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Using models of the ocean's mean dynamic topography to identify errors in coastal geodetic levelling

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Abstract

Identifying errors (blunders and systematic errors) in coastal geodetic levelling networks has often been problematic. This is because (1) mean sea level (MSL) at tide gauges cannot be directly compared to height differences from levelling because the geoid/quasigeoid and MSL are not parallel, being separated by the ocean's mean dynamic topography (MDT) and (2) the lack of redundancy at the edge of the levelling network. This paper sets out a methodology to independently identify blunders and/or systematic errors (over long distances) in geodetic levelling using MDT models to account for the separation between the geoid/quasigeoid and MSL at tide gauges. This method is then tested in a case study using an oceanographic MDT model, MSL observations, GNSS data and a quasigeoid model. The results are significant because the errors found could not be detected by standard levelling misclosure checks alone, with supplementary data from an MDT model, with cross-validation from GNSS-quasigeoid allowing their detection. In addition, it appears that an oceanographic-only MDT is as effective as GNSS and a quasigeoid model for detecting levelling errors, which could be particularly useful for countries with coastal levelling errors in their levelling networks that cannot be identified by conventional levelling closure checks.

Keywords: coastal mean dynamic topography, geodetic levelling, mean sea level, CARS2009, geoid, Australia

Introduction

The ocean's time-mean dynamic topography (MDT; also referred to as dynamic ocean topography, or sea surface topography in geodetic literature) is the spatially variable height difference between the geoid and mean sea level (MSL) corrected for the inverse barometer response (IBR) (Tapley and Kim 2001), which is the ocean's response to atmospheric pressure loading. The magnitude of MDT can reach 1.5 m, but unlike the geoid (or quasigeoid; see next), which is subject to relatively small changes over time (cf. Rangelova and Sideris 2008), MDT can be dependent on the period over which the observations are taken. This is due largely to the time-variation of MSL, which comprise non-linear tides and long- and medium-term sea level variability that are collectively denoted v by Penna et al. (2013), such that $h_T = \zeta + (MDT + IBR) + v$, where h_T is the ellipsoidal height of MSL at a tide gauge, and ζ is the quasigeoid height anomaly (Fig. 1). Over the oceans, the geoid and quasigeoid are coincident, so because this study is confined to the ocean and low-lying coastal regions where geoid-quasigeoid differences are assumed negligible, and that a quasigeoid model is used in the case study, the term quasigeoid (compatible with ζ) will be used for the remainder of this paper.

Bowie (1929) first found that height differences from geodetic levelling on land (herein, levelling will refer to geodetic levelling on land, unless otherwise stated) did not agree with MSL at different tide gauges, with subsequent comparisons confirming that levelling and MSL were not coincident, or parallel to the quasigeoid by the magnitude of MDT (plus IBR and v; Fig. 1). A further enigma emerged when it was found that oceanographic levelling (also referred to as dynamic or steric levelling) often did not agree with levelling on land (e.g., Sturges 1967), despite oceanographic levelling theoretically accounting for MDT and thus should provide the same height differences relative to the quasigeoid as levelling. The uncertainty was compounded by levelling being connected to MSL at coastal tide gauges (usually located in harbours) which are subject to different physical processes (e.g., Merry and Vaníček 1983) than the open ocean where oceanographic levelling is conducted.

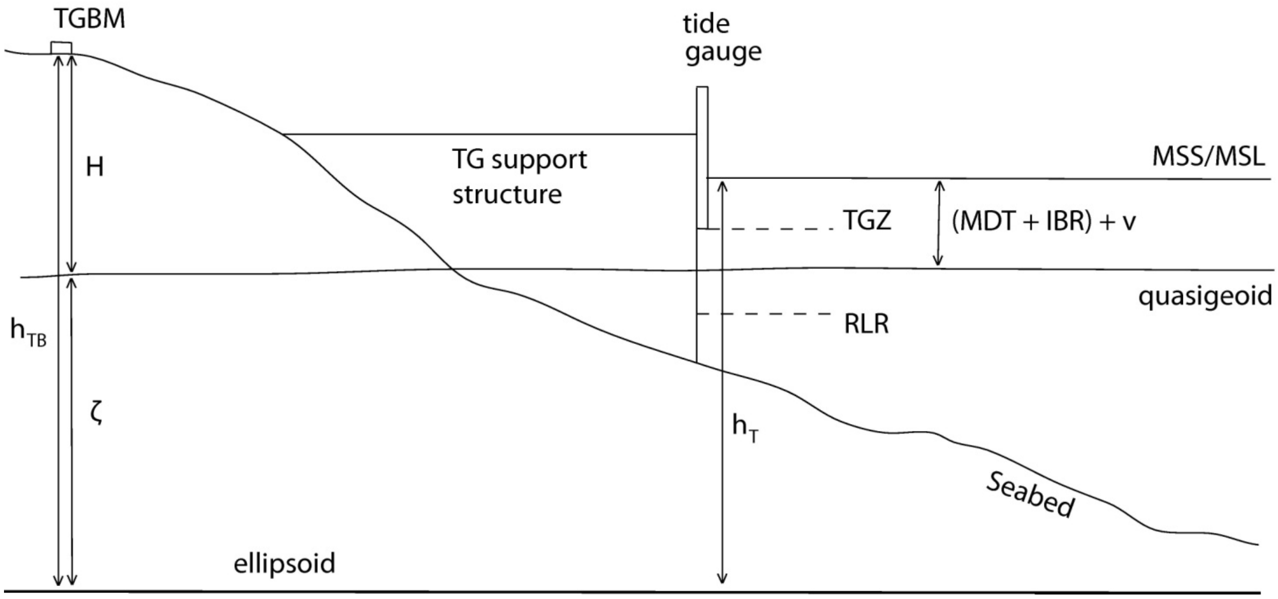

Fig. 1: Diagram relating MDT to other values. H is the normal height of the TGBM, ζ is the quasigeoid height anomaly, h_{TB} is the ellipsoidal height of the TGBM, h_T is the ellipsoidal height of MSL at the tide gauge, TGZ is the tide gauge zero, RLR is the revised local reference (Woodworth and Player 2003) and MSS/MSL is the mean sea surface/level observed at the tide gauge. The IBR is included within the MDT, along with all other effects v.

In these instances, the reliability of levelling over long distances was questioned, although the cause for this apparent defect was not obvious. Levelling errors considered in this paper are blunders (field observation or booking mistakes) and systematic errors, many of which are difficult to identify and quantify, e.g., refraction (e.g., Strange 1981), magnetic errors in automatic levels (e.g., Strange 1985), staff settlement (e.g., Entin 1959), staff expansion (e.g., Rüeger 1997), Earth tides (e.g., Bretreger 1986), and staff calibration (e.g., Craymer and Vaníček 1995), among others (for an overview of levelling, see e.g., Vaníček et al. 1980).

The standard method for detecting levelling blunders and some types of systematic errors is by (1) two-way levelling (i.e., forward and reverse levelling runs between two endpoints) and (2) summation of the (preferably) two-way levelled height differences to form levelling loops that close back to their start point (cf. Fig. 3). For (1), the difference (referred to as the misclosure [ε]) between the two-way levelling runs is expected to close within a maximum allowable misclosure (MAM), as are the closed levelling loops in (2).

Applying height corrections (e.g., orthometric or normal corrections; see Heiskanen and Moritz 1967, Ch. 4) to the levelling to account for the non-parallelism of the Earth's equipotential surfaces should theoretically result in the loop ε being zero (Sansò and Vaníček 2006). On land, orthometric corrections/heights (e.g., Helmert 1890) are compatible with the geoid, while normal corrections/heights (Molodensky et al. 1962) are compatible with the quasigeoid. However, due to an accumulation of random errors, ε will generally be non-zero, but if blunder-free, is expected to be less than MAM, which for levelling is traditionally calculated according to the quality of the levelling technique (c) multiplied by the square root of the distance along the levelling route (d; in km), i.e., $c\sqrt{d} < \varepsilon$ or the loop is rejected and must be re-observed (ICSM 2007).

Redundancy in inland levelling allows adjacent loop ε to assist in locating errors greater than MAM, but coastal levelling lacks this redundancy because there is no adjoining loop on the ocean side of the network. Compensating errors are a further problem, where loops extending hundreds of km can contain multiple errors of opposing sign such that $\varepsilon <$ MAM despite the loop containing errors. This paper demonstrates that coastal levelling traverses can be formed into loops comprising the land levelling component and a modelled MDT component, validating levelling lines that are connected to MSL at tide gauges, and identifying levelling errors that cannot be discovered by standard levelling-only closure checks. This is analogous with the concept presented by Filmer and Featherstone (2009) of using GNSS-derived ellipsoidal heights (h) - ζ to form loops with inland levelling as a way of detecting levelling errors.

Using MDT models to identify errors in coastal levelling is possible because of significant improvements in MDT models (e.g., Dunn and Ridgway 2002; Rio et al. 2011) in recent years. When used in a relative sense (i.e. the difference between MDT at two tide gauges; see Fig. 2) MDT can potentially identify levelling blunders larger than the relative errors in the MDT, and systematic levelling errors over a longer distance because levelling precision propagates with respect to $\sqrt{d}$. For example, the MAM for third-order levelling (c=12; ICSM 2007) over 100 km is 120 mm. Featherstone and Filmer (2012) tested five different MDT models using MSL at 30 tide gauges around Australia and a minimally constrained least-squares adjustment of the Australian national levelling network (ANLN). These models included oceanographic-only, combined oceanographic-geodetic, and geodetic-only, with all having a standard deviation (SD) of the levelling –MSL-MDT differences at tide gauges of around ±200 mm. The internal precision of the adjusted levelling is ~±100-120 mm at the tide gauges, suggesting that a crude uncertainty estimate for the MDT models could be ~±100 mm, or perhaps less, given that the levelling network was also affected by regional distortions.

An earlier study by Filmer and Featherstone (2012) using MDT models and GNSS h and quasigeoid models to estimate MDT spatial variability between tide gauges in Tasmania and the south-eastern Australian mainland also found ~±100 mm to be a realistic error estimate for modelled MDT at the coast. In both studies, oceanographic-only models performed better than geodetic-only models because the quasigeoid and mean sea surface models used to realise geodetic-only MDT models contain larger uncertainties in coastal regions (Featherstone and Filmer 2012). This error estimate includes any error resulting from extrapolating MDT values from offshore to the tide gauge and subsumes the unknown magnitude v. These empirical MDT uncertainty estimates agree broadly with further work conducted by the author and colleagues (not yet ready for publication) using variance component estimation with MDT, GNSS-quasigeoid and levelling in Australia. Based on this empirical evidence, it is postulated that relative modelled MDT at the coast could identify levelling blunders >100 mm and systematic errors at distances >100 km for third-order levelling. This is tested further in the case study (see later).

Combinations of levelling, GNSS, quasigeoids and various MDT models have been used in the past to test MDT models or identify apparent tilts in vertical datums (e.g., Hipkin et al. 2004; Featherstone and Filmer 2012, Penna et al. 2013), but this paper takes a new approach as it seeks to use MDT models specifically to isolate sections of coastal levelling to detect levelling errors that have previously been unidentified, but corrupt levelling based vertical datums. There are numerous levelling networks around the world that are the basis for national vertical datums, but which may benefit from this method to detect levelling errors along their coasts. This method is complementary to the use of GNSS h-ζ with levelling to form loops (e.g., Filmer and Featherstone 2009), as shown later in the case study.

Method

Three data types are required for this method: levelling, MSL observations at tide gauges, and modelled MDT. The levelling network must be connected to MSL at the tide gauges. The addition of GNSS and quasigeoid data adds robustness to the results through independent validation. Fig. 1 shows these different quantities and how they relate to each other.

Methodology

A levelling-MDT model loop (referred to as an LM loop in this paper) with misclose (ε_{LM}) can be formed as (also refer to Fig. 2)

$$\Delta H_{1-2} + \Delta MDT_{2-1} = \varepsilon_{LM} \quad (1)$$

where ΔH_{1-2} is the levelled height difference (with normal correction applied) from MSL at tide gauge 1 to MSL at tide gauge 2. ΔMDT_{2-1} is the difference from MDT_2 to MDT_1, which are the modelled MDT values at tide gauge 2 and tide gauge 1 respectively. Modelled MDT converts MSL to the quasigeoid, thus cancelling spatially variable MDT between tide gauges, and eliminating the systematic misclose that exists when levelling between MSL at different tide gauges. This is comparable with the use of a quasigeoid model to reduce GNSS h to the quasigeoid (cf. Eq. (3)). Using the LM loop formed between URAN and BUND in Fig. 3 (Case study section) as an example, ΔH_{1-2} is the levelled height difference (black line on land) from URAN to BUND, with MDT_1 and MDT_2 the MDT values at URAN and BUND respectively.

MDT values at each tide gauge (Fig. 2) needed in Eq. (1) are extrapolated from a MDT model grid to the location of the tide gauge. While acknowledging that MDT in coastal regions may differ significantly to MDT further offshore, modelled MDT error estimates reported earlier (~±100 mm) in this paper from Featherstone and Filmer (2012) and Filmer and Featherstone (2012) include any extrapolation error from modelled MDT values close to the shore, or from further offshore where there are coastal gaps in MDT models. This suggests that extrapolation error is relatively small for these previous studies.

ΔH_{1-2} in Eq. (1) comprises (see Fig. 2): (1) the levelled height difference between the TGBM near tide gauge 1 to the TGBM near tide gauge 2 and (2) the levelled connection from each TGBM to MSL (ΔH_M). (1) is routine, but (2) can be problematic, because ΔH_M can be variable due to changes in sea level over different time-scales (e.g., Holgate 2007). This is particularly so for short time-period tide gauge records. The treatment of the IBR (e.g., Wunsch and Stammer 1997) prior to computing Eq. (1) also needs to be considered and is dependent on the MDT model used. This is because oceanographic-only MDT models generally contain the IBR component within the modelled MDT value, while geodetic-only MDT models usually do not (see Andersen and Knudsen 2009; Fig 1 in this paper). This may necessitate the computation of IBR and its application to the MDT at tide gauges to be compatible with MSL observations that contain the IBR.

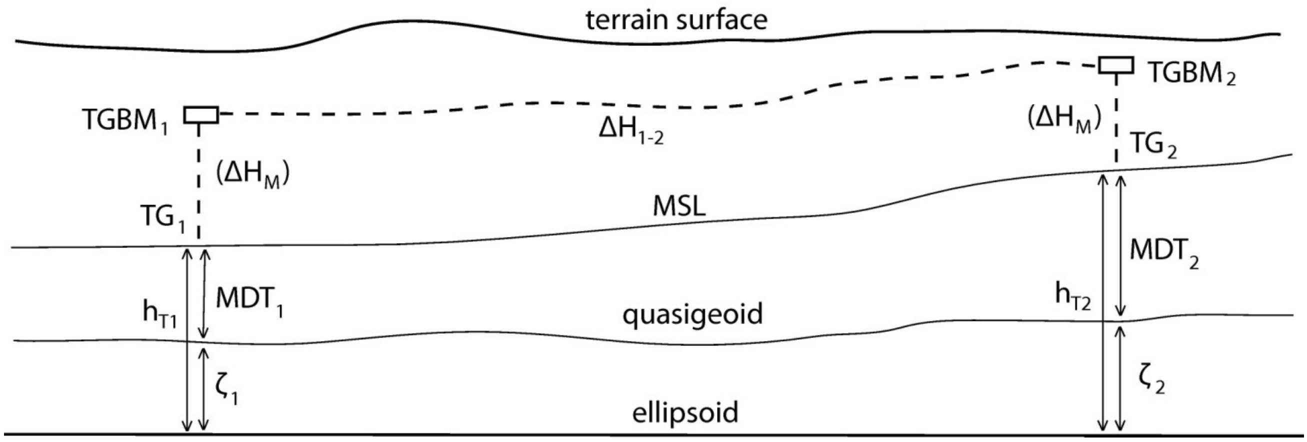

Fig. 2: Profile of an LM loop (Eq. (1)) and LG loop (Eq. (3)) showing tide gauge 1 (TG_1) and tide gauge 2 (TG_2). ζ_1 and ζ_2 are the height anomalies at TG_1 and TG_2 respectively, h_{T1} and h_{T2} are the ellipsoidal heights of MSL at TG_1 and TG_2 respectively (Eq. (4); cf. Fig. 1), ΔH_{1-2} is the levelled height difference (with normal correction) from MSL at TG_1 to MSL at TG_2, while ΔH_M is the height difference between TGBM and MSL (Eq. (2)), at each tide gauge, which are included within ΔH_{1-2}.

ΔH_M is (see Fig 2; cf. Hipkin 2004)

$$\Delta H_M = \Delta H_Z + MSL_Z \tag{2}$$

ΔH_Z is the levelled height difference between the TGBM and the TGZ (tide gauge zero) and MSL_Z is the observed height of MSL above the TGZ from tide gauge records (Fig. 1).

Levelling-GNSS-quasigeoid loops (referred to as LG loops for this paper) can also be used to detect certain levelling errors (Filmer and Featherstone 2009), and in this paper will be used to cross-validate the LM loops. LG loops are defined as (Fig 2)

$$\Delta H_{1-2} + (\Delta h_{2-1} - \Delta\zeta_{2-1}) = \varepsilon_{LM} \tag{3}$$

The levelled height difference component of the LG loop is the same as that for the LM loop, Δh_{2-1} is the difference between h_T at tide gauge 2 to tide gauge 1, and $\Delta\zeta_{2-1}$ is the ζ difference from tide gauge 2 to tide gauge 1. Eq. (3) relies on the relative accuracy of the quasigeoid, which is generally inferior in coastal regions due to the dearth of gravity data over the coast. Featherstone and Filmer (2012) suggest that AGQG09 (Featherstone et al. 2011) can have relative uncertainties at tide gauges in the ~±50-100 mm range. To compute Δh_{2-1} in Eq. (3), h_T is

$$h_T = h_{TB} - \Delta H_M \quad (4)$$

where h_{TB} is h at the TGBM (Fig. 1). This assumes that the TGBM is near the tide gauge (ideally on the tide gauge, although cf. Bevis et al. 2002) so that the ellipsoid and quasigeoid are parallel over this short distance (Hipkin et al., 2004). Thus ζ at the TGBM (ζ_{TB}) is equal to ζ at the tide gauge (ζ_T) and ΔH_M is equivalent to Δh_M (Δh from TGBM to MSL).

In many instances, vertical datums are fixed to local MSL at the tide-gauges (e.g., Roelse et al. 1971; Zilkoski et al. 1992), thus neglecting MDT, which becomes the local offset between the quasigeoid and the vertical datum (Filmer and Featherstone 2012). Under this assumption, the normal height of the TGBM above the vertical datum can be used in place of ΔH_M in Eq. (4) (cf. Hipkin et al. 2004) as this is effectively the levelled height difference between the TGBM and MSL. The assumptions made for Eq. (4) will become less valid as the distance between the TGBM and tide gauge become larger (i.e., $\zeta_{TB} \neq \zeta_T$), contaminating ε_{LG} from the h-ζ component of LG loops.

Limitations of the method

MDT uncertainty will limit the effectiveness of this method. A rigorous accuracy assessment is problematic for coastal MDT, with empirical testing against independent data sets (i.e. levelling v MDT v GNSS-quasigeoid) the most practical form of validation (Andersen and Knudsen 2009). Relative uncertainties of <~100 mm may be obtained from modern MDT (as indicated from empirical testing against levelling in Featherstone and Filmer 2012; Filmer and Featherstone 2012), which permit the identification of blunders in levelling traverses of all lengths (>~100 mm), and for systematic errors at scales > ~100 km. The additional effects of temporal MDT variability v are subsumed in the uncertainty estimates described above. The case study (following) will contribute to the empirical estimation of these uncertainties.

The other significant limitations of the LM loop method are obtaining reliable levelling connections between the TGBM and MSL (cf. Woodworth and Player 2003). It is difficult because over time TGBMs can be destroyed or disturbed and the original connection between MSL and the levelling network lost. This can be exacerbated by temporal variability in MSL (e.g., Meyssignac and Cazenave 2012), changes in TGZ, tide gauge malfunction and/or equipment changes and vertical land motion at the tide gauge (e.g., Bouin and Wöppelmann 2010). Poor record keeping, or the records not being updated when the connection changes add to the uncertainty in this critical component of the LM levelling loop. Inconsistent tide gauge observation periods and tide systems in the different data sets (see later) can also introduce errors that reduce the effectiveness of this method.

Case study

The case study examines levelling traverses along the east coast of Australia, using the LM loop method described in the Method Section, with supplementary information provided by LG loops where GNSS h is available at tide gauges. Tide gauges will be referred to by their four letter code given in Table 1 and Fig 3 for the remainder of the paper. This study limits itself to the coastal levelling between COFF (30° 18'S, 153° 08'E) and COOK (15° 28'S, 145° 15'E) (Fig. 3). Pre-1971 levelling (Australian third-order; MAM = $12\sqrt{d}$; c=12; cf. ICSM 2007) used in the Australian Height Datum (AHD; Roelse et al. 1971) indicated an increase in MSL of +1.45 m (relative to the quasigeoid) from COFF to COOK, with +0.989 m of this increase from COFF to CAIR (Roelse et al. 1971, Annex C). A study by Hamon and Grieg (1972), found that while offshore oceanographic levelling agreed that MSL increased from COFF to COOK, it could only account for ~0.3 m of the magnitude inferred by the geodetic levelling. In 1975-1976, a first-order levelling traverse (MAM = $4\sqrt{d}$; c=4; cf. ICSM 2007) was conducted between COFF and CAIR in an attempt to solve this enigmatic discrepancy between oceanographic and geodetic levelling on land (Coleman et al. 1979; Morgan 1992).

Morgan (1992) describes the 1975-76 levelling and methods: a 'rapid' one-way first-order levelling technique was used from BUND to CAIR, and from half way between BRIS and BALL (referred to as BRIS/BALL) to COFF (see Fig. 3), while the traverse between BUND and BRIS/BALL used a 'rapid' two-way first-order levelling technique. Both methods differed from conventional (Australian) first-order levelling in that the maximum sight length was 80 m rather than 40 m, but both retain first-order MAM of $4\sqrt{d}$. The conventional two-way technique allows identification of blunders and systematic errors through each section being levelled in opposite directions at different times. The one-way technique used two sets of coded staves with one instrument per set up (hence, two observations taken in the same direction at the same time), with alternate sub-sections run in opposite directions. Morgan (1992) reports that while the one-way technique can detect blunders through the double observations, the identical conditions under which the two observations were taken makes it unable to detect systematic errors such as refraction or staff calibration errors.

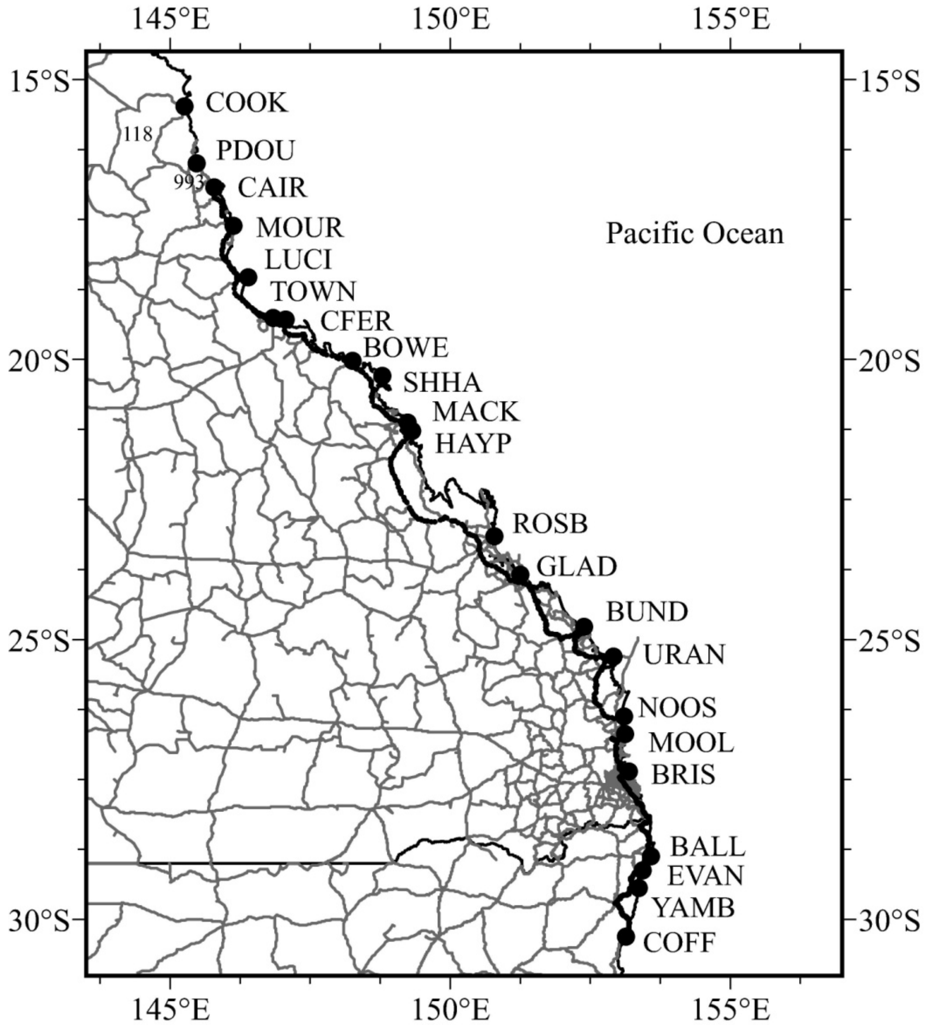

Fig. 3: Northeast coastline of Australia showing the location of 22 tide gauges (black circles) used in this study. Thin grey lines show the levelling network (ANLN), with the thick black line along the coast between COFF and CAIR representing the route of the 1975-76 first-order re-levelling. Loop 118 and 993 adjacent COOK and PDOU are used for the CAIR-COOK part of the case study. See Table 1 for full names of tide gauges. Mercator projection.

The 1975-1976 levelling indicated a negative MSL (northward) slope of -0.505 m (with respect to the quasigeoid) from COFF to CAIR compared to the previous positive MSL (northward) slope of +0.989 m from the original third order levelling (Coleman et al. 1979; Morgan 1992). This discrepancy warrants revisiting, firstly to test the utility of MDT for detecting levelling errors, but also to investigate this anomaly. Further to this, the large increase in levelling-MSL difference of +0.460 m between CAIR and COOK (Roelse et al. 1971, Annex C) is also investigated. Significantly, neither of these levelling errors can be detected by the levelling closures, so cannot be proven without the additional information provided by MDT models, with validation from GNSS-quasigeoid.

Data used for case study

The levelling required for both case studies is contained in the Australian national levelling network (ANLN; supplied by Geoscience Australia; GA; G. Johnston 2007, pers. comm.). The ANLN predominately comprises pre-1971 third-order levelling (see Filmer and Featherstone 2009 for full details), but has received several updates (e.g., Morgan 1992; Wellman and Tracey 1987).

MSL records at the tide gauges (Fig. 3 and Table 1) were downloaded from the Permanent Service for Mean Sea Level (PSMSL) website (accessed 31 August 2012; http://www.psmsl.org/data/obtaining/), as were the derived levelling connections to RLR (Revised Local Reference) or to the TGZ for metric data (see Woodworth and Player 2003). Supplementary evidence was supplied by the Australian Hydrographic Office (AHO; Z. Jayaswal 2012, pers. comm.). RLR records are those which PSMSL can reliably reference to a TGBM during the period of tide gauge observation using the data provided by the local tide gauge authority. RLR replaces the TGZ as the local MSL datum when available (cf. Fig 1.). RLR records also include corrections for any offsets in the record from tide gauge instrument change/malfunction, and/or a change in the TGZ. Records where the TGBM reference for the tide gauge is uncertain are referred to as 'metric' records (Woodworth and Player 2003). Mostly RLR tide gauge records were used, but it was decided to use five records that had only metric data; the trade off in using less reliable data was the extra redundancy of using all tide gauges.

CAIR was flagged in the PSMSL web site as possibly being subject to vertical land motion (cf. Ostanciaux et al. 2012). Trends were computed for all the tide gauge records using linear regression and CAIR showed a relative sea level change (SLC) trend similar to all other tide gauges with observation periods (1966-2010; Table 1) of ~+1.5 mm/yr. On the basis that similar relative SLC trends at other tide gauges indicate a stable tide gauge, CAIR was retained as it was important to the case study. The different sea level records

(Table 1) are problematic because (1) some records are very short (less than four years) and; (2) some records cover different time periods. Short sea level records are particularly susceptible to aliasing from temporal variability in sea level (e.g., Woodworth et al. 2009), while MSL records covering different periods are subject to the unique variability within the periods that define them (e.g., Meyssignac and Cazenave 2012). However, this variability will reduce over time so that longer records (>50 years; Douglas 2001) over different periods should have smaller biases.

Tide gauge name	Abbreviation	MSL epoch	Record type	Levelled distance from COFF (km)
Coffs Harbour*	COFF	1956-1970	RLR	0
Yamba	YAMB	1989-2010	RLR	167
Evans Head*	EVAN	1968-1970	RLR	212
Ballina*	BALL	1959-1964	RLR	242
Brisbane*	BRIS	1966-2010	RLR	489
Mooloolaba	MOOL	1979-2009	Metric	598
Noosa Head	NOOS	1970-1973	Metric	648
Urangan	URAN	1958-1962	Metric	852
Bundaberg*	BUND	1966-2010	RLR	959
Gladstone	GLAD	1978-2010	RLR	1180
Rosslyn Bay*	ROSB	1993-2011	RLR	1341
Hay Point	HAYP	1985-2010	Metric	1657
Mackay*	MACK	1966-2010	RLR	1672
Shute Harbour	SHHA	1983-2010	RLR	1834
Bowen	BOWE	1986-2010	RLR	1863
Cape Ferguson	CFER	1992-2011	RLR	2049
Townsville*	TOWN	1959-2010	RLR	2073
Lucinda	LUCI	1985-2010	RLR	2217
Mourilyan Harbour	MOUR	1985-2009	RLR	2349
Cairns*	CAIR	1966-2010	RLR	2441
Port Douglas	PDOU	1987-2009	RLR	
Cooktown*	COOK	1966-1968	Metric	

Table 1: List of the 22 tide gauges along the first order re-levelling of the northeastern Australian coast in 1975-76. PDOU and COOK are added because they are used in the CAIR-COOK case study, although the 1975-76 re-levelling did not extend past CAIR. Tide gauges marked with * indicate that a GNSS observation is available for that location.

It is assumed that long-term SLC (e.g., White et al. 2005) has a relatively small effect for this study. It was decided to use all the available records so that maximum length records and maximum number of tide gauges can be used given that an accuracy of several cm in MSL is expected to be sufficient for the purpose in this paper. MSL was computed as the mean of available mean monthly sea level records which eliminates most aliasing due to monthly tidal changes (Pugh 1987, p. 303). Monthly means with more than 10 days of data missing were discarded from the long-term means to avoid possible aliasing for specific monthly means.

The 10 TGBM GNSS *h* used (indicated by * in Table 1) were extracted from a dataset of 1,052 3D coordinates supplied by GA (N. Brown 2009, pers. comm.) and processed by Hu (2009) in the International Reference Frame 2005 (ITRF2005; Altamimi et al. 2007). The TGBMs were mostly < 1 km from the tide gauge. These data were observed for at least five continuous days by Australian State/Territory geodetic agencies in circa 2000 using dual-frequency geodetic GNSS receivers. Internal error estimates from GNSS processing software tend to be over-optimistic by a factor of 5-10 (e.g., Rothacher 2002). When scaled by 10, (conservative) error estimates for *h* are ~±5 mm for eight TGBM, but increasing to ±29 mm and ±43 mm at BALL and EVAN respectively.

The quasigeoid model used is AGQG09 (Featherstone et al. 2011), which uses the zero-tide version of EGM2008 (Pavlis et al. 2012) to degree 2190 as its reference field. AGQG09 height anomalies were bi-cubically interpolated from the 1'x1' grid to the location of the TGBMs. Featherstone et al. (2011) tested AGQG09 using a fixed LSA (to MSL-MDT at 30 tide gauges) of the ANLN, finding the SD of differences at ~1000 GNSS/ANLN benchmarks to be ~±130 mm, of which more than half is likely to be attributable to the ANLN. Hence, a crude GNSS *h*- AGQG09 ζ uncertainty could be ~±50 mm, which is comparable to the ~±50 to ±100 mm reported by Featherstone and Filmer (2012).

The MDT model used in this study is CARS2009 (Commonwealth Scientific and Industrial Research Organisation (CSIRO) Atlas of Regional Seas 2009; CSIRO Marine Laboratories; Ridgway et al. 2002; Dunn and Ridgway 2002) which was obtained from http://www.marine.csiro.au/~dunn/cars2009/. CARS2009 is a high resolution seasonal climatology computed using buoy and hydrographic cast data from the BLUElink Ocean Archive (Dunn 2008) compiled between 1950 and 2009, although weighted towards the more recent time period. CARS2009 includes schemes that adjust the weightings for data points to allow for the influence of ocean bottom topography and land barriers, to map more realistic ocean properties in coastal regions (Dunn and Ridgway 2002). The oceanographic-only CARS2009 MDT model is realised from this climatology using the principle of steric levelling. Although there are other MDT models available, CARS2009 has been found by Featherstone and Filmer (2012) and Filmer and Featherstone (2012) to be the best performing MDT model in the Australian region. Although not defined in the literature, it is assumed that because CARS2009 is an oceanographic-only model which describes the ocean's physical surface, it contains the IBR, so is compatible with MSL observed at tide gauges without corrections for the IBR (cf. Andersen and Knudsen 2009).

So that the treatment of the permanent tide was consistent between datasets, both GNSS h (provided in the tide-free (non-tidal) system; Hu 2012, pers. comm.) and AGQG09 ζ (zero-tide) were converted to the mean-tide system using the equations from Ekman (1989) (as re-formulated in Penna et al. 2013), so as to be compatible with CARS2009 and the levelling, which, although neither are in a specified tide system, are probably closest to the mean-tide system (e.g., Mäkinen and Ihde 2009).

The ocean adjacent to the northern east coast of Australia (Fig. 3) where this study is conducted is rather complex, due to the presence of the Great Barrier Reef (GBR) and the East Australia Current (EAC) e.g., Ridgway and Dunn (2003). A lack of oceanographic data and depths <2000 m on the coastal side of the GBR means that CARS2009 does not contain MDT values near the coast, necessitating extrapolation from the ocean outside the GBR to tide gauge positions (latitude, longitude). The GMT (Wessel and Smith 1998) interpolation routine **surface** (Smith and Wessel 1990) was used to extrapolate CARS2009 to the tide gauges. The uncertainty in the tide gauge MDT values resulting from extrapolating from offshore values is not well known, as estimation of these uncertainties is problematic. Comparison with independent height data from levelling and/or GNSS-quasigeoid provide empirical estimates of tide gauge MDT values. CARS2009 has been tested using GNSS-quasigeoid data along the northern east coast of Australia in Featherstone and Filmer (2012), and confirm MDT error estimates to be generally <100 mm, but typically ~50 mm. Further comparisons with levelling for the central east coast (presented later in this paper) provide further validation for the CARS2009 extrapolated tide gauge MDT.

1975-1976 first-order levelling

The 1975-76 first-order levelling was extracted from the ANLN file, summing the height differences from MSL at COFF to MSL at all 19 other tide gauges along the coast to CAIR, computing the TGBM to TGBM levelling component for ΔH_{1-2}. Mean monthly MSL observations were used in Eq. (2) to compute the ΔH_M component of ΔH_{1-2} so that 19 LM loops (all related to COFF) were then formed (Eq. (1)) using CARS2009 for the ΔMDT_{2-1} component. Normal corrections were applied to the levelled height differences using

EGM2008-derived gravity at BMs as per the methods described in Filmer et al. (2010). This accounts for the non-parallelism of the Earth's equipotential surfaces, which is a predominately north-south effect.

ε_{LM} are plotted in Fig. 4 with respect to the distance of the levelled component of each individual LM loop from COFF. Also plotted are first-order levelling MAM ($4\sqrt{d}$; dotted line) and third-order levelling MAM ($12\sqrt{d}$; dashed line) from COFF (0 km) to CAIR (2441 km). Although ΔMDT are not likely to propagate with respect to $\sqrt{d}$, it is convenient to use levelling MAM as the allowable limit of difference compared to levelling for this study. Except for the COFF–YAMB ε_{LM} (167 km from COFF), all CARS2009 ε_{LM} are within first-order MAM, suggesting CARS2009 modelled MDT can match the precision of first-order levelling along the coast. The apparent outlier at YAMB is more likely to be caused by the differences in MSL period between YAMB (1981-2010) and its neighbouring tide gauges (mostly 1960-1970; Table 1), or ΔH_Z rather than CARS2009 or the levelling. Fig. 4 indicates that after BUND (959 km), ε_{LM} systematically increases in magnitude (northward) to be outside third-order MAM before CAIR. This is significant because Morgan (1992) identifies that after BUND, the levelling technique changes from two-way 'rapid' first-order to 'rapid' one-way.

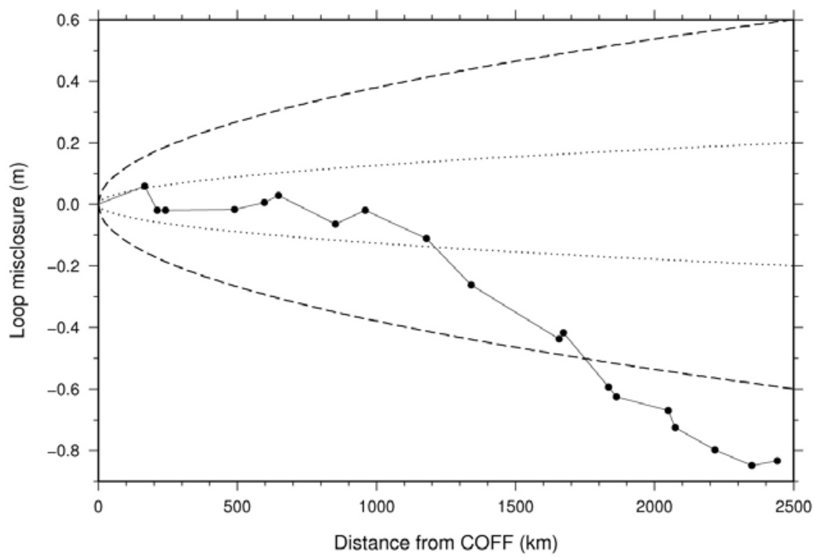

Figure 4: CARS2009 ε_{LM} (circles) for 19 loops between COFFS and each tide gauge up to CAIR using the 1975-76 first-order levelling traverse as the levelling component of the loops. The dashed line is first-order MAM and the dotted line is third-order MAM. BUND tide gauge is at the 959 km mark from COFF (0 km).

This suggests that the 'rapid' one-way levelling technique has caused a systematic levelling error after BUND to incorrectly indicate MSL decreased relative to the quasigeoid. The 'rapid' one-way levelling method was also used between COFFS and BALL/BRIS (~360 km), but does not appear to have caused the same errors. Morgan (1992) considers the technique to be susceptible to undetectable systematic errors because the two observations for the one-way rapid levelling are taken at the same time and in the same direction. The lack of independence between the two observations can result in ε < MAM for each levelling section (analysed by Morgan 1992) so that the levelling appears to be reliable, but Fig. 4 indicates a bias in the BUND to CAIR one-way levelling, corroborating Morgan's (1992) analysis. The likely culprits are refraction (e.g., Strange 1981) and staff calibration errors (Craymer and Vaníček 1995), although this cannot be investigated further as the original field observations are not available (only the mean of the two observations are provided in the ANLN).

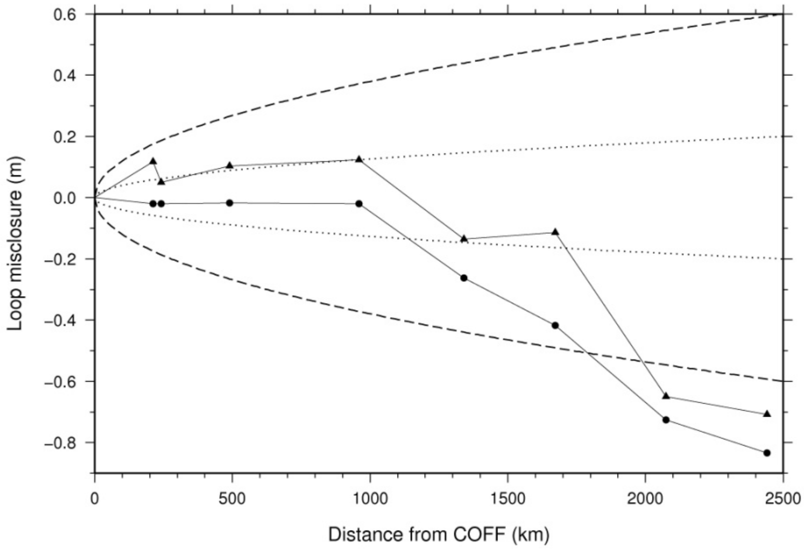

Fig. 5: CARS2009 ε_{LM} (circles) and GNSS h – AGQG09 ζ ε_{LG} (triangles) for eight loops between COFFS and each tide gauge up to CAIR with a GNSS h available (Table 1). The dashed line is first-order MAM and the dotted line is third-order MAM. BUND tide gauge is at the 959 km mark from COFF (0 km).

The good agreement (< first-order MAM) between CARS2009 and the first-order levelling between COFF and BUND suggest this section of first-order levelling is reliable, and can be used as a validation measure for the extrapolated CARS2009 tide gauge MDT for this section, although acknowledging that this is a small sample of nine tide gauges, but covering almost 1,000 km of the central east Australian coast. The mean of the differences between the levelling for the COFFS to BUND section and CARS2009 MDT is 5 mm, the SD is ±35 mm, maximum +64 mm and minimum -59 mm. These differences include error components from the MSL observation (and any temporal bias by the length and epoch of observation) and the levelling. Although crude, adopting an error estimate (1σ) for CARS2009 MDT at tide gauges of ±30 mm to ±50 mm may be realistic. This should be tempered by the knowledge that CARS2009 MDT values are reasonably close to the coast between COFF and BUND, but further north they tend to stop much further offshore due to the GBR and shallow depths adjacent to the coast.

To cross-validate the LM loops, GNSS h- AGQG09 ζ at nine tide gauges were formed into eight LG loops (all relative to COFF) using Eq. (4) and plotted with LM loops in Fig. 5. The LG loops largely support the results for the LM loops, with small ε_{LG} for loops up to BUND (exception of EVAN; at 212 km), but increasing in magnitude northwards after that. This largely substantiates the CARS2009 ε_{LM}, adding weight to the likelihood that systematic errors in the 'rapid' one-way levelling north of BUND are causing the large ε_{LM}. The almost constant offset between ε_{LM} and ε_{LG} indicate a disagreement between CARS2009 and GNSS h - AGQG09 ζ, or an error in ΔH_M at COFF, which is the common origin for the comparison. Noticeable differences at EVAN (212 km) and MACK (1,672 km) in Fig. 5, also indicate disagreement among CARS2009, GNSS h-ζ, ΔH_M at these tide gauges.

An additional cross-validation was conducted, with LM and LG loops formed for all possible loops from the nine tide gauges with GNSS h, computing ε_{LM} and ε_{LG} for each of the 36 loops formed. Differences were taken between ε_{LM} and ε_{LG} for common loops, and these are plotted in Fig. 6 with first-order and third-order MAM for the distance of the levelling component of the loop. Levelling MAM is only a proxy for permissible differences, as it can be seen in Fig. 6 that the differences between the GNSS-quasigeoid and MDT components of the loops do not propagate with respect to distance as is the expectation with levelling.

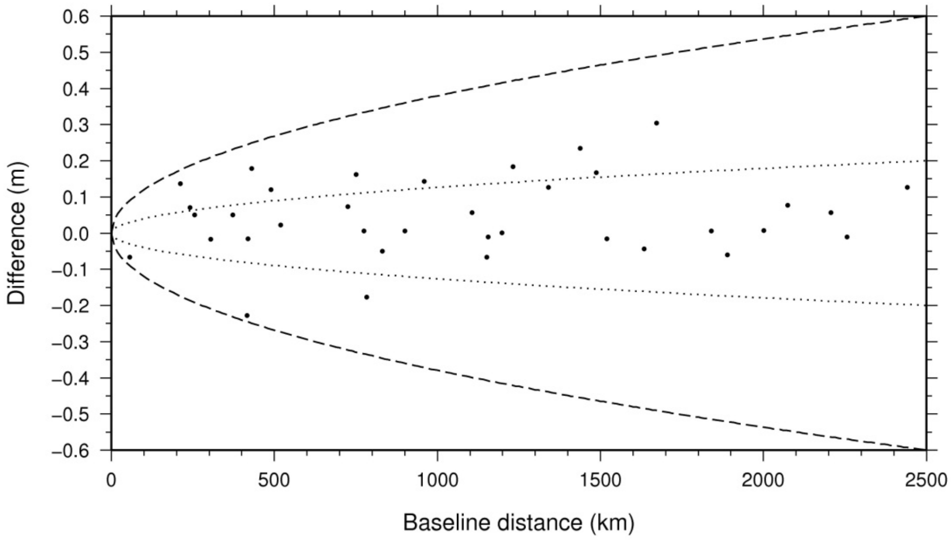

Fig. 6: Differences between CARS2009 ε_{LM} and GNSS h – AGQG09 ζ ε_{LG} for 36 loops plotted against the distance between the tide gauges in each loop. The dotted black line is the MAM for first-order levelling and the dashed line for third-order levelling, with respect to the distance between the tide gauges in each loop.

Statistics for ε_{LM} and ε_{LG} (Fig. 6) are maximum 0.305 m, minimum -0.228 m, mean 0.044 m, and SD 0.116 m, with 13 differences above the first-order MAM, but all within third-order MAM. All of the largest differences involve MACK, with the remaining differences outside of first-order MAM involving COFF or EVAN. This suggests an error in either h, ζ, or ΔH_Z (cf. ε_{LM} and ε_{LG} for COFF, EVAN and MACK in Fig. 5) based on the large (apparently site-dependent) deviation from the smoother CARS2009 ε_{LM}, independent tests with other MDT models (not shown here), and that quasigeoid models tend to be less reliable over coastal boundaries (e.g., Hipkin 2000).

Third-order levelling loop CAIR-COOK

The second part of the case study applies the LM loop method to the suspected third-order levelling error causing the levelling-MSL difference to jump 0.46 m between COOK and CAIR tide gauges (Roelse et al. 1971, Annex C). This suspected error remains apparent in Featherstone and Filmer (2012), but the location of any levelling error cannot be easily determined, because this is an example of a levelling loop with no adjoining loop on the coastal side (loop 118 in Fig. 3), which would otherwise provide a misclose covering the common section. There is no direct levelling section along the coast between PDOU and COOK. Loop 118 levelling ε is -0.241 m, which is a lesser magnitude than the third-order levelling MAM (±0.317) for its 698 km traverse perimeter, initially suggesting loop 118 does not contain a blunder.

Six CARS2009 LM loops were formed using permutations of the eastern and western sections of levelling-only loops 118 and 993 (Fig. 3; Table 2). LM loops between CAIR and COOK are supplemented by LG loops as a GNSS h is available at COOK and CAIR TGBMs.

Loop #	TGs used	CARS2009 ε_{LM}	h-AGQG09 ζ ε_{LG}	Third-order MAM	Levelled distance
993E	PDOU-CAIR	-0.063	NA	±0.105	77
993W	PDOU-CAIR	-0.085	NA	±0.159	176
118E	PDOU-COOK	0.410	NA	±0.202	284
118W	PDOU-COOK	0.170	NA	±0.300	624
993-118E	CAIR-COOK	0.444	0.483	±0.222	343
993-118W	CAIR-COOK	0.204	0.243	±0.314	683

Table 2: LM and LG loops in northern Queensland. Loops with E indicate east section of the levelling loop, W indicates western section of the loop. Units for ε and MAM in metres; for distance in km.

The results in Table 2 show ε_{LM} and ε_{LG} for CAIR-PDOU-COOK. ε_{LM} using the western and eastern levelling sections for loop 993 between CAIR and PDOU are both within third-order MAM, indicating that they do not contain the error. Likewise, ε_{LM} for the LM loop using the eastern route around loop 118 from PDOU-COOK is within third-order MAM, as is the CAIR-COOK LM loop following the western route. The result for this LM loop is supported by the ε_{LG} for the same loop (within 39 mm). The levelling error has been isolated to the eastern section of loop 118 (118E), identified by the PDOU-COOK LM loop as ~ 0.40 m, supported by the LM and LG loops for CAIR-COOK that follow the same eastern section of loop 118 and indicate the error to be between 0.44 and 0.48 m. Allowing some uncertainty in the MDT and GNSS h-ζ component of these loops (probably <0.10 m based empirically on the COFF-BUND comparison), it is likely that the apparent 0.46 m levelling-MSL jump between CAIR and COOK has been located in the eastern (coastward) section of loop 118. This error has been previously undetected because the third-order levelling-only loop 118 ε was < third-order MAM, due to a compensating misclose of the western section of loop 118. Redundant information from MDT and h-ζ has made it possible to detect this error emphasising the potential of this method to find errors that were undetectable using standard levelling loop closures.

4. Discussion and conclusions

The method and supporting case study presented here demonstrates the potential for MDT models to be used to detect and identify blunders (> the MDT uncertainty) in coastal levelling that are undetectable using standard levelling checks. This paper uses individual levelling lines rather than heights from an adjusted levelling network, which tend to mask errors in individual levelling traverses. The method described here may be of benefit to countries that are looking to locate/correct levelling errors in coastal regions, but do not have GNSS observations and/or sufficiently precise quasigeoid coverage to use h-ζ loops, or seek to supplement these with MDT loops. The availability of local tide gauges with a sea level record >three years (preferably longer, but this is not always possible) and reliable connections to the levelling to be investigated is sufficient to employ this method. There are a number of oceanographic MDTs available for this purpose, with good global coverage, that are independent of quasigeoid models and altimetry and the uncertainties associated with them in the coastal zone.

The apparently reliable section of levelling between COFF-BUND provides an insight into the performance of CARS2009 and AGQG09, albeit from small samples. CARS2009, despite being extrapolated from offshore, agrees slightly better with the levelling than AGQG09 (assuming GNSS h errors to be negligible by comparison) in this region of Australia, apparently being able to close first-order levelling within MAM of $4\sqrt{d}$. A relative uncertainty for CARS2009 MDT at tide gauges of ~±50 mm appears to be realistic based on empirical evidence in this and previous studies, meaning that levelling blunders >~50 mm can potentially be identified, as can systematic errors over longer distances, although this is dependent on the MDT uncertainty and the quality (hence MAM) of the levelling.

The good agreement between the LM and LG loops provide sufficient evidence to prove the systematic errors in the 1975-1976 first-order levelling campaign north of BUND up to CAIR. The one-way first-order levelling campaign in 1975-76 between BUND and CAIR contains large, apparently systematic errors, and is fundamentally flawed, inferring the sea slope to be in the opposite direction to its true gradient relative to the quasigeoid. This confirms the assertion by Morgan (1992) that the one-way first-order levelling employed along the north east coast is susceptible to systematic errors. It also shows that considerable caution is needed when adopting 'rapid'-type levelling methods that eschew conventional wisdom that levelling must be conducted in two directions as suggested by Morgan (1992) and Filmer and Featherstone (2009) for the entire ANLN.

The CAIR-COOK example showed that inland levelling sections can also be assessed using this method. Previous analysis of these inland loops had failed to find this error because the loop closures were within MAM, and unable to detect the errors that were masked by compensating errors.

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