Fitting gravimetric geoid models to vertical deflections

W.E. Featherstone (✉)
Western Australian Centre for Geodesy & The Institute for Geoscience Research,
Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia
Tel: +61 8 9266-2734; Fax: +61 8 9266-2703; Email: W.Featherstone@curtin.edu.au

D.D. Lichti
Department of Geomatics Engineering & The Centre for Bioengineering Research and Education, Schulich School of Engineering, The University of Calgary,
2500 University Drive NW, Calgary, Alberta, T2N 1N4, Canada
Tel: +1 403 210-9495; Fax: +1 403 284 1980; Email: ddlichti@ucalgary.ca

Abstract. Regional gravimetric geoid and quasigeoid models are now commonly fitted to GPS-levelling data, which simultaneously absorbs levelling, GPS and quasi/geoid errors due to their inseparability. We propose that independent vertical deflections are used instead, which are not affected by this inseparability problem. The formulation is set out for geoid slopes and changes in slopes. Application to 1080 astrogeodetic deflections over Australia for the AUSGeoid98 model shows that it is feasible, but the poor quality of the historical astrogeodetic deflections led to some unrealistic values.

Keywords: Gravimetric geoid errors, vertical deflections, vertical datum errors

1. Introduction

Fitting regional gravimetric geoid or quasigeoid models to GPS-levelling data has become a widespread practice. A principal objection to this is the inseparability of errors among the levelling and local vertical datum (LVD), GPS and gravimetric quasi/geoid model (cf. Featherstone 2004). While numerous different parameterisations have been devised for this fitting (e.g., Milbert 1995; Jiang and Duquenne 1996; Forsberg 1998, Kotsakis and Sideris 1999, Fotopoulos 2005; Featherstone and Sproule 2006; Soltanpour et al. 2006, etc.), it only ever models the reference surface of the LVD for GPS-based levelling, rather than the classical quasi/geoid (cf. Featherstone 1998, 2006b).

On the other hand, astrogeodetically observed deflections (or deviations) of the vertical (i.e., from precisely timed observations to the stars) provide a source of terrestrial gravity field information that is independent of errors in the LVD (e.g., Featherstone 2006a). Also, Jekeli (1999), Kütreiber (1999), Hirt and Flury (2007), Hirt et al. (2007), Hirt and Seeber (2008), Kühtreiber and Abd-Elmotaal (2007), Marti (2007) and Müller et al. (2007b) demonstrate the utility of vertical deflections for gravity field determination and validation. Moreover, modern digital zenith cameras can now observe astrogeodetic vertical deflections to 0.1 arc-second in about 20 mins (e.g., Hirt and Bürki 2002, Hirt and Seeber 2007, Müller et al. 2007a). As such, vertical deflections will probably become more important for gravity field model validation (cf. Jekeli 1999; Featherstone and Morgan 2007, Pavlis et al. 2008).

In this short note, we propose that astrogeodetic vertical deflections are used to 'correct/control' errors in regional gravimetric quasi/geoid models, as a preferable alternative to the widespread use of using only GPS-levelling data because of the inseparability problem. This is akin to the classical orientation of a reference ellipsoid to a regional geodetic datum (e.g., Mather 1970, Mather and Fryer 1970). We present functional models for the two-, three- and four-parameter vertical deflection fitting (essentially geoid slopes and changes in slopes), which are then applied to 1080 historical astrogeodetic vertical deflections and vertical deflections derived from AUSGeoid98 (Featherstone et al. 2001) over Australia.

2. Background & Definitions

Vertical deflections can either be absolute or relative, depending respectively on whether a geocentric or local reference ellipsoid (and datum) is used in their definition (Jekeli 1999; Featherstone and Rüeger 2000). Here, we will only deal with absolute vertical deflections since modern gravimetric quasi/geoid models refer to a geocentric reference ellipsoid, and geodetic coordinates (used to compute the astrogeodetic vertical deflections; see below) are directly or indirectly (i.e., by datum transformation) on a geocentric datum and geocentric reference ellipsoid.

2.1 Astrogeodetic deflections

Astrogeodetic observations to the stars lead to natural/astronomic coordinates (latitude Φ, longitude Λ) of a point on or just above the Earth's surface, which when compared with geocentric geodetic coordinates (latitude ϕ, longitude λ) of the same point yield absolute Helmert (i.e., at the Earth's surface; cf. Jekeli 1999) north-south (ξ) and east-west (η) deflections according to (e.g., Bomford 1980):

$$\xi_H = \Phi - \phi \tag{1}$$

$$\eta_H = (\Lambda - \lambda)\cos\phi \tag{2}$$

where subscript H is used to distinguish these as Helmert deflections. Sign conventions mean that the deflection in the meridian ξ is positive north and negative south, and the deflection in the prime vertical η is positive east and negative west.

2.2 Gravimetric deflections

Absolute Pizzetti deflections (i.e., deflections at the geoid; cf. Jekeli 1999) can be computed directly by Vening-Meinesz's integral (e.g., Heiskanen and Moritz 1967), or can be computed indirectly from horizontal gradients of a gravimetric geoid model by (e.g., Torge 1991)

$$\xi_P = \frac{-\Delta N}{\rho\,\Delta\phi} \quad (3)$$

$$\eta_P = \frac{-\Delta N}{\nu\,\Delta\lambda\cos\phi} \quad (4)$$

where subscript P is used to distinguish these as Pizzetti deflections. The same sign conventions as for astrogeodetic deflections also apply here. In Eqs. (3) and (4), ΔN is the change in the geoid height between grid nodes of latitude spacing ($\Delta\phi$) and longitude spacing ($\Delta\lambda$), ρ is the radius of curvature of the [geocentric] reference ellipsoid in the meridian,

$$\rho = \frac{a(1-e^2)}{\left(\sqrt{1-e^2\sin^2\phi}\right)^3} \quad (5)$$

and ν is the radius of curvature in the prime vertical

$$\nu = \frac{a}{\sqrt{1-e^2\sin^2\phi}} \quad (6)$$

where e is the first numerical eccentricity and a is the semi-major axis length of the reference ellipsoid; GRS80 (Moritz 1980) is used here.

2.3 Curvature and torsion of the plumbline

The curvature and torsion of the plumbline (cf. Grafarend 1997) cause a [small] angular difference between Helmert and Pizzetti deflections, which is a function of 3D position.

However, the curvature and torsion are rather difficult to estimate accurately because they require detailed knowledge of the shape of and mass-density distribution in the topography (e.g., Heiskanen and Moritz 1967; Bomford 1980). Here, they are assumed to be small (less than one arc-second) and thus neglected in the sequel, but in order to achieve the best results in terms of theoretical consistency, they should be computed and applied to the [astrogeodetic] Helmert deflections to give Pizzetti deflections consistent with the geoid model.

3. Functional Model

A common mathematical model used to fit regional gravimetric quasi/geoids to GPS-levelling has been a bias (simultaneously accounting for the deficient zero-degree term in the quasi/geoid, LVD offsets and other constant biases (cf. Prutkin and Klees 2007)) and two orthogonal tilts (simultaneously accounting for the deficient first-degree terms in the quasi/geoid, long-wavelength quasi/geoid errors, long-wavelength distortions in the LVD and other tilts between the data). These all reflect the inseparability problem.

The origin of this popular four-parameter functional model can be traced back to Heiskanen and Moritz (1967, Sects 2-18 and 2-19), where the scale and origin deficiencies in a gravimetric geoid model δN, due to the inadmissible zero- and first-degree terms, may be determined using external geometrical control via

$$\delta N = N_0 + \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi \qquad (7)$$

where N_0 is the zero-degree term in the geoid representing the scale deficiency, and $\Delta X, \Delta Y, \Delta Z$ are the three orthogonal origin shifts of the geocentre from the centre of the reference ellipsoid (Heiskanen and Moritz 1967). This model is analogous with a four-parameter geodetic datum transformation (cf. Kotsakis 2008).

Equation (7) has often been recast in the simpler equivalent form of a biased, tilted and warped plane (cf. Forsberg 1998), giving

$$\delta N = A + B\phi + C\lambda + D\phi\lambda \qquad (8)$$

where A is the bias term (equivalent to N_0 in Eq. (7)), B and C describe the tilted plane in ϕ and λ, and D allows for the tilted plane to be warped into a hyperbolic paraboloid (e.g., Farin 2001, p.246).

The difference between astrogeodetic and geoid-derived deflections is parameterised similarly here to give for the north-south (N-S) component

$$\delta\xi = a_{00} + a_{10}\phi + a_{01}\lambda + a_{11}\phi\lambda \qquad (9)$$

and for the east-west (E-W) component

$$\delta\eta = b_{00} + b_{10}\phi + b_{01}\lambda + b_{11}\phi\lambda \qquad (10)$$

where $\delta\xi = \delta\xi_{\text{astro}} - \delta\xi_{\text{grav}}$ and $\delta\eta = \delta\eta_{\text{astro}} - \delta\eta_{\text{grav}}$ are the N-S and E-W deflection differences, respectively. Simplifications of these models down to two and three parameters will be tested later.

Since vertical deflections are second derivatives of the Earth's disturbing potential, the interpretation of the parameters in Eqs. (9) and (10) is slightly different to that for Eqs. (7) or (8). Firstly, the zero-degree term in the geoid (or LVD offset or other constant biases) is indeterminate from vertical deflections; since they are angular measures, they are insensitive to a scale change. The bias terms a_{00} and b_{00} in Eqs. (9) and (10) represent the average difference in N-S and E-W tilts between the gravimetric geoid and the [orthogonal] astrogeodetic deflections. The higher order terms in Eqs. (9) and (10) represent latitudinal and longitudinal changes in the differences, thus permitting medium-wavelength errors in the gravimetric geoid model to be controlled by the approach proposed.

4. Data

1080 astrogeodetic deflections (Fig. 1) were compiled from data held by Geoscience Australia and Landgate (the Western Australian geodetic agency). Most of these historical data were observed over 40 years ago so as to provide azimuth control on the long-line traverses for the Australian Geodetic Datum 1966 (Bomford 1967); also see Featherstone (2006) and Featherstone and Morgan (2007). No digital zenith camera observations are yet available in Australia.

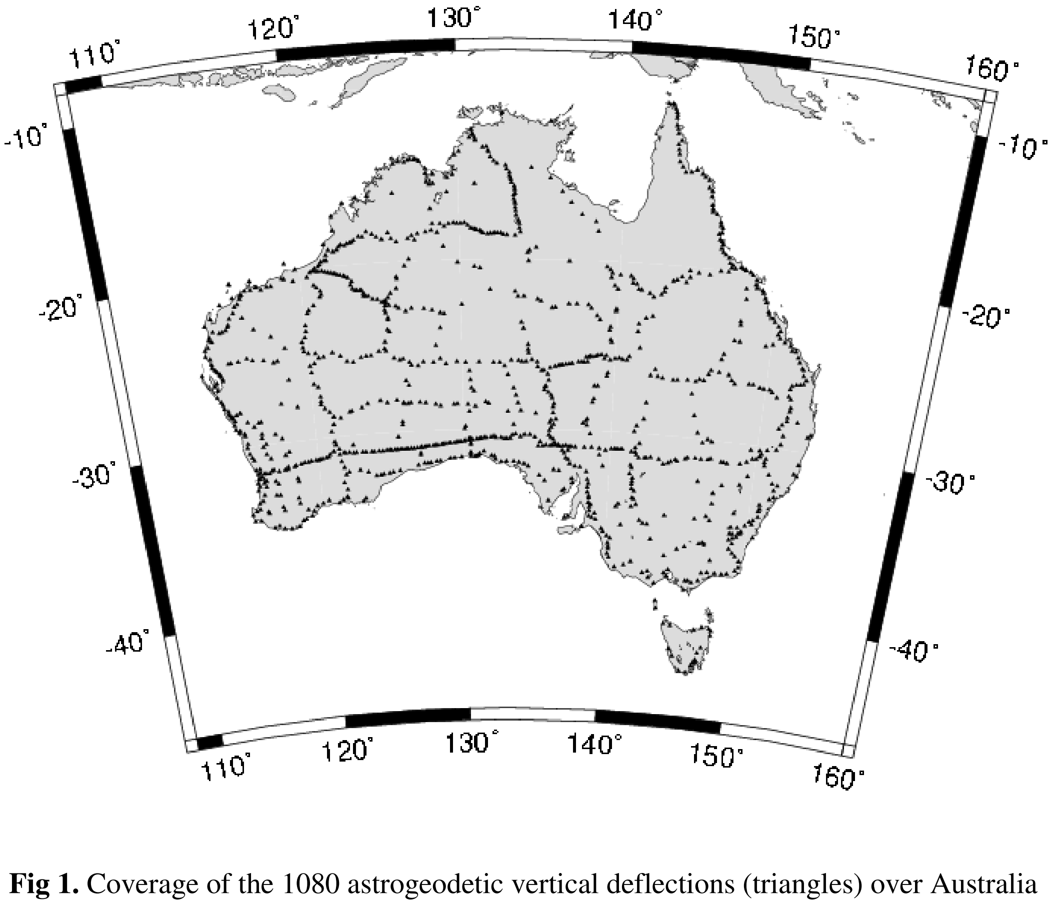

Fig 1. Coverage of the 1080 astrogeodetic vertical deflections (triangles) over Australia [Lambert projection]

The accuracy of the Australian astrogeodetic deflections is very difficult to ascertain because original records appear to be unavailable. Given the era of the observations, the main limiting factors are precise timing and the accuracy of the star catalogues then available, which will be substantiated later in Fig 2 by a larger spread in the E-W deflections. Using crude hand-waving arguments, as well as comparisons with AUSGeoid98, the accuracy of these astrogeodetic deflections is cautiously estimated to be one arc-second (Featherstone and Rüeger 1999; Featherstone 2006; Featherstone and Morgan 2007); also see Kearsley (1976). The geodetic coordinates are on the Geocentric Datum of Australia 1994, thus yielding absolute Helmert deflections (Eqs 1 and 2).

	All 1080 stations		After removal of 39 outliers	
	N-S ($\delta\xi$)	E-W ($\delta\eta$)	N-S ($\delta\xi$)	E-W ($\delta\eta$)
Max	17.83	9.11	2.92	3.00
Min	–7.76	–12.65	–3.36	–3.62
Mean	–0.25	–0.17	–0.25	–0.16
STD	±1.28	±1.36	±0.80	±1.05

Table 1. Statistics (in arc-seconds) of the difference between AUSGeoid98-derived and astrogeodetic deflections. Outlier detection used Baarda's (1968) data-snooping technique.

The Pizzetti vertical deflections were derived from AUSGeoid98 (Featherstone et al. 2001) using Eqs. (3) to (6) for GRS80. The accuracy of these deflections is also difficult to ascertain, but they are also cautiously estimated to be around one arc-second (Featherstone 2006; Featherstone and Morgan 2007). However, this becomes immaterial if the astrogeodetic vertical deflections are to be used as control. The AUSGeoid98-derived deflections were bi-cubically interpolated from a pre-computed grid (Featherstone 2001), then subtracted from the astrogeodetic deflections. Bi-cubic interpolation proved to be better than bi-linear interpolation, which is consistent with expectation because vertical deflections contain more power in the high frequencies. The statistics of these differences are in Table 1, before and after rejection of 39 outliers that were identified with Baarda's (1968) data-snooping test at 99.9% confidence (cf. Kuang 1996). Descriptive statistics are acceptable metrics because the differences are reasonably normally distributed (Fig. 2).

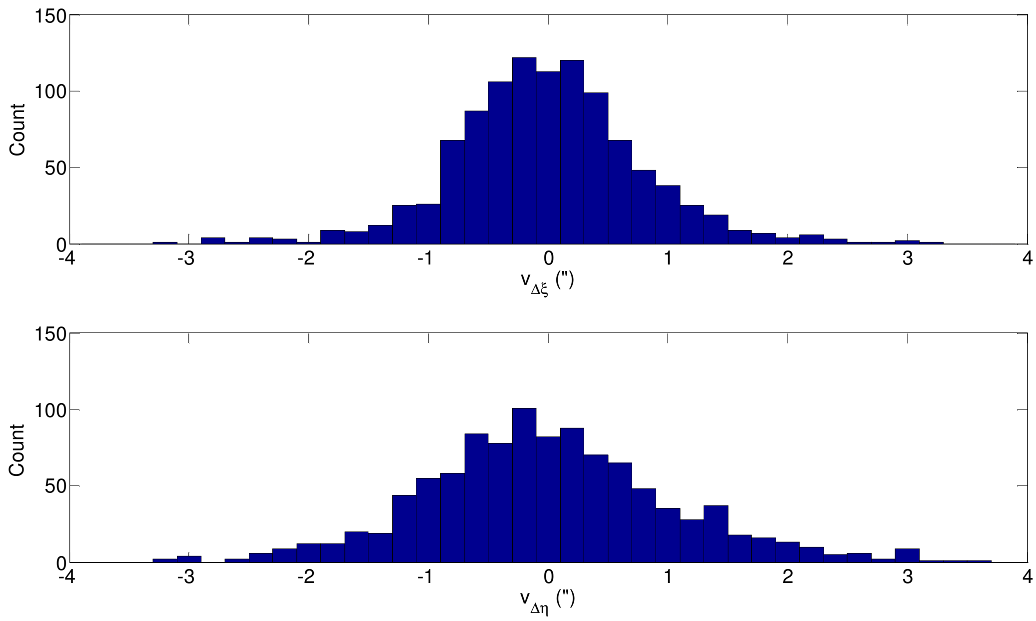

Fig 2. Histograms (in arc-seconds) of the difference between AUSGeoid98-derived and astrogeodetic deflections (top: N-S; bottom: E-W). The larger spread in the E-W deflection differences probably reflects the poorer astrogeodetic measurements due to timing and star-catalogue errors in these historical data.

5. Results

Equations (9) and (10) were applied to the differences between the AUSGeoid98-derived and astrogeodetic deflections, but in stages to determine the relative statistical significance of each of the parameters. This involved a two-, three- and four-parameter model variants of Eqs. (9) and (10) for each deflection component (Sect. 5.1).

Standard parametric least-squares was used to estimate the parameters in each case with the stochastic models $C_{\delta\xi} = \sigma_{\delta\xi}^2 I$ and $C_{\delta\eta} = \sigma_{\delta\eta}^2 I$, where $\sigma_{\delta\xi} = \sigma_{\delta\eta} = \pm 1''$ based on the earlier crude estimate of the accuracy of the astrogeodetic deflection data. All data were first reduced to their 2D centroid (i.e., mean ϕ and mean λ of the stations in Fig. 1) to improve the conditioning of the normal equation matrices.

5.1 Adjustment cases

In the first case tested, Eqs. (9) and (10) reduce to

$$\delta\xi = a_{00} + a_{10}\phi \tag{11}$$

$$\delta\eta = b_{00} + b_{01}\lambda \tag{12}$$

while for the second case, they reduce to

$$\delta\xi = a_{00} + a_{01}\lambda \tag{13}$$

$$\delta\eta = b_{00} + b_{10}\phi \tag{14}$$

For the three-parameter model, Eqs. (9) and (10) reduce to

$$\delta\xi = a_{00} + a_{10}\phi + a_{01}\lambda \tag{15}$$

$$\delta\eta = b_{00} + b_{10}\phi + b_{01}\lambda \tag{16}$$

The least-squares parameter estimates, without the 39 outliers, from these cases (Eqs. 11 to 16) as well as the four-parameter model (Eqs. 9 and 10) are given in Table 2. Only significant parameters are reported. Significance was evaluated by testing the ratio of the parameter estimate and its estimated standard deviation at 95% confidence for which the critical value was taken from the Gaussian distribution tables due to the high redundancy of the fitting and the distribution of the deflection differences (Fig 2). Individual testing of terms is valid due to the low correlation among parameters: the largest correlation coefficient magnitude was 0.29 from the four-term model. The statistics of the post-fit residuals are in Table 3 (cf. Table 1).

Table 2 shows that in both two-parameter cases, only the bias term is significant in the N-S deflection differences, while the bias and linear term are both significant in the E-W deflection differences (discussed later in Sect 5.2). The significant terms in the three-parameter model are the same as for the two-parameter models. The additional longitudinal parameters (a_{01} and b_{01}) are insignificant, which is also reflected in the post-fit residuals, where the values are very similar (Table 3). The additional parameterisation is not warranted here, mostly because of the data quality (discussed later in Sect 5.2). In the four-parameter case, the significance of the parameters is consistent with the two- and three-parameter models, with the exception of the latitude-longitude cross term (b_{11}) for the E-W vertical deflection difference.

deflection	parameter	2-term model	2-term model	3-term model	4-term model
		Eqs. (11, 12)	Eqs. (13, 14)	Eqs. (15, 16)	Eqs. (9, 10)
N-S ($\delta\xi$)	a_{00} (″)	–0.245±0.031	–0.245±0.031	–0.245±0.031	–0.249±0.031
	a_{10} (″/rad)	--	n/a	--	--
	a_{01} (″/rad)	n/a	--	--	--
	a_{11} (″/rad^2)	n/a	n/a	n/a	--
E-W ($\delta\eta$)	b_{00} (″)	–0.161±0.031	–0.161±0.031	–0.161±0.031	–0.173±0.031
	b_{10} (″/rad)	n/a	–1.214±0.274	–1.158±0.275	–0.879±0.289
	b_{01} (″/rad)	0.381±0.159	n/a	--	--
	b_{11} (″/rad^2)	n/a	n/a	n/a	–5.181±1.596

Table 2. Summary of the significant parameter estimates for the two-, three- and four-parameter deflection fitting models (n/a = not applicable; -- = insignificant)

	2-term model		2-term model		3-term model		4-term model	
	Eqs. (11, 12)		Eqs. (13, 14)		Eqs. (15, 16)		Eqs. (9, 10)	
	N-S ($\delta\xi$)	E-W ($\delta\eta$)	N-S ($\delta\xi$)	E-W ($\delta\eta$)	N-S ($\delta\xi$)	E-W ($\delta\eta$)	N-S ($\delta\xi$)	E-W ($\delta\eta$)
Max	3.11	3.52	3.11	3.29	3.10	3.20	3.10	3.14
Min	–3.16	–3.16	–3.19	–3.22	–3.18	–3.26	–3.14	–3.17
STD	±0.80	±1.04	±0.80	±1.04	±0.80	±1.04	±0.79	±1.03

Table 3. Residual statistics for the two-, three- and four-parameter deflection model fits (in arc-seconds) after rejection of 39 outliers

5.2 Deflection-derived geoid corrections and discussion

Only the statistically significant parameter estimates in Table 2 will be used to attempt to apply 'corrections' to the gravimetric model. For the N-S deflection differences, only the first term (a_{00}) is significant for all parameterisations tested, which consistently shows an N-S-oriented misalignment of ~-0.25 arc-seconds between the astrogeodetic and geoid-derived deflections. For the E-W deflection differences, the first term (b_{00}) is also significant for all parameterisations, showing an E-W-oriented misalignment of ~-0.16 arc-seconds.

The first of the two-parameter models for the E-W deflection differences shows a significant longitudinal term (b_{01}), but which is not significant in the three- and four-parameter models (Table 2). This is explained when seeing that the latitudinal term (b_{10}) is significant in the other two-parameter model, as well as in the three- and four-parameter models, and a significant latitude-longitude term (b_{11}) occurs in the four-parameter model. Therefore, the longitudinal term in the two-parameter model is actually a part of the latitude-longitude dependency (b_{11}) that becomes evident in the four-parameter model for the E-W deflection difference.

We now use these parameter estimates to apply 'corrections' to the gravimetric geoid model, akin to the use of GPS-levelling. The first terms (a_{00} and b_{00}) are straightforward to apply; they represent N-S and E-W tilts that should be applied to the gravimetric geoid model. Applying the estimated a_{00} and b_{00} terms over the data ranges of $\Delta\phi$=0.5948rad (34.0810° or ~3783km) and $\Delta\lambda$=0.7059rad (40.4449° or ~4489km) gives a N-S tilt of –(4.49±0.02)m and an E-W tilt of –(3.50±0.02)m.

These values are much larger than could realistically be expected. For instance, comparisons of AUSGeoid98 with GPS-levelling data do not show such large tilts (e.g., Featherstone et al. 2001; Featherstone and Sproule 2006; Soltanpour et al. 2006), especially not in the E-W direction, though there is evidence for a ~–1-2 m N-S-oriented tilt (using the same sign convention) in the Australian Height Datum (e.g., Featherstone 2004; 2006a). This exemplifies the problem of the inseparability when using GPS-levelling data. The only plausible reason for these unrealistically large N-S and E-W tilts comes from the poor quality of the historic astrogeodetic deflections over Australia.

Recall that their accuracy was estimated to be one arc-second, which is substantially larger than the parameter estimates summarised in Table 10. Applying this one arc-second uncertainty over the N-S and E-W data ranges, gives uncertainties in the tilts of ±18.34m and ±21.55m respectively. Accordingly, the above-estimated tilts of –4.49m and –3.50m are statistically insignificant when considering the quality of these historical deflection data. Therefore, very accurately known astrogeodetic deflections would be needed to utilise this method over a very large area like Australia. However, this accuracy requirement will be lessened over a smaller area, so may be attractive in geographically smaller countries.

6. Summary and Conclusion

We have presented an alternative and new method with which to control gravimetric geoid model errors using astrogeodetic deflections of the vertical. This is a preferable alternative to the current widespread use of GPS-levelling data, which suffers from the inseparability of height-related errors in that data combination strategy. Two-, three- and four-parameter functional models have been formulated here, but other parameterisations are possible, as has been the case for the GPS-levelling combination strategy. These are left for future work.

Numerical experiments with 1080 historical astrogeodetic deflections over Australia and AUSGeoid98 show that the approach presented is indeed feasible, but the poor quality of the astrogeodetic deflections, coupled with the size of the study area, causes unrealistically large values for the deflection-derived geoid corrections. However, using modern digital zenith cameras would provide much better results.

Acknowledgements: WEF thanks the Australian Research Council for research grant DP0663020. Thanks also go to Jim Steed (Ret.) of Geoscience Australia and Linda Morgan of Landgate for providing the astrogeodetic deflections. Figure 1 was produced with the Generic Mapping Tools (Wessel and Smith, 1998; http://gmt.soest.hawaii.edu/). This is TIGeR publication number **xx**.

References

Baarda W (1968) A testing procedure for use in geodetic networks, Publications on Geodesy, New Series, 2(5), Netherlands Geodetic Commission, Delft

Bomford AG (1967) The geodetic adjustment of Australia 1963-66, Surv Rev 19(144):52-71

Bomford G (1980) Geodesy (fourth edition), Oxford Univ Press, Oxford

Farin GE (2001) Curves and Surfaces for CAGD: A Practical Guide, 5th ed., Morgan Kaufmann, San Francisco

Featherstone WE (1998) Do we need a gravimetric geoid or a model of the Australian height datum to transform GPS Heights in Australia? Austral Surv 43(4):273-280

Featherstone WE (2001) Absolute and relative testing of gravimetric geoid models using Global Positioning System and orthometric height data, Comput & Geosci 27(7):807-814, doi: 10.1016/S0098-3004(00)00169-2

Featherstone WE (2004) Evidence of a north-south trend between AUSGeoid98 and AHD in southwest Australia, Surv Rev 37(291):334-343

Featherstone WE (2006a) Yet more evidence for a north-south slope in the AHD, J Spatial Sci 51(2):1-6; corrigendum in 52(1):65-68

Featherstone WE (2006b) The pitfalls of using GPS and levelling data to test gravity field models, EGU General Assembly, Vienna, Austria, April

Featherstone WE, Kirby JF, Kearsley AHW, Gilliand JR, Johnston J, Steed R, Forsberg R, Sideris MG (2001) The AUSGeoid98 geoid model of Australia: data treatment, computations and comparisons with GPS/levelling data, J Geod 75(5-6):313-330, doi: 10.1007/s001900100177

Featherstone WE, Rüeger JM (2000) The importance of using deviations of the vertical in the reduction of terrestrial survey data to a geocentric datum, Trans-Tasman Surv 1(3):46-61. Erratum in Austral Surv 47(1):7

Featherstone WE, Sproule DM (2006) Fitting AUSGeoid98 to the Australian Height Datum using GPS data and least squares collocation: application of a cross validation technique, Surv Rev 38(301): 573-582

Featherstone WE, Morgan L (2007) Validation of the AUSGeoid98 model in Western Australia using historic astrogeodetically observed deviations of the vertical, J Royal Soc Western Australia 90(3): 143-149.

Forsberg R (1998) Geoid tailoring to GPS - with example of a 1-cm geoid of Denmark, Finnish Geodetic Institute Report 98(4):191-198

Fotopoulos G (2005) Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetric geoid height data, J Geod 79(1-3):111-123, doi: 10.1007/s00190-005-0449-y

Grafarend EW (1997) Field lines of gravity, their curvature and torsion, the Lagrange and the Hamilton equations of the plumbline, Annals of Geophys 40(5):1233-1247

Heiskanen WA, Moritz H (1967) Physical Geodesy, Freeman, San Francisco

Hirt C, Bürki B (2002) The digital zenith camera – a new high-precision and economic astrogeodetic observation system for real-time measurement of deflections of the vertical, in: Tziavos IN (ed) Gravity and Geoid 2002, Department of Surveying and Geodesy, Aristotle University of Thessaloniki, pp 389-394

Hirt C, Flury J (2007) Astronomical-topographic levelling using high-precision astrogeodetic vertical deflections and digital terrain model data, J Geod 82(4-5):231-248, 10.1007/s00190-007-0173-x

Hirt C, Denker H, Flury J, Lindau A, Seeber G (2007) Astrogeodetic validation of gravimetric quasigeoid models in the German Alps – first results, in: Kiliçoğlu A, Forsberg R (eds) Gravity Field of the Earth, General Command of Mapping, Ankara, pp

Hirt C, Seeber G (2007) High-resolution local gravity field determination at the sub-millimetre level using a digital zenith camera system, in Tregoning P, Rizos C (eds) Dynamic Planet, Springer, Berlin Heidelberg New York, pp 316-321

Hirt C, Seeber G (2008) Accuracy analysis of vertical deflection data observed with the Hannover Digital Zenith Camera System TZK2-D, J Geod 82(6):347-356, 10.1007/s00190-007-0184-7

Jekeli C (1999) An analysis of vertical deflections derived from high-degree spherical harmonic models, J Geod 73(1):10-22, doi: 10.1007/s001900050213

Jiang Z, Duquenne H (1996) On the combined adjustment of a gravimetrically determined geoid and GPS levelling stations, J Geod 70(8):505-514, doi: 10.1007/s001900050039

Kearsley AHW (1976) The computation of deflections of the vertical from gravity anomalies, Unisurv Rep S15, School of Surveying, Univ of New South Wales, Sydney

Kotsakis C (2008) Transforming ellipsoidal heights and geoid undulations between different geodetic reference frames, J Geod 82(4-5), 249-260, doi: 10.1007/s00190-007-0174-9

Kotsakis C, Sideris MG (1999) On the adjustment of combined GPS/levelling/geoid networks, J Geod 73(8):412-421, doi: 10.1007/s001900050261

Kuang S (1996) Geodetic network analysis and optimal design: concepts and applications, Ann Arbor Press, Chelsea

Kütreiber N (1999) Combining gravity anomalies and deflections of the vertical for a precise Austrian geoid, Bollettino di Geofisica Teorica ed Applicata 40(3-4):545-553

Kühtreiber N, Abd-Elmotaal HA (2007) Ideal combination of deflection components and gravity anomalies for precise geoid computation, in: Tregoning P, Rizos C (eds) Dynamic Planet, Springer, Berlin Heidelberg New York, pp 259-265

Marti U (2007) Comparison of high precision geoid models in Switzerland, in: Tregoning P, Rizos C (eds) Dynamic Planet, Springer, Berlin Heidelberg New York,, 377-382

Mather RS (1970) The geodetic orientation vector for the Australian Geodetic Datum, Geophys J Royal Astron Soc 22(1):55-81, doi: 10.1111/j.1365-246X.1971.tb03583.x

Milbert DG (1995) Improvement of a high resolution geoid model in the United States by GPS height on NAVD88 benchmarks, Int Geoid Serv Bull 4:13-36

Moritz H (1980) Geodetic reference system 1980, Bull Géod 54(4):395-405

Müller A, Bürki B, Kahle HG, Hirt C, Marti U (2007a) First results from new high-precision measurements of deflections of the vertical in Switzerland, in: Jekeli C, Bastos L, Fernandes J (eds) Gravity Geoid and Space Missions, Springer, Berlin Heidelberg New York, pp 143-148

Müller A, Bürki B, Limpach P, Kahle HG, Grigoriadis VN, Vergos GS, Tziavos IN (2007b) Validation of marine geoid models in the North Aegean Sea using satellite altimetry, marine GPS data and astrogeodetic measurements, in: Kiliçoğlu A, Forsberg R (eds) Gravity Field of the Earth, General Command of Mapping, Ankara, pp

Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2008) An Earth gravitational model to degree 2160: EGM2008, EGU General Assembly, Vienna, Austria, April

Prutkin I, Klees R (2007) On the non-uniqueness of local quasi-geoids computed from terrestrial gravity anomalies, J Geod 82(3):147-156, doi: 10.1007/s00190-007-0161-1

Soltanpour A, Nahavandchi H, Featherstone WE (2006) The use of second-generation wavelets to combine a gravimetric geoid model with GPS-levelling data, J Geod 80(2):82-93, doi: 10.1007/s00190-006-0033-0

Torge W (2001) Geodesy, third edition, de Gruyter, Berlin

Wessel P, Smith WHF 1998 New, improved version of Generic Mapping Tools released, EOS - Trans AGU 79(47):579

Zhong D (1997) Robust estimation and optimal selection of polynomial parameters for the interpolation of GPS geoid heights, J Geod 71(9):552-561, doi: 10.1007/s001900050123