Simplified MMSE Precoding Design in Interference Two-Way MIMO Relay Systems

Khoa Xuan Nguyen, *Student Member, IEEE*, Yue Rong, *Senior Member, IEEE*, and Sven Nordholm, *Senior Member, IEEE*

***Abstract*—We investigate the transceiver design for interference two-way amplify-and-forward multiple-input multiple-output relay communication systems. A novel algorithm with a closed-form solution is developed to optimize the relay precoding matrix based on its optimal structure and a modified transmission power constraint at the relay node. An iterative algorithm is proposed to minimize the sum mean-squared error of the signal waveform estimation. Simulation results demonstrate that the proposed algorithm achieves a better performance-complexity tradeoff compared with existing techniques.**

***Index Terms*—Interference channel, MIMO relay, MMSE.**

I. Introduction

Two-way multiple-input multiple-output (MIMO) relay communications have attracted much research interest recently [1]. Thanks to the technique of analog network coding [2], two-way information exchange can be achieved in two time slots with half-duplex relay node(s). For single user two-way amplify-and-forward (AF) MIMO relay systems with a single relay node, the optimal source and relay matrices have been developed in [3] to maximize the achievable weighted sum rate. An iterative source and relay matrices design algorithm has been proposed in [4] by solving convex quadratically constrained quadratic program (QCQP) problems. A unified framework has been developed in [5] to optimize the source and relay matrices for a broad class of frequently used objective functions. The impact of mean-squared error (MSE) constraints on two-way MIMO relay systems has been studied in [6]. For a single-user two-way MIMO relay system with multiple parallel relay nodes, a gradient descent based transceiver design algorithm has been proposed in [7].

Zero-forcing (ZF) and minimum MSE (MMSE) based transceiver design algorithms have been developed in [8] for interference two-way MIMO relay systems, where each user transmits a single data stream. An MMSE based iterative transceiver design algorithm has been proposed in [9] where each user may transmit multiple data streams. In [10], a projection based separation of multiple operators (ProBaSeMo) relay transmit strategy has been developed which provides a significant gain in terms of the sum rate. In [11], a general cellular two-way relay network has been investigated which includes many two-way relay networks as special cases.

In this letter, we investigate the transceiver design for interference two-way MIMO relay systems where multiple two-way links communicate simultaneously with the aid of a single relay node. Due to its attractive features [12], [13], the MMSE is chosen as the design criterion. We propose an iterative algorithm to optimize the source, relay, and receiver matrices to suppress the interference and minimize the sum MSE (SMSE) of the signal waveform estimation at the receivers, subjecting to transmission power constraints at the source and relay nodes.

The contributions of this letter compared with existing works such as [8]-[11] are: (1) We derive the optimal structure of the relay precoding matrix. (2) By modifying the power constraint at the relay node, we propose a novel relay precoding matrix optimization algorithm with a closed-form solution. (3) The proposed iterative transceiver design algorithm provides a better performance-complexity tradeoff which is very useful for practical interference two-way MIMO relay communication systems.

II. System Model And Problem Formulation

We study an interference two-way MIMO relay communication system where K user pairs communicate simultaneously with the aid of a single relay node as shown in Fig. 1. For simplicity, the direct links between user pairs are ignored as they undergo much larger path attenuation compared with the links via the relay node. The kth node at site 1 and site 2 is equipped with $N_{k,1}$ and $N_{k,2}$ antennas, respectively, and the number of antennas at the relay node is N_r.

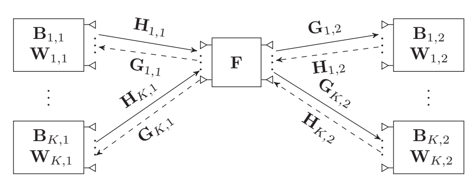

Fig. 1. Block diagram of an interference two-way MIMO relay system.

We assume that the relay node works in the practical half-duplex mode so the communication between the user pairs is completed in two time slots. In the first time slot, the kth nodes at site $i = 1, 2$, encodes the $d \times 1$ information-carrying symbol vector $\mathbf{s}_{k,i}$ with the $N_{k,i} \times d$ source precoding matrix $\mathbf{B}_{k,i}$ before transmitting the $N_{k,i} \times 1$ precoded signal vector

$$\mathbf{x}_{k,i} = \mathbf{B}_{k,i}\mathbf{s}_{k,i}, \quad k = 1, \cdots, K, \quad i = 1, 2 \tag{1}$$

to the relay node. The received signal vector at the relay node is given by

$$\mathbf{y}_r = \sum_{k=1}^{K}\sum_{i=1}^{2}\mathbf{H}_{k,i}\mathbf{x}_{k,i} + \mathbf{n}_r \tag{2}$$

where $\mathbf{H}_{k,i}$ is the $N_r \times N_{k,i}$ up-link MIMO channel matrix between the kth node at site i and the relay node, $\mathbf{n}_r$ is the $N_r \times 1$ additive white Gaussian noise (AWGN) vector at the relay node with zero mean and covariance matrix $E[\mathbf{n}_r\mathbf{n}_r^H] = \sigma_r^2\mathbf{I}_{N_r}$. Here $(\cdot)^H$ denotes matrix Hermitian transpose, $E[\cdot]$

Manuscript received September 21, 2015. The associate editor coordinating the review of this letter and approving it for publication was L. Sanguinetti.

This work was supported under the Australian Research Council's Discovery Projects funding scheme (DP110100736, DP140102131).

The authors are with the Department of Electrical and Computer Engineering, Curtin University of Technology, Bentley, WA 6102, Australia, e-mails: khoa.x.nguyen@ieee.org, y.rong@curtin.edu.au, s.nordholm@curtin.edu.au.

stands for the statistical expectation, and $\mathbf{I}_n$ denotes the $n \times n$ identity matrix.

In the second time slot, the relay node amplifies the received signal vector with the $N_r \times N_r$ precoding matrix $\mathbf{F}$ as

$$\mathbf{x}_r = \mathbf{F}\mathbf{y}_r. \quad (3)$$

The precoded signal vector $\mathbf{x}_r$ is broadcast back to the nodes at site $i = 1, 2$. The received signal vector at the kth node of site i is given by

$$\mathbf{y}_{k,i} = \mathbf{G}_{k,i}\mathbf{F}\mathbf{y}_r + \mathbf{n}_{k,i}, \quad k = 1, \cdots, K, \quad i = 1, 2 \quad (4)$$

where $\mathbf{G}_{k,i}$ is the $N_{k,i} \times N_r$ down-link MIMO channel matrix between the relay node and the kth node at site i, $\mathbf{n}_{k,i}$ is the $N_{k,i} \times 1$ AWGN vector at the kth node at site i with zero mean and covariance matrix $E[\mathbf{n}_{k,i}\mathbf{n}_{k,i}^H] = \sigma_{k,i}^2 \mathbf{I}_{N_{k,i}}$.

Due to their simplicity, linear receivers are used to retrieve the transmitted signals. The estimated signal vector at the kth node of site i can be written as

$$\bar{\mathbf{s}}_{k,i} = \mathbf{W}_{k,i}^H \mathbf{y}_{k,i}, \quad k = 1, \cdots, K, \quad i = 1, 2 \quad (5)$$

where $\mathbf{W}_{k,i}$ is an $N_{k,i} \times d$ receiver matrix at the kth node of site i. As each node has the knowledge of its own transmitted signal vector, the self-interference (SI) $\mathbf{W}_{k,i}^H \mathbf{G}_{k,i}\mathbf{F}\mathbf{H}_{k,i}\mathbf{B}_{k,i}\mathbf{s}_{k,i}$ in (5) can be easily canceled. From (1)-(5), the estimated signals after removing the SI become

$$\hat{\mathbf{s}}_{k,i} = \underbrace{\mathbf{W}_{k,i}^H \mathbf{G}_{k,i}\mathbf{F}\mathbf{H}_{k,\bar{i}}\mathbf{B}_{k,\bar{i}}\mathbf{s}_{k,\bar{i}}}_{\text{desired signal}} + \underbrace{\mathbf{W}_{k,i}^H \bar{\mathbf{n}}_{k,i}}_{\text{noise}}$$
$$\underbrace{+\mathbf{W}_{k,i}^H \mathbf{G}_{k,i}\mathbf{F}\sum_{m \neq k}^{K}(\mathbf{H}_{m,\bar{i}}\mathbf{B}_{m,\bar{i}}\mathbf{s}_{m,\bar{i}} + \mathbf{H}_{m,i}\mathbf{B}_{m,i}\mathbf{s}_{m,i})}_{\text{interference}} \quad (6)$$

where $\bar{i} = 1$ for $i = 2$, $\bar{i} = 2$ for $i = 1$, and $\bar{\mathbf{n}}_{k,i} = \mathbf{G}_{k,i}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{k,i}$ is the total noise at the kth node of the ith site.

The signal vector transmitted from each source node and the relay node must satisfy the transmission power constraints

$$tr\big(\mathbf{F}E[\mathbf{y}_r\mathbf{y}_r^H]\mathbf{F}^H\big) \leq P_r \quad (7)$$
$$tr\big(\mathbf{B}_{k,i}E[\mathbf{s}_{k,i}\mathbf{s}_{k,i}^H]\mathbf{B}_{k,i}^H\big) \leq P_{k,i}, \ k = 1, \cdots, K, \ i = 1, 2 \quad (8)$$

where $tr(\cdot)$ stands for matrix trace, $P_{k,i}$ and P_r denote the power budget at the kth node of site i and the relay node, respectively, $E[\mathbf{s}_{k,i}\mathbf{s}_{k,i}^H] = \mathbf{I}_d$ is the covariance matrix of the information-carrying symbol vectors, and $E[\mathbf{y}_r\mathbf{y}_r^H] = \sum_{i=1}^2 \sum_{k=1}^K \mathbf{H}_{k,i}\mathbf{B}_{k,i}\mathbf{B}_{k,i}^H\mathbf{H}_{k,i}^H + \sigma_r^2 \mathbf{I}_{N_r}$ is the covariance matrix of the received signal vector at the relay node.

The aim of this work is to optimize the source precoding matrices $\{\mathbf{B}_{k,i}\} = \{\mathbf{B}_{k,i}, k = 1, \cdots, K, i = 1, 2\}$, the relay precoding matrix $\mathbf{F}$, and the receiver matrices $\{\mathbf{W}_{k,i}\} = \{\mathbf{W}_{k,i}, \ k = 1, \cdots, K, i = 1, 2\}$, to minimize the SMSE of the signal waveform estimation at the receivers under transmission power constraints at the source and relay nodes. From (6), the MSE of the signal waveform estimation at the kth node of site i can be calculated for $k = 1, \cdots, K$, $i = 1, 2$ as

$$\begin{aligned}\mathrm{MSE}_{k,i} &= tr(E[(\hat{\mathbf{s}}_{k,i} - \mathbf{s}_{k,\bar{i}})(\hat{\mathbf{s}}_{k,i} - \mathbf{s}_{k,\bar{i}})^H]) \\ &= tr((\mathbf{W}_{k,i}^H \mathbf{L}_{k,i} - \mathbf{I}_d)(\mathbf{W}_{k,i}^H \mathbf{L}_{k,i} - \mathbf{I}_d)^H \\ &\quad + \mathbf{W}_{k,i}^H(\mathbf{N}_{k,i} + \mathbf{\Xi}_{k,i})\mathbf{W}_{k,i})\end{aligned} \quad (9)$$

where $\mathbf{L}_{k,i}$ is the equivalent MIMO channel matrix of the kth site $\bar{i}$-site i user pair, $\mathbf{N}_{k,i} = E[\bar{\mathbf{n}}_{k,i}\bar{\mathbf{n}}_{k,i}^H]$ is the covariance matrix of the equivalent noise, and $\mathbf{\Xi}_{k,i}$ is the covariance matrix of interference at the kth node of site i. They are given for $k = 1, \cdots, K$, $i = 1, 2$ as

$$\begin{aligned}\mathbf{L}_{k,i} &= \mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}} \\ \mathbf{N}_{k,i} &= \sigma_r^2 \mathbf{G}_{k,i}\mathbf{F}\mathbf{F}^H\mathbf{G}_{k,i}^H + \sigma_{k,i}^2 \mathbf{I}_{N_{k,i}} \\ \mathbf{\Xi}_{k,i} &= \mathbf{G}_{k,i}\mathbf{F}\sum_{j=1}^{2}\sum_{m \neq k}^{K}\bar{\mathbf{H}}_{m,j}\bar{\mathbf{H}}_{m,j}^H\mathbf{F}^H\mathbf{G}_{k,i}^H\end{aligned}$$

where $\bar{\mathbf{H}}_{k,i} = \mathbf{H}_{k,i}\mathbf{B}_{k,i}$ is the equivalent MIMO channel matrix between the kth source node of site i and the relay node. From (7)-(9), the optimal source, relay, and receiver matrices design problem can be written as

$$\min_{\{\mathbf{W}_{k,i}\},\{\mathbf{B}_{k,i}\},\mathbf{F}} \ \sum_{i=1}^{2}\sum_{k=1}^{K}\mathrm{MSE}_{k,i} \quad (10)$$
$$s.t. \ tr\big(\mathbf{B}_{k,i}\mathbf{B}_{k,i}^H\big) \leq P_{k,i}, k = 1, \cdots, K, i = 1, 2 \quad (11)$$
$$tr\big(\mathbf{F}E[\mathbf{y}_r\mathbf{y}_r^H]\mathbf{F}^H\big) \leq P_r. \quad (12)$$

III. Proposed Algorithm

The problem (10)-(12) is nonconvex with matrix variables, and a globally optimal solution is intractable to obtain. We propose an iterative transceiver design algorithm. In each iteration, we first optimize $\{\mathbf{W}_{k,i}\}$ based on $\{\mathbf{B}_{k,i}\}$ and $\mathbf{F}$ from the previous iteration. Then using $\{\mathbf{B}_{k,i}\}$ from the previous iteration, we optimize $\mathbf{F}$. We derive a closed-form solution of $\mathbf{F}$ based on its optimal structure and the modified power constraint at the relay node. Finally, we optimize $\{\mathbf{B}_{k,i}\}$ based on $\{\mathbf{W}_{k,i}\}$ and $\mathbf{F}$ obtained from this iteration.

As the power constraints (7) and (8) are independent of $\mathbf{W}_{k,i}$, with given $\{\mathbf{B}_{k,i}\}$ and $\mathbf{F}$, the optimal $\mathbf{W}_{k,i}$ which minimizes $\mathrm{MSE}_{k,i}$ in (9) is the MMSE receiver [14]

$$\mathbf{W}_{k,i} = \big(\mathbf{L}_{k,i}\mathbf{L}_{k,i}^H + \mathbf{N}_{k,i} + \mathbf{\Xi}_{k,i}\big)^{-1}\mathbf{L}_{k,i} \quad (13)$$

where $(\cdot)^{-1}$ denotes matrix inversion. Substituting (13) back into (9), $\mathrm{SMSE} = \sum_{i=1}^2 \sum_{k=1}^K \mathrm{MSE}_{k,i}$ can be rewritten as a function of $\mathbf{F}$ as

$$\begin{aligned}\mathrm{SMSE} &= \sum_{i=1}^{2}\sum_{k=1}^{K} tr\big(\mathbf{I}_d - \mathbf{L}_{k,i}^H(\mathbf{L}_{k,i}\mathbf{L}_{k,i}^H + \mathbf{N}_{k,i} + \mathbf{\Xi}_{k,i})^{-1}\mathbf{L}_{k,i}\big) \\ &= \sum_{i=1}^{2}\sum_{k=1}^{K} tr\big(\mathbf{I}_d - \bar{\mathbf{H}}_{k,\bar{i}}^H\mathbf{F}^H\mathbf{G}_{k,i}^H(\mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}}\bar{\mathbf{H}}_{k,\bar{i}}^H \\ &\quad \times \mathbf{F}^H\mathbf{G}_{k,i}^H + \mathbf{N}_{k,i} + \mathbf{\Xi}_{k,i})^{-1}\mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}}\big).\end{aligned} \quad (14)$$

Let us denote

$$\mathbf{H} = [\bar{\mathbf{H}}_{1,2}, \cdots, \bar{\mathbf{H}}_{K,2}, \bar{\mathbf{H}}_{1,1}, \cdots, \bar{\mathbf{H}}_{K,1}] = \mathbf{U}_h\mathbf{\Lambda}_h\mathbf{V}_h^H \quad (15)$$
$$\mathbf{G} = [\mathbf{G}_{1,1}^T, \cdots, \mathbf{G}_{K,1}^T, \mathbf{G}_{1,2}^T, \cdots, \mathbf{G}_{K,2}^T]^T = \mathbf{U}_g\mathbf{\Lambda}_g\mathbf{V}_g^H \quad (16)$$

as the singular value decompositions (SVDs) of the equivalent first-hop channel $\mathbf{H}$ and the equivalent second-hop channel $\mathbf{G}$. The dimensions of $\mathbf{U}_h$, $\mathbf{\Lambda}_h$, $\mathbf{V}_h$ are $N_r \times L_1$, $L_1 \times L_1$, $2Kd \times L_1$, respectively and the dimension of $\mathbf{U}_g$, $\mathbf{\Lambda}_g$, $\mathbf{V}_g$ are $N_d \times L_2$, $L_2 \times L_2$, $N_r \times L_2$, respectively, where $N_d = \sum_{i=1}^{2}\sum_{k=1}^{K} N_{k,i}$, $L_1 = \min(2Kd, N_r)$ and $L_2 = \min(N_d, N_r)$. Based on (15) and (16), we have

$$\mathbf{H}_{k,i}\mathbf{B}_{k,i} = \mathbf{U}_h\mathbf{\Lambda}_h\mathbf{V}_{hk,i}^H, \quad \mathbf{G}_{k,i} = \mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{V}_g^H \tag{17}$$

where $\mathbf{V}_{hk,i}$ and $\mathbf{U}_{gk,i}$ have dimensions of $d \times L_1$, $N_{k,i} \times L_2$ such that $\mathbf{V}_h = [\mathbf{V}_{h1,2}^T, \cdots, \mathbf{V}_{hK,2}^T, \mathbf{V}_{h1,1}^T, \cdots, \mathbf{V}_{hK,1}^T]^T$, $\mathbf{U}_g = [\mathbf{U}_{g1,1}^T, \cdots, \mathbf{U}_{gK,1}^T, \mathbf{U}_{g1,2}^T, \cdots, \mathbf{U}_{gK,2}^T]^T$. Here $(\cdot)^T$ denotes matrix transpose.

It can be proven similar to [15] that the optimal structure of the relay precoding matrix is

$$\mathbf{F} = \mathbf{V}_g\mathbf{A}\mathbf{U}_h^H \tag{18}$$

where $\mathbf{A}$ is an $L_2 \times L_1$ matrix that remains to be optimized. Let us introduce

$$\mathbf{\Lambda}_g\mathbf{A} = \mathbf{U}_g^H\mathbf{C} = \sum_{i=1}^{2}\sum_{k=1}^{K}\mathbf{U}_{gk,i}^H\mathbf{C}_{k,i} \tag{19}$$

where $\mathbf{C} = [\mathbf{C}_{1,1}^T, \cdots, \mathbf{C}_{K,1}^T, \mathbf{C}_{1,2}^T, \cdots, \mathbf{C}_{K,2}^T]^T$ and $\mathbf{C}_{k,i}$ is an $N_{k,i} \times L_1$ matrix. Since $\mathbf{U}_g^H\mathbf{U}_g = \mathbf{I}_{L_2}$, for any $\mathbf{A}$, we have $\mathbf{C} = \mathbf{U}_g\mathbf{\Lambda}_g\mathbf{A}$. Thus, instead of optimizing $\mathbf{A}$, we can optimize $\mathbf{C}$. Substituting (19) back into (18), we have

$$\mathbf{F} = \mathbf{V}_g\mathbf{\Lambda}_g^{-1}\mathbf{U}_g^H\mathbf{C}\mathbf{U}_h^H. \tag{20}$$

By substituting (17) and (20) back into (14), we have $\mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}} = \mathbf{C}_{k,i}\mathbf{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^H$, $\mathbf{N}_{k,i} = \sigma_r^2\mathbf{C}_{k,i}\mathbf{C}_{k,i}^H + \sigma_{k,i}^2\mathbf{I}_{N_{k,i}}$. Thus, we obtain the SMSE as a function of $\mathbf{C}_{k,i}$ as

$$\text{SMSE} = \sum_{i=1}^{2}\sum_{k=1}^{K} q_{k,i}(\mathbf{C}_{k,i}) \tag{21}$$

where

$$\begin{aligned} q_{k,i}(\mathbf{C}_{k,i}) = {} & tr(\mathbf{I}_d - \mathbf{V}_{hk,\bar{i}}\mathbf{\Lambda}_h\mathbf{C}_{k,i}^H(\mathbf{C}_{k,i}\mathbf{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^H\mathbf{V}_{hk,\bar{i}}\mathbf{\Lambda}_h\mathbf{C}_{k,i}^H \\ & + \sigma_r^2\mathbf{C}_{k,i}\mathbf{C}_{k,i}^H + \mathbf{C}_{k,i}\sum_{j=1}^{2}\sum_{m\neq k}^{K}\mathbf{\Lambda}_h\mathbf{V}_{hm,j}^H\mathbf{V}_{hm,j}\mathbf{\Lambda}_h\mathbf{C}_{k,i}^H \\ & + \sigma_{k,i}^2\mathbf{I}_{N_{k,i}})^{-1}\mathbf{C}_{k,i}\mathbf{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^H). \end{aligned} \tag{22}$$

It can be seen from (21) and (22) that the MSE of the signal waveform estimation at the kth node of site i is a function of $\mathbf{C}_{k,i}$ only. In other words, the objective function (21) is decomposed in terms of the optimization variables.

From (19), the transmission power constraint at the relay node in (7) can be rewritten as

$$tr(\mathbf{F}E[\mathbf{y}_r\mathbf{y}_r^H]\mathbf{F}^H) = tr(\mathbf{C}^H\mathbf{\Pi}\mathbf{C}\mathbf{\Psi}) \leq P_r \tag{23}$$

where $\mathbf{\Pi} = \mathbf{U}_g\mathbf{\Lambda}_g^{-2}\mathbf{U}_g^H$ and $\mathbf{\Psi} = \mathbf{\Lambda}_h^2 + \sigma_r^2\mathbf{I}_{L_1}$. It can be seen from (23) that $\mathbf{C}_{k,i}, k = 1, \cdots, K, i = 1, 2$, are coupled through the power constraint. We propose to modify (23) by applying the inequality of $tr(\mathbf{AB}) \leq tr(\mathbf{A})tr(\mathbf{B})$. The transmission power at the relay node becomes

$$tr(\mathbf{C}^H\mathbf{\Pi}\mathbf{C}\mathbf{\Psi}) \leq tr(\mathbf{C}\mathbf{\Psi}\mathbf{C}^H)tr(\mathbf{\Pi}). \tag{24}$$

Then the power constraint in (23) is modified to be

$$\sum_{j=1}^{2}\sum_{k=1}^{K} tr(\mathbf{C}_{k,i}\mathbf{\Psi}\mathbf{C}_{k,i}^H) \leq P_r/tr(\mathbf{\Lambda}_g^{-2}). \tag{25}$$

In fact, (25) imposes a stricter transmission power constraint at the relay node, i.e., if (25) holds, the original power constraint (23) is also satisfied.

Based on (21) and (25), the modified relay precoding matrix optimization problem can be written as

$$\min_{\mathbf{C}} \sum_{i=1}^{2}\sum_{k=1}^{K} q_{k,i}(\mathbf{C}_{k,i}) \tag{26}$$

$$s.t. \sum_{j=1}^{2}\sum_{k=1}^{K} tr(\mathbf{C}_{k,i}\mathbf{\Psi}\mathbf{C}_{k,i}^H) \leq \bar{P}_r \tag{27}$$

where $\bar{P}_r = P_r/tr(\mathbf{\Lambda}_g^{-2})$ is the modified power budget at the relay node. We can see from (26) and (27) that the relay precoding matrix optimization problem can be decomposed into $2K$ subproblems where the (k,i)-th subproblem, $k = 1, \cdots, K$, $i = 1, 2$, is to optimize $\mathbf{C}_{k,i}$ as

$$\min_{\mathbf{C}_{k,i}} q_{k,i}(\mathbf{C}_{k,i}) \tag{28}$$

$$s.t. \; tr(\mathbf{C}_{k,i}\mathbf{\Psi}\mathbf{C}_{k,i}^H) \leq P_{rk,i} \tag{29}$$

where $P_{rk,i} \geq 0$ and $\sum_{i=1}^{2}\sum_{k=1}^{K} P_{rk,i} = \bar{P}_r$.

Let us introduce the following matrices for $k = 1, \cdots, K$ and $i = 1, 2$

$$\mathbf{J}_{rk} = \sum_{j=1}^{2}\sum_{m\neq k}^{K}\mathbf{\Lambda}_h\mathbf{V}_{hm,j}^H\mathbf{V}_{hm,j}\mathbf{\Lambda}_h + \sigma_r^2\mathbf{I}_{L_1} \tag{30}$$

$$\mathbf{X}_{k,i} = \mathbf{J}_{rk}^{-\frac{1}{2}}\mathbf{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^H, \quad \mathbf{Y}_{k,i} = \mathbf{C}_{k,i}\mathbf{J}_{rk}^{\frac{1}{2}}. \tag{31}$$

Then $q_{k,i}(\mathbf{C}_{k,i})$ in (22) becomes

$$\begin{aligned} f_{k,i}(\mathbf{Y}_{k,i}) = {} & tr(\mathbf{I}_d - \mathbf{X}_{k,i}^H\mathbf{Y}_{k,i}^H(\mathbf{Y}_{k,i}\mathbf{X}_{k,i}\mathbf{X}_{k,i}^H\mathbf{Y}_{k,i}^H \\ & + \mathbf{Y}_{k,i}\mathbf{Y}_{k,i}^H + \sigma_{k,i}^2\mathbf{I}_{N_{k,i}})^{-1}\mathbf{Y}_{k,i}\mathbf{X}_{k,i}). \end{aligned} \tag{32}$$

Using (32), the problem (28)-(29) can be rewritten as

$$\min_{\mathbf{Y}_{k,i}} f_{k,i}(\mathbf{Y}_{k,i}) \tag{33}$$

$$s.t. \; tr(\mathbf{Y}_{k,i}(\mathbf{X}_{k,i}\mathbf{X}_{k,i}^H + \mathbf{I}_{L_1})\mathbf{Y}_{k,i}^H) \leq P_{rk,i} \tag{34}$$

where (34) is obtained by substituting (30)-(31) back into (29). Interestingly, the problem (33)-(34) is the MMSE-based relay precoding matrix optimization problem for a single-user two-hop MIMO relay system [16] with the first-hop channel $\mathbf{X}_{k,i}$, the relay matrix $\mathbf{Y}_{k,i}$ and the second-hop channel $\mathbf{I}_{N_{k,i}}$. It can be shown similar to [16] that the optimal structure of $\mathbf{Y}_{k,i}$ is

$$\mathbf{Y}_{k,i} = \left[\mathbf{I}_d, \; \mathbf{0}_{d\times(N_{k,i}-d)}\right]^T\mathbf{\Lambda}_{yk,i}\mathbf{U}_{xk,i}^H \tag{35}$$

where $\mathbf{X}_{k,i} = \mathbf{U}_{xk,i}\mathbf{\Lambda}_{xk,i}\mathbf{V}_{xk,i}^H$ is the SVD of $\mathbf{X}_{k,i}$ and $\mathbf{\Lambda}_{yk,i}$ is a $d \times d$ diagonal matrix. The dimensions of $\mathbf{U}_{xk,i}$, $\mathbf{\Lambda}_{xk,i}$, and $\mathbf{V}_{xk,i}$ are $L_1 \times d$, $d \times d$, and $d \times d$, respectively.

By substituting (35) back into (33)-(34), the relay precoding matrix optimization problem (26)-(27) can be equivalently rewritten as the following problem with scalar variables

$$\min_{\{\lambda_{yk,i,j}\}} \sum_{i=1}^{2}\sum_{k=1}^{K}\sum_{j=1}^{d}\left(1 + \frac{\lambda_{xk,i,j}^2\lambda_{yk,i,j}^2}{\lambda_{yk,i,j}^2 + \sigma_{k,i}^2}\right)^{-1} \tag{36}$$

$$s.t. \sum_{i=1}^{2}\sum_{k=1}^{K}\sum_{j=1}^{d}\lambda_{yk,i,j}^2(\lambda_{xk,i,j}^2 + 1) \leq \bar{P}_r \tag{37}$$

$$\lambda_{yk,i,j} \geq 0, k = 1, \cdots, K, i = 1, 2, j = 1, \cdots, d \tag{38}$$

where $\lambda_{xk,i,j}$ and $\lambda_{yk,i,j}$, $j = 1, \cdots, d$, are the jth diagonal elements of $\mathbf{\Lambda}_{xk,i}$ and $\mathbf{\Lambda}_{yk,i}$, respectively, and $\{\lambda_{yk,i,j}\} = \{\lambda_{y1,1,1}, \cdots, \lambda_{yK,2,d}\}$. The problem (36)-(38) has the well-known water-filling solution given by

$$\lambda_{yk,i,j} = \sqrt{\frac{1}{\lambda^2_{xk,i,j}+1}\left[\sqrt{\frac{\sigma^2_{k,i}\lambda^2_{xk,i,j}}{(\lambda^2_{xk,i,j}+1)\mu}} - \sigma^2_{k,i}\right]^{\dagger}}$$
$$k = 1, \cdots, K,\ i = 1, 2,\ j = 1, \cdots, d \qquad (39)$$

where $[x]^{\dagger} = \max(x, 0)$, and $\mu > 0$ can be obtained by substituting (39) back into (37) and solving a nonlinear equation using the bisection method [17]. Finally, $\mathbf{F}$ can be obtained from (20), (31), (35) and (39).

With given receiver matrices $\{\mathbf{W}_{k,i}\}$ and relay matrix $\mathbf{F}$, the SMSE can be rewritten as a function of $\{\mathbf{B}_{k,i}\}$ as

$$\mathrm{SMSE} = \sum_{i=1}^{2}\sum_{k=1}^{K} tr\Big((\bar{\mathbf{G}}_{k,i}\mathbf{H}_{k,\bar{i}}\mathbf{B}_{k,\bar{i}} - \mathbf{I}_d)(\bar{\mathbf{G}}_{k,i}\mathbf{H}_{k,\bar{i}}\mathbf{B}_{k,\bar{i}} - \mathbf{I}_d)^H$$
$$+ \bar{\mathbf{G}}_{k,i}\sum_{j=1}^{2}\sum_{m\neq k}^{K}\mathbf{H}_{m,j}\mathbf{B}_{m,j}\mathbf{B}^H_{m,j}\mathbf{H}^H_{m,j}\bar{\mathbf{G}}^H_{k,i}\Big) + t_2 \qquad (40)$$

where $t_2 = \sum_{i=1}^{2}\sum_{k=1}^{K} tr(\mathbf{W}^H_{k,i}\mathbf{N}_{k,i}\mathbf{W}_{k,i})$ can be ignored as it is independent of $\{\mathbf{B}_{k,i}\}$, and $\bar{\mathbf{G}}_{k,i} = \mathbf{W}^H_{k,i}\mathbf{G}_{k,i}\mathbf{F}$. Using (40), the source matrices optimization problem is given by

$$\min_{\{\mathbf{B}_{k,i}\}} \ \mathrm{SMSE} \qquad (41)$$
$$s.t.\ tr\big(\mathbf{B}_{k,i}\mathbf{B}^H_{k,i}\big) \leq P_{k,i}, k = 1, \cdots, K, i = 1, 2 \quad (42)$$
$$tr\Big(\mathbf{F}\sum_{i=1}^{2}\sum_{k=1}^{K}\mathbf{H}_{k,i}\mathbf{B}_{k,i}\mathbf{B}^H_{k,i}\mathbf{H}^H_{k,i}\mathbf{F}^H\Big) \leq \tilde{P}_r \quad (43)$$

where $\tilde{P}_r = P_r - \sigma^2_r tr(\mathbf{F}\mathbf{F}^H)$. The problem (41)-(43) is a convex QCQP problem and can be solved by the CVX MATLAB toolbox [18] for disciplined convex programming.

Since the dimension of $\{\lambda_{y,k,i,j}\}$ is $2Kd$, the computational complexity of solving the problem (36)-(38) is $\mathcal{O}(Kd)$. Assuming $2Kd \leq N_r$, the SVD of $\mathbf{X}_{k,i}$ has a complexity of $\mathcal{O}(Kd^3)$. Therefore, the complexity of the simplified relay matrix design is $\mathcal{O}(K^2d^3)$, which is lower than the complexity of updating the relay matrix in [9] ($\mathcal{O}(N_r^6)$) and [11] ($\mathcal{O}(KN_r(K-1)^2)$). Moreover, we observed through simulations that the proposed algorithm typically converges within three iterations. Therefore, the overall computational complexity of the proposed transceiver design algorithm is lower than those of [9] and [11].

IV. Numerical Examples

We simulate an interference two-way MIMO relay system where all transmitters and receivers have the same number of antennas, i.e., $N_{k,i} = 2$, $i = 1, 2$, $k = 1, \cdots, K$, and the relay node has $N_r = 10$ antennas. We assume that all source nodes have the same power budget as $P_{k,i} = 15$dB, $i = 1, 2$, $k = 1, \cdots, K$. All channel matrices have i.i.d. complex Gaussian entries with zero-mean and unit variance. The noises are i.i.d. Gaussian with zero mean and unit variance. QPSK constellations are used to modulate the source symbols and the simulation results are averaged over 5×10^5 independent channel realizations. The proposed algorithm is

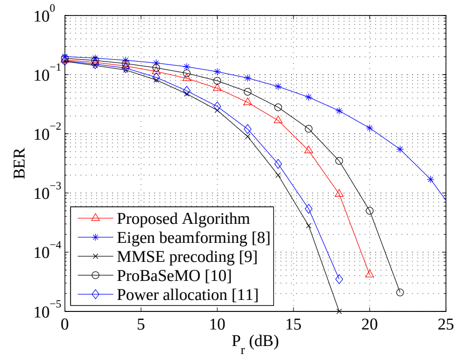

Fig. 2. Example 1: BER of five algorithms versus P_r.

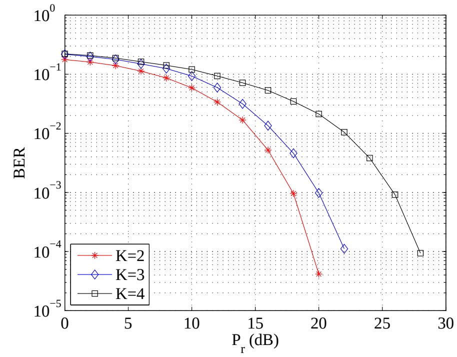

Fig. 3. Example 2: BER of the proposed algorithm at various K.

initialized with $\{\mathbf{B}^{(0)}_{k,i} = \sqrt{P_{k,i}/N_{k,i}}\mathbf{I}_{N_{k,i}}\}$ and $\mathbf{F}^{(0)} = \sqrt{P_r/tr(\sum_{i=1}^{2}\sum_{k=1}^{K}\mathbf{H}_{k,i}\mathbf{B}_{k,i}\mathbf{B}^H_{k,i}\mathbf{H}^H_{k,i} + \sigma^2_r\mathbf{I}_{N_r})}\mathbf{I}_{N_r}$.

In the first example, we compare the performance of the proposed algorithm with the transceiver design algorithms in [8]-[11] for a MIMO relay system with $K = 2$ two-way link pairs. For a fair comparison with [8], we set $d = 1$. Fig. 2 shows the bit-error-rate (BER) performance of the five algorithms tested versus P_r. It can be seen that while the proposed algorithm outperforms the eigen-beamforming algorithm in [8] and the ProBaSeMO scheme in [10], the MMSE precoding algorithm in [9] and the power allocation algorithm in [11] yield a lower BER than the proposed algorithm. However, the algorithms in [9] and [11] have the highest computational complexity among five algorithms, and the computational complexity of the algorithms in [8] and [10] is lower than the other three algorithms. Therefore, the proposed algorithm provides a better performance-complexity tradeoff than those in [8]-[11], which is very useful for practical interference two-way MIMO relay communication systems.

In the second example, we study the BER performance of the proposed algorithm at various K. It can be seen from Fig. 3 that as expected, the system BER increases with K.

V. Conclusion

We have developed a novel algorithm for jointly optimizing the source, relay, and receiver matrices of interference two-way MIMO relay systems. By exploiting the optimal structure of the relay precoding matrix and modifying the power constraint at the relay node, the computational complexity of optimizing the relay precoding matrix is significantly reduced with only a small performance degradation.

REFERENCES

[1] L. Sanguinetti, A. A. D'Amico, and Y. Rong, "A tutorial on the optimization of amplify-and-forward MIMO relay systems," *IEEE J. Selet. Areas Commun.*, vol. 30, no. 8, pp. 1331-1346, Sep. 2012.

[2] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *Proc. ACM SIGCOMM*, Aug. 2007.

[3] S. Xu and Y. Hua, "Optimal design of spatial source-and-relay matrices for a non-regenerative two-way MIMO relay system," *IEEE Trans. Wireless Commun.*, vol. 10, no. 5, pp. 1645-1655, May 2011.

[4] R. Wang, M. Tao, and Y. Huang, "Linear precoding designs for amplify-and-forward multiuser two-way relay systems," *IEEE Trans. Wireless Commun.*, vol. 11, no. 12, pp. 4457-4469, Dec. 2012.

[5] Y. Rong, "Joint source and relay optimization for two-way linear nonregenerative MIMO relay communications," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6533-6546, Dec. 2012.

[6] Z. He, Z. Lang, Y. Rong, and S. Qu, "Joint transceiver optimization for two-way MIMO relay systems with MSE constraints," *IEEE Wireless Commun. Lett.*, vol. 3, no. 6, pp. 613-616, Dec. 2014.

[7] K.-J. Lee, H. Sung, E. Park, and I. Lee, "Joint optimization for one and two-way MIMO AF multiple-relay systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3671-3681, Dec. 2010.

[8] J. Joung and A. H. Sayed, "Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1833-1846, Mar. 2010.

[9] H. Yi, J. Zou, H. Luo, H. Yu, and J. Ma, "Joint MMSE precoding design in multi-user two-way MIMO relay systems," in *Proc. IEEE Int. Conf. Wireless Commun. Signal Process.*, Nanjing, China, Nov. 2011.

[10] J. Zhang, F. Roemer, and M. Haardt, "Relay assisted physical resource sharing: Projection based separation of multiple operators (ProBaSeMO) for two-way relaying with MIMO amplify and forward relays," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4834-4848, Sep. 2012.

[11] M. Wang, P. Wang, Y. Li, Z. Zhong, F. Wang, and B. Vucetic, "Transceiver design for multi-user cellular two-way relay networks," *IEEE Trans. Signal Process.*, vol. 63, no. 15, pp. 4065-4078, Aug. 2015.

[12] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2381-2401, Sep. 2003.

[13] Z. Wang and A. C. Bovik, "Mean squared error: Love it or leave it? A new look at signal fidelity measures," *IEEE Signal Process. Magazine*, vol. 26, no. 1, pp. 98-117, Jan. 2009.

[14] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cilffs, NJ: Prentice Hall, 1993.

[15] Y. Rong, "Linear non-regenerative multicarrier MIMO relay communications based on MMSE criterion," *IEEE Trans. Commun.* vol. 58, no. 7, pp. 1918-1923, Jul. 2010.

[16] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear nonregenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Process.*, vol. 57, no. 12, pp. 4837-4851, Dec. 2009.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U. K.: Cambridge University Press, 2004.

[18] M. Grant and S. Boyd, "Cvx: Matlab software for disciplined convex programming (webpage and software)," [Online]. Available: http://cvxr.com/cvx, Apr. 2010.

[19] Y. Nesterov and A. Nemirovski, *Interior Point Polynomial Algorithms in Convex Programming*. Philadelphia, PA: SIAM, 1994.