FRM-Based FIR Filters with Minimum Coefficient Sensitivities

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***Abstract*—A method for optimizing FRM-based FIR filters with optimum coefficient sensitivity is presented. This technique can be used in conjunction with nonlinear optimization techniques to design very sharp filters that do not only have very sparse coefficient values but also very low coefficient sensitivity.**

I. INTRODUCTION

THE frequency response masking (FRM) technique [1-20] has received much attention for the synthesis of very sharp digital filters with very sparse coefficients. It has found applications in diverse fields including the synthesis of various types of filters such as half-band filters [21]-[23], 2D filters [24], IIR filters [25]-[28], filter banks [29]-[34], decimators and interpolators [35], [36], and Hilbert transformers [37], [38], FPGA implementations [39-41], transmultiplexer design [42], ECG signal processing [43], hearing aids [44], digital audio [45]-[49] application and analysis, speech recognition [50], array beamforming [51], software radio [52], and noise thermometer [53].

Fig.1 shows the structure of an FIR filter synthesized using the FRM technique. A filter with z-transform transfer function $H(z)$ is synthesized using a network of sub-filters $H_a(z^M)$, $H_{Ma}(z)$, $H_{Mc}(z)$, and $z^{-\bullet}$ where $z^{-\bullet}$ represents an appropriate negative integer power of z and M is an integer [1]; all the sub-filters have low arithmetic complexities.

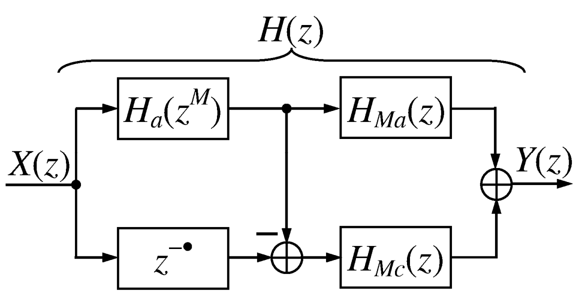

Fig.1 The structure of a filter synthesized using the FRM technique.

Many different optimization techniques have been developed for optimizing the sub-filters of Fig.1. For a given set of frequency response requirements imposed on $H(z)$, there is a wide range of sub-filter frequency responses that can meet the requirement. The coefficient sensitivity of $H(z)$ depend on the frequency responses of the sub-filters and may differ by many orders of magnitudes for different choice of $H_a(z^M)$, $H_{Ma}(z)$, and $H_{Mc}(z)$. Section II shows an example of an FRM-based filter optimized only for overall frequency response performance under infinite precision condition disregarding the coefficient sensitivity. Thus, it is important to steer the optimization algorithm during the course of optimization so as to produce a design with desirable coefficient sensitivity. This paper addresses the issue of designing FRM-based digital filters with minimum coefficient sensitivity.

II. COEFFICIENT SENSITIVITY OF SUB-FILTERS DESIGNED FOR OPTIMAL FREQUENCY RESPONSE PERFORMANCE

Many powerful nonlinear optimization techniques have been developed for the design of the sub-filters. These advanced non-linear optimization techniques jointly optimize all the sub-filters for obtaining the optimum overall frequency response. The frequency response of the overall filter obtained using these nonlinear optimization techniques is significantly better than that obtained by optimizing the sub-filters separately using linear optimization technique. Unfortunately, even though the filter designed using these advanced techniques has good overall frequency response under infinite precision arithmetic condition; its coefficient sensitivity may be undesirable; the coefficient values may be very large. We shall illustrate this problem by means of an example.

Consider the design of a low-pass filter with band edges at $0.3f_s$ and $0.305f_s$, respectively, where f_s is the sampling frequency. The allowed peak ripple magnitude is 0.01. When the peak ripple magnitude is used as the objective function for minimization, there are a large number of solutions with almost the same objective function values. The optimization algorithm may converge to any one of the solutions if no further criterion is imposed. The coefficient values for a typical solution are shown in Table I. The value of M in $H_a(z^M)$ is 9. As can be seen from Table I, the coefficients of $H_a(z)$ have very large magnitude resulting in serious coefficient sensitivity problem.

III. COEFFICIENT SENSITIVITY

The frequency response of the overall filter $H(e^{j\omega})$ is given by

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TABLE I
COEFFICIENT VALUES FOR THE EXAMPLE OF SECTION II.

$h_{Ma}(-13) =$	-0.000000924108	$= h_{Ma}(13)$
$h_{Ma}(-12) =$	0.000000260414	$= h_{Ma}(12)$
$h_{Ma}(-11) =$	0.000001052353	$= h_{Ma}(11)$
$h_{Ma}(-10) =$	-0.000001157981	$= h_{Ma}(10)$
$h_{Ma}(-9) =$	-0.007413359428	$= h_{Ma}(9)$
$h_{Ma}(-8) =$	0.001620873527	$= h_{Ma}(8)$
$h_{Ma}(-7) =$	0.016104361571	$= h_{Ma}(7)$
$h_{Ma}(-6) =$	-0.023711885463	$= h_{Ma}(6)$
$h_{Ma}(-5) =$	-0.008454089406	$= h_{Ma}(5)$
$h_{Ma}(-4) =$	0.066353136586	$= h_{Ma}(4)$
$h_{Ma}(-3) =$	-0.042077991579	$= h_{Ma}(3)$
$h_{Ma}(-2) =$	-0.103044888697	$= h_{Ma}(2)$
$h_{Ma}(-1) =$	0.293337987854	$= h_{Ma}(1)$
$h_{Ma}(0) =$	0.616801115467	

$h_{Mc}(-9) =$	-0.007413178906	$= h_{Mc}(9)$
$h_{Mc}(-8) =$	0.001618924470	$= h_{Mc}(8)$
$h_{Mc}(-7) =$	0.016105518119	$= h_{Mc}(7)$
$h_{Mc}(-6) =$	-0.023710366562	$= h_{Mc}(6)$
$h_{Mc}(-5) =$	-0.008456582979	$= h_{Mc}(5)$
$h_{Mc}(-4) =$	0.066353689510	$= h_{Mc}(4)$
$h_{Mc}(-3) =$	-0.042074860240	$= h_{Mc}(3)$
$h_{Mc}(-2) =$	-0.103047683414	$= h_{Mc}(2)$
$h_{Mc}(-1) =$	0.293336561926	$= h_{Mc}(1)$
$h_{Mc}(0) =$	0.616804933802	

$h_a(-22) =$	112.01	$= h_a(22)$
$h_a(-21) =$	-239.28	$= h_a(21)$
$h_a(-20) =$	-62.89	$= h_a(20)$
$h_a(-19) =$	169.90	$= h_a(19)$
$h_a(-18) =$	4.74	$= h_a(18)$
$h_a(-17) =$	-291.36	$= h_a(17)$
$h_a(-16) =$	115.66	$= h_a(16)$
$h_a(-15) =$	317.12	$= h_a(15)$
$h_a(-14) =$	-283.84	$= h_a(14)$
$h_a(-13) =$	-336.54	$= h_a(13)$
$h_a(-12) =$	513.53	$= h_a(12)$
$h_a(-11) =$	229.61	$= h_a(11)$
$h_a(-10) =$	-780.66	$= h_a(10)$
$h_a(-9) =$	-10.84	$= h_a(9)$
$h_a(-8) =$	1080.06	$= h_a(8)$
$h_a(-7) =$	-415.84	$= h_a(7)$
$h_a(-6) =$	-1335.71	$= h_a(6)$
$h_a(-5) =$	1163.59	$= h_a(5)$
$h_a(-4) =$	1587.93	$= h_a(4)$
$h_a(-3) =$	-2702.65	$= h_a(3)$
$h_a(-2) =$	-1882.04	$= h_a(2)$
$h_a(-1) =$	7204.87	$= h_a(1)$
$h_a(0) =$	2243.27	

$$H(e^{j\omega}) = H_a(e^{j\omega M})H_{Ma}(e^{j\omega}) + \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega}) \quad (1)$$

Let the n^{th} coefficient values of $H_a(z^M)$, $H_{Ma}(z)$, and $H_{Mc}(z)$ be $h_a(n)$, $h_{Ma}(n)$, and $h_{Mc}(n)$, respectively. When the coefficient values are made discrete, round off errors are introduced into the coefficient values. Let the errors introduced into $h_a(n)$, $h_{Ma}(n)$, and $h_{Mc}(n)$ be $\Delta h_a(n)$, $\Delta h_{Ma}(n)$, and $\Delta h_{Mc}(n)$, respectively. We shall investigate the change in $H(e^{j\omega})$ caused by small values of $|\Delta h_a(n)|$, $|\Delta h_{Ma}(n)|$, and $|\Delta h_{Mc}(n)|$ where $|x|$ denotes "the magnitude of x". Suppose that $H(e^{j\omega})$, $H_a(e^{j\omega M})$, $H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$ become $H(e^{j\omega}) + \Delta H(e^{j\omega})$, $H_a(e^{j\omega M}) + \Delta H_a(e^{j\omega M})$, $H_{Ma}(e^{j\omega}) + \Delta H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega}) + \Delta H_{Mc}(e^{j\omega})$, respectively, when $h_a(n)$, $h_{Ma}(n)$, and $h_{Mc}(n)$ become $h_a(n) + \Delta h_a(n)$, $h_{Ma}(n) + \Delta h_{Ma}(n)$, and $h_{Mc}(n) + \Delta h_{Mc}(n)$. We shall assume that $|\Delta H(e^{j\omega})|$, $|\Delta H_a(e^{j\omega M})|$, $|\Delta H_{Ma}(e^{j\omega})|$, and $|\Delta H_{Mc}(e^{j\omega})|$ are small. Thus, neglecting the second order terms, we have

$$\Delta H(e^{j\omega}) = \{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}\Delta H_a(e^{j\omega M}) + H_a(e^{j\omega M})\Delta H_{Ma}(e^{j\omega}) + \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}\Delta H_{Mc}(e^{j\omega}) \quad (2)$$

It can be seen from (2) that the magnitudes of the sensitivities of $H(e^{j\omega})$ with respect to changes in $H_a(e^{j\omega M})$, $H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$ are $|H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})|$, $|H_a(e^{j\omega M})|$, and $|e^{-j\omega\bullet} - H_a(e^{j\omega M})|$, respectively. Thus, $|H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})|$, $|H_a(e^{j\omega M})|$, and $|e^{-j\omega\bullet} - H_a(e^{j\omega M})|$ should be minimized for good robustness against changes in $H_a(e^{j\omega M})$, $H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$. Squaring both sides of (2) leads to

$$\begin{aligned}\{\Delta H(e^{j\omega})\}^2 &= \{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2\{\Delta H_a(e^{j\omega M})\}^2 \\ &+ (H_a(e^{j\omega M}))^2\{\Delta H_{Ma}(e^{j\omega})\}^2 + \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2\{\Delta H_{Mc}(e^{j\omega})\}^2 \\ &+ 2\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}H_a(e^{j\omega M})\Delta H_a(e^{j\omega M})\Delta H_{Ma}(e^{j\omega}) \\ &+ 2\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}\Delta H_a(e^{j\omega M})\Delta H_{Mc}(e^{j\omega}) \\ &+ 2H_a(e^{j\omega M})\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}\Delta H_{Ma}(e^{j\omega})\Delta H_{Mc}(e^{j\omega}) \quad (3)\end{aligned}$$

Let $E\{x\}$ denotes the expected value of x. Taking the expected values for both sides of (3) and taking note that $E\{\Delta H_a(e^{j\omega M})\Delta H_{Ma}(e^{j\omega})\} = E\{\Delta H_a(e^{j\omega M})\Delta H_{Mc}(e^{j\omega})\} = E\{\Delta H_{Ma}(e^{j\omega})\Delta H_{Mc}(e^{j\omega})\} = 0$, we have

$$\begin{aligned}E\{(\Delta H(e^{j\omega}))^2\} &= \{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 E\{(\Delta H_a(e^{j\omega M}))^2\} \\ &+ (H_a(e^{j\omega M}))^2 E\{(\Delta H_{Ma}(e^{j\omega}))^2\} \\ &+ \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2 E\{(\Delta H_{Mc}(e^{j\omega}))^2\} \quad (4)\end{aligned}$$

Depending on the parity of N_x, the frequency response $H_x(e^{j\omega})$ of a symmetrical impulse response FIR filter with length N_x and coefficient values $h_x(n)$, $n = 0, \ldots, N_x - 1$ is given by either (5a) or (5b)

$$H_x(e^{j\omega}) = e^{-j\frac{N_x-1}{2}\omega}\sum_{n=1}^{N_x/2} 2h_x\left(\tfrac{N_x}{2} - n\right)\cos(\omega(n - \tfrac{1}{2})) \quad (5a)$$

$$H_x(e^{j\omega}) = e^{-j\frac{N_x-1}{2}\omega}\left\{h_x\left(\tfrac{N_x-1}{2}\right) + 2\sum_{n=1}^{\frac{N_x-1}{2}} h_x\left(\tfrac{N_x-1}{2} - n\right)\cos(\omega n)\right\} \quad (5b)$$

Suppose that changing $h_x(n)$ to $h_x(n) + \Delta h_x(n)$ causes $H_x(e^{j\omega})$ to

change to $H_x(e^{j\omega}) + \Delta H_x(e^{j\omega})$. Thus, depending on the parity of N_x, we have either (6a) or (6b)

$$\Delta H_x(e^{j\omega}) = e^{-j\frac{N_x-1}{2}\omega}\sum_{n=1}^{N_x/2} 2\Delta h_x\left(\tfrac{N_x}{2} - n\right)\cos(\omega(n - \tfrac{1}{2})) \quad (6a)$$

$$\Delta H_x(e^{j\omega}) = e^{-j\frac{N_x-1}{2}\omega}\left\{\Delta h_x\left(\tfrac{N_x-1}{2}\right) + 2\sum_{n=1}^{\frac{N_x-1}{2}} \Delta h_x\left(\tfrac{N_x-1}{2} - n\right)\cos(\omega n)\right\} \quad (6b)$$

Assume that for $i \neq j$,

$$E\{\Delta h_x(i)\, \Delta h_x(j)\} = 0 \quad (7)$$

Define the quantity ε^2 as

$$\varepsilon^2 = E\{(\Delta h_x(i))^2\} \quad (8)$$

Define

$$\|\Delta H_x(e^{j\omega})\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} E\{(\Delta H_x(e^{j\omega}))^2\}\,\mathrm{d}\omega \quad (9)$$

From (6), (7), (8) and (9), we have

$$\|\Delta H_x(e^{j\omega})\|^2 = N_x\, E\{(\Delta h_x(n))^2\} = N_x\, \varepsilon^2 \quad (10)$$

Although $\Delta H_x(e^{j\omega})$ is a function of ω for a given filter, $E\{(\Delta H_x(e^{j\omega}))^2\}$ for a large number of independent filters is a constant independent of ω if $\Delta h_x(i)$ has flat spectrum. Thus,

$$E\{(\Delta H_x(e^{j\omega}))^2\} = N_x\, \varepsilon^2 \quad (11)$$

Applying the result of (11), (4) becomes

$$E\{(\Delta H(e^{j\omega}))^2\} = \varepsilon^2\{N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 + N_{Ma}H_a(e^{j\omega M})^2 + N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2\} \quad (12)$$

where N_a, N_{Ma}, and N_{Mc} are the numbers of coefficients of $H_a(z)$, $H_{Ma}(z)$, and $H_{Mc}(z)$, respectively.

We have

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(H_x(e^{j\omega})\right)^2\mathrm{d}\omega = \sum_{n=0}^{N_x-1}\left(h_x(n)\right)^2 \quad (13)$$

Define

$$\|h_a\|^2 = \sum_{n=0}^{N_a-1}\left(h_a(n)\right)^2 \quad (14a)$$

$$\|1 - h_a\|^2 = \left(1 - h_a\left(\frac{N_a - 1}{2}\right)\right)^2 + 2\sum_{n=0}^{\frac{N_a-1}{2}}\left(h_a(n)\right)^2 \quad (14b)$$

$$\|h_{Ma} - h_{Mc}\|^2 = \sum_{n=0}^{N_{Ma}-1} \left(h_{Ma}(n) - h_{Mc}(n)\right)^2 \quad (14c)$$

For $H_{Ma}(z)$ and $H_{Mc}(z)$ to produce the same phase shifts so that their outputs can be summed correctly, $H_{Ma}(z)$ and $H_{Mc}(z)$ must have the same order [1], i.e. $N_{Ma} = N_{Mc}$. From (12), (13), and (14), we have

$$\|\Delta H(e^{j\omega})\|^2 = \{N_a \|h_{Ma} - h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1-h_a\|^2 \varepsilon^2 \quad (15)$$

Let

$$S^2 = N_a \|h_{Ma} - h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1-h_a\|^2 \quad (16)$$

From (15) and (16), we have

$$\|\Delta H(e^{j\omega})\|^2 = S^2\varepsilon^2 \quad (17)$$

From (17), and since $\varepsilon^2 = \mathrm{E}\{(\Delta h_x(i))^2\}$ by definition, it is clear

that S^2 is a coefficient sensitivity measure. In order to minimize S^2, the peak ripple magnitude of $H(e^{j\omega})$, denoted as δ, may be set as a constraint and S^2 becomes the objective for minimization as in (18).

$$\text{Minimize } S^2 = N_a \|h_{Ma} - h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1-h_a\|^2 \quad (18a)$$

$$\text{subject to } \delta \le \delta_0 \quad (18b)$$

In (18b), δ_0 is a predefined constant.

IV A MINIMUM SENSITIVITY EXAMPLE

We choose the design of a low-pass filter with the same band edges, M value for $H_a(z^M)$, and sub-filter lengths as the filter shown in Section II as an example to illustrate the superiority of this new design technique. The peak ripple magnitude is relaxed by 2% to 0.0102 and (18a) is used as the objective function. The coefficient values of this new

TABLE II
COEFFICIENT VALUES FOR $H_a(z)$, $H_{Ma}(z)$, AND $H_{Mc}(z)$ FOR THE FILTER OBTAINED BY MINIMIZING S^2.

$h_{Ma}(-13) = -0.03100 = h_{Ma}(13)$	$h_a(-22) = 0.00306 = h_a(22)$
$h_{Ma}(-12) = 0.01063 = h_{Ma}(12)$	$h_a(-21) = -0.00567 = h_a(21)$
$h_{Ma}(-11) = 0.03332 = h_{Ma}(11)$	$h_a(-20) = -0.00148 = h_a(20)$
$h_{Ma}(-10) = -0.03671 = h_{Ma}(10)$	$h_a(-19) = 0.00702 = h_a(19)$
$h_{Ma}(-9) = -0.01144 = h_{Ma}(9)$	$h_a(-18) = 0.00010 = h_a(18)$
$h_{Ma}(-8) = 0.06737 = h_{Ma}(8)$	$h_a(-17) = -0.00783 = h_a(17)$
$h_{Ma}(-7) = -0.02291 = h_{Ma}(7)$	$h_a(-16) = 0.00280 = h_a(16)$
$h_{Ma}(-6) = -0.07251 = h_{Ma}(6)$	$h_a(-15) = 0.00962 = h_a(15)$
$h_{Ma}(-5) = 0.07348 = h_{Ma}(5)$	$h_a(-14) = -0.00874 = h_a(14)$
$h_{Ma}(-4) = 0.04921 = h_{Ma}(4)$	$h_a(-13) = -0.00995 = h_a(13)$
$h_{Ma}(-3) = -0.14608 = h_{Ma}(3)$	$h_a(-12) = 0.01540 = h_a(12)$
$h_{Ma}(-2) = -0.01013 = h_{Ma}(2)$	$h_a(-11) = 0.00703 = h_a(11)$
$h_{Ma}(-1) = 0.34265 = h_{Ma}(1)$	$h_a(-10) = -0.02315 = h_a(10)$
$h_{Ma}(0) = 0.49052$	$h_a(-9) = -0.00080 = h_a(9)$
$h_{Mc}(-9) = -0.00775 = h_{Mc}(9)$	$h_a(-8) = 0.03195 = h_a(8)$
$h_{Mc}(-8) = 0.00006 = h_{Mc}(8)$	$h_a(-7) = -0.01301 = h_a(7)$
$h_{Mc}(-7) = 0.01675 = h_{Mc}(7)$	$h_a(-6) = -0.03934 = h_a(6)$
$h_{Mc}(-6) = -0.02333 = h_{Mc}(6)$	$h_a(-5) = 0.03402 = h_a(5)$
$h_{Mc}(-5) = -0.01018 = h_{Mc}(5)$	$h_a(-4) = 0.04775 = h_a(4)$
$h_{Mc}(-4) = 0.06650 = h_{Mc}(4)$	$h_a(-3) = -0.08291 = h_a(3)$
$h_{Mc}(-3) = -0.04176 = h_{Mc}(3)$	$h_a(-2) = -0.05549 = h_a(2)$
$h_{Mc}(-2) = -0.10389 = h_{Mc}(2)$	$h_a(-1) = 0.21111 = h_a(1)$
$h_{Mc}(-1) = 0.29163 = h_{Mc}(1)$	$h_a(0) = 0.07998$
$h_{Mc}(0) = 0.61760$	

design are shown in Table II. The value of S^2 is 26.43 and the peak ripple magnitudes of the overall filter become 0.01029 if the coefficient quantization step sizes for each coefficient in Table II are 2^{-14}.

For the purpose of comparison, the value of $N_a \|h_{Ma} - h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1-h_a\|^2$ (i.e. the equivalent S^2value) for the filter whose coefficients are shown in Table I is 6.78×10^9. High coefficient sensitivity is expected. To achieve a peak ripple magnitude of about 0.0103 the coefficient quantization step size for the filter shown in Table I should not be larger than 2^{-27}.

V CONCLUSIONS

In this paper, we present a technique that includes coefficient sensitivity measure into the objective function of the optimization algorithm used for designing FRM based FIR filters. It results in filters with low coefficient sensitivity.

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