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Sequential decisions in the Diamond-Dybvig banking model

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Abstract

We study the Diamond-Dybvig model of financial intermediation (JPE, 1983) under the assumption that depositors have information about previous decisions. Depositors decide sequentially whether to withdraw their funds or continue holding them in the bank. If depositors observe the history of all previous decisions, we show that there are no bank runs in equilibrium independently of whether the realized type vector selected by nature is of perfect or imperfect information.

Keywords: Bank Run, Imperfect Information, Perfect Bayesian Equilibrium

JEL Classiffication: C72, D82, G21

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Szekvenciális döntések a Diamond–Dybvig-bankmodellben

Markus Kinateder – Kiss Hubert János

Összefoglaló

A Diamond–Dybvig-modellt (JPE, 1983) vizsgáljuk azon feltevés mellett, hogy a betétesek ismerik a korábbi döntéseket. A betétesek egymás után döntenek arról, hogy kivegyék-e a pénzüket a bankból vagy továbbra is a bankban hagyják. Ha a betétesek megfigyelik az összes korábbi döntést, akkor egyensúlyban nem történhet bankroham, függetlenül attól, hogy a betétesek likviditási típusa megfigyelhető-e vagy sem.

Tárgyszavak: bankroham, tökéletlen információ, tökéletes bayesi egyensúly

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Sequential decisions in the Diamond-Dybvig banking model

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24 October 2013

Abstract

We study the Diamond-Dybvig model of financial intermediation (JPE, 1983) under the assumption that depositors have information about previous decisions. Depositors decide sequentially whether to withdraw their funds or continue holding them in the bank. If depositors observe the history of all previous decisions, we show that there are no bank runs in equilibrium independently of whether the realized type vector selected by nature is of perfect or imperfect information.

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1 Introduction

Several banks and other financial institutions experienced sudden and massive withdrawals of deposits and other funding sources during the recent financial crisis. Examples include the retail bank Northern Rock in the UK, the investment bank Bear Stearns in the US and the DSB Bank in the Netherlands. It is often claimed that not only the deterioration of fundamental variables led to the run of withdrawals, but that there was also a substantial self-fulfilling component to the behavior of depositors. Depositors may rush to the bank to withdraw fearing that other depositors' withdrawals will cause the bank to fail. This idea is borne out by the words of Anne Burke, an ordinary customer of Northern Rock, who—while queuing up to withdraw money from the bank—said: "It's not that I disbelieve Northern Rock, but everyone is worried and I don't want to be the last one in the queue. If everyone else does it, it becomes the right thing to do."[1] Actually, the run on Northern Rock started after the Bank of England announced that it would provide the necessary emergency liquidity support to prevent fundamental problems in the bank. The above quote illustrates that depositors react to other depositors' observed decisions.

As further evidence consider Kelly and O Grada (2000), Starr and Yilmaz (2007), and Iyer and Puri (2012) who empirically analyze real-world bank runs and illustrate that depositors' actions are affected by observable decisions of their peers. Experimental evidence also suggests that observability plays an important role in the emergence of bank runs (see, for example, Garratt and Keister, 2009; Schotter and Yorulmazer, 2009; Kiss et al. 2012). Motivated by the relevance of observability of depositors' decisions, we modify the canonical Diamond-Dybvig model (1983) by assuming that depositors perfectly observe the sequence of actions taken by those who precede them. Therefore, we model a sequential-move game with a finite number of depos-

[1] See http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aeypCkzcRlU4

itors who contact the bank in an exogenously given fixed order to communicate whether to leave the money deposited or to withdraw it. Moreover, following Diamond and Dybvig (1983) and recent models, such as Ennis and Keister (2009), we assume that there is aggregate certainty about liquidity types.[2]

Converting the original Diamond-Dybvig setup in which depositors decide simultaneously into a sequential-move game yields interesting results. When liquidity types and actions are perfectly observed, then no bank run occurs and the Pareto efficient allocation is the unique equilibrium outcome. Our main contribution is to extend this result to the case when the sequence of liquidity types is of imperfect information, that is, a depositor's liquidity type is her private information.

Under perfect information, our result is obtained by backward induction. Waiting dominates withdrawal for the last patient depositor if enough depositors before her waited. Anticipating this decision, the next to last patient depositor's decision is of the same nature, and by moving backwards, all patient depositors wait.

Under imperfect information, the liquidity type vector is randomly selected by nature and is unobservable to the depositors and the bank. Every depositor, as it is her turn to decide, observes previous decisions and forms beliefs about which type vector was selected, or in other words, whether before her withdrawals were due to impatient depositors only or patient ones withdrew as well. Based on her observation, on her belief and on the strategy profile, a depositor determines whether it is optimal for her to withdraw or not. Perfect Bayesian Equilibrium, as defined by Fudenberg and Tirole (1991), imposes a strong rationality criterion on the strategy profile and belief system. This enables us to obtain a unique prediction on depositors' behavior which coincides with the solution under perfect information. On

[2]Banks usually know how much funds are withdrawn on average a day, and any larger withdrawal frequently has to be announced by the depositor to the bank in order for the bank to make the corresponding funds available.

the equilibrium path, patient depositors wait and impatient ones withdraw.

The last decade saw an immense increase in the information flow between people and depositors' decisions are no exception. Hence, a considerable amount of information is available through traditional (TV, radio and newspapers) and social media (Facebook, Twitter, etc.) about how depositors decide. It is of utmost importance for policymakers to know how this abundance of information affects depositors' behavior. This paper attempts to be a step in this understanding by showing that when all previous decisions are observed, bank runs resulting from coordination problems do not occur.

Although we focus on banks, run-like phenomena occur in other institutions and markets as well (such as money-market, hedge or pension funds) and our analysis applies analogously to them.

Related literature

In the classic Diamond-Dybvig framework multiple equilibria exist, and the Pareto efficient outcome of no bank run is no unique equilibrium. This suggests that banks are intrinsically fragile and susceptible to self-fulfilling runs. The literature attempts to identify elements that lead to this kind of fragility and conditions to mitigate it. There are two approaches in the literature: one is game theoretic and the other based on mechanism design.

Given certain constraints, the mechanism design strand of the literature studies how to optimally assign consumption to depositors depending on their announcements.[3] For example, Green and Lin (2003) add aggregate uncertainty about liquidity needs to the Diamond-Dybvig framework and assume that depositors know the order in which they have an opportunity to withdraw. The bank updates its belief about the type distribution after each decision and optimizes the contract accordingly. As a result, complex contracts arise that are contingent on the exact sequence of announcements

[3] Usually a direct revelation mechanism is studied: when contacting the bank, depositors tell the bank their type. When a depositor announces to be impatient, the bank assigns her an optimal consumption based on the available information.

and payments to depositors may be fairly variable. Nevertheless, the Pareto efficient allocation is shown to be the unique equilibrium outcome.

Regarding the game theoretic approach, the first step is to find the Pareto efficient allocation which a social planner would choose if she knew the type vector in the economy. Then, the outcomes of a game are studied assuming that types are imperfect information. In the Diamond-Dybvig setup with aggregate certainty about liquidity types the first best yields an optimal simple demand deposit contract that determines how much money the bank should pay to those who withdraw in the early period and together with the number of early withdrawals consumption in the second period is determined. If the game is specified as a simultaneous-move game, then a bank run and a no bank run equilibrium arises. However, Diamond and Dybvig (1983) show that even if the optimal simple demand deposit contract is maintained, the Pareto efficient allocation becomes the unique equilibrium outcome if the simultaneous-move game is complemented by a suspension of payments clause. It stipulates that, after a certain number of withdrawals, payment to subsequent depositors is suspended. The mere expectation of suspension is enough to rule out bank runs.

Our paper applies the game theoretic approach. We build on the original Diamond-Dybvig setup and use their optimal simple demand deposit contract. As Ennis and Keister (2010) point out, the contract that implements the optimal allocation in the Green-Lin model is highly contingent on the available information, and hence results in volatile payments to depositors of the same liquidity type. However, in reality, we observe stable payments to depositors and the face value of deposits is respected most of times. These features are more akin to the simple demand deposit contract á la Diamond and Dybvig (1983). We maintain this optimal simple demand deposit contract and change the game by allowing the exact sequence of previous actions to be observed, though not the liquidity type vector.

As already mentioned, observability is an essential element of a depositor's decision. The empirical studies cited above suggest that, at least to some extent, both waiting and withdrawal may be observed. At first it appears questionable how waiting can be observed. However, Kelly and O Grada (2000) show that although there were other factors, the most important one determining whether an individual panicked or not in New York during a bank run episode in the 19th century was his county of origin in Ireland. This common origin presumably had an effect since immigrants from the same county lived in the same neighborhood and observed each other. Iyer and Puri (2012) stress the importance of observing decisions of both sorts in one's social network when studying a bank run that occurred in India in 2001. In Starr and Yilmaz (2007), small and medium-sized depositors of an Islamic bank in Turkey seemed to observe only withdrawals of their peers during a bank run incident in 2001, but the behavior of large depositors appears to be driven by observing both actions.

Our paper shows that even without a suspension-of-convertibility clause, no bank run is the unique equilibrium outcome if the history of previous actions is observed. The paper closest to ours regarding the emphasis put on observability is Andolfatto et al. (2007) who are the first to assume the observability of the history of announcements. In the spirit of Green and Lin (2003), they use a mechanism design approach in an environment characterized by aggregate uncertainty about liquidity needs and show that any allocation that is implementable is also strongly implementable. The role of observability is intricate. On the one hand, knowing the complete history allows a depositor to condition her action on it. This strengthens the incentive compatibility constraints, implying that fewer allocations are implementable. On the other hand, a depositor prefers to announce her type truthfully if she believes that those who follow her will do so as well. In that sense, observing previous decisions does not affect the optimal decision. Contrary to Andolfatto et al. (2007), we assume that the bank pays the same amount of money to withdrawing depositors as long as it has funds left. In our model, the bank may run out of funds, leaving depositors who wish to withdraw unpaid. This cannot happen in Green and Lin (2003) or Andolfatto et al. (2007). Depositors condition their choice on the history of previous decisions and we consider all possible histories after which a depositor is asked to decide. Sometimes, given certain histories and independently of what subsequent depositors do even a patient depositor is strictly better off to withdraw. We determine all histories that imply truthful reporting for subsequent depositors and find that as the game unfolds only these histories arise. In a Perfect Bayesian Equilibrium, early depositors correctly anticipate to be on the equilibrium path, take the optimal decision and lead the game down the path to the unique equilibrium outcome with no bank run.

The paper is organized as follows. Section 2 introduces notation and defines the model. In section 3, we provide examples and the general results, while section 4 concludes the paper. All proofs are relegated to the appendix.

2 The model

There are three time periods denoted by $t = 0, 1, 2$, and a finite set of depositors denoted by $I = \{1, ..., N\}$, where $N > 2$. The consumption of depositor $i \in I$ in period $t = 1, 2$ is denoted by $c_{t,i} \in \mathbb{R}^0_+$, and her liquidity type by θ_i. This is a binomial random variable with support given by the set of liquidity types $\Theta = \{0, 1\}$. If $\theta_i = 0$, depositor i is called *impatient*, that is, she only cares about consumption at $t = 1$. If $\theta_i = 1$, depositor i is called *patient*. Given $\theta_i \in \{0, 1\}$, each depositor i's utility function is given by

$$u_i(c_{1,i}, c_{2,i}, \theta_i) = u_i(c_{1,i} + \theta_i c_{2,i}).$$

It is assumed to be strictly increasing, strictly concave, twice continuously differentiable and to satisfy the Inada conditions. The relative risk-aversion coefficient, $-c_i u_i''(c_i)/u_i'(c_i)$, is assumed to be strictly larger than 1, for all $c_i \in \mathbb{R}_+$, and all $i \in I$.

At $t = 0$, each depositor $i \in I$ has one unit of a homogeneous good which she deposits in the bank. The bank has access to a constant-return-to-scale productive technology which pays a gross return of one unit for each endowment liquidated at $t = 1$, and a fixed return of $R > 1$ for each endowment liquidated at $t = 2$. It offers a simple demand deposit contract which pays c_1^*, which is determined below, to any depositor i who withdraws at $t = 1$, as long as the bank has funds left, and the same pro rata share of funds available to all depositors who wait until $t = 2$.

The number of patient depositors is assumed to be constant and given by $p \in \{1, ..., N\}$ and the remaining depositors are impatient. The number of patient and impatient depositors is common knowledge. However, each depositor's type is only realized at $t = 1$.

Let $\Theta^N = \{0, 1\}^N$, and $\theta^N = (\theta_1, ..., \theta_N)$ denote the sequence of depositors, also called (liquidity) type vector. The set of sequences of length N with p patient depositors is given by

$$\Theta^{N,p} = \{\theta^N \in \Theta^N : \sum_{i=1}^{N} \theta_i = p\}.$$

There are $\binom{N}{p}$ possible type vectors. At $t = 1$, one is selected randomly by a process which selects each of them with equal probability. Under imperfect information, the realized liquidity type vector is unobserved both by the depositors and the bank, while it is observable under perfect information.

Next, the Pareto efficient allocation is derived. A social planner could maximize the sum of depositors' utilities (which are assumed to be identical, except of the liquidity type) with respect to $c_{1,i}$ and $c_{2,i}$ subject to a resource constraint and to the commonly known number of patient and impatient depositors, p and $N - p$, respectively. The first best allocation solves

$$\max_{c_{1,i}, c_{2,i}} (N - p) u_i(c_{1,i}) + p u_i(c_{2,i})$$

$$\text{s. t. } (N - p) c_{1,i} + \tfrac{p}{R} c_{2,i} = N.$$

The solution to this problem is

$$u'(c_1^*) = Ru'(c_2^*),$$

which, as in Diamond and Dybvig (1983), implies that $R > c_2^* > c_1^* > 1$. In the first best allocation, all impatient depositors consume c_1^* at $t = 1$, and all patient ones c_2^* at $t = 2$. Hence, patient depositors receive a higher consumption than impatient ones.

2.1 Strategies and equilibrium concept

A sequential service constraint is assumed to hold, that is, at $t = 1$, the depositors contact the bank sequentially in the order given by θ^N, and the payment to any withdrawing depositor only depends on the history, but not on the decisions of subsequent depositors, as will be specified below.

Depositor i's strategy $\mathbf{s}_i \in \{0, 1\}$ is to announce a type from Θ. When type 0 or type 1 is announced, the depositor wishes to withdraw or wait, respectively. Anonymity is assumed, that is, the depositors' indexes do not reveal any information. Each depositor i is assumed to observe the entire history of previous type announcements $s^{i-1} = (s_1, ..., s_{i-1})$, where $s^{i-1} \in \mathbf{\Theta}^{i-1}$. Depositor i's strategy is conditional on the history and her type. It is defined as $\mathbf{s}_i : \mathbf{\Theta}^{i-1} \times \Theta \rightarrow \Theta$. Let $\mathbf{S} = \{0, 1\}^N$ be the game's strategy space, and let $\mathbf{s} \in \mathbf{S}$ be a strategy profile, that is, $\mathbf{s} = (\mathbf{s}_1, ..., \mathbf{s}_N)$. In order to emphasize depositor i's strategy, $\mathbf{s}$ is sometimes written as $(\mathbf{s}_i, \mathbf{s}_{-i})$.

Given strategy profile $\mathbf{s} \in \mathbf{S}$, depositor i's consumption is specified by $c_i = (c_{1,i}; c_{2,i})$, where $c_{1,i} : \mathbf{\Theta}^i \rightarrow \mathbb{R}_+^0$, and $c_{2,i} : \mathbf{\Theta}^N \rightarrow \mathbb{R}_+^0$. The consumption of all depositors is feasible if $\sum_{i=1}^N (c_{1,i} + \frac{c_{2,i}}{R}) \leq N$. Depositor i's period-1 consumption is then defined as

$$c_{1,i} = \begin{cases} c_1^*, & \text{if } s_i = 0 \text{ and } N - \sum\limits_{j=1}^{i-1} s_j c_1^* \geq c_1^*, \\ y, & \text{if } s_i = 0 \text{ and } 0 < N - \sum\limits_{j=1}^{i-1} s_j c_1^* < c_1^*, \\ 0, & \text{otherwise,} \end{cases}$$

where $y = N - \sum_{j=1}^{i-1} s_j c_1^*$: until the bank runs out of funds, any depositor who announces to be impatient receives a positive consumption c_1^* or y.

Let $\eta \in \{0, ..., p\}$ be the number of depositors who wait at $t = 1$, that is, each of them announces to be of type 1.[4] Given $\eta = \sum_{i=1}^{N} s_i \geq 0$, all players who wait at $t = 1$, obtain the same consumption at $t = 2$, namely,

$$c_2(\eta) = \max\{0, \tfrac{R(N-(N-\eta)c_1^*)}{\eta}\}.$$

If $\eta = p$, that is, only impatient depositors withdraw at $t = 1$, then $c_2(\eta) = c_2^* > c_1^*$, and patient depositors enjoy a higher consumption than impatient ones.

The consumption in both periods depends on the strategy profile and determines each depositor's utility. For any $i \in I$, and any $\mathbf{s} \in \mathbf{S}$, this is denoted by $u_i(\mathbf{s})$. Thus, u_i is a mapping from $\mathbf{S}$ to $\mathbb{R}_+^0$. Let the tuple $(I, \mathbf{S}, u)$ be the *bank run game*, where $u = (u_1, ..., u_N)$.

Any depositor i observes history s^{i-1}, knows her type θ_i and the commonly known parameters p and N. However, under imperfect information, she does not observe the realized type vector and both patient and impatient depositors may choose to withdraw. Therefore, given the available information, she forms beliefs about the type vector that was selected by nature. Let $\mu_i \equiv \mu_i(\theta^N \mid s^{i-1}, \theta_i)$ denote depositor i's belief about the *true* type vector. This belief is conditional on the history and i's type and is updated according to Bayes' rule whenever possible. The belief together with a strategy profile defines a Perfect Bayesian Equilibrium.

Definition 1. *Given a bank run game. Then, strategy profile* $\mathbf{s} \in \mathbf{S}$ *and belief system* $\mu = (\mu_1, ...\mu_N)$ *are a Perfect Bayesian Equilibrium (PBE) if, and only if, for all* $i \in I$, *given* θ_i, s^{i-1} *and any* $\tilde{\mathbf{s}}_i \in \{0, 1\}$,

[4] Note that η is restricted to be equal to p or smaller since an impatient depositor has a dominant strategy to withdraw, and thus, not more than p depositors will wait.

$$\sum_{\theta^N \in \Theta^N} \mu_i(\theta^N \mid s^{i-1}, \theta_i) u_i(\mathbf{s}) \;\; \geq \;\; \sum_{\theta^N \in \Theta^N} \mu_i(\theta^N \mid s^{i-1}, \theta_i) u_i(\tilde{\mathbf{s}}_i, \mathbf{s}_{-i}),$$

where $\mu_i(\theta^N \mid s^{i-1}, \theta_i)$ is consistent with Bayes' rule whenever possible.

A strategy profile and a system of beliefs are a PBE if, and only if, the strategy is sequentially rational for all players and the belief is consistent with the strategy (see Fudenberg and Tirole, 1991, and Myerson, 1997). Moreover, there are consistency requirements on the beliefs that arise from the fact that p and N are commonly known and also since an impatient depositor's dominant strategy is to withdraw. These are discussed in more depth in the section on imperfect information below.

3 Results

The simple demand deposit contract defined above yields the Pareto efficient allocation (see Diamond and Dybvig, 1983). Our goal is to show that this allocation is the unique PBE outcome of the bank run game.

Given p, N and c_1^* it is possible to determine how many patient depositors have to wait in order for waiting to be an optimal strategy for each of them. In Lemma 1, one part of this threshold is derived,[5] namely, the one (denoted as $\bar{\eta}$) such that $c_{2,i} > c_1^*$, for every patient depositor i who waits at $t = 1$. If some patient depositor declares to be impatient, then the bank spends funds on her which it would otherwise have kept until $t = 2$. Recall that η is the number of patient depositors that wait.

Lemma 1. *Given p, N and c_1^*, there is a unique $\bar{\eta}$ such that $1 \leq \bar{\eta} \leq p$, and for every patient depositor i for whom $\mathbf{s}_i = 1$, $c_{2,i}(\eta) < c_1^*$, for all $\eta \leq \bar{\eta}$, and $c_{2,i}(\eta) > c_1^*$, for all $\eta > \bar{\eta}$.*

The proof of Lemma 1 can be found in Appendix A.

[5] The other part is a technical detail which is derived below in Proposition 1's proof.

3.1 The type vector is perfect information

The benchmark case with perfect information is studied next. The depositors commonly know the number of patient depositors p, and each depositor's type, or in other words, the type vector selected randomly by nature.

Any impatient depositor i has a dominant strategy to withdraw, and thus, $s_i(s^{i-1}, \theta_i = 0) = 0$ given any s^{i-1}. By eliminating uncertainty about the type vector we can apply standard backward induction arguments to find the equilibrium in Proposition 1.[6]

Proposition 1. *Given a bank run game. Suppose that the type vector is perfect information. Then, the Pareto efficient allocation is the unique PBE outcome and depositors tell the truth.*

The proof of Proposition 1 can be found in Appendix B. Since the type vector is of perfect information, the concepts of subgame perfect equilibrium and PBE coincide. Intuitively, the last patient depositor's optimal decision is to wait if enough preceding patient depositors waited so that her consumption in period 2 is higher than that received upon immediate withdrawal. Anticipating this decision, the next to last patient depositor's optimal decision is to wait, and by moving backward all patient depositors wait.

Apart from the unique PBE outcome derived in Proposition 1, the set of Nash Equilibria of this bank run game contains strategy profiles in which all patient depositors withdraw, and therefore, a bank run occurs. However, these equilibria are not subgame perfect and are eliminated when requiring the additional rigor of PBE or subgame perfectness imposed by sequential rationality and backward induction, respectively.

[6] Note that if types are observable, then the bank could impose the first best allocation by force, denying to pay to patient depositors. However, we disregard this possibility and show that the depositors' decisions lead to the first best allocation.

3.2 The type vector is imperfect information

When the type vector is not observable, depositors cannot apply the previous reasoning. Nevertheless and as before, they commonly know p and N, and that nature selects each type vector with equal probability. Moreover, each depositor knows her own type and observes the history. This is referred to as *available information*. Given the available information, a depositor forms beliefs about the type vector selected by nature and, by sequential rationality, anticipates how subsequent depositors behave. In this environment of imperfect information, sequential rationality plays a similar role as backward induction in games of perfect information (see Myerson, 1997). Before proving the general result, the difficulties that arise are illustrated in an example.

3.2.1 Example

Suppose that there are four depositors: one is impatient and the other three are patient. Before the game begins, nature selects each of the four possible type vectors with equal probability. Once the type vector is selected, each depositor observes her type but not any other's. Then, they take decisions in a sequential order. Also, all three patient depositors have to keep the money in the bank in order to make waiting worthwhile for all of them.

We depict the corresponding extensive form of the game in Figure 1, where p stands for patient, i for impatient, *keep* for wait, and *wi* for withdraw, and where the outcome of each branch of the game is depicted, that is, bank run and no bank run, respectively. There are several information sets that are not singletons: for instance, a patient depositor in position 2 who observes a withdrawal does not know whether she is in a type vector that starts with a patient depositor who decided to withdraw or in the type vector that begins with the impatient depositor. Finally, in order to simplify the figure, we suppose that each impatient depositor withdraws since this is a dominant strategy for her, as is shown below. We will go through all possible strategy profiles and show that no bank run is the unique PBE outcome.

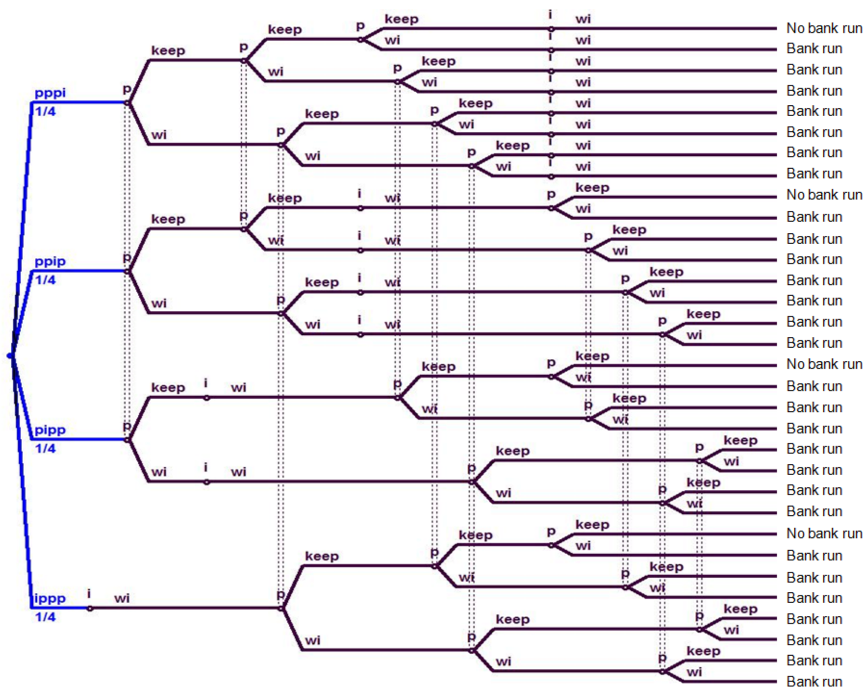

Figure 1: Extensive form game of the example

Belief updating and consistency of beliefs

Since each type vector is selected by nature with equal probability, a patient depositor in position $k \in \{1, 2, 3, 4\}$ assigns the same probability to each sequence in which the impatient depositor is at position $j \in \{1, 2, 3, 4\}$ such that $j \neq k$. That is, k's updated prior belief assigns a probability of $\frac{1}{3}$ to any of these sequences. Posterior belief updating takes into account the observed decisions.

Consider first a separating equilibrium, in which depositors' strategies reveal their type. Since an impatient depositor is never strictly better off to wait we focus on a separating equilibrium in which any patient depositor is asked to wait and the impatient one to withdraw. For instance, consider a patient depositor in position 2. Her updated prior assigns a belief of $\frac{1}{3}$ to type vectors $(0,1,1,1)$, $(1,1,0,1)$ and $(1,1,1,0)$, respectively. On the equilibrium path, a depositor's observation of the previous depositors' decisions reveals their types and she also knows her own type. Suppose now that she observes that depositor 1 waits. Then, her updated posterior, μ_2, assigns probability $\frac{1}{2}$ each to type vectors $(1,1,1,0)$ and $(1,1,0,1)$. However, if a withdrawal is observed, then μ_2 assigns probability 1 to type vector $(0,1,1,1)$. Depositor 2 thus updates her belief in a Bayesian way and belief updating is consistent with the available information and the proposed strategy profile.

On any off-equilibrium path, at least one patient depositor withdraws. Consider now a patient depositor in position 3. Given her available information, her updated prior assigns probability $\frac{1}{3}$ to each sequence $(1,1,1,0)$, $(1,0,1,1)$ and $(0,1,1,1)$. After observing two withdrawals, she knows that she is off the equilibrium path. Since this history has an ex ante 0-probability, her belief updating is unconstrained and her posterior belief μ_3 may assign any probability that is consistent with her observation. As it turns out, on an off-equilibrium path, it is enough for depositors to take into account the observed history and their strategy is optimal given any consistent belief—and there are several since the history has an ex ante 0-probability.

In a pooling equilibrium, observations are not informative since both types of depositors take the same decision. Since an impatient depositor is never better off to wait, we focus on the case in which some patient depositor withdraws. Suppose that a patient depositor 1 should withdraw according to her strategy, that is, independently of her type, depositor 1 is asked to withdraw. On the equilibrium path, a patient depositor 2 observes a withdrawal and her posterior belief μ_2 assigns equal probability to each type vector in which depositor 2 is patient. Suppose that on an off-equilibrium path, a patient depositor 2 observes a waiting. This is off-equilibrium since depositor 1 is asked to withdraw independently of her type. In this case, depositor 2's posterior belief updating is unconstrained since this history has an ex ante

0-probability. Depositor 3 also realizes to be on an off-equilibrium path and updates her belief again in a Bayesian way based on depositor 2's updated posterior belief.

PBE requires that the belief system is consistent with the strategy profile. Moreover, it has to be consistent with the available information, that is, a depositor's type, her observed history, the commonly known parameters p and N, and that each type vector is selected by nature with equal probability.

Deriving the PBE

Consider the following *PBE candidate*: independently of the realized type vector, on the equilibrium path, each patient depositor waits, while the single impatient depositor withdraws. Each depositor's prior belief is obtained as stated above. On the equilibrium path, each depositor obtains her posterior belief by updating her prior in a Bayesian way, taking into account the available information. In particular, any depositor before her who waited is believed to be patient, while a depositor who withdrew is believed to be impatient (as long as there was at most one withdrawal). Each order among the remaining patient and impatient depositors that she believes are behind her in the queue is equally likely. The depositors' off-equilibrium path strategies and beliefs are derived below for each depositor.

Since an impatient depositor has a dominant strategy to withdraw, her strategy is optimal after any history and given any belief.[7] She has no profitable deviation from withdrawing and, given that she is impatient, her updated prior belief assigns probability 1 to the true type vector selected by nature since the other three depositors are patient.

Consider now the patient depositors' complete strategy and belief system: *Depositor 4:* On the equilibrium path, she identifies the type vector since, given the strategy profile, there are two patient depositors before her who waited and an impatient one who withdrew. By Bayesian belief updating,

[7] Though on an off-equilibrium path on which the bank ran out of funds she is indifferent to wait or not since her utility is 0 anyway, and thus, her strategy is not strictly dominant.

μ_4 assigns probability 1 to the true type vector selected by nature.

In case she waits, she receives $u_4(\theta_4 c_2^*)$, while she receives $u_4(c_1^*)$ if she withdraws. Since she is patient, that is, $\theta_4 = 1$, it holds that $u_4(\theta_4 c_2^*) > u_4(c_1^*)$, and she is strictly better off to wait. Thus, her strategy is sequentially rational given her belief and her belief is consistent with the strategy.

Suppose that she observes an off-equilibrium path history. Then, her belief and her corresponding off-equilibrium path strategy are as follows:

After observing history (1, 0, 0) or (0, 1, 0), her belief updating is unconstrained since both histories have an ex ante 0-probability. After history (0, 0, 1) or (0, 0, 0), depositor 4 updates μ_4 according to Bayes' rule, though depositor 3 updated μ_3 after an unexpected history in an unconstrained way. In any of the four cases, depositor 4 observed two or three withdrawals before her. Since by assumption waiting is worthwhile if, and only if, all three patient depositors wait, she cannot deviate profitably from withdrawing. If she withdraws, then she receives a higher utility than if she waits, unless the bank ran out of funds. Her decision is sequentially rational given any consistent belief and any belief is consistent with the strategy since an off-equilibrium path is reached and belief updating is unconstrained.

Suppose next that depositor 4 observes history (1, 1, 1), that is, all three preceding depositors wait. Then, depositor 4 is the first to find herself on an off-equilibrium path and her belief updating is unconstrained. However, independently of her updated posterior belief, her expected utility is $u_4(\theta_4 c_2^*)$ if she waits and $u_4(c_1^*)$ if she withdraws. It holds again that $u_4(\theta_4 c_2^*) > u_4(c_1^*)$, and she is strictly better off to wait. Given any belief she may have, her strategy is sequentially rational, and even more, it is optimal given the observed history and her belief is consistent with the strategy.

Depositor 3: There are four possible histories a patient depositor 3 can observe. After observing histories (1, 1), (1, 0) and (0, 1), she believes to be on the equilibrium path and Bayesian updating assigns a posterior belief of probability 1 to type vectors (1, 1, 1, 0), (1, 0, 1, 1) and (0, 1, 1, 1), respectively. While in the first case, depositor 3 is the third depositor that waits, in both others, she is the second one that waits. However, in the last two cases, she believes with probability 1 that the last depositor is patient and by sequential rationality anticipates that this depositor will wait.

If she waits, her expected utility is $u_3(\theta_3 c_2^*)$, since her updated posterior belief assigns probability 1 to the true type vector, while it is $u_3(c_1^*)$ if she withdraws. Since she is patient, that is, $\theta_3 = 1$, it holds that $u_3(\theta_3 c_2^*) > u_3(c_1^*)$, and she is strictly better off to wait. Her strategy is sequentially rational given her belief, and her belief is consistent given the strategy.

Note that if a patient depositor in position 1 or 2 withdraws, and the other depositor waits, then the game reached an off-equilibrium path, though for depositor 3 the observed history is consistent with being on the equilibrium path and she behaves as if this were the case. Given her consistent belief, she is strictly better off to wait.

Finally, suppose that she observes history (0, 0), that is, the only possible off-equilibrium path history a patient depositor 3 can identify. Since this history has an ex ante 0-probability, belief updating is unconstrained. In any case, if she withdraws, then she receives a higher utility than if she waits, unless the bank ran out of funds. Her strategy is optimal given the observed history and there are several consistent beliefs.

Depositor 2: She either observes that depositor 1 waits or withdraws. In both cases, her observation is consistent with the equilibrium path (since there is one impatient and two patient depositors apart from her), even if an impatient depositor 1 waits or a patient one withdraws. After observing a waiting, consistency of beliefs requires that her updated posterior belief μ_2 assigns probability $\frac{1}{2}$ respectively to type vectors (1, 1, 0, 1) and (1, 1, 1, 0), and after observing a withdrawal, that it assigns probability 1 to type vector (0, 1, 1, 1). Both updated posterior beliefs are obtained by Bayes' rule and are consistent with the strategy profile in which all patient depositors wait.

If she observes a waiting and waits herself, by sequential rationality, she

anticipates that the last patient depositor behind her waits (after observing two waitings), and if she observes a withdrawal, then she believes that the two remaining depositors after her in the queue are patient. By sequential rationality she anticipates that both of them wait. In any case, her expected utility by waiting is $u_2(\theta_2 c_2^*)$. If she withdraws instead, then her expected utility is $u_2(c_1^*)$. Since she is patient, that is, $\theta_2 = 1$, it yields her a higher utility to wait. Depositor 2's updated posterior belief μ_2 is consistent with the strategy profile which is sequentially rational given her belief.
Depositor 1: She knows that she is patient and μ_1, the updated prior, assigns an equal probability of $\frac{1}{3}$ to type vectors $(1,1,1,0)$, $(1,1,0,1)$ and $(1,0,1,1)$, respectively. By sequential rationality, she anticipates that, on the equilibrium path, the other two patient depositors wait, and her expected utility by waiting is $u_1(\theta_1 c_2^*)$, while it is $u_1(c_1^*)$ if she withdraws. Since she is patient, it yields her a higher utility to wait. Belief μ_1 is consistent with the strategy profile which is sequentially rational given her belief.

Given this strategy, any depositor's decision on the equilibrium path is fully revealing for the subsequent depositors and no bank run is a PBE.

Are there other consistent belief systems?
On the equilibrium path, any other belief system is not consistent with the strategy profile since it assigns a positive probability to a depositor who withdraws to be patient or to a depositor who waits to be impatient or both. Even more, it is not consistent with the available information. However, on some off-equilibrium path, while the depositors' strategies are optimal given the observed history, a depositor's posterior belief updating is unconstrained and there are several possible consistent beliefs. Thus, the consistent belief system is not unique. Yet, the strategy profile is a PBE and yields no bank run as outcome.

Uniqueness of PBE outcome
We are only interested in showing that the unique PBE outcome is no bank

run, though there are multiple PBE which differ by beliefs and strategies of depositors off the equilibrium path. Once the bank ran out of funds, any depositor, whether patient or not, is indifferent to wait or withdraw, and this gives rise to multiple off-equilibrium path strategies compatible with the unique PBE outcome (on the equilibrium path). However, since no depositor has a profitable deviation, and thus, no off-equilibrium path is ever reached, it holds that no bank run is the unique PBE outcome as long as there are no other PBE in which there is a bank run.

In any other PBE candidate, either a patient depositor withdraws or an impatient one waits on the equilibrium path or both. Since an impatient depositor's dominant strategy is to withdraw, we only consider all strategy profiles in which one or more patient depositors withdraw.

Suppose first that all three patient depositors withdraw. We show next that this is no PBE since some patient depositor has a profitable deviation. Later we do the same for the strategy profiles in which one or two patient depositors withdraw. In this way, we consider all possible strategy profiles, and after showing that a bank run is never a PBE outcome, it follows that no bank run is the unique PBE outcome.

PBE candidate: all three patient depositors withdraw

The impatient depositor's dominant strategy is to withdraw. Moreover, given her knowledge of her own type and the common knowledge that the other three depositors are patient, her updated prior and posterior belief assigns probability 1 to the true type vector selected by nature.

Consider now the decision of any patient depositor. A patient depositor 4 observes three withdrawals on the equilibrium path. Given the available information, she cannot update her prior belief about the type vector selected by nature since the strategies of the preceding depositors do not reveal their type. Since all three patient depositors have to keep the money in the bank in order for waiting to be worthwhile, she is either indifferent to wait or to withdraw if the bank ran out of funds, or she is strictly better off to withdraw if the bank still has funds left. Thus, she withdraws. This behavior is sequentially rational and her belief is consistent given her strategy.

In order to analyze her off-equilibrium behavior, we focus on a specific history for which we will show that some depositor has a profitable deviation.

Suppose that on an off-equilibrium path, a patient depositor 4 observes that two depositors waited before her. Then, by waiting she receives a strictly higher utility than by withdrawing and it is sequentially rational for her to wait given the observed history. The depositor who observes the first waiting updates her belief in an unconstrained way since she finds herself on an off-equilibrium path. However, it is consistent with the available information for a patient depositor 4 to believe that both depositors that waited are patient. Thus, μ_4 assigns probability 1 to the type vector in which the depositor that withdrew is impatient. This belief is consistent with this off-equilibrium path strategy which in turn is sequentially rational given her belief.

Consider now a patient depositor 3. On the equilibrium path, she observes two withdrawals and cannot update her belief from the prior. She is strictly better off to withdraw than to wait as long as the bank has still funds left.

Suppose that on an off-equilibrium path she observes that two depositors waited before her. Then, by waiting she receives a strictly higher utility than by withdrawing and it is sequentially rational for her to wait given the observed history. Similarly as before, one consistent updated belief μ_3 is that both depositors that chose to wait are patient. In this case, μ_3 assigns probability 1 to the type vector in which the last depositor is impatient. This belief is consistent with the available information and the strategy which in turn is sequentially rational given her belief.

If depositor 3 observed one waiting and one withdrawal, then she understands that the depositor before her that waited brought the game onto an off-equilibrium path. If the first depositor withdrew and the second waited, then she is not sure whether depositor 1 is patient and complied with the equilibrium strategy or is impatient. However, if depositor 1 waits, then it is consistent with the available information for depositor 3 to believe that depositor 1 is patient and that depositor 2 who withdrew is impatient since a patient depositor 2—after observing a waiting—would have waited as well.[8] Then, depositor 3 receives a strictly higher utility by waiting than by withdrawing given depositor 4's off-equilibrium path strategy, that is, she believes that depositor 4 is patient and will wait as well. This belief is consistent with the strategy which is sequentially rational given her belief.

Consider now a patient depositor 2. On the equilibrium path, she observes one withdrawal and cannot update her belief from the updated prior. Thus, she is strictly better off to withdraw than to wait.

Suppose that on an off-equilibrium path she observes that depositor 1 waits. Then, she understands that depositor 1 brought the game onto an off-equilibrium path. Since this history has an ex ante 0-probability her belief updating is unconstrained. However, it is consistent with the available information for a patient depositor 2 to believe that it is equally likely for the impatient depositor to be in the third or fourth position, respectively, and for the third patient depositor to be in the other. By sequential rationality, she anticipates that, if she waits, then the last patient depositor will wait as well. This yields her a higher expected utility and she waits. Her belief is consistent with the strategy which in turn is sequentially rational given her belief.

Consider now a patient depositor 1. On the equilibrium path, she should withdraw. However, she can increase her expected utility by deviating, that is, by waiting. By sequential rationality she anticipates that then the second patient depositor waits independently of whether she is in position 2 or 3. Given this, the third patient depositor (in position 3 or 4) waits as well. Hence, there is no bank run. Depositor 1 cannot update her prior apart form taking into account that she is patient. This belief is consistent given the

[8]A patient depositor 2 would wait since upon observing two waitings the remaining patient depositor either in position 3 or 4 would wait by sequential rationality.

strategy profile and is obtained by Bayesian updating.

Given this strategy profile and a type vector in which the first depositor is patient, there is no bank run since depositor 1 has a profitable deviation to lead the game onto an off-equilibrium path on which—based on the previous arguments—all patient depositors wait. Therefore, for all depositors to withdraw is no PBE.

The behavior of patient depositors on an off-equilibrium path is sequentially rational given the observed history. Even if they formed other beliefs than stated—and whenever the history has an ex ante 0-probability, belief updating is unconstrained—, then for all depositors to withdraw is no PBE since a patient depositor 1 has a profitable deviation to wait.

Finally, note that patient depositors in positions 2 to 4 have profitable deviations if they were asked to withdraw on some off-equilibrium path on which they observe a certain number of waitings before them. More concretely, depositor 3 and 4 wait if they observe two waitings and depositor 2 waits after a waiting. Similarly, depositor 3 waits even after observing a waiting followed by a withdrawal since using sequential rationality she can infer that depositor 2 was impatient, and hence, by waiting she can induce the last patient depositor to wait as well.

Similarly, a strategy profile in which one or two patient depositors are asked to withdraw is no PBE. To see this, consider first that two patient depositors are asked to wait and one to withdraw. This is no PBE since there is a type vector in which depositors 1 and 2 are patient and wait, and the third patient depositor in the queue who is asked to withdraw has a profitable deviation to wait after observing two waitings before her.

Suppose next that two patient depositors are asked to withdraw and one to wait. Then, analogously as above, it is possible to construct off-equilibrium paths such that the first patient depositor in the queue who is asked to withdraw has a profitable deviation to wait, and any other patient depositor behind her will follow suit. If depositor 1 waits, then by sequential rationality a patient depositor 2 waits as well (even if she were asked to withdraw) since after observing two waitings she anticipates that the last patient depositor will wait as well. Then, the third patient depositor in the queue also waits.

Hence, in this example, no bank run is the unique PBE outcome since given any other strategy profile some depositor has a profitable deviation.

3.3 The general case

The arguments in the previous subsection are generalized in order to find the set of PBE for any bank run game and the unique PBE outcome is that no bank run occurs which is the Pareto efficient allocation.

Proposition 2. *Given a bank run game. Suppose that the type vector is imperfect information. Then, the Pareto efficient allocation is the unique PBE outcome.*

The proof of Proposition 2 can be found in Appendix C. Intuitively, in any PBE, given the available information, it is consistent for a patient depositor to believe to be on the equilibrium path as long as there are $N - p$ or less withdrawals, that is, unless she observes a history which is incompatible with being on the equilibrium path. She waits and anticipates, by sequential rationality, that all other patient depositors behind her will wait as well. Since to wait yields each of them a strictly larger consumption, for each of them, it is optimal to wait. This in turn generates a history which induces all other patient depositors to wait, while all impatient ones withdraw.

No bank run is the unique PBE outcome. However, there are several PBE which all are identical on the equilibrium path, though they differ on off-equilibrium paths. To see this, consider any history in which the bank ran out of funds since there were too many withdrawals. Then, any depositor is indifferent to wait or to withdraw since her utility is zero in any case, and she has no profitable deviation. Therefore, it is possible to construct multiple

PBE that differ, for example, by the depositors' optimal behavior after the bank went bankrupt. However, on the equilibrium path, such a history never occurs: patient depositors always wait and impatient ones always withdraw, and the unique PBE outcome is no bank run. In Proposition 2's proof it is shown that any other strategy profile that can arise is no PBE. Analogously to the example provided above, given any other strategy profile, there is a type vector for which some patient depositor has a profitable deviation and by leading the game down an off-equilibrium path, all patient depositors are better off to wait. By doing this, each of them receives a higher payoff.

Finally, note that other Bayesian Nash Equilibria exist. However, all of them are based on incredible threats, and thus, are no PBE. For example, for all depositors to withdraw is a Bayesian Nash Equilibrium of the game which results in a bank run. However, in a PBE, each patient depositor upon being called to decide is better off to wait than to withdraw, unless she would receive a lower period-2 consumption. Since all patient depositors' reasoning is identical and, by sequential rationality, each of them anticipates that all subsequent patient depositors will wait as well, in a PBE, each of them is better off to wait. The rigor of PBE makes no bank run the unique outcome.

There is some experimental evidence supporting our finding. Kiss et al. (2012) find that in small-scale experimental banks with unknown type vector the occurrence of bank runs is significantly lower in sequential setups (as in our model) than in simultaneous setups (resembling the Diamond-Dybvig framework).

4 Conclusion

Descriptions of bank runs suggest that depositors' behavior depends crucially on other depositors' observed behavior. Existing theoretical models in the Diamond-Dybvig tradition, without aggregate uncertainty about liquidity needs, do not incorporate this idea, sequentiality is missing from them.[9] We attempt to fill this gap and assume that depositors observe all previous strategies. We show that bank runs do not occur in equilibrium, even though the type of preceding depositors is not observed. This result contrasts starkly with the findings of previous models, and suggests that the insensitivity of the Diamond-Dybvig contract to aggregate liquidity needs does not lead necessarily to bank runs as an equilibrium outcome. If all previous decisions are observed, in our model, bank runs are no equilibrium outcome.

Two elements of the model contribute to the absence of bank runs. First, aggregate certainty serves as a kind of coordination device to signal all patient depositors that it is in their best interest to wait, that is, it is commonly known that a bank run never occurs if all of them wait. Moreover, this assumption is quite realistic most of times. Second, the sequentiality of moves together with the perfect observability of previous decisions ensure that this is the unique PBE outcome. This equilibrium concept imposes strong rationality requirements on the depositors in terms of beliefs and sequential rationality.

Our model helps to understand depositor behavior in the polar case when information is highly detailed. Arguably, in reality depositors do not have the amount of information assumed in our model. Thus, it remains to be studied exactly how much information is needed to prevent bank runs due to coordination failure. For example, suppose that only withdrawals can be observed. Then, obviously, in the example with four depositors the reasoning which makes no bank run the unique PBE outcome breaks down, since deviations from the run strategy cannot be observed, so a patient depositor by waiting cannot induce subsequent depositors to wait. This suggests that observing waitings is necessary to prevent bank runs, though it is an open question to what extent. Advancing our understanding in these issues en-

[9] However, there are models that follow the spirit of Green and Lin (2003) and allow depositors to observe previous strategies to a certain extent, such as Ennis and Keister (2011).

ables policymakers to implement better rules and create environments that reduce the occurence of bank runs.

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Appendix A

Appendix A contains the proof of Lemma 1.

Proof. In order to derive the threshold value $\bar{\eta}$, a condition is found such that c_1^* is strictly smaller than period-2 consumption, that is,

$$c_1^* < \frac{R(N-(N-\eta)c_1^*)}{\eta}, \tag{1}$$

where the right-hand-side is period-2 consumption if η depositors wait at $t = 1$. Solving this inequality for η yields

$$\eta > \frac{RN(c_1^*-1)}{c_1^*(R-1)}. \tag{2}$$

Denote by $[x]$ the integer part of any $x \in \mathbb{R}$. Since η is a natural number, the previous condition becomes

$$\eta > \left[\frac{RN(c_1^* - 1)}{c_1^*(R-1)} \right] \equiv \bar{\eta}. \tag{3}$$

The right-hand side of (3) defines the threshold value $\bar{\eta}$. This value is unique since the bank pays to every depositor who withdraws c_1^*, and therefore, loses funds monotonically. If there are too many withdrawals by patient depositors, then the bank only pays $c_{2,i} < c_1^*$ to every depositor i which waits until $t = 2$. If the number of patient depositors that wait η is not larger than $\bar{\eta}$, as derived in (3), then period-2 consumption is strictly below c_1^*. □

Appendix B

Appendix B contains the proof of Proposition 1.

Proof. We show that under perfect information, in the unique PBE which in this case is the unique subgame perfect equilibrium each depositor announces her type truthfully.

First, conditions are derived under which period-1 consumption is strictly larger than period-2 consumption and a patient depositor is better off to withdraw at $t = 1$, that is, she declares to be impatient, and does not announce her type truthfully. Thereafter, it is shown that this never occurs in equilibrium, the depositors' equilibrium strategies are derived and shown to be a PBE, and finally, uniqueness is established.

As shown in Lemma 1, if $\bar{\eta}$ or less patient depositors wait, then $c_{2,i} < c_1^*$ and a patient depositor is better off to withdraw as long as the bank pays her c_1^*. However, if there are further withdrawals, then at some point the bank cannot pay c_1^* any more, but rather has $0 < y < c_1^*$ of funds left which she would pay to the last depositor that declares to be impatient. Then, there are two possibilities. Either period-2 consumption is larger than or equal to

y, or it is strictly smaller. In the first case, any patient depositor is better off to wait if, and only if, there is no more impatient depositor left in the queue (since she would withdraw y). If there is some impatient depositor left who would withdraw y, then the patient depositor is better off to take y at $t = 1$, rather than to get 0 at $t = 2$, and she withdraws all remaining funds. In this case, the depositor whose turn it is, once the bank has left y of funds, declares to be impatient independently of her type.

Now the complete strategy for all depositors is derived: an impatient depositor always withdraws and a patient depositor withdraws if, and only if, her period-1 consumption is strictly larger than her expected period-2 consumption. Otherwise, she waits. This is a subgame perfect equilibrium and a PBE of the bank run game if no depositor's deviation from this strategy profile is profitable. Consider first any impatient depositor i's deviation and suppose that she waits at $t = 1$. Then, she receives the same or a lower utility since for her $\theta_i = 0$, and thus $u_i = 0$, and this deviation is not profitable. Consider now any patient depositor's unilateral deviation. If she withdraws instead of waiting, then her consumption is $c_1^* < c_2^*$, and this deviation is not profitable for her given that all other patient depositors wait under the proposed strategy profile. Similarly, suppose that too many depositors before her withdrew already and that her strategy prescribes her to withdraw at $t = 1$. Then, she cannot deviate profitably either: she would receive 0 by waiting and at least the same amount by withdrawing.

This subgame perfect equilibrium is found by backward induction. Since any impatient depositor has a dominant strategy to withdraw at $t = 1$, the argument focuses on patient depositors. The last patient depositor waits if enough patient depositors waited since then her consumption is $c_{2,i} > c_1^*$ or $c_{2,i} > y$. The next to last patient depositor waits if enough patient depositors waited anticipating (by backward induction) that then also the last patient depositor is strictly better off to wait. By induction, this is true for all previous patient depositors. Finally, by induction, also the first patient depositor waits, that is, all of them wait and each receives $c_2^* > c_1^*$. Applying backward induction, each depositor's decision is unique on the equilibrium path since none of them is ever indifferent. However, on any off-equilibrium path on which the bank ran out of funds, any depositor is indifferent to wait or withdraw since her utility is 0 independently of her strategy. Hence, there is a unique subgame perfect equilibrium and PBE outcome, while there are several subgame perfect equilibria and PBE which differ by the depositors' behavior on irrelevant off-equilibrium paths. □

Appendix C

Appendix C contains the proof of Proposition 2.

Proof. We prove Proposition 2 in various steps. First, we give a complete description of all possible types or classes of equilibrium strategy profile candidates and their corresponding belief system. Then, we show for each class whether the corresponding strategy profiles are PBE or not, and finally, it follows as a corollary that the unique equilibrium outcome is no bank run since the only class of strategy profiles that are PBE is the one that yields no bank run as outcome. For all other classes it is shown that the strategy profiles belonging to them are no PBE.

Since any depositor can only wait or withdraw, there are the following classes of equilibrium strategy profile candidates: on the equilibrium path, all depositors wait or withdraw (these candidates correspond to pooling equilibria since all depositors are prescribed the same strategy), all patient depositors wait and all impatient ones withdraw or vice versa (these candidates correspond to separating equilibria since all depositors of one type are prescribed a different strategy than those of the other), and finally, any other mixture of strategies among patient and impatient depositors.

Next it is shown that any strategy profile in which impatient depositors are prescribed to wait on the equilibrium path is no PBE. Consider first a pooling equilibrium in which all depositors are asked to wait. Then, there is a type vector in which some depositor has a profitable deviation. Suppose that depositor 1 is impatient. Then, she gets a utility of 0 if she waits and c_1^* if she withdraws. Thus, she withdraws. In general, waiting yields an impatient depositor always a utility of 0, and withdrawing at least 0, and strictly more if the bank still has funds left. Therefore, withdrawing is a weakly dominant strategy for an impatient depositor, and thus, for all depositors to wait is no PBE. By an identical argument, it follows immediately that a separating equilibrium candidate in which on the equilibrium path impatient depositors wait and any mixture candidate in which some impatient depositor is asked to wait on the equilibrium path are no PBE either.

Thus, a separating equilibrium candidate is left in which patient depositors wait and impatient ones withdraw on the equilibrium path, a pooling equilibrium candidate in which all depositors withdraw on the equilibrium path, and any mixture equilibrium candidate in which, on the equilibrium path, all impatient depositors and at least one but less than p patient depositors should withdraw, while the remaining patient ones are asked to wait.

Belief updating

For the three remaining classes of strategy profiles, belief updating is illustrated in detail. Each type vector is selected by nature with equal probability. Since there are N depositors of which p are patient and this is commonly known, there are $\binom{N}{p}$ possible type vectors. Each is selected with probability $\frac{1}{\binom{N}{p}}$.

After knowing her type, a depositor updates her belief assigning equal probability to all type vectors in which the depositor in her position has her type. In particular, a patient depositor in the n-th position believes that each type vector in which a patient depositor occupies the n-th position is selected with probability $\frac{1}{\binom{N-1}{p-1}}$. Similarly, an impatient depositor in the n-th position believes that each type vector in which an impatient depositor occupies the n-th position is selected with probability $\frac{1}{\binom{N-1}{p}}$. We sometimes refer to this as updated prior belief. Further belief updating depends on the proposed strategy profile, the observed history, the commonly known parameters p and N, that each order is equally likely to be selected by nature and on a depositor's realized type. We refer to this as *available information.* Beliefs are updated in a Bayesian way (whenever possible) taking into account the available information and requiring consistency of beliefs.

Finally, since impatient depositors are always weakly better of to withdraw, consistency of belief updating for the three remaining equilibrium strategy profile candidates requires that a depositor who observes that some other before her waits believes that this depositor is patient. All remaining patient and impatient depositors are equally likely to occupy the remaining spots in the queue (unless the observed history has 0-probability to occur and belief updating is unconstrained).

Deriving the PBE

After eliminating any equilibrium candidate in which some impatient depositor waits on the equilibrium path, three classes of equilibrium strategy profile candidates are left. First it is shown that a pooling equilibrium strategy profile candidate in which all depositors withdraw on the equilibrium path and any mixture candidate in which on the equilibrium path all impatient and at least one but not all patient depositors withdraw are no PBE.

No PBE

Pooling equilibrium candidate: On the equilibrium path, any depositor is asked to withdraw. In this case, a depositor's strategy does not reveal her type (at least on the equilibrium path), and thus, a depositor keeps her updated prior as belief. On an off-equilibrium path, it is consistent for her to believe that someone who waits is patient given that any impatient depositor's weakly dominant strategy is to withdraw.

In order to show that this strategy profile is no PBE, we will focus on a specific type vector selected by nature but unknown to the depositors and a specific off-equilibrium path history for which some depositor's deviation is profitable. Suppose that a patient depositor towards the end of the queue observes a history with $p-1$ waitings. Then, she concludes that she is off-equilibrium, that the $p-1$ depositors that waited before her are patient and that she is the last patient depositor in the queue. Updating her belief in a Bayesian way, she believes that all remaining depositors in the queue are impatient. She is strictly better off to wait since she is patient and by waiting receives c_2^*, while she get c_1^* if she withdraws. Hence, she waits. If her strategy prescribes her to withdraw, then she has a profitable deviation to wait, and she complies with her strategy if she is asked to wait after this history. Note that the same is true for any history starting with $p-1$ waitings and followed by at most $N-p$ withdrawals.

Consider next a patient depositor who observes $p-2$ waitings and knows to be on an off-equilibrium path. Updating her belief in a Bayesian way, she assigns an equal probability to each type vector in which one more patient depositor occupies each of the remaining positions behind her in the queue. By sequential rationality she anticipates that this depositor will wait upon observing $p-1$ waitings. In any case, as long as there were enough waitings such that $c_{2,i} > c^*_{1,i}$, then she is strictly better off to wait given the observed history. Analogously any patient depositor i before her who observed enough waitings such that $c_{2,i} > c_{1,i}$, is strictly better off to wait. Updating her belief in a Bayesian way, depositor i believes that before her those depositors that waited are patient and all others impatient. Thus, any distribution among the remaining depositors is equally likely. In any case, there were enough waitings such that waiting is strictly better for i. Hence, on this off-equilibrium path, she either has a profitable deviation from withdrawing or is prescribed to wait and this yields her a higher payoff.

Consider any history starting with $p-2$ waitings. Given that a patient depositor observing $p-1$ waitings will wait, a patient depositor after $p-2$ waitings waits. Thus, if a withdrawal is observed after $p-2$ waitings, then it is due to an impatient depositor. As a consequence, a patient depositor observing a history that begins with $p-2$ waitings and is followed by a withdrawal is strictly better off to wait. By the same reasoning, when observing any history that starts with $p-2$ waitings and is followed by at most $N-p$ withdrawals, a patient depositor waits.

Going backwards in the queue, consider now the patient depositor i who observes just enough waitings such that $c_{2,i} < c_{1,i}$ if she withdraws, but $c_{2,i} > c_{1,i}$ if she waits since then there were enough waitings for waiting to be a dominant strategy for any remaining patient depositor in the queue. After observing this history, it is a strictly dominant strategy for her to wait. One consistent belief of her is that all depositors before her who waited are patient and that the remaining patient ones are equally likely to occupy the remaining spots in the queue. As before, she has a profitable deviation if she is asked to withdraw.

Consider now a patient depositor towards the beginning of the queue. Suppose that she observes that the first depositor in the queue waits. Then, she knows that this depositor brought the game onto an off-equilibrium path by deviating since the first depositor (as all others) are asked to withdraw on the equilibrium path. In case she observed further waitings, then she concludes that all depositors that waited are patient (since an impatient one would never wait). Her belief assigns equal probability to all remaining patient depositors, less the ones that waited before her and herself, to occupy the remaining positions behind her in the queue. By sequential rationality, she anticipates that all of them will wait upon observing this history. Then, weighting her payoff by her belief she is strictly better off to wait rather than to withdraw. She deviates profitably if she is asked to withdraw.

Going backwards, this argument applies analogously to all patient depositors in the queue but the first one. The first depositor knows that she is patient and that by waiting she leads the game onto an off-equilibrium path on which it is then sequentially rational for all remaining patient depositors in the queue to wait as well. Provided that nature selects a type vector in which the first depositor in the queue is patient, then this depositor has a profitable deviation and no bank run is the outcome.

Thus, we have shown that there are type vectors such that a pooling strategy profile is no PBE since a patient depositor 1 who is prescribed to withdraw has a profitable deviation. In case some patient depositors are asked to withdraw on certain off-equilibrium paths, then it is possible to find other profitable deviations. Therefore, this strategy profile is no PBE.

Any remaining mixture equilibrium candidate: In this case, at least one and at most $p-1$ patient depositors and all impatient ones are asked to withdraw.

Then, there are two possibilities. Suppose first that only few patient depositors are asked to withdraw such that $c_{2,i} > c_1^*$ for any patient depositor i who waits, and provided that all of them comply with the prescribed strategy profile. Then, obviously, any patient depositor i who is asked to withdraw has a profitable deviation to wait as well since $c_{2,i} > c_1^*$. Hence, we focus on a strategy profile in which enough patient depositors are asked to withdraw such that $c_{2,i} < c_1^*$ for any patient depositor i who waits indeed. Similarly as before, the type of a depositor who withdraws is not revealed, though it is consistent for a depositor to believe that those who wait are patient.

As before, there are type vectors selected by nature such that a patient depositor is first in the queue and is asked to withdraw. She has a profitable deviation to wait, and by doing this, leads the game down an off-equilibrium path whose outcome is no bank run. Then, by sequential rationality, all patient depositors are better off to wait. Therefore, any type of strategy profile in this class yields no PBE either.

PBE

We are left to show that the *separating equilibrium candidate* predicting no bank run is a PBE. In order to do this, first the corresponding strategy profile and belief updating are described in detail. Then, it is shown that no player's deviation is profitable. This shows existence. However, there are other off-equilibrium paths than the one proposed such that no depositor has a profitable deviation either. Hence, it follows immediately that there is no unique PBE, though the unique PBE outcome is no bank run since, as shown above, given any other strategy profile, some depositor has a profitable deviation.

On the equilibrium path, any patient depositor waits and any impatient one withdraws. In fact, an impatient depositor is asked to withdraw after any history, though she is indifferent to wait or not once the bank ran out of funds. On the same off-equilibrium paths, this indifference holds analogously for patient depositors. Therefore, there are multiple PBE strategy profile candidates that differ on off-equilibrium paths.

On the equilibrium path, every depositor's strategy perfectly reveals her type and it is consistent for a depositor to believe that those who wait are patient and those who withdraw are impatient. Thus, she believes that the remaining patient and impatient depositors (less the ones she observed before her, including herself) are behind her in the queue. By Bayesian updating, each distribution of remaining depositors is equally likely.

On an off-equilibrium path, belief updating in a PBE depends on whether the observed history has an ex ante 0-probability or not. As long as it is consistent with being on the equilibrium path, a depositor's belief is identical to the one she would have if all depositors complied with the prescribed strategy profile since there is no information that indicated her to be on an off-equilibrium path. If some patient depositor before her withdrew, then she only observes this depositor's strategy and not her type. Therefore, a depositor believes to be on the equilibrium path if, and only if, the total number of withdrawals—taking into account her own type—is not larger than the total number of impatient depositors and, in this case, belief updating and consistency of beliefs are identical as above.

Consider now any other off-equilibrium path strategy profile and belief system. The first depositor who knows to be on an off-equilibrium path has to be an impatient one who observes $N - p$ withdrawals. While a patient depositor who observes $N - p$ withdrawals believes that all remaining depositors in the queue are patient, an impatient one who observes this history knows to be on an off-equilibrium path. Given her type, this history has an ex ante 0-probability and her belief updating is unconstrained. In this case, there are several possible ways to update beliefs consistently with the available information and we propose one of them.

Suppose that she believes that all withdrawals she observes are due to patient depositors, unless there are more withdrawals than patient depositors. In this case, suppose that she believes that the remaining withdrawals were made by impatient depositors. Hence, she anticipates that all depositors behind her are impatient. Finally, it could be that there were less withdrawals than patient depositors: then she believes that all withdrawals are due to patient depositors, and that all impatient and the remaining patient depositors except of herself are behind her in the queue.

Consider the first patient depositor i in the queue who is asked to take a decision after this transition to the off-equilibrium path. She updates her belief in a Bayesian way. In the first two cases, the number of withdrawals that depositor i believes are due to patient depositors is such that $c_{2,i} < c_{1,i}$, even if i keeps the money in the bank, while in the last case it could be that $c_{2,i} > c_{1,i}$, if i and all remaining patient depositors that i believes to be behind her in the queue wait. Then, i is prescribed to withdraw, unless she is in the last case, in which she is asked to wait.

In the last case, there could be another patient depositor $j \neq i$, at a later position in the queue who observed enough additional withdrawals such that $c_{2,j} < c_{1,j}$, provided that j believes that all withdrawals were due to patient depositors. Then, there is a transition from the off-equilibrium path on which $c_{2,i} > c_{1,i}$ to the one on which $c_{2,j} < c_{1,j}$,[10] and j is prescribed to withdraw.

Belief updating in this transition from one off-equilibrium path to the other is unconstrained since the observed history has an ex ante 0-probability, and a patient depositor believes that enough if not all other patient depositors were before her in the queue and withdrew, and thus, she anticipates that all remaining depositors behind her in the queue will withdraw as well.[11]

Finally, there is a third (but less important) type of off-equilibrium path: in case there are enough withdrawals for the bank to run out of funds, then any depositor is indifferent to wait or not (independently of her type and belief) since her utility is 0 anyway. In this case, there are several beliefs that are consistent with the strategy profile of the remaining depositors, and any strategy is sequentially rational given any of these beliefs (since both waiting and withdrawing yields any depositor a utility of 0). Belief updating is not restricted in the transition from the history in which the bank is not

[10] Lemma 1 fully describes all situations and conditions in which $c_{2,j} < c_{1,j}$ since on any off-equilibrium path, given the proposed belief system, it cannot happen that the condition derived in the proof of Proposition 1 applies: that is, it cannot happen that there is y of funds left and that on an off-equilibrium path, given the proposed belief system, a patient depositor believes that there are only patient depositors left behind her in the queue.

[11] Nevertheless, on any off-equilibrium path, once a depositor has done the belief transition after a 0-probability event, all others update their belief in a Bayesian way as long as the observed history has a positive conditional probability to occur.

yet bankrupt to the one in which it ran out of funds.

No profitable deviation

We show next that no depositor has a profitable deviation from this strategy profile. Obviously, no impatient depositor can deviate profitably.

Consider next any patient depositor and the different possible histories: on the equilibrium path, no patient depositor i can deviate profitably since she is strictly better off to wait. Her belief to be on the equilibrium path is consistent with the strategy profile and the available information. Given her belief, it is sequentially rational for i to wait (since $c_2^* > c_1^*$) and her expected utility weighted by her belief is $u_i(\theta_i c_2^*) > u_i(c_1^*)$ since $\theta_i = 1$.

Consider now the different off-equilibrium paths. Suppose that for a patient depositor i who finds herself on an off-equilibrium path it holds that $c_{2,i} > c_{1,i}$, even if i believes that all withdrawals were due to patient depositors. Then, given her belief, it is sequentially rational for i to wait and this is consistent with her belief—i believes that there are enough patient depositors behind her in the queue for $c_{2,i} > c_{1,i}$ to hold, provided that all of them wait. Her deviation to withdraw is not profitable.

On any other off-equilibrium path, a patient depositor i believes that enough withdrawals were due to patient depositors such that $c_{2,i} < c_{1,i}$. Hence, it is sequentially rational for i to withdraw and her belief is consistent with her strategy. Her deviation to wait is not profitable. All subsequent depositors observe a series of withdrawals, but not the depositors' types, and thus, each's belief is consistent with her strategy and the observed history.

Finally, and as mentioned above, on an off-equilibrium path on which the bank went bankrupt already, no depositor has a profitable deviation and any remaining depositor's utility is 0 independently of her strategy and belief.

Therefore, we found a strategy profile such that no depositor's deviation is ever profitable. This shows existence and concludes the proof that no bank run is a PBE outcome. Note again that there are other off-equilibrium path belief systems and strategies. Any such strategy profile and belief system that is sequentially rational and consistent, respectively, is also a PBE. Hence, there is no unique PBE strategy profile.

A unique PBE outcome

After completely describing all possible strategy profiles that can be constructed given the set of depositors, their types and available strategy choices (wait and withdraw), we proved that any other type of candidate is no PBE, except for the separating equilibrium candidate in which patient depositors wait and impatient ones withdraw, and therefore, no bank run occurs. Since this is the only outcome arising from a PBE, it follows that the unique equilibrium outcome is no bank run. □