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## COMPUTER AIDED BUILDING PHYSICAL MODELING

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# PARAMETER UNCERTAINTY INVESTIGATION USING LINEARIZED MODEL 

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#### Abstract

During the design of new technical systems, their manufacturing tolerances (i.e. system parameter uncertainties) should be determined basically by working requirements of the system and technological possibilities of the manufacturers. Therefore the determination of manufacturing tolerances can be a very important and difficult task. It is possible that system output parameter values are inadequate, but every equipment of the system meets its own requirements, because their manufacturing tolerances (allowable uncertainties) of elements have been determined incorrectly. It is possible too, that the working requirements of the designed system have to determine strictly the system parameter tolerances. For example, these system requirements can be velocity or acceleration of the piston of aircraft hydraulic servo actuator, from the flight mechanical point of view. These external parameters should require very strictly the tolerances of internal parameters of the given system. Using linearized mathematical diagnostic model of the given system, the problems mentioned above can be investigated and solved. This paper will show the methodology of the linearized mathematical-diagnostic model's usage to investigate effects of manufacturing parameter uncertainties.


Keywords: parameter uncertainty, linearized model,

## 1. INTRODUCTION

During the design of new technical systems, their manufacturing tolerances (i.e. parametrical uncertainties of its model) should be determined basically by working requirements of the system and technological possibilities of the manufacturers. Therefore the determination of manufacturing tolerances can be a very important and difficult task. It is possible that system output parameter values are inadequate, but every units of the system meets its own requirements, because their manufacturing tolerances (allowable uncertainties) of elements have been determined incorrectly. It is possible too, that the working requirements of the designed system have to determine strictly the system parameter tolerances. For example, these system requirements can be velocity or acceleration of the piston of aircraft hydraulic servo actuator, from the flight mechanical point of view. These external parameters should require very strictly the tolerances of internal parameters of the given system.

Using linearized mathematical diagnostic model of the given system, the problems mentioned above can be investigated and solved [3].

Uncertainty analysis is a systematic study in which "a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified".

In the eyes of Ferson and Tucker there is an interval way to effect such a study which is to bound the neighborhood with interval ranges [1]. Another natural way is to ascribe a probability distribution to the elements in the neighborhood.

Muzzioli, and Reynaerts clarified the link between interval linear systems and fuzzy linear systems. The authors have introduced a simple algorithm, which boils down to an
interval technique, in order to find the vector solution to the system [2].
The aim of this paper is to show the methodology of the linearized mathematicaldiagnostic model's usage to investigate effects of manufacturing parameter uncertainties.

The outline of the paper is as follows. Section 2 shows the basis methodology of mathematical diagnostic modeling. Section 3 words linearized mathematical model based. investigation of manufacturing anomalies. Section 4 presents the inverse method of investigation shortly.

## 2. METHODOLOGY OF THE MATHEMATICAL DIAGNOSTICS MODELING

The setting up mathematical model should start from splitting up of the investigated system into its functional units. These units should be examined and the interdependencies between their $\mathbf{x}$ input and $\mathbf{y}$ output parameters should be established mathematically [4]. This model can be written in general case:

$$
\begin{equation*}
f(\mathbf{y})=g(\mathbf{x}) \tag{1}
\end{equation*}
$$

For setting up of the linear diagnostic model, the mathematical model should be linearized. For linearization logarithmic linearization can be used, when firstly, the natural logarithm (to e base) of both sides of the general non-linear equation(s) should be formed. Then the total differential of the logarithmic equation should be determined. Its usage is admissible basically in case of exponential (thermodynamical) equations.

System of equation got in this way describes interdependencies between relative changes of independent ( $\delta \mathbf{x}$ ) and dependent ( $\delta \mathbf{y}$ ) parameters in point of view of given investigation. This model can be written in the following matrix formula:

$$
\begin{equation*}
\mathbf{A} \delta \mathbf{y}=\mathbf{B} \delta \mathbf{x} \tag{2}
\end{equation*}
$$

Using the

$$
\begin{equation*}
\mathbf{D}=\mathbf{A}^{-1} \mathbf{B} \tag{3}
\end{equation*}
$$

diagnostic matrix, the equation

$$
\begin{equation*}
\delta \mathbf{y}=\mathbf{D} \delta \mathbf{x} \tag{4}
\end{equation*}
$$

can be used for diagnostic investigations [3].

## 3. INVESTIGATION OF EFFECTS OF MANUFACTURING ANOMALIES

Using mathematical diagnostic model of the given system parameter uncertainty, in technical practice which is manufacturing anomalies, problems can be investigated and solved.

Manufacturing anomalies of internal parameters can be characterized by their maximum and minimum values.

In case of interval investigation of manufacturing anomalies, the linearized diagnostic model - see equation (4) - of the investigated system should be modified. The so-called "positive diagnostic matrix" and "negative diagnostic matrix" should be introduced. Elements of the first one are the positive-sign, and another one's elements are negative-sign elements of the original diagnostic matrix (or zero):

$$
\mathbf{D}_{+}=\left[d_{i j+}=\left\{\begin{array}{ccc}
d_{i j} & \text { if } & d_{i j} \geq 0  \tag{5}\\
0 & \text { if } & d_{i j}<0
\end{array}\right] \quad, \quad \mathbf{D}_{-}=\left[d_{i j-}=\left\{\begin{array}{cll}
d_{i j} & \text { if } & d_{i j} \leq 0 \\
0 & \text { if } & d_{i j}>0
\end{array}\right]\right.\right.
$$

Knowing the above mentioned matrices, the vectors of relative maximum and minimum values of the external parameters:

$$
\begin{equation*}
\delta \mathbf{y}_{\max }=\mathbf{D}_{+} \delta \mathbf{x}_{\max }+\mathbf{D}_{-} \delta \mathbf{x}_{\min } \quad \text { and } \quad \delta \mathbf{y}_{\min }=\mathbf{D}_{+} \delta \mathbf{x}_{\min }+\mathbf{D}_{-} \delta \mathbf{x}_{\max } . \tag{6}
\end{equation*}
$$

For the sake of the inverse method, the following hyper-matrix equation should be devised:

$$
\left[\begin{array}{l}
\delta \mathbf{y}_{\max }  \tag{7}\\
\delta \mathbf{y}_{\min }
\end{array}\right]=\left[\begin{array}{ll}
\mathbf{D}_{+} & \mathbf{D}_{-} \\
\mathbf{D}_{-} & \mathbf{D}_{+}
\end{array}\right]\left[\begin{array}{l}
\delta \mathbf{x}_{\max } \\
\delta \mathbf{x}_{\min }
\end{array}\right] .
$$

Knowing the relative maximum and minimum external parameter values can be determined.

## 4. THE INVERSE METHOD

If the task and work of the investigated (designed) system limits the output parameter values and their tolerances strictly, the manufacturing tolerances of internal parameters should be determined or estimated depend on required output parameter tolerances. This task can be solved by the inverse method of investigation mentioned above.

For estimation of required maximum and minimum values of internal parameters can be determined on basis of hyper-matrix equation (7). The vector which satisfies the scalarvector equation

$$
\left(\left[\begin{array}{l}
\delta \mathbf{y}_{\max }  \tag{8}\\
\delta \mathbf{y}_{\min }
\end{array}\right]-\left[\begin{array}{ll}
\mathbf{D}_{+} & \mathbf{D}_{-} \\
\mathbf{D}_{-} & \mathbf{D}_{+}
\end{array}\right]\left[\begin{array}{l}
\delta \mathbf{x}_{\max } \\
\delta \mathbf{x}_{\min }
\end{array}\right]\right)^{2}=0
$$

should be estimated using any search of optimum method.
On the basis of vector estimated above, the required real (measurable) values of internal parameters can be determined.

It is very important to mention that these data have to be investigated from the technological and the manufacturing points of view. If the technological possibilities do not meet the required quality, on the basis of the practicable tolerance zones of the internal parameters should be determined and the base investigation should be performed once more while the external system parameters will meet the requirements [3].

It is also important to mention that this method does not give the unambiguous solution of the above-mentioned technical problem. Because of this investigation uses any estimation process. This method is „only" an effective adjuvancy to determine the most practicable manufacturing tolerances of the internal parameters during the design of the system.

## 5. SUMMARY

The writer of this paper would like to arouse readers' interest in possibilities of use of mathematical diagnostic modeling to investigate of technical systems during their
design. In this paper the methodology of mathematical diagnostics and investigation of influences of manufacturing anomalies has been shown.

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