A COMPARISON OF TECHNIQUES FOR THE INTEGRATION OF SATELLITE ALTIMETER AND SURFACE GRAVITY DATA FOR GEOID DETERMINATION

J F Kirby
School of Surveying and Land Information,
Curtin University of Technology, GPO Box U1987,
Perth, WA 6845, Australia.

R Forsberg
Geodetic Section, KMS, Rentemestervej 8,
Copenhagen NV, Denmark DK 2400.

Abstract

Two methods are tested whereby satellite altimeter measurements of the geoid height are combined with surface measurements of the free-air gravity anomaly. The study area comprises the oceans around the Australian continent. The first method involves draping a grid of the free-air anomaly from satellite data onto a grid of the ship and land data. The second method utilises grids of the altimeter-derived geoid height, combining these with the surface data in an iterative superposition. Preliminary results show that the draping method yields a fit of 5.4 mGal between the satellite and marine data, while the iterative procedure returns 8.1 mGal. Further work can be done, however, to improve these results. The impact of the combined marine gravity data sets is illustrated by comparing the effects on an Australia-wide spherical-FFT geoid solution.

Introduction

Although there exists an almost global coverage of satellite altimetry data, with the newly released Geosat/GM data and ERS-1 data greatly improving resolution, there is also an extensive global marine gravity data set. This data set should not be ignored as it can quite easily be incorporated with satellite altimetry data to give a more accurate map of the geoid than presumably could ever be obtained from purely altimetric data.

A method extensively used to combine heterogeneous gravity data is least squares collocation (LSC) [Moritz (1980)]. However, LSC is notoriously costly in computer execution time. To combine M data points, a matrix of size $M \times M$ must be inverted; "inversions of this size obviously present time problems even on a supercomputer and results will suffer from round-off errors" [Schwarz *et al.* (1990)].

Other procedures to transform gravity anomalies to geoid heights are based directly upon the classical Stokes integral [Heiskanen and Moritz (1967)]. These include direct numerical integration of the anomalies through ring integration methods [e.g., Kearsley (1985)] or quadrature integration; Fourier-domain approximations of the Stokes integral, a number of different techniques being summarised in Tziavos (1996); and a method by Sideris (1995) to construct a geoid from irregularly spaced gravity data using Fourier techniques.

This paper tests two approaches to the integration of satellite and surface data: a straightforward "draping" of the marine gravity onto the altimeter field; and an iterative scheme whereby the two heterogeneous data sets are integrated through the fast Fourier transform (FFT).

The Data

The two methods were tested on data over and around the Australian continent. The gravity data were supplied by the Australian Geological Survey Organisation (AGSO), revalidated by Featherstone *et al.* (1997). They comprise 111,396 free-air anomalies at sea and 526,091 on land. Both data sets were gridded together at 6 minutes using the GMT software [Wessel and Smith (1995)].

The altimeter data were also gridded at 6 minutes from the global 2 minute grid of Sandwell *et al.* (1995). Both data sets had the EGM96 geopotential model to degree and order 360 removed.

The resulting geoid models were compared with GPS/levelling data at 34 stations of the Australian Fiducial and National Networks (AFN/ANN) around the coast of Australia, where a geometric control geoid height could be determined.

The Iterative FFT Method

The algorithm combining heterogeneous gravity data sets [Kirby (1996)] makes use of the wavenumber domain relationship between geoid height and free-air anomaly, and can thus employ the fast Fourier transform to carry out rapid conversions between these various forms of potential field data. The algorithm presented here has been given the name IFC, for Iterative Fourier Combination.

The procedure utilises the planar 2-D Fourier-domain representation of the boundary value problem of physical geodesy:

$$\mathcal{F}\{\Delta g\} = \gamma\, k\, \mathcal{F}\{N\} \tag{1}$$

after Hipkin (1988), where k is the two-dimensional wavenumber.

The IFC routine requires not only a grid of gravity values, but also a grid of weights for the data set. This grid should reflect the relative influence which the gridnode corresponding to the data point has in the data set combination. This depends upon the distance of the node from the actual observation points.

A provisional gravity field model, h, is created which is improved upon in successive iterations of a weighted superposition with grids of geoid or free-air anomaly measurements. Conversion is always performed through equation (1), while the superpositions take the forms:

$$h \rightarrow \frac{h + \omega_{terr} \Delta g_{terr}}{1 + \omega_{terr}} \tag{2}$$

for gravity anomaly combination, and

$$h \rightarrow \frac{h + \omega_{sat} N_{sat}}{1 + \omega_{sat}} \tag{3}$$

for geoid undulation combination. Here, ω represents the weight grids for the terrestrial ($terr$) and satellite (sat) data sets.

This procedure can be iterated any number of times until the provisional model, h, stabilises. Stabilisation occurs at the iteration count when the RMS difference between successive 'like' provisional models reaches a previously specified value, indicating convergence.

The Draping Method

The second method of combination tested involved "draping" the terrestrial gravity data onto the Sandwell altimeter grid. This was achieved by frequency filtering the satellite grid with a 200 km high-pass filter. The difference between this and the marine gravity was then gridded using the Geosoft collocation package, and added back to the filtered altimeter grid only where no marine data existed.

A geoid model was then produced from this combined gravity field using the 2-D multiband spherical FFT [Forsberg and Sideris (1993)], with equation:

$$\mathcal{F}\{N\} = \frac{R}{4\pi\gamma} \, \mathcal{F}\{\Delta g \cos\phi\} \, \mathcal{F}\{S(\psi)\} \tag{4}$$

where $S(\psi)$ is the Stokes function.

Results

To assess their accuracy, the models were compared with the original data by interpolation at the observation locations. Table 1 shows the statistics of the difference between the original data observations, and values interpolated from the indicated models at their locations.

The accuracy of the GMT gridding procedure was assessed by comparing the AGSO ship and land data with their gridded counterpart. Rows 1 and 5 in Table 1 show the accuracy for gridding the marine data is 4.341 mGal RMS, and 3.675 mGal RMS for the land data.

Points of interest in this table are the large RMS difference between the marine gravity and the altimeter data: 10.617 mGal, indicating that one of these data sets has gross errors.

data	model	max	min	mean	std dev	RMS
	AGSO	74.015	-170.035	-0.050	4.341	4.341
AGSO marine	Altimeter	147.783	-239.579	-1.676	10.483	10.617
(mGal)	IFC	136.966	-233.471	-1.181	8.060	8.146
	Draped	95.536	-221.555	-0.103	5.447	5.448
AGSO land	AGSO	82.154	-62.403	-0.032	3.675	3.675
(mGal)	IFC	92.462	-69.082	-0.206	4.665	4.670
	Draped	97.617	-96.074	-1.113	6.795	6.885
Altimeter	IFC	98.650	-90.113	-0.211	2.357	2.357
(mGal)	Draped	159.166	-104.167	-0.084	6.158	6.159
AFN/ANN	IFC	1.133	-0.671	0.054	0.375	0.379
(metres)	Draped	1.196	-0.396	0.065	0.332	0.338

Table 1: Statistics of the difference between the data observations in the first column, and the values interpolated from the indicated model in the second column.

model	max	min	mean	std dev	RMS
gravity (mGal)	159.514	-110.888	-0.388	7.085	7.095
geoid (metres)	4.384	-3.798	-0.023	0.303	0.304

Table 2: Statistics of the difference between the IFC and draped potential fields.

The draped free-air field shows a better accuracy at sea than on land, 5.448 mGal against 6.885 mGal; while the reverse is true for the IFC gravity field: 4.670 mGal on land versus 8.146 mGal at sea.

The comparisons between the combined models and the 6 minute altimeter grid are also shown in Table 1. The IFC field shows a marked improvement over the draped field, with an RMS difference of 2.357 mGal against 6.159 mGal.

Finally, the geoid models from the two methods were compared with the coastal AFN/ANN data. The improvement of the draped solution over the IFC solution is slight: 33.8 cm against 37.9 cm RMS.

Figures 1 and 2 show the resultant combined geoid models. Note that the IFC solution has more successfully integrated the ship track data with the altimeter field to produce a smoother grid. However, the draped gravity field shows better agreement with the AGSO marine data. This discrepancy is most probably due to errors in the marine data.

Table 2 shows the statistics of the difference between the IFC and draped gravity field and geoid models.

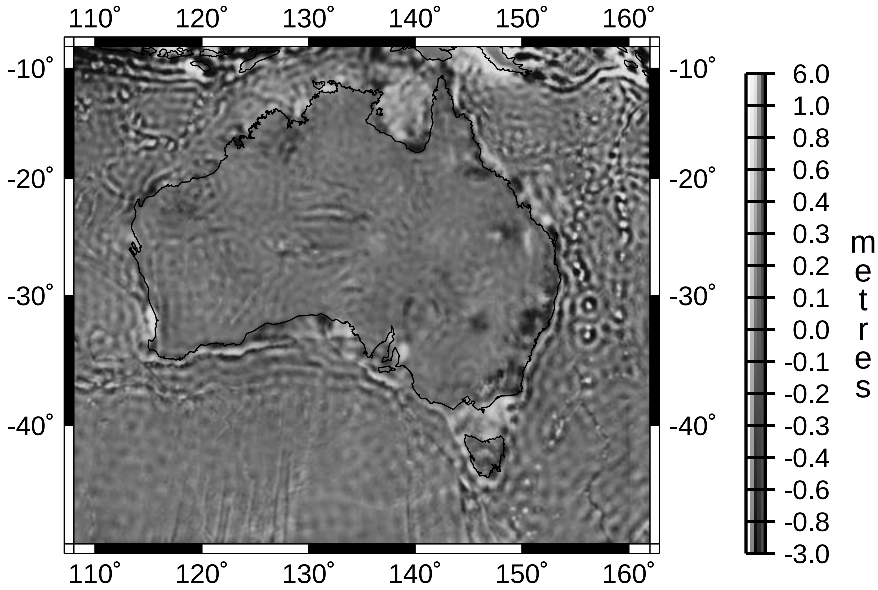

Figure 1: The geoid from the IFC method.

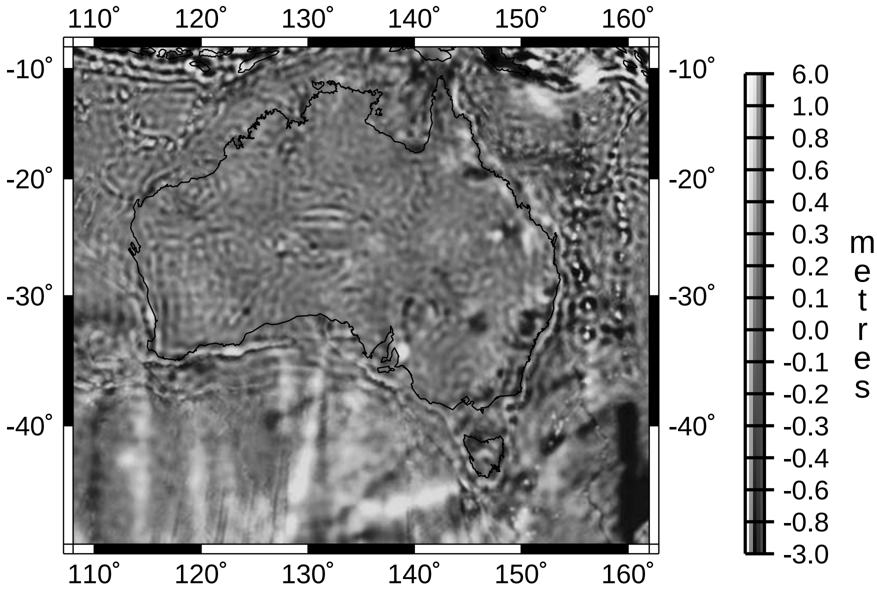

Figure 2: The geoid from the draping method.

Discussion and Conclusions

From Table 1, it can be seen that the majority of the differences occur at sea, where gravity data is usually ill constrained. Indeed, the ship data shows a very poor fit to the altimeter gravity field. However, both combination solutions have succeeded in blending the two data sets together, and, given accurate ship data, the benefits of this process would be multiple.

Further work should look at combining the benefits of both techniques, and this is indeed the subject of a proposal by the first author.

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