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The effect of EGM2008-based normal, normal-orthometric and Helmert orthometric height systems on the Australian levelling network

M.S. Filmer · W.E. Featherstone · M. Kuhn

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M.S. Filmer

Western Australian Centre for Geodesy & The Institute for Geoscience Research,

Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia

Tel.: +61-8-9266-2218

Fax: +61-8-9266-2703

E-mail: M.Filmer@curtin.edu.au

W.E Featherstone

Western Australian Centre for Geodesy & The Institute for Geoscience Research,

Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia

Tel.: +61-8-9266-2734

Fax: +61-8-9266-2703

E-mail: W.Featherstone@curtin.edu.au

M. Kuhn

Western Australian Centre for Geodesy & The Institute for Geoscience Research,

Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia

Tel.: +61-8-9266-2603

Fax: +61-8-9266-2703

E-mail: M.Kuhn@curtin.edu.au

Abstract This paper investigates the normal-orthometric correction used in the definition of the Australian Height Datum (AHD), and also computes and evaluates normal and Helmert orthometric corrections for the Australian National Levelling Network (ANLN). Testing these corrections in Australia is important to establish which height system is most appropriate for any new Australian vertical datum. An approximate approach to assigning gravity values to ANLN benchmarks (BMs) is used, where the EGM2008-modelled gravity field is used to 're-construct' observed gravity at the BMs. Network loop closures (for first- and second-order levelling) indicate reduced misclosures for all height corrections considered here, particularly in the mountainous regions of south eastern Australia. Differences between Helmert orthometric and normal-orthometric heights reach 44 cm in the Australian Alps, and differences between Helmert orthometric and normal heights are about 26 cm in the same region. Normal-orthometric heights differ from normal heights by up to 18 cm in mountainous regions > 2000 m. This indicates that the quasigeoid is not compatible with normal-orthometric heights.

Keywords AHD · height systems · EGM2008 · vertical datums

1 Introduction

The Australian Height Datum (AHD) is Australia's first and only official height datum. On the mainland, it was established in 1971 from the least-squares adjustment (fixed to mean sea level (MSL) = zero at 30 tide gauges around Australia) of the (then named) Australian Levelling Survey (ALS) comprising approximately 97,000 km of 'primary' levelling (Roelse et al. 1971). Due to the lack of computing power at that time, the basic (or primary) network was adjusted on a State-by-State basis before the

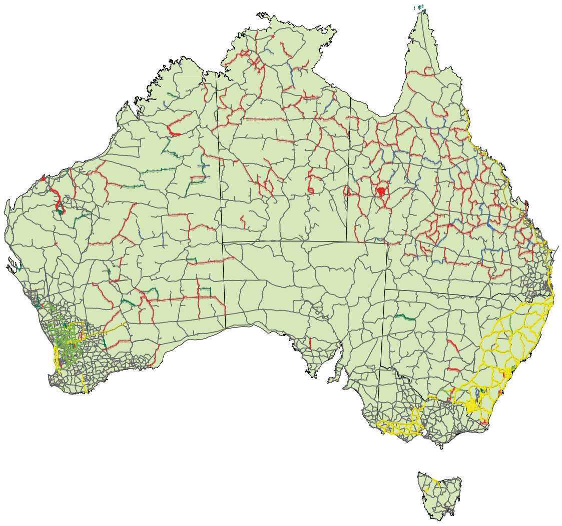

Fig. 1 The Australian National Levelling Network (ANLN). First order sections are in yellow, second order sections in light green, third order in thin grey, fourth order in dark green, one-way (third order) in red and two-way (order undefined; Steed 2006, pers. comm.) in blue. Lambert projection. ANLN data courtesy of Geoscience Australia.

full national adjustment was conducted. Approximately 80,000 km of 'supplementary' levelling were subsequently least-squares adjusted to the basic junction point (JP) heights (held fixed), which were defined in the full national adjustment (Roelse et al. 1971). The ANLN in Tasmania was adjusted in 1983 (NMC 1986), with MSL at two tide-gauges held fixed at zero in the adjustment. An offset between AHD(mainland) and AHD(Tas) thus exists which is estimated to be between 10 cm and 20 cm (Rizos et al. 1991; Featherstone 2000).

A number of issues have arisen with regards to the AHD, with numerous studies investigating different deficiencies in the datum (e.g., Featherstone 2001). The major areas of concern in the AHD include the fixing of the levelling network (now referred to as the Australian National Levelling Network; ANLN, see Fig. 1) to 30 tide gauges using MSL as the zero reference surface (e.g., Hamon and Greig 1972; Mitchell 1973b; Coleman et al. 1979), the quality of the levelling (e.g., Morgan 1992; Kearsley et al. 1993; Filmer and Featherstone 2009) and the omission of gravity based height corrections (e.g., Mitchell 1973a; Allister and Featherstone 2001).

The levelling errors in the ANLN (data provided by Geoscience Australia; GA; G. Johnston pers. comm. 2007) are particularly problematic. The cause of the largest regional distortions in the AHD are gross levelling errors, chiefly in the interior of the continent (Filmer and Featherstone 2009). The magnitude of the above-tolerance loop misclosures often exceed 0.5 m in central Australia, with the maximum reaching 0.93 m. Note that the perimeter length of some of these loops are $>$ 2000 km (Fig. 1).

When the AHD was defined in 1971, insufficient gravity observations were available to apply gravimetric height corrections. Instead, a truncated version of the so-called normal-orthometric correction of Rapp (1961) was applied; there is no requirement for observed gravity whatsoever in this correction. As such, the AHD should be considered a normal-orthometric height system (Holloway 1988; Featherstone and Kuhn 2006). However, sufficient gravity data now exists in Australia to investigate the effects of gravimetric height corrections to the ANLN (cf. Mitchell 1973a).

The problems with the AHD have become considerably more apparent with the emergence of GNSS heighting and regional gravimetric quasigeoid models (e.g., Featherstone et al. 2001). Various methods of overcoming the incompatibility of GNSS-derived ellipsoid heights and AUSGeoid98 (Featherstone et al. 2001) with the AHD have been investigated (e.g., Featherstone 1998; Featherstone and Sproule 2006; Soltanpour et al. 2006). However, the long-term solution to this problem, plus the expected requirement for accurate physical heights from new generation DEMs, is a scientifically rigorous new Australian vertical datum. This includes correct treatment of the ANLN with respect to gravity.

This paper investigates the effects of introducing three different height systems and the heights resulting from least-squares adjustments of the ANLN. Helmert (1890) orthometric, Molodensky et al. (1962, loc. cit.) normal and normal-orthometric heights (here, we use the Rapp (1961) version) are all in common use around the world today. For example, the United States uses Helmert orthometric heights (Zilkoski et al. 1992), the former U.S.S.R and eastern European countries use normal heights (Vaníček and Krakiwsky 1982, p. 371) and Australia, New Zealand and the United Kingdom use normal-orthometric heights (Roelse et al. 1971; Amos and Featherstone 2009; Ziebart et al. 2008).

The formulas for each height system are first summarised, followed by a description of a method for obtaining gravity values at ANLN BMs using EGM2008 (Pavlis et al. 2008) modelled gravity. Results are then presented showing the effects of the three height systems on levelling loop closures and also on heights after minimally constrained adjustments of the ANLN.

2 Height systems

2.1 Helmert orthometric heights

The orthometric height H^O is defined by Heiskanen and Moritz (1967 p. 166)

$$H^O = \frac{C}{\bar{g}} \tag{1}$$

where C is the geopotential number and $\bar{g}$ is the integral mean of gravity along the plumbline between the Earth's surface and the geoid. The orthometric height is thus defined as the distance along the (curved and torsioned) plumbline between the surface point P and the point P_0 (Fig. 2).

However, a true orthometric height cannot be computed exactly (e.g., Jekeli 2000), because $\bar{g}$ is inside the topography and cannot be measured (cf. Strange 1982; Tenzer et al. 2005). There are a number of different methods of approximating $\bar{g}$, resulting in several variants of orthometric heights (e.g., Helmert 1890; Neithammer 1932; Mader 1954; Strang van Hees 1992; Kao et al. 2000; Hwang and Hsiao 2003; Tenzer et al. 2005). The simplest of these is the Helmert (1890) orthometric height which uses the simplified gravity reduction of Poincaré-Prey (hereafter referred to as SPP) to approximate $\bar{g}$ (Heiskanen and Moritz 1967 p. 164),

$$g_{Q^O} = g_P + 0.0848(H_P - H_{Q^O}) \tag{2}$$

where g_{Q^O} is the value of gravity (Gals) at the midpoint along the plumbline $P - P_0$, g_P (Gals) is the value of observed gravity at P, H_P (km) is the orthometric height of P and H_{Q^O} (km) is the Helmert orthometric height of Q^O (approximate mid-point of plumbline; see Fig. 2). The SPP reduction makes a number of approximations, including the neglect of terrain effects and variations in the Earth's mass-density (cf. Neithammer 1932; Mader 1954; Hwang and Hsiao 2003; Tenzer et al. 2005).

The practical application of Helmert orthometric corrections HOC is usually made to the levelled height differences δn. This is (e.g., Heiskanen and Moritz 1967 p. 168)

$$HOC_{1-2} = \sum_{1}^{2} \frac{g - \gamma_0}{\gamma_0} \delta n + \frac{\bar{g}_1 - \gamma_0}{\gamma_0} H_1 - \frac{\bar{g}_2 - \gamma_0}{\gamma_0} H_2 \tag{3}$$

with the SPP reduction (Eq. 2) determining the mean value of gravity along the plumbline, at BM 1 $\bar{g}_1$ and BM 2 $\bar{g}_2$. H_1 is the orthometric height at BM 1, likewise for BM 2. γ_0 is the value of normal gravity at 45 degrees latitude.

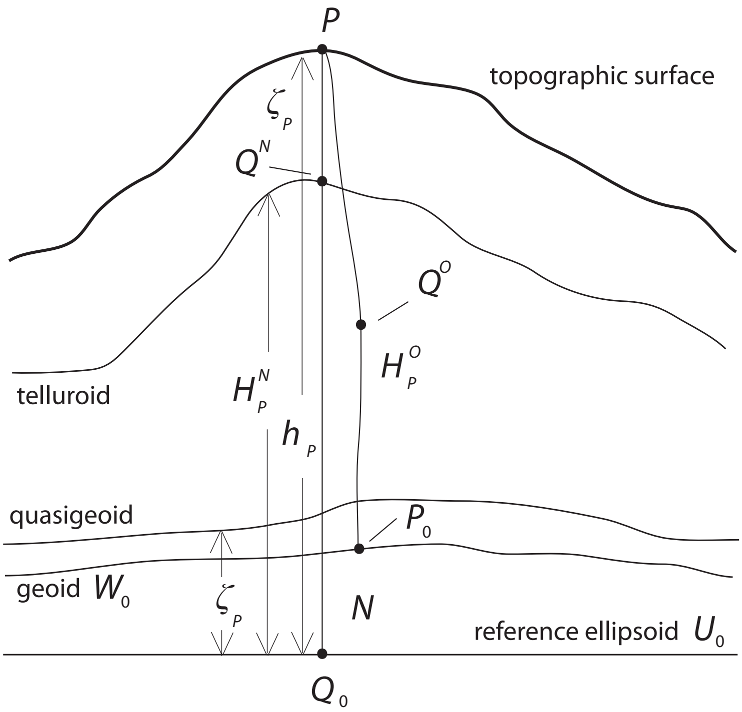

Fig. 2 The orthometric height of point P (H_P^O) is the distance along the curved and torsioned plumbline P-P_0. The normal height of point P (H_P^N) is the distance along the slightly curved (not straight as drawn for convenience) normal plumbline Q^N - Q_0. The ellipsoidal height of P (h_P) is the length of the straight ellipsoid normal between P and the reference ellipsoid Q_0. The geoid-ellipsoid separation N allows H_P^O and h_P to be related, while the height anomaly ζ_P relates H_P^N and h_P. The point Q^N is on the telluroid ($W_P = U_{Q^N}$) and Q^O is the approximate midpoint along the plumbline $P - P_0$.

2.2 Normal heights

In 1945, Molodensky (Molodensky et al. 1962, loc. cit.) introduced the concept of the normal height system H^N (Heiskanen and Moritz 1967, p. 291). Here, $\bar{g}$ is replaced by the integral mean of normal gravity along the normal plumbline $\bar{\gamma}$ between the ellipsoid and telluroid (Heiskanen and Moritz 1967, p. 171)

$$H^N = \frac{C}{\bar{\gamma}}. \tag{4}$$

Figure 2 shows the relationship between components of the normal height system. The normal height at P, H_P^N is thus defined as the distance along the normal plumbline between Q^N on the telluroid and Q_0 on the ellipsoid. Although the height anomaly ζ is defined between the telluroid and topographic surface, ζ can be plotted above the reference ellipsoid to map the quasigeoid (Fig. 2). The quasigeoid is not an equipotential surface (nor is the telluroid) in either the normal or actual gravity field (Jekeli 2000), so has lesser physical meaning. Thus, for practical purposes, H^N is the normal height of P above the quasigeoid in analogy to H^O (Sect. 2.1).

The formula to compute $\bar{\gamma}$ is derived from the second order free-air gravity correction (Heiskanen and Moritz, 1967 p. 170)

$$\bar{\gamma} = \gamma \left[1 - (1 + f + m - 2f \sin^2 \phi_{BM}) \frac{H^N}{a} + \left(\frac{H^N}{a} \right)^2 \right] \tag{5}$$

where γ is normal gravity on the ellipsoid at the point of computation, f is the geometrical flattening of the ellipsoid, m is the geodetic parameter (ratio of gravitational and centrifugal forces at the equator) and ϕ_{BM} is the geodetic latitude at the BM. Equation (5) can be used to compute $\bar{\gamma}$ in the same way that the SPP reduction is used to compute $\bar{g}$. The critical difference is that $\bar{\gamma}$ can be computed analytically and without assumptions and approximations, whereas $\bar{g}$ requires assumptions to be made regarding the topographic masses (cf. Sect. 2.1).

Normal heights can also be computed by a correction to measured height differences, which corresponds to Eq. (3), with $\bar{\gamma}$ replacing $\bar{g}$

$$NC_{1-2} = \sum_1^2 \frac{g - \gamma_0}{\gamma_0} \delta n + \frac{\bar{\gamma}_1 - \gamma_0}{\gamma_0} H_1 - \frac{\bar{\gamma}_2 - \gamma_0}{\gamma_0} H_2. \tag{6}$$

2.3 Normal-orthometric heights

Normal-orthometric heights are, like normal and orthometric heights, generally computed through a correction applied to the levelling observations. There are numerous versions of the normal-orthometric correction (NOC), including Rapp (1961; hereafter referred to as NOC_R), Bomford (1980), New Zealand (e.g., Amos and Featherstone 2009) and Heck (1995). However, only the NOC_R correction will be investigated here, as it is the height system used in the AHD.

The general concept of normal-orthometric heights is that the normal gravity field completely replaces the actual gravity field, with the geopotential numbers replaced by normal potential numbers C^N (e.g., Rapp 1961) which can be defined as (cf. Jekeli 2000)

$$C^N = U_P - U_{N\text{-}O} \tag{7}$$

where U_P is normal potential on the topographic surface and $U_{N\text{-}O}$ is normal potential on the zero reference surface for $H^{N\text{-}O}$ (discussed in Sect. 2.5). In analogy to H^N and H^O, normal-orthometric heights $H^{N\text{-}O}$ are

$$H^{N\text{-}O} = \frac{C^N}{\bar{\gamma}}. \tag{8}$$

Note that normal-orthometric corrected loop closures are dependant on the levelling route taken (Featherstone and Kuhn 2006).

Normal-orthometric heights were intended as an approximation of H^O in areas where insufficient gravity observations are available to implement H^O or H^N. As such, H^{N-O} have no physical meaning. Many of the vertical datums using $H^{N\text{-}O}$ are often incorrectly termed orthometric heights, despite not using gravity in their realisation (e.g., Holloway 1988; Ziebart et al. 2008).

2.4 Rapp's 1961 normal-orthometric correction

The NOC_R reads (cf. Rapp 1961, p.16)

$$NOC_R = (A\bar{H}_{1\text{-}2} + B\bar{H}_{1\text{-}2}^2 + C\bar{H}_{1\text{-}2}^3)\phi_{1\text{-}2} \tag{9}$$

where $\bar{H}_{1\text{-}2}$ is the average normal-orthometric height between BM 1 and BM 2, with $\phi_{1\text{-}2}$ the latitude difference between BM 1 and BM 2 (arc minutes). The coefficients A, B and C are computed using (Rapp 1961, p. 17)

$$A = 2\sin 2\bar{\phi}\alpha\prime(1 + \cos 2\bar{\phi}\left(\alpha\prime - \frac{2\kappa}{\alpha\prime}\right) - 3\kappa\cos^2 2\bar{\phi})Q \tag{10}$$

$$B = 2\sin 2\bar{\phi}\,\alpha\prime\, t_2\left(t_3 + \frac{t_4}{2\alpha\prime} + \cos 2\bar{\phi}\left(\frac{3}{2}t_4 + 2\alpha\prime t_3 - \frac{2\kappa t_3}{\alpha\prime}\right)\right)Q \tag{11}$$

$$C = 2\sin 2\bar{\phi}\alpha\prime\, t_2^2\, t_3\left(t_3 + \frac{t_4}{2\alpha\prime} + \cos 2\bar{\phi}\left(2t_4 - \frac{2\kappa t_3}{\alpha\prime}\right)\right)Q \tag{12}$$

where Q is 1 arc minute in radians and $\bar{\phi}$ is the mid-latitude between BM 1 and BM 2. The constants $\alpha\prime$, κ, t_2, t_3 and t_4 are computed using (Rapp 1961, p. 11,14)

$$\alpha\prime = \frac{\beta}{2 + \beta + 2\epsilon} \tag{13}$$

$$\kappa = \frac{-2\epsilon}{2 + \beta + 2\epsilon} \tag{14}$$

$$t_2 = \frac{2(1 + \alpha + c\prime)}{a\left(1 + \frac{\beta}{2} + \epsilon\right)} \tag{15}$$

$$t_3 = \frac{1 - d_3}{2} = 1 - t_4 \tag{16}$$

$$t_4 = 1 - t_3 \tag{17}$$

with a the semi-major axis of the reference ellipsoid, β the gravity flattening and ϵ described by Rapp (1961, p. 7) as a constant in the normal gravity formula and by Moritz (1980) as f_4 in the Chebyshev series approximation of the gravity formula 1980. Rapp (1961, p. 13) defines d_3 as,

$$d_3 = \frac{(3\alpha - 2.5\,c\prime)}{2} \tag{18}$$

and (Rapp 1961, p. 12)

$$c\prime = \frac{\omega^2 a^3}{GM}, \tag{19}$$

with ω the angular velocity of the Earth's rotation and GM the geocentric gravitational constant.

The AHD uses the Geodetic Reference System 1967 (GRS67; IAG 1967) parameters (Roelse et al. 1971), which has now been superseded by GRS80 (Moritz 1980). The effect of these two systems on the NOC_R was tested by evaluating the NOC_R at two points in Australia using both the GRS67 and GRS80 parameters (Australian mainland 2120 m, $\phi = \sim 36°$S; Tasmania 1180 m, $\phi = \sim 42°$S). The differences at both test sites were < 0.001 mm, indicating that the NOC_R difference is negligible when using GRS67 or GRS80 parameters are in Australia. GRS80 parameters have been used for this study.

Only the first two terms in Eq. (9) (containing A and B coefficients) were computed and applied to the AHD, as it was then considered that the third term in Eq. (9) (containing the C coefficient) was negligible (Roelse et al. 1971). We re-computed NOC_R values, firstly using all terms in Eq. (9) and then only the first two terms, confirming that the truncation effect (neglecting the C coefficient) introduced by Roelse et al. (1971) is negligible (< 0.001 mm per 1 arc minute of latitude at $H^{N\text{-}O} = 2228$ m, which is mainland Australia's highest point, Mt Kosciuszko). Despite this, the full Eq. (9) has been used for this study, as the additional computation is not excessive with modern computing power.

2.5 A remark on the compatibility of $H^{N\text{-}O}$ and gravimetric quasigeoid models

In terms of compatibility with quasigeoid models, AHD $H^{N\text{-}O}$ has been considered similar to H^N for practical purposes, such as using AUSGeoid98 to transform GNSS derived ellipsoidal heights to the AHD (e.g., Featherstone and Kirby 1998; Featherstone and Kuhn 2006). However, despite the closeness of the quasigeoid and the zero reference surface for $H^{N\text{-}O}$, they are inconsistent, as follows.

Observed surface gravity g appears in the Helmert orthometric (Eq. 3) and normal corrections (Eq. 6) and the geopotential number C in H^O (Eq. 1) and H^N (Eq. 4). In contrast, no relation to actual gravity or geopotential appear in the NOC_R (Eqs. 9 - 19). Therefore, the difference between H^N and $H^{N\text{-}O}$ is of interest as it leads to a theoretical incompatibility between $H^{N\text{-}O}$ and gravimetric quasigeoid models. Different versions of the normal-orthometric correction will refer to slightly different surfaces (e.g., Rapp 1961; Heck 1995; Bomford 1980).

The difference between H^N and $H^{N\text{-}O}$ can be derived through Eqs. (4) and (8). Thus,

$$H^N - H^{N\text{-}O} = \frac{C}{\bar{\gamma}} - \frac{C^N}{\bar{\gamma}} = \frac{W_P - W_0}{\bar{\gamma}} - \frac{U_P - U_{N\text{-}O}}{\bar{\gamma}} \tag{20}$$

which becomes

$$H^N - H^{N\text{-}O} = \frac{(W_P - U_P) - (W_0 - U_{N\text{-}O})}{\bar{\gamma}}. \tag{21}$$

If we let the value of normal potential at the $H^{N\text{-}O}$ reference surface $U_{N\text{-}O}$ be equal to $U_R + \delta U$ (normal potential on the quasigeoid U_R plus the unknown normal potential difference between the quasigeoid and $H^{N\text{-}O}$ reference surface δU) and knowing the disturbing potential at point P, $T_P = W_P - U_P$, we get

$$H^N - H^{N\text{-}O} = \frac{T_P - (W_0 - (U_R + \delta U))}{\bar{\gamma}} \tag{22}$$

which becomes

$$H^N - H^{N\text{-}O} = \frac{T_P - (W_0 - U_R)}{\bar{\gamma}} + \frac{\delta U}{\bar{\gamma}}. \tag{23}$$

According to Molodensky et al. (1962, loc.cit.), W_P is equal to U_{Q^N} (the value of normal potential at the telluroid for point P; see Fig. 2), so

$$T_P = U_{Q^N} - U_P. \tag{24}$$

As the height anomaly ζ computed at the surface is mapped from the ellipsoid (U_0) to realise the quasigeoid and assuming changes in the gradient of normal potential $\frac{\partial U}{\partial h}$ between the telluroid and ellipsoid are negligible (the gradient changes linearly by $\sim$ 0.003 m^2 s^{-2}, or 0.3 mm per 1000 m height), we can say that

$$(U_0 - U_R) = (U_{Q^N} - U_P). \tag{25}$$

As the potential values (considered on a global basis) $W_0 = U_0$ (whereby, the g term effectively drops out), we now find from Eqs. (22 - 25)

$$\frac{T_P - (W_0 - U_R)}{\bar{\gamma}} = 0, \tag{26}$$

thus, Eq. (26) reduces to

$$H^N - H^{N\text{-}O} = \frac{\delta U}{\bar{\gamma}}. \tag{27}$$

This shows that H^N and $H^{N\text{-}O}$ are not coincident, but depend on δU. If the value of δU were known, the difference between H^N and $H^{N\text{-}O}$ could be computed using Eq. (27). However, this value is not known; we have only been able to estimate this difference from the ANLN (see Sect. 4.2.3).

As such, we have shown that $H^{N\text{-}O}$ does not refer to the quasigeoid, but to another, poorly defined, surface. This distinction is important for those attempting to use GPS-derived ellipsoid heights and gravimetric quasigeoid models to realise accurate (cm level) normal-orthometric heights, particularly in mountainous regions (e.g., Sect. 4.2.3).

3 EGM2008 re-constructed gravity data

Gravity values g and hence C are not always known at BMs and in some regions are too sparse for interpolation to BMs. An alternative method for computing gravity-related heights in these areas is to synthesise the EGM2008 (Pavlis et al. 2008) gravity field as a substitute for observed gravity.

EGM2008 is a combined global geopotential model of the Earth's gravity field that performs well over Australia (Claessens et al. 2008). We have used the tide-free release of EGM2008 to 're-construct' gravity (cf. Featherstone and Kirby 2000) at all ANLN BMs for the computation of height corrections. The method requires the computation of the gravity disturbance δg from EGM2008 and normal gravity γ at the ANLN BM on the topographic surface (details later).

The horizontal datum of ANLN BM coordinates is probably Australian Geodetic Datum (AGD66) (G. Holloway 2009, pers. comm.), although recent additions to the ANLN could be in Geodetic Datum of Australia (GDA94). A horizontal error of approximately 190 m is introduced with respect to GDA94/WGS84 (Featherstone 1995). The maximum effect of this datum error on the EGM2008 δg has been tested and is $\sim$ 2 mGal, but generally much less (cf. Featherstone 1995; error in γ from datum effect is $\sim$ 0.1 mGal). The ANLN coordinates were scaled from 1:250,000 maps (Roelse et al. 1971) to the nearest arc minute resulting in an accuracy of no better than $\sim$ 900 m. As the datum difference is a relatively small component of the total benchmark positional uncertainty (and the possibility that some recent data may already be GDA94), the given ANLN coordinates will be assumed the best currently available for this study.

3.1 Method of BM gravity re-construction

The scalar gravity disturbance at a BM δg_{BM} is the difference between the magnitude of observed gravity g_{BM} and normal gravity γ_{BM}, both at the BM (cf. Heiskanen and Moritz 1967 p. 84)

$$\delta g_{BM} = g_{BM} - \gamma_{BM} \tag{28}$$

and can be used to infer gravity at the BM

$$g_{BM} = \gamma_{BM} + \delta g_{BM}. \tag{29}$$

The FORTRAN77 program **harmonic_synth.f77** (Holmes and Pavlis 2008) is used to compute the spherical approximation of the radial component of the EGM2008 δg

$$\delta g_{BM} = \frac{\partial T}{\partial r} |_{BM} \tag{30}$$

where T_{BM} is the disturbing potential at the BM and r_{BM} is the radial distance at the BM increasing outward from the geocentre (**harmonic_synth.f77** computes δg with the sign opposite to usual convention). The inclusion of the derived ellipsoidal height h_D of the ANLN benchmarks (with respect to the GRS80 ellipsoid) allows δg to be computed at the ANLN BM on the topographic surface (see Fig. 2). h_D is computed as

$$h_D = H^{N\text{-}O} + \zeta \tag{31}$$

where $H^{N\text{-}O}$ is the AHD normal-orthometric height and ζ is the EGM2008 height anomaly. To compute r_{BM}, (which is required to compute ζ and δg), **harmonic_synth.f77** converts the geodetic latitude ϕ_{BM} (geodetic longitude is the same as geocentric longitude) to geocentric latitude at the ANLN BM $\bar{\theta}_{BM}$ for spherical approximation of the required values. The formula for geodetic to geocentric conversion used by **harmonic_synth.f77** is (Holmes and Pavlis 2008)

$$\bar{\theta}_{BM} = \arctan \frac{(\nu(1-e^2)+h_D)\sin\phi_{BM}}{(\nu+h_D)\cos\phi_{BM}} \tag{32}$$

where ν is the prime vertical radius of curvature

$$\nu = \frac{a}{\sqrt{1-e^2\sin^2\phi_{BM}}} \tag{33}$$

and e^2 is the square of the first eccentricity. Note that $\bar{\theta}$ is the geocentric latitude in spherical polar coordinates, not the co-latitude θ.

To compute γ_{BM}, γ must first be computed on the GRS80 ellipsoid surface at the ANLN BM latitude (Moritz 1980)

$$\gamma = \gamma_e \frac{1+k\sin^2\phi_{BM}}{\sqrt{1-e^2\sin^2\phi_{BM}}} \tag{34}$$

where γ_e is the value of normal gravity at the equator and (Moritz 1980),

$$k = \frac{b\gamma_p}{a\gamma_e} - 1 \tag{35}$$

with b the semi-minor axis of the GRS80 ellipsoid and γ_p normal gravity at the poles. Values of the GRS80 parameters required to evaluate Eq. (34) can be found in Moritz (1980).

Next, we use the second order free-air gravity correction δg_{F2} (cf. Hackney and Featherstone 2006)

$$\delta g_{F2} = \frac{2\gamma}{a}\left(1+f+m-2f\sin^2\phi\right)h_D + \frac{3\gamma}{a^2}h_D^2. \tag{36}$$

Normal gravity at γ_{BM} is thus

$$\gamma_{BM} = \gamma - \delta g_{F2} \tag{37}$$

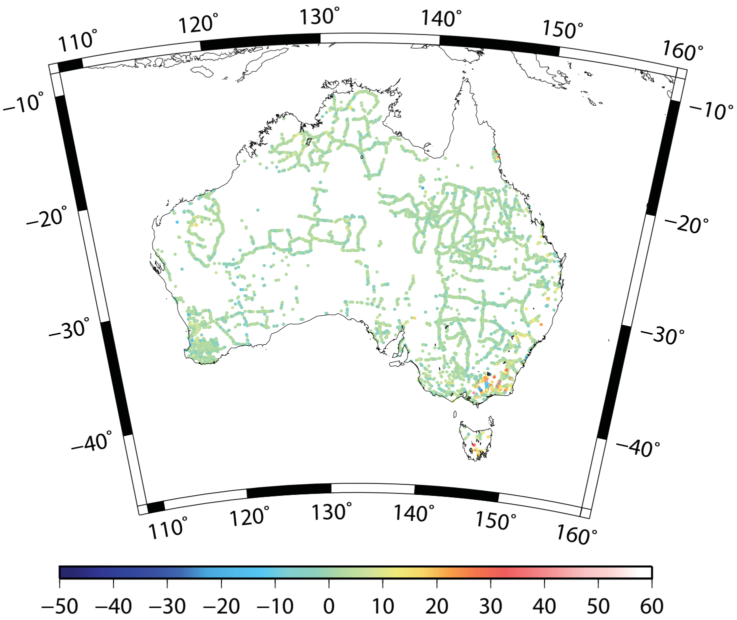

Fig. 3 Differences between EGM2008 're-constructed' gravity and 2007GAgrav at 9527 ANLN BMs (cf. Fig. 1). Statistics for these differences are: maximum 56.75 mGal, minimum -41.33 mGal, average 1.87 mGal, STD 4.96 mGal. Note the cluster of large differences in the Southern Alpine region. Units in mGal.

and the 're-constructed' gravity at the benchmark g_{BM} is computed using Eq. (29) or, in full from the previous equations

$$g_{BM} = \gamma - \delta g_{F2} + \delta g_{BM}(EGM). \tag{38}$$

3.2 Validation of EGM2008 re-constructed gravity

The 're-constructed' EGM2008 gravity can be partially validated (cf. Claessens et al. 2008) using the 2007 release of the GA gravity database (hereafter referred to as 2007GAgrav). This is possible because 9527 of the $\sim$ 90,000 BMs in the ANLN can be matched with terrestrial gravity observations in 2007GAgrav that were taken directly on the BMs. Note that later releases from GA (http://www.geoscience.gov.au/gadds) omit this metadata that was only provided for a short while in mid-2007.

The spatial distribution of the 9527 ANLN-2007GAgrav benchmarks can be seen in Fig. 3 (cf. Fig. 1). Most of the differences between EGM2008 're-constructed' and 2007GAgrav at the 9527 ANLN benchmarks are within $\pm$ 10 mGal (see Fig. 4). However, a cluster of larger differences is evident in Fig. 3 in the Southern Alpine region centred at $\sim 37°$S, and $\sim 147°$E (cf. Claessens et al. 2008, Figs. 9 and 11). Several other points with differences of this magnitude are seen in Tasmania (Fig. 3; in mountains, $\sim 42°$S, $\sim 147°$E), along the Great Dividing Range in NSW and the north Queensland coast ($\sim 18°$S).

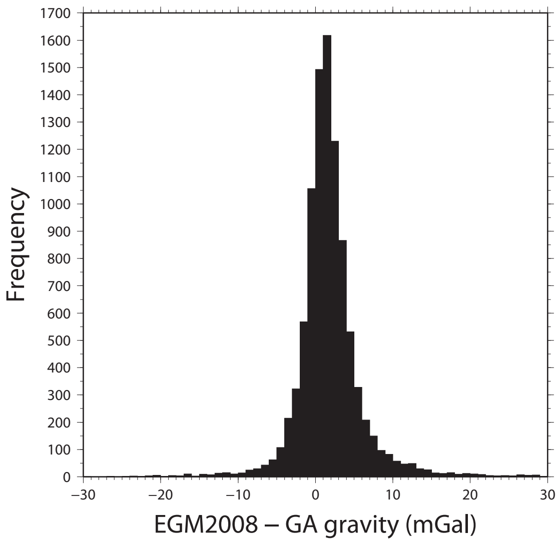

Fig. 4 Histogram of differences between EGM2008 're-constructed' gravity and 2007GAgrav at 9527 ANLN BMs. Statistics for these differences are: maximum 56.75 mGal, minimum -41.33 mGal, average 1.87 mGal, STD 4.96 mGal. Note the high kurtosis (kurtosis value of +18.92 for this set). Units in mGal.

The cause of these differences is not completely clear; Claessens et al. (2008) suggest the differences in these areas could be attributed to EGM2008 (problems modelling the variable gravity field in mountainous areas and omission error). However, while Sproule et al. (2006) indicate that large errors in the GA gravity database are not common, they cannot be completely excluded as a cause of some of the larger errors (particularly if regional biases are present). Uncertainty in the horizontal positions of ANLN BMs (described at the start of this section) will also contribute to these differences, especially in mountainous areas.

4 Results

4.1 Loop closure analysis

A first indication of the effectiveness of the height corrections may be seen in comparisons of levelling loop closures. Gravimetric height corrections should theoretically reduce the loop misclosures.

We base our loop closure assessment on the equation (cf. ICSM 2007)

$$c = \frac{\varepsilon}{\sqrt{d}} \cdot 1000 \tag{39}$$

where ε (m) is the loop misclosure, d (km) is the length of the loop perimeter and c (mm) is an estimate of the levelling precision (1 σ) per $\sqrt{d}$ km of levelling (multiplying by 1000 converts c into mm). Kao et al. (2000) make the point that improved loop closures do not necessarily result in more accurate heights on benchmarks in the loop. However, as there are no 'known' heights to check against, loop closures are a useful test for the different height corrections.

Table 1 Computed c in mm / $\sqrt{d}$ km (Eq. 39; cf. ICSM 2007) for 1366 supplementary and basic ANLN loop closures (see Filmer and Featherstone 2009) with different height corrections applied. The column No HC contains computed c when no height correction is applied; differences to these values are shown for the respective height correction. Positive values indicate higher computed c compared to No HC; negative indicate lower computed c (i.e. reduced misclosure).

	# loops	No HC	HOC	NC	NOC_R
All loop types	1366	5.210	0.000	-0.001	0.005
First-order	56	2.517	-0.106	-0.110	-0.052
Second-order	20	2.879	-0.088	-0.085	-0.040
Third-order	975	4.213	0.004	0.004	0.004
Fourth-order	37	6.275	0.042	0.031	0.031
Third-order one-way	256	9.185	0.002	0.000	0.000
Two-way	8	10.318	0.311	0.267	0.267
Third-order one-way/fourth-order	14	10.274	-0.075	-0.077	-0.077

We first assess the entire ANLN (Table 1), including separate treatment of the different levelling types (cf. Filmer and Featherstone 2009), and then compare the closures for 18 first-order loops in the Australian Alpine region ($\sim 37°$S, $\sim 147°$E). Heights in this region are often over 1000 m, rising to 2228 m at Australia's highest point Mt Kosciuzko (36.5°S, 147.25°E).

The differences between computed c when no height correction is applied and when the HOC, NC and NOC_R are applied are shown in Table 1. The computed c for all 1366 ANLN loops shows no real reduction for the HOC and NC compared to applying no height correction, while the NOC_R makes the closures slightly larger. However, the first-order levelling was ~ 0.1 mm per $\sqrt{d}$ km less (cf. Allister and Featherstone 2001) and the second-order levelling almost 0.09 mm per $\sqrt{d}$ km less for both the HOC

Table 2 Computed c (in mm / $\sqrt{d}$ km) for 18 first order ANLN loop closures in the Australian Alpine region with different height corrections applied. The column No HC contains computed c when no height correction is applied.

	No HC	HOC	NC	NOC_R
Min	0.367	0.193	0.180	0.146
Max	14.464	11.630	11.629	13.865
Mean	3.540	3.068	3.072	3.319

(cf. Ramsayer 1959; Hwang and Hsiao 2003) and NC loops. The NOC_R reduces loop misclosure, but only about half that of the gravity-based corrections (cf. Rapp 1961, Table 15 p. 88; Kao et al. 2000).

The third-order (and third-order one-way) loop misclosures were only slightly reduced compared to no correction, with fourth-order indicating that the corrections have made the loop closures larger. However, these are lower quality levelling observations, so do not provide a sound test of small height corrections. ICSM (2007) does not recommend applying height corrections to levelling with precision lower than class LC (equivalent to third-order shown here) as the random levelling noise is larger than the magnitude of the corrections. As such, the results from the fourth-order, two-way and third-order one-way/fourth-order shall not be considered further.

A comparison of maximum, minimum and mean computed c for the 18 first order loops in the Alpine region is shown in Table 2. Note that use of standard deviation is not appropriate here as computed c is not normally distributed (Morgan 1992; Filmer and Featherstone 2009). The minimum computed c for these 18 loops is the NOC_R (0.146 mm$\sqrt{d}$), but the HOC and NC (0.170 mm$\sqrt{d}$ and 0.169 mm$\sqrt{d}$, respectively) are only slightly larger.

The lowest maximum computed c is the HOC and NC (11.616 mm$\sqrt{d}$ and 11.646 mm$\sqrt{d}$ respectively), with the NOC (13.865 mm$\sqrt{d}$) and no correction (14.464 mm$\sqrt{d}$) being slightly larger. The mean computed c also shows the different corrections in this order. This demonstrates a small decrease after applying height corrections.

4.2 Differences among height systems across Australia

To enable height comparisons, minimally constrained least-squares adjustments of the ANLN were conducted. Four identical adjustments were run, the only variation being the different height correction applied: no correction, HOC, NC and NOC_R. The Survey Network Adjustment Program (SNAP) developed at Land Information New Zealand (LINZ; http://www.linz.govt.nz/geodetic/software-downloads) was used for this task, with GA (Steed 2006 pers. comm.) *a priori* error estimates adopted for these adjustments (cf. Filmer and Featherstone 2009). All adjustments were fixed to MSL (held at zero) at the Albany tide-gauge (Western Australia; 35° 02'S, 117° 53'E). Thus, as the levelling errors propagate similarly in each adjustment, any variations in adjusted heights is the result of the different height correction applied.

4.2.1 Effect of not applying height corrections

Figure 5 demonstrates the effect of applying no height correction compared to applying the NOC_R to the ANLN (a difference of about 0.5 m over the continent). The convergence of the equipotential surfaces towards the pole dominates the differences (cf. Rapp 1961). Despite the lack of clear improvement from applying the NOC_R, NC or HOC to the entire ANLN in the loop closures (Table 1), the requirement for the

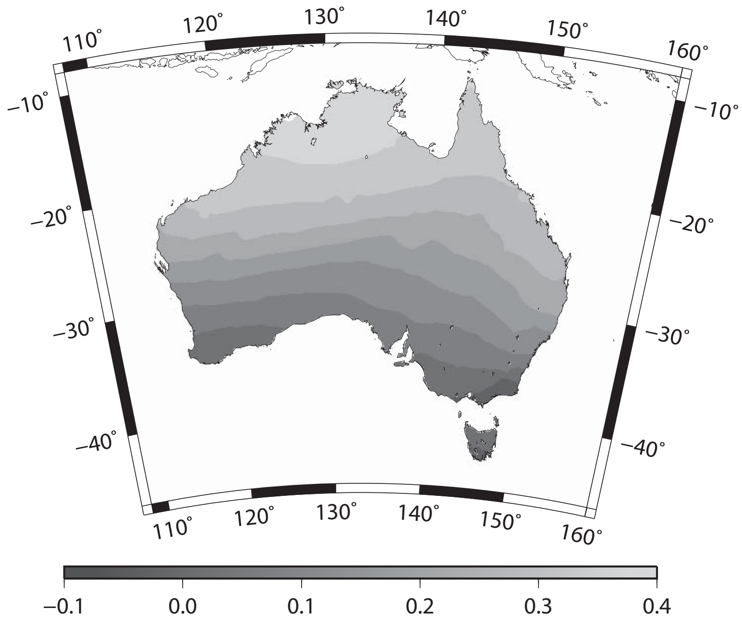

Fig. 5 Minimally constrained adjustment of the ANLN with NOC_R applied minus no height correction applied. Comparisons are made at 4247 ANLN supplementary and basic JPs. Units in metres.

height correction in the north-south direction remains, as it is a systematic error that accumulates.

4.2.2 Differences between Helmert orthometric and normal heights

Figure 6 shows the magnitude of differences between Helmert H^O and H^N over Australia resulting from minimally constrained least-squares adjustments of the ANLN with Helmert orthometric (Sect. 2.1; Eqs. 2 and 3) or normal corrections applied (Sect. 2.2; Eqs. 5 and 6) using EGM2008 're-constructed' gravity (Sect. 3.1; Eq. 38). The most notable features are the differences in central Australia around the MacDonnell Ranges (up to about 10 cm), where heights range up to ~ 1000 m and contain large changes in

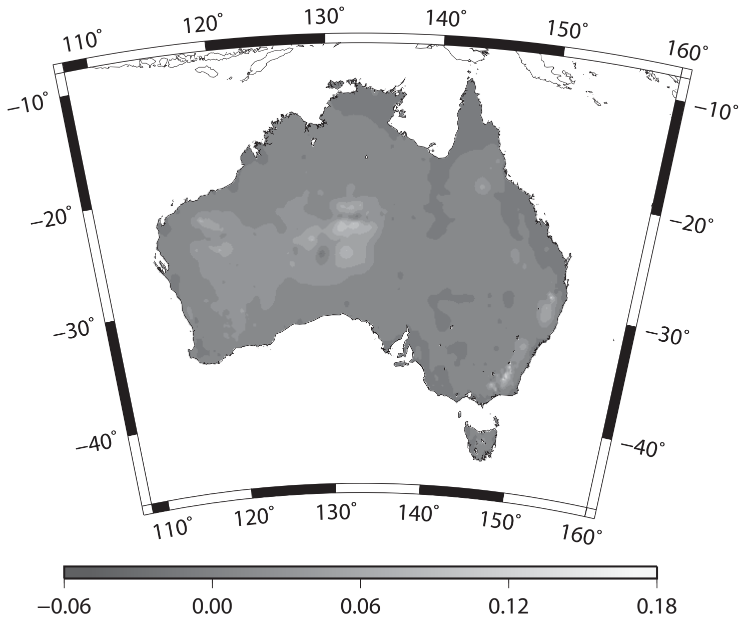

Fig. 6 Difference over Australia between Helmert H^O minus H^N using EGM2008 predicted gravity. Comparisons are made at 4247 ANLN supplementary and basic JPs. Units in metres.

gravity, and the differences along the Great Dividing Range where the height reaches 2228 m at Mt Kosciuszko.

Table 3 contains the statistics of the differences between Helmert H^O and H^N over Australia and over the Alpine region in the south east of the country (also see Fig. 7). The differences in the east range up to 10 cm in places along the Great Dividing Range (Fig. 6), with a sharp spike in the Southern Alpine region (Fig. 7) of up to $\sim$ 26 cm. This differs from the $\sim$ 15 cm difference Featherstone and Kirby (1998) found in the MacDonnell Ranges, but they used an approximate relation. Marti and Schlatter (2002) computed H^O - H^N differences of $\sim$ 48 cm at about 2500 m in Switzerland, while Flury and Rummel (2009) found differences of $\sim$ 24 cm and $\sim$ 48 cm at the Zugspitze (Germany; 2941 m) and Grossglockner (Austria; 3798 m) Alpine regions, respectively.

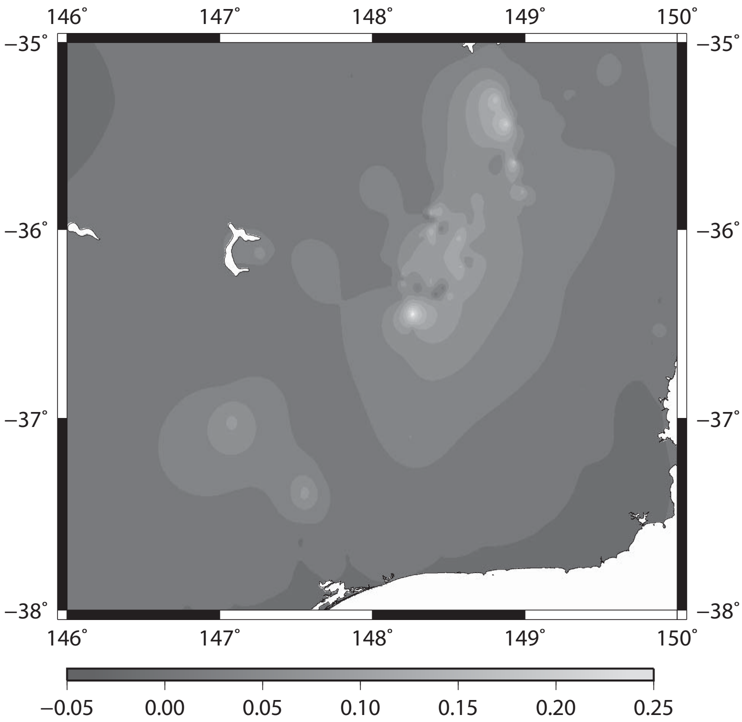

Fig. 7 Difference in the Australian Alpine region between Helmert H^O minus H^N using EGM2008 predicted gravity. Comparisons are made at 241 ANLN supplementary and basic JPs. Units in metres.

Table 3 Statistics showing differences between normal, Helmert orthometric and normal-orthometric heights over Australia and the mountainous Alpine region. Column (a) is Helmert H^O minus H^N; (b) is Helmert H^O minus $H^{N\text{-}O}$ and (c) is H^N minus $H^{N\text{-}O}$. Comparisons are made for 4247 (Australia) and 241 (Alpine region) ANLN supplementary and basic JPs. Units in cm.

	Australia			Alpine Region		
Stat.	(a)	(b)	(c)	(a)	(b)	(c)
Min	-2.28	-2.66	-2.38	-0.90	-2.45	-2.38
Max	26.27	43.97	17.70	26.27	43.97	17.70
Mean	0.74	0.35	-0.40	3.43	5.34	1.92
STD	1.67	2.63	1.22	4.34	6.95	2.86

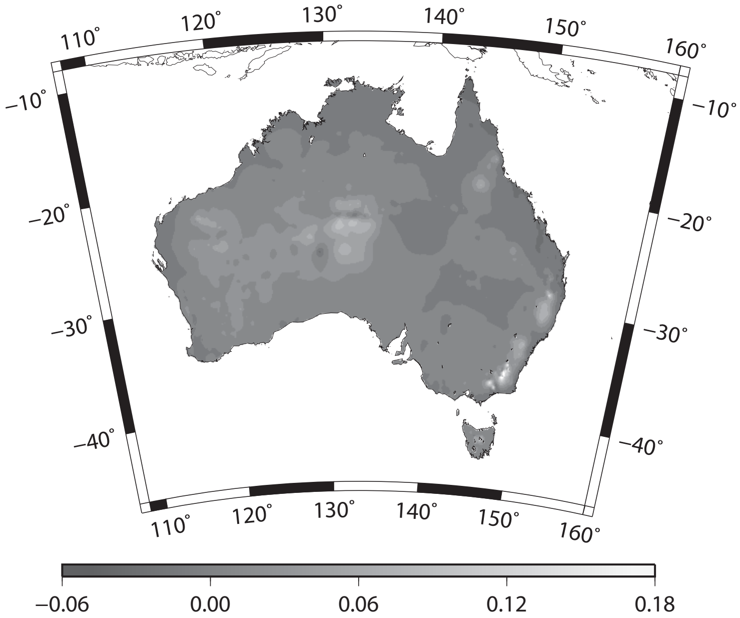

Fig. 8 Difference over Australia between Helmert H^O using EGM2008 gravity minus $H^{N\text{-}O}$. Comparisons are made at 4247 ANLN supplementary and basic JPs. Units in metres.

4.2.3 Differences with normal-orthometric heights

The differences between Helmert H^O and $H^{N\text{-}O}$ (Sect. 2.3; Eq. 9) and between H^N and $H^{N\text{-}O}$ over all Australia and in the Alpine subset are presented in Table 3. The maximum for Helmert H^O minus $H^{N\text{-}O}$ over all Australia is almost 44 cm; in the Alpine subset, the maximum is the same, but with the STD much higher at nearly 7 cm. This indicates (not unexpectedly) much larger differences between the Helmert H^O and $H^{N\text{-}O}$ in mountainous regions (cf. Rapp 1961, Kao et al. 2000). For the H^N minus $H^{N\text{-}O}$ difference, the maximum is almost 18 cm.

Given that H^N and $H^{N\text{-}O}$ have previously been considered the same for practical purposes (see Sect. 2.5), a smaller difference than between Helmert H^O and $H^{N\text{-}O}$ is not unexpected. However, it confirms that the two height systems (H^N and $H^{N\text{-}O}$)

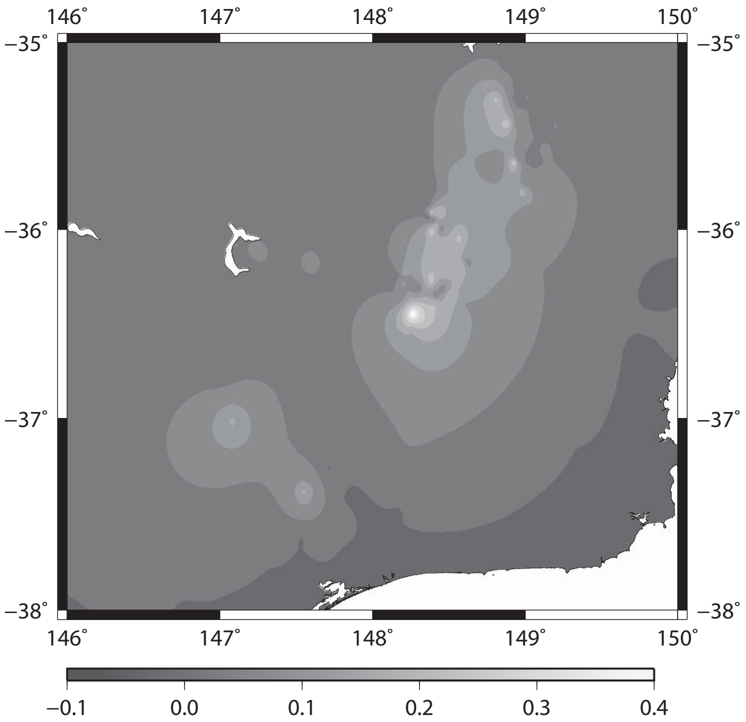

Fig. 9 Difference in the Australian Alpine region between Helmert H^O using EGM2008 gravity minus $H^{N\text{-}O}$. Comparisons are made at 241 ANLN supplementary and basic JPs. Units in metres.

are not consistent (cf. Eq. 27) and in mountainous regions > 1000 m can differ by > 5 cm. This empirical evidence backs up the derivation and discussion in Sect. 2.5.

Figure 8 shows Helmert H^O minus $H^{N\text{-}O}$ differences over Australia, with Fig. 9 showing the Helmert H^O minus $H^{N\text{-}O}$ differences in the Australian Alpine region. The differences approach 10 cm in Central Australia (cf. ~ 15 cm for Featherstone and Kirby 1998), with the maximum differences (44 cm) in the Alpine region, but which were not observed by Featherstone and Kirby (1998). The H^N minus $H^{N\text{-}O}$ differences are much less over most of Australia (Fig. 10), generally no more than 2-3 cm in most places, but increase to about 5 cm or more in higher-elevated regions in Tasmania,

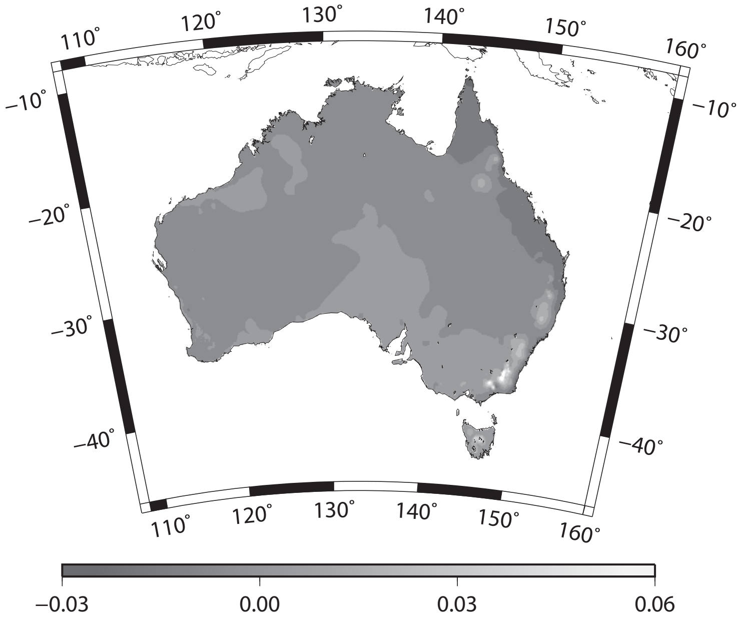

Fig. 10 Difference over Australia between H^N using EGM2008 gravity minus $H^{N\text{-}O}$. Comparisons are made at 4247 ANLN supplementary and basic JPs. Units in metres.

and also along the Great Dividing Range (Fig. 11) . The maximum differences in the Alpine region spike in the Mt Kosciuszko area. The surrounding mountainous areas show differences less than 25 cm for Helmert H^O minus $H^{N\text{-}O}$ (Fig. 9) and less than 10 cm for H^N minus $H^{N\text{-}O}$ (Fig. 11).

5 Conclusions

It has been demonstrated that gravimetric height systems can be applied to the ANLN using gravity predicted from EGM2008 (Sect. 3). Despite differences of up to 50 mGals from 2007GAgrav in the mountainous Australian Alpine region, it is generally a sufficiently accurate representation of the gravity field to compute Helmert orthometric and normal corrections. Omission errors in EGM2008 and the positional uncertainty of the

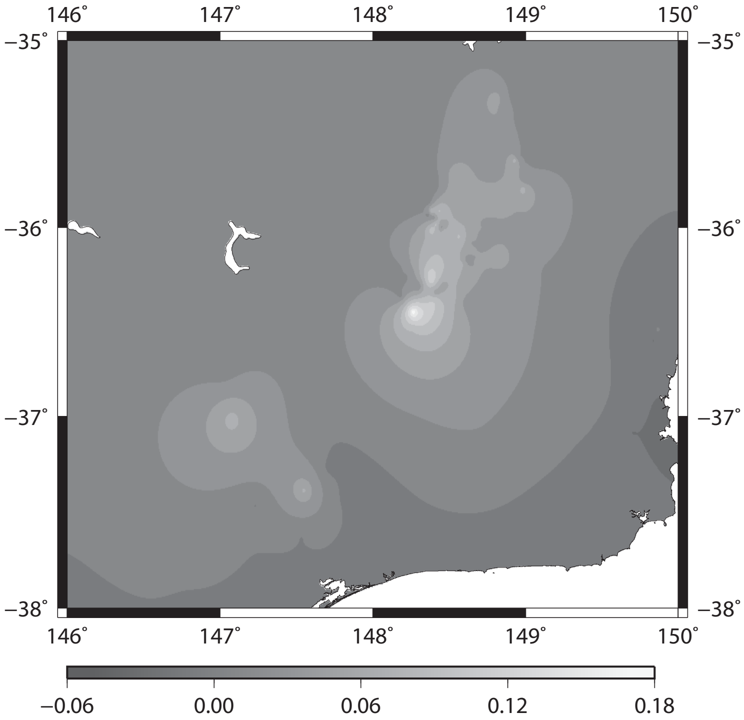

Fig. 11 Difference in the Australian Alpine region between H^N using EGM2008 gravity minus $H^{N\text{-}O}$. Comparisons are made at 241 ANLN supplementary and basic JPs. Units in metres.

ANLN benchmarks contribute to the differences between EGM2008 and 2007GAgrav in the Alpine regions, although this requires further investigation.

The differences between normal and normal-orthometric heights is only a couple of centimetres over much of Australia, but in the mountainous southeast can reach nearly 18 cm. This indicates that AHD normal-orthometric heights are not fully compatible with height anomalies from global or regional gravimetric quasigeoid models, but which has been implied previously (e.g., Featherstone and Kirby 1998; Featherstone and Kuhn 2006). Differences between Helmert orthometric and normal-orthometric heights reach 44 cm for heights over 2000 m in the Australian Alpine region. Differences of > 10

cm appear in several locations around Australia. The differences between Helmert orthometric and normal heights are similar, but are < 30 cm in the Alpine region.

In view of the sensitivity of Helmert orthometric heights (effect of using the actual gravity gradient) to gravity errors, it is recommended that until the Australian gravity data are improved (e.g., new terrestrial gravity observations and accurate benchmark positions), a normal height system should be implemented in any new Australian vertical datum. Further research is required on rigorous orthometric heights with a view to implementation in the future when the gravity data is improved. While orthometric heights with the geoid as the datum is a preferred option, current difficulties with accurately computing this system make normal heights a more realistic option for any new Australian vertical datum.

Finally, a normal-orthometric height system does not relate to the quasigeoid, differing from normal heights by over 10 centimetres in mountains above 1000 m. Furthermore, the normal-orthometric zero reference surface is difficult to define and will not be compatible with quasigeoid models.

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Figure 5 colour

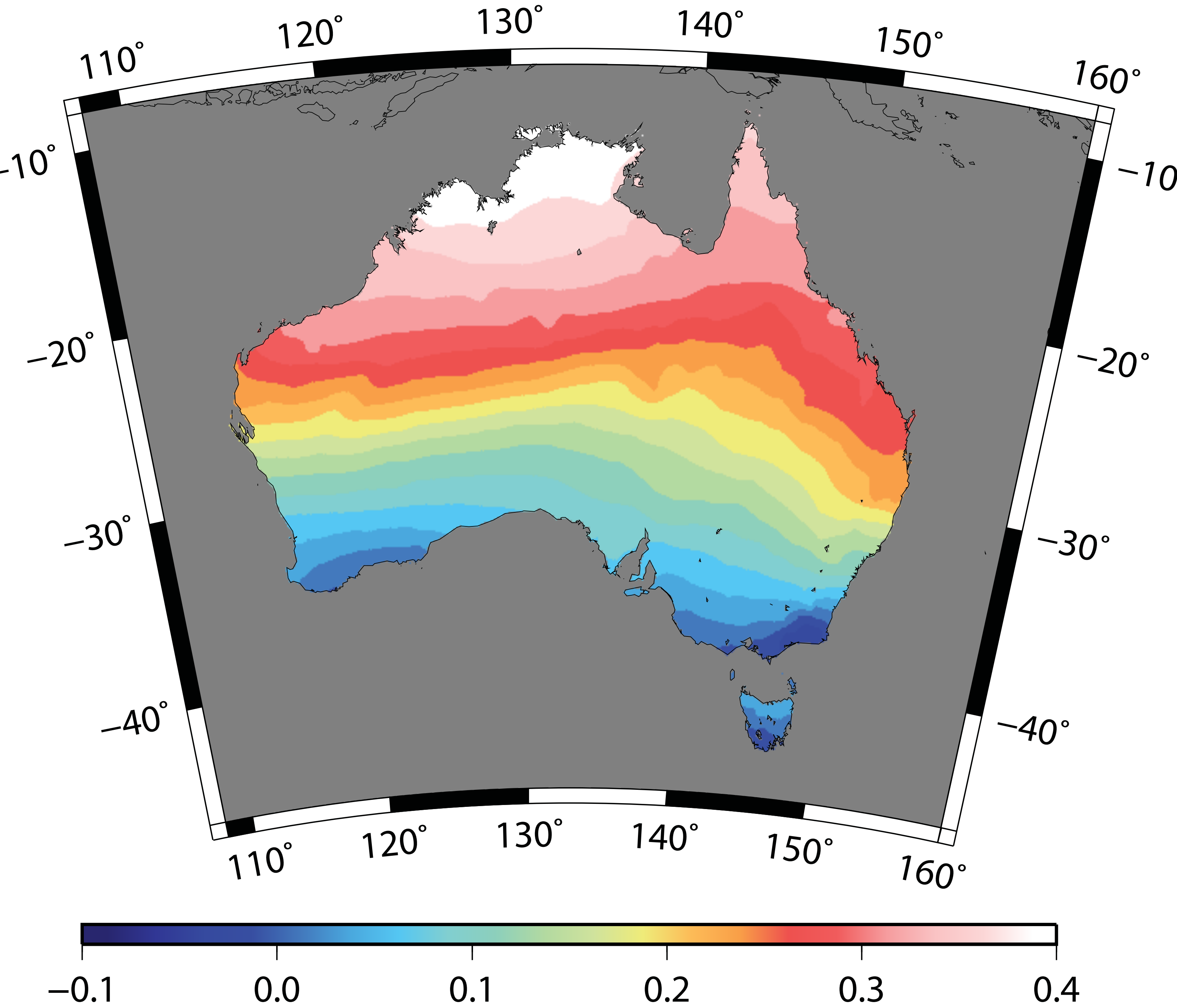
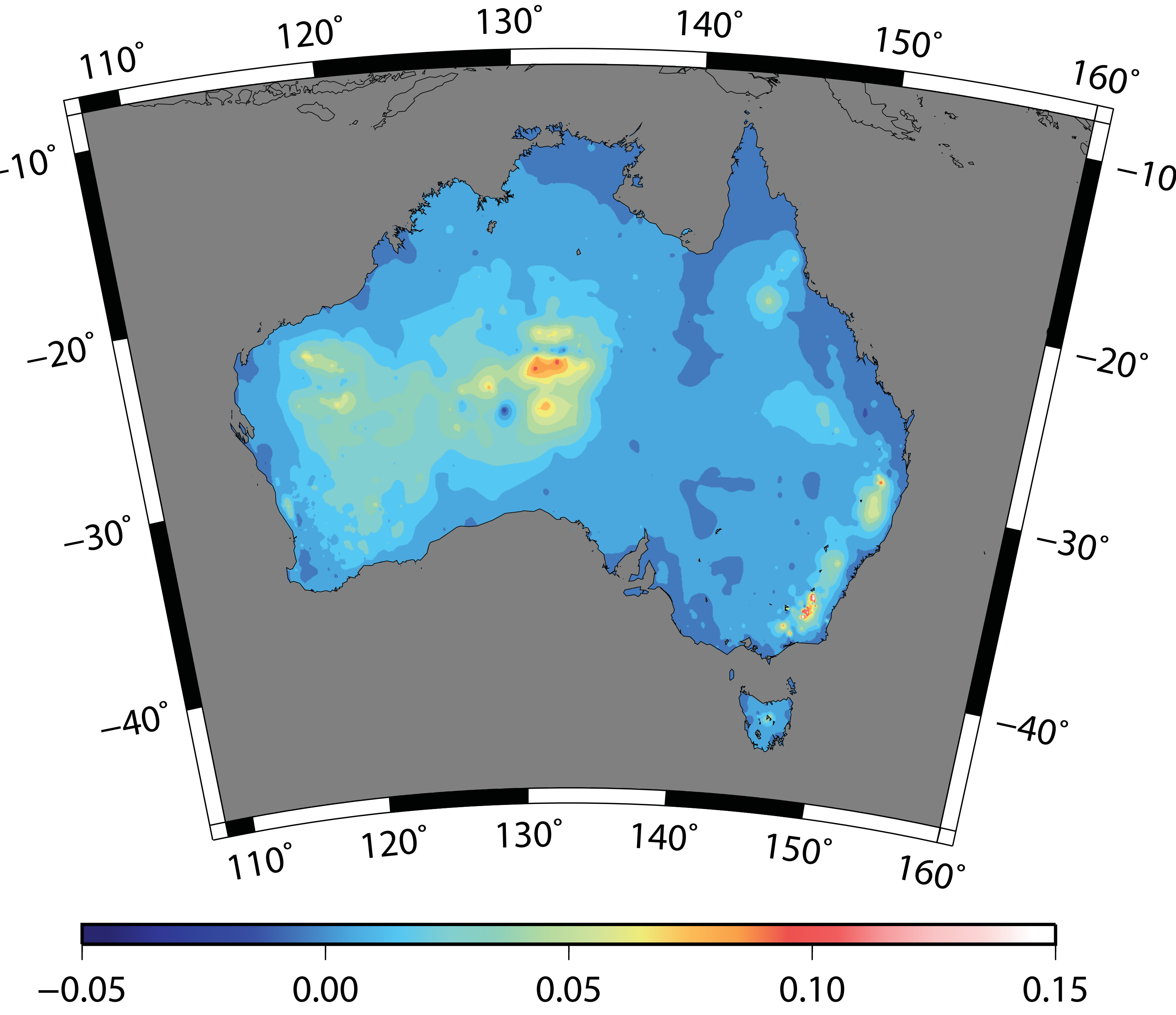


Figure 6 colour



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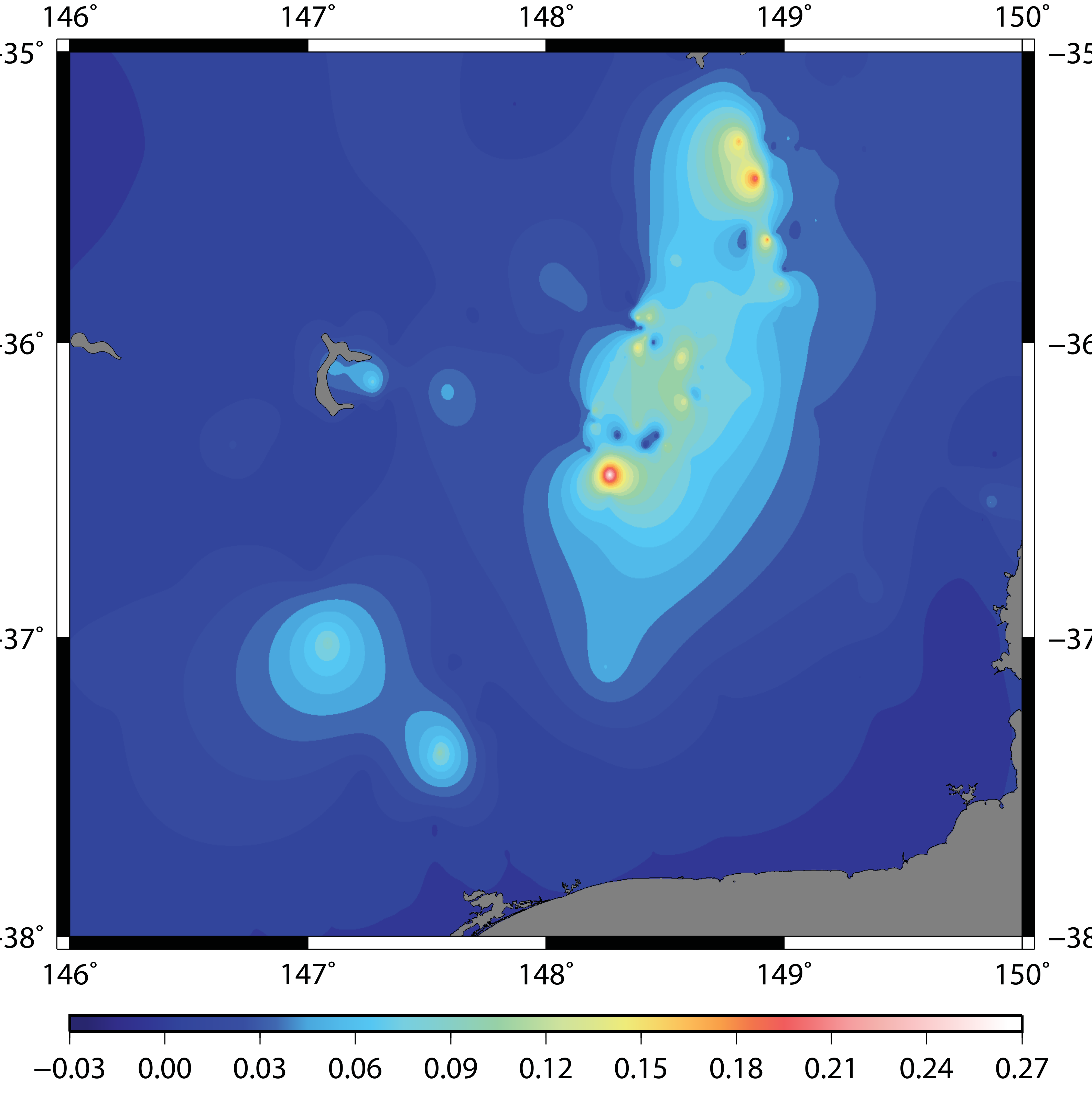

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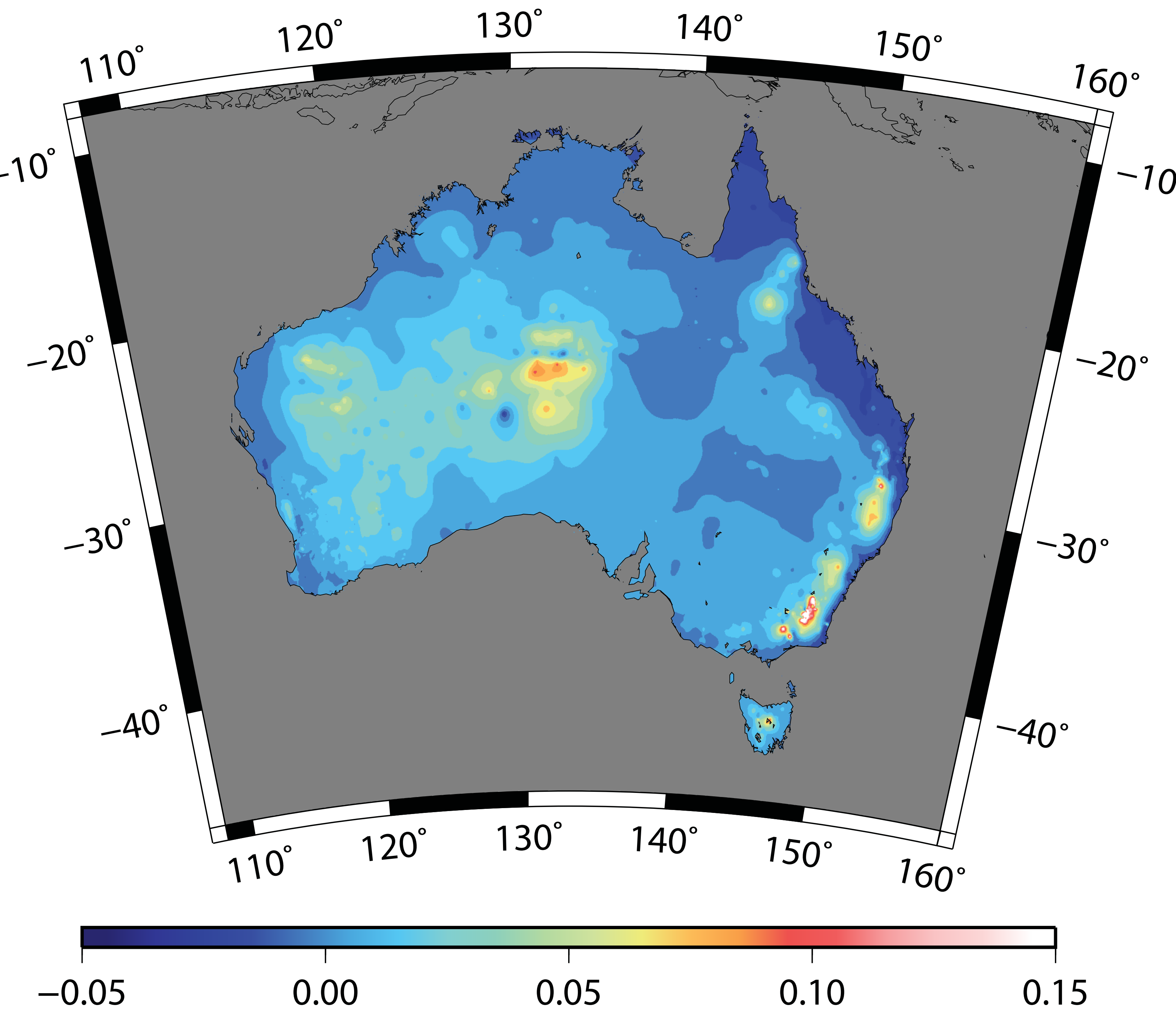

Figure 9 colour

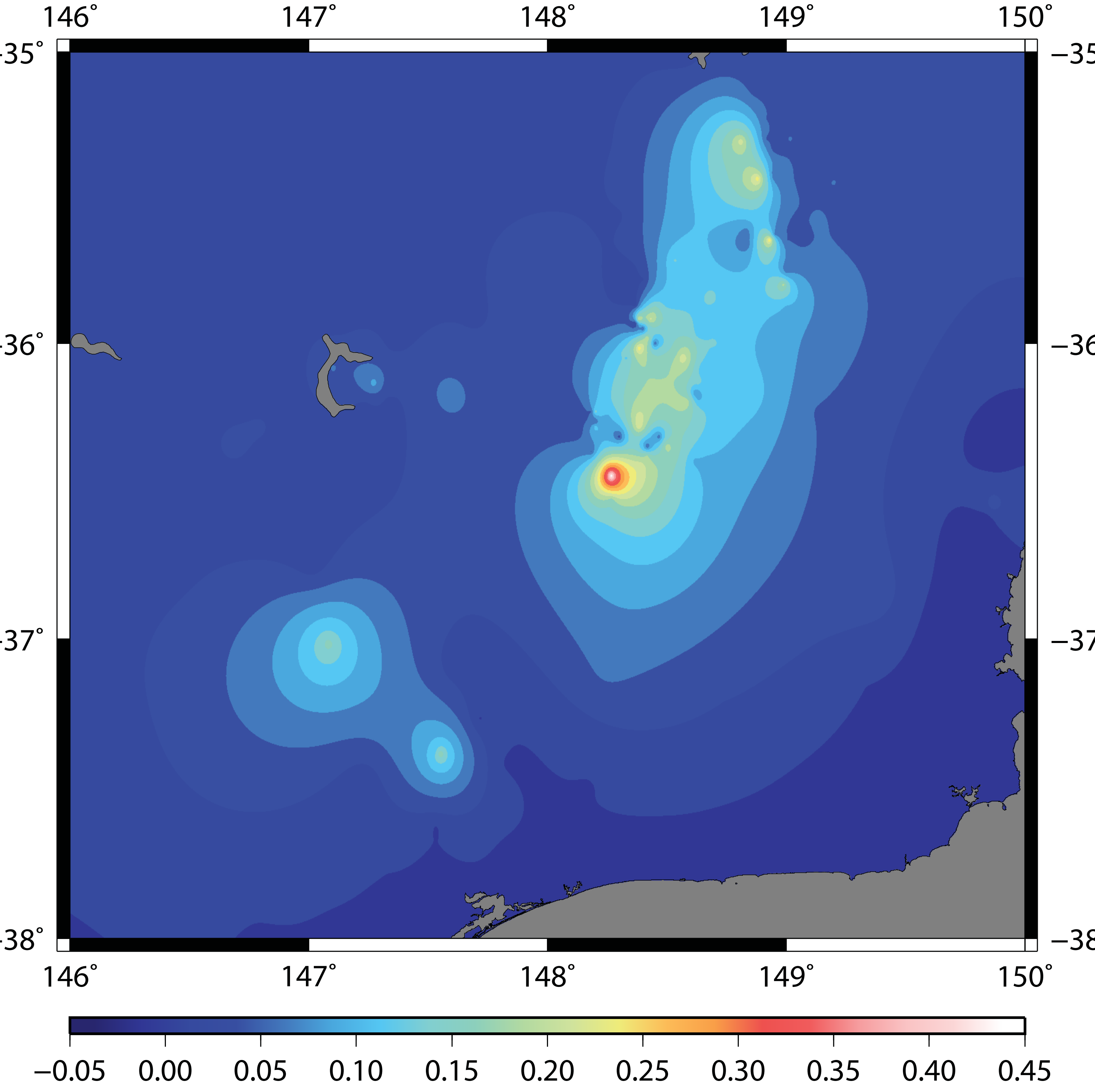

Figure 10 colour

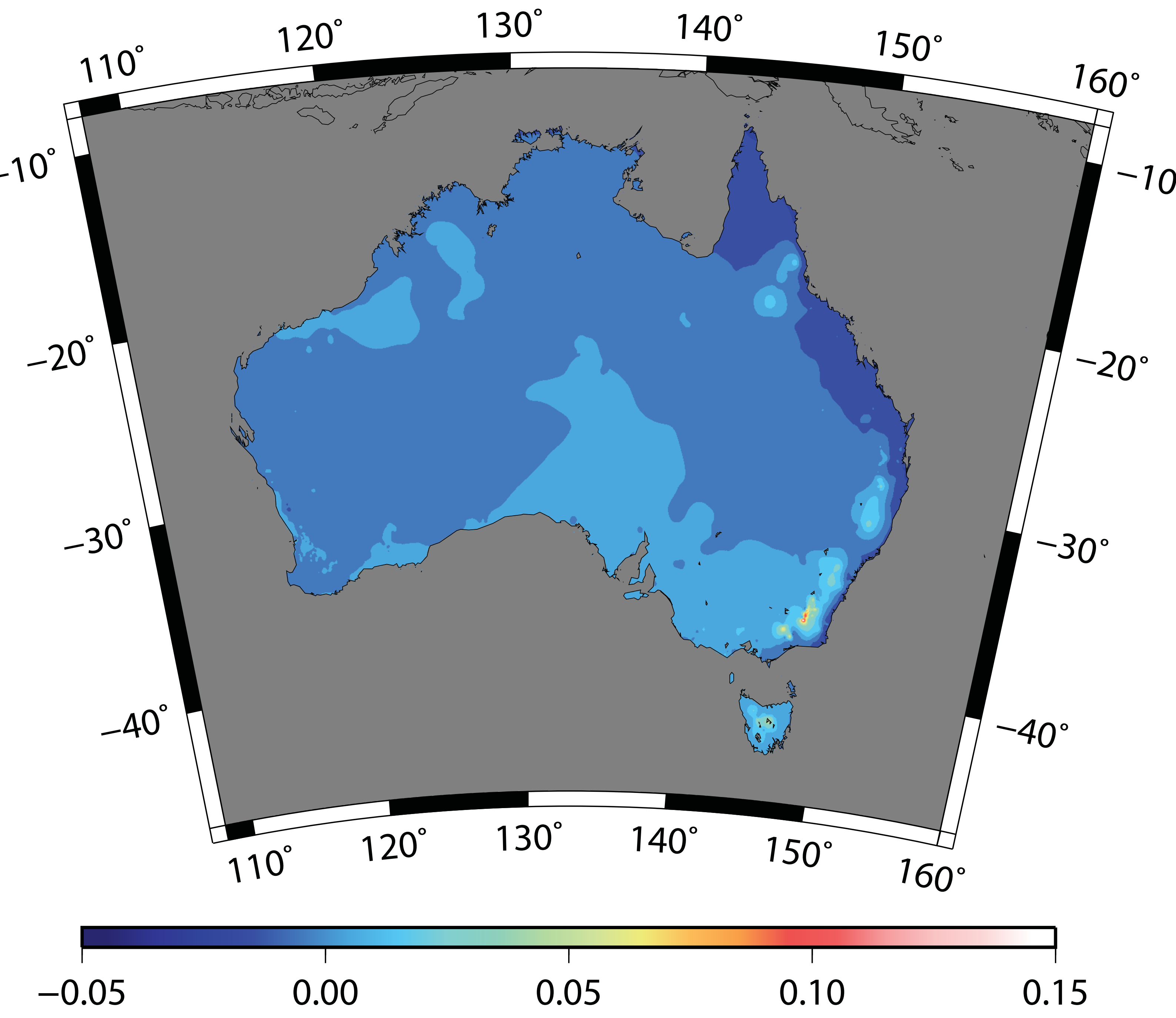

Figure 11 colour