CONTACT STRUCTURES ON $M \times S^2$

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ABSTRACT. We show that if a manifold M admits a contact structure, then so does $M \times S^2$. Our proof relies on surgery theory, a theorem of Eliashberg on contact surgery and a theorem of Bourgeois showing that if M admits a contact structure then so does $M \times T^2$.

1. INTRODUCTION

One of the most important questions in contact topology is to determine which odd dimensional oriented manifolds admit contact structures. Recall that a (positive, coorientable) contact structure on an oriented manifold M of dimension $2q+1$ is a hyperplane distribution $\xi \subset TM$ which can be given as $\ker \alpha$ for a 1-form $\alpha \in \Omega^1(M)$ satisfying

$$\alpha \wedge (d\alpha)^q > 0.$$

The 2-form $d\alpha$ defines a symplectic form on $\ker \alpha$, which determines an almost complex structure J on the sub-bundle $\xi \subset TM$, unique up to contractible choice. Therefore the existence of a contact structure implies that TM decomposes as the sum of a q-dimensional complex bundle and a trivial real line bundle. The pair $(\xi \subset TM, J)$ is called an *almost contact structure* on M. It is equivalent to a reduction of the structure group of TM from $SO(2q+1)$ to $U(q) \times 1$. Now the above existence question can be refined as follows: Which almost contact manifolds admit contact structures?

The answer to this question is positive for open manifolds (by an application of Gromov's h-principle), in dimension three (by Lutz [12] and Martinet [13]) and in dimension five (by Casals-Presas-Pancholi [3] and Etnyre [6]). (For further results see [7].) Less is known for higher dimensional closed manifolds, but so far no example of an almost contact manifold with no contact structure has been found. According to a beautiful result of Bourgeois [1], for a closed contact manifold (M, ξ) the product $M \times \Sigma_g$ also admits a contact structure provided $g \geq 1$. (Here Σ_g denotes the closed orientable surface of genus g.) This construction relies on the theory of compatible open book decompositions of Giroux-Mohsen [8], and provides a contact structure on $M \times T^2$ with the property that for each $p \in T^2$ the submanifold $M \times \{p\} \subset M \times T^2$ is contact and indeed contactomorphic to (M, ξ).

The purpose of the present article is to prove that the result of Bourgeois holds for $g = 0$ as well. (The $g = 0$ case is expected to play a key role in the positive resolution of the general existence problem.)

Theorem 1.1. *Suppose that (M, ξ) is a closed, contact manifold. Then, the product $M \times S^2$ admits a contact structure.*

Furthermore, with a little more care we prove a relative version of this result, which answers a question posed by F. Presas:

Theorem 1.2. *Suppose that (M, ξ) is a closed, contact manifold and let $p \in S^2$. Then the product $M \times S^2$ admits a contact structure such that the submanifold $M \times \{p\}$ is contact and the natural map to (M, ξ) is a contactomorphism.*

Remark 1.3. The proofs of Theorems 1.1 and 1.2 generalise so that we may replace S^2 with other even-dimensional manifolds including any even-dimensional sphere; see Theorem 4.1.

Remark 1.4. Theorem 1.1 has also been obtained independently in [9] using different methods.

The idea of the proof is the following: fix a contact structure on M^{2q+1} and consider the contact structure on $M \times T^2$ provided by the construction of Bourgeois. Let the corresponding almost contact structure be denoted by φ. Then we claim that there is a smooth $(2q + 4)$-dimensional cobordism Y from $M \times T^2$ to $M \times S^2$ which admits an almost complex structure extending φ and a Morse function with critical points of indices $\leq q+2$. By work of Eliashberg and Weinstein [5, 15], for $q \geq 1$ such a cobordism gives rise to a sequence of contact surgeries on $M \times T^2$, inducing a contact structure on $M \times S^2$. The existence of the cobordism Y, on the other hand, can be naturally studied in the framework of stable complex surgery.

2. Preliminaries

Let α be a contact form on a closed $(2q+1)$-manifold M with associated contact structure ξ and let φ be the induced almost contact structure. Then φ naturally induces a stable complex structure ζ_φ on the stable tangent bundle $\tau_M := TM \oplus \underline{\mathbb{R}}^k$ of M (where $\underline{\mathbb{R}}^k$ denotes the trivial real k-plane bundle).

Define the manifold W as $(D^2 \times S^1) - \text{Int}(D^3)$, the solid 3-dimensional torus with a small open 3-ball removed from its interior. Observe that W admits a map $c \colon W \to S^2$ which is a diffeomorphism on the boundary component $\partial_1 W$ diffeomorphic to S^2 (and has degree one on the boundary component $\partial_0 W$ diffeomorphic to T^2).

Consider the stable complex structure ζ on the solid torus which comes from the splitting $T(D^2 \times S^1) = TD^2 \times TS^1$ and from a choice of an almost complex structure on TD^2. Let $\zeta_W := \zeta|_W$ denote the restriction of ζ to W, and let ζ_{S^2} and ζ_{T^2} denote the stable complex structures induced by the complex structures on S^2 and T^2. Note that by construction ζ_{T^2} is homotopic to the stabilisation of an almost complex structure on T^2. Since $W \simeq S^1 \vee S^2$, after a choice of trivialisation homotopy classes of stable complex structures on TW can be identified with $H^2(S^1 \vee S^2; \pi_2(SO/U)) \cong \pi_2(SO/U)$; in particular, in follows that the pull-back stable complex structure $c^*(\zeta_{S^2})$ is homotopic to ζ_W.

We now consider a general stably complex n-manifold (X, ζ_X). (In our immediate applications (X, ζ_X) will be either (W, ζ_W), (S^2, ζ_{S^2}) or (T^2, ζ_{T^2}); in Section 4 we will consider more general stably complex manifolds.) The stable tangent bundle of $M \times X$ is the exterior Whitney sum

$$\tau_{M \times X} = \tau_M \times \tau_X,$$

and therefore it admits the stable complex structure $\zeta_\varphi \times \zeta_X$. In particular, the products $M \times W$, $M \times S^2$ and $M \times T^2$ admit the stable complex structures $\zeta_\varphi \times \zeta_W$, $\zeta_\varphi \times \zeta_{S^2}$ and $\zeta_\varphi \times \zeta_{T^2}$. We will view $M \times W$ as a cobordism from $M \times T^2$ to $M \times S^2$ so that as oriented manifolds

$$\partial(M \times W) = -(M \times T^2) \sqcup (M \times S^2),$$

where the orientations on all manifolds are those induced by the stable complex structure on $T(M \times W)$.

The map $g \colon M \times W \to M \times S^2$ given by $\mathrm{id}_M \times c$ is covered by a map of stable tangent bundles $\bar{g} \colon \tau_{M \times W} \to \tau_{M \times S^2}$, and $g^*(\zeta_\varphi \times \zeta_{S^2})$ is homotopic to $\zeta_\varphi \times \zeta_W$. The above manifolds and maps fit in the commutative diagram

(1)
$$
\begin{array}{ccccc}
M \times T^2 & \xrightarrow{\ i_0\ } & M \times W & \xleftarrow{\ i_1\ } & M \times S^2 \\
& {\scriptstyle f_0}\searrow & \downarrow{\scriptstyle g} & \swarrow{\scriptstyle f_1} & \\
& & M \times S^2, & &
\end{array}
$$

where the maps i_0, i_1 are the embeddings of the boundary components, $f_1 = \mathrm{id}_M \times c|_{\partial_1 W}$ is a diffeomorphism and $f_0 = \mathrm{id}_M \times c|_{\partial_0 W}$ has degree one. In addition, the bundle map $\bar{g}$ above restricts to give bundle maps $\bar{f}_0 \colon \tau_{M \times T^2} \to \tau_{M \times S^2}$ and $\bar{f}_1 \colon \tau_{M \times S^2} \to \tau_{M \times S^2}$ covering f_0 and f_1 respectively. (As always, a bundle map is an isomorphism of a bundle with the pullback of the target bundle.)

3. A contact structure on $M \times S^2$

In this section we prove Theorems 1.1 and 1.2; the proofs will be simple consequences of Propositions 3.1, 3.2, 3.3 and 3.4 below.

Our first proposition, Proposition 3.1, is an analogue of Kreck's [10, Proposition 4]. Whereas Kreck works with bundle maps from the stable normal bundle, we work with bundle maps from the stable tangent bundle, since this better reflects the contact geometry involved. The modifications from stable normal surgery to stable tangential surgery are standard: for example, stable tangential surgery is treated in [11, Theorem 3.59] in the case where the target of the surgery is a Poincaré pair. However, the techniques for surgery below the middle dimension, which are all that we use, do not rely on the target being a Poincaré pair. For the sake of completeness, we give the proof which involves making minor modifications to the proof of [10, Proposition 4] which arise in the stable tangential setting. Recall that M is a closed smooth $(2q+1)$-dimensional manifold, hence $M \times W$ is a compact $(2q+4)$-manifold with boundary.

Proposition 3.1. *The manifold $M \times W$ can be modified by a finite sequence of surgeries in its interior to obtain a manifold Y with the following properties:*

- *Y fits into the following commutative diagram:*

(2)
$$
\begin{array}{ccccc}
M \times T^2 & \xrightarrow{\ i_0\ } & Y & \xleftarrow{\ i_1\ } & M \times S^2 \\
& {\scriptstyle f_0}\searrow & \downarrow{\scriptstyle g_Y} & \swarrow{\scriptstyle f_1} & \\
& & M \times S^2. & &
\end{array}
$$

- *The map g_Y is a $(q + 2)$-equivalence, that is, $(g_Y)_*\colon \pi_i(Y) \to \pi_i(M \times S^2)$ is an isomorphism for $i \leq q + 1$ and a surjection for $i = q + 2$.*
- *There is a bundle map $\bar{g}_Y\colon \tau_Y \to \tau_{M \times S^2}$ covering g_Y which restricts to the bundle maps $\bar{f}_0$ and $\bar{f}_1$ on the boundary of Y. Hence Y admits a stable complex structure ζ_Y such that (Y, ζ_Y) is a stable complex bordism from $(M \times T^2, \zeta_\varphi \times \zeta_{T^2})$ to $(M \times S^2, \zeta_\varphi \times \zeta_{S^2})$.*

Proof. Let $B := M \times S^2$, let $\tau_B\colon M \times S^2 \to BO$ be the classifying map of the stable tangential bundle of B and let $g\colon M \times W \to M \times S^2$ be the map described in Section 2.

We proceed by induction on homotopy groups π_i starting from $g\colon M \times W \to M \times S^2$. Since both $M \times W$ and $M \times S^2$ are connected, we have an isomorphism for $i = 0$. Let $\pi = \pi_1(M \times S^2) = \pi_1(M)$. Note that $g_*\colon \pi_1(M \times W) \to \pi_1(M \times S^2)$ is isomorphic to the projection $\pi \times \mathbb{Z} \to \pi$, hence g_* is surjective on π_1. Now consider the following commutative diagram:

(3)
$$
\begin{array}{ccc}
 & & M \times S^2 = B \\
 & \nearrow{\scriptstyle g_X} & \downarrow{\scriptstyle \tau_B} \\
X & \xrightarrow{\ \tau_X\ } & BO
\end{array}
$$

where X is a bordism from $M \times T^2$ to $M \times S^2$. Suppose that the map g_X induces an isomorphism between the homotopy groups $\pi_j(X) \to \pi_j(B)$ for $j < i \leq q+1$ and a surjection on π_i. We first kill the kernel of $(g_X)_*\colon \pi_i(X) \to \pi_i(B)$. Since $\pi_j(B, g_X(X)) = 0$ for $j < i$, by the Hurewicz Theorem we have that $\pi_i(B, g_X(X)) \cong H_i(B, g_X(X); \mathbb{Z}[\pi])$, hence the kernel of $(g_X)_*$ is finitely generated over $\mathbb{Z}[\pi]$. Suppose that $S^i \to X$ represents a generator of the kernel of $(g_X)_*$. For dimensional reasons we can assume that S^i is embedded. For any i the stable tangent bundle τ_{S^i} is stably trivial and $\tau_X|_{S^i}$ is the pull-back from B along a homotopically trivial map, hence $\tau_X|_{S^i} = \nu_{S^i \subset X} \oplus \tau_{S^i}$ implies that the normal bundle $\nu_{S^i \subset X}$ of S^i in X is stably trivial. Since the rank of $\nu_{S^i \subset X}$ is greater than i, if follows that $\nu_{S^i \subset X}$ is trivial. In order to kill the class represented by S^i, we attach a $(2q + 5)$-handle to X along $D^{m-i} \times S^i \subset X$, where $m := 2q + 4$. For a particular choice of framing the map g_X will extend over the attached handle in such a way that analogue of diagram (3) above for the induced cobordism remains commutative [10, Lemma 2 (ii)]; that is, the bundle map $\bar{g}_X$ extends over the trace of the surgery to classify the stable tangent bundle of this trace. Since we are free to choose the framing, we choose this particular one. After finitely many surgeries we can kill the kernel on π_i and maintain the stable tangential bundle maps.

Now we must arrange that the map g_X is surjective on π_{i+1} for $i \geq 1$. Since B is a finite CW-complex, $\pi_{i+1}(B)$ is finitely generated over $\mathbb{Z}[\pi]$. For each element x_j of a generating set $\{x_1, \ldots, x_k\}$ of the cokernel of $(g_X)_*\colon \pi_{i+1}(X) \to \pi_{i+1}(B)$, we consider a twisted bundle $S^{m-i} \widetilde{\times}_{\alpha_j} S^{i+1}$, where the twisting α_j is determined by the image of $(g_X)_*(x_j)$ in $\pi_{i+1}(BO)$. The map g_X can be extended from X to the interior connected sum of X with this twisted bundle in such a way that the commutativity of diagram (3) is preserved. As a result, we obtain a new map $g_X\colon X \to B$ such that g_X induces a surjective map on $\pi_{i+1}(X)$ and is covered by a map of the stable tangent bundle of X. Inductively repeating this procedure for $i \leq q+1$ we obtain a manifold Y and a map $g_Y\colon Y \to M \times S^2$ with the desired properties. $\square$

Consider now the cobordism Y between $M \times T^2$ and $M \times S^2$ given by Proposition 3.1.

Proposition 3.2. *For $q \geq 1$ the cobordism Y^{2q+4} admits a handle decomposition with handles of index at most $q + 2$ attached to $(M \times T^2) \times [0, 1]$.*

Proof. In the terminology of [14] we shall show that Y (as a cobordism built on $M \times S^2$) is *geometrically* $(q+1)$-connected. Indeed, according to [14, Theorem 3], this property follows once we can show that the cobordism is $(q+1)$-connected, that is, the relative homotopy groups $\pi_i(Y, M \times S^2)$ vanish for $i \leq q+1$. Notice, however, that the portion

$$
\begin{array}{ccc}
Y & \xleftarrow{\ i_1\ } & M \times S^2 \\
\downarrow{\scriptstyle g_Y} & \swarrow{\scriptstyle f_1} & \\
M \times S^2 & &
\end{array}
$$

of diagram (2) implies that $(i_1)_*$ is an isomorphism for $i \leq q+1$, since $(g_Y)_*$ is an isomorphism in all these dimensions, and f_1 is a diffeomorphism. The long exact sequence of homotopy groups for the pair $(Y, M \times S^2)$ shows that in the dimensions $i \leq q+1$ we have vanishing relative homotopy groups and so by [14, Theorem 3], Y has a handle decomposition relative to $M \times S^2$ with handles of index $q+2$ and higher. Hence Y has a handle decomposition relative to $M \times T^2$ with handles of index at most $q+2$. $\qquad\square$

Proposition 3.3. *Suppose that (M, ξ) is a contact manifold and let $\mathrm{pr}_1 \colon M \times T^2 \to M$ and $\mathrm{pr}_2 \colon M \times T^2 \to T^2$ denote the projections. Then there is a contact structure ξ' on $M \times T^2$ such that, the induced almost contact structure is homotopic in $T(M \times T^2)$ to the complex sub-bundle $\mathrm{pr}_1^*(\xi, J) \oplus \mathrm{pr}_2^*(T(T^2), J_{T^2})$, where J_{T^2} is an almost complex structure on T^2. In particular, the stable complex structure induced by ξ' is homotopic to the stable complex structure $\zeta_\varphi \times \zeta_{T^2}$ of Section 2.*

Proof. In [1] a contact structure on $M \times T^2$ was given by the following formula: if ξ on M is defined as $\ker \alpha$ then we define

$$
\alpha' = \mathrm{pr}_1^*(\alpha) + f(r)(\cos\theta\, dx_1 + \sin\theta\, dx_2),
$$

where θ is the angular coordinate coming from an open book decomposition of M compatible with α, r is the radial coordinate in a small neighbourhood of the binding of the open book, f is a suitable function and x_1, x_2 are coordinates on T^2. (For further details of this construction see [1].) The contact structure $\xi' = \ker \alpha'$ intersects the sub-bundle $\mathrm{pr}_1^*(TM)$ in $\mathrm{pr}_1^*(\xi)$, therefore as symplectic vector bundles

$$
(4) \qquad\qquad (\xi', d\alpha'|_{\xi'}) \cong (\mathrm{pr}_1^*(\xi),\ \mathrm{pr}_1^*(d\alpha)) \oplus (E, d\alpha'|_E),
$$

where $E = \mathrm{pr}_1^*(\xi)^{\perp d\alpha'}$ denotes the symplectic complement of $\mathrm{pr}_1^*(\xi)$ with respect to the symplectic form $d\alpha'|_{\xi'}$. Since projection to the second factor maps $\mathrm{pr}_1^*(\xi)^{\perp d\alpha'}$ to the trivial sub-bundle tangent to the T^2 fibers, we obtain the splitting described in (4) above. This symplectic splitting then determines a product complex structure whose stabilisation is homotopic to $\zeta_\varphi \times \zeta_{T^2}$. $\qquad\square$

The final ingredient we need in the proof of Theorem 1.1 is the following result of Eliashberg [5], which realises certain cobordisms via what have become known as Weinstein handle attachments [15].

Proposition 3.4 ([5]). *Suppose that (Y, J) is a compact $(2q+2)$-dimensional almost complex cobordism from M_1 to M_2, where M_1 and M_2 are closed manifolds. Suppose furthermore that $q \geq 2$ and Y admits a handle decomposition with handles of indices $\leq q+1$, and M_1 admits a contact structure with induced almost contact structure being equal to the restriction of J along M_1. Then, the manifold M_2 admits a contact structure.* $\qquad\square$

With these preparatory results at our disposal, we now turn to the proofs of Theorems 1.1 and 1.2.

Proof of Theorem 1.1. For $q = 0$ the manifold M is diffeomorphic to S^1, and $S^1 \times S^2$ is known to admit a contact structure. Consider now a closed contact manifold (M, ξ) of dimension $2q + 1 \geq 3$, and apply the result of Bourgeois [1] to equip $M \times T^2$ with a contact structure. By Proposition 3.3 the almost contact structure induced by this contact structure on $M \times T^2$ is $\zeta_\varphi \times \zeta_{T^2}$ considered in Section 2.

Now consider the cobordism Y given by Proposition 3.1. The stable complex structure may be destabilised to an almost complex structure, which extends the almost contact structure given by the the contact structure constructed by Bourgeois on $M \times T^2$ (and is described in Proposition 3.3). Indeed, since Y is given by attaching handles of index at most $q + 2$ to $M \times T^2$, the obstruction for extending the stable complex structure from $M \times T^2$ coincides with the obstruction for extending the (unstable) complex structure. (This last claim follows from the fact that the embedding $SO(2q + 4)/U(q + 2) \rightarrow SO/U$ induces isomorphisms on the homotopy groups of dimensions $i \leq q + 1 \leq 2q + 2$.)

By Proposition 3.2 the cobordism Y has a handle decomposition with handles of index at most $q + 2$ (as a cobordism built on $M \times T^2$) and it admits an almost complex structure extending the one on $M \times T^2$ supporting a contact structure, therefore Proposition 3.4 implies the claimed existence result. $\square$

Proof of Theorem 1.2. To prove the relative case we consider the cobordism $\widehat{W}$ given by removing a ball from the solid torus $D^2 \times S^1$ and removing an open neighbourhood of an embedded arc joining the boundary components and intersecting S^2 in the point p. The product $M \times \widehat{W}$ is then a cobordism between $M \times (S^2 - D^2)$ and $M \times (T^2 - D^2)$ and there is a natural map $\widehat{W} \rightarrow (S^2 - D^2)$ that is a diffeomorphism on the boundary component corresponding to $S^2 - D^2$.

Then the same argument shows that Proposition 3.1 holds when S^2 and T^2 are replaced by $S^2 - D^2$ and $T^2 - D^2$, respectively. Moreover, the results of Wall [14] apply to cobordisms between manifolds with boundary, when the cobordism between the boundaries of the boundary manifolds is a product, which is the case for the manifold $\widehat{Y}$ that is obtained from $M \times \widehat{W}$ via surgery as in Proposition 3.1. Thus the argument of Proposition 3.2 applies and we conclude that $M \times (S^2 - D^2)$ can be obtained from $M \times (T^2 - D^2)$ via handle attachments of index at most $q + 2$. Now we glue in a copy of $M \times D^2 \times [0, 1]$ along part of the boundary of $\widehat{Y}$ to obtain a bordism from $M \times T^2$ to $M \times S^2$. In order to realise these handle attachments via Weinstein handles [15], one must first apply an h-principle to find an isotopy of the spheres to attaching spheres that are isotropic. Since this can be done in a C^0-small fashion (cf. [4]), we see that all the Weinstein handles can be attached along spheres that are disjoint from $M \times \{p\}$, where $p \in D^2 \subset S^2$. Finally, the contact structure found by Bourgeois on $M \times T^2$ has the additional property that for any $p \in T^2$ the submanifold $M \times \{p\}$ is contact and contactomorphic to (M, ξ). Since contact surgery preserves the contact structure outside a small neighbourhood of the surgery sphere, the result follows. $\square$

4. Final remarks

We point out that one can actually show that all almost contact structures on $M \times S^2$ inducing the stable complex structure $\zeta_\varphi \times \zeta_{S^2}$ admit contact structures: the details will appear in [2]. In addition, the arguments used to prove the existence of a contact structure on $M \times S^2$ actually show the following:

Theorem 4.1. *Suppose that (M, ξ) is a closed contact manifold inducing the stably complex manifold (M, ζ_φ), and that (X^{2k}, ζ_X) is a closed stably complex manifold. Suppose furthermore that (X, ζ_X) satisfies the following conditions:*

- *There is a closed stably complex manifold (Z^{2k}, ζ_Z) and a stably complex cobordism (W, ζ_W) between (X, ζ_X) and (Z, ζ_Z) which admits a map $c \colon W \to X$ restricting to a diffeomorphism along $X \subset W$.*
- *The product manifold $M \times Z$ admits a contact structure ξ' compatible with $\zeta_\varphi \times \zeta_Z$.*

Then $M \times X$ admits a contact structure compatible with $\zeta_\varphi \times \zeta_X$. Moreover, the contact structure on $M \times X$ can be chosen in such a way that for a fixed point $x \in X$ the submanifold $M \times \{x\}$ is a contact submanifold and the natural projection restricted to $M \times \{x\}$ is a contactomorphism to (M, ξ). $\qquad\square$

Example 4.2. It is not hard to see that the manifold $X = S^{i_1} \times \ldots \times S^{i_n}$ with $\sum_{j=1}^n i_j = 2k$ satisfies the conditions of Theorem 4.1, when one further chooses Z^{2k} to be the $2k$-dimensional torus T^{2k}.

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