Improved engineering model for interfacial shearing stress analysis of a heated three layered structure in Electronic Packaging

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Abstract

An improved solution of a heated three layered structure for shearing stresses subjected to uniform temperature change is presented in this paper. Earlier solutions of three layered structure were proposed by Schmidt in 1999 and Suhir in 2003. However, there exist some contradictions and inconsistencies in both the solutions of Schmidt and Suhir. The contradiction arises in consideration of the exponent parameter *k* in the characteristic equation. Both Schmidt and Suhir showed that the exponent parameter *k* in the shearing stresses contains two roots. But for both cases considered only one root for *k* and as a consequence it leads to a mathematical inconsistency in the solution. In the present approach both the roots for the exponent parameter μ of the characteristic solution are considered in the characteristic equation which eliminates the mathematical inconsistencies in the earlier solution. The contradictions in Schmidt's and Suhir's solutions are highlighted in this paper. The analytical computation of shearing stress based on the present, Schmidt's and Suhir's model are presented in graphical form and compared with results from numerical simulation. The numerical example is carried out for a known three layered electronic packaging case (die-die attach-substrate), which was used by Suhir for comparison. The comparison between the present improved model and the finite element solution shows reasonably good agreement.

Key Words
Three layered structure, Shearing stress, Uniform temperature change, Exponent parameter, Electronic packaging

Introduction

Thermo-mechanical analysis of Tri-material assembly in electronic packages follows essentially the same lines as the well-known bi-material assembly analysis method, the only difference being the need for enforcing compatibility at more than one interface. But there are some inconsistencies as highlighted below:

(a) Schmidt [1] addressed the problem on the above lines and obtained the governing equations in the form of a coupled integral equation for the two interface shearing stresses. However, the solution was incomplete and ambiguous, because the characteristic equation for the exponent parameter *k* in the shearing stresses had two roots , whereas the assumed solution form had unknown constants enough in number to account for only one root. It was not stated which root had to be considered. The reason for discarding any one of the roots was also not clear. Even if any one root was considered for any reason, there were more equations to be satisfied than the number of unknowns. These anomalies are highlighted in [3]

(b) Suhir [2] provided another formulation for the tri-material assembly. The governing equations obtained were in the form of a coupled second order differential equations for the axial forces in the top and bottom layers, rather than the interface shearing stresses. As the interface shearing stress is first derivative of the axial force, Suhir's and Schmidt's formulations were mathematically equivalent. Suhir, like Schmidt, also considered only one root for *k* for no particular reason, but the difference was that Suhir was specific about which root be considered. Again, as in Schmidt's solution, there were more equations than unknowns and there is a contradiction in the solution obtained. The anomalies are also highlighted in [3]

Sujan et al [4] extended Schmidt's [1] uniform temperature shearing stress model for three layered assembly primarily by introducing two temperature ratio parameters $m_1(=\Delta T_2/\Delta T_1)$ and $m_2(=\Delta T_3/\Delta T_2)$ to account for differential temperatures developed in the layers. Subsequently a model was proposed for peeling stress at the interfaces in a three layered structure accounting for differential temperatures in the layers. But these extensions were based on inconsistent uniform temperature model proposed by Schimdt [1] and Suhir [2]. It is therefore seen that the existing solution [1, 2, 4] for the tri-material problems are not satisfactory and have inherent flaws. Both the analyses [1, 2] use incomplete solutions to the governing equations which lead to contradictions. In the present work, the complete solution using both the roots for *k* is considered.

Developing model of shearing stress with uniform temperature changes in the layers.

Figure 1 shows the model with the three layers designated as 1, 2, and 3 and a free body diagram for a cut at some arbitrary *x* location.

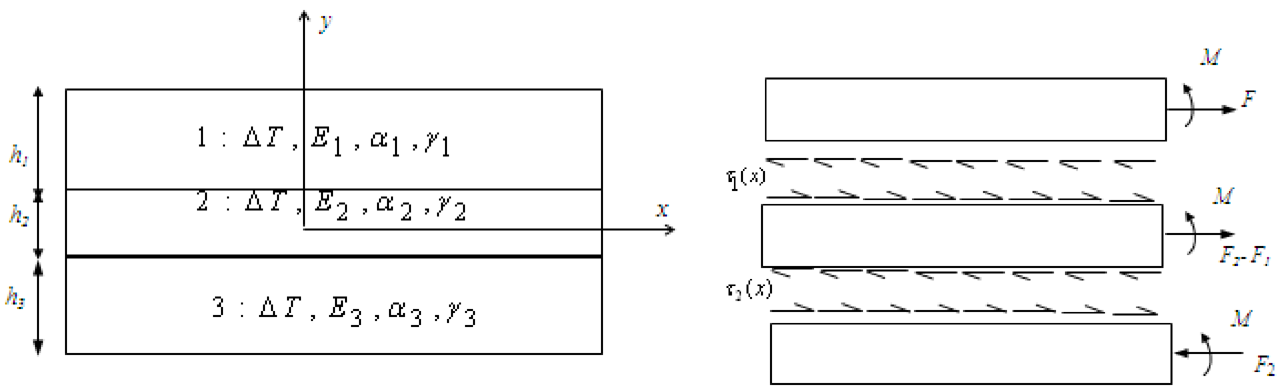

Figure 1: Geometrica and material parameters, and free-body diagram of the present model.

To develop the analytical model of shearing stress, we can refer to Sujan's [4] until basic governing equations (6). The summary of the important equations are presented from equation 1 to equation 4 as below:

The condition at the two interfaces can be written as

$$\epsilon_{x(1)}^{B}=\epsilon_{x(2)}^{T} \text{ and } \epsilon_{x(2)}^{B}=\epsilon_{x(3)}^{T}, \tag{1}$$

The expression for radius of curvature is

$$\frac{1}{R}=(\frac{h_1+h_2}{2D})F_1+(\frac{h_2+h_3}{2D})F_2 \tag{2}$$

The axial strains at the interfaces of the uniformly heated three layered structure take the form,

$$
\left.\begin{aligned}
\in_{x(1)}{}^{B} &= \alpha_1 \Delta T + \lambda_1 F_1 + \frac{h_1}{2R} - K_1 \frac{\partial \tau_1}{\partial x} \\
\in_{x(2)}{}^{T} &= \alpha_2 \Delta T + \lambda_2 (F_2 - F_1) - \frac{h_2}{2R} + K_2 \frac{\partial \tau_1}{\partial x} \\
\in_{x(2)}{}^{B} &= \alpha_2 \Delta T + \lambda_2 (F_2 - F_1) + \frac{h_2}{2R} - K_2 \frac{\partial \tau_2}{\partial x} \\
\in_{x(3)}{}^{T} &= \alpha_3 \Delta T - \lambda_3 F_2 - \frac{h_3}{2R} + K_3 \frac{\partial \tau_2}{\partial x}
\end{aligned}\right\} \tag{3}
$$

Substituting (3) into (1) and using (2), can get

$$
\left.\begin{aligned}
K_{12} \frac{\partial \tau_1}{\partial x} - \lambda_{12} F_1 + \lambda_{20} F_2 &= \Delta T (\alpha_1 - \alpha_2) \\
K_{23} \frac{\partial \tau_2}{\partial x} + \lambda_{20} F_1 - \lambda_{23} F_2 &= \Delta T (\alpha_2 - \alpha_3)
\end{aligned}\right\}, \tag{4}
$$

where $K_{ij} = K_i + K_j$, $\lambda_{20} = \lambda_2 - \frac{(h_1 + h_2)(h_2 + h_3)}{4D}$, and $\lambda_{ij} = \lambda_i + \lambda_j + \frac{(h_i + h_j)^2}{4D}$.

The solution of the equation (4) is assumed to be of the form:

$$
\tau_i = A_i^{(1)} sinh(k_1 x) + A_i^{(2)} sinh(k_2 x) , \tag{5}
$$

where $A_i^{(1)}$ and $A_i^{(2)}$ are arbitrary constants and k_1 and k_2 are, as we see later, roots of a certain characteristic equation.

It is at this point that the present approach deviates from both Schmidt's[1] and Suhir's[2] solutions. It may be seen that both the terms on the right hand side of eq. (5) are mathematically similar. Both Schmidt and Suhir considered only one of the terms on the right hand side of eq. (5). In other words, they considered only one root k of the characteristic equation, whereas in the present case both the roots k_1 and k_2 have been considered. The need for considering both the roots was already highlighted in the introduction and discussed in detail.

Differentiating (5), $\frac{\partial \tau_i}{\partial x} = k_1 A_i^{(1)} cosh(k_1 x) + k_2 A_i^{(2)} cosh(k_2 x)$ (6)

and integrating (5),

$$
F_i = \int_{-L}^{x} \tau_i dx = \frac{A_i^{(1)}}{k_1} \left\{ cosh(k_1 x) - cosh(k_1 L) \right\} + \frac{A_i^{(2)}}{k_2} \left\{ cosh(k_2 x) - cosh(k_2 L) \right\} \tag{7}
$$

Using (6) and (7), eqs. (4) becomes,

$$
\left.\begin{array}{l}
\left(k_1 K_{12} A_1^{(1)} - \lambda_{12}\dfrac{A_1^{(1)}}{k_1} + \lambda_{20}\dfrac{A_2^{(1)}}{k_1}\right) cosh(k_1 x) + \\
\left(k_2 K_{12} A_1^{(2)} - \lambda_{12}\dfrac{A_1^{(2)}}{k_2} + \lambda_{20}\dfrac{A_2^{(2)}}{k_2}\right) cosh(k_2 x) + \\
\left(\dfrac{1}{k_1}\right)\left\{\lambda_{12} A_1^{(1)} - \lambda_{20} A_2^{(1)}\right\} cosh(k_1 L) + \\
\left(\dfrac{1}{k_2}\right)\left\{\lambda_{12} A_1^{(2)} - \lambda_{20} A_2^{(2)}\right\} cosh(k_2 L) = (\alpha_1 - \alpha_2)\Delta T \\
\\
\left(k_1 K_{23} A_2^{(1)} + \lambda_{20}\dfrac{A_1^{(1)}}{k_1} - \lambda_{23}\dfrac{A_2^{(1)}}{k_1}\right) cosh(k_1 x) + \\
\left(k_2 K_{23} A_2^{(2)} + \lambda_{20}\dfrac{A_1^{(2)}}{k_2} - \lambda_{23}\dfrac{A_2^{(2)}}{k_2}\right) cosh(k_2 x) + \\
\left(\dfrac{1}{k_1}\right)\left\{-\lambda_{20} A_1^{(1)} + \lambda_{23} A_2^{(1)}\right\} cosh(k_1 L) + \\
\left(\dfrac{1}{k_2}\right)\left\{-\lambda_{20} A_1^{(2)} + \lambda_{23} A_2^{(2)}\right\} cosh(k_2 L) = (\alpha_2 - \alpha_3)\Delta T
\end{array}\right\} \quad (8)
$$

Eq. (8) must be valid for all x, which imposes the requirement that the coefficients of $cosh(k_1 x)$ and $cosh(k_2 x)$, as well as the constant terms on either sides of the above two equations (8) must be equal. Employing these conditions gives the following three sets of algebraic equations

$$
\left.\begin{array}{l}
\left(k_1^2 K_{12} - \lambda_{12}\right) A_1^{(1)} + \lambda_{20} A_2^{(1)} = 0 \\
\left(k_1^2 K_{23} - \lambda_{23}\right) A_2^{(1)} + \lambda_{20} A_1^{(1)} = 0
\end{array}\right\} \quad (9)
$$

$$
\left.\begin{array}{l}
\left(k_2^2 K_{12} - \lambda_{12}\right) A_1^{(2)} + \lambda_{20} A_2^{(2)} = 0 \\
\left(k_2^2 K_{23} - \lambda_{23}\right) A_2^{(2)} + \lambda_{20} A_1^{(2)} = 0
\end{array}\right\} \quad (10)
$$

$$\left.\begin{aligned}
&\left(\frac{1}{k_1}\right)\left\{\lambda_{12}A_1^{(1)} - \lambda_{20}A_2^{(1)}\right\} cosh(k_1L) + \\
&\left(\frac{1}{k_2}\right)\left\{\lambda_{12}A_1^{(2)} - \lambda_{20}A_2^{(2)}\right\} cosh(k_2L) = (\alpha_1 - \alpha_2)\Delta T \\
&\left(\frac{1}{k_1}\right)\left\{-\lambda_{20}A_1^{(1)} + \lambda_{23}A_2^{(1)}\right\} cosh(k_1L) + \\
&\left(\frac{1}{k_2}\right)\left\{-\lambda_{20}A_1^{(2)} + \lambda_{23}A_2^{(2)}\right\} cosh(k_2L) = (\alpha_2 - \alpha_3)\Delta T
\end{aligned}\right\} \tag{11}$$

Equation sets (9) and (10) are similar and can be replaced by a single set as follows:

$$\left.\begin{aligned}
&\left(k_i^2K_{12} - \lambda_{12}\right)A_1^{(i)} + \lambda_{20}A_2^{(i)} = 0 \\
&\left(k_i^2K_{23} - \lambda_{23}\right)A_2^{(i)} + \lambda_{20}A_1^{(i)} = 0
\end{aligned}\right\} \text{ where } i = 1, 2. \tag{12}$$

For a solution other than $A_1^{(i)} = A_2^{(i)} = 0$, the determinant of the coefficients of $A_1^{(i)}$ and $A_2^{(i)}$ in eq. (12) must be zero. This means

$$(k_i^2K_{12} - \lambda_{12})(k_i^2K_{23} - \lambda_{23}) - \lambda_{20}^2 = 0 \tag{13}$$

Solving eq. (13) for k_i^2,

$$k_i^2 = \frac{\left\{r \pm \left[r^2 - 4K_{12}K_{23}s\right]^{\frac{1}{2}}\right\}}{2K_{12}K_{23}}, \tag{14}$$

where $r = \lambda_{12}K_{23} + \lambda_{23}K_{12}$ and $s = \lambda_{12}\lambda_{23} - \lambda_{20}^2$

So there are two roots for k_i^2 which are interpreted as k_1^2 and k_2^2. For each of these values of k_i^2, there are two values of k_i, equal in magnitude but of opposite sign. Here only one root is needed to be considered because the solution for shearing stress τ_i involves $sinh(k_ix)$ and the solution involving $sinh(-k_ix)$ is not independent.

Now to determine all the four constants $A_i^{(1)}$ and $A_i^{(2)}$ for i =1, 2 in eq. (5), there are six equations available namely eq. (9), (10), and (11). One may wonder that there are six equations for four unknowns. At first sight, it may appear that something is wrong. Actually, it is not so. The second of each of equations (9) and (10) is same as the first one in each case. The values of k_1 and k_2 are determined from that condition only (i.e., the condition of equivalence of equations (9) and (10)). So actually we have only four independent equations, namely, the first of eq. (9), the first of eq. (10) and the two eqs. (11). Now the four constants $A_i^{(1)}$ and $A_i^{(2)}$ are solved by reducing the problem to two linear simultaneous equations by carefull manipulation of the algebra. Eqs. (12) can be written as,

$$\lambda_{12}A_1^{(i)} - \lambda_{20}A_2^{(i)} = k_i^2 K_{12} A_1^{(i)} \qquad \text{where } i = 1, 2. \tag{15}$$

$$\lambda_{23}A_2^{(i)} - \lambda_{20}A_1^{(i)} = k_i^2 K_{23} A_2^{(i)}, \tag{16}$$

Using eq. (15) into the first of the eqs. (11), gives

$$k_1\, cosh(k_1 L)\, A_1^{(1)} + k_2\, cosh(k_2 L)\, A_1^{(2)} = \frac{(\alpha_1 - \alpha_2)\Delta T}{K_{12}} \tag{17}$$

Similarly, using eq. (16) into the second of eqs. (11), gives

$$k_1\, cosh(k_1 L)\, A_2^{(1)} + k_2\, cosh(k_2 L)\, A_2^{(2)} = \frac{(\alpha_2 - \alpha_3)\Delta T}{K_{23}} \tag{18}$$

But it can be observed from the homogeneous eqs. (12) that $A_2^{(i)}$ bears a known ratio to $A_1^{(i)}$ for i =1 and 2. This ratio can be obtained from either of the two identical eqs. (12).

Using the first of the eqs. (12),

$$A_2^{(i)} = \left[(\lambda_{12} - k_i^2 K_{12}) / \lambda_{20}\right] A_1^{(i)} \qquad \text{for } i = 1, 2. \tag{19}$$

Using eq. (19), eq. (18) becomes,

$$Z_1 A_1^{(1)} + Z_2 A_1^{(2)} = \frac{(\alpha_2 - \alpha_3)\Delta T}{K_{23}}, \tag{20}$$

$$\text{where } Z_i = \frac{k_i(\lambda_{12} - k_i^2 K_{12})}{\lambda_{20}} cosh(k_i L) \qquad \text{for } i = 1, 2 \tag{21}$$

Now it needs to solve eqs. (17) and (20) for $A_1^{(1)}$ and $A_1^{(2)}$ and then determine $A_2^{(1)}$ and $A_2^{(2)}$ using eq. (19).
Eqs. (17) and (20) can be represented in the following form:

$$L_1\, A_1^{(1)} + L_2\, A_1^{(2)} = B_1 \tag{22}$$

and

$$Z_1 A_1^{(1)} + Z_2 A_1^{(2)} = B_2, \tag{23}$$

where $L_i = k_i\, cosh(k_i L)$, $B_1 = \dfrac{(\alpha_1 - \alpha_2)\Delta T}{K_{12}}$, and $B_2 = \dfrac{(\alpha_2 - \alpha_3)\Delta T}{K_{23}}$

Solving eqs. (22) and (23) for $A_1^{(2)}$ and substituting into eq. (22) results in

Substituting $A_1^{(2)}$ into eq. (22) results in

$$A_1^{(1)} = \frac{\Delta T\left[(\alpha_1 - \alpha_2)\dfrac{(-\lambda_{12} + k_2^2 K_{12})}{K_{12}} + (\alpha_2 - \alpha_3)\dfrac{\lambda_{20}}{K_{23}} \right]}{k_1 K_{12}(k_2^2 - k_1^2)\cosh(k_1 L)}$$

Substituting the above expression for $A_1^{(1)}$ and $A_1^{(2)}$ in eq. (19), $A_2^{(1)}$ and $A_2^{(2)}$ are also determined.

Finally results in the following:

$$\left.\begin{aligned} A_1^{(1)} &= \frac{\Delta T\left[(\alpha_1 - \alpha_2)\beta_2 + (\alpha_2 - \alpha_3)\gamma \right]}{k_1 K_{12}(k_2^2 - k_1^2)\cosh(k_1 L)} \\ A_1^{(2)} &= \frac{\Delta T\left[(\alpha_1 - \alpha_2)\beta_1 - (\alpha_2 - \alpha_3)\gamma \right]}{k_2 K_{12}(k_2^2 - k_1^2)\cosh(k_2 L)} \\ A_2^{(1)} &= \beta_3 \frac{\Delta T\left[(\alpha_1 - \alpha_2)\beta_2 + (\alpha_2 - \alpha_3)\gamma \right]}{k_1 K_{12}(k_2^2 - k_1^2)\cosh(k_1 L)} \\ A_2^{(2)} &= \beta_4 \frac{\Delta T\left[(\alpha_1 - \alpha_2)\beta_1 - (\alpha_2 - \alpha_3)\gamma \right]}{k_2 K_{12}(k_2^2 - k_1^2)\cosh(k_2 L)} \end{aligned}\right\}, \qquad (24)$$

where $\beta_1 = \dfrac{(\lambda_{12} - k_1^2 K_{12})}{K_{12}}$, $\beta_2 = \dfrac{(-\lambda_{12} + k_2^2 K_{12})}{K_{12}}$ $\beta_3 = \dfrac{(\lambda_{12} - k_1^2 K_{12})}{\lambda_{20}}$, $\beta_4 = \dfrac{(\lambda_{12} - k_2^2 K_{12})}{\lambda_{20}}$, and $\gamma = \dfrac{\lambda_{20}}{K_{23}}$

Shear stresses τ_1 and τ_2 at the two interfaces are now determined from eq. (5) written for i = 1 and 2, as all the arbitrary constants appearing in the equation are determined and given by eq. (24).

Results and Discussions

Analytical computation of shearing stress based on the present, Schmidt's and Suhir's model are presented in graphical form and compared with numerical simulation. Analytical and numerical results of peeling stress based on the present model are presented in table form. The numerical example is carried out for the same electronic packaging case as used by Suhir [2] for comparison purpose. The package consists of a silicon chip (layer 2) which is attached to a printed circuit board (layer 1) and is over-molded by an epoxy molding compound (layer 3). The following input data is used: E_1 = 34.5x10^3 MPa, ν_1 = 0.33, α_1 = 15.0x10^{-6} /°C, h_1 = 1.0 mm, E_2 = 120.7x10^3 MPa, ν_2 = 0.24, α_2 = 3.2x10^{-6} /°C, h_2 = 0.508 mm, E_3 = 6.9x10^3 MPa, ν_3 = 0.35, α_3 = 12.0x10^{-6} /°C, h_3 = 1.2 mm. The length of the assembly is 2L = 0.02 m. In this computation ΔT is taken as -120°C. For numerical simulation a 3D model is considered and one quarter of the model is analyzed due to the condition of symmetry. The size of the quarter assembly is 10 mm x 5 mm x 2.708 mm. Lengthwise, from x/L =0 to 0.6, course meshing is applied and from x/L = 0.6 to 1, fine meshing is applied. The number of elements for each layer is as follows:

Results are presented from x/L= 0.52 to 1 only, since the stress values are significantly small beyond that limit.

From eq. (14), considering positive sign of the square root component, produces $k_1 = \left[\dfrac{r + r_1}{2K_{12}K_{23}} \right]^{\frac{1}{2}}$ = 2307 and considering negative sign, produces $k_2 = \left[\dfrac{r - r_1}{2K_{12}K_{23}} \right]^{\frac{1}{2}}$ = 1011, where $r_1 = \left[r^2 - 4K_{12}K_{23}s \right]^{\frac{1}{2}}$.

Table 1: Shearing stress along layers 1-2 interface(*MPa*)

x/L	[1]Schmidt	[2]Schmidt	*Suhir	*Present	*FEM
1	-41.15	-18.03	-41.15	-23.82	-15.61
0.99	-32.67	-16.30	-32.67	-20.69	-17.91
0.98	-25.94	-14.73	-25.94	-17.99	-17.05
0.97	-20.60	-13.31	-20.60	-15.66	-15.63
0.96	-16.35	-12.03	-16.35	-13.66	-13.45
0.94	-10.31	-9.83	-10.31	-10.42	-10.41
0.92	-6.50	-8.03	-6.50	-7.99	-7.78
0.90	-4.10	-6.56	-4.10	-6.16	-6.13
0.88	-2.58	-5.36	-2.58	-4.78	-5.03
0.86	-1.63	-4.38	-1.63	-3.72	-3.66
0.84	-1.03	-3.58	-1.03	-2.91	-3.03
0.82	-0.65	-2.92	-0.65	-2.29	-2.16
0.80	-0.41	-2.39	-0.41	-1.81	-1.81
0.76	-0.16	-1.59	-0.16	-1.15	-1.27
0.72	-0.06	-1.06	-0.06	-0.74	-0.89
0.68	-0.03	-0.71	-0.03	-0.48	-0.62
0.64	-0.01	-0.47	-0.01	-0.32	-0.42
0.60	-0.004	-0.32	-0.004	-0.21	-0.27
0.56	-0.002	-0.21	-0.002	-0.14	-0.15
0.52	-0.001	-0.14	-0.001	-0.09	-0.09

Table 1 represents shearing stress comparison along the interface of layers 1 and 2. Here [1]Schmidt represents shearing stress using positive sign root of k_1=2307 and [2]Schmidt represents shearing stress computation using negative sign root of k_2=1011. For Suhir, positive sign root k_1=2307 is used and for *present, shearing stress is computed using both roots as shown in the proposed accurate model. For column *FEM, the finite element results are presented for numerical simulation using ANSYS. The comparison shows reasonably good agreement between columns *present and *FEM at all *x/L* locations except near the vicinity of the free end possibly indicating free surface effect.

Table 2: Shearing stress along layers 2-3 interface (*MPa*)

x/L	[1]Schmidt	[2]Schmidt	*Suhir	*Present	*FEM
1	-25.83	-11.32	-25.83	9.16	4.18
0.99	-20.51	-10.23	-20.51	8.51	8.74
0.98	-16.28	-9.25	-16.28	7.90	9.69
0.97	-12.93	-8.36	-12.93	7.31	9.01
0.96	-10.26	-7.55	-10.26	6.77	8.18
0.94	-6.47	-6.17	-6.47	5.76	5.61
0.92	-4.08	-5.04	-4.08	4.89	4.43
0.90	-2.57	-4.12	-2.57	4.13	5.0
0.88	-1.62	-3.36	-1.62	3.48	3.32
0.86	-1.02	-2.75	-1.02	2.93	2.93
0.84	-0.64	-2.25	-0.64	2.45	2.58
0.82	-0.41	-1.83	-0.41	2.05	2.24
0.80	-0.26	-1.50	-0.26	1.72	1.94
0.76	-0.10	-1.00	-0.10	1.19	1.06
0.72	-0.04	-0.67	-0.04	0.83	0.75
0.68	-0.02	-0.45	-0.02	0.57	0.52
0.64	-0.006	-0.30	-0.006	0.40	0.36
0.60	-0.003	-0.20	-0.003	0.27	0.23
0.56	-0.001	-0.13	-0.001	0.19	0.16
0.52	-0.0004	-0.09	-0.0004	0.13	0.13

Table 2 represents similar information to the table 1 except the shearing stress is computed along the interface of layers 2 and 3. Comparison again shows good agreement between columns *present and *FEM at all *x/L* locations except one location near the free end as expected.

Conclusion

In the present analysis the authors have presented the complete solution of tri-material material assembly for shearing stress. Both the roots for the exponential parameter are considered which leads to the correct solution. The comparison between the corrected proposed model and the finite element solution suggests reasonably good agreement.

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Appendix

<u>Some symbols and their meanings:</u>

i= Material layer no. as subscript, =1, 2, and 3; E = Young's modulus : h_i = Thickness
α_i = Coefficient of thermal Expansion ; ν_i = poison's ratio; R = Radius of curvature

Shear modulus, $G_i = \dfrac{E_i}{2(1+\nu_i)}$; Flexural rigidity, $D_i = \dfrac{E_i h_i^3}{12(1-\nu_i^2)}$ where $D = D_1 + D_2 + D_3$

Axial compliance, $\lambda_i = \dfrac{(1-\nu_i^2)}{E_i h_i}$; Coefficient of interfacial compliance, $K_i = \dfrac{h_i}{3G_i}$

Biographies

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