Robust Stability Analysis of Guaranteed Cost Control for Impulsive Switched Systems

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***Abstract*—This correspondence is concerned with the robust stability for a class of impulsive switched systems under the LQ guaranteed cost control. Some results on robust stability for this class of impulsive switched systems are obtained. Sufficient conditions for the existence of a guaranteed cost control law are also given. Subject to these sufficient conditions, the closed-loop uncertain impulsive switched system under the guaranteed cost control law is robustly stable with a guaranteed cost value.**

***Index Terms*—Impulsive switched system, linear quadratic (LQ) guaranteed cost control, robust stability.**

I. Introduction

Studies of dynamical systems with impulsive effects and switching phenomena attract more and more attention recently. These dynamical systems are usually called impulsive switched systems, which are encountered in a wide range of disciplines such as drug administration in cancer chemotherapy and insulin injection [9], orbital transfer of satellites [10], forest ecosystems management [11], neural networks [12], switched systems [14], and many other problems arising in areas such as engineering, economics, and biology. For example, in some circuit systems, the circuits' units switching is one of the main factors that can cause abrupt changes of system states in the transmission of signals. The electrical current changes produced by faulty circuit elements can be regarded as impulse events. Since abrupt changes at certain moments of time due to impulses or switching behaviors previously mentioned are often the cause for instability of these nonlinear systems, the stability problem has become an important issue, both theoretically and practically. Recently, there are some interesting results on robust stability of impulsive switched systems that are reported in the literature. For example, in [1], robust stabilization problems with definite attenuation of uncertain impulsive switched systems are studied. Some sufficient conditions for ensuring the existence of feedback control laws are derived and expressed in terms of linear matrix inequalities (LMIs). In [2], sufficient conditions for uniformly asymptotical stability of uncertain impulsive switched systems are derived through solving some LMIs. In [3], results for the design of robust state feedback controllers and steady state robust state estimators for a class of uncertain linear systems are given. In [4], a design method for robust static and dynamic output feedback controllers is developed for a class of uncertain linear systems with norm-bounded uncertainties. In [5], [6], [8], and [13], guaranteed cost control problems for various uncertain dynamical systems are considered. Their approaches are based on extensions of the Riccati equation or the LMI approach by incorporating the robust performance with the linear quadratic (LQ) cost function to form a new performance index. The quadratic guaranteed cost control laws are constructed via solving a parameter dependent algebraic Riccati equation or LMIs. In this correspondence, the guaranteed cost control problem for a class of impulsive switched systems with norm-bounded uncertainties is considered. The aim is to study robust stability performance for this impulsive switched system with an LQ guaranteed cost control law, and to obtain conditions for ensuring the existence of an LQ guaranteed cost control law.

The rest of this correspondence is organized as follows. In Section II, two definitions on guaranteed cost control and LQ guaranteed cost control is introduced. The LQ guaranteed cost control problem involving uncertain impulsive switched systems with norm-bounded uncertainties is then formulated. In Section III, conditions on the state feedback gain matrix such that the closed-loop uncertain impulsive switched system is quadratically stable and an upper bound on the LQ performance index is minimized is then presented. Conditions for ensuring the existence of an LQ guaranteed cost control law are derived. Section IV contains some concluding remarks.

II. Problem Statement

Consider the following uncertain impulsive switched system:

$$\dot{x}(t) = (A_{i_k} + \Delta A_{i_k})x(t) + (B_{i_k} + \Delta B_{i_k})u(t), \qquad t \neq t_k \quad \text{(1a)}$$

$$\Delta x(t) = C_k x(t), \qquad t = t_k \quad \text{(1b)}$$

$$x(t_0) = x_0 \quad \text{(1c)}$$

where $x(t) \in R^n$ is the state, and $u(t) \in R^m$ is the control input. $A_{i_k} \in R^{n\times n}$, $B_{i_k} \in R^{n\times m}$, $C_k \in R^{n\times n}$ are constant real matrices, $k = 1, 2, \ldots, i_k \in \{1, 2, \ldots, M\}$, and M is a positive integer. $\Delta A_{i_k}(\cdot)$ and $\Delta B_{i_k}(\cdot)$ are unknown real norm-bounded matrix valued functions, representing time-varying parameter uncertainties. The admissible uncertainties are assumed to be of the form

$$\begin{bmatrix}\Delta A_{i_k}(t) & \Delta B_{i_k}(t)\end{bmatrix} = E_{i_k}\Psi_{i_k}(t)[H_{i_k} \quad \hat{H}_{i_k}] \quad (2)$$

where E_{i_k}, H_{i_k}, $\hat{H}_{i_k}$ are known real constant matrices, $\Psi_{i_k}(t)$ is an unknown real time-varying matrix satisfying $\|\Psi_{i_k}(t)\| < 1$, t_k is an impulsive switching point, $k = 1, 2, \ldots, t_0 < t_1 < t_2 < \cdots < t_k < \ldots (k \to \infty,\ t_k \to \infty)$. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^-) = x(t_k) = \lim_{h\to 0+} x(t_k - h)$, $x(t_k^+) = \lim_{h\to 0+} x(t_k + h)$. Associated with system (1) is the cost function given by

$$J = \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \left[x^{\mathrm{T}}(t)R_{i_k}x(t) + u^{\mathrm{T}}(t)Q_{i_k}u(t)\right] dt \quad (3)$$

where $R_{i_k} > 0$ and $Q_{i_k} > 0$ are positive definite symmetric matrices.

Definition 1: The uncertain impulsive switched system (1) is considered. Suppose that there exist a control law $u(t) = F_{i_k}x(t)$ and a positive real number J^* such that for all admissible uncertainties, the closed-loop system is stable and the corresponding value of the cost function (3) satisfies $J \leq J^*$. Then, J^* is said to be a guaranteed cost, and $u(t)$ is said to be a guaranteed cost control law for the uncertain impulsive switched system (1).

Definition 2: System (1) is said to be quadratically stable when $u(t) = 0$ if there exists a positive definite symmetric matrix P_{i_k} such that there exists a quadratic function $V(t) = x^{\mathrm{T}}(t)P_{i_k}x(t)$ such that it

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decreases along every nonzero trajectory of system (1). $V(t)$ is called a quadratic Lyapunov function.

Definition 3: Suppose that there exist a control law $u(t) = F_{i_k}x(t)$ and a positive real number J^* such that the value of the cost function (3) satisfies $J \le J^*$, and the following conditions hold:

(i) $$R_{i_k} + F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k} + 2P_{i_k}(\hat{A}_{i_k} + \Delta A_{i_k} + \Delta B_{i_k} F_{i_k}) < 0$$

where $\hat{A}_{i_k} = A_{i_k} + B_{i_k}F_{i_k}$ (see [3, Sec. II] for the interpretation of this condition) and the associated cost matrix for the system (1) satisfies $P_{i_k} = P_{i_k}^{\mathrm{T}} > 0$

(ii) $$0 < \beta_k < 1$$

where $\beta_k = \lambda_{\max}[P_{i_{k-1}}^{-1}(I + C_k)^{\mathrm{T}} P_{i_k}(I + C_k)]$, $i_k = 1, 2, \ldots, M$. Here, $\lambda_{\max}[\cdot]$ denotes the maximum eigenvalue of the matrix inside the square brackets. Then, $u(t) = F_{i_k}x(t)$ is said to be an LQ guaranteed cost control law for the uncertain impulsive switched system (1).

Remark 1: In Definitions 1, 2, and 3, the definitions of a guaranteed cost control law reported in [3] are extended to the case of impulsive switched systems.

III. Main Results

In this section, results on sufficient conditions for quadratic stability of the closed-loop uncertain system under a linear state piecewise control law are stated and proven.

Theorem 1: Consider the impulsive switched system (1) with the cost function (3). Suppose that the control law $u(t) = F_{i_k}x(t)$ is an LQ guaranteed cost control law with the positive definite cost matrix $P_{i_k} = P_{i_k}^{\mathrm{T}} > 0$. Then, the closed-loop uncertain system is quadratically stable. Furthermore, the corresponding value of the cost function (3) satisfied $J < x_0^{\mathrm{T}} P_{i_0} x_0$ for all admissible uncertainties Ψ_{i_k}. Conversely, if a control law $u(t) = F_{i_k}x(t)$ exists such that the resulting closed-loop system is quadratically stable, then this control law is an LQ guaranteed cost control law with some cost matrix $\hat{P}_{i_k} > 0$ for all positive definite symmetric matrices R_{i_k} and Q_{i_k}, where $\hat{P}_{i_k} = (1/\mu)P_{i_k}$, and μ is any positive real number.

Proof: Suppose that the control law $u(t) = F_{i_k}x(t)$ is an LQ guaranteed cost control law with some cost matrix $P_{i_k} > 0$. Then, it follows from condition (i) of Definition 3 that

$$x^{\mathrm{T}}\left[R_{i_k} + F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k}\right] x + 2x^{\mathrm{T}} P_{i_k}[\hat{A}_{i_k} + \Delta A_{i_k} + \Delta B_{i_k} F_{i_k}]\, x < 0 \tag{4}$$

for all admissible uncertainties.

First, the case when the impulsive switched system is defined on $t \in (t_k, t_{k+1}]$ is considered

$$V(t) = x^{\mathrm{T}}(t) P_{i_k} x(t). \tag{5}$$

Differentiating the function along the trajectory of the closed-loop system and then using (4), the following is obtained:

$$\begin{aligned}\dot{V}(t) &= \dot{x}^{\mathrm{T}}(t)P_{i_k}x(t) + x^{\mathrm{T}}(t)P_{i_k}\dot{x}(t)\\ &= x^{\mathrm{T}}(t)(A_{i_k} + B_{i_k}F_{i_k} + \Delta A_{i_k} + \Delta B_{i_k}F_{i_k})^{\mathrm{T}} P_{i_k} x(t)\\ &\quad + x^{\mathrm{T}}(t)P_{i_k}(A_{i_k} + B_{i_k}F_{i_k} + \Delta A_{i_k} + \Delta B_{i_k}F_{i_k})x(t)\\ &= x^{\mathrm{T}}(t)\left[(\hat{A}_{i_k} + \Delta A_{i_k} + \Delta B_{i_k}F_{i_k})^{\mathrm{T}} P_{i_k}\right.\\ &\qquad \left. + P_{i_k}(\hat{A}_{i_k} + \Delta A_{i_k} + \Delta B_{i_k}F_{i_k})\right] x(t)\\ &< -x^{\mathrm{T}}(t)\left[R_{i_k} + F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k}\right] x(t) < 0 \end{aligned} \tag{6}$$

for all $x(t) \neq 0$.

Next, the case when the impulsive switched system is at the impulsive and switching time point t_k is considered

$$V\left(t_k^+\right) = x^{\mathrm{T}}\left(t_k^+\right) P_{i_k} x\left(t_k^+\right). \tag{7}$$

Then, by (1b) and noting that $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$

$$\begin{aligned} V\left(t_k^+\right) &= \left[(I + C_k)x\left(t_k^-\right)\right]^{\mathrm{T}} P_{i_k}\left[(I + C_k)x\left(t_k^-\right)\right]\\ &= x^{\mathrm{T}}\left(t_k^-\right)(I + C_k)^{\mathrm{T}} P_{i_k}(I + C_k)x\left(t_k^-\right). \end{aligned} \tag{8}$$

In view of Lemma 2 of [1] (i.e., $x^{\mathrm{T}}Qx \le \lambda_{\max}(S^{-1}Q)x^{\mathrm{T}}Sx$ if S is a positive definite matrix and Q is a symmetric matrix) and condition (ii) of Definition 2, it follows from (7) that

$$\begin{aligned} V\left(t_k^+\right) &\le \lambda_{\max}\left[P_{i_{k-1}}^{-1}(I+C_k)^{\mathrm{T}} P_{i_k}(I+C_k)\right] x^{\mathrm{T}}(t_k)P_{i_{k-1}}x(t_k)\\ &\le \beta_k x^{\mathrm{T}}(t_k)P_{i_{k-1}}x(t_k) = \beta_k V\left(t_k^-\right) \end{aligned} \tag{9}$$

where β_k, $0 < \beta_k < 1$, is defined in condition (ii) of Definition 2 and $P_{i_{k-1}}$ is the cost matrix at the time t_k^-. Clearly, from (6) and (9), we conclude that the closed-loop system of the uncertain impulsive switched system (1) is quadratically stable.

Now, by substituting $u(t) = F_{i_k}x(t)$ into (3), we obtain

$$\begin{aligned} J &= \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \left[x^{\mathrm{T}}(t)R_{i_k}x(t) + u^{\mathrm{T}}(t)Q_{i_k}u(t)\right] dt\\ &= \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} x^{\mathrm{T}}(x)\left[R_{i_k} + F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k}\right] x(t)dt. \end{aligned} \tag{10}$$

Then, from condition (i) of Definition 2, (6), and (9), it follows that

$$\begin{aligned} J &= \lim_{k\to\infty} J_k\\ &< -\lim_{k\to\infty} \sum_{j=0}^{k} \int_{t_j}^{t_{j+1}} 2x^{\mathrm{T}} P_{i_j}(\hat{A}_{i_j} + \Delta A_{i_j} + \Delta B_{i_j} F_{i_j})x(t)dt\\ &= -\lim_{k\to\infty} \sum_{j=0}^{k} \int_{t_j}^{t_{j+1}} \frac{dV(t)}{dt} dt\\ &= -\lim_{k\to\infty} \left\{-V(0) + V\left(t_1^-\right) - V\left(t_1^+\right) + V\left(t_2^-\right) - V\left(t_2^+\right)\right.\\ &\qquad \left. + \ldots - V\left(t_k^-\right) + V\left(t_k^+\right) - V\left(t_{k+1}^-\right)\right\}\\ &= \lim_{k\to\infty} \left\{V(0) + \sum_{j=1}^{k}\left\{V\left(t_j^+\right) - V\left(t_j^-\right)\right\} + V\left(t_{k+1}^-\right)\right\}\\ &= V(0) + \lim_{k\to\infty} \sum_{j=1}^{k} (\beta_j - 1)V\left(t_j^-\right) - \lim_{k\to\infty} V\left(t_{k+1}^-\right). \end{aligned} \tag{11}$$

From (5), (6), and (9), we know that

$$\lim_{k\to\infty} V\left(t_{k+1}^-\right) = 0. \tag{12}$$

Since $0 < \beta_j < 1$ for all $j = 1, 2, \ldots$, it is clear that

$$\lim_{k\to\infty} \sum_{j=1}^{k} (\beta_j - 1)V(t_j) < 0. \tag{13}$$

Substituting (12) and (13) into (11), and then using (5) with $t = 0$, we obtain

$$J < V(0) = x_0^{\mathrm{T}} P_{i_0} x_0. \tag{14}$$

Conversely, if there exists a control law $u(t) = F_{i_k} x(t)$ such that the resulting closed-loop system is quadratically stable, then there exist a matrix $P_{i_k} = P_{i_k}^{\mathrm{T}} > 0$ and a constant $\mu > 0$ such that

$$\mu x^{\mathrm{T}}\left[R_{i_k}+F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k}\right] x + 2x^{\mathrm{T}} P_{i_k}\left[\hat{A}_{i_k}+ \Delta A_{i_k}+ \Delta B_{i_k} F_{i_k}\right] x < 0 \tag{15}$$

for all nonzero $x \in R^n$ and $\|\Psi_{i_k}(t)\| < 1$. Hence, this control law is an LQ guaranteed cost control law with the cost matrix $\hat{P}_{i_k} = (1/\mu) P_{i_k} > 0$. ■

Remark 2: Theorem 1 extends Theorem 2.1 given in [3] to the case of impulsive switched systems. The next theorem presents another sufficient condition for the existence of an LQ guaranteed cost control law which can ensure the closed-loop system (1) is quadratically stable.

Theorem 2: Suppose there exist $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $P_{i_k} = P_{i_k}^{\mathrm{T}} > 0$ such that the following conditions are satisfied:

(i) $$\varepsilon_1 I - E_{i_k}^{\mathrm{T}} P_{i_k} E_{i_k} > 0 \tag{16}$$

(ii) $$\begin{aligned} &A_{i_k}^{\mathrm{T}} A_{i_k} + A_{i_k}^{\mathrm{T}} E_{i_k}\left(\varepsilon_1 I - E_{i_k}^{\mathrm{T}} E_{i_k}\right)^{-1} E_{i_k}^{\mathrm{T}} A_{i_k} + \varepsilon_1 H_{i_k}^{\mathrm{T}} H_{i_k} \\ &\quad + F_{i_k}^{\mathrm{T}} B_{i_k}^{\mathrm{T}} P_{i_k} + P_{i_k} B_{i_k} F_{i_k} + \varepsilon_2 P_{i_k} E_{i_k} E_{i_k}^{\mathrm{T}} P_{i_k} \\ &\quad + \frac{1}{\varepsilon_2} F_{i_k}^{\mathrm{T}} \hat{H}_{i_k}^{\mathrm{T}} \hat{H}_{i_k} F_{i_k} + P_{i_k}^2 + R_{i_k} + F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k} < 0 \end{aligned} \tag{17}$$

(iii) $$\beta_k = \lambda_{\max}\left[P_{i_{k-1}}^{-1}(I + C_k)^{\mathrm{T}} P_{i_k}(I + C_k)\right], \quad (i_k = 1, 2, \ldots, M) \tag{18}$$

where $0 < \beta_k < 1$. Then, there exists an LQ guaranteed cost control law $u(t) = F_{i_k} x(t)$ and the closed-loop system (1) is quadratically stable.

Proof: Since condition (16) hold, if follows from Proposition 2.2 given in [7] that

$$\begin{aligned} &(A_{i_k}+\Delta A_{i_k})^{\mathrm{T}} P_{i_k} + P_{i_k}(A_{i_k}+\Delta A_{i_k}) \\ &\quad \le (A_{i_k}+\Delta A_{i_k})^{\mathrm{T}}(A_{i_k}+\Delta A_{i_k}) + P_{i_k}^2 \le A_{i_k}^{\mathrm{T}} A_{i_k} \\ &\qquad + A_{i_k}^{\mathrm{T}} E_{i_k}\left(\varepsilon_1 I - E_{i_k}^{\mathrm{T}} E_{i_k}\right)^{-1} E_{i_k}^{\mathrm{T}} A_{i_k} + \varepsilon_1 H_{i_k}^{\mathrm{T}} H_{i_k} + P_{i_k}^2 \end{aligned} \tag{19}$$

$$\begin{aligned} &(B_{i_k} F_{i_k} + \Delta B_{i_k} F_{i_k})^{\mathrm{T}} P_{i_k} + P_{i_k}(B_{i_k} F_{i_k} + \Delta B_{i_k} F_{i_k}) \\ &\quad = F_{i_k}^{\mathrm{T}} B_{i_k}^{\mathrm{T}} P_{i_k} + P_{i_k} B_{i_k} F_{i_k} + F_{i_k}^{\mathrm{T}} \Delta B_{i_k}^{\mathrm{T}} P_{i_k} + P_{i_k} \Delta B_{i_k} F_{i_k} \\ &\quad \le F_{i_k}^{\mathrm{T}} B_{i_k}^{\mathrm{T}} P_{i_k} + P_{i_k} B_{i_k} F_{i_k} + \varepsilon_2 P_{i_k} E_{i_k} E_{i_k}^{\mathrm{T}} P_{i_k} \\ &\qquad + \frac{1}{\varepsilon_2} F_{i_k}^{\mathrm{T}} \hat{H}_{i_k}^{\mathrm{T}} \hat{H}_{i_k} F_{i_k}. \end{aligned} \tag{20}$$

From (17), we recall that

$$\begin{aligned} &A_{i_k}^{\mathrm{T}} A_{i_k} + A_{i_k}^{\mathrm{T}} E_{i_k}\left(\varepsilon_1 I - E_{i_k}^{\mathrm{T}} E_{i_k}\right)^{-1} E_{i_k}^{\mathrm{T}} A_{i_k} + \varepsilon_1 H_{i_k}^{\mathrm{T}} H_{i_k} \\ &\quad + P_{i_k}^2 + F_{i_k}^{\mathrm{T}} B_{i_k}^{\mathrm{T}} P_{i_k} + P_{i_k} B_{i_k} F_{i_k} + \varepsilon_2 P_{i_k} E_{i_k} E_{i_k}^{\mathrm{T}} P_{i_k} \\ &\quad + \frac{1}{\varepsilon_2} F_{i_k}^{\mathrm{T}} \hat{H}_{i_k}^{\mathrm{T}} \hat{H}_{i_k} F_{i_k} + R_{i_k} + F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k} < 0. \end{aligned} \tag{21}$$

Combining (19) and (20), and then applying condition (21) to the right-hand side of the resulting inequality, we obtain

$$\begin{aligned} &R_{i_k} + F_{i_k}^{\mathrm{T}} Q_{i_k} F_{i_k} + (A_{i_k} + \Delta A_{i_k})^{\mathrm{T}} P_{i_k} + P_{i_k}(A_{i_k} + \Delta A_{i_k}) \\ &\quad + (B_{i_k} F_{i_k} + \Delta B_{i_k} F_{i_k})^{\mathrm{T}} P_{i_k} + P_{i_k}(B_{i_k} F_{i_k} + \Delta B_{i_k} F_{i_k}) < 0. \end{aligned} \tag{22}$$

This shows that condition (i) of Definition 2 is satisfied. Finally, by (18), we see that condition (ii) of Definition 2 is also satisfied. Thus, it is clear upon Definition 2 that $u(t) = F_{i_k} x(t)$ is an LQ guaranteed cost control law for the uncertain impulsive switched system (1). Furthermore, by virtue of Theorem 1, the closed-loop uncertain system is quadratically stable. The proof is complete. ■

The sufficient conditions (16)–(18) given in Theorem 2 can be expressed in terms of LMIs as detailed in the following corollary.

Corollary 1: Suppose there exist $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $P_{i_k} = P_{i_k}^{\mathrm{T}} > 0$ such that the following conditions are satisfied:

(i) $$\begin{bmatrix} \varepsilon_1 I & E_{i_k}^{\mathrm{T}} P_{i_k} \\ P_{i_k} E_{i_k} & P_{i_k} \end{bmatrix} > 0 \tag{23}$$

(ii) $$\begin{bmatrix} S_{i_k} & P_{i_k} \\ P_{i_k} & -W_{i_k}^{-1} \end{bmatrix} < 0 \tag{24}$$

where

$$\begin{aligned} S_{i_k} &= A_{i_k}^{\mathrm{T}} A_{i_k} + A_{i_k}^{\mathrm{T}} E_{i_k}\left(\varepsilon_1 I - E_{i_k}^{\mathrm{T}} E_{i_k}\right)^{-1} E_{i_k}^{\mathrm{T}} A_{i_k} \\ &\quad + \varepsilon_1 H_{i_k}^{\mathrm{T}} H_{i_k} + R_{i_k} \end{aligned} \tag{25}$$

$$\begin{aligned} W_{i_k} &= L_{i_k}^{\mathrm{T}} B_{i_k}^{\mathrm{T}} + B_{i_k} L_{i_k} + \varepsilon_2 E_{i_k} E_{i_k}^{\mathrm{T}} + \frac{1}{\varepsilon_2} L_{i_k}^{\mathrm{T}} \hat{H}_{i_k}^{\mathrm{T}} \hat{H}_{i_k} L_{i_k} \\ &\quad + I + L_{i_k}^{\mathrm{T}} Q_{i_k} L_{i_k} \end{aligned} \tag{26}$$

while L_{i_k} is an appropriate given matrix and $F_{i_k} = L_{i_k} P_{i_k}$

(iii) $$\begin{bmatrix} P_{i_{k-1}} & (I + C_k)^{\mathrm{T}} P_{i_k} \\ P_{i_k}(I + C_k) & P_{i_k} \end{bmatrix} > 0. \tag{27}$$

Then, there exists an LQ guaranteed cost control law $u(t) = L_{i_k} P_{i_k} x(t)$, and the closed-loop system (1) is quadratically stable [3].

Proof: Using Schur complements, we can easily show that (11)–(13) of Theorem 2 are equivalent to (23)–(27). Thus, by virtue of Theorem 2, the conclusion of the corollary follows readily. This completes the proof. ■

Remark 3: The feasibility of the LMIs presented in Corollary 1 can be solved by using the feasp command in the LMI toolbox within the MATLAB environment. Once a solution of these LMIs is feasible, the required LQ guaranteed cost control law, that ensures the quadratic stability of the closed-loop system, can be contracted readily.

Remark 4: We can see that the system model considered in this correspondence is more general than those studied in [1] and [2]. Thus, some of our results cover those existing stability criteria obtained in [1] and [2].

IV. Example

We shall illustrate our results through an example. Consider the impulsive switched system (1) with the following specifications:

$$A_1 = \begin{bmatrix} 1 & 3.6 \\ 0.7 & -0.8 \end{bmatrix} \quad B_1 = \begin{bmatrix} 1.4 & 1.8 \\ 1.2 & 2 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 0.5 & 0.6 \\ 1.2 & 0.4 \end{bmatrix} \quad \hat{H}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1.01 & 0 \\ 0 & 1.01 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0.8 & 3.7 \\ 0.6 & -0.7 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1.3 & 1.5 \\ 1.2 & 1.8 \end{bmatrix} \quad H_2 = \begin{bmatrix} 0.4 & 0.4 \\ 1 & 0.5 \end{bmatrix}$$

$$\hat{H}_2 = \begin{bmatrix} 0.8 & 0.1 \\ 0.1 & 1.2 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1.01 & 0 \\ 0 & 1.01 \end{bmatrix}$$

$$C_k = -0.6 \quad \Psi_1 = \Psi_2 = \sin(15 * t).$$

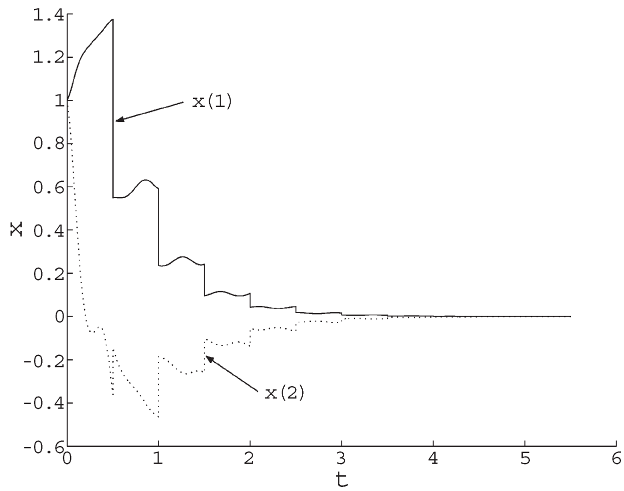

Fig. 1. Time series of the state $x(t)$ when $\Psi_1(t) = \Psi_2(t) = \sin(15t) * I$.

For the cost function (3), let Q_1, Q_2, R_1, R_2 be the identity matrices with appropriate dimensions. Let $\Delta t_k \equiv 0.5$. Suppose that the impulsive switched system starts to operate using subsystem (1) from the initial point $[1, 1]^{\mathrm{T}}$. Assume that the system switches between using the subsystem (1) and subsystem (2) under a switching law. Select $L_1 = \begin{bmatrix} -14 & 5 \\ 3 & -14 \end{bmatrix}$, and $L_2 = \begin{bmatrix} -13 & -14 \\ -7 & 6 \end{bmatrix}$. By Corollary 1, we can see the following results which satisfy LMIs (23)–(27):

$$P_1 = \begin{bmatrix} 0.3000 & 0.1243 \\ 0.1243 & 0.5881 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0.2398 & 0.0433 \\ 0.0433 & 0.6167 \end{bmatrix}.$$

Furthermore, through $F_{i_k} = L_{i_k} P_{i_k}$, we obtain

$$F_1 = \begin{bmatrix} -3.5784 & -1.2007 \\ -0.8400 & -7.8610 \end{bmatrix} \quad F_2 = \begin{bmatrix} -5.6398 & -9.8496 \\ -1.3542 & 2.6588 \end{bmatrix}.$$

It means that there exists an LQ guaranteed cost control law that makes the closed-loop system quadratically stable. Then, the guaranteed cost of impulsive switched systems is obtained, which is $J^* = 1.1367$.

Fig. 1 and Fig. 2 show the time series behavior and the phase portrait of the impulsive switched systems. We see that $x(1)$ and $x(2)$ approach equilibrium after the system evolve 5 s from the starting points.

V. Conclusion

This correspondence studied the robust stability for a class of impulsive switched system with guaranteed cost control. Two new definitions on the guaranteed cost control and LQ guaranteed cost control for this class of systems are introduced. New sufficient conditions for the existence of an LQ guaranteed cost controller are obtained. These results guarantee both robust quadratic stability performance and LQ guaranteed cost performance.

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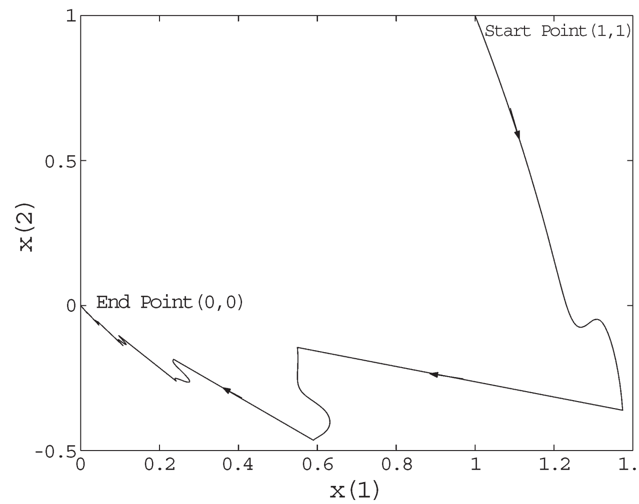

Fig. 2. Phase portrait of uncertain impulsive switched systems when $\Psi_1(t) = \Psi_2(t) = \sin(15t) * I$.

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