Origami and its Applications in Automotive Field

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Abstract— This paper presents the fundamentals of Origami engineering and its application in motorcycle wind shield design and construction. Several mathematical approaches are developed by mathematicians, scientists and engineers. *Tree Theorem* introduced by Robert Lang is the focus in this research. *Rigid origami* is applicable as material with thickness is used in designing a foldable windshield for motorcycle. Wind shield, on the other hand, is a common product worldwide for many motorcyclists. However, due to its bulk size and the requirement of permanent installation to the body of the motorcycle, many other motorcyclist choose not to use the wind shield where in fact this product provides full protection from flying debris and wind when the motorcycle is moving forward. It is in hope that this design project would initiate certain level of interest among the automobile manufacturers in Malaysia to consider Origami engineering as a new leaf of technology to boost the technology application in their field.

Keywords- Origami, origami engineering, rigid origami, wind shield

I. INTRODUCTION

Origami is the art of folding originated from Japan and has been commonly practiced worldwide. Traditionally square papers are folded into 2 or 3 dimensional figures as resemblances of existing matters: birds, mammals, trees, furniture, etc, regardless if the origami (folded paper) is made of polygon shape or not. As the application is being introduced to the scientists and engineers, Origami has become a useful tool in design and fabrication of appliances. Till date the folding is no longer being restricted to square papers. Many industry applications use origami technique in designing new products. The *Eyeglass* space telescope as shown in Fig. 1 designed by Robert Lang is a great example in merging origami and engineering.

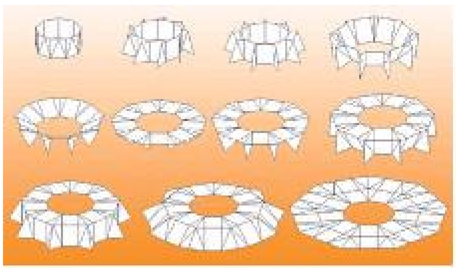

FIG.1 EYEGLASS DESIGNED BY ROBERT LANG
(FIGURE ADOPTED FROM ORIGAMI-RESOURCE-CENTER.COM)

II. ORIGAMI SCIENCE

A. *The Fundamentals*

Origami originated as a trial-and-error art design for making paper(s) to appear like real object by folding them. Later on several mathematical approaches were developed to understand the phenomena on the paper generated by the folding and also to estimate the outlook of the origami (folded paper).

Mathematical approaches for Origami, to name a few, are *Geometry*, *Topology* (explained by Thomas Hull [1]), Robert Lang's *Tree Theorem* [9] and Maekawa's *String-to-beads* method [8]. In this paper Lang's *Tree Theorem* would be discussed and applied in the windshield design.

The mentioned mathematical approaches are surrounding three main fundamentals: *Huzita-Hatori axioms*, *Kawasaki* and *Haga's theorems*.

Firstly discovered by Jacques Justin in 1989, the *Huzita-Hatori axioms* consist of 7 axioms were improved by Humiaki Huzita in 1991, and finalized by Koshiro Hatori, Justin and Robert Lang in 2001. The axioms [2] are:

1. Given two points P_1 and P_2, there is a unique fold that passes through both of them.
2. Given two points P_1 and P_2, there is a unique fold that places P_1 onto P_2.
3. Given two lines l_1 and l_2, there is a fold that places l_1 onto l_2.
4. Given a point P_1 and a line l_1, there is a fold that places P_1 onto l_2.
5. Given two points P_1 and P_2 and a line l_1, there is a fold that places P_1 onto l_1 and passes through P_2
6. Given two points P_1 and P_2, and two lines l_1 and l_2, there is a fold that places P_1 onto l_1 and P_2 onto l_2.
7. Given one point P_1 and two lines l_1 and l_2, there is a fold that places P_1 onto l_1 and is perpendicular to l_2.

Well explored by Toshikazu Kawasaki, the Kawasaki Theorem has proven that for a given crease pattern is foldable only if all the sequences of angles surrounding each (interior) vertex can be summed to 180° [3]. For example, for the following crease pattern in Fig. 2 on a square paper to be foldable, the summation of the angles, α has to be 180°.

$$\alpha_1 + \alpha_1 + \alpha_{1+}\ \alpha_{1+} \ldots\ldots + \alpha_n = 180° \qquad (1)$$

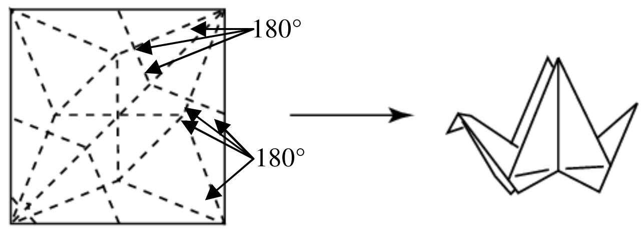

FIG. 2 APPLICATION OF KAWASAKI'S THEOREM IN A FOLDABLE PAPER CRANE (FIGURE ADOPTED FROM [3])

Haga's theorems explain impossibilities of area division on a piece of paper using Compass and Straight Edge Construction method (SE&C) [4]. Illustrated in the following in Fig. 3 and descriptions in Table 1, the length of the desired side of the square paper with 1 unit side length (say 2/3 unit for BT) can be achieved by altering the length of AP to 1/2 unit [7], which is impossible using SE&C.

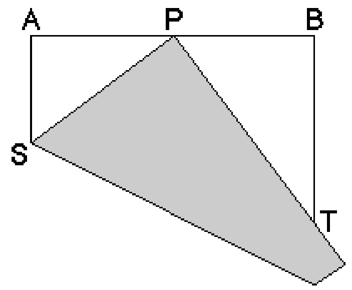

FIG. 3 HAGA'S THEOREM EXPLANATION IN DIVIDING THE PAPER INTO SECTIONS WHICH IS NOT POSSIBLE IN SE&C APPROACH (FIGURE ADOPTED FROM [7])

TABLE 1 SUMMARY OF HAGA'S THEOREM (TABLE ADOPTED FROM [7])

AP/AB, AB = 1 unit	BT/AB, AB = 1 unit
1/2	2/3
1/4	2/5
3/4	6/7
1/8	2/9
3/8	6/11
5/8	10/13
7/8	14/15

$n/2^r$	$2n/(2^r+n)$

The notations of Origami have been standardized by scientists, to name a few, including Lang, Huffman, Clowes, Waltz, Takeo Kanade, and Akira Yoshizawa [5] & [9]. In summary, the notations being widely used in the Origami practice are made of lines, arrows, and terms as illustrated in Fig. 4 and Fig. 5.

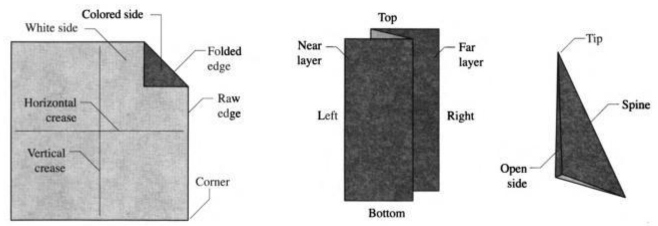

FIG. 4 TERMS USED IN A TYPICAL ORIGAMI FOLDING (FIGURE ADOPTED FROM [9])

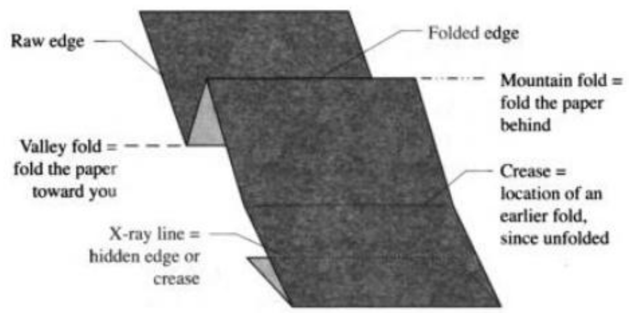

FIG. 5 TYPES OF LINES USED IN ORIGAMI (FIGURE ADOPTED FROM [9])

The basic folding methods in the industry are the Cupboard Base, Windmill Base, Waterbomb Base, and Preliminary Fold [5] as illustrated below in Fig. 6.

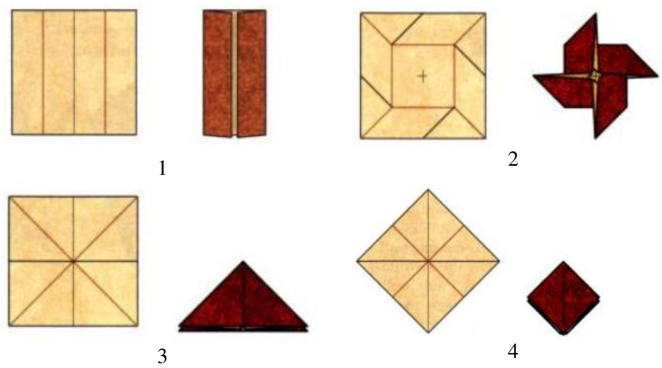

FIG. 6 BASES OF FOLDING METHODS: (1) CUPBOARD, (2) WINDMILL, (3) WATER BOMB, (4) PRELIMINARY FOLD (FIGURES ADOPTED FROM [9])

According to Lang [9], there are 3 types of flaps: corner, edge and middle flaps as given in Fig. 7. The names of the flap come after the location of the tip of the flap falls on the square. Flaps are distinguished to determine the amount of material used in folding. Middle flap consumes the most part of a material as compare to corner flap which consumes the least amount of material. To prove this, Lang proposed that each the tips (of the flaps) forms the centre of a circle on a flat material. As illustrated in Fig. 8, the circles are formed by narrowing the flaps.

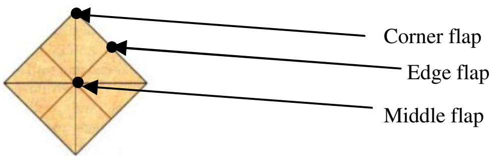

FIG. 7 TYPES OF FLAP

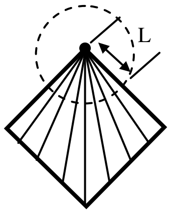

FIG. 8 CIRCLE FORMED AROUND A CORNER FLAP

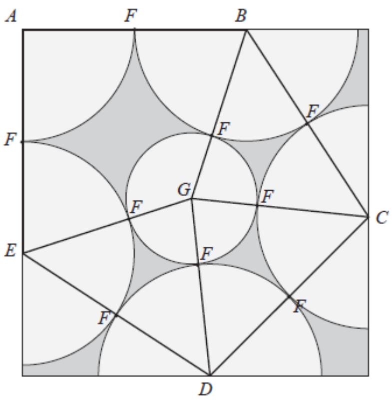

FIG.9 CIRCLE PACKING FOR TIPS OF FLAPS ON A PIECE OF SQUARE FLAT MATERIAL WITH EACH ACTIVE PATHS (B-G, G-E, G-D, ETC) BEING INDICATED (FIGURES ADOPTED FROM [8])

Lang suggested that all circles surrounding the tips, may it be quarter-circle (for corner flap), semi-circle (for edge flap), or full circle (for middle flap), the radius of the circle is determined by the length of the flap, L. He said, "The amount of paper consumed doesn't depend on the angle of the tip of the flap, only its length and location." Thus, circles-packing where circles do not overlap each other, is used in determining the sizes of the circles – the size of the flap. This approach is seem to be essential in *Rigid Origami*, as hard materials in engineering applications are not capable to recover from the crease formed by folding. *Rigid Origami* is explained in Section C.

B. Tree Theorem by Robert Lang

The components of *Tree Theorem* are: molecules and stick figure.

Molecules are basic tile patterns of an origami. The molecules are *triangle molecules, quadrilateral molecules, waterbomb molecules, arrowhead molecules, Gusset molecule,* and *rivers* [9].

Stick figure is also referred as tree graph [8]. A tree graph is basically the top projection of an origami, displaying its shadow. Tree graph which is made of nodes and edges (Fig. 11) is used to determine the number of flaps, their lengths, and how they are connected to each other [9]. Fig. 10 illustrates the tree graph as a top projection of an origami.

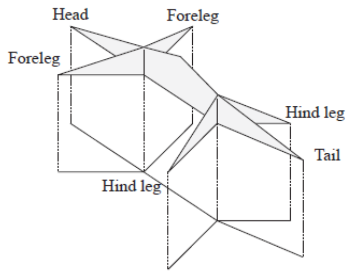

FIG. 10 TOP PROJECTION OF AN ORIGAMI PROVIDES A TREE GRAPH (FIGURE ADOPTED FROM [8])

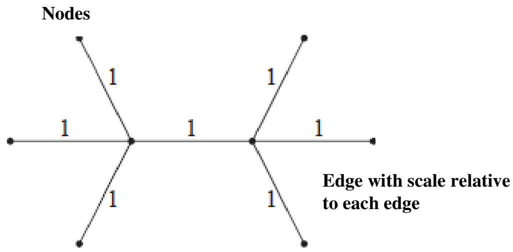

FIG.11 TERMS APPLICABLE ON A TREE GRAPH (FIGURE ADOPTED FROM [8])

Each terminal node (node that is connected to another node by only one edge) is the tip of the flap. In which by using the circle packing method, there are a total of 6 circles (2 corner flaps, 3 edge flaps and a middle flap) formed in the tree graph in Fig. 11.

Transforming the tree graph to actual creases and paths on a flat material require the following equations (2) - (4).

The minimum length of a path connecting two nodes [9]

$$l_{ij} = |u_i - u_j| \tag{2}$$

$$|u_i - u_j| \le m l_{ij} \tag{3}$$

Where u_i *and* u_j are each the local coordinates of node *i* and *j*. The distance between the nodes is l_{ij}. *m* is the scale factor which is constrained by the rules: Condition with equation (2) is satisfied and all nodes lie within a unit square [9].

An active path is a path of its actual length is equal to its minimum length l_{ij}.

If the nodes are separated by edges with length *a* and *b*, the actual distance is governed by the Eqn. 4 on the basis one circle formed on each node.

$$|u_a - u_b| \geq m(a+b) \tag{4}$$

In completing the transformation of tree graph to actual folding the concept of *universal molecule* is used. Referring to Fig. 12, by increasing the length of h a reduced polygon is generated. The boundary condition for h is

$$|A'_i - A'_j| \geq l'_{ij} \tag{5}$$

where l'_{ij} is a reduced path length described in Eqn. 6.

$$l'_{ij} = l_{ij} - h(\cot\alpha_i + \cot\alpha_j) \tag{6}$$

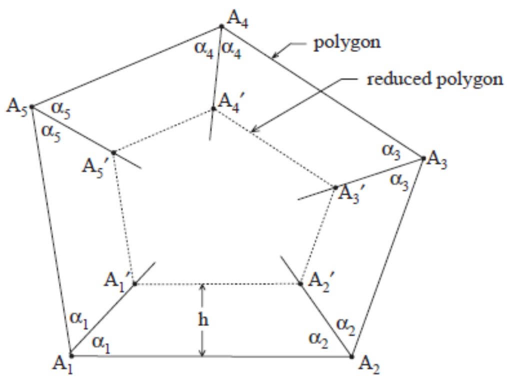

FIG.12 AN ACTIVE POLYGON WITH ITS REDUCED POLYGON (DASH LINE) AT AN INSET OF DISTANCE H (FIGURE ADOPTED FROM [8])

C. Rigid Origami

Rigid origami applies in folding materials with thickness. Car airbags, large solar panel arrays for space satellites (using *Miura-fold*), paper shopping bags are amongst the studies in *Rigid origami*.

Having thickness in the material has disabled some fundamentals of origami in *Rigid Origami* such as *Huzita-Hatori axioms*. However Kawasaki's Theorem and Haga's Theorem are still valid in real material folding [12].

Tomohiro Tachi mentioned in his research that *Rigid origami* consists of rigid panels connected by hinges constrained around vertices [11]. The origami configuration is represented by fold angles between the adjacent panels [6]. Intensive Mathematical model for 3D folding was presented in his paper [11] and it was used in his written software *RigidOrigami*.

Britney Gallivan has developed a loss function for origami folding with thick material and the function sounds like this:

$$L = \frac{\pi t}{6}(2^n + 4)(2^n - 1) \tag{7}$$

where *L* is the minimum length of the material, *t* is the thickness of the material, and *n* is the number of folds possible [6].

It appears that there is no direct connection between Lang's *Tree Theorem* and Tachi's *Rigid Origami*. However, the former could be used in designing the folding based on a desired final outlook of an origami where else the latter could provide calculations on the folding mechanisms with materials with thickness.

D. Origami Design Methodology

This Origami design approach is mentioned by Lang in his book [9] and it is adopted with some customizations based on the suitability of the design and construct project.

1) Decide the final outlook (folded form) of the windshield. Draw a tree graph of the base with the desired length of the edges.

2) Find a pattern of terminal nodes on the rectangle that satisfies the tree theorem.

3) Identify the active paths by marking the paths which actual length is equal to their minimum length, using equation (2) - (4).

4) Using *Tree Maker to* apply the *universal molecule* method to locate the folding patterns.

III. WIND SHIELD DESIGN CONSIDERATIONS

A. Types of Wind Shield

The focus on type of windshield in this paper is for standard motorcycle. It provides full protection from flying debris and wind when the motorcycle is moving forward.

There are mainly two types of wind shield for motorcycle: one allows the rider to look through the windshield while the other not. The types of windshield are illustrated in Fig. 13. In this project, the latter type is chosen due to its suitability for warm climate [10] as it is in Malaysia. Also it is because of the consideration of providing a minimal disturbed view to the rider on the road.

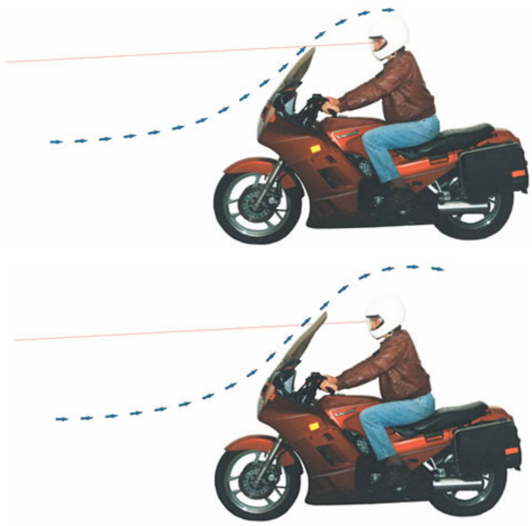

FIG. 13 TYPES OF MOTORCYCLE WINDSHIELD (FIGURE ADOPTED FROM [10])

B. *Wind Shield Design Parameters*

- Height

  For a windshield to be shorter than the rider after installation, a methodology introduced by Mark Lawrence [10] is used to determine the dimension of the windshield for this project.

  The desired dimensions of the windshield are found to be 40cm x 60cm (width x height). These dimensions have included the mounting requirement.

- Weight (Material)

  In practice the two most common materials for motorcycle windshield are polycarbonate (Lexan) and acrylic (Plexiglas) [10]. They are chosen due to their great transparency degree, strength of materials, and durability in molding the plastic.

  With the application of *Rigid origami*, the panels (pieces of polycarbonate) selection would based on the available thicknesses from the manufacturer.

- Optical Consideration

  The windshield should not be an obstacle to the visibility of the rider on the road (e.g. the road mark should not appear to be bent to the rider). Transparent material is to be used with suitable coating for sunlight filtration.

- Folding Process

  At this stage the design of the windshield involves manual unfolding and folding process. *Miura* fold is to be considered before further design modification is done using *TreeMaker* and *RigidOrigami*.

- Attach Method

  It is proposed for the unfolded windshield to be attached to the rack using firm clip-on method.

CONCLUSION AND FUTURE WORKS

This paper has presented the findings on Origami science which mainly focus on the Robert Lang's *Tree Theorem*, the fundamentals, *Rigid origami* and one of its applications. Beyond this paper are works on detailed designing a foldable (and usable) motorcycle windshield with all the studied Origami fundamentals and aiding software.

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