Assessment of EGM2008 over Sri Lanka, an area where 'fill-in' data were used in EGM2008

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Abstract. The tide-free EGM2008 combined global geopotential model is compared with land and marine gravity observations and co-located GPS-levelling on and around Sri Lanka (formerly Ceylon). Not all these data are in the public domain, so offer an informative test of how the 'fill-in' methodology used in EGM2008 performs versus observed data. Sri Lanka is also in an area where the geoid exhibits its lowest elevation with respect to a geocentric reference ellipsoid. A -1.75 m bias between the GPS-levelling and EGM2008 led to an investigation into the Sri Lankan geodetic datums, showing a bias in the ellipsoidal heights. After rejection of 15 outliers, the standard deviation of the difference between 207 Sri Lankan GPS-levelling points and EGM2008 is ± 0.184 m. The difference between the gravity anomalies and EGM2008 showed that the Sri Lankan gravity data is based on the old Potsdam datum. The Sri Lankan land gravity data, after rejection of outliers, yielded standard deviations of ±6.743 mGal for 20 GPS-coordinated gravity points on fundamental benchmarks, ±14.704 mGal for 42 gravity points on fundamental benchmarks but with coarse locations, and ±6.367 mGal for 1032 digitised and reconstructed free-air anomalies from a Bouguer anomaly map. The ship-track gravity data have not been crossover adjusted, and yield a standard deviation of ±43.683 mGal. Importantly, the ability of EGM2008 to identify datum deficiencies is an implicit validation and leads to its application in other areas to search for datum deficiencies.

1. INTRODUCTION

From Pavlis et al. (2008), Sri Lanka (called Ceylon before 1972) is one of the regions where 'fill-in' 5-arc-minute mean terrestrial gravity anomalies were used to generate EGM2008. To our understanding, the fill-in procedure takes Bouguer gravity anomalies from commercially sensitive or confidential data sources, then 'reconstructs' free-air anomalies using the elevations from the DEM2006.0 digital elevation model (Pavlis et al. 2006), which is derived principally from the Shuttle Radar Topography Mission (SRTM; Werner 2001). This reconstruction procedure is described in Lemoine et al. (1998); also see Featherstone and Kirby (2000).

In this assessment of EGM2008, we use previously unavailable GPS-levelling data across Sri Lanka, gravity observations at fundamental benchmarks (FBMs) of the Sri Lankan geodetic levelling network (Udayakantha and Tennakone 1993), shiptrack gravity anomalies offshore (NGDC 1999), and gravity anomalies digitised from a Bouguer gravity anomaly map of Sri Lanka (Hatherton and Ranasinghe 1972). The GPS-levelling comparisons show a -1.75 m bias (standard deviation of ± 0.18 m), which is attributed mostly to a problem with the origin of the ellipsoidal heights. The gravity comparisons show that the Sri Lankan data are referred to the old Potsdam gravity datum, but the gravity anomalies used in EGM2008 appear to use the International Gravity Standardisation Network 1971 (IGSN71).

2. DESCRIPTION OF THE SRI LANKAN DATA

2.1 Land and ship-track gravity data

2.1.1 International gravity links to Sri Lanka

The first recorded gravity observations in Sri Lanka were made by Glennie (1935), of the Survey of India, which involved a set of 21 pendulum gravity observations. These observations resulted in two map compilations showing Hayford anomalies and 'crustal warp' anomalies (Hatherton and Ranasinghe 1972). A definition of crustal warp anomalies could not be found, but it is likely they are isostatic gravity anomalies. However, the original data are not now available for further analysis.

There were several later surveys that made gravity observations to connect Sri Lanka to international gravity networks. Relative observations were made at two airports: Ratmalana (Colombo) and Katunayake. Woollard and Rose (1963) made observations at Ratmalana airport, giving a value of 978132.3 mGal. Gravity observations at both airports were made again in 1969 when the British Institute of Geologi-

cal Sciences used a LaCoste & Romberg (L&R) gravimeter (serial number G97) to give gravity differences among New Delhi, Yangon (Rangoon), Singapore and Colombo.

The gravity difference between Singapore University (Geography Department) and Katunayake airport was 40.78 mGal and between Singapore and Ratmalana airport was 50.14 mGal. The absolute values were assigned 978122.24 mGal at Katunayake and 978131.6 mGal at Ratmalana based on the Singapore value of 978081.5 mGal (Hatherton at al. 1975), which was on the Potsdam gravity datum (J Mäkinen 2008, pers. comm.). These two Potsdam-related gravity values were referred as Evans's values in subsequent Sri Lankan gravity surveys (e.g., Hatherton and Ranasinghe 1972; Udayakantha and Tennakone 1993); described in Section 2.1.2. As such, Sri Lankan gravity data are offset from IGSN71 (Morelli et al. 1974) by around 14 mGal, which will be shown later in Sections 3.2.3 and 3.2.4.

Another control gravity survey was carried out in 1973 by Evans (Hatherton at al. 1975, Appendix) using the same G97 L&R meter at the base stations occupied by Hatherton and Ranasinghe (1972) (Section 2.1.2) and connected to Evans's 1969 value (978131.6 mGal) at Ratmalana airport. The IGSN71 value at Ratmalana was later calculated by using the IGSN71-derived calibration factor for the L&R G97 difference measured between the same Singapore and Ratmalana points in 1969. The IGSN71 value at Ratmalana is 978116.81 mGal based on the IGSN71 value (978066.68 mGal) at Singapore University (Hatherton et al. 1975).

Ratmalana airport has also been tied to IGSN71 by NAVOCEANO (formerly the U.S. Naval Oceanographic Office), giving the value as 978116.900 mGal. While this point was also located at Ratmalana, it is not at the same ground mark observed by Evans. As such, the datum of the Sri Lankan gravity anomalies described next is based on the old Potsdam gravity datum, even though IGSN71 values are available. This will lead to an expected bias of around 14 mGal between Sri Lankan gravity anomalies and any gravity anomalies referred to IGSN71 (Section 3.2).

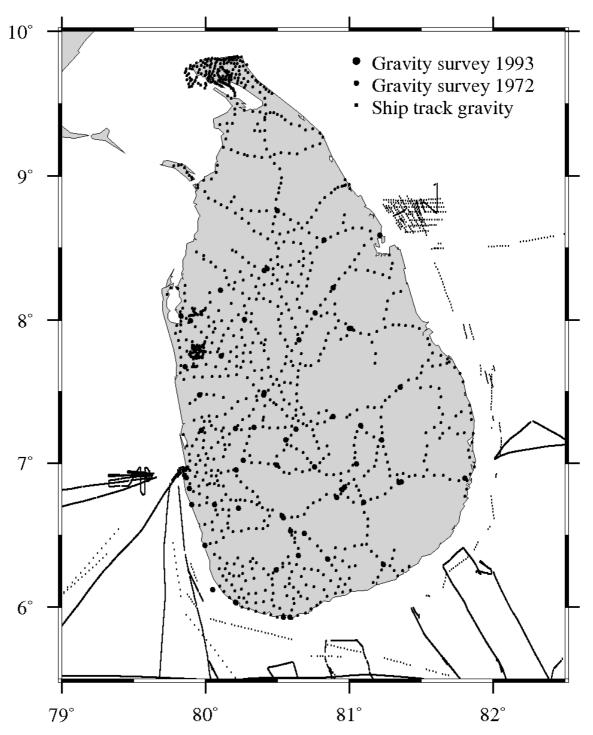


Fig 1 Locations of the Sri Lankan gravity data (Mercator projection): the digitised 1972 land gravity survey of Hatherton and Ranasinghe (1972) (1,070 points), the 1993 land gravity survey of Udayakantha and Tennakone (1993) (52 points), and ship-track gravity from NGDC (1999) (12,192 points)

2.1.2 National terrestrial gravity surveys

a) 1971 survey of 1170 points

A significant gravity survey was conducted over the whole of Sri Lanka in 1971, and led to two maps showing Bouguer anomalies (Hatherton and Ranasinghe 1972, Hatherton et al. 1975) and isostatic anomalies (Hatherton and Hutchings 1972). According to Hatherton and Ranasinghe (1972), the survey was to produce a gravity anomaly map of the whole country, together with detailed surveys of several areas to use the gravity method for studying the geology of the country.

The observations were made using a Worden gravimeter (serial number W283), which was a high-drift meter (~0.35 mGal/day). The survey was carried out by establishing 19 base stations relative to Evans's 1969 Potsdam-related gravity value (978131.6 mGal) at Ratmalana airport (Hatherton and Ranasinghe 1972). Unfortunately, the original data are no longer available. However, Hatherton and Ranasinghe (1972) provide a contour map of complete Bouguer anomalies with the locations of the gravity observations, which we have manually digitised (Section 3.2.2).

A total of 1170 gravity observations were performed during this 1971 survey relative to these 19 base stations, with 87% of observations in areas where the altitude is less than 500 ft (~150m). Very few observations were made at higher altitudes in the central southern mountainous region (Fig. 2) due to instrumental limitations (the travel time in mountains versus the need to regularly occupy a base station to correct for the gravimeter's drift, and avoiding calibrating the extended range dial in the gravimeter). Also, this 1971 survey did not establish base stations on permanent monuments, and the method of adjustment of the network was not made clear. The 19 base stations were reoccupied and remeasured in 1973 with the L&R G97 meter, and the differences were found to agree within 0.25 mGal, except for two stations where the difference was in excess of 1 mGal.

The horizontal positions and the heights of these 1971 gravity observations were taken from the Sri Lankan one-inch topographic map series (1:63,360). The observations were made approximately every four miles (~6.5km) along roads (Hatherton and Ranasinghe 1972, Hatherton and Hutchings 1972); see Fig. 1.

The complete Bouguer anomalies in Hatherton and Ranasinghe's (1972) contour map were based on the International Gravity Formula 1930 with a standard rock density of 2670 kg/m³. Terrain corrections were computed out to Hammer's (1939) zone M, and were found to be small, except in the highlands. The largest terrain cor-

rection was 11.9 mGal at the southern scarp of the highlands (cf. Fig. 2). The complete Bouguer gravity anomalies were in the range of -75 mGal to +45 mGal (Hatherton and Ranasinghe 1972).

Since the accuracy of the locations and heights are rather coarse and the datums implicit to the computation of gravity anomalies are somewhat ambiguous, this survey might not reflect the actual gravity field over Sri Lanka and thus not be such a strong validation of EGM2008. Digitising these data from a contour map also adds further uncertainty to the veracity of this dataset to validate EGM2008. The digitisation process is described in Section 3.2.2.

b) 1993 survey of 52 points

Udayakantha and Tennakone (1993) established a new systematic gravity network by making observations with a newer L&R gravimeter (serial number D186) at 52 permanent monuments, which included 49 points on FBMs of the Sri Lankan geodetic levelling network (Price 1932). [The geodetic datums in Sri Lanka will be described in Section 2.2]. This 1993 gravity survey was also based on Evans's 1969 Potsdam-related gravity value (978131.6 mGal) at Ratmalana airport, and the network was least-squares adjusted (Udayakantha and Tennakone 1993).

Nine identifiable base stations from Hatherton and Ranasinghe (1972) were also remeasured and compared. The surveys agreed to within 0.32 mGal, except one station with a difference of ~1 mGal. However, these nine stations were not adjusted in the 1993 survey because they were not on permanent monuments (Udayakantha and Tennakone 1993). As well as the adjusted 52 points, an additional 11 unadjusted points were available from this gravity survey, but are not used in this assessment of EGM2008 because they are unadjusted.

The horizontal positions of the FBMs are not very accurately known as they were compiled from topographic maps available at that time, so are probably accurate to around 0.5-1.0 km. However, 20 of the FBMs have now been incorporated in the national GPS-based geodetic network (Section 2.2.1) and hence their locations are known to a few centimetres (Geodetic Survey Unit 2000), but there remains some uncertainty around their ellipsoidal heights, which will be discussed in Section 2.2.1.

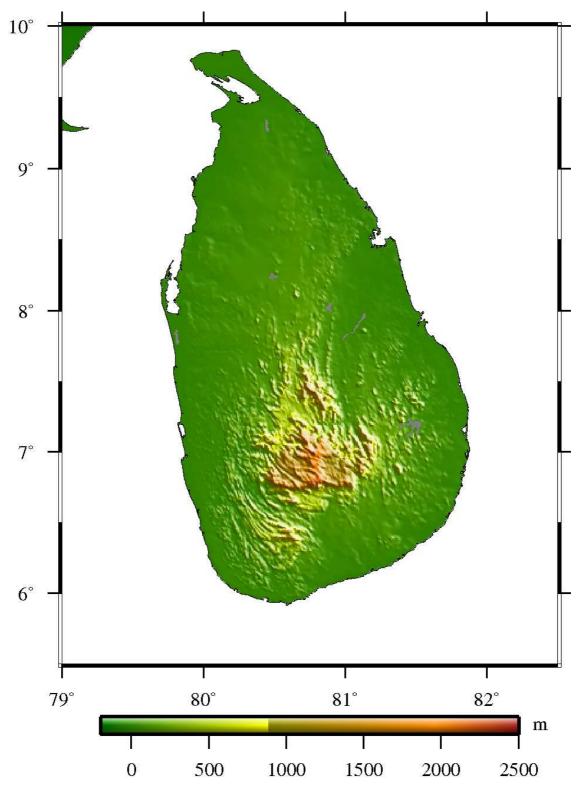


Fig 2 Sri Lankan topography from a 1 km resolution DEM based on digitised contours from the Survey Department of Sri Lanka (Mercator projection). The maximum elevation is 2,524 m in the central southern mountains.

2.1.3 Ship-track gravity

As well as the land gravity data, some ship track gravity data are also available around Sri Lanka (NGDC 1999; Fig. 1). The NGDC database contains 12,192 sea surface gravity observations between 79.0°E to 83.0°E and 5.0°N to 10.0°N. According to the metadata, only 643 measurements (one cruise) were crossover adjusted for gravimeter drift, and no evidence could be found for tide corrections for the whole set of data. As such, these data will not be able to provide a useful validation of EGM2008 around Sri Lanka.

2.2 The geodetic datums of Sri Lanka

2.2.1 Horizontal datums

The principal triangulation network of Sri Lanka began in 1857 and was completed by 1885 for the Topographical Survey of Ceylon, which was on the scale of half mile to an inch (Geodetic Survey Unit 2000). The most significant revision to the network is documented in Jackson (1933). It involved re-measuring the distance and azimuth of each baseline at Negombo (Kandawala to Halgastota) and Batticaloa (Tavelamunai to Vaunativu) with invar tapes and a Gautier 5" micrometer theodolote.

The two baselines were about ~5.5 miles (~8.8 km) long and separated by 127 miles (~205km). The triangulated value of the Negombo base computed from the Batticaloa base was found to be 1:116,000 (Jackson 1933). Astronomical coordinates of Kandawala and the astronomical azimuth of the Negombo baseline were kept fixed to form the astronomical datum of Sri Lanka. This Sri Lankan horizontal datum is referred to as Kandawala in most of the literature (e.g., Jackson 1933; Geodetic Survey Unit 2000; NIMA 2000).

The Sri Lankan horizontal geodetic control network has continuously been upgraded by using newer technology and computational methodologies. A major breakthrough took place in 1993 for densification of the network and upgrading its accuracy by using GPS (Geodetic Survey Unit 2000). The new control network was used to form the Sri Lanka Datum 1999 (SLD99), and consists of one base station (ISMD, Institute of Surveying and Mapping Diyatalawa), 10 secondary base stations, 48 existing Kandawala triangulation stations, 20 FBMs (cf. Price 1932) and 194 new control stations. SLD is not a geocentric datum.

The GPS baselines were observed with Leica 300 receivers and processed with SKI v1.2. The network was least-squares adjusted with GPSENV v3.32 in the

Geolab v2.6a software under a minimal constraint by fixing the 3D coordinates of ISMD (see the discussion below). The average ppm precision of the 1,265 baselines in the network was 0.127229, and the computed distance accuracy was 1:7,900,000. The highest (A) and the lowest (2-I) rank of FGCC (United States' Federal Geodetic Control Committee) order (Bossler 1984) of the baselines were found to be 5 each, while 496 and 759 baselines were ranked as B and 1 respectively (Geodetic Survey Unit 2000).

The old triangulation stations were used to determine datum transformation parameters from WGS84 to Kandawala and from WGS84 to SLD99. The transformation parameters were determined based on the seven-parameter Bursa-Wolf model for each datum (Geodetic Survey Unit 2000). SLD99 was made available for use in Sri Lanka since 2000, and the national grid coordinates (SL_GRID_99) are computed using the transverse Mercator projection on the Everest 1830 (1937 Adjustment) ellipsoid, which was also used for the Kandawala datum (e.g., NIMA 2000). Both horizontal geodetic datums are still being used separately in Sri Lanka.

Prompted by a significant bias of -1.75 m in the GPS-levelling comparisons (Section 3.1.1), we scrutinised the GPS ellipsoidal height and spirit-levelling datums. We were satisfied that the MSL-based spirit-levelling datum (Section 2.2.2) could not account for this bias, so focussed on the GPS ellipsoidal height datum. According to Geodetic Survey Unit (2000), the starting point for the GPS surveys was the DORIS (Doppler Orbit Determination and Radiopositioning Integrated on Satellite) station at Colombo (COLA; DOMES ID 23501S001), which was active from 1991 to 2004.

Since the ground mark beneath the ~3-m-high DORIS beacon could not be occupied by GPS, Geodetic Survey Unit (2000) occupied another GPS base station nearby. The coordinates in Geodetic Survey Unit (2000) are not the same as the GPS marker (DOMES ID: 23501M001) listed at the IDS (International DORIS Service) website (http://ids.cls.fr/), though we have been advised that that same GPS marker was occupied. The coordinates of this GPS base station at the Survey General's Office (SGO), Colombo, are latitude: 6°N 53' 30.8699", longitude 79°E 52' 26.3102", ellipsoidal height: -76.238 m.

The site-log for COLA at the IDS website gives the tie vector between the DORIS and their GPS marker, which gives the ITRF2000 coordinate as latitude 6°N 53' 30.8611", longitude 79°E 52' 26.3146", ellipsoidal height -75.692 m. The DOMES 23501M001 GPS mark was local-tied to COLA during the Epoch'92 GPS

campaign by IGN (Institut Géographique National) using conventional geodetic techniques (total station), and no GPS observations were made on the DORIS ground mark (H Fagard 2008, pers. comm.). Apparently, the GPS points are physically the same, but different coordinates seem to have been inaccurately incorporated into the GPS survey. This introduces a bias in the ellipsoidal heights of 0.456 m.

The uncertainty of the origin of the GPS survey is compounded further by the fact that the origin point was chosen to be ISMD, which is approximately 120 km east of COLA. From Geodetic Survey Unit (2000), two GPS baselines (i.e., radiations) were used to establish this site from SGO over an ellipsoidal height difference of ~1.2 km. The differential tropospheric delays in this near-equatorial region are probably poorly modelled by commercial software in single baseline mode, resulting in a height error (e.g., Rothacher 2002). The coordinates of the ISMD origin are: latitude: 6°N 49' 02.68716", longitude: 80°E 57' 40.88000", ellipsoidal height: 1164.366 m.

Geodetic Survey Unit (2000) also occupied ISMD for seven days and determined a sequential code GPS position solution using Leica's SKI v2.1 software (note that this site was occupied before GPS selective availability was discontinued). This sequential code GPS position was 3.555 m higher than that from the ~120 km baseline from SGO, though the latitude and longitude were comparatively closer to each other. This indicates (neglecting the effect of selective availability) that the differential tropospheric bias over this ~1.2 km height difference is an additional problem that will cause the GPS ellipsoidal heights to be biased.

While the realisation of ellipsoidal heights in this way is not of concern to geodetic surveyors in Sri Lanka (because relative GPS surveys will be insensitive to any bias), it will cause a problem for an absolute GPS-levelling evaluation (cf. Featherstone 2001) of EGM2008. At this stage, and before further investigations can be completed, we believe the combination of the ambiguous connection to the COLA DORIS site and the long GPS baseline observed over a substantial height difference to ISMD could account for much of the -1.75 m mean difference in the GPS-levelling comparisons (Section 3.1). In addition, COLA was identified as one of 17 stations of having poor antenna stability at the end of 1999 by the IDS (Fagard 2006) and was not recommended for DORIS core network for ITRF2005 (Willis et al. 2005).

Importantly, and as a validation of EGM2008, this uncertainty in the ellipsoidal height datum would not have been investigated so thoroughly if we had not found a large bias with a small standard deviation between the GPS-levelling and

EGM2008. As such, it could be argued that EGM2008 is already contributing to vertical datum unification.

2.2.2 Vertical datum

The vertical geodetic control network of Sri Lanka (originally termed as the geodetic levelling of Ceylon) was established between 1926 and 1930 using parallel glass plate micrometer attachments mounted on precise levels with staves graduated to fiftieths of a foot on a strip of invar fixed at one end (Price 1932).

For the datum's origin, tidal observations of mean sea level (MSL) were carried out between 1923 and 1933 at two harbours, Colombo and Trincomalee, using self-recording tide gauges. Beforehand, MSL was determined at three harbours by the Great Trigonometrical Survey of India using self-recording tide gauges over the following periods: 1884-1889 at Colombo and Galle; and 1889-1896 at Tricomalee (Jackson 1936).

Initially, the levelling network was supposed to be fixed at the Colombo and Trincomalee tide gauges, with redetermination of the MSL by the newer observations. However, since the tidal observations were underway at Colombo and Trincomalee when the levelling network was adjusted in 1932, the MSL at Colombo as determined by the Great Trigonometric Survey of India during 1884-1889 was used instead. However, after the local tidal observations were completed at Colombo and Tricomalee during 1929-1933, a small rise of MSL of 0.074 ft (~0.023 m) at Colombo and 0.199 ft (~0.061 m) at Tricomalee relative to the 1884-1889 and 1889-1896 values was seen (Price 1932, Jackson 1936).

According to Price (1932), the main part of the Sri Lankan levelling network comprises 27 circuits of ~2,400 miles (~4,300 km) of levelling in total. Originally, there were 53 FBMs, though some have been destroyed or disturbed since then. The probable error of the levelling is ± 0.42 mm/ \sqrt{km} and the accuracy of determination is ± 0.0005 ft (~ ± 0.02 mm). However, these values are for spirit-levelling conducted over 70 years ago, so are probably too overoptimistic.

Normal-orthometric corrections (quoted incorrectly as orthometric corrections in Price 1932) were applied to observed height differences to account for the non-parallelism of level surfaces using the formula $-0.005302(\Delta\phi)H\sin2\phi_{M}$ ft (Bomford

1971, chapter 3), where H is the mean height of the section of levelling; ϕ_M is the mean latitude and $\Delta \phi$ is the difference in latitude of the terminal points of the section.

The network was adjusted using least squares, holding the MSL at Colombo fixed to zero. Dynamic heights were also calculated (e.g. Bomford, 1971, chapter 3) for the adjusted benchmarks by considering 7°N as the standard latitude along with normal gravity at the mean latitude of the terminals of the levelling section. The whole network was based on precise geodetic levelling observations, and no evidence was found that observed gravity data were used in the computations.

From Price and Grice (1932), the height system of Sri Lanka is a largely geometrical system, which does not reflect the actual Sri Lankan gravity field, as would be more of the case for an orthometric-type height system. Since the corrections were computed with a normal gravity field, it is more appropriate to say that the Sri Lankan heights are based on normal orthometric height system (cf. Featherstone and Kuhn 2006).

However, Sri Lanka has claimed to use an orthometric height system since establishing the geodetic levelling network of the country til present by almost every user of heights (e.g., Price 1932, Price and Grice (1932), Geodetic Survey Unit 2000) and the Sri Lankan State authorities. This basic misunderstanding surrounding height systems leads to confusion (cf. Bomford, 1971, Chapters 3 & 7; Featherstone and Kuhn 2006; Heiskanen and Moritz 1967, Chapters 4 & 8; Jekeli 2000).

In actuality, the Sri Lankan vertical datum is based on five years of MSL observations at Colombo and an approximation of the normal orthometric height system.

2.2.3 GPS-levelling data

There are 222 GPS-levelling points in Sri Lanka, comprising 20 FBMs and 202 new points, which have been used for this assessment of EGM2008 (Fig. 3). The [normal-orthometric] heights of the FBMs relative to MSL at Colombo were taken from Price and Grice (1932) and the heights of new points were obtained by spirit levelling (Geodetic Survey Unit 2000). No control points have been used for this study whose heights were derived by trigonometric levelling techniques.

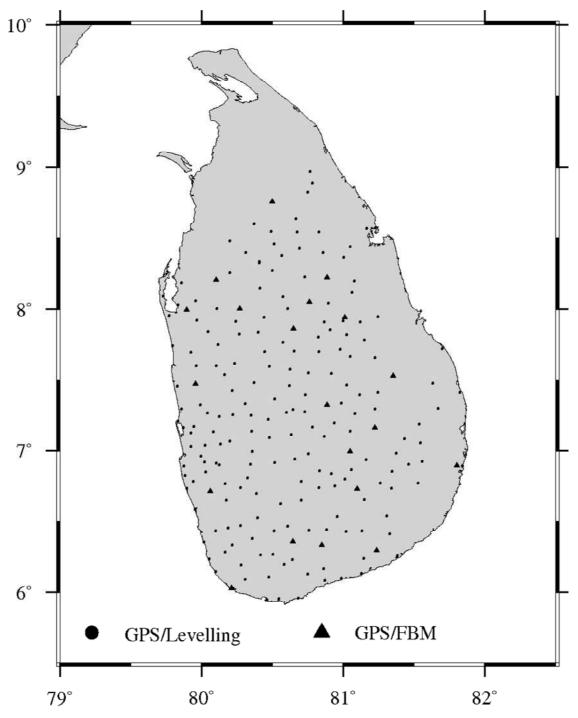


Fig 3 Coverage of the 207 GPS-levelling points in Sri Lanka (Mercator projection). Spurious heights were removed as blunders after initial investigation (Section 3.1)

However, the accuracy of the levelled heights at the new control points is uncertain, except for the 20 FBMs (Section 2.1.2). According to the levelling standards of Sri Lanka (Goonewardena 1970), the detailed levelling has the lowest accuracy with a 24 mm/ \sqrt{km} allowable misclose, and the misclosure factors for minor levelling and third-order levelling are 14 mm/ \sqrt{km} and 8 mm/ \sqrt{km} , respectively.

Due to uncertainty of the heights at these new control points, eight were identified and rejected after initial investigation as blunders. The procedure for selecting the control points used in the assessment is described in Section 3.1.1. Figure 3 shows the 207 points used for the assessment after the outliers were removed.

3 RESULTS AND DISCUSSION

All gravity-field-related quantities computed from EGM2008 used the HARMONIC_SYNTH.f FORTRAN-77 software provided by the EGM development team. GRS80 parameters were set in the 'parameter input' so that the zero-degree term and scaling of the even-degree coefficients were computed according to the algorithm in Lemoine et al. (1998).

3.1 Comparisons with GPS-levelling data

The Sri Lankan GPS-levelling data have not been compared with previously available geopotential models, which are considerably lower degree and order than EGM2008. We have not traced any publication on testing geopotential models for the study area.

GPS-levelling data can be used to assess geoid and quasigeoid models in absolute or relative modes (Featherstone 2001). The absolute tests can tell something about biases in EGM2008 or the local vertical datum, though these are inseparable (cf. Featherstone 2004). The relative tests can tell something about slopes in EGM2008 or the local vertical datum, but again being inseparable.

3.1.1 Absolute GPS-levelling tests

Height anomalies at the GPS-levelled points were compared with synthesised values from EGM2008 by using HARMONIC_SYNTH.f in scattered point computation mode. The differences are summarised statistically in Table 1 after the outliers $(\pm 3\sigma)$ were removed. The analysis classified the available GPS-levelling data by the perceived level of accuracy of the normal orthometric heights on the Sri Lankan vertical datum. Since the height accuracy is not exactly known at points other than the FBMs, these

two categories are assessed separately (Table 1). For comparison, the statistics were also computed for EGM96 (Table 2).

Data type	Min (m)	Max (m)	Mean (m)	STD (m)	
20 (FBMs)	-1.885	-1.528	-1.727	±0.098	
187 (levelled)	-2.465	-0.991	-1.763	±0.190	
207 (All)	-2.465	-0.991	-1.760	±0.184	

Table 1 Statistics of the differences of height anomalies between GPS-levelling and EGM2008. There are 20 FBMs where the height is known accurately and 187 points were spirit levelled but with unknown height accuracy.

Data type	Min (m)	Max (m)	Mean (m)	STD (m)	
20 (FBMs)	-2.571	-0.751	-1.902	±0.398	
187 (levelled)	-2.929	-0.724	-1.888	±0.423	
207 (All)	-2.929	-0.724	-1.890	±0.420	

Table 2 Statistics of the differences of height anomalies between GPS-levelling and EGM96. There are 20 FBMs where the height is known accurately and 187 points were spirit levelled but with unknown height accuracy.

Eight points were removed as clear (>5 m) blunders from the comparisons due to spurious differences of height anomalies, which were separated from the main cluster in Fig. 4, which shows the differences before the blunders were identified. The positional accuracy of the GPS data was confirmed by the Geodetic Survey Unit (2000), so the blunders are most likely due to inferior levelled heights. This left 214 usable out of 222 GPS-levelled points, which gave a mean difference of -1.737 m and a standard deviation of ± 0.249 m before detection of outliers. Using $\pm 3\sigma$, a further seven points were rejected, leaving 207 usable GPS-levelled points (Fig. 3).

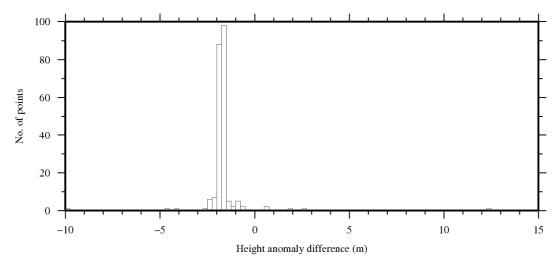


Fig 4 Histogram of differences of height anomalies for the whole set of GPS-levelled points (222). Most of points are clustered (between -2.75 m and -0.5 m) with a bias of -1.76 m. The eight isolated points are identified as blunders due to inferior levelled heights.

A large mean difference (-1.760 m for all points) in Table 1 is larger than the difference of MSL values used for the vertical datum of Sri Lanka (Section 2.2.2). The mean dynamic topography (MDT) from the DNSC08 model, which has been derived from the DNSC08 mean sea surface and EGM2008 (Andersen and Knudsen 2008), around the island is ~ +0.7 m above EGM2008. According to the results in Table 1, the geometric quasigeoid associated with the observed MSL at the Colombo tide gauge is 1.760 m below EGM2008.

The relatively small standard deviation of ± 0.184 m for the whole data set (± 0.098 m for the 20 FBMs) therefore suggests that there is a bias between EGM2008 and the Sri Lankan GPS-levelling data. The $\sim +0.7$ m MDT cannot explain this -1.760 m offset (it is in the opposite direction), so it is more likely to originate from the ellipsoidal heights starting from a point that is not properly connected to the COLA DORIS GPS mark at SGO and the 'unstable' radiated GPS baseline vector from SGO to ISMD (Section 2.2.1). This is an implicit validation of EGM2008, as it led to the investigation of the ellipsoidal height datum in Sri Lanka (Section 2.2.1).

The calculated height anomaly differences are plotted against the normal orthometric height of the point in Fig. 5 to determine if there is any correlation with height. Unfortunately, there is only a limited number of points available in the mountainous areas above 1,000 m, with most GPS-levelling points located below 250 m. The correlation coefficient for the linear regression in Fig. 5 is -0.195 and hence no significant height-dependent trend in the differences is observed.

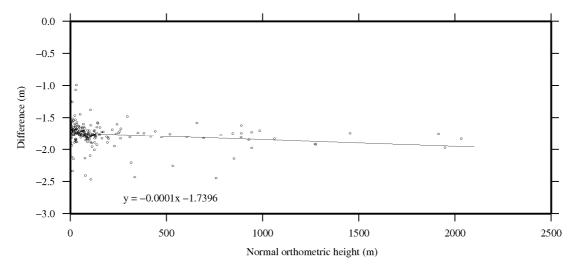


Fig 5 Difference of height anomalies versus normal orthometric height. The correlation coefficient is -0.195 and no significant trend is seen.

Figure 6 shows a map of the height anomaly difference derived by subtracting the geometric GPS-levelled height anomalies and EGM2008-synthesised height anomalies (in scattered point mode). The surface was created by gridding with continuous curvature splines in tension (Smith and Wessel 1990) available in Generic Mapping Tools (GMT) open-source software (Wessel and Smith 1998). From Fig. 6, some spurious points appear to remain that were not rejected by the $\pm 3\sigma$ threshold. All these are not at FBMs, indicating that levelling errors occur that these points too, but they could not be justifiably rejected by statistical outlier detection. Further investigation (and re-levelling) would be needed to properly isolate these. Also, there is no correlation of the differences with the southern central mountains (cf. Fig. 2), as already seen in Fig. 5.

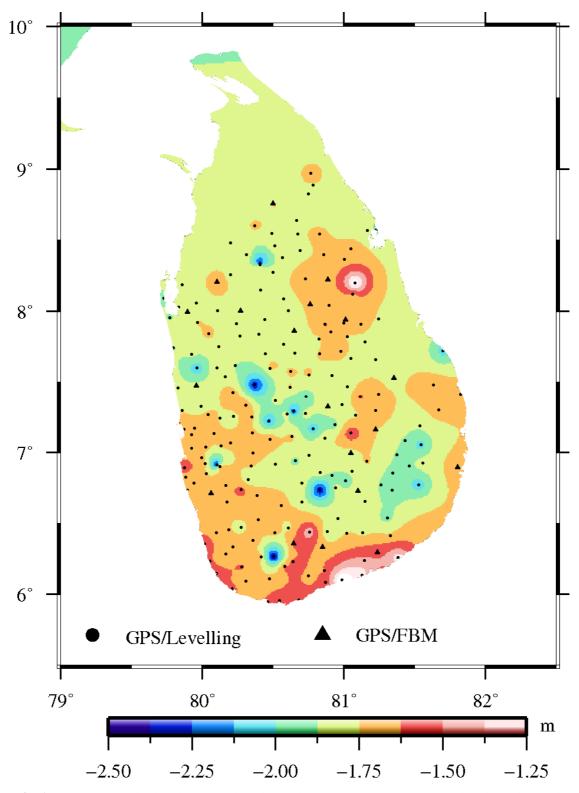


Fig 6 Difference of the height anomalies derived from geometric quasigeoid and synthesised values from EGM2008 (Mercator projection). The majority of points have differences near to the mean (-1.760 m). A few points show relatively larger differences, indicating that levelling errors remain.

North-south and east-west trends in the differences of height anomalies were calculated by performing linear regressions in latitude (Fig. 7) and longitude (Fig. 8). The north-south tilt equates to ~0.41 mm/km when converting degrees to kilometres (one degree is ~111 km at the equator). The east-west tilt equates to ~0.04 mm/km. However, the correlation coefficient for the north-south tilt is ~-0.025 and for the east-west tilt is ~-0.013, so these slopes are not significant.

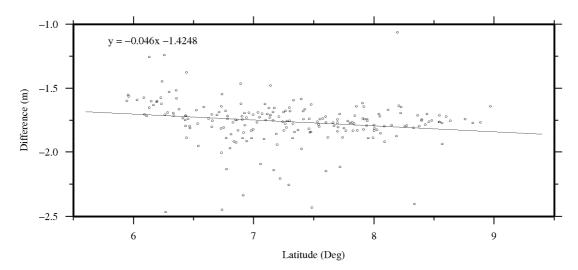


Fig 7 Linear regression of the height anomaly differences between GPS-levelling and EGM2008 in latitude. The north-south tilt equates to ~0.41 mm/km and the linear correlation coefficient is ~-0.025. No significant slope is observed.

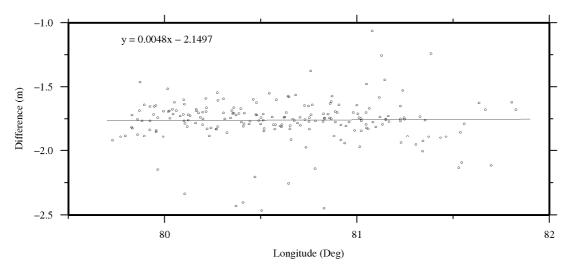


Fig 8 Linear regression of the height anomaly differences between GPS-levelling and EGM2008 in longitude The east-west tilt equates to ~0.04 mm/km and the linear correlation coefficient is ~-0.013. No significant slope is observed.

3.1.2 Relative GPS-levelling tests

In addition to the absolute tests, the relative (i.e., quasigeoid slope) differences between EGM2008 and the GPS-levelling have been obtained from all 21,321 possible baselines between the 207 GPS-levelling stations (i.e., after the outlier rejection in Section 3.1.1.) using the GEOID_REL_TESTER.f software in Featherstone (2001). Geometric height anomalies were determined from the GPS-levelling data and EGM2008 height anomalies at these points were interpolated from an equiangular 1 arc-minute (synthesised height anomalies) grid using bi-cubic methods.

Table 3 was created from the output of <code>GEOID_REL_TESTER.f</code> and shows the statistics of the differences of the relative height anomalies or gradients in height anomaly differences. The statistics of absolute differences are in Table 3 are similar to the values in Table 1. The small differences are due to interpolation error, but they are much smaller than the expected errors in the levelling. As such, the absolute differences in Table 1 should be interpreted as the more accurate values.

	Baseline length (km)	Absolute differences (m)	Relative differences (m)	Misclosure (ppm)
Max	336.157	-0.927	1.557	653.796
Min	0.912	-2.484	-1.504	0.000
Mean	121.964	-1.764	0.026	2.284
STD	±60.625	±0.218	±0.282	±5.937
RMS	±136.201	±1.778	±0.283	±6.361

Table 3 Statistics of the baseline analysis. Absolute and relative differences were calculated from the geometric quasigeoid and a one arc min equiangular EGM2008 quasigeoid grid.

The large maximum misclosure value is due to shorter baselines.

In relative quasigeoid testing, it is immaterial where the heights are tied to, because any constant bias cancels on differencing (e.g., Featherstone 2001). This results in a relatively small mean difference (0.026 m), where the -1.760 m bias has cancelled (cf. Figs. 9 and 10). However, the standard deviation of ± 0.282 m is a small increase on the absolute differences (± 0.184 m in Table 1) because uncorrelated errors do not cancel on differencing. The misclosure is calculated in mm per km (ppm) by dividing

the relative difference by the length (geodesic distance) of the baseline. The mean ppm value shows that, on average, EGM2008 can recover normal orthometric height differences with a precision of about ~2.3 ppm (mm/km).

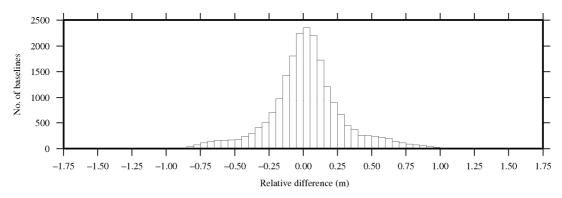


Fig 9 Histogram showing the unbiased distribution of the relative differences of height anomalies among 21,321 baselines.

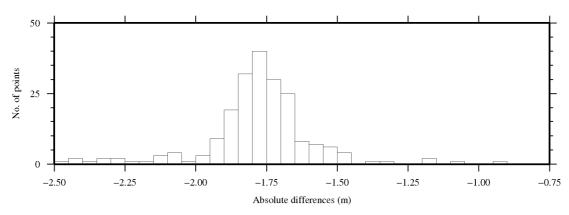


Fig 10 Histogram showing the absolute differences calculated from GEOID_REL_TESTER.f

The allowable misclose of third order levelling (A; 8 mm/ \sqrt{km}), minor levelling (B; 14 mm/ \sqrt{km}) and detail levelling (C; 24 mm/ \sqrt{km}) of Sri Lankan standards (Goonewardena 1970) are represented by curves in Fig. 11. 34% of the relative differences are under curve A, ~54% are under curve B, and ~72% are under curve C, which is the lowest accuracy (detail) levelling standard generally used for colleting coarse height information for elevation contours.

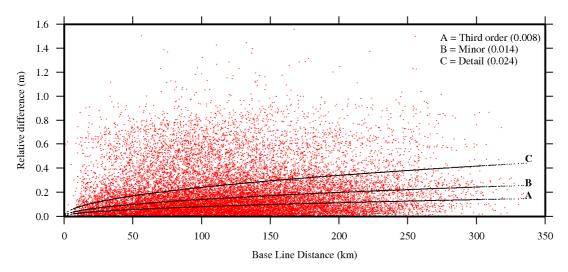


Fig 11 Relative differences of the height anomalies from GPS-levelling and EGM2008 computed over 21,321 possible baselines. Misclosure standards of third order, minor levelling and detail levelling are shown as A, B and C.

3.2 Comparison with Sri Lankan gravity data

3.2.1 Computation of gravity anomalies

The Sri Lankan free air gravity anomalies were re-calculated with respect to the Geodetic Reference System 1980 (Moritz 1980). The linear free air correction, F, was computed with normal gravity γ_{1980} from the International Gravity Formula 1980 with

$$F = -\frac{\partial \gamma}{\partial H}H = +0.3086H \ mGal \tag{1}$$

$$\gamma_{1980} = 978 \ 032.7 \left(1 + 0.005 \ 302 \ 4 \sin^2 \phi - 0.000 \ 005 \ 8 \sin^2 2\phi \right) \ mGal \tag{2}$$

No atmospheric correction was applied, but which is only 0.871 mGal at sea level (Moritz 1980). The linear free-air correction was considered sufficient because most of the gravity observations are made below ~250m (Section 2.1.1)

These were compared with synthesised values from harmonic_sysnth.f software, which gives the gravity anomaly (i.e., Molodensky free air gravity anomalies) at the observation point. For the synthesis, the 3D location with respect to the geometrical surface of the reference ellipsoid (GRS80 was used for this Sri Lankan assessment) is needed. Therefore, the geometric quasigeoid was interpolated in order to find the height anomalies for the non GPS-coordinated points in Table 4, except for the marine gravity 9described later). Harmonic_sysnth.f was used to synthesise free air anomalies in scattered point mode in this comparison.

3.2.2. Digitised and reconstructed gravity anomalies

Gravity data were digitised from the complete/refined Bouguer anomaly map of Hatherton and Ranasinghe (1972). The locations of gravity observations (marked on the map) were compiled by onscreen digitisation and transformed to a geocentric datum using ESRI's ArcMap v9.2. The Bouguer anomalies were interpolated from the 5-mGal-interval contours to the observation locations on the map. The International Gravity Formula 1924 was used to compute the Bouguer anomalies in Heatherton and Ranasinghe (1972). Therefore, the normal gravity field was converted from International 1924 (γ_{1924}) to GRS80 (γ_{1980}) for latitude ϕ using (e.g., Li and Götze 2001).

$$\gamma_{1980} - \gamma_{1930} = -16.3 + 13.7 \sin^2 \phi \quad \text{mGal}$$
 (3)

To check the reliability of the digitised values, simple Bouguer anomalies were calculated at the 20 GPS-coordinated FBMs (Section 2.2.1) and compared with values obtained by interpolating the digitised Bouguer anomalies to the FBMs using GMT. The mean difference was -0.476 mGal with a standard deviation of ± 1.815 mGal, showing the digitisation to be reasonably accurate. However, terrain corrections had been applied out to Hammer's (1939) zone M on Hatherton and Ranasinghe's (1972) map, whereas they were not applied at the 20 FBMs. Only a small difference can be expected because most of the digitised observations were in low-lying regions.

Next it was necessary to 'reconstruct' free-air anomalies (actually Faye anomalies that have a terrain correction applied to a free-air anomaly) from the digitised complete Bouguer anomalies (cf. Featherstone and Kirby 2000). The normal orthometric heights (H) required to calculate the Bouguer slab correction ($2\pi G\rho$) were bicubically interpolated from a 100 m DEM (Fig. 2) and subsequently added to the Bouguer anomalies (Δg_B) to find the free air anomaly (Δg_F)

$$\Delta g_F = \Delta g_B + 2\pi G \rho H \tag{4}$$

with the standard rock density ρ of 2670kg/m³. These 1055 digitised and reconstructed gravity anomalies will be compared with EGM2008.

3.2.3 Results for land gravity

The results presented in Table 4 for the Sri Lankan gravity observations, not all of which are available in the public domain, indicates how EGM2008 agrees with the

local gravity field. The terrestrial gravity data are grouped as (a), (b) and (c) in descending order the level of perceived accuracy of location, height and gravity (Section 2.2). The statistics of differences between the Sri Lankan and EGM2008-synthesised free air anomalies are shown in Table 4 after removing outliers $(\pm 3\sigma)$.

The mean difference for all 63 points under (a) and (b) is 13.736 mGal. Only one value was an outlier with the standard deviation of ± 8.485 mGal used for the outlier detection (cf. Fig. 12). For dataset (c), 23 points were found to be outliers for the 1055 digitised points (Fig. 13) with a prior mean of 12.578 mGal and standard deviation of ± 7.984 mGal used for the outlier detection.

Gravity data	# points	Min	Max	Mean	Std
a) GPS-coordinated gravity on FBMs	20	-7.589	19.430	10.194	±6.743
b) FBM and permanent positions with coarse locations	42	-26.039	35.799	8.711	±14.704
c) Digitised and reconstructed free-air anomalies from a Bouguer anomaly map	1032	-10.532	33.660	12.841	±6.367
d) Ship track gravity	12,192	-96.768	107.198	9.552	±43.683

Table 4 Agreement of EGM2008 with Sri Lankan free air anomalies after removing outliers $(\pm 3\sigma)$. Points under (a) are where gravity was measured with a L&R meter and also belong to the geodetic levelling network of Sri Lanka. Points under (b) have coarse locations (compiled from maps). Points under (c) are reconstructed (digitised) free air anomalies from a Bouguer anomaly map of Ceylon. Points under (d) are marine gravity anomalies. All values in mGal.

The results in Table 4 are all based on the old Potsdam gravity datum (Ratmalana airport) as it is the datum of previous national gravity surveys in Sri Lanka (Section 2.1.2). The conversion (about -14 mGal) between the Potsdam and IGSN71 gravity datums (e.g., Morelli et al. 1974; Hatherton et al. 1975) has not been applied, and thus remains present in the mean differences for EGM2008 over the region. The positive biases (~10 to 15 mGal in the means) in Table 4 are consistent with this gravity datum offset. This shows that EGM2008 has used ISGN71-based gravity anoma-

lies over Sri Lanka, unlike some previous geopotential models that used Potsdam (e.g., Kim and Rapp 1990).

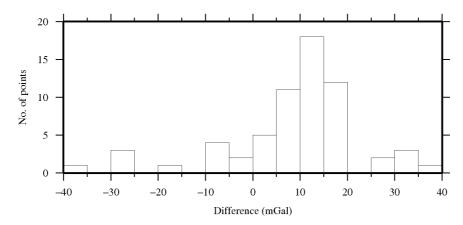


Fig 12 Distribution of differences between free air anomalies of 63 terrestrial gravity observations from subsets (a) and (b) and EGM2008. The mean is 8.485 mGal and the standard deviation (1σ) is ± 13.736 mGal for the whole set of data. One point was outside $\pm 3\sigma$.

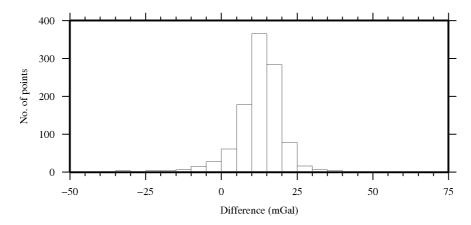


Fig 13 Distribution of differences between free air anomalies of 1055 terrestrial gravity observations from subset (c) and EGM2008. The mean is 12.578 mGal and the standard deviation (1σ) is ± 7.984 mGal for the whole set of data. Twenty three points were outside $\pm 3\sigma$.

The frequency distribution of the differences for datasets (a) and (b) combined spans over ~-40 to ~+40 mGal (Fig. 12). The 20 GPS-coordinated gravity points (a) have a smaller standard deviation (±6.743 mGal) than the gravity points (b) whose locations were scaled from maps (±15.997 mGal). The reason is the coarse locations of the latter. This inaccurate location would give an uncertainty in height, especially in the hilly areas, and therefore the EGM2008 free air anomaly will not be synthesised at the same 3D location by HARMONIC_SYNTH.f. This effect is shown in Fig 14,

where the differences for the non-GPS-coordinated points have a fairly large deviation from the mean for heights greater than 500m.

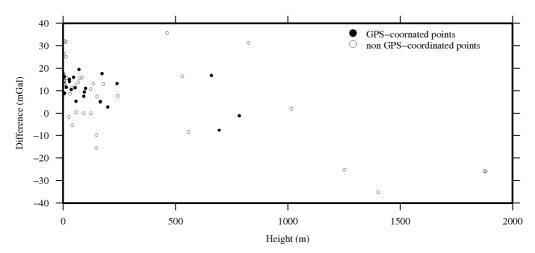


Fig 14 Differences of free air anomalies for (a) and (b) with height. Non GPS-coordinated points have large deviation than GPS-coordinated points at higher altitudes.

Figure 15 shows the difference between free air anomalies from Sri Lankan gravity and EGM2008. Though the most of differences are consistent with the ~14 mGal gravity datum bias (Section 2.1.1), the central southern mountain region (cf. Fig. 2) shows notable extreme values. The poorer Sri Lankan gravity positions would provide some explanation for this, but the omission error and downward continuation corrections in EGM2008 may also be responsible. From Pavlis et al. (2008), the standard deviation of the 'fill-in' gravity anomalies on and around Sri Lanka is about 3-5 mGal, with the larger values being in the mountainous central south of the island.

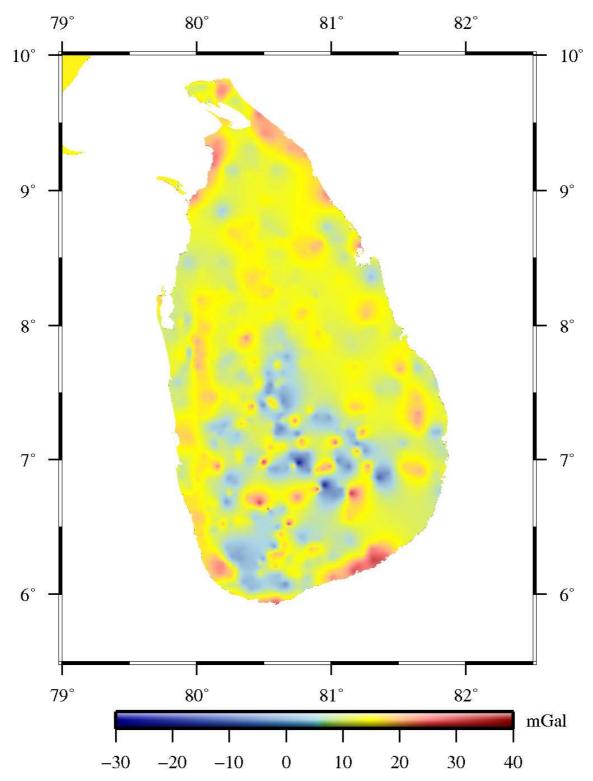


Fig 15 Difference between free air anomalies of Sri Lankan gravity and EGM2008 (Mercator projection). The larger differences are seen in the central southern mountains.

3.2.3 Results for ship-track gravity

Figure 16 shows a histogram of the differences between the NGDC (1999) ship-track gravity data and EGM2008, and Fig. 17 shows a chart of the differences along the cruise lines. To enable the harmonic_synth.f software to compute gravity anomalies on the mean sea surface where the ship-track gravity observations are taken, the ellipsoidal heights of the ship-track gravity points were obtained by applying the MDT model from the Danish National Space Institute (DNSC) (Andersen and Knudsen 2008). The 2 arc-minute grid was interpolated to the locations of the ship-track observations, then harmonic_sysnth.f was run in scattered point mode, where is added the EGM2008 height to the MDT to compute the gravity anomaly on the mean sea surface.

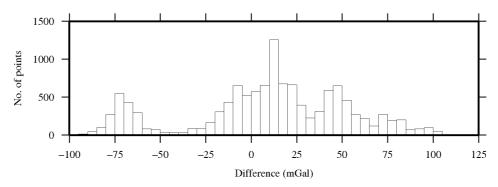


Fig 16 Frequency distribution of differences in marine free air gravity anomalies and EGM2008. Clusters indicate that the data are not crossover adjusted. The largest central peak is consistent with the datum bias in the terrestrial gravity anomalies.

The offsets in the differences among the ship tracks appear as widely spread and clustered peaks in the histogram (Fig. 16), confirming further that they have not been crossover adjusted. We did not attempt a crossover adjustment because the tracks are widely spaced with few crossovers, rendering any adjustment ill conditioned. Also, the majority of differences are around 10–15 mGal which correlates with the bias in the terrestrial gravity observations due to the Ratmalana datum being tied to Potsdam.

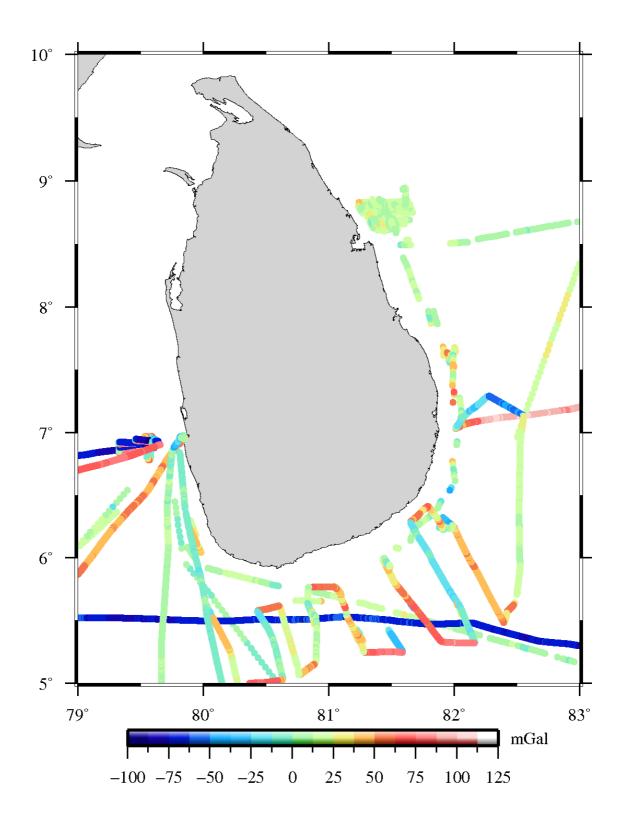


Fig 17 Differences between marine free air gravity anomalies and EGM2008. The significant offsets among overlapping tracks shows that the cruise lines have not been crossover adjusted. In addition, there is a consistent bias of ~14 mGal in the cruises coming from Colombo (~7N, 80E), indicating that they were tied to the Potsdam datum via Ratmalana airport. (Mercator projection)

4. CONCLUSIONS

We have used Sri Lankan gravity and GPS-levelling data to assess EGM2008, which is where 'fill-in' gravity anomalies were used in EGM2008. The analysis eventually proved to be more useful for detecting problems in the Sri Lankan data, particularly with respect to the datums. From the initial quick-look comparisons, we detected a ~1.7 m bias between the GPS-levelling and EGM2008, which led to an in-depth investigation into the Sri Lankan vertical geodetic datums. This uncovered a probable bias in the ellipsoidal heights used due to a combination of an incorrect local tie at the COLA DORIS site and a ~120-km-long radiated GPS baseline over a 1.2 km height difference. The difference between the gravity anomalies and EGM2008 showed that the Sri Lankan data, tied to an absolute value at Ratmalana airport, is based on the old Potsdam datum, which is offset from IGSN71 by about -14 mGal. This is similar to the mean differences observed, indicating that EGM2008 uses IGSN71-referenced gravity anomalies over Sri Lanka, unlike some earlier geopotential models (e.g. Kim and Rapp 1990).

After these datum-related biases are neglected, the standard deviations of the differences become more informative. These were calculated for different subsets of the Sri- Lankan GPS-levelling and gravity data according to their perceived reliability. After rejection of 15 outliers, the standard deviation of the difference between 207 Sri Lankan GPS-levelling points and EGM2008 is ± 0.184 m, which reduces to ± 0.098 m for a subset of 20 stations at fundamental benchmarks. Over baselines, the standard deviation was ± 0.282 m, showing the presence of uncorrelated errors. The Sri Lankan land gravity data, after rejection of outliers, yielded standard deviations of ± 6.743 mGal for 20 GPS-coordinated gravity points on fundamental benchmarks, ± 14.704 mGal for 42 gravity points on fundamental benchmarks but with coarse locations, and ± 6.367 mGal for 1032 digitised and reconstructed free-air anomalies from a Bouguer anomaly map. The ship-track gravity data have not been crossover adjusted, and yield a standard deviation of ± 43.683 mGal, but showed that several tracks originating from Colombo had been tied to the Potsdam datum at Ratamala airport.

This study has therefore implicitly validated EGM2008 because the standard deviations of the differences are reasonable, and the large biases can be explained by peculiarities in the Sri Lankan geodetic and gravity datums. Indeed, this is an auxiliary application of EGM2008, where datum deficiencies can be detected and then in-

vestigated further. The detection of ellipsoidal height bias shows that EGM2008 is already making a contribution to vertical datum unification.

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