Validation of the AUSGeoid98 model in Western Australia using historic astrogeodetically observed deviations of the vertical

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Abstract

AUSGeoid98 is the national standard quasigeoid model of Australia, which is accompanied by a grid of vertical deviations (angular differences between the Earth's gravity vector and the surface-normal to the reference ellipsoid). Conventionally, co-located Global Positioning System (GPS) and spirit-levelling data have been used to assess the precision of quasigeoid models. Here, we instead use a totally independent set of 435 vertical deviations, observed at astrogeodetic stations across Western Australia before 1966, to assess the AUSGeoid98 gravimetrically modelled vertical deviations. This point-wise comparison shows that (after three-sigma rejection of 15 outliers) AUSGeoid98 can deliver vertical deviations with a precision (standard deviation) of around one arc-second, which is generally adequate for the reduction of current terrestrial-geodetic survey data in this State.

Keywords: geodesy, vertical deviations, quasigeoid, geodetic surveying, geodetic astronomy

Introduction

Gravimetric quasigeoid models are commonly validated on land using co-located Global Positioning System (GPS) and spirit-levelling data (e.g., Featherstone, 1999, Featherstone and Guo, 2001, Featherstone et al., 2001, Amos and Featherstone, 2003). However, this approach suffers from correlations among the data and deficiencies in the local vertical datum, which is especially the case for the Australian Height Datum (Featherstone, 1998, 2004, 2006, Featherstone and Stewart, 1998, Featherstone and Kuhn, 2006). A better validation can be achieved by using deviations of the vertical (cf. Jekeli, 1999; Hirt and Seeber, 2007), which are observed using different principles and thus are totally independent of the vertical datum (cf. Featherstone, 2006).

The deviation (or sometimes deflection) of the vertical is the angle between the Earth's gravity vector and the surface-normal to the reference ellipsoid (Bomford, 1980 and Figure 1). Since the plumblines (field lines) of the Earth's gravity field have both curvature and torsion, due varying mass-density distributions inside the Earth, the deviation of the vertical is a function of 3D position. The two main sub-classes of vertical deviation are (Jekeli, 1999): Pizetti deviations at the geoid (essentially the undulating mean sea level surface; Featherstone, 1999), and Helmert deviations at the Earth's surface.

The total vertical deviation (θ) in Figure 1 is further decomposed into north-south (ξ) and east-west (η) components. These are needed in the reduction of terrestrial-geodetic survey data to the reference ellipsoid (Featherstone and Rüeger, 2000)

Figure 1 near here

Vertical deviations can either be observed geodetically or computed from gravity data. Helmert vertical deviations are observed from the difference between astronomical latitude (Φ) and longitude (Λ) and geodetic latitude (φ) and longitude (λ), with the latter scaled by meridional convergence. Astronomical or natural coordinates are derived from timed angular measurements to the stars (e.g., Bomford, 1980; Hirt and Seeber, 2007). Geodetic coordinates are derived from geodetic surveying observations, e.g., angles, distances and GPS. Pizetti deviations can be computed from gravity data using Vening-Meinesz's integral (e.g., Heiskanen and Moritz, 1967) or from horizontal gradients of a geoid model (cf. Figure 1). All the relevant formulas are given in Featherstone and Rüeger (2000).

Vertical deviations are of practical importance in high-precision terrestrial-geodetic surveying (Featherstone and Rüeger, 2000), which has now become more important because of the introduction of the Geocentric Datum of Australia (GDA94) (ICSM, 2002). The AUSGeoid98 gravimetric quasigeoid model (Featherstone et al., 2001) is accompanied by a regular two-arc-minute grid of vertical deviations, which were computed from horizontal quasigeoid gradients in the north-south and east-west directions.

Strictly, the Pizetti vertical deviations should be computed from the horizontal gradients of a geoid, not quasigeoid, model because a quasigeoid does not model equipotential (level) surfaces of the Earth's gravity field (cf. Jekeli, 1999). The differences are correlated with height and Bouguer gravity anomaly (e.g., Rapp, 1997). However, AUSGeoid98 is not strictly a quasigeoid model because some terms were approximated, notably the Molodensky G1 term (e.g., Heiskanen and Moritz, 1967) by the linear Morizian terrain correction (Featherstone et al., 2001). The difference between the geoid and quasigeoid over Australia only reaches 15 cm and varies relatively smoothly (Featherstone and Kirby, 1998). Therefore, this effect on the vertical deviations will be small, probably less than one arc-second (discussed later).

Comparing observed and computed vertical deviations is an independent way of validating the latter (cf. Featherstone, 2006, 2007). In this paper, we use a recently released set of additional vertical deviations over Western Australia to validate the performance of the AUSGeoid98 gravimetric vertical deviations. As pointed out in Featherstone (2007), most of the Western Australian data were omitted in Featherstone (2006). Of the 435 vertical deviations across Western Australia, only 96 were used by Featherstone (2006).

Data, Methods and Results

Observed astronomic-Helmert vertical deviations

A set of 339 vertical deviations has recently been released by Landgate (formerly the Western Australian Department of Land Information). These are from the State's geodetic network at sites that have co-located geodetic and astronomic observations. The astronomic observations were made before 1966 to provide azimuth control (orientation) to the long-line traverses used to establish the old Australian Geodetic Datum 1966 (Bomford, 1967).

Landgate extracted the GDA94 geodetic coordinates of these points, which allowed the computation of the vertical deviations with a fairly good geographical distribution across the State (Figure 2). The formulas for computing vertical deviations from astronomical latitude (Φ) and longitude (Λ) and geodetic latitude (φ) and longitude (λ) are given in, e.g., Featherstone and Rüeger (2000) and Jekeli (1999) so will not be duplicated here. Since the astronomic observations are made at the Earth's surface, this yields Helmert deviations.

Figure 2 near here

The accuracy of these astrogeodetic deviation data is difficult to ascertain (cf. Featherstone, 2006), principally because of errors in timing measurements of the astronomic longitude observations collected over four decades ago. A crude estimate of the standard deviation in each of the north-south (ξ) and east-west (η) vertical deviation components is about one arc-second. Unfortunately, little information remains about the original observations, but most were probably collected with high-precision Kern DKM3 theodolites available before 1966.

Computed AUSGeoid98 vertical deviations

AUSGeoid98 (Featherstone et al., 2001) vertical deviations are provided in the data files released by Geoscience Australia, as well as the primary dataset of quasigeoid heights. An accompanying public-domain Windows™ program, WINTER v5.08, bicubically interpolates these vertical deviations from the regular two arc-minute grid to the points of interest. WINTER and the AUSGeoid98 data files are freely available from Geoscience Australia (http://www.ga.gov.au/geodesy/ausgeoid/).

Figures 3 and 4 near here

Figures 3 and 4 show the vertical deviations computed from AUSGeoid98. Since they are derived from regional gravity data, geological features are evident (cf. Featherstone, 1997), most noticeably the Darling Fault close to the Western Australian south west coast (~116°E in Figure 3), the eastern portion of the Albany-Fraser Orogen (from ~33°S, ~122°E to ~29°S, ~125°E in Figures 3 and 4) and the western MacDonald Ranges (~25°S, ~128°E in Figure 4). Other geological features are visible, but this is not the aim of this article; see Featherstone et al. (2000).

The AUSGeoid98-derived vertical deviations refer to the quasigeoid. Therefore, they are not strictly Pizetti deflections, as discussed earlier, but the difference is probably less than one arc-second. The difference between Helmert and Pizetti deviations is due to the curvature and torsion of the plumbline through the topography, which depends on the height of the observation point (Jekeli, 1999). As discussed in Featherstone (2006), since the topography in Australia is generally benign, the curvature and torsion effect is likely to be less than one arc-second, which is less than the estimated precision of the astronomically observed deviations.

Thus, for the purposes of this evaluation, plumbline curvature and torsion and differences between quasigeoid-derived and geoid-derived Pizetti deviations are neglected. This assumption will be validated later.

Comparisons

The observed astronomic-Helmert deviations were compared with the AUSGeoid98-derived deviations. The astronomic-Helmert deviations were computed from coordinates in Landgate's database according to the formulas in Featherstone and Rüeger (1999). The GDA94 geodetic coordinates of these points were used to bicubically interpolate the AUSGeoid98 vertical deviations using the WINTER v5.08 software.

Table 1 shows descriptive statistics of the differences (astronomic minus gravimetric), both with (Table 1a) and without (Table 1b) 15 outliers as detected by the three-sigma test assuming a normal distribution of the deviation differences. Figures 5 and 6 show histograms of the differences (including outliers), which are near-normally distributed, thus justifying the use of the Z-score for outlier rejection. If one deviation component was determined as an outlier, then both components were rejected. However, around 10 of the outliers were in both deviation components.

Figures 5 and 6 near here

Table 1 near here

Discussion

The results in Tables 1a and 1b largely mirror those in Featherstone (2006); after the removal of outliers (based on the three-sigma criterion), the precision of AUSGeoid98-derived vertical deviations in Western Australia is roughly one arc-second. Indeed, this is commensurate with the estimated precision of the astronomically determined deviations. From the error analysis in Featherstone and Rüeger (2000), this is adequate for the reduction and post-processing of current terrestrial-geodetic survey observations. As such, geodetic surveyors in Western Australia are well-served by AUSGeoid98.

The outliers, acknowledging the simplicity of the three-sigma test, should not be treated blindly. It is conceivable that the curvature and torsion of the plumbline is larger than anticipated. For instance, in areas of complex geology or high elevation, it is conceivable that mass-density contrasts will cause large curvature and torsion in the plumbline or steep gravity field gradients that are not be modelled by AUSGeoid98. A key example is in the proximity of the Darling Fault, where low-density sediments juxtapose the high-density Yilgarn Craton.

In order to test this and to ascertain any effect of using quasigeoid, rather than geoid, gradients to approximate Pizetti vertical deviations, the north-south and east-west deviation differences (astronomic minus gravimetric) are plotted as a function of Australian Height Datum (AHD) height of the astrogeodetic stations (Figures 6 and 7). Unweighted linear regression, coupled with the correlation coefficient (R-squared statistic), show that the differences are uncorrelated with AHD height. This observation justifies the earlier assumptions that the curvature and torsion of the plumbline are negligible in Western Australia and the use of the quasigeoid as opposed to geoid to compute Pizetti vertical deflections is acceptable, certainly in relation to the expected one arc-second precision of the astrogeodetic vertical deflections.

Figures 7 and 8 near here

Given that the assumptions about the plumbline and quasigeoid have no appreciable effect, timing errors in the original (before 1966) astronomical longitudinal observations are a more plausible cause of the observed differences. This is implied in Table 1b, where –

even after outlier detection – the longitudinal (east-west) deviation discrepancies are larger than the latitudinal (north-south) differences.

Summary and Conclusion

In this short paper, we have used a recently released set of historic (pre-1966) astronomically observed vertical deviations to independently verify the AUSGeoid98-computed vertical deviations across Western Australia. Our results agree with earlier studies (e.g., Jekeli, 1999, Featherstone, 2006), showing that vertical deviations are a useful independent validation of a gravimetric quasigeoid model, but the vintage of the astrogeodetic data, particularly in longitude/time, is a limiting factor.

We have shown that the AUSGeoid98-computed vertical deviations are generally of sufficient precision (one arc-second standard deviation) to support the reduction of terrestrial-geodetic survey data in Western Australia. A new Australia-wide gravimetric quasigeoid model is currently in preparation, awaiting the release of new datasets from dedicated satellite gravimetry, which should improve the situation yet further.

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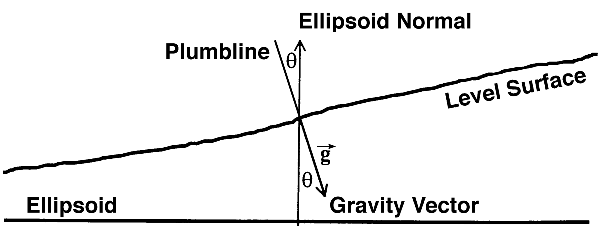

Figure 1. A generalised schematic of the deviation of the vertical, where the plumbline is perpendicular to the level surface, thus the deviation is a measure of the slope of the level surface with respect to the ellipsoid

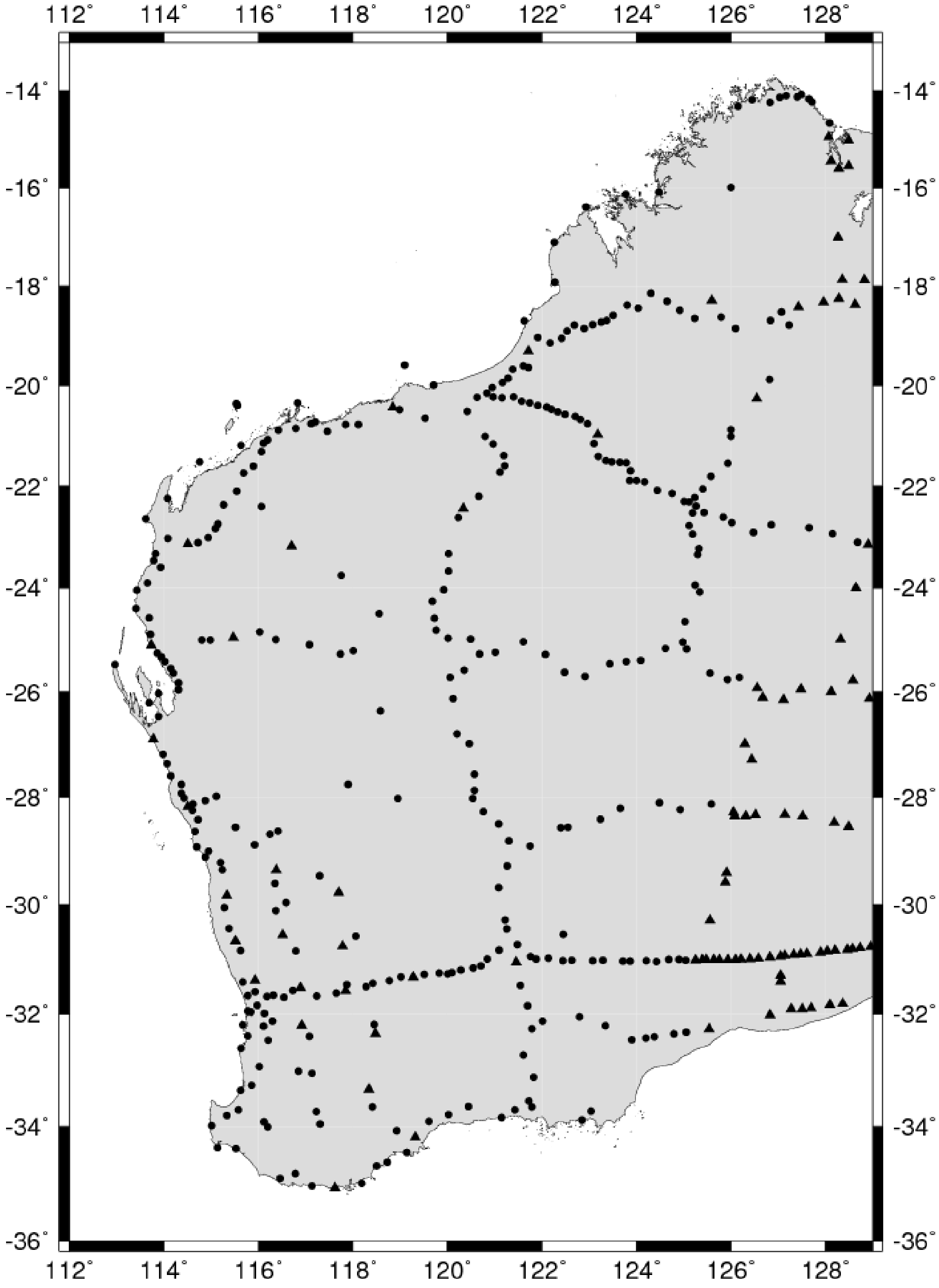

Figure 2. Locations of the observed astronomic-Helmert vertical deviations across Western Australia (Mercator projection). Triangles denote sites used by Featherstone (2006); circles denote the new sites.

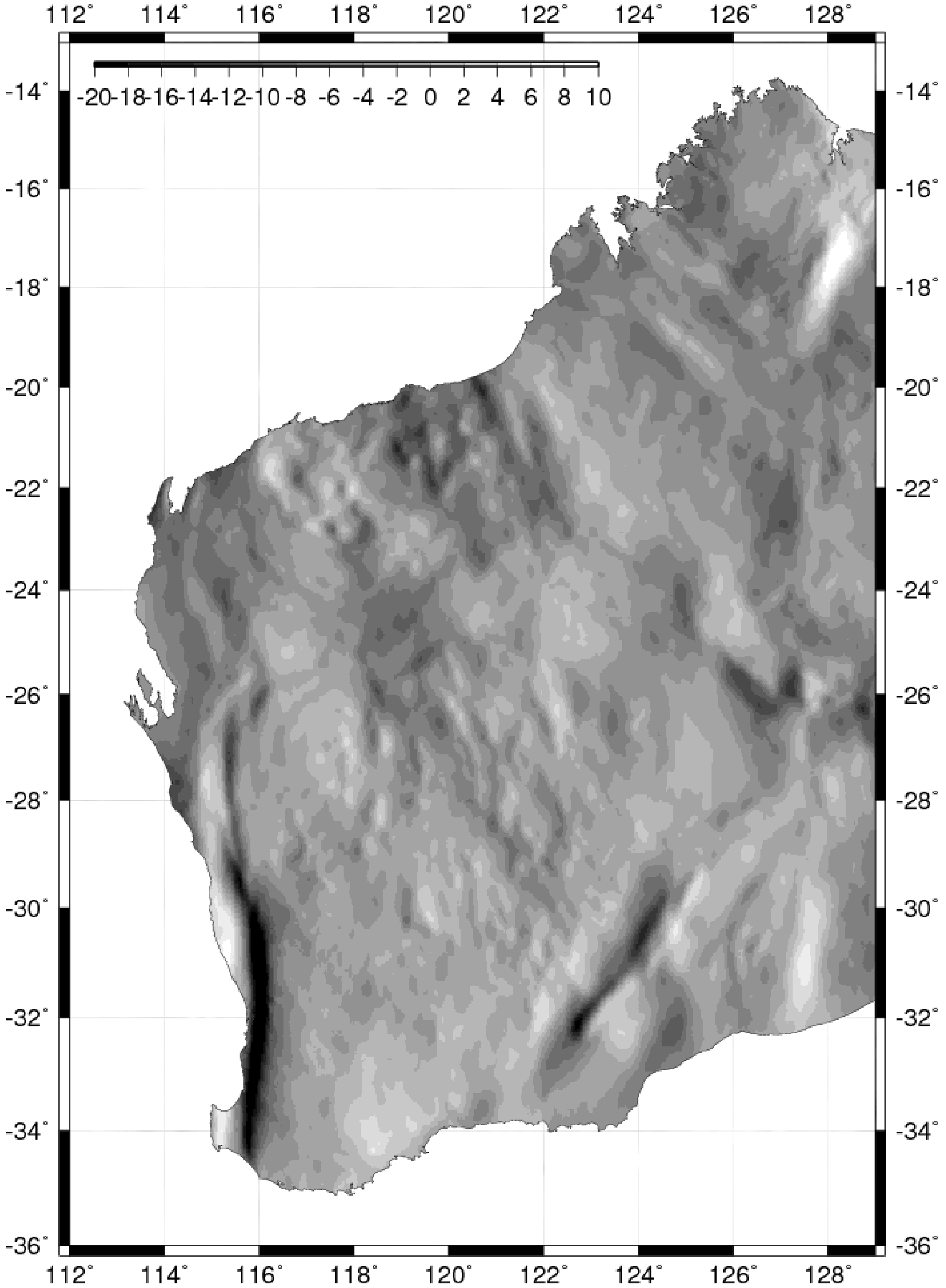

Figure 3. Computed AUSGeoid98 east-west vertical deviations across Western Australia (Mercator projection; Units in arc-seconds)

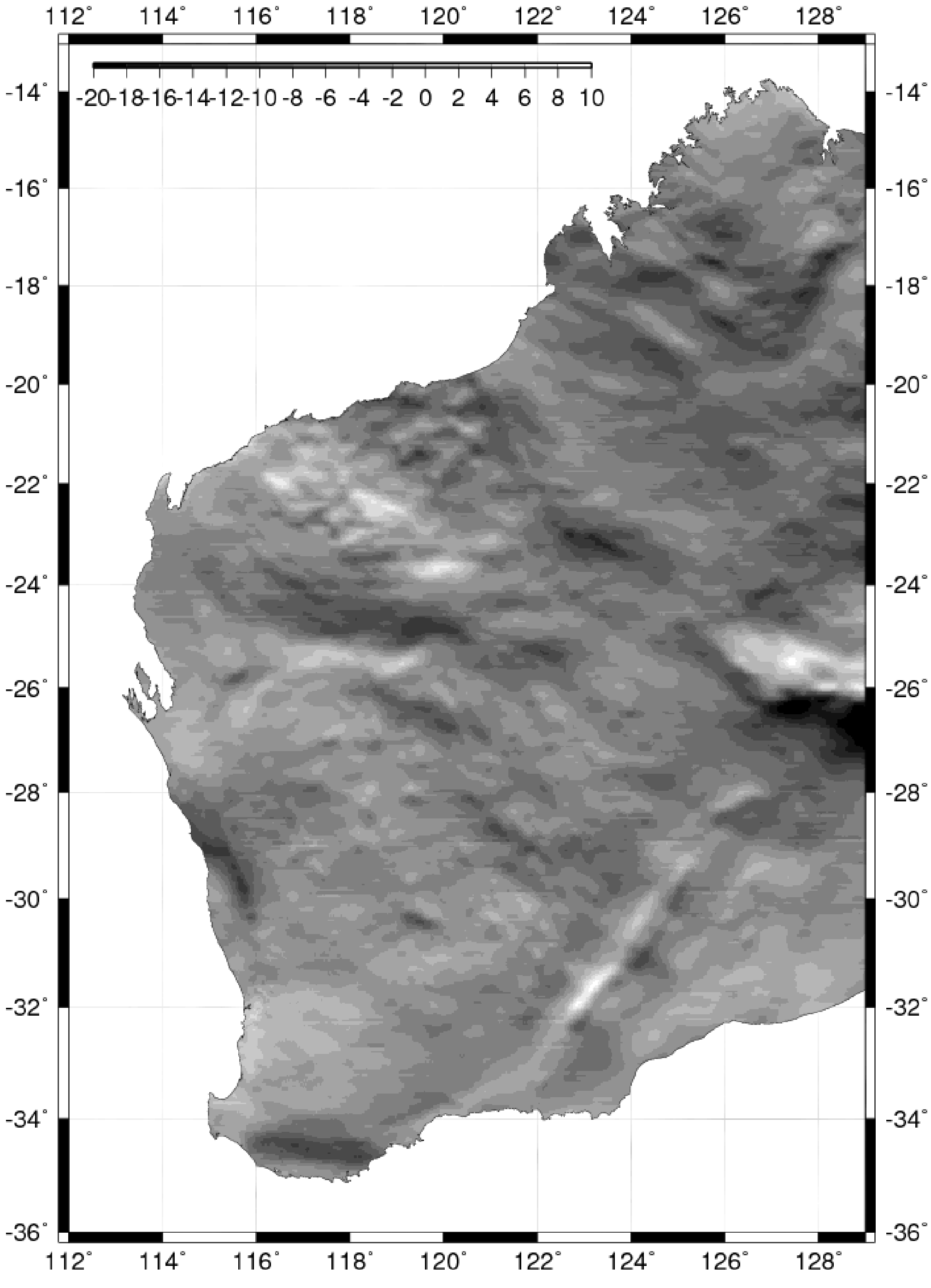

Figure 4. Computed AUSGeoid98 north-south vertical deviations across Western Australia (Mercator projection; Units in arc-seconds)

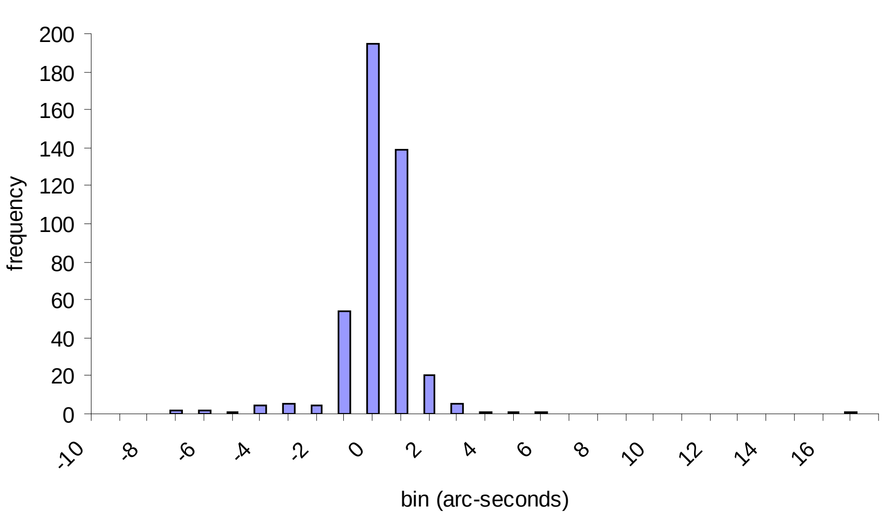

Figure 5: Histogram of the differences between astronomic-Helmert deviations and AUSGeoid98-derived deviations in the north-south component (435 points). Units in arc-seconds

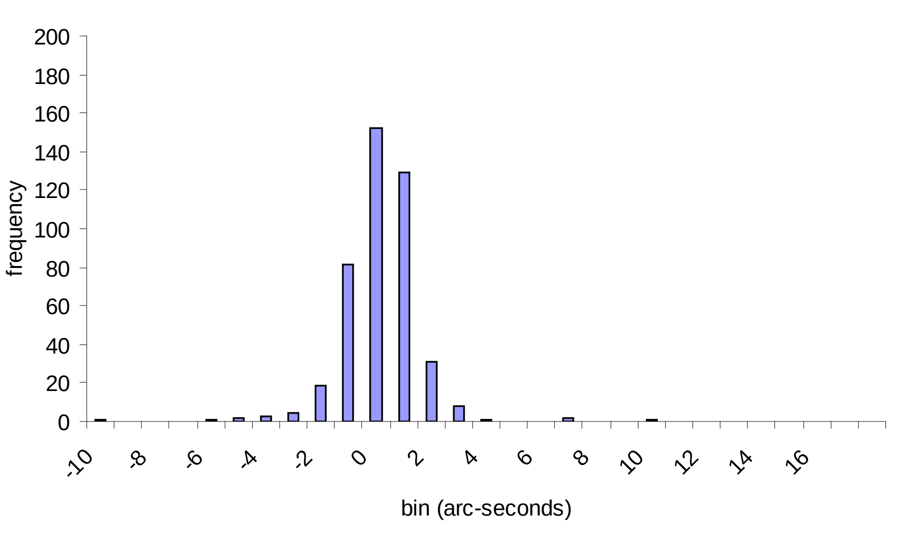

Figure 6: Histogram of the differences between astronomic-Helmert deviations and AUSGeoid98-derived deviations in the east-west component (435 points). Units in arc-seconds

	North-south (ξ)	east-west (η)
Maximum	16.9	9.1
Minimum	-7.8	-10.9
Mean	-0.3	-0.3
STD	1.5	1.5

Table 1a. Descriptive statistics of the differences between astronomic-Helmert deviations and AUSGeoid98-derived deviations (435 points, including 15 outliers). Units in arc-seconds

	North-south (ξ)	east-west (η)
Maximum	3.0	3.3
Minimum	-4.7	-4.6
Mean	-0.2	-0.3
STD	0.9	1.1

Table 1b. Descriptive statistics of the differences between astronomic-Helmert deviations and AUSGeoid98-derived deviations (420 points, excluding 15 outliers). Units in arc-seconds

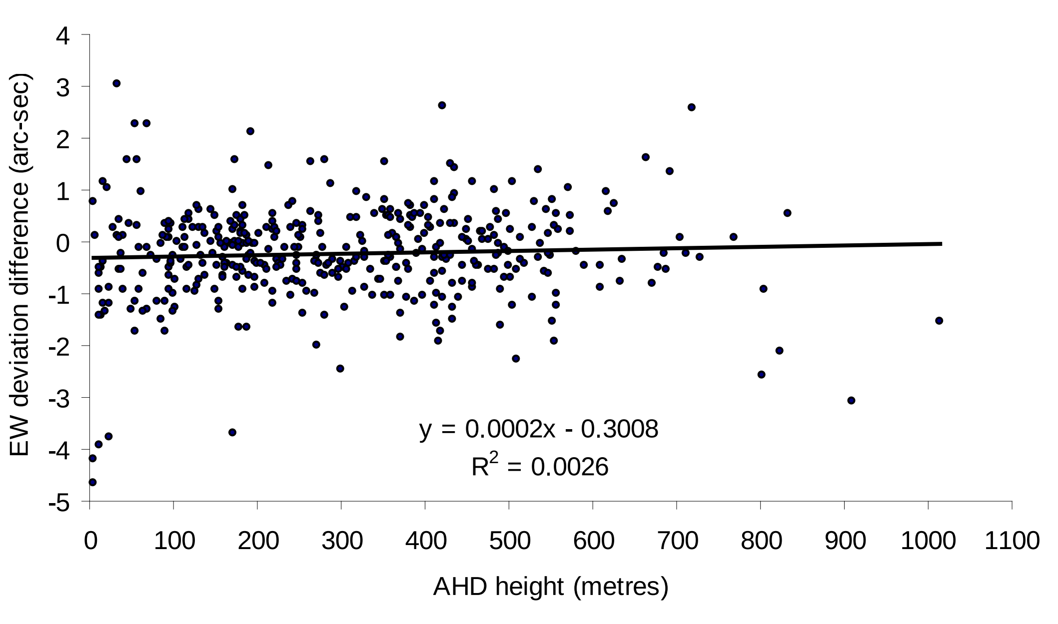

Figure 7: Differences between astronomic-Helmert deviations and AUSGeoid98-derived deviations in the east-west component as a function of AHD height (420 points).

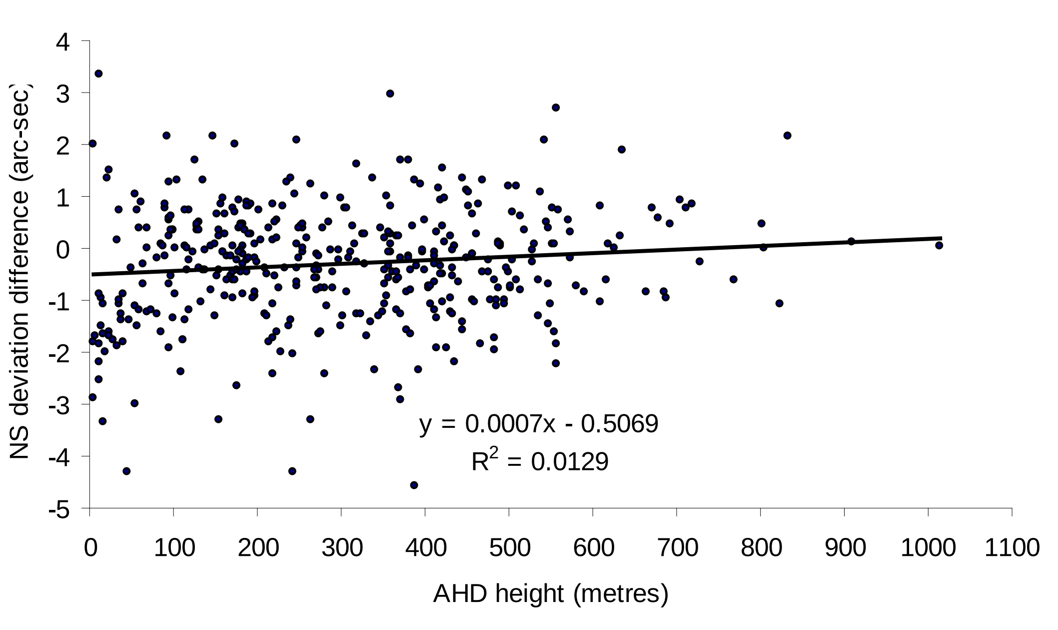

Figure 8: Differences between astronomic-Helmert deviations and AUSGeoid98-derived deviations in the north-south component as a function of AHD height (420 points).