs further. Considering this gives a more accurate reflection of the true optimum scenario during system operation.

Once the scheduling is determined to cater to the highest probability scenarios, the system operator can use the probability distribution to determine the expected shortfalls or excess. Based on this contingencies can be planned as required. In a smart grid environment with increased monitoring and automation, implementing such technologies will be easier.

## V. CONCLUSION

In this paper, stochastic optimization of generator scheduling was done considering the uncertainty associated with real time transmission capacity which is usually a function of environmental factors. A Monte Carlo based simulation method was used to generate multiple scenarios of available wind power and transmission capacity and optimization was carried out for each scenario using quadratic programming.

It was observed that when stochastic line ratings were considered, the probability of requiring a large amount of reserves was approximately 60% lower. The probability of a high optimized cost was also reduced by 49%. From this analysis it can be concluded that optimization using probabilistic line ratings as a dynamic constraint, yield a solution that is closer to the true optimum scenario during operation as compared to those using fixed constraints on line capacity. Conventional line ratings are conservative and may be suitable for planning purposes but the real time operational situation may be quite different. Thus, stochastic methods should be used to consider the real time ratings of transmission capacity.

## ACKNOWLEDGMENT

The authors gratefully acknowledge the support of Verve Energy, Western Australia and Western Power Corporation, Western Australia in providing data for the Albany wind farm.

## REFERENCES

- [1] R. Barth, P. Meibom, and C. Weber, "Simulation of short-term forecasts of wind and load for a stochastic scheduling model," in Power and Energy Society General Meeting, 2011 IEEE, 2011, pp. 1-8.
- [2] S. J. Kazempour, M. P. Moghaddam, M. R. Haghifam, and G. R. Yousefi, "Dynamic self-scheduling of a fuel and emission constrained power producer under uncertainties," in Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES, 2009, pp. 1-10.
- [3] P. Meibom, R. Barth, B. Hasche, H. Brand, C. Weber, and M. O'Malley, "Stochastic Optimization Model to Study the Operational Impacts of High Wind Penetrations in Ireland," Power Systems, IEEE Transactions on, vol. 26, pp. 1367-1379, 2011.
- [4] A. Sturt and G. Strbac, "Efficient Stochastic Scheduling for Simulation of Wind-Integrated Power Systems," Power Systems, IEEE Transactions on, vol. 27, pp. 323-334, 2012.
- [5] J. B. Cardell and C. L. Anderson, "Analysis of the System Costs of Wind Variability Through Monte Carlo Simulation," in System Sciences (HICSS), 2010 43rd Hawaii International Conference on, 2010, pp. 1-8.
- [6] H. Falaghi, M. Ramezani, C. Singh, and M. Haghifam, "Probabilistic Assessment of TTC in Power Systems Including Wind Power Generation," Systems Journal, IEEE, vol. 6, pp. 181-190, 2012.

- [7] R. A. Jabr and B. C. Pal, "Intermittent wind generation in optimal power flow dispatching," Generation, Transmission & Distribution, IET, vol. 3, pp. 66-74, 2009.
- [8] L. Shi, C. Wang, L. Yao, Y. Ni, and M. Bazargan, "Optimal Power Flow Solution Incorporating Wind Power," Systems Journal, IEEE, vol. 6, pp. 233-241, 2012.
- [9] Z. Shuo, L. Gengyin, and Z. Ming, "Calculation and analysis of capacity credit of wind farms based on Monte-Carlo simulation," in Power and Energy Society General Meeting, 2010 IEEE, 2010, pp. 1-6.
- [10] A. F. K. Kamba, S. Voller, and J. F. Verstege, "Congestion management in transmission systems with large scale integration of wind energy," in Integration of Wide-Scale Renewable Resources Into the Power Delivery System, 2009 CIGRE/IEEE PES Joint Symposium, 2009, pp. 1-1.
- [11] A. Vergnol, J. Sprooten, B. Robyns, V. Rioux, and J. Deuse, "Real time grid congestion management in presence of high penetration of wind energy," in Power Electronics and Applications, 2009. EPE '09. 13th European Conference on, 2009, pp. 1-10.
- [12] J. Fu, D. J. Morrow, S. Abdelkader, and B. Fox, "Impact of Dynamic Line Rating on Power Systems," Universities' Power Engineering Conference (UPEC), Proceedings of 2011 46th International, pp. 1-5, 2011.
- [13] J. Hosek, P. Musilek, E. Lozowski, and P. Pytlak, "Effect of time resolution of meteorological inputs on dynamic thermal rating calculations," Generation, Transmission & Distribution, IET, vol. 5, pp. 941-947, 2011.
- [14] A. K. Kazerooni, J. Mutale, M. Perry, S. Venkatesan, and D. Morrice, "Dynamic thermal rating application to facilitate wind energy integration," in PowerTech, 2011 IEEE Trondheim, 2011, pp. 1-7.
- [15] M. Matus, D. Saez, M. Favley, C. Suazo-Martinez, J. Moya, G. Jimenez-Esteviz, R. Palma-Behnke, G. Olguin, and P. Jorquera, "Identification of Critical Spans for Monitoring Systems in Dynamic Thermal Rating," Power Delivery, IEEE Transactions on, vol. 27, pp. 1002-1009, 2012.
- [16] H. Taha, Operations research : an introduction / Hamdy A. Taha.
- [17] J. B. Cardell and C. L. Anderson, "Estimating the system costs of wind power forecast uncertainty," in Power & Energy Society General Meeting, 2009. PES '09. IEEE, 2009, pp. 1-4.
- [18] X.-F. Wang, Y.-H. Song, and M. Irving, Modern power systems analysis : Power electronics and power systems, 2008.