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The apparent British sea slope is caused by systematic errors in the levelling-based vertical datum

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SUMMARY

The spirit-levelling–based British vertical datum (Ordnance Datum Newlyn) implies a south–north apparent slope in mean sea level of up to 53 mm deg^{-1} latitude, due to the datum falling on heading northwards. Although this apparent slope has been investigated since the 1960s, explanations of its origin have remained inconclusive. It has also been suggested that, rather than a slope, the British vertical datum includes a step of about 240 mm affecting all sites north of about 53°N. In either case, the British vertical datum may be of limited use for any study requiring accurate heights or changes in heights, such as testing geoid models, groundwater and hydrocarbon extraction, the calibration and validation of satellite-based digital terrain models, and the unification of vertical datums internationally. Within the last decade, however, based on an apparent reduction in the slope to only -12 mm deg^{-1} latitude with respect to recent geoid models, it has been claimed that the British vertical datum does provide a physically meaningful surface for use in scientific applications. In this paper, we reinvestigate the presence of apparent south–north sea slopes around Britain and reported slopes in the vertical datum, using the EGM2008 global gravitational model, together with mean sea level and GPS data from British tide gauges, GPS ellipsoidal heights of 178 fundamental benchmarks across mainland Britain, and vertical deflection observations at 192 stations. We demonstrate a south–north slope in the British vertical datum of $-(20\text{–}25)$ mm deg^{-1} latitude with respect to both mean sea level (corrected for the ocean's mean dynamic topography and the inverse barometer response to atmospheric pressure loading) and the EGM2008 quasigeoid model, while EGM2008 is shown to exhibit a negligible slope of (2 ± 4) mm deg^{-1} with respect to mean sea level. It is clear, therefore, that the slope can only arise from systematic errors in the levelling, although we are unable to isolate their exact origin. Using an offset detection method based on a penalized likelihood maximization using the Schwarz Information Criterion, we do not detect a step in the vertical datum affecting all sites north of 53°N, but do identify regional distortions that we attribute to the inhomogeneity in both the levelling data used and the least squares adjustment procedures used to realize the datum. We conclude that the British vertical datum remains unsuitable for scientific purposes.

Key words: Spatial analysis; Reference systems; Space geodetic surveys.

1 INTRODUCTION

Vertical geodetic datums provide the reference or zero level for physically meaningful heights on land, and usually take local mean sea level (MSL) that has been observed by one or more tide gauges over a particular time span as the zero point(s) (e.g. Vaníček 1991). Traditionally, national, regional and continental vertical datums have been realized point-wise via a series of benchmarks, with their heights above local MSL at the tide gauge(s) transferred using differential spirit-levelling, after correction for the non-parallelism of the equipotential surfaces of the Earth's gravity field. Theoretically, the zero point(s) of a vertical datum should coincide with MSL, after applying corrections for the many effects that cause local MSL to depart from the geoid (e.g. Pugh 1986). Though time-consuming, spirit levelling provides the most precise means of determining physically meaningful height differences on land (e.g. Vaníček *et al.* 1980).

A vertical datum principally provides the framework for topographic mapping and is even legislated in some countries for use in surveying and land titling, but it is also implicit in many scientific

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studies that require heights or changes in heights. For instance, it can provide a reference level for monitoring land subsidence or uplift (e.g. Kelsey 1972; Vaníček *et al.* 1980; Zerbini *et al.* 2007; Chen *et al.* 2011) due to processes such as groundwater or hydrocarbon extraction (e.g. Chi & Reilinger 1984), earthquakes (e.g. Wellman & Tracey 1987; Guglielmino *et al.* 2011; Cheloni *et al.* 2012), tectonic processes (e.g. Holdahl 1982; Schlatter *et al.* 2005) or glacial isostatic adjustment (GIA, e.g. Mäkinen & Saaranen 1997; Vestøl 2006). Levelled topographic heights can also be used for the validation of satellite-sensed digital elevation models (e.g. Hirt *et al.* 2010), to test regional and global geoid models (e.g. Ziebart *et al.* 2008; Pavlis *et al.* 2012) and to detect apparent sea slopes (e.g. Sturges 1967, 1974; Castle & Elliott 1982). If there are deficiencies in a vertical datum or the levelling used to establish it, all these activities will be compromized.

Some quite large disparities have been found worldwide between the heights of MSL relative to the vertical datum (or levelling) at the connected tide gauges, which theoretically should be zero. For North America, Sturges (1967) reported south–north apparent MSL slopes of around 30 mm deg^{-1} latitude on both the Pacific and Atlantic coasts. Furthermore, the MSL values at tide gauges on the Pacific coast appeared systematically 600–700 mm higher than those of similar latitude on the Atlantic coast, which was shown by Sturges (1974) to be caused by a combination of Pacific Ocean and Atlantic Ocean density differences and boundary current effects. For Brazil, an apparent MSL slope of 12 mm deg^{-1} latitude with respect to the levelling was reported by Rodiguez (1970), while for Australia, Featherstone & Filmer (2012) showed that an MSL slope of 26 mm deg^{-1} latitude was caused by neglecting mean dynamic topography (MDT) effects at the tide gauges fixed to realize the Australian Height Datum. For Ireland, an apparent MSL slope of 98 mm deg^{-1} is deduced from Dixon (1979), which occurred using levelling data observed in 1900 and 1970, but its origin has not yet been explained.

Discussion and debate regarding the disparity between levelled height differences and MSL at tide gauges in Britain have waxed and waned for nearly a century, with a recent example being by Philip (2008). An apparent slope in British levelling was even commented on after the completion of the Second Geodetic Levelling (SGL) in relation to the First Geodetic Levelling (FGL), but dismissed as insignificant at that time (Jolly & Wolff 1921). An apparent south–north MSL slope for Britain of 53 mm deg^{-1} latitude was reported by Thompson (1980), who suggested levelling errors were the likely cause as it could not be explained oceanographically, and this ‘apparent sea slope’ (or equivalently, a slope in the vertical datum of opposite sign, i.e. here a fall of 53 mm deg^{-1} latitude on heading northwards) has since been investigated extensively, as follows.

Ashkenazi *et al.* (1990) used GPS to compute the heights of MSL above the EDIN89 geoid model (Hipkin 1995) at five east coast tide gauges to independently check the slope, but the results were inconclusive due to the small geographical data coverage (only 400 km south–north) and large GPS and geoid model errors. Hipkin *et al.* (2004) used a similar approach to Ashkenazi *et al.* (1990) but with more GPS stations and the more recent EDIN2000 geoid model, suggesting that there was not a slope but instead a step in the vertical datum of about 240 mm at 53°N, which they postulated was caused by levelling errors. Featherstone & Olliver (1994) used GPS and the OX92 geoid model to deduce a slope of −58 mm deg^{-1} latitude. However, Ziebart *et al.* (2008) compared the British vertical datum with the OSGM02 and OSGM05 gravimetric geoid models, finding respective south–north slopes of −34 and −12 mm deg^{-1} latitude and concluded that the British vertical datum was indeed a physically meaningful, unbiased surface that improved geoid models converge towards (although they suggested the remaining −12 mm deg^{-1} latitude slope could be due to levelling errors).

Any slope or step in the British vertical datum will compromise its use in the height studies and applications mentioned earlier, as well as stifling the development of unified international vertical datums such as EUVN_DA (Kenyeres *et al.* 2010) and other efforts based on the boundary-value approach. For instance, Gerlach & Rummel’s (2013) boundary-value approach assigns a constant 50 mm height offset of the British mainland with respect to continental Europe, but this really should have been formulated as a bias and a slope.

The sometimes-contradictory magnitudes and unresolved explanations for the cause of the slope in the levelling-based British vertical datum, called Ordnance Datum Newlyn (ODN), warrant its further investigation and is the topic of this paper. Our study is timely given the relatively recent suggestion that ODN is a suitable reference for testing geoid models (Ziebart *et al.* 2008). Furthermore, the approach taken by the Ordnance Survey to access ODN using GPS has been to compute a gravimetric geoid but then to warp this to agree with ODN using least squares collocation (Forsberg *et al.* 2003; Iliffe *et al.* 2003). This study seeks to conclusively isolate the cause of the apparent sea slope in Britain, by taking advantage of a combination of heterogeneous datasets including EGM2008 (Pavlis *et al.* 2012, 2013), continuous GPS at tide gauges, access to GPS and levelling-based ODN heights of 178 fundamental benchmarks (FBMs), long-term (up to 37 yr) tide gauge records of MSL, and modelling of the ocean’s MDT.

We first present in Section 2 an introduction to the geodetic concepts of height systems and the terminology used throughout this paper. In Section 3, we describe the data sources used and their treatment, which includes a summary and critique of the different British geodetic levellings and their least squares adjustments. Section 4 details comparisons of south–north trends using levelling, MSL, GPS, geoid, MDT and inverse barometer response (IBR) data, with new regional distortion maps presented and discussed in Section 5. Finally, Section 6 provides discussion and conclusions.

2 CONCEPTS

2.1 Height systems

It is instructive for the later discussions that we briefly cover some of the subtle concepts related to heights (e.g. Heiskanen & Moritz 1967, chapter 4; Jekeli 2000). Fig. 1 shows the relationships among the relevant height systems: all are reckoned positively above their respective reference surfaces and negatively below. Ellipsoidal heights (h) refer to the surface of the ellipsoid and are taken along the ellipsoidal surface normal. The geoid (N) is an equipotential surface, whereas, in the presence of topography, the quasigeoid (ζ) is not, but the two surfaces do coincide over the oceans. On land, orthometric heights (H) refer to the geoid and are taken along the curved and torsioned plumbline of the Earth’s gravity field. Normal heights (H^*) refer to the quasigeoid and are taken along the normal plumbline generated by the normal ellipsoid that only has meridional curvature and no torsion. Thus:

$$h \approx H + N \quad (1a)$$

$$h \approx H^* + \zeta. \quad (1b)$$

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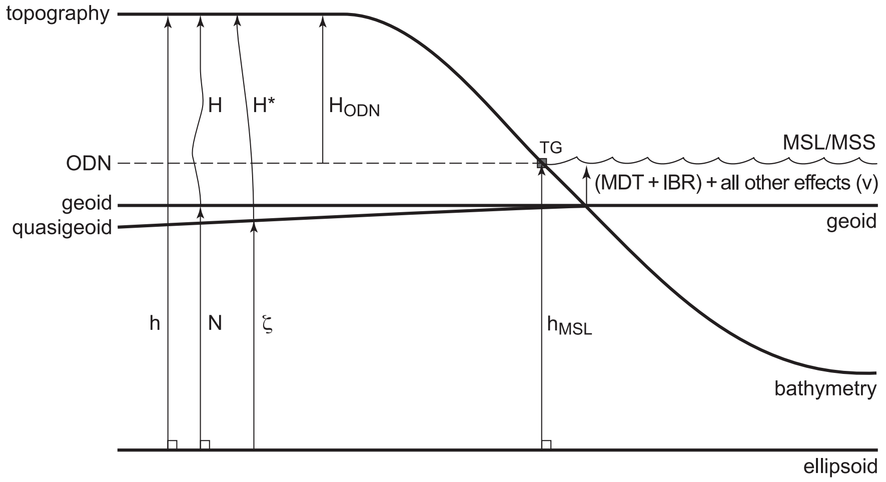

Figure 1. Schematic of height systems, surfaces and datums. Due to (MDT+IBR) and all other effects (v), ODN is not coincident with the geoid or quasigeoid.

The approximations in eq. (1) arise because the heights are taken along different (some curved and some straight-line) paths (see Fig. 1). However, this approximation error is less than a millimetre in Britain (*cf.* Jekeli 2000), particularly for the benchmarks used in this study, which have median and maximum ODN heights (H_{ODN}) of 94 and 368 m, respectively.

To a first-order approximation, the difference between the geoid and the quasigeoid (or equivalently the difference between a normal height and an orthometric height) is given by (e.g. Heiskanen & Moritz 1967; Rapp 1997):

$$N - \zeta = H^{*} - H \approx \frac{\Delta g_B}{\bar{\gamma}} H, \qquad (2)$$

where Δg_B is the Bouguer gravity anomaly and $\bar{\gamma}$ is a mean value of normal gravity. Higher orders of approximation can be found in, for example, Flury & Rummel (2009).

In this regard, there is a misconception in several previous geoid computations for Britain (e.g. Featherstone & Olliver 1994; Hipkin *et al.* 2004), and notably with the OSGM02 gravimetric geoid model (Forsberg *et al.* 2003). Specifically, they have computed the geoid from an interim quasigeoid using eq. (2), whereas the quasigeoid is probably more appropriate. This is because the normal-orthometric height system is more akin to the normal height system (e.g. Filmer *et al.* 2010). However, the subsequent fitting of the OSGM02 gravimetric geoid model to the FBMs (Iliffe *et al.* 2003) will have masked some of this effect, but it will become more pronounced in mountainous regions. From Forsberg *et al.* (2003, table 1), the effect will range between –60 and +40 mm, which may become significant when determining ODN heights from GPS. We also note that Ziebart *et al.* (2008) use the term OSGM02 interchangeably for the gravimetric geoid and fitted product that is released by the Ordnance Survey.

The above definitions of the various heights are largely conceptual, and practical realization necessitates approximations. If differential levelling is undertaken without gravity observations along the traverses, as has occurred in Britain, the measured height differences cannot be converted to geopotential numbers and are thus non-holonomic (i.e. they will not theoretically give a zero closure). A pragmatic yet approximate solution has been to apply what have been called normal-orthometric corrections (NOCs). These do not require geopotential numbers and instead use normal gravity to account for the divergence of the equipotential surfaces towards the equator. However, normal-orthometric heights do not refer to a well-defined or equipotential reference surface (Filmer *et al.* 2010, p. 505), but are expected to be more closely aligned with the quasigeoid than the geoid. The NOC used for the British levelling is given in Burnett & Carmody (1960) as:

$$\mathrm{NOC} = 2\alpha \overline{H}_{\mathrm{ODN}} \sin 2\bar{\phi} \sin 1'' \qquad (3)$$

where $2\alpha = 0.005302$, $\overline{H}_{\mathrm{ODN}}$ is the mean ODN height of the levelling section, and $\bar{\phi}$ is the mean geodetic latitude of the levelling section. Although Burnett & Carmody (1960) and many others (e.g. Macdonald & Christie 1991; Christie 1994; Iliffe *et al.* 2003) call this an orthometric correction, strictly it is an NOC. Therefore, ODN heights are not strictly orthometric heights but normal-orthometric heights.

2.2 MSL and the geoid

Fig. 1 also shows the relationships among the geoid, MSL or the mean sea surface (MSS), the ocean's MDT and the IBR of the MSS to atmospheric pressure loading. The practical realization of a vertical datum that has its zero point set to local MSL observed at a tide gauge over some time span means that it can be offset from the geoid (or quasigeoid). This is due primarily to the ocean's steady-state circulation, which has an additional MSL signature—commonly referred to as the MDT—of up to 1.5 m, and the ocean's surface IBR of up to 0.1 m. In addition, non-linear tides and long- and medium-term sea level variability due to ocean and atmospheric processes (collectively denoted by *v*), mean that MSL will—to some extent—reflect the time span over which it is computed. Therefore:

$$h_{\mathrm{MSL}} = N + (\mathrm{MDT} + \mathrm{IBR}) + v. \qquad (4)$$

Here, h_{MSL} has been derived from the ellipsoidal height of a tide gauge GPS station, a local levelling connection to the tide gauge benchmark (TGBM) and the MSL record referenced to the TGBM. As such, it is therefore possible to [only partially because *v* is neglected] assess a geoid model using GPS connected to a tide gauge and an (MDT + IBR) model (*cf.* Fig. 1). It is pertinent to point out that some previous assessments of the British levelling or the testing of geoid models have failed to account for the (MDT+IBR) component of MSL (e.g. Ashkenazi *et al.* 1990; Bingley *et al.* 2002).

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Table 1. Contributions to the south–north apparent sea slope caused by different permanent tide systems on heights.

Height	Tide system	Slope over Britain (mm deg^{-1} latitude)
Orthometric	Zero	1.6
Orthometric	Mean	− 3.3
Geoid	Zero	1.4
Geoid	Mean	6.3
Ellipsoidal	Zero	3.0
Ellipsoidal	Mean	3.0

2.3 Tide systems

Another factor that can affect the proper detection of slopes in levelling data is the role of the permanent tide in all the heights summarized in Fig. 1 (e.g. Ekman 1989; Poutanen *et al.* 1996; Mäkinen & Ihde 2009). Global and regional reference frames such as the International Terrestrial Reference Frame (ITRF) and realizations of the European Terrestrial Reference System 1989 (ETRS89) and hence GPS solutions usually adopt the tide-free ('non-tidal') system (Poutanen *et al.* 1996), so this will be used as the reference tidal system for this study. However, it is not known (or not documented publicly) which tide system has been used in the British levellings. Therefore, we use the following equations from Ekman (1989) to determine the differences among tide systems across the south–north extent of Britain, noting that a minus sign is absent in Ekman (1989, eq. 20), but which has been corrected for in the equations below. The subscripts denote zero tide (Z), mean tide (M) and tide-free (F) and the numerical values are based on the Love numbers $k = 0.29$ and $\gamma = 0.68$:

$$\Delta H_F \approx \Delta H_Z + 95(\sin^2\phi_N - \sin^2\phi_S)[\text{mm}] \quad (5)$$

$$\Delta H_F \approx \Delta H_M - 201(\sin^2\phi_N - \sin^2\phi_S)[\text{mm}] \quad (6)$$

$$\Delta N_F \approx \Delta N_Z + 86(\sin^2\phi_N - \sin^2\phi_S)[\text{mm}] \quad (7)$$

$$\Delta N_F \approx \Delta N_M + 382(\sin^2\phi_N - \sin^2\phi_S)[\text{mm}] \quad (8)$$

$$\Delta h_F \approx \Delta h_Z + 181(\sin^2\phi_N - \sin^2\phi_S)[\text{mm}] \quad (9)$$

$$\Delta h_F \approx \Delta h_M + 181(\sin^2\phi_N - \sin^2\phi_S)[\text{mm}], \quad (10)$$

where ϕ_N and ϕ_S denote northern and southern latitude, respectively.

Table 1 shows the south–north slope effects of the different permanent tide systems across the British mainland, referred to the tide-free system usually used for GPS heights, including in this study. It can be seen that the largest slope is 6.3 mm deg^{-1} latitude, which cannot explain an apparent sea slope of up to 53 mm deg^{-1} latitude. The nodal tide with 18.61 yr period also has the potential to impact on the slope. We therefore calculated the effect of this on the slope using Woodworth (2012, eq. 1 and table 1), but it only contributes 0.067 mm deg^{-1} latitude, and hence also cannot explain the apparent sea slope.

3 DATA SOURCES AND THEIR TREATMENT

To investigate the apparent sea slope, we collated ODN heights of benchmarks, GPS ellipsoidal heights of the same benchmarks, MSL from tide gauges and modelled (MDT+IBR) values at the tide gauges. The stations used per data type are shown in Fig. 2, and further details of these data types, including their treatment and errors, are described next, together with EGM2008 syntheses.

3.1 ODN and British levelling errors

ODN is provided by the heights of benchmarks published by the Ordnance Survey, following the observation and analysis of three geodetic levelling campaigns, termed the FGL, the SGL and the Third Geodetic Levelling (TGL). Exact details of the levelling of the British mainland and analysis are sometimes rather sketchy and are scattered among a variety of reports and papers (see Table 2). We shall only attempt to summarize the information which is most pertinent to our investigation of the south–north slope. The FGL is not discussed as these values are no longer used. Since the SGL, the zero point of ODN has been defined as coincident with MSL as measured by a tide gauge at Newlyn over the period 1915 May 1 to 1921 April 30 (Ordnance Survey 2010).

The SGL (Jolly & Wolff 1921) was conducted in England and a part of Wales between 1912 and 1921, with the south-east of England levelled between 1946 and 1951. Southern Scotland was levelled between 1936 and 1941 and northern Scotland between 1942 and 1952. The coverage of the traverses and the years over which they were levelled are shown in Kelsey (1972, Fig. 2). Deep-seated FBMs were established on geologically stable ground at around 50 km intervals, at the locations shown in Fig. 2(a) (a few FBMs shown have been added since the SGL and a few FBMs are now disused or have been destroyed; Christie (1994)). NOCs were applied to the SGL, and only the levelling network in parts of England and Wales was least-squares adjusted under a minimal constraint, with the MSL height of Newlyn held to a fixed height to yield ODN. The northern English and Scottish levelling was adjusted separately as four separate subnetworks, holding four junction points fixed to their heights from the previous minimally constrained adjustment (Christie 1994).

During the SGL, tide gauges were established at Dunbar, south-east Scotland [1913], Newlyn, south-west England [1915] and Felixstowe, south-east England [1917]. A discrepancy between the levelling and MSL between Newlyn and Dunbar of ~250 mm was observed, which was larger than could be explained by oceanographic phenomena alone, pointing even back then to a systematic error in the levelling. Analysis of the misclosures of the levelling loops gave a probable levelling error of 1.8 mm$\sqrt{}$km for the SGL (e.g. Kelsey 1972). However, levelling misclosures are unable to detect compensating errors and are thus insensitive to some systematic errors. As a simplistic example, a −40 mm deg^{-1} latitude south–north levelling slope (commensurate with the Newlyn and Dunbar discrepancy) will manifest as that in a northerly direction and as +40 mm deg^{-1} latitude in a southerly direction, thus cancelling out in the closure analysis.

The discrepancy between the SGL and MSL led the Ordnance Survey to undertake the TGL, which was conducted between 1951 and 1959. It followed many of the same traverses as the SGL (some new FBMs were installed) and used similar instrumentation and techniques (Kelsey 1972). The complete coverage is shown in Kelsey (1972, Fig. 4) and Christie (1994, Fig. 2), and the temporal progression of the network is in Kelsey (1970, Fig. 1). The TGL also connected to more tide gauges (ibid.). The principal observation from comparisons of the TGL and SGL was a slope and some regional distortions. From Kelsey (1972, Fig. 5), the differences range from −49 mm near Newlyn to +260 mm in north-east Scotland, which Kelsey (1972) equated to an apparent land uplift rate

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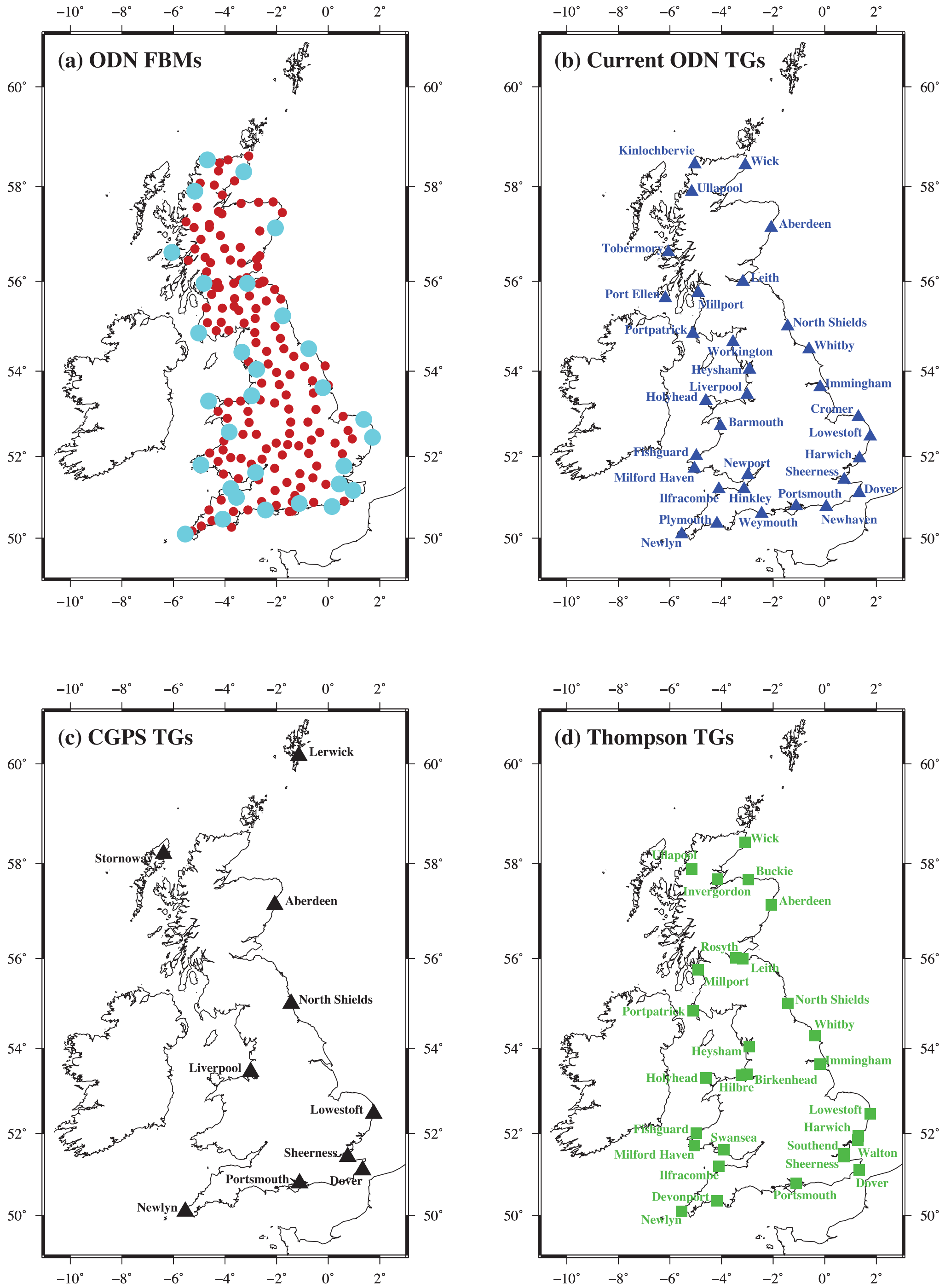

Figure 2. Maps showing stations per data set considered. (a) Locations of the 178 FBMs, with the cyan circles denoting the subset of FBMs nearest the current tide gauges (TGs) shown in panel (b); (b) Current ODN TGs used to assess the slope; (c) TGs with colocated CGPS used to assess the slope with respect to EGM2008; (d) TGs used by Thompson (1980).

of ∼5 mm yr^{-1} in northern Scotland. This is implausible, however, given GPS observations and models of GIA, which show a differential change from south to north, that is, Scotland rising with respect to southern England, of only ∼1–2 mm yr^{-1} (Milne *et al.* 2006; Teferle *et al.* 2006, 2009; Bradley *et al.* 2011). This will be discussed further in Section 6.

A problem in the 1950s and 1960s was the lack of reliable and long-term MSL tide gauge records to attempt to discriminate which, if any, levelling network was less affected by systematic south–north errors. Ideally, the TGL should have superseded the SGL completely because it was conducted over a shorter time, so is less subject to GIA effects, but the Ordnance Survey decided that the TGL should be 'in sympathy' with the SGL (Kelsey 1959; Christie 1994). Scientifically, this is unsatisfactory, especially as the SGL was questionable in relation to MSL at Newlyn and Dunbar, and it also showed a slope with respect to the FGL (Jolly & Wolff 1921).

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Table 2. Levellings of the British mainland (compiled from Jolly & Wolff (1921); Edge (1959); Kelsey (1959, 1972); Burnett & Carmody (1960); Proctor & Richards (1975); Macdonald & Christie (1991); Christie (1994)). NOC is the normal-orthometric correction.

	Areas covered	Observed	Datum point	Adjustment	NOC
First Geodetic Levelling	England, Wales, Ireland, Scotland	1840–1860	Liverpool (MSL over 10 days in 1884 March)	Least squares, minimal constraint	No
Second Geodetic Levelling	England, Wales	1912–1921, 1946–1951	Newlyn (MSL between 1915 May 1 and 1921 April 30)	Least squares, minimal constraint	Yes
	Scotland	1936–1941, 1942–1952		Least squares, fixed-network with junction points from above adjustment	
Third Geodetic Levelling	England, Wales Scotland England, Wales, Scotland	1951–1956 1956–1959 1959	Newlyn (MSL between 1915 May 1 and 1921 April 30)	Least squares, fixed-network with FBM heights from the Second Geodetic Levelling	Unknown

This is where the realization of the current ODN becomes convoluted. Though it is connected to MSL at Newlyn, it is not the result of a single minimally constrained adjustment. Instead, the TGL was least squares adjusted while holding the heights of the FBMs to their values from the SGL (Kelsey 1959, section 5.2). This is a constrained adjustment, thus allowing for distortions to be introduced and for a slope in the SGL to affect the current ODN heights. Also, it is not stated explicitly in any publication concerning the TGL whether NOCs were applied to the TGL before it was adjusted on to the SGL; if not, this is conceptually incorrect.

The exact source of the postulated systematic levelling error and its effect on ODN remains enigmatic. Halliday (1959) investigated unequal thermal expansion of the levelling staves used, but this was inconclusive. The use of the rather approximate NOC could be a contributor, but it is not possible to estimate what effect this may have without access to the original levelling observations. From Featherstone & Filmer (2012, Fig. 3), the NOC can contribute 15 mm deg^{-1} latitude of slope if not applied, but which is much less than the largest British apparent sea slope of 53 mm deg^{-1} latitude reported, so cannot explain it fully. Entin (1959) lists the principal error sources in spirit levelling, but it would only be speculation as to which could cause the slope in ODN. However, a systematic error in the observations or instruments and their calibrations remains the most likely candidate.

The least squares adjustment strategy used for the SGL is also significant in the context of the results presented in Section 4. The minimally constrained adjustment of the England and Wales levelling, followed by the constrained adjustment of the northern English and Scottish levelling (Christie 1994), effectively produces a heterogeneous network. This is compounded further in the adjustment of the TGL, whereby fixing all FBM height values from the SGL adjustment allowed systematic errors from the SGL to dominate the current ODN. It could be argued that—at the very least—a readjustment of the various levellings should be conducted on a scientific basis, but the Ordnance Survey has changed its approach to the provision of heights in Britain. It now relies on a continuous GPS (CGPS) network and the OSGM02 regional geoid model that has been fitted to 178 FBMs using least-squares collocation (Forsberg *et al.* 2003; Iliffe *et al.* 2003). [Ziebart *et al.* (2008) presented a more recent regional geoid model OSGM05, although this has yet to be released.] Nevertheless, the heights provided by GPS and OSGM02 will still contain a slope, rendering ODN heights incompatible with the quasigeoid. The fitting of geoid models to the ODN simply masks the problem. Though very expensive (Christie 1994), a Fourth Geodetic Levelling (as proposed by Proctor & Richards (1975)) could be used to remove the slope and distortions, improve the quality of Britain's spatial data infrastructure, and allow it to contribute to the applications identified in Section 1.

3.2 MSL data and (MDT+IBR) modelling

Annual MSL values for 32 British stations (connected to ODN) from the current UK National Tide Gauge Network (Fig. 2b) were obtained from the Permanent Service for Mean Sea Level (PSMSL; www.psmsl.org). To average out the 18.6 yr nodal tide and sea level variability (e.g. Amin 1988; Volkov & van Aken 2003), annual MSL data from 1974.0 to 2011.0 were collated per tide gauge wherever possible (Note that for Liverpool, due to localized subsidence at the tide gauge, only data up to 2004.0 were considered despite more recent data being available). Four other tide gauges connected to ODN (Llandudno, Mumbles, Bournemouth and Avonmouth) were not considered due to their short MSL data records. Factors such as station maintenance, instrumentation failures, local works and new installations meant that annual means were not available for all 37 yr for all stations, with the average and median number of complete years of MSL data available over this time span being 22.5 and 21 respectively per station.

To enable MSL to be compared with ODN and the quasigeoid, (MDT+IBR) values from the Ocean Circulation and Climate Advanced Modelling project model (OCCAM; Marsh *et al.* 2009) from the National Oceanography Centre, Southampton, were subtracted from the MSL values. The run we considered (run 401) was at 1/12-degree horizontal resolution, with 66 depth levels, ranging in thickness from 5 m at the surface to 300 m at the bottom. This high horizontal and vertical resolution, together with OCCAM's partial bottom cell scheme, which allows for a more faithful representation of sea-bottom topography, should aid the ability of OCCAM to model the small-scale MDT surrounding the British Isles. Moreover, since the model has a free-surface, its time-mean sea level naturally includes the IBR to atmospheric pressure, provided by the National Center for Environmental Prediction (NCEP) reanalysis products (Kalnay *et al.* 1996) with which the model is forced. Following an initial 4 yr of spin-up, the run covers the 20-yr time span from 1985 to the end of 2004. The (MDT+IBR) values used were computed over the 19-yr time span from 1986.0 to 2005.0.

3.3 GPS ellipsoidal heights

For the 178 FBMs on the British mainland, GPS data from two 4-hr static occupations were collected in 1999 March. ETRS89 (epoch

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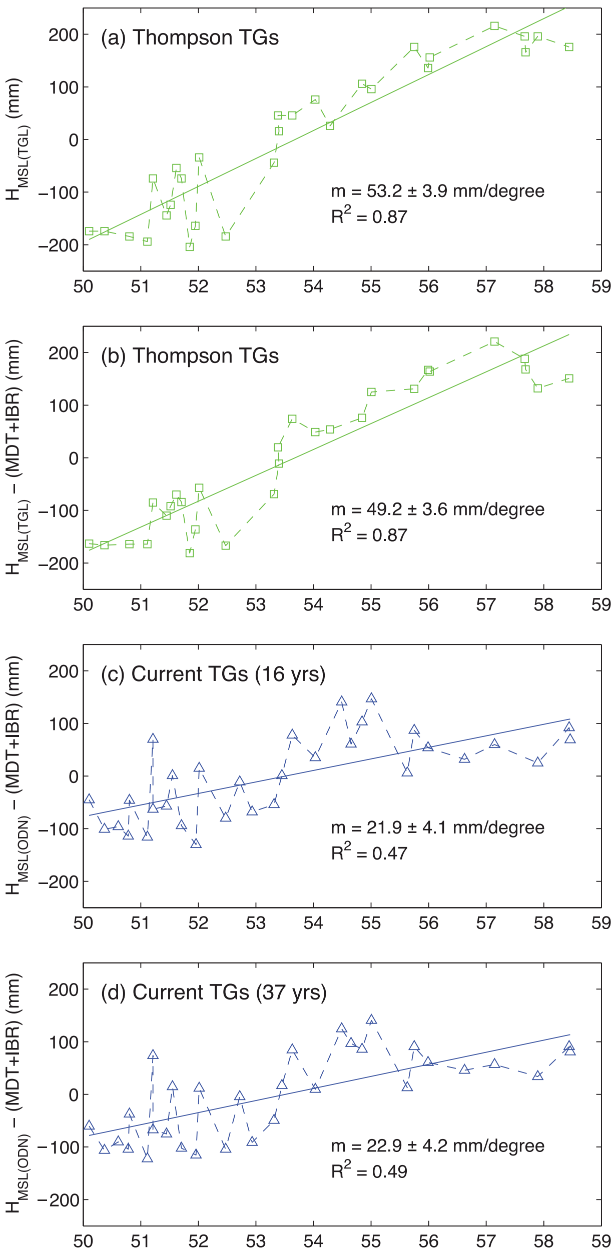

Figure 3. Demonstration of apparent sea slope with respect to levelling-based datums, plotted against latitude. (a) Reproduction of figure 7(a) of Thompson (1980), that is, height of MSL with respect to the TGL (SSA); (b) As for (a) but with (MDT+IBR) removed using the OCCAM model; (c) Height of MSL above current ODN using up to 16 yr of MSL data from current tide gauges from 1995.0 to 2011.0, to be commensurate with the approach of Thompson (1980); (d) As for (c) but using, wherever possible, 37 yr of MSL data from 1974.0 to 2011.0. The mean has been removed in all cases, thus eliminating the constant term in eq. (11).

1989.0) tide-free ellipsoidal heights on the GRS80 ellipsoid (Moritz 1980) with a precision of around ±15 mm were computed by Penna *et al.* (2002), and have been used in this study. Ten stations from the current UK National Tide Gauge Network include a co-located CGPS station (Fig. 2c), which together with local GPS antenna to tide gauge connection data obtained from www.sonel.org, Dayoub (2010) and Newcastle University levelling at North Shields and Liverpool, enabled discrete estimates of the geoid-ellipsoid separation to be obtained, after application of (MDT+IBR), but ignoring the other effects v that cause MSL to depart from the geoid (eq. 4).

All available CGPS data from 2006.0 to 2007.0 (chosen because equipment changes and data gaps were minimal) were processed in precise point positioning mode as described in Williams & Penna (2011), except GIPSY 6.1.2 software was used and the fiducial-free outputs were transformed to the International GNSS Service (IGS) realization of ITRF2008 (Altamimi *et al.* 2011). To ensure consistency with the FBM coordinates, a second transformation to ETRS89 epoch 1989.0 was then undertaken using http://www.epncb.oma.be/_productsservices/coord_trans/, and the mean ellipsoidal height per station computed. The standard deviation of the mean of each of the 10 CGPS tide gauge ellipsoidal heights was about ±0.5 mm.

3.4 EGM 2008 syntheses

The EGM2008 global gravitational model (Pavlis *et al.* 2012, 2013) to degree 2190 was chosen as currently the best-available model of the Earth's external gravitational field, but also because we do not have access to the OSGM02 (Forsberg *et al.* 2003) and OSGM05 (Ziebart *et al.* 2008) gravimetric geoid models. Nevertheless, EGM2008 is already a very good model and regional refinements in other countries have only made marginal improvements upon it (e.g. Featherstone *et al.* 2011; Wang *et al.* 2012). By inference, it is likely that any attempt to improve upon EGM2008 in Britain will face a similar fate. Also, OSGM02 and OSGM05 should not strictly be used because they are geoid, not quasigeoid, models.

Spherical harmonic synthesis can generate various gravity field functionals from the 4π fully normalized coefficients of a global gravitational model. The two functionals of interest here are quasigeoid heights (ζ) and the meridional absolute Helmert vertical deflection (ξ). The public-domain *hsynth_WGS84.f* program from http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html was used for the syntheses (tide-free EGM2008 coefficients were used in order to be compatible with the GPS ellipsoidal heights). This software version also computes the zero-degree bias term that accounts for the difference in mass and potential between EGM2008 and the GRS80 normal ellipsoid (noting that GRS80 and the WGS84 ellipsoid are identical at the sub-mm level). To ensure that the functionals are evaluated at the correct locations, this software requires the 3-D geocentric (here ETRS89) geodetic coordinates (ϕ, λ, h) of the computation points, that is, here the FBM and CGPS coordinates.

As ODN heights from the British levelling do not refer to a well-defined or equipotential reference surface (*cf.* Filmer *et al.* 2010, p. 505), but are expected to be more closely aligned with the quasigeoid than the geoid, we used quasigeoid heights (ζ). However, to determine whether the difference between the geoid and quasigeoid has any significant effect on our investigation of the apparent sea slope, we used eq. (2) and gravity values measured between 1964 and 1971 (i.e. after the TGL was completed) at 160 of the 178 FBMs. The mean difference was 0.2 mm (standard deviation, STD, ± 23 mm) because most of the FBMs have ODN heights <200 m. Computing the same for EGM2008 gave 0.9 mm (STD ± 41 mm). As such, this conceptual difference (*cf.* Section 2) cannot contribute significantly to the slope.

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4 ASSESSMENT OF SOUTH–NORTH SLOPES

4.1 Demonstration of ODN/apparent sea slope

The largest apparent south–north sea slope reported for Britain is 53 mm deg^{-1} latitude (Thompson 1980), based on linear regression of the height of MSL above ODN (determined from the Second Scientific Adjustment [SSA] of the TGL, i.e. a minimally constrained adjustment with only Newlyn held fixed) at the 29 tide gauges shown in Fig. 2(d). Thompson's MSL was determined over a 16 yr time span (1960.0–1976.0), with the average data span per tide gauge being 10 yr. Using the MSL heights listed in (ibid., Table 1), we have reproduced this apparent sea slope in Fig. 3(a), obtaining the same slope and uncertainty using the fit routine of Press *et al.* (1992, p 459) of (53 ± 4) mm deg^{-1} with a coefficient of determination of 0.87. That is, the levelled and least-squares-adjusted heights fall below the MSL heights at this rate on heading northwards.

While Thompson (1980) considered the contribution of the IBR, he did not consider the MDT effects needed for a conceptually closer comparison with the geoid (Section 2). Therefore, in Fig. 3(b), we show the slope obtained after correcting the (ibid.) values for (MDT+IBR) using the OCCAM model. The slope reduces to (49 ± 4) mm deg^{-1} latitude, with a coefficient of determination of 0.87, demonstrating that (MDT+IBR) alone cannot explain the apparent sea slope, and adds weight to Thompson's suggestion that errors in the TGL (SSA) were the cause.

As the current values of ODN benchmarks are not those of the TGL (SSA) used by Thompson (1980), but are based on a combination of the SGL and TGL (with the FBMs all fixed to their values from the SGL; Section 3.1), we recomputed the slope using the current ODN benchmark heights for 32 stations of the current UK National Tide Gauge Network (Fig. 2b). We used up to 16 yr of MSL data (1995.0–2011.0) where possible (average number of data years per tide gauge was 12), to use roughly the same length of time span as Thompson (1980), and also corrected for (MDT+IBR). This leads to a slope of (22 ± 4) mm deg^{-1} latitude with a coefficient of determination of 0.47 (Fig. 3c), that is, still a substantial south–north slope but only about half the magnitude of Thompson's.

To try to average out MSL variability and the 18.6 yr nodal tide around Britain, we recomputed the slope using, where possible, 37 yr of MSL data (1974.0–2011.0) and found a similar slope of (23 ± 4) mm deg^{-1} latitude with a coefficient of determination of 0.49, as shown in Fig. 3(d). This demonstrates that the variation in longer term MSL around Britain is not very sensitive to the time span considered, and means that the effect of the different time spans used to compute MSL can be considered negligible in relation to the apparent sea slope.

From Fig. 3, there is still a substantial apparent sea slope of around (23 ± 4) mm deg^{-1} latitude when using the current ODN benchmark heights, even after correcting MSL for (MDT+IBR) effects (*cf.* eq. 4). This strongly suggests that there is a systematic error in the levelling-based ODN.

4.2 Demonstration of no sea slope and no quasigeoid slope

To further investigate the suggestion that the apparent sea slope arises predominantly from levelling errors in ODN, we computed geoid estimates from CGPS ellipsoidal heights, 37 yr of MSL at British tide gauges and (MDT+IBR) using the OCCAM model, and compared them with EGM2008. From eqs (2) and (4) (also see Fig. 1) and ignoring v as it cannot be quantified with any reliability:

$$\varepsilon_\zeta = (h_{\mathrm{MSL}} - (\mathrm{MDT{+}IBR})) - \zeta_{\mathrm{EGM2008}} + c, \qquad (11)$$

where c is a constant that absorbs offsets among the data sets, for example, due to the zero-degree term in the quasigeoid and/or the reference level of the MDT model. Eq. (11) was evaluated at the 10 CGPS stations shown in Fig. 2(c), and—most importantly—is independent of ODN.

Fig. 4 shows a linear regression of ε_ζ versus latitude for the 10 CGPS stations shown in Fig. 2(c). The two northernmost stations (Stornoway and Lerwick) are not directly connected to ODN, but this is immaterial because the comparison is independent of the levelling-based ODN, and is being used to assess any apparent sea slope with respect to EGM2008. The mean has been subtracted to eliminate the constant term in eq. (11). The important finding seen in Fig. 4 is that there is no significant slope in ε_ζ (only –(2 ± 4) mm deg^{-1} latitude with a coefficient of determination of 0.04), especially compared with the slopes shown in Fig. 3, and the STD of the ε_ζ values themselves is ±40 mm. This gives us confidence that we have successfully separated the terms in eq. (1) with regard to apparent slopes, that there is no significant slope in EGM2008, and that the slopes shown in Fig. 3 must only be caused by errors in ODN.

The ε_ζ series in Fig. 4 includes a data point at Liverpool (latitude 53.4°N) that only used MSL data to end of 2003, due to localized subsidence of the tide gauge (uncorrected PSMSL MSL readings from 2004 onwards appear high). While the local CGPS to tide gauge levelling was carried out in 2012 after the subsidence had started, GIPSY 6.1.2 precise point positioning processing (as described in Williams & Penna (2011)) of 13 yr of data (1999–2011) from the Liverpool CGPS station led to a vertical velocity estimate of ∼1 mm yr^{-1}, demonstrating negligible subsidence of the CGPS station. There was also no visual subsidence evidence (such as cracking) of a nearby auxiliary benchmark that was also spirit levelled to. Therefore the CGPS to auxiliary benchmark connection, and the ODN heights of the auxiliary benchmark and the TGBM (when both last verified by the Ordnance Survey in 1991 before the subsidence started) have been used to generate the Liverpool data point shown in Fig. 4.

If the Liverpool data point were excluded, together with the Lowestoft (latitude 52.5°N) data point, whose local CGPS to tide gauge connection was determined by a combination of local levelling and previous episodic GPS measurements, the value obtained for the

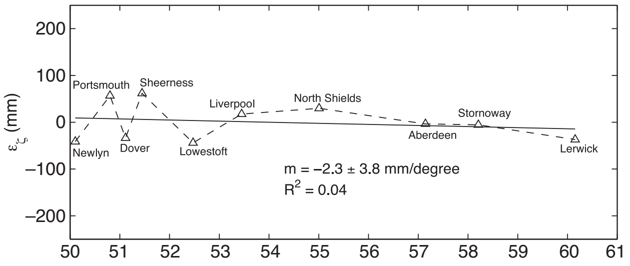

Figure 4. Demonstration of no EGM2008 slope by plotting ε_ζ versus latitude for the CGPS tide gauge stations shown in Fig. 2(c). The mean has been removed in all cases, thus eliminating the constant term in eq. (11).

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slope in ε_ζ changes to –(3 ± 4) mm deg^{-1} latitude. Thus a clear demonstration of no slope in ε_ζ is given, irrespective of the inclusion or not of the Liverpool and Lowestoft data points, which have slightly greater uncertainties than the other eight tide gauges.

Our ε_ζ series is more precise (although we used fewer stations) than equivalent plots produced by Bingley *et al.* (2002) and Hipkin *et al.* (2004), who obtained STDs of about ±100 mm when using the EGG97 geoid (Denker & Torge 1998), although we note that Bingley *et al.* (2002) did not correct for the (MDT+IBR) components of MSL. While Hipkin *et al.* (2004) obtained a STD of ±30 mm when using the EDIN2000 geoid, this was only after removing two outliers including Newlyn, which they attributed to local geoid model errors. However, Newlyn (latitude 50.10°N) does not show up anomalously in our results (Fig. 4). We therefore conclude that the breakthrough obtained in resolving the ODN slope has largely arisen through the improved quality of EGM2008.

Another method for detecting slopes in geoid models is to compare them with astrogeodetic vertical deflections as these are measures of geoid gradients (*cf.* Featherstone 2006, 2007). J. G. Olliver (2012, personal communication) provided us with 192 astrogeodetic vertical deflections on the British mainland referred to WGS84 and used in Olliver (1992), and which are compatible with EGM2008 (*cf.* Jekeli 1999). This test is however not as powerful as the ε_ζ test above because of the ±0.3″ estimated precision of the deflections (Dean 1980). We restricted the comparison to the meridional component, and computed

$$\varepsilon_\xi = \xi_{\mathrm{astro}} - \xi_{\mathrm{EGM2008}}, \tag{12}$$

where ξ_{astro} is the astrogeodetic meridional deflection and $\xi_{\mathrm{EGM\ 2008}}$ is the EGM 2008 meridional deflection synthesized at the 3-D geocentric geodetic coordinates of the astrogeodetic stations. If there is no slope in the geoid with respect to the vertical deflections, then the expectation is $\varepsilon_\xi = 0$.

After rejection of one outlier, the mean value of ε_ξ is 0.10″, which translates to a south–north slope of 54 mm deg^{-1} latitude. While seemingly large and commensurate with Thompson (1980), this is completely insignificant in relation to the STD of the differences (±1.15″ or 810 mm deg^{-1} latitude) or the expected precision of the astrogeodetic deflections (±0.3″ or ±162 mm deg^{-1} latitude). Nevertheless, it does not contradict the observation from Fig. 4 that there is no significant slope in EGM2008.

4.3 Comparison of EGM2008 with ODN

To provide further confirmation that the apparent south–north sea slope is due to errors in ODN, we compared EGM2008 quasigeoid heights, ζ_{EGM2008} (shown in the previous section to have no slope) with ζ_{ODN}, which is defined as the difference between the FBM's GPS ellipsoidal and current ODN heights (eq. 1). We first considered only the FBMs nearest to each of the 32 tide gauges used in generating the slope shown in Fig. 3(d) (the median tide gauge to FBM distance was 10.6 km, with the maximum distance 37 km), with the differences ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) plotted in Fig. 5. The equivalent plots of MSL–(MDT+IBR) using 37 yr of MSL data above ODN are also reproduced in Fig. 5 for ease of comparison.

A south–north slope of (25 ± 3) mm deg^{-1} latitude in ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) is shown in Fig. 5(b) (coefficient of determination 0.64), which is in very good agreement with the MSL–(MDT+IBR) slope of (23 ± 4) mm deg^{-1} latitude (Fig. 5a). This, together with the presence of similar visual trends in the two data series, such as increases around 55°N and then slight decreases at around 57–58°N, provide further evidence that EGM2008 and MSL–(MDT+IBR) are in close agreement (and not just from using the 10 tide gauges with colocated CGPS shown in Fig. 4) and, again, that systematic errors in the levelling-based ODN are the cause of the slope.

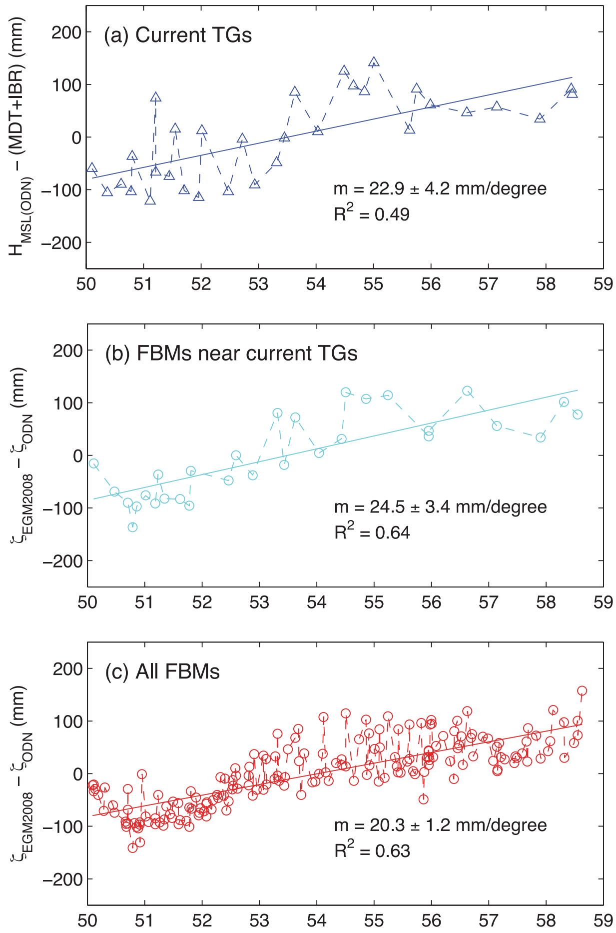

Figure 5. Comparisons with ODN. (a) Height of MSL–(MDT+IBR) above ODN at current tide gauges (TGs) using 37 yr of data, that is, a reproduction of Fig. 3(d) for ease of comparison; (b) ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) at the FBMs nearest to the TGs used in (a); (c) ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) at 177 of the 178 FBMs. The mean has been removed in all cases, thus eliminating the constant term in eq. (11).

To assess the slope across the whole of the British mainland, rather than just at or near the tide gauges, the values of ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) for 177 of the 178 FBMs (one, being greater than five times the median absolute deviation, was deemed an outlier and thus omitted) are plotted in Fig. 5(c). Linear regression of these differences demonstrates a south–north slope of (20 ± 1) mm deg^{-1} latitude (coefficient of determination 0.63), which is slightly less than that obtained when just using the FBMs nearest the 32 tide gauges or MSL–(MDT+IBR) at the tide gauges themselves. This is due to the regional distortions in ODN, which will be presented and discussed in Section 5. That a similar slope in ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) is obtained for the entire British mainland as when considering the tide gauges reinforces that systematic errors in the levelling-based ODN are the cause of the slope.

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Importantly, having found no slope in EGM2008 with respect to MSL–(MDT+IBR), it is clear that the substantial $(\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}})$ south–north $(20 \pm 1)\,\mathrm{mm\,deg^{-1}}$ latitude slope represents a fall in ODN on heading northwards, and confirms the warning of Christie (1994) that ODN should not be used for scientific purposes. It contradicts the claim by Ziebart *et al.* (2008) that ODN is a physically meaningful reference that improved geoid models should converge towards (although they did not provide details of the ODN version used—we assume the current ODN values for the FBMs). Apparent from Fig. 5(c) however are deviations from a linear trend, with a slight dip at around 51–52°N, a steeper slope from around 51.5–53.5°N, increased scatter around 53–56°N, and a slight tail-off around 57–59°N. We consider these to be due to regional distortions, which are addressed in the next section.

5 REGIONAL DISTORTIONS

While several authors who have used or investigated British levelling and ODN have concentrated on the apparent south–north sea slope (e.g. Thompson 1980; Ashkenazi *et al.* 1990; Hipkin *et al.* 2004), the scatter in Figs 3 and 5 indicates some spatial dependency. Regional distortions in vertical datums are not uncommon (e.g. Featherstone & Filmer 2008), and can come from errors in the levelling (e.g. Entin 1959) or the way in which they were adjusted (e.g. Featherstone & Filmer 2012). In the British case, the SGL and TGL followed some different traverses (*cf.* Kelsey 1972, Figs 3 and 4), so will be affected by the non-holonomity of levelling (because the NOC is not a complete account of the non-parallelism of equipotential surfaces), but this effect is likely to be small in comparison with undetected blunders and systematic errors in the levelling observations.

Fig. 6 presents a contour map of the differences between the TGL and SGL from the values given at common FBMs in Kelsey (1972, Fig. 5). Not all areas are covered because of a combination of (i) no SGL data in south-east England, most of Wales and along the north-east coast of England (Kelsey 1972, Fig. 3); (ii) destruction of some FBMs and the establishment of new FBMs since 1959 that now have GPS ellipsoidal heights; (iii) using *ibid.*, it was not possible to reliably colocate FBMs in some areas where there are several, notably near Dunbar and Newlyn; and (iv) some FBMs were ambiguous so were simply excluded.

In Fig. 6, there is a general south–north trending difference, with a greater apparent uplift of the land relative to the TGL than the SGL on heading northwards. This is equivalent to the TGL zero level becoming progressively lower than the SGL zero level on moving northwards. Kelsey (1972) and others describe heights from the TGL as being higher than those from the SGL, but this is not strictly correct. Instead, the systematic error in the TGL places its zero increasingly beneath that of the SGL on heading northwards, making the heights only appear to be higher. In Fig. 6, regional distortions are also present, with some differences larger than could be expected from the closure tolerances estimated for each levelling (around $2\,\mathrm{mm}\sqrt{\mathrm{km}}$).

We have also illustrated the south–north trend in Fig. 7, and linear regression of this estimates the slope as $(26 \pm 1)\,\mathrm{mm\,deg^{-1}}$ latitude. It is not possible to determine which of the SGL and TGL is more accurate from these comparisons alone, and it is not explicitly stated by Kelsey (1972) whether these are raw height differences or are the result of two separate minimally constrained adjustments, or if NOCs were applied to the TGL. Nevertheless, the $26\,\mathrm{mm\,deg^{-1}}$ latitude difference between the TGL and SGL slopes is remarkably similar to the $26\,\mathrm{mm\,deg^{-1}}$ latitude difference apparent from Fig. 3 between the $49\,\mathrm{mm\,deg^{-1}}$ latitude slope obtained when using MSL above the TGL (SSA) that were corrected for (MDT+IBR), and the equivalent of $23\,\mathrm{mm\,deg^{-1}}$ latitude above ODN, whose heights are dominated by the SGL.

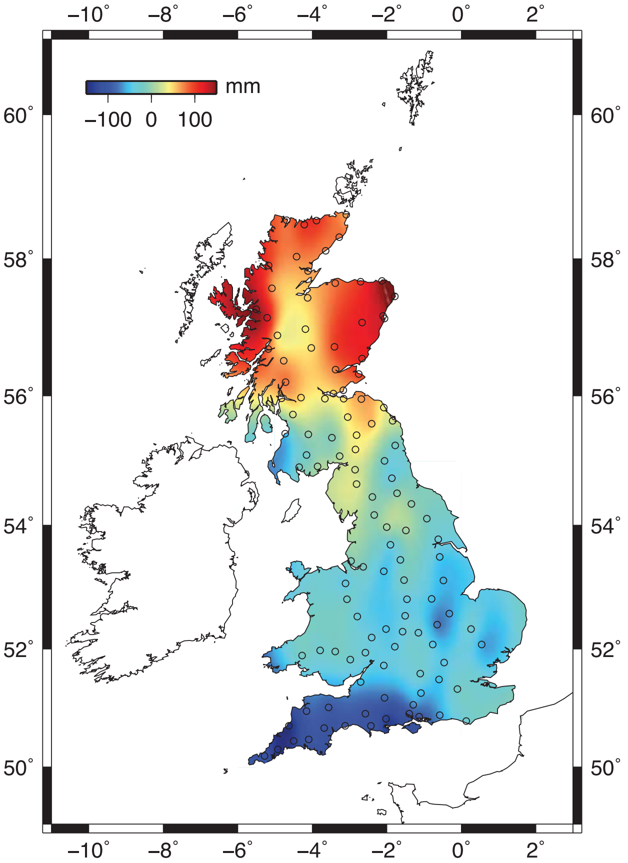

Figure 6. Contours of TGL minus SGL height differences (mean removed): spline-interpolated from values taken from Kelsey (1972, figure 5) at the FBMs denoted by the circles. The coloured landmass and islands denote the extent of ODN, that is, some (groups of) islands have their own local vertical datums.

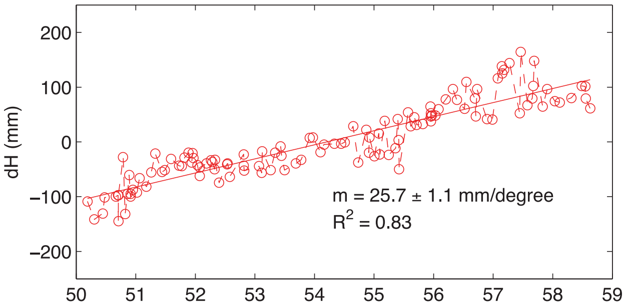

Figure 7. TGL minus SGL height differences (dH) versus latitude (mean removed). Data taken from Kelsey (1972, figure 5).

Many of the regional distortions in Fig. 6 are defined by multiple FBMs, again pointing to systematic levelling errors. Since we do not have access to the original levelling observations, we cannot investigate these distortions. However, as a further illustration of regional distortions in the levelling and hence ODN, Fig. 8 presents a contour map of the $(\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}})$ differences (spline-interpolated

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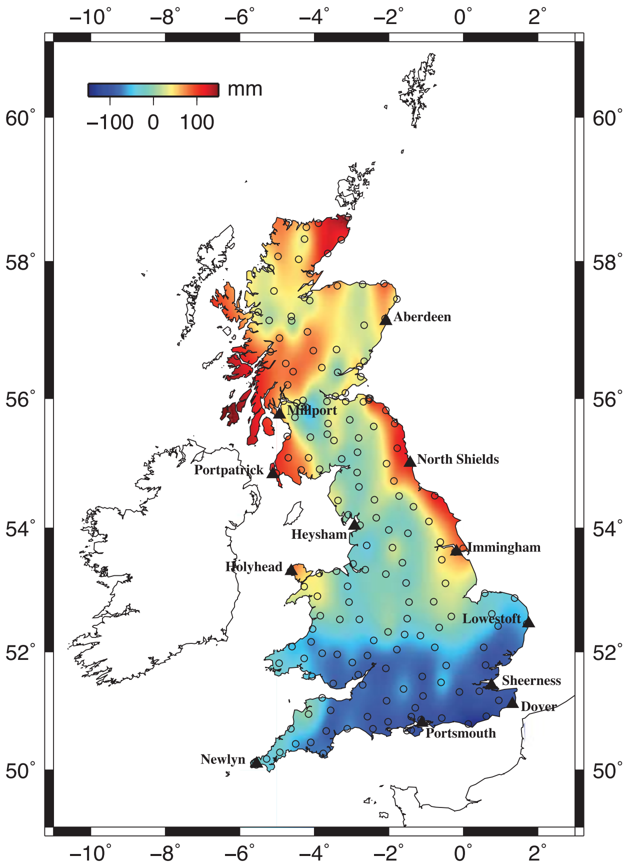

Figure 8. Contours of ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) (mean removed): spline-interpolated from values at the FBMs denoted by the circles. Also marked by the black triangles are the tide gauges considered by Hipkin *et al.* (2004), from which an ODN step of around 240 mm, affecting all stations north of 53°N, was suggested. The coloured landmass and islands denote the extent of ODN, that is, some (groups of) islands have their own local vertical datums.

from the values at the FBMs). It is effectively an update to Ziebart *et al.* (2008, Figs 3 and 4) using the EGM2008 quasigeoid and not the OSGM geoids, and is a geographical illustration of the linear regression for these data that is shown in Fig. 5(c). The regional distortions shown in Fig. 8 are more relevant than the above differences between the SGL and TGL as they represent the distortions in the current ODN heights. Many of these distortions are defined by multiple FBMs, again pointing to systematic errors in the levelling-based ODN, notably the reddened contours along, for example, the north-east coast of England. These can be explained because only the TGL was used to provide heights in this region (*cf.* Kelsey 1972, Fig. 3) whose zero level falls on heading northwards at a greater rate than the SGL, and these TGL data were 'bolted on' to the central-spine FBMs whose ODN heights are dominated by the SGL.

Contrary to that reported in Hipkin *et al.* (2004), in Fig. 8 there is no obvious step in ODN of around 240 mm affecting all areas north of 53°N, or any substantial west–east trend. The presence of such a step was postulated by *ibid*. based on seemingly only a visual inspection of plots comprising the 12 tide gauge stations labelled in Fig. 8, with no physical explanation or rigorous detection of an offset provided. The tide gauges north of 53°N shown in Fig. 8 are mainly located, by chance, in the 'red' zones, which seemingly led to the Hipkin *et al.* (2004) misconception regarding the presence of a step.

To further investigate the presence of a step, we undertook a penalized likelihood maximization (PLM) using the Schwarz Information Criterion analysis (or Bayesian Information Criterion: Schwarz (1978)), with a single offset and linear trend on the 177 ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) data points and their variation with latitude, as shown in Fig. 9(a). However, no offset was detected. To validate this technique, an artificial offset of 120 mm was then applied to all points north of 53°N and the test repeated, as shown in Fig. 9(b). One offset (at 53°N where applied) was detected, confirming the reliability of PLM for detecting any offsets in this particular data series, especially as the artificial offset applied is half the magnitude of the 240 mm datum step at 53°N suggested by Hipkin *et al.* (2004).

PLM, as with most other offset detection methods, is based on the calculation of the variance of the signal. The use of shortened data series tends to result in spurious offsets being detected, for example, here we found that a false offset was detected in the data series when 30–35 ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) data points rather than 177 were used. Therefore, we did not attempt to detect or quantify offsets in the shorter data series shown in Figs 3 and 4. This also adds weight to the claim that Hipkin *et al.* (2004) were misled by the use of only 12 data points. However, our analysis of the 177 ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) data points suggests that there is no step in ODN, but instead there is a general south–north slope, coupled with the regional distortions illustrated in Fig. 8.

6 DISCUSSION AND CONCLUSIONS

This study has reinvestigated the slope in the spirit-levelling–based British vertical datum, that is, ODN. We first considered the difference between MSL–(MDT+IBR) and current ODN heights at 32 TGBMs, finding a substantial apparent south–north sea slope of (23 ± 4) mm deg^{-1} latitude. This is roughly half of the (53 ± 4) mm deg^{-1} latitude slope reported by Thompson (1980), which we have shown cannot be explained by (MDT+IBR) effects, since applying OCCAM model values only reduces the slope to (49 ± 4) mm deg^{-1}.

The much greater slope of 49 mm deg^{-1} latitude using the Thompson (1980) MSL–(MDT+IBR) heights than the 23 mm deg^{-1} latitude slope found here arises because ODN is dominated by fixing all FBMs to their values from the staged/mixed least squares adjustments of the SGL, whereas Thompson (1980) used only a minimally constrained adjustment of the TGL. We showed in Figs 6 and 7 that this 26 mm deg^{-1} latitude slope difference is very similar to the trend taken from the differences in Kelsey (1972) between the heights from the SGL and TGL, that is, the TGL appears to contain larger systematic errors than the SGL. We have independently demonstrated a very similar slope of (25 ± 3) mm deg^{-1} latitude by comparing EGM2008 quasigeoid heights with ζ_{ODN} at each FBM nearest to the 32 tide gauges considered, and a slope of (20 ± 1) mm deg^{-1} latitude using 177 FBMs across the entire British mainland. These two values differ because of the regional distortions shown in Fig. 8.

Given that we considered MSL–(MDT+IBR) versus EGM2008 quasigeoid values at 10 tide gauges colocated with CGPS and found no significant slope of –(2 ± 4) mm deg^{-1} latitude (coefficient of determination 0.04), it is demonstrable that there is neither a slope in the EGM2008 quasigeoid nor a slope in MSL when corrected for (MDT+IBR). EGM2008 was also compared with astrogeodetic

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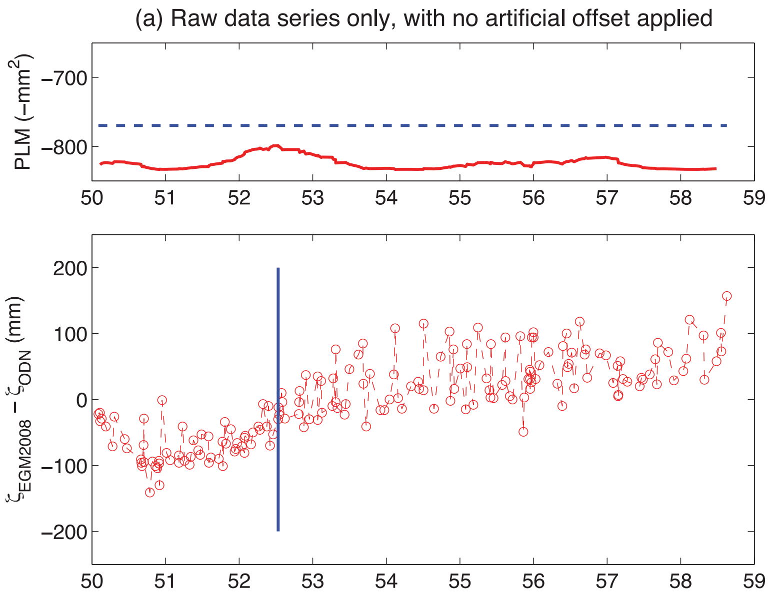
(a) Raw data series only, with no artificial offset applied

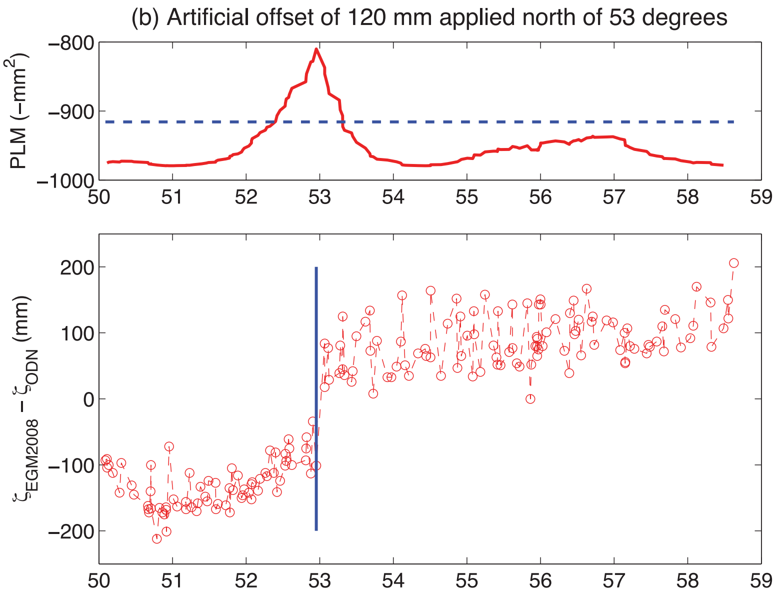
(b) Artificial offset of 120 mm applied north of 53 degrees

Figure 9. (a) The upper pane shows offset detection based on a penalized likelihood maximization (PLM) using the Schwarz Information Criterion, applied to the FBM ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) data series that is plotted against latitude in the lower pane. In the upper pane, the blue dashed line represents the penalized likelihood calculated under the hypothesis that there is no offset in the data set. The red solid line shows, at each latitude, the value of the penalized likelihood, under the alternative hypothesis that there is an offset at the latitude considered. The red solid line rising above the blue threshold denotes that the alternative hypothesis is more likely than the initial hypothesis. The blue vertical line in the lower pane denotes the likelihood maximum; (b) As for (a), but having applied an artificial offset of 120 mm to all ($\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}}$) data points north of 53°N.

observations of the deflection of the vertical at 191 stations, with no significant slope prevailing (albeit with much noisier data). There can therefore be little doubt that the British 'apparent sea slope' is caused by a south–north downwards slope in the vertical datum (i.e. the zero of ODN drops below the geoid on heading northwards, making heights appear higher than they should be). This can only arise from systematic levelling errors. However, we are unable to isolate the source of this levelling error as we do not have access to the original observations, but can state that ODN is a questionable datum for scientific purposes.

Levelling errors causing the apparent sea slope has been postulated in previous studies (e.g. Thompson 1980; Hipkin *et al.* 2004; Ziebart *et al.* 2008) but until now not as conclusively demonstrated. We also contradict some of these previous studies, instead supporting the warning of Christie (1994) that ODN should not be used for scientific purposes. Ziebart *et al.* (2008) suggested that the slope in the vertical datum could be −12 mm deg^{-1} latitude, and that ODN is a physically meaningful reference. Comparing this with the ODN slope of −(20–25) mm deg^{-1} latitude that we have confirmed, could indicate that the OSGM05 gravimetric geoid model contains a slope of around 10–15 mm deg^{-1} latitude. Hipkin *et al.* (2004) suggested the presence of a step in ODN of about 240 mm at 53°N, but using many more data points (177 compared with their 12) and an offset detection method based on PLM using the Schwarz Information

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Criterion, we did not find such a step. Instead, there is simply a slope coupled with regional distortions.

In terms of quantifying other contributors to the slope, on correcting the MSL values for (MDT+IBR) effects using the OCCAM model and comparing with current ODN heights, we still isolate a slope of (23 ± 4) mm deg^{-1} latitude, confirming that (MDT+IBR) are not the main contributors (together they only account for ∼3 mm deg^{-1}). The EGM2008 quasigeoid and GPS observations used here both refer to the tide-free system and, while the tide system used for the SGL and TGL is unknown, we showed in Table 1 that the maximum slope that any different tide system could contribute is only ∼6 mm deg^{-1} latitude. We have used ETRS89 at epoch 1989.0, but the difference between this and ITRF2008 equates to only ∼1 mm deg^{-1}. The contribution of the nodal tide to the slope is negligible (0.067 mm deg^{-1} latitude). As such, levelling error remains the only plausible explanation for the slope and distortions in ODN.

It is also necessary to comment on the effects of relative vertical land movement in Britain due to GIA, particularly in relation to the different epochs of observations considered herein. GPS data (Teferle *et al.* 2006, 2009) and GIA models (Milne *et al.* 2006; Bradley *et al.* 2011) suggest an approximate 1–2 mm yr^{-1} rise of land heights in northern Britain (Scotland) relative to southern Britain. We have shown in Section 4 that there is a ∼200 mm total height difference between the values of MSL–(MDT+IBR) (averaged from c1974 to c2011; ∼37 yr) and the ODN between southern England and northern Scotland (dominated by the SGL and observed from c1912 to c1951; ∼39 years). These suggested 1–2 mm yr^{-1} GIA effects, which raise the land heights with respect to MSL over time, may have caused a reduction in the south–north height difference of ∼40–80 mm (and hence the ODN slope) compared with using ∼37 yr of MSL observations from the same mid-epoch as the SGL observations, that is, c1930. However, there are insufficient long-term tide gauge or CGPS records to test this hypothesis, but this could be tested in the future. It is nevertheless clear that GIA effects are not the dominant cause of the apparent sea slope in Britain. Furthermore, the minimally constrained adjustment of the TGL (SSA), which shows a south–north height difference of ∼400 mm, is based on levelling observed over a 9 yr period only, during which GIA contributions amount to only ∼10–20 mm.

It is noteworthy that the $(\zeta_{\mathrm{EGM2008}} - \zeta_{\mathrm{ODN}})$ slope from 177 FBMs is about 20 mm deg^{-1} latitude, that is, a total height difference of nearly 200 mm, from GPS ellipsoidal heights observed in 1999. This is very similar to the 23 mm deg^{-1} latitude MSL–(MDT+IBR) slope (∼37 yr, with mid-epoch 1992.5) with respect to ODN, which could be expected since relative GIA should affect both slopes by roughly the same amount. It is interesting however that the 26 mm deg^{-1} slope arising from the TGL-SGL FBM height differences (mid epochs of c1955 and c1930, respectively) is roughly the same as the difference between the slopes of MSL–(MDT+IBR) above the TGL (SSA) (49 mm deg^{-1}) and above current ODN (23 mm deg^{-1}). That is, GIA effects are not so apparent, but this statement should be tempered by the veracity of the levelling data that we bring into question here.

Although we have demonstrated that the geodetic levelling is responsible for the slope in ODN, we do not yet have an explanation for the exact source of the errors. This would be best assessed using the original observations, but we do not have these data. A fourth geodetic levelling is recommended to realize a more accurate British vertical datum, to enable it to be used for scientific purposes, though Christie (1994) states that the cost would be prohibitive. Alternatively, a new vertical datum, based on GPS together with the EGM2008 geoid model, which has been shown to have no slope (or some regional refinement using gravity data that have been corrected for the slope) would overcome the current problems in Britain. This would also avoid the need to distort gravimetric geoid models to fit ODN heights at FBMs as is done at present (Forsberg *et al.* 2003; Iliffe *et al.* 2003).

In closing, ODN is a vertical datum that contains a significant south–north slope of around –(20–25) mm deg^{-1} latitude (i.e. a fall in the zero of the vertical datum on heading northwards) and smaller regional distortions with respect to the geoid and local MSL, both caused by erroneous levelling observations. It should not be used for scientific purposes.

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