Stress dependency of elastic properties of shales: the effect of uniaxial stress

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Summary

Understanding seismic anisotropy in shales is important for quantitative interpretation of seismic data, 4D monitoring and pore pressure prediction. Along with intrinsic anisotropy caused by preferred mineral orientation that is common in shales, anisotropic stress is an important factor that affects shale elastic response. While variations of elastic coefficients with anisotropic stress have been the subject of experimental studies, theoretical insight is still largely lacking. Here we suggest a new model that allows parameterization of the stress dependency of elastic coefficients of shales under anisotropic stress conditions. We show that the parameterization requires four parameters, namely, specific tangential compliance of a single crack, the ratio of normal to tangential compliances, characteristic pressure and a crack orientation anisotropy parameter. These parameters can be estimated from experimentally measured stress sensitivity of elastic coefficients in shales to isotropic stress.

Introduction

The effect of stress on elastic properties of shales is also important for understanding of depositional trends especially at the upper 2000-3000 meters where the compaction is mostly mechanical. Despite the importance of the effects of isotropic and especially anisotropic stress on elastic properties of shales, little work has been done on theoretical understanding and predicting such properties and generally for the case of isotropic stress. All the existing theoretical approaches to the problem of elastic stress sensitivity are based on the analysis of orientation distribution of discontinuities and their normal, B_N, and shear, B_T, compliances. Sayers (1999) studied stress-dependent seismic anisotropy of shales using ultrasonic measurements on two fully saturated shales of Jurassic age reported by Hornby et al. (1994), two air-dry shales from the Millboro and Braillier members of the Devonian-Mississipian Chattanooga Formation (Johnston and Christensen, 1993) and air-dry mature, kerogen-rich shale (Vernik, 1993). Sayers used the formalism presented in Sayers and Kachanov (1995) that takes into account extra compliance induced by fractures and cracks. Sayers (1999) calculated B_N/B_T ratios for each point of confining stress assuming that the discontinuities are perfectly aligned. Analyzing the experimentally obtained stress dependencies of the five elastic coefficients, Sayers (1999) concluded that the contacts between clay particles are more compliant in shear than in compression and the ratios of normal to tangential compliances of individual cracks are higher for the air-dry shales compared to those in saturated shales.

Ciz and Shapiro (2009) studied the variations of elastic compliances and anisotropic parameters of Jurassic North Sea shale with isotropic stress reported in Prioul et al. (2004). They applied the so-called porosity deformation model (also know as dual porosity model, stress-sensitivity or piezo-sensitivity) initially developed by Shapiro (2003) for dry isotropic rock and later extended by Shapiro and Kaselow (2005) to the case of orthorhombic symmetry. Ciz and Shapiro (2009) described stress dependency of all elastic coefficients apart from S_{13}, which is independent of stress in the porosity deformation model. As laboratory measurements commonly show noticeable variations in S_{13} with pressure, this is a limitation of the porosity deformation model. The Sayers-Kachanov model seems to be more universal.

Using the Sayers-Kachanov model, Gurevich et al. (2011) suggested a new analytical model of stress induced anisotropy caused by application of uniaxial stress to an isotropic cracked medium. Using similar approach, Pervukhina et al. (2010, 2011) developed a new model for stress dependency of transversely isotropic (TI) media which predicts stress sensitivity behavior of all five elastic coefficients using four physically plausible parameters. These are the specific tangential compliance of a single crack, the ratio of normal to tangential compliances, the characteristic pressure and a crack orientation anisotropy parameter (Pervukhina et al., 2010, 2011). The model has been used to parameterize elastic properties of about 20 shales. The four fitting parameters showed moderate to good correlations with the depth from which the shale was extracted. With increasing depth, the tangential compliance exponentially decreases. The crack orientation anisotropy parameter broadly increases with the depth for most of the shales, indicating that cracks are getting more aligned in the bedding plane. The ratio of normal to shear compliance and characteristic pressure decrease with the depth to 2500 m, and then increase in the depth range of 2500-3600 m. The suggested model also allows prediction of the stress dependency of all five elastic compliances of a TI medium, even if only some of them are known.

This study extends the model of Pervukhina et al. (2010, 2011) to anisotropic (uniaxial) stresses. We show that at small uniaxial stresses, these effects can be described using the same four parameters. Moreover, the values of these parameters can be extracted from the experimental measurements obtained at isotropic stress. Predictions of the model are compared with experimental measurements.

Modeling the effect of anisotropic stress on elastic coefficients of shales

Following Pervukhina et al. (2011), we model the shale as an intrinsically transversely isotropic medium that is permeated with discontinuities (cracks or fractures). We assume an anisotropic orientation distribution of discontinuities for which the probability density for a particular orientation can be written as

$$W(\theta,\phi)=\frac{1+\eta\cos^2\theta}{\int_0^{2\pi}\int_0^{\pi}\left(1+\eta\cos^2\theta\right)\sin\theta d\theta d\phi}=\frac{1+\eta\cos^2\theta}{4\pi\left(1+\eta/3\right)} \quad (1)$$

where θ is an angle between the z-axis and the normal to the crack surface (range [0, π]), ϕ determines the rotation about the z-axis (range [0, 2π]) and η is the crack orientation anisotropy parameter. An isotropic distribution of cracks corresponds to the case where $\eta=0$ and in the case when η is large, there is a strong alignment of cracks. One can check that the probability density defined by equation 1 satisfies the normalization condition:

$$\int_0^{2\pi}\int_0^{\pi}W(\theta,\phi)\sin\theta d\theta d\phi=1 \quad (2)$$

Both here and below, the *z*-axis is chosen as a symmetry axis of the TI medium. The exact geometry of individual cracks is not specified. Instead, the behavior of cracks is defined by a ratio *B* of the normal B_N to tangential B_T excess crack compliances. All cracks are assumed identical; thus *B* is the same for all cracks.

The medium can undergo a zero or nonzero isotropic stress. The effect of the nonzero isotropic stress on shale can be taken into account by assuming an exponential reduction of crack area $A^{(r)}$ (and specific area of cracks $s=A^{(r)}/V$) with effective pressure *P* (confining pressure minus pore pressure) as follows:

$$s=s^0\exp\left(-P/P_c\right) \quad (3)$$

where s^0 is specific area of all the cracks at zero pressure and P_c is a characteristic crack closing pressure (Schoenberg, 2002; Shapiro, 2003; Shapiro and Kazelow, 2005; Vlastos et al., 2006).

When the rock is subjected to a small uniaxial compressive stress (σ) in addition to an isotropic stress, the density of cracks along a particular plane is reduced in proportion to the normal stress traction acting on that plane. To model closure of cracks due to application of anisotropic stress, we can assume that $B_N^{(r)}$ and $B_T^{(r)}$ are the same for all orientations of cracks, while $A^{(r)}$ (and specific area of cracks $s=A^{(r)}/V$) varies with direction of the crack normal, depending on the normal stress acting in that direction,

$$s=s^0\exp\left(-\sigma/P_c\right). \quad (4)$$

When uniaxial stresses are small compared to P_c, the exponential expression in equation 4 can be approximated by a linear expression

$$s=s^0\left(1-\sigma/P_c\right) \quad (5)$$

The uniaxial stress does not change the TI symmetry of the shale if it is applied along its symmetry axis (*z* axis in our case). The variation of elastic compliances with the applied isotropic and uniaxial stresses can be calculated using Sayers-Kachanov model as follows

$$\Delta S_{ijkl}\equiv S_{ijkl}-S_{ijkl}^0=\frac{1}{4}\left(\delta_{ik}\alpha_{jl}+\delta_{il}\alpha_{jk}+\delta_{jk}\alpha_{il}+\delta_{jl}\alpha_{ik}\right)+\beta_{ijkl} \quad (6)$$

$$\alpha_{ij}=\frac{1}{V}\sum_r B_T^{(r)}n_i^{(r)}n_j^{(r)}A^{(r)} \quad (7)$$

$$\beta_{ijkl}=\frac{1}{V}\sum_r\left(B_N^{(r)}-B_T^{(r)}\right)n_i^{(r)}n_j^{(r)}n_k^{(r)}n_l^{(r)}A^{(r)} \quad (8)$$

Here, ΔS_{ijkl} is the excess compliance caused by the presence of compliant cracks, S_{ijkl}^0 are compliances at high stress with all soft cracks closed and S_{ijkl} are the compliances at some intermediate stress; δ_{ij} is the Kronecker delta; *r* is the number of planar discontinuities with surface area $A^{(r)}$ and $n_i^{(r)}$ and $n_j^{(r)}$ are *i*th and *j*th components of the unit vector that is normal to the surface of the r^{th} grain boundary in volume *V*; finally, B_N and B_T are the normal and tangential compliances of an individual crack.

Substituting equations 1, 3 and 5 together into equations (6-8), we can obtain variations of five elastic compliances for a TI fractured medium that is subject to both pressure (P) and uniaxial stress (σ) parallel to the symmetry axis of the TI medium. The derivation of the equations can be done following the procedure that is described in detail in Pervukhina et al. (2011).

Here we restrict our study to the case of constant effective pressure P and calculate variations of elastic compliances with variation of only anisotropic stress σ. The formulae below express the variations in terms of the four parameters as follows:

$$\Delta S_{11}^{an} = \frac{\sigma}{105P_c} B_{T0} e^{-\frac{P}{P_c}} \left(4 + 2\eta + 3B + B\eta\right) \quad (9)$$

$$\Delta S_{33}^{an} = \frac{\sigma}{315P_c} B_{T0} e^{-\frac{P}{P_c}} \left(18 + 10\eta + 45B + 35B\eta\right) \quad (10)$$

$$\Delta S_{13}^{an} = \frac{\sigma}{315P_c} B_{T0} e^{-\frac{P}{P_c}} \left(-9 - 5\eta + 9B + 5B\eta\right) \quad (11)$$

$$\Delta S_{44}^{an} = \frac{2\sigma}{315P_c} B_{T0} e^{-\frac{P}{P_c}} \left(24 + 17\eta + 18B + 10B\eta\right) \quad (12)$$

$$\Delta S_{66}^{an} = \frac{2\sigma}{315P_c} B_{T0} e^{-\frac{P}{P_c}} \left(15 + 7\eta + 6B + 2B\eta\right). \quad (13)$$

Here we employ the conventional matrix notation (e.g., Nye, 1985), which supposes that

$S_{mn} \to S_{ijkl}$ when m and n are 1, 2, or 3;
$S_{mn} \to 2S_{ijkl}$ when m or n is 4, 5, or 6;
$S_{mn} \to 4S_{ijkl}$ when both m and n are 4, 5, or 6.

Note that the four parameters in equations 9-13 are the same as in the case of isotropic stress and hence can be determined from the isotropic experiments. Thus, the variations of elastic properties of shales if an anisotropic stress is applied can be predicted from the variations that occur when the shale is subjected to an isotropic stress. Here we show the applicability of this approach using experimental measurements made on a shale from the onshore Officer Basin in Western Australia (Kuila et al., 2010).

Validation on experimental data

We compare ultrasonic velocities that were measured in a multistage triaxial test on a shale sample from the Officer Basin with predictions from the theoretical model. The Officer Basin shales are red shales of low porosity, comprising mainly illite, orthoclase and quartz. These shales are laminated and in parts are rigid grain supported. The sample of interest is extracted from the depth of 603 m and has a porosity of 6%, clay fraction of 17% and clay content of 41%. It consists of quartz (25%), orthoclase (29%), illite (35%), albite (4%), chlorite (4%), kaolinite (2%) and haematite (2%). The sample is cut perpendicular to the bedding and thus, the application of a uniaxial stress normal to the bedding does not change the TI symmetry of the sample.

The ultrasonic experiments were done on a preserved, saturated shale sample with controlled pore pressure. First a confining pressure is increased to some level, for instance 20 MPa, and pore pressure is equilibrated at, for instance, 5 MPa, then the axial stress is increased by increments, for instance, to 1, 2, 5, 8 and 15 MPa. The details of the experimental procedure and sample preparation can be found in Kuila et al. (2010).

The theoretical predictions made using formulas 9-13 and model parameters B = 0.2, B_T = 7 MPa^{-1} η = 4 and P_c = 57 MPa were obtained from isotropic measurements (see Pervukhina et al., 2011, for further details). In Figure 1, the measured and predicted variations in P- and S-wave velocity due to variations of axial stress are shown for effective pressures of ~10 MPa. In Figure 2 (a, b), P- and S-wave velocities are shown as a function of angle of incidence for the same effective stress and axial stresses of 1, 8 and 15 MPa.

Discussion

Both theoretical predictions and measurements show moderate dependency of velocities with the uniaxial stress: this dependency gets weaker with an increase of effective stress. While uniaxial stress increases from 0 to 8 MPa, V_P and V_S increase by ~20 m/s at effective stresses of 10-20 MPa and even by a smaller amount at higher stresses. The model predicts maximal variations in V_P when the wave propagates perpendicular to the bedding plane, while V_S increases in both directions (parallel and perpendicular to the bedding plane). Variations in the uniaxial stress have the smallest effect on acoustic wave propagation in oblique directions. However, these conclusions are valid for this particular shale and need to be verified on other shales before application.

The developed model predicts well the variations in velocities due to the application of uniaxial stress at small values (< 10 MPa) of the uniaxial stress. At higher values of uniaxial stress, the model overestimates experimentally measured velocities, which show no further increase and can even decrease with increasing uniaxial stress. This is probably caused by opening of cracks parallel to the applied direction of the uniaxial stress which is known to reduce velocities in shale samples (Sayers, 1988; Dewhurst and Siggins, 2006; Kuila et al., 2010). The model presented here is not designed for prediction of such effects and is thus applicable to small axial stress variations ($\sigma << P_c$) only, due to the assumed linear closure of discontinuity areas with stress (equation 5).

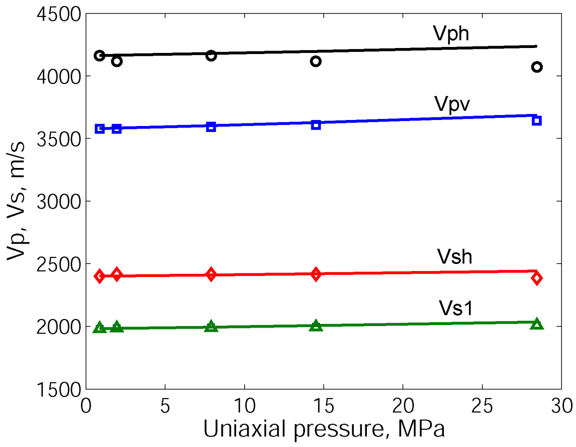

Figure 1: *V*ph (circles), *V*sh (diamonds), *V*pv (squares) and Vs1(triangles) velocities measured at 10 MPa of effective pressure in Officer Basin shale compared with model predictions (solid lines).

It is worth emphasizing again that the four parameters used in the model are derived from the dependency of the elastic properties of the shale on *isotropic* stress and are nevertheless shown to the given an adequate prediction of variation in elastic properties under uniaxial stress. This fact implies that the model captures the essence of the stress dependency mechanism and the proposed four parameters can be used to parameterize stress sensitivity of different rock types.

The model can be used to analyze stress-related anisotropy in shales *in situ* where stress perturbations are small and the corresponding variation in elastic moduli can be assumed linear. For the general case of large stresses, when density of the cracks cannot be considered reducing linearly, an analytical solution is not feasible and numerical solution is required. However, experimental measurements in shales show that for many samples, linear variation of compliances with stress is a reasonable approximation and our model can be used to analyze the experimental data for a moderate range of applied uniaxial stresses. The stress range that can be investigated increases with shale strength.

Conclusions

A new model for prediction of elastic properties variation due to application of uniaxial stress for TI media with discontinuities has been developed. The model has been tested on experimental data obtained for a shale sample from the Officer Basin in Western Australia. The predicted variations in elastic velocities are in a good agreement with experimentally measured data. The developed model can be used for prediction of elastic properties response to application of a small uniaxial stress normal to the bedding plane when only variation of elastic properties with isotropic stress is known.

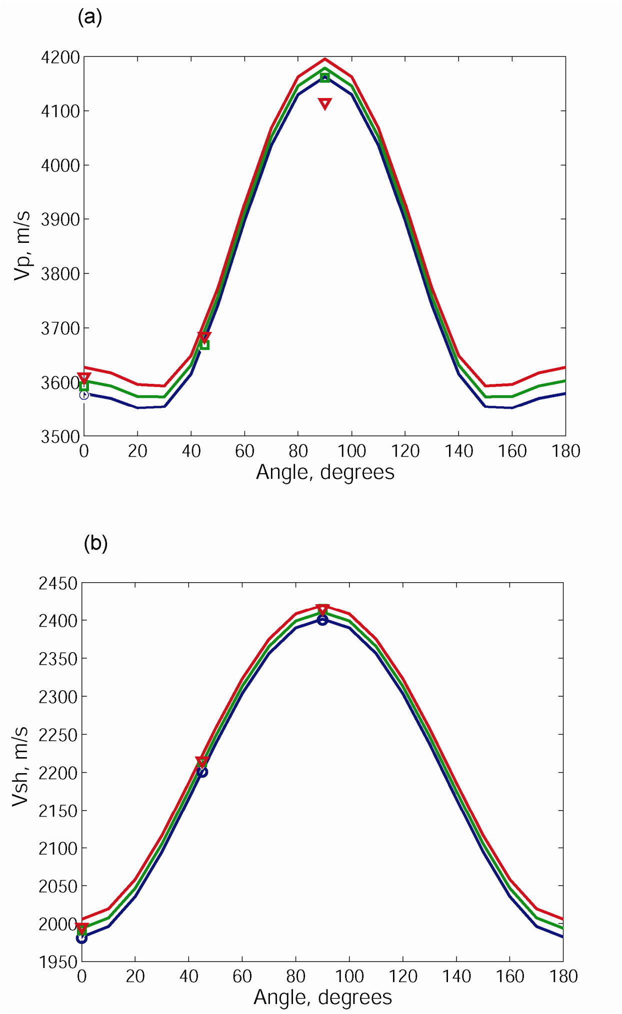

Figure 2: Experimentally measured and predicted velocities in Officer Basin shale at effective pressures of 10 MPa vs. angle between the direction of the wave propagation and the normal to the bedding plane: (a) V_P, (b) V_{SH}. Predicted angular dependencies for 1 MPa, 8 MPa and 15 MPa are shown by blue, green and red lines, respectively. Experimentally measured velocities for 1 MPa, 8 MPa and 15 MPa are shown by blue circles, green squares and red triangles, respectively.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2011 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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