School of Electrical Engineering and Computing
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Improving the Performance of MIMO Relay Networks Using Parallel Relays

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Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature: .....

Date: .....

To

my parents, my husband Lujiber, and my kids Aurilia and Edward

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Abstract

Recently, multiple-input multiple-output (MIMO) relay communication systems have attracted much research interest, and provided significant improvement in terms of both spectral efficiency and link reliability. This thesis aims at improving the performance of MIMO relay communication systems using parallel relays and successive interference cancellation (SIC) technique.

In the first part of the thesis, we propose the optimal transmit beamforming vector and the relay amplifying factors for a MIMO relay communication system with distributed single-antenna relay nodes when a single data stream is transmitted from the source to destination. The proposed joint source and relay beamforming algorithm provides an improved bit-error-rate (BER) performance.

The second part of the thesis focuses on parallel MIMO relay communication systems where each relay has multiple antennas and multiple data streams are transmitted from the source to the destination. We derive the optimal structure of relay amplifying matrices when a linear minimal mean-squared error (MSE) receiver is used at the destination node and each relay node is subject to power constraint. Simulation results demonstrate the effectiveness of the proposed source and relay matrices design which minimizes the MSE of the signal waveform estimation using the projected gradient (PG) approach.

To reduce the complexity of the algorithm in the second part of the thesis, we propose a simplified source and relay matrices design by first relaxing the power constraint at each relay node to the power constraint at the output of the second-hop channel. After solving the relaxed optimisation problem, the relay matrices are then scaled to satisfy the individual power constraint at each relay node. Simulation results show a good performance-complexity tradeoff of the simplified algorithm.

In the fourth part of the thesis, we investigate the optimal structure of the source precoding matrix and the relay amplifying matrices for MIMO relay communication systems with parallel relay nodes when a nonlinear MMSE-DFE receiver is used at the destination node. The MMSE criterion is used to detect the transmitted signal at each stream. We show that the optimal source precoding matrix and the optimal relay amplifying matrices have a beamforming structure. By using a DFE receiver, we can remove the effect of interferences of data stream we have already recovered from the subsequent streams. Therefore, introducing a nonlinear MMSE-DFE receiver at the destination node yields further improvement in the system BER performance compared with the parallel MIMO relay communication systems using a linear MMSE receiver.

Author's Note

Parts of this thesis and concepts from it have been previously published in the following journal and/or conference papers.

Journal Papers

[1] A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Optimization for Parallel MIMO Relay Networks", *EURASIP Journal on Advances in Signal Processing*, 2012:174.

[2] A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Design for MIMO Multi-Relay Systems Using Projected Gradient Approach", *EURASIP Journal on Wireless Commun. Network.*, submitted, Aug. 2013.

Conference Papers

[1] A. Toding, M. R. A. Khandaker, and Y. Rong, "Optimal Joint Source and Relay Beamforming for Parallel MIMO Relay Networks", in *Proc. 6th Int. Conf. Wireless Commun., Networking and Mobile Computing (WiCOM'2010)*, Chengdu, China, Sep. 23-25, 2010.

[2] A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Optimization for Parallel MIMO Relays Using MMSE-DFE Receiver", in *Proc. 16th Asia-Pacific Conference on Communications (APCC'2010)*, Auckland, New Zealand, Nov. 1-3, 2010, pp. 12-16.

[3] A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Optimization for Distributed MIMO Relay System", in *Proc. 17th Asia-Pacific Conference on Communications (APCC'2011)*, Sabah, Malaysia, Oct. 2-5, 2011, pp. 604-608.

[4] A. Toding and Y. Rong, "Investigating Successive Interference Cancellation in MIMO Relay Network", in *Proc. IEEE TENCON*, Bali, Indonesia, Nov. 21-24, 2011, pp. 359-363.

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List of Acronyms

AF	amplify and forward
BER	bit-error-rate
BPSK	binary phase shift keying
BS	base station
CF	compress and forward
CSI	channel state information
DF	decode and forward
DFE	decision feedback equalizer
EVD	eigenvalue decomposition
i.i.d.	independent and identically distributed
JSR-PG	joint source and relay optimization algorithm using the projected gradient
LMMSE	linear minimum mean-squared error
MIMO	multiple-input multiple-output
MMSE	minimum mean-squared error
MMSE-DFE	minimum mean-squared error-decision feedback equalizer
MSE	mean-squared error
NAF	naive amplify-and-forward

ORO	optimal relay only
ORO-PG	optimal relay-only algorithm using the projected gradient
PG	projected gradient
PSD	positive semi-definite
SDP	semi-definite programming
SIC	successive interference cancellation
SISO	single-input single-output
SNR	signal-to-noise ratio
SVD	singular value decomposition
ZF	zero-forcing

Chapter 1

Introduction

This thesis aims at developing and studying advanced algorithms for multiple-input multiple-output (MIMO) relay communication systems using parallel relays and successive interference cancellation (SIC) technique. In this introductory chapter, we briefly present necessary background on MIMO relay communication systems using parallel relays and SIC technique and overview the contributions of the thesis.

1.1 Background

In order to establish a reliable wireless transmission, one needs to compensate for the effects of signal fading due to multi-path propagation and strong shadowing. One way to address these issues is to transmit the signal through one or more relays [1–23], which can be accomplished via a wireless network consisting of geographically separated nodes. And then the basic motivation behind the use of cooperative communications lies in the exploitation of spatial diversity provided by the network nodes [9, 14–18], as well as the efficient use of power resources [19, 20, 24–29] which can be achieved by a scheme that simply receives and forwards a given information, yet designed under certain optimality criterion.

Wireless relaying systems have lots of advantages over traditional direct transmission systems and have drawn considerable interest from both the academic and industrial communities [30–34]. Relays can help improve the system capacity by decreasing the per-hop transmitter-receiver distance, decrease the infrastructure cost since they are much cheaper than base station (BS), and extend the network coverage [9, 13, 14, 35–39].

A three-terminal relay channel model was first studied in the 1970's as in [1, 2]. Relay schemes can be broadly categorized into three general groups: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF). In the AF scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination node [6, 21, 37, 40–47]. In the DF scheme, the relay nodes first decode the received signals and then forward the re-encoded signals toward the destination node [5, 22, 32, 48–50]. In the CF method, the relay nodes compress the received signals by exploiting the statistical dependencies between the signals at the nodes [32, 51–53].

When nodes in the relay system are installed with multiple antennas, we call such relay system MIMO relay communication system. Recently, MIMO relay communication systems have attracted much research interest and provided significant improvement in terms of both spectral efficiency and link reliability [7, 12, 37, 38, 41, 54–72]. In the meantime, work is being carried out on the factors affecting MIMO relay channels. Some proposed designs of the transmitter, receiver, and relay matrices show that increasing the number of relay antennas can reduce the system cost of building BS, increase the reliability of signals that arrive at the receiver, increase the channel capacity, improve the coverage area and increase the system diversity.

The main issue in a MIMO relay wireless communication system is to build the communication when there are some obstacles between transmitter and receiver. An efficient way to combat the problem is to use multiple parallel relays between the transmitter and the receiver which can provide more diversity and larger coverage area and overcome any possible shadowing effect [73–75].

Two types of receiver architectures are considered at the destination node, namely the linear receiver and the nonlinear receiver. For complexity reduction reasons, there has been great interest in the class of linear receivers such as zero-forcing (ZF) and minimum mean-squared error (MMSE) receivers [24, 27, 57, 58, 68, 76–86]. The ZF receiver first inverts the channel matrix so that each data stream can be recovered via a single-input single-output (SISO) decoding algorithm. The MMSE receiver does the same except with a regularized channel inverse that accounts for possible noise amplification. It has been shown in many recent works [26, 49, 87–104] that nonlinear receivers yield better performance compared to linear receivers at the cost of increased computational complexity. Even the simplified nonlinear receivers such as the decision feedback equalizer (DFE) based on the MMSE/ZF criterion can achieve performance gain. Both the MMSE and ZF schemes permit the use of powerful channel codes that

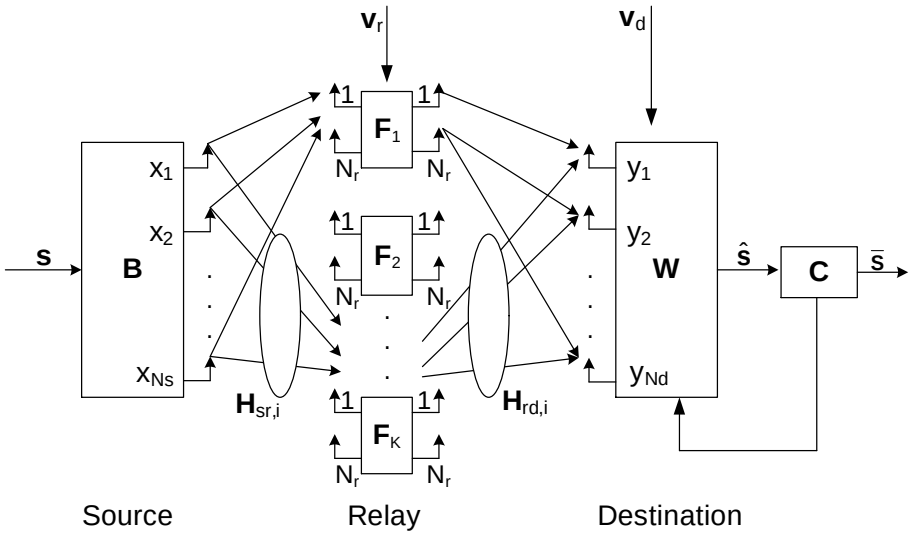

Figure 1.1: Block diagram of a parallel MIMO relay communication system using SIC technique.

can achieve high data rates at practically-relevant signal-to-noise ratio (SNR). The DFE technique is also well-known as the SIC technique. A nonlinear DFE receiver recovers the source signals successively by exploiting the finite alphabet property of the source signals. By using a DFE receiver we can remove the effect of interferences of the data streams we have already recovered from the subsequent streams. In this thesis, both linear and nonlinear methods have been proposed in parallel MIMO relay communication systems to separate the spatially multiplexed inputs at the receiver. A typical MIMO relay communication system with the DFE receiver is illustrated in Fig. 1.1, where $\mathbf{s}$ is an $N_b \times 1$ source symbol vector, $\mathbf{B}$ is an $N_s \times N_b$ source precoding matrix, $\mathbf{F}_1, \cdots, \mathbf{F}_K$ are $N_r \times N_r$ amplifying matrices at the relay nodes, $\mathbf{W}$ is an $N_d \times N_b$ receive weight matrix, $\mathbf{C}$ is an $N_b \times N_b$ strictly upper-triangle feedback matrix of the DFE receiver, and $\bar{\mathbf{s}}$ is the estimated signal waveform vector. This thesis aims at optimizing $\mathbf{B}$, $\mathbf{F}_1, \cdots, \mathbf{F}_K$, $\mathbf{W}$, and $\mathbf{C}$ to minimize the MSE of the signal waveform estimation at the destination node.

1.2 Notations

The notations used in this thesis are as follows: Lower case letters are used to denote scalars, e.g. s, n. Bold face lower case letters denote vectors, e.g. $\mathbf{s}$, $\mathbf{n}$. Bold face upper

case letters are reserved for matrices, e.g. $\mathbf{S}$, $\mathbf{N}$. For matrices, $(\cdot)^T, (\cdot)^*, (\cdot)^H, (\cdot)^{-1}$, and $(\cdot)^+$ denote transpose, complex conjugate, Hermitian transpose, inversion, and pseudo-inverse operations, respectively. rank$(\cdot)$ and tr$(\cdot)$ denote the rank and trace of matrices, respectively. diag$(\mathbf{a})$ stands for a diagonal matrix with the vector $\mathbf{a}$ as the main diagonal and zero elsewhere. E$[\cdot]$ represents the statistical expectation, bd$[\cdot]$ stands for a block-diagonal matrix, and $\|\cdot\|$ denotes the maximum among the absolute value of all elements in the matrix. An N dimensional identity matrix is denoted as $\mathbf{I}_N$, $(\cdot)^n$ denotes the variable at the nth iteration, and $[x]^\dagger \triangleq \max(x, 0)$. Note that the scope of any variable in each chapter is limited to that particular chapter.

1.3 Receiver Algorithms for MIMO Systems

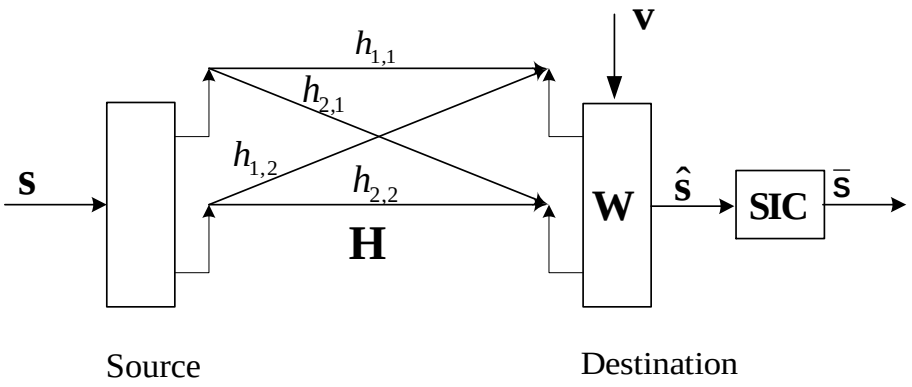

Figure 1.2: Block diagram of the equivalent MIMO channel.

In the following, we briefly review receiver algorithms for a single-hop MIMO system. For simplicity, we consider here a 2×2 MIMO transmission which essentially means both the transmitter and the receiver are equipped with two antennas (see Fig. 1.2). Thus, the received signal on the first receive antenna is

$$\begin{aligned} y_1 &= h_{1,1}\, s_1 + h_{1,2}\, s_2 + v_1 \\ &= \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + v_1 \end{aligned} \tag{1.1}$$

and that on the second receive antenna is

$$\begin{aligned} y_2 &= h_{2,1}\, s_1 + h_{2,2}\, s_2 + v_2 \\ &= \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + v_2 \end{aligned} \tag{1.2}$$

where y_1 and y_2 are the received symbols on the first and the second antennas respectively, $h_{j,i}$ is the channel from ith transmit antenna to jth receive antenna, s_1 and s_2 are the transmitted symbols, and v_1 and v_2 are the noises on first and second receive antennas respectively. For convenience, the above two equations can be combined in matrix notation as

$$\mathbf{y}_d = \mathbf{H}\mathbf{s} + \mathbf{v} \tag{1.3}$$

where $\mathbf{y}_d \triangleq \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is the received signal vector, $\mathbf{H} \triangleq \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}$ is the MIMO channel matrix, $\mathbf{s} \triangleq \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$ is the source symbol vector, and $\mathbf{v} \triangleq \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is the additive Gaussian noise vector.

The ZF Algorithm

The first decoding technique to be described in this section is ZF. The ZF linear detector meeting the constraint $\mathbf{W}^H\mathbf{H} = \mathbf{I}_2$ is given by

$$\mathbf{W} = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}. \tag{1.4}$$

$\mathbf{W}^H$ is also known as the pseudo-inverse of $\mathbf{H}$. Then the estimate for the transmit signal vector is

$$\hat{\mathbf{s}} = \mathbf{W}^H\mathbf{y}_d. \tag{1.5}$$

Using the ZF equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols s_1 and s_2 as

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{1.6}$$

where $\hat{s}_1$ and $\hat{s}_2$ are the estimates of s_1 and s_2 respectively.

The MMSE Algorithm

To estimate $\mathbf{s}$ in (1.3) the MMSE approach tries to find a weight matrix $\mathbf{W}$ that minimizes the statistical expectation of the signal estimation error given by

$$\mathrm{tr}\left(\mathrm{E}[(\hat{\mathbf{s}} - \mathbf{s})(\hat{\mathbf{s}} - \mathbf{s})^H]\right) \tag{1.7}$$

Substituting (1.5) into (1.7), we find that the $\mathbf{W}$ which minimizes (1.7) can be written as

$$\mathbf{W} = (\mathbf{H}\mathbf{H}^H + \mathbf{C}_v)^{-1}\mathbf{H} \tag{1.8}$$

where $\mathbf{C}_v$ is the noise covariance matrix. Using the MMSE equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols as

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \mathbf{C}_v)^{-1} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \tag{1.9}$$

The ZF and MMSE with SIC Algorithm

In the classical SIC algorithm, the receiver arbitrarily takes one of the estimated symbols, and subtracts its effect from the received symbols y_1 and y_2 (Fig. 1.3). However, a better BER performance can be achieved by choosing whether we should subtract the effect of $\hat{s}_1$ first or $\hat{s}_2$ first. To make this decision, let us find out the transmit symbol which comes at higher power at the receiver. The received power at both antennas corresponding to the transmitted symbol s_1 is

$$P_{s_1} = |\ h_{1,1}|^2 + |\ h_{2,1}|^2 \tag{1.10}$$

and the received power at both antennas corresponding to the transmitted symbol s_2 is

$$P_{s_2} = |\ h_{1,2}|^2 + |\ h_{2,2}|^2. \tag{1.11}$$

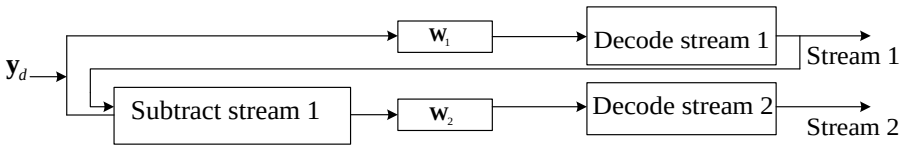

Figure 1.3: Successive interference cancellation technique.

Now, if $P_{s_1} > P_{s_2}$ then the receiver decides to remove the effect of $\hat{s}_1$ from the received signals y_1 and y_2 and obtain

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,1}\hat{s}_1 \\ y_2 - h_{2,1}\hat{s}_1 \end{bmatrix}. \tag{1.12}$$

If there is no error propagation we can express the above equation in matrix notation as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,2} \\ h_{2,2} \end{bmatrix} s_2 + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1.13}$$

or equivalently as

$$\hat{\mathbf{r}}_2 = \mathbf{h}_2\ s_2 + \mathbf{v}. \tag{1.14}$$

From (1.14), we can re-estimate s_2 as

$$\bar{s}_2 = \frac{\mathbf{h}_2^H \hat{\mathbf{r}}_2}{\mathbf{h}_2^H \mathbf{h}_2}. \tag{1.15}$$

Similarly, if $P_{s_2} > P_{s_1}$ then the receiver decides to remove the effect of $\hat{s}_2$ from the received signals y_1 and y_2 and then re-estimates $\hat{s}_1$ as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2}\hat{s}_2 \\ y_2 - h_{2,2}\hat{s}_2 \end{bmatrix}. \tag{1.16}$$

If there is no error propagation we can express the above equation in matrix notation as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} s_1 + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1.17}$$

or equivalently as

$$\hat{\mathbf{r}}_1 = \mathbf{h}_1 \, s_1 + \mathbf{v}. \tag{1.18}$$

From (1.18), we can re-estimate s_1 as

$$\bar{s}_1 = \frac{\mathbf{h}_1^H \hat{\mathbf{r}}_1}{\mathbf{h}_1^H \mathbf{h}_1}. \tag{1.19}$$

In this thesis, we focus on the MMSE receiver, since it performs better than the ZF receiver. We performed all the simulations in Matlab and used CVX toolbox wherever necessary [105].

1.4 Thesis Overview and Contributions

In this thesis, advanced algorithms for MIMO relay communication systems using parallel relays and SIC technique are presented considering non-regenerative relays. In Chapter 2, we study transmit beamforming and relay precoding design problem for parallel MIMO relay communication systems with distributed single-antenna relay nodes when a single data stream is transmitted from the source to destination node. In Chapter 3, we propose jointly optimal source precoding matrix and relay amplifying matrices for two-hop parallel MIMO relay communication systems with a linear minimum mean-squared error (LMMSE) receiver applied at the destination node. The projected gradient (PG) approach is applied to optimise the relay matrices where each relay has multiple antennas and multiple data streams are transmitted from the source to the destination node. To reduce the complexity of the algorithm in Chapter 3, we propose a simplified source and relay matrices design in Chapter 4 by relaxing power constraints at each relay node

to the power constraint at the output of the second-hop channel. In Chapter 5, we investigate ZF and MMSE receiver performance with and without using SIC technique in MIMO relay communication systems. We also investigate the optimal structure of the source precoding matrix and the relay amplifying matrices for parallel MIMO relay communication systems with multiple data streams when a nonlinear MMSE decision feedback equalizer (MMSE-DFE) receiver is used at the destination node. Chapter 6 summarizes the thesis and gives the outlook to some interesting future works.

Chapter 2: MIMO Transceiver Design with Distributed Relay Nodes
In this chapter, we study the optimal transmit beamforming vector and the relay amplifying factors design problem of parallel MIMO relay communication systems with distributed single-antenna relay nodes when a single data stream is transmitted from the source to destination node. We propose a jointly optimal iterative source and relay beamforming algorithm which minimizes the mean-squared error (MSE) of the signal waveform estimation for single-antenna relay nodes in a parallel MIMO relay communication system. Compared with the existing techniques, the proposed algorithm performs much better in terms of bit-error-rate (BER) and MSE of signal waveform estimation at the destination node.

The material in Chapter 2 is based on the conference publication:

- A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Optimization for Distributed MIMO Relay System", in *Proc. 17th Asia-Pacific Conference on Communications (APCC'2011)*, Sabah, Malaysia, Oct. 2-5, 2011, pp. 604-608.

In Chapters 3-5, we investigate optimal source and relay matrices design for MIMO relay communication systems with multiple parallel relay nodes when multiple parallel data streams are transmitted from source to destination node.

Chapter 3: Parallel MIMO Relay Design Based on Linear Receiver
We propose jointly optimal source precoding matrix and relay amplifying matrices for two-hop parallel MIMO relay communication systems with multiple parallel data streams when a linear receiver is applied at the destination node considering individual transmission power constraint at each relay node. We derive the structure of the optimal relay matrices and develop an iterative algorithm to jointly optimize the source and relay matrices to minimize the MSE of the signal waveform estimation at the destination node using the PG approach.

The material in Chapter 3 is based on the journal submission:

- A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Design for MIMO Multi-Relay Systems Using Projected Gradient Approach", *EURASIP Journal on Wireless Commun. Network.*, submitted, Aug. 2013.

Chapter 4: Simplified Source and Relay Matrices Design

To reduce the complexity of the algorithm in Chapter 3, we first relax the power constraint at each relay node to the power constraint at the output of the second-hop channel. After solving the relaxed optimisation problem, the relay matrices are then scaled to satisfy the individual power constraint at each relay node. Compared with the second part of the thesis, the algorithm in Chapter 4 has an improved performance-complexity tradeoff.

The material in Chapter 4 is based on the journal publication:

- A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Optimization for Parallel MIMO Relay Networks", *EURASIP Journal on Advances in Signal Processing*, 2012:174.

and the conference publication:

- A. Toding, M. R. A. Khandaker, and Y. Rong, "Optimal Joint Source and Relay Beamforming for Parallel MIMO Relay Networks", in *Proc. 6th Int. Conf. Wireless Commun., Networking and Mobile Computing (WiCOM'2010)*, Chengdu, China, Sep. 23-25, 2010.

Chapter 5: Parallel MIMO Relay Design Based on Nonlinear Receiver

We investigate the optimal structure of the source precoding matrix and the relay amplifying matrices for parallel MIMO relay communication systems with multiple parallel data streams when a nonlinear MMSE-DFE receiver is used at the destination node. The MMSE criterion is used to detect the transmitted signal at each stream. We show that the optimal source precoding matrix and the optimal relay amplifying matrices have a beamforming structure. By using such optimal source and relay matrices and the MMSE-DFE receiver, a joint source and relay power loading algorithm is developed to minimize the MSE of the signal waveform estimation.

The material in Chapter 5 is based on the journal publication:

- A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Optimization for Parallel MIMO Relay Networks", *EURASIP Journal on Advances in Signal Processing*, 2012:174.

and the conference publications:

- A. Toding, M. R. A. Khandaker, and Y. Rong, "Joint Source and Relay Optimization for Parallel MIMO Relays Using MMSE-DFE Receiver", in *Proc. 16th Asia-Pacific Conference on Communications (APCC'2010)*, Auckland, New Zealand, Nov. 1-3, 2010, pp. 12-16.

- A. Toding and Y. Rong, "Investigating Successive Interference Cancellation in MIMO Relay Network", in *Proc. IEEE TENCON*, Bali, Indonesia, Nov. 21-24, 2011, pp. 359-363.

Chapter 2

MIMO Transceiver Design with Distributed Relay Nodes

In this chapter, we study the optimal transmit beamforming vector and the relay amplifying factors design problem for a parallel MIMO relay communication system with distributed single-antenna relay nodes when a single data stream is transmitted from the source to destination node. By using the optimal beamforming vector, an iterative joint source and relay beamforming algorithm is developed to minimize the MSE of the signal waveform estimation. In Section 2.1, we give a brief overview of existing works. The system model of a parallel MIMO relay communication system with distributed single-antenna relay nodes is introduced in Section 2.2. In Section 2.3, we develop the jointly optimal source and relay algorithm. Section 2.4 shows the simulation results. The chapter is briefly summarized in Section 2.5.

2.1 Overview of Single-Antenna Relay Technique

In wireless communications, signal fading and shadowing effects are very common phenomena that degrade the system performance. An efficient way to address this issue is to transmit signals through one or multiple relays [1–23]. Introducing multiple antennas at transmitting and receiving ends, which we call MIMO relay communication systems, can provide further improvement in terms of both spectral efficiency and link reliability [37, 47]. Many works have studied the optimal relay amplifying matrix for MIMO relay channels. In [12] and [54], the optimal relay amplifying matrix is designed to maximize

the MI between the source node and the destination node, assuming that the source covariance matrix is an identity matrix. In [57] and [58], the optimal relay amplifying matrix was designed to minimize the MSE of the signal waveform estimation at the destination node.

However, few researches have studied the jointly optimal source precoding matrix and the relay amplifying matrix for the source-relay-destination channel. In [60], a unified framework was developed to jointly optimize the source precoding matrix and the relay amplifying matrix for a broad class of objective functions. In [59], the source covariance matrix and the relay amplifying matrix were jointly designed to maximize the source-destination MI. All the works [12, 37, 47, 54, 57–60] focus on MIMO relay systems with a single relay node at each hop. In [38] and [89], the optimal source and relay matrices were designed for a multi-hop MIMO relay network with serial relays.

MIMO relay communication systems with multiple parallel relay nodes have been investigated in [73, 74]. In [74], the authors investigated the jointly optimal structure of the source precoding matrix and the relay amplifying matrices considering a linear MMSE receiver at the destination node. On the other hand, a distributed relay network is investigated in [29] where multiple users and relays, each having a single-antenna, are considered.

In this chapter, we propose a jointly optimal source and relay beamforming algorithm which minimizes the MSE of the signal waveform estimation for single-antenna relay nodes in a MIMO relay communication system. In contrast to [73, 74], where the receive power constraint at the destination node is considered, we consider in this chapter the sum transmit power constraint throughout all relay nodes.

2.2 Distributed MIMO Relay System Model

Fig. 2.1 illustrates a two-hop parallel MIMO relay communication system consisting of one source node, K parallel relay nodes, and one destination node. We assume that the source and the destination nodes have N_s and N_d antennas, respectively, whereas each relay node has a single-antenna. Due to its merit of simplicity, we consider the amplify-and-forward scheme at each relay. The communication process between the source and destination nodes is completed in two time slots. In the first time slot, the modulated symbol s is linearly precoded as

$$\mathbf{x} = \mathbf{b}s \tag{2.1}$$

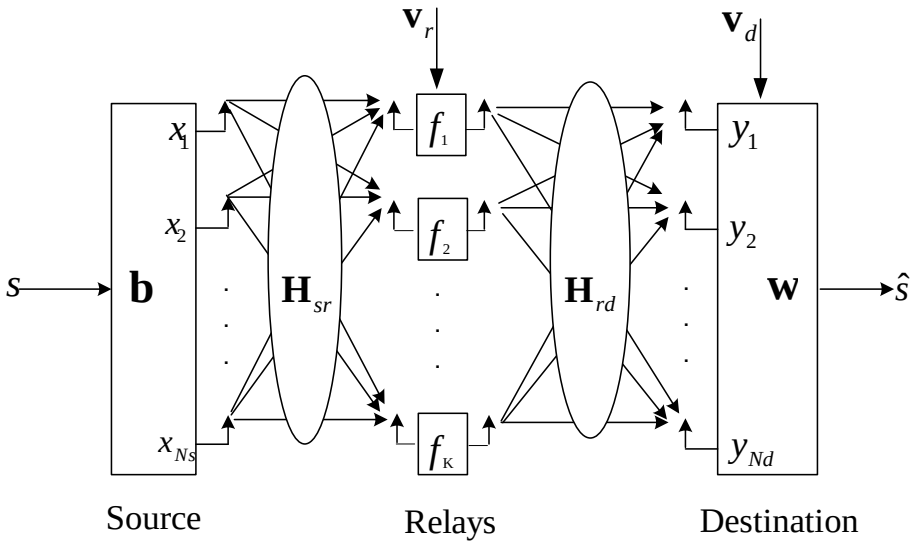

Figure 2.1: Block diagram of a parallel MIMO relay communication system with a single data stream.

where $\mathbf{b}$ is an $N_s \times 1$ transmit beamforming vector. The precoded vector $\mathbf{x}$ is then transmitted to the relay nodes. The received signal at the ith relay node can be written as

$$y_{r,i} = \mathbf{h}_{sr,i}\mathbf{x} + v_{r,i}, \qquad i = 1, \cdots, K \tag{2.2}$$

where $\mathbf{h}_{sr,i}$ is the $1 \times N_s$ channel vector between the source and the ith relay node, $y_{r,i}$ and $v_{r,i}$ are the received signal and the additive Gaussian noise at the ith relay node, respectively.

In the second time slot, the source node is silent, while each relay node transmits the amplified signal to the destination node as

$$x_{r,i} = f_i y_{r,i}, \qquad i = 1, \cdots, K \tag{2.3}$$

where f_i is the amplifying coefficient at the ith relay node. The received signal vector at the destination node can be written as

$$\mathbf{y}_d = \sum_{i=1}^{K} \mathbf{h}_{rd,i} x_{r,i} + \mathbf{v}_d \tag{2.4}$$

where $\mathbf{h}_{rd,i}$ is the $N_d \times 1$ channel vector between the ith relay and the destination node, $\mathbf{y}_d$ and $\mathbf{v}_d$ are the received signal and the additive Gaussian noise vector at the destination node, respectively.

Substituting (2.1)-(2.3) into (2.4), we have

$$\begin{aligned}\mathbf{y}_d &= \sum_{i=1}^{K}(\mathbf{h}_{rd,i}f_i\mathbf{h}_{sr,i}\mathbf{b}s + \mathbf{h}_{rd,i}f_i v_{r,i}) + \mathbf{v}_d \\ &= \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{b}s + \mathbf{H}_{rd}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d \end{aligned} \tag{2.5}$$

where we define

$$\begin{aligned}\mathbf{H}_{sr} &\triangleq \left[\mathbf{h}_{sr,1}^T, \mathbf{h}_{sr,2}^T, \cdots, \mathbf{h}_{sr,K}^T\right]^T \\ \mathbf{H}_{rd} &\triangleq [\mathbf{h}_{rd,1}, \mathbf{h}_{rd,2}, \cdots, \mathbf{h}_{rd,K}] \\ \mathbf{F} &\triangleq \mathrm{diag}\left([f_1, f_2, \cdots, f_K]^T\right) \\ \mathbf{v}_r &\triangleq \left[v_{r,1}, v_{r,2}, \cdots, v_{r,K}\right]^T.\end{aligned}$$

Here $\mathbf{H}_{sr}$ is a $K \times N_s$ channel matrix between the source node and all relay nodes, $\mathbf{H}_{rd}$ is an $N_d \times K$ channel matrix between all relay nodes and the destination node, $\mathbf{F}$ is a $K \times K$ diagonal equivalent relay matrix, and $\mathbf{v}_r$ is obtained by stacking the noise terms at all the relays. We assume that all noises are independent and identically distributed (i.i.d.) with zero mean and unit variance.

The diagram of the equivalent MIMO relay system with a single data stream described by (2.5) is shown in Fig. 2.2. The received signal vector at the destination node can be equivalently written as

$$\mathbf{y}_d = \bar{\mathbf{h}}s + \bar{\mathbf{v}}$$

where we define $\bar{\mathbf{h}} \triangleq \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{b}$ as the effective MIMO channel vector of the source-relay-destination link, and $\bar{\mathbf{v}} \triangleq \mathbf{H}_{rd}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d$ as the equivalent noise vector.

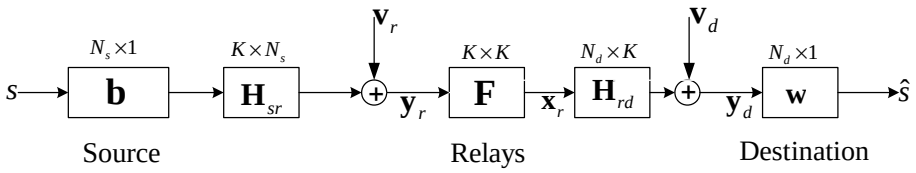

Figure 2.2: Block diagram of the equivalent MIMO relay system with a single data stream.

2.3 Optimal Source and Relay Design

In this section we develop the optimal transmit beamforming vector $\mathbf{b}$ and the relay amplifying matrix $\mathbf{F}$ to minimize the MSE of the signal waveform estimation. By using

a linear receiver, the estimated signal waveform vector at the destination node is given by

$$\hat{s} = \mathbf{w}^H \mathbf{y}_d \tag{2.6}$$

where $\mathbf{w}$ is an $N_d \times 1$ weight vector.

The MMSE approach tries to find a weight vector $\mathbf{w}$ that minimizes the statistical expectation of the signal waveform estimation given by

$$\text{MSE} = \mathrm{E}\big[|\hat{s} - s|^2\big]. \tag{2.7}$$

We assume that the source signal satisfies $\mathrm{E}[|s|^2] = 1$. Substituting (2.6) into (2.7), we find that the $\mathbf{w}$ which minimizes (2.7) can be written as

$$\mathbf{w} \;=\; (\bar{\mathbf{h}}\bar{\mathbf{h}}^H + \mathbf{C}_{\bar{v}})^{-1}\bar{\mathbf{h}} \tag{2.8}$$

where $\mathbf{C}_{\bar{v}}$ is the equivalent noise covariance matrix given by

$$\begin{aligned} \mathbf{C}_{\bar{v}} \;&=\; \mathrm{E}\big[\bar{\mathbf{v}}\bar{\mathbf{v}}^H\big] \\ &=\; \mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{I}_{N_d}. \end{aligned} \tag{2.9}$$

Substituting (2.8) back into (2.7), we obtain the minimal MSE as a function of $\mathbf{b}$ and $\mathbf{F}$, given by

$$\text{MSE}=1-\bar{\mathbf{h}}^H(\bar{\mathbf{h}}\bar{\mathbf{h}}^H + \mathbf{C}_{\bar{v}})^{-1}\bar{\mathbf{h}}. \tag{2.10}$$

Applying the matrix inversion lemma $(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{DA}^{-1}\mathbf{B}+\mathbf{C}^{-1})^{-1}\mathbf{DA}^{-1}$, (2.10) can be written as

$$\text{MSE}=\big(1+\bar{\mathbf{h}}^H\bar{\mathbf{C}}_{\bar{v}}^{-1}\bar{\mathbf{h}}\big)^{-1}. \tag{2.11}$$

From (2.3), the total transmission power consumed by all the relay nodes can be expressed as

$$\mathrm{tr}(\mathrm{E}[\mathbf{x}_r\mathbf{x}_r^H]) = \mathrm{tr}(\mathbf{F}\big[\mathbf{H}_{sr}\mathbf{b}\mathbf{b}^H\mathbf{H}_{sr}^H + \mathbf{I}_K\big]\mathbf{F}^H). \tag{2.12}$$

By using (2.12), the joint source and relay optimization problem can be formulated as

$$\min_{\mathbf{F},\mathbf{b}} \quad \text{MSE} \tag{2.13}$$

$$\text{s.t.} \quad \mathbf{b}^H\mathbf{b} \le P_s \tag{2.14}$$

$$\mathrm{tr}\big(\mathbf{F}\big[\mathbf{H}_{sr}\mathbf{b}\mathbf{b}^H\mathbf{H}_{sr}^H + \mathbf{I}_K\big]\mathbf{F}^H\big) \le P_r \tag{2.15}$$

where (2.14) is the transmit power constraint at the source node, and (2.15) is the sum transmit power constraint throughout all relay nodes. Here $P_r > 0$ and $P_s > 0$ are the corresponding power budget. The problem (2.13)-(2.15) is highly nonconvex and a closed-form expression of the optimal $\mathbf{F}$ and $\mathbf{b}$ is intractable. In the following, we develop an iterative algorithm to optimize $\mathbf{F}$ and $\mathbf{b}$.

2.3.1 Optimal Relay Factors

For given beamforming vector $\mathbf{b}$ satisfying (2.14), we optimize the relay matrix $\mathbf{F}$ by solving the following optimization problem

$$\min_{\mathbf{F}} \quad \text{MSE} \tag{2.16}$$

$$\text{s.t.} \quad \text{tr}\big(\mathbf{F}\big[\mathbf{H}_{sr}\mathbf{b}\mathbf{b}^H\mathbf{H}_{sr}^H+\mathbf{I}_K\big]\mathbf{F}^H\big) \le P_r. \tag{2.17}$$

Let us introduce

$$\bar{\mathbf{h}}_s \triangleq \mathbf{H}_{sr}\mathbf{b}. \tag{2.18}$$

Substituting (2.18) back into (2.16)-(2.17), we can rewrite the optimization problem as

$$\min_{\mathbf{F}} \big[1+\bar{\mathbf{h}}_s^H\mathbf{F}^H\mathbf{H}_{rd}^H(\mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H+\mathbf{I}_{N_d})^{-1}\mathbf{H}_{rd}\mathbf{F}\bar{\mathbf{h}}_s\big]^{-1} \tag{2.19}$$

$$\text{s.t.} \sum_{i=1}^{K}|f_i|^2\big(|\bar{h}_{s,i}|^2+1\big) \le P_r. \tag{2.20}$$

The problem (2.19)-(2.20) is equivalent to

$$\max_{\mathbf{F}} \ \bar{\mathbf{h}}_s^H\mathbf{F}^H\mathbf{H}_{rd}^H(\mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H+\mathbf{I}_{N_d})^{-1}\mathbf{H}_{rd}\mathbf{F}\bar{\mathbf{h}}_s \tag{2.21}$$

$$\text{s.t.} \ \sum_{i=1}^{K}|f_i|^2\big(|\bar{h}_{s,i}|^2+1\big) \le P_r. \tag{2.22}$$

Since the objective function (2.21) is still a complicated function of $\mathbf{F}$, in the following, we optimize an upper-bound of (2.21). The problem can be rewritten as

$$\max_{\mathbf{F}} \ \bar{\mathbf{h}}_s^H\mathbf{F}^H\mathbf{H}_{rd}^H\mathbf{H}_{rd}\mathbf{F}\bar{\mathbf{h}}_s \tag{2.23}$$

$$\text{s.t.} \ \sum_{i=1}^{K}|f_i|^2\big(|\bar{h}_{s,i}|^2+1\big) \le P_r. \tag{2.24}$$

Let $\mathbf{f} \triangleq [f_1, f_2, \cdots, f_K]^T$ denote the diagonal elements of $\mathbf{F}$ and define $\mathbf{D}_s \triangleq \text{diag}(\bar{\mathbf{h}}_s)$, so that

$$\mathbf{F}\bar{\mathbf{h}}_s = \mathbf{D}_s\mathbf{f}. \tag{2.25}$$

Now by substituting (2.25) in (2.23)-(2.24), we can express the maximization problem as follows

$$\max_{\mathbf{f}} \quad \mathbf{f}^H \mathbf{D}_s^H \mathbf{H}_{rd}^H \mathbf{H}_{rd} \mathbf{D}_s \mathbf{f} \tag{2.26}$$

$$\text{s.t.} \quad \mathbf{f}^H \mathbf{A} \mathbf{f} \leq P_r \tag{2.27}$$

where $\mathbf{A} \triangleq \mathbf{D}_s \mathbf{D}_s^H + \mathbf{I}_K$. Defining $\bar{\mathbf{f}} \triangleq \mathbf{A}^{\frac{1}{2}} \mathbf{f}$, the problem (2.26)-(2.27) can be equivalently written as

$$\begin{aligned} \max_{\bar{\mathbf{f}}} \quad & \bar{\mathbf{f}}^H \mathbf{A}^{-\frac{H}{2}} \mathbf{D}_s^H \mathbf{H}_{rd}^H \mathbf{H}_{rd} \mathbf{D}_s \mathbf{A}^{-\frac{1}{2}} \bar{\mathbf{f}} \\ \text{s.t.} \quad & \bar{\mathbf{f}}^H \bar{\mathbf{f}} \leq P_r. \end{aligned}$$

Introducing $\mathbf{Z} \triangleq \mathbf{A}^{-\frac{H}{2}} \mathbf{D}_s^H \mathbf{H}_{rd}^H \mathbf{H}_{rd} \mathbf{D}_s \mathbf{A}^{-\frac{1}{2}}$, we obtain

$$\max_{\bar{\mathbf{f}}} \quad \bar{\mathbf{f}}^H \mathbf{Z} \bar{\mathbf{f}} \tag{2.28}$$

$$\text{s.t.} \quad \bar{\mathbf{f}}^H \bar{\mathbf{f}} \leq P_r. \tag{2.29}$$

The Lagrangian of the optimization problem (2.28)-(2.29) can be written as

$$\mathcal{L} = -\bar{\mathbf{f}}^H \mathbf{Z} \bar{\mathbf{f}} + \mu_1 (\bar{\mathbf{f}}^H \bar{\mathbf{f}} - P_r) \tag{2.30}$$

where $\mu_1 \geq 0$ is the Lagrangian multiplier associated with the constraint (2.29). Taking the derivative of $\mathcal{L}$ with respect to $\bar{\mathbf{f}}^H$ and letting the result be 0, it can be shown that the optimal $\bar{\mathbf{f}}$ satisfies the following equation

$$\mathbf{Z} \bar{\mathbf{f}} = \mu_1 \bar{\mathbf{f}}.$$

Thus $\bar{\mathbf{f}} = \sqrt{P_r}\text{eig}(\mathbf{Z})$, where $\text{eig}(\mathbf{Z})$ stands for the principal eigenvector of $\mathbf{Z}$.

2.3.2 Joint Source and Relay Optimization Algorithm

For a fixed $\mathbf{F}$, the problem (2.13) - (2.15) can be expressed as below to optimize the beamforming vector $\mathbf{b}$

$$\min_{\mathbf{b}} \quad \left(1 + \mathbf{b}^H \mathbf{\Psi}_1 \mathbf{b}\right)^{-1} \tag{2.31}$$

$$\text{s.t.} \quad \mathbf{b}^H \mathbf{b} \leq P_s \tag{2.32}$$

$$\mathbf{b}^H \mathbf{\Psi}_2 \mathbf{b} \leq \bar{P}_r \tag{2.33}$$

where we define

$$\begin{aligned}\mathbf{\Psi}_1 &\triangleq \mathbf{H}_{sr}^H \mathbf{F}^H \mathbf{H}_{rd}^H \mathbf{H}_{rd} \mathbf{F} \mathbf{H}_{sr} \\ \mathbf{\Psi}_2 &\triangleq \mathbf{H}_{sr}^H \mathbf{F}^H \mathbf{F} \mathbf{H}_{sr} \\ \bar{P}_r &\triangleq P_r - \mathrm{tr}(\mathbf{F}\mathbf{F}^H).\end{aligned}$$

The problem (2.31)-(2.33) is equivalent to

$$\max_{\mathbf{b}} \quad \mathbf{b}^H \mathbf{\Psi}_1 \mathbf{b} \tag{2.34}$$

$$\text{s.t.} \quad \mathbf{b}^H \mathbf{b} \leq P_s \tag{2.35}$$

$$\mathbf{b}^H \mathbf{\Psi}_2 \mathbf{b} \leq \bar{P}_r. \tag{2.36}$$

The Lagrangian function associated with the problem (2.34)-(2.36) can be written as

$$\mathcal{L} = -\mathbf{b}^H \mathbf{\Psi}_1 \mathbf{b} + \mu_2(\mathbf{b}^H \mathbf{b} - P_s) + \mu_3(\mathbf{b}^H \mathbf{\Psi}_2 \mathbf{b} - \bar{P}_r). \tag{2.37}$$

Here $\mu_2 \geq 0$ and $\mu_3 \geq 0$ are the Lagrangian multipliers associated with the constraints (2.35) and (2.36), respectively. The problem (2.34)-(2.36) can be solved by using KKT conditions [106] that can be expressed as

$$\frac{\partial \mathcal{L}}{\partial \mathbf{b}^H} = 0 \tag{2.38}$$

$$\mu_2(\mathbf{b}^H \mathbf{b} - P_s) = 0 \tag{2.39}$$

$$\mu_3(\mathbf{b}^H \mathbf{\Psi}_2 \mathbf{b} - \bar{P}_r) = 0 \tag{2.40}$$

$$\mathbf{b}^H \mathbf{b} \leq P_s \tag{2.41}$$

$$\mathbf{b}^H \mathbf{\Psi}_2 \mathbf{b} \leq \bar{P}_r \tag{2.42}$$

$$\mu_2 \geq 0, \quad \mu_3 \geq 0. \tag{2.43}$$

Now we set out to solve (2.38)-(2.43). If $\mu_2 > 0$ and $\mu_3 = 0$, then the optimization problem (2.34)-(2.36) can be rewritten as

$$\max_{\mathbf{b}} \quad \mathbf{b}^H \mathbf{\Psi}_1 \mathbf{b} \tag{2.44}$$

$$\text{s.t.} \quad \mathbf{b}^H \mathbf{b} = P_s. \tag{2.45}$$

Thus we solve $\mathbf{b}$ as

$$\mathbf{b} = \sqrt{P_s}\, \mathrm{eig}(\mathbf{\Psi}_1). \tag{2.46}$$

If $\mathbf{b}$ in (2.46) satisfies the constraint (2.36), then (2.46) is the optimal solution to the problem (2.34)-(2.36). Otherwise, if $\mu_3 > 0$ and $\mu_2 = 0$, then the optimization problem (2.34)-(2.36) can be rewritten as

$$\max_{\mathbf{b}} \quad \mathbf{b}^H \mathbf{\Psi}_1 \mathbf{b} \tag{2.47}$$

$$\text{s.t.} \quad \mathbf{b}^H \mathbf{\Psi}_2 \mathbf{b} = \bar{P}_r. \tag{2.48}$$

Then we solve $\mathbf{b}$ as

$$\mathbf{b} = \alpha \, \text{eig}(\mathbf{\Psi}_2^{-1} \mathbf{\Psi}_1) \tag{2.49}$$

where $\alpha = \sqrt{\bar{P}_r / (\text{eig}^H(\mathbf{\Psi}_2^{-1}\mathbf{\Psi}_1)\mathbf{\Psi}_2 \text{eig}(\mathbf{\Psi}_2^{-1}\mathbf{\Psi}_1))}$. If $\mathbf{b}$ in (2.49) satisfies the constraint (2.35), then (2.49) is the optimal solution to the problem (2.34)-(2.36).

Finally, if $\mu_2 > 0$ and $\mu_3 > 0$, then we have from (2.38) that

$$\mathbf{\Psi}_1 \mathbf{b} = \mu_2 \mathbf{b} + \mu_3 \mathbf{\Psi}_2 \mathbf{b}$$

which can be equivalently written as

$$\left(\mathbf{I}_{N_s} + \frac{\mu_3}{\mu_2} \mathbf{\Psi}_2 \right)^{-1} \mathbf{\Psi}_1 \mathbf{b} = \mu_2 \mathbf{b}. \tag{2.50}$$

Thus the optimal $\mathbf{b}$ can be obtained as

$$\mathbf{b} = \sqrt{P_s} \, \text{eig}((\mathbf{I}_{N_s} + \lambda \mathbf{\Psi}_2)^{-1} \mathbf{\Psi}_1) \tag{2.51}$$

where $\lambda > 0$ can be found by substituting (2.51) back into (2.48) and solving the obtained nonlinear equation.

Table 2.1: Procedure of solving the problem (2.13) - (2.15) by the proposed iterative algorithm.

1. Initialize the algorithm with random $\mathbf{b}^{(0)}$; Set $n = 0$.
2. Solve the subproblem (2.26)-(2.27) using given $\mathbf{b}^{(n)}$ to obtain $\mathbf{f}^{(n)}$.
3. Solve the subproblem (2.34)-(2.36) using $\mathbf{F}^{(n)}$ to obtain $\mathbf{b}^{(n)}$.
4. If max abs $\|\mathbf{b}^{(n+1)} - \mathbf{b}^{(n)}\| \leq \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to step 2.

Now the original problem (2.13)-(2.15) can be solved in an iterative fashion. In each iteration, we first fix $\mathbf{b}$ and update $\mathbf{f}$ by solving the problem (2.26)-(2.27). Then we update $\mathbf{b}$ with $\mathbf{F}$ fixed through solving the problem (2.34)-(2.36). The procedure of the proposed iterative algorithm is listed in Table 2.1, where ε is a small positive number close to zero. Since each update may only reduce or maintain, but can not increase the MSE, a monotonic convergence of this iterative algorithm follows directly from this observation.

2.4 Numerical Examples

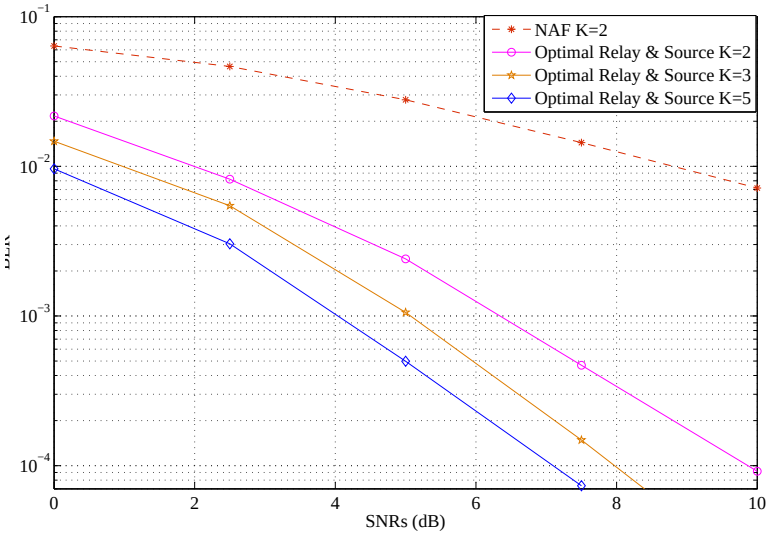

Figure 2.3: Example 2.1 BER versus $\mathrm{SNR_s}$ with varying K. $N_s = N_d = 3$, $\mathrm{SNR_r} = 20\mathrm{dB}$.

In this section, we study the performance of the proposed optimal transmit beamforming vector and the relay amplifying factors for a parallel MIMO relay communication system with distributed single-antenna relay nodes when a single data stream is transmitted from the source to destination node. We compare the performance of the proposed algorithm with the naive amplify-and-forward (NAF) scheme in terms of BER. For the NAF scheme, we define source and relay matrices as scaled identity matrices satisfying the power constraints. All simulations are conducted in a flat Rayleigh fading environment using binary phase shift keying (BPSK) constellations, and the noises are i.i.d. Gaussian random variables with zero mean and unit variance. The channel

matrices have zero-mean entries with variances σ_s^2/N_s and σ_r^2/K for $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. We vary the SNR in the source-to-relay link ($\mathrm{SNR_s}$) while fixing the SNR in the relay-to-destination link ($\mathrm{SNR_r}$) to 20dB. We transmit 1000 randomly generated bits in each channel realization, and the BER results are averaged through 200 random channel realizations.

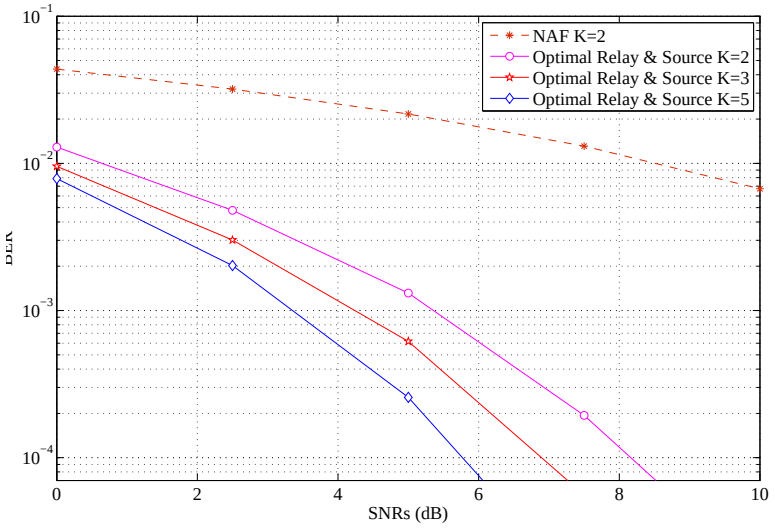

Figure 2.4: Example 2.2 BER versus $\mathrm{SNR_s}$ with varying K. $N_s = N_d = 5$, $\mathrm{SNR_r} = 20\mathrm{dB}$.

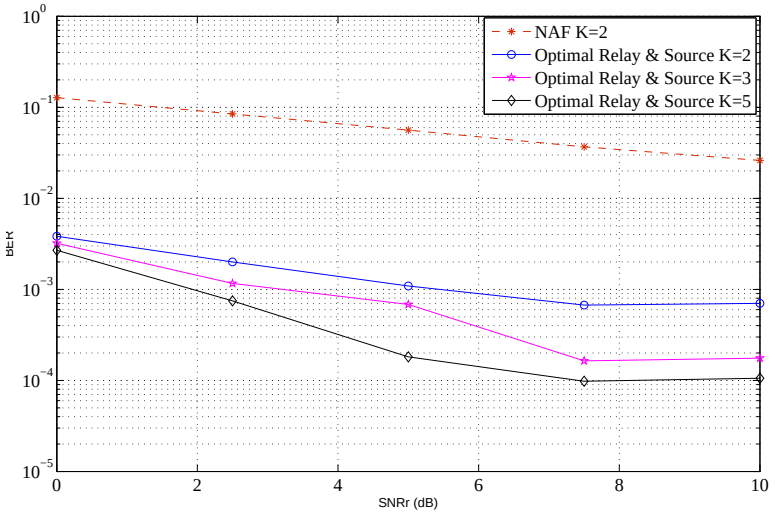

Figure 2.5: Example 2.3 BER versus $\mathrm{SNR_r}$ with varying K. $N_s = N_d = 3$, $\mathrm{SNR_s} = 20\mathrm{dB}$.

In the first example, we study the effect of the number of relays to the system BER

performance using the proposed algorithm. We choose $N_s = N_d = 3$. Fig. 2.3 shows the BER performance with $K = 2$, 3, and 5. It can be seen that at BER = 10^{-3}, we achieve a 2.5dB gain by increasing from $K = 2$ to $K = 5$.

In the second example, we simulate a MIMO relay communication system with distributed single-antenna relay nodes with $N_s = N_d = 5$. Fig. 2.4 shows the BER performance with $K = 2$, 3, and 5. It can be seen that at BER = 10^{-3}, we achieve a 2dB gain by increasing from $K = 2$ to $K = 5$.

In the last example, we study the effect of the number of relays to the system BER performance versus SNR_r. We choose $N_s = N_d = 3$. Fig. 2.5 shows the BER performance of the algorithms with $K = 2$, 3, and 5. It can be seen that at BER = 10^{-3}, we achieve a 3dB gain by increasing from $K = 2$ to $K = 5$.

Table 2.2 indicates how fast the proposed iterative algorithm converges. The number of iterations required in a typical run to converge up to $\varepsilon = 10^{-3}$ is shown in the table. Here we set $N_s = N_d = 3$, $K = 2$.

Table 2.2: Iterations required till convergence in the proposed algorithm

SNR_s (dB)	0	2	4	6	8	10
Iterations required	2	3	3	4	5	5

2.5 Chapter Summary

In this chapter, we have studied the optimal transmit beamforming vector and the relay amplifying factors design problem of a parallel MIMO relay communication system with distributed single-antenna relay nodes when a single data stream is transmitted from the source to destination node. The proposed joint source and relay beamforming algorithm provides an improved performance.

Chapter 3

Parallel MIMO Relay Design Based on Linear Receiver

In this chapter, we develop the optimal source precoding matrix and relay amplifying matrices for two-hop non-regenerative MIMO multi-relay systems with multiple data streams when a linear MMSE receiver is used at the destination node. We provide an overview of existing works in Section 3.1. In Section 3.2, the parallel MIMO relay system model is introduced. We show in Section 3.3 that the optimal relay amplifying matrices have a beamforming structure. Exploiting the structure of relay matrices, an iterative joint source and relay matrices optimization algorithm is developed to minimize the MSE of the signal waveform estimation at the destination node using the PG approach. The performance of the proposed algorithm is demonstrated through numerical simulations in Section 3.4. Finally, the chapter is summarized in Section 3.5.

3.1 Existing Works with Linear MMSE Receiver

Recently, MIMO relay communication systems have attracted much research interest and provided significant improvement in terms of both spectral efficiency and link reliability [3, 12, 38, 54, 57–60, 69, 73, 75]. Many works have studied the optimal relay amplifying matrix for the source-relay-destination channel. In [12, 54], the optimal relay amplifying matrix maximizing the mutual information (MI) between the source and destination nodes was derived assuming that the source covariance matrix is an identity

matrix. In [57, 69], the optimal relay amplifying matrix was designed to minimize MSE of the signal waveform estimation at the destination node.

A few researches have studied the joint optimization of the source precoding matrix and the relay amplifying matrix for the source-relay-destination channel. In [59], both the source and relay matrices were jointly designed to maximize the source-destination MI. In [38, 60], source and relay matrices were developed to jointly optimize a broad class of objective functions. The author of [75], investigated the joint source and relay optimization for two-way MIMO relay systems using the projected gradient (PG) approach. All the works in [3, 12, 38, 54, 57–60, 69, 75] considered a single relay node at each hop. The authors of [73] developed the optimal relay amplifying matrices with multiple relay nodes. In Chapter 2, we proposed a jointly optimal source and relay beamforming algorithm which minimizes the MSE of the signal waveform estimation for single-antenna relay nodes in a MIMO relay communication system.

In this chapter, we propose a jointly optimal source precoding matrix and relay amplifying matrices design for a two-hop non-regenerative MIMO relay communication system with multiple relay nodes using the PG approach. We show that the optimal relay amplifying matrices have a beamforming structure. This result generalizes the optimal source and relay matrices design from a single relay node case [60] to multiple parallel relay nodes scenario. Exploiting the structure of relay matrices, an iterative joint source and relay matrices optimization algorithm is developed to minimize the MSE of the signal waveform estimation. Different to Chapter 2, in this chapter, we study the optimal source and relay matrices which minimize the MSE of the signal waveform estimation of multiple data streams with multi-antenna relay nodes. Simulation results demonstrate the effectiveness of the proposed iterative joint source and relay matrices design algorithm with multiple parallel relay nodes using the PG approach.

3.2 Parallel MIMO Relay System

In this section, we introduce the model of a two-hop parallel MIMO relay communication system consisting of one source node, K parallel relay nodes, and one destination node with multiple data streams as shown in Fig. 3.1. We assume that the source and destination nodes have N_s and N_d antennas, respectively, and each relay node has N_r antennas. The generalization to systems with different number of antennas at each

relay node is straightforward. Due to its merit of simplicity, a linear non-regenerative strategy is applied at each relay node.

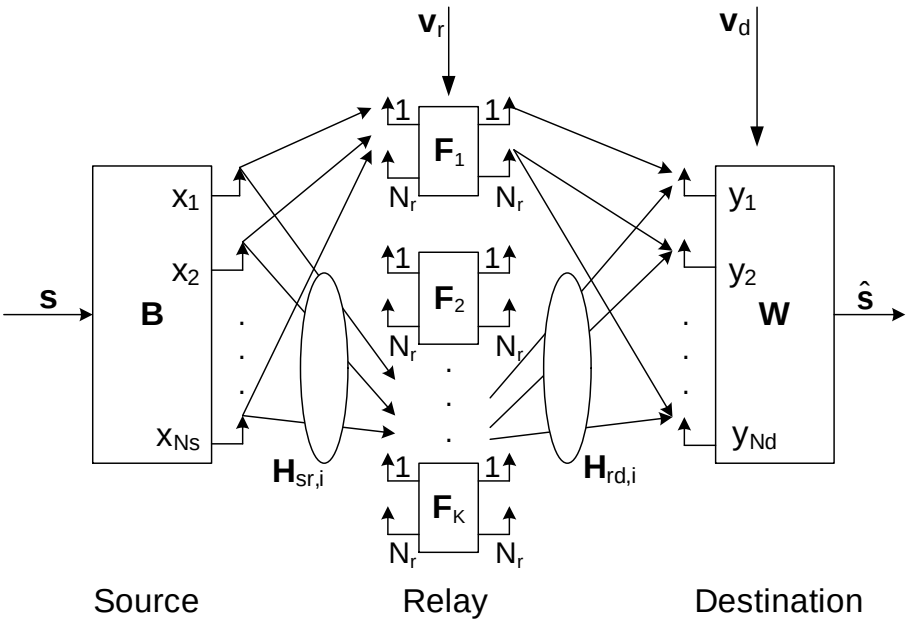

Figure 3.1: Block diagram of a parallel MIMO relay communication system with multiple data streams.

The communication process between the source and destination nodes is completed in two time slots. In the first time slot, the $N_b \times 1$ modulated source symbol vector $\mathbf{s}$ is linearly precoded as

$$\mathbf{x} = \mathbf{B}\,\mathbf{s} \tag{3.1}$$

where $\mathbf{B}$ is an $N_s \times N_b$ source precoding matrix. We assume that the source signal vector satisfies $\mathrm{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_{N_b}$. The precoded vector $\mathbf{x}$ is transmitted to K parallel relay nodes. The $N_r \times 1$ received signal vector at the ith relay node can be written as

$$\mathbf{y}_{r,i} = \mathbf{H}_{sr,i}\mathbf{x} + \mathbf{v}_{r,i}, \qquad i = 1, \cdots, K \tag{3.2}$$

where $\mathbf{H}_{sr,i}$ is the $N_r \times N_s$ MIMO channel matrix between the source and the ith relay nodes and $\mathbf{v}_{r,i}$ is the additive Gaussian noise vector at the ith relay node.

In the second time slot, the source node is silent, while each relay node transmits the linearly amplified signal vector to the destination node as

$$\mathbf{x}_{r,i} = \mathbf{F}_i\,\mathbf{y}_{r,i}, \qquad i = 1, \cdots, K \tag{3.3}$$

where $\mathbf{F}_i$ is the $N_r \times N_r$ amplifying matrix at the ith relay node. The received signal vector at the destination node can be written as

$$\mathbf{y}_d = \sum_{i=1}^{K} \mathbf{H}_{rd,i}\mathbf{x}_{r,i} + \mathbf{v}_d \tag{3.4}$$

where $\mathbf{H}_{rd,i}$ is the $N_d \times N_r$ MIMO channel matrix between the ith relay and the destination nodes, $\mathbf{v}_d$ is the additive Gaussian noise vector at the destination node. Substituting (3.1)-(3.3) into (3.4), we have

$$\begin{aligned}\mathbf{y}_d &= \sum_{i=1}^{K}(\mathbf{H}_{rd,i}\mathbf{F}_i\mathbf{H}_{sr,i}\mathbf{B}\mathbf{s} + \mathbf{H}_{rd,i}\mathbf{F}_i\mathbf{v}_{r,i}) + \mathbf{v}_d \\ &= \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B}\mathbf{s} + \mathbf{H}_{rd}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d \\ &\triangleq \bar{\mathbf{H}}\mathbf{s} + \bar{\mathbf{v}}\end{aligned} \tag{3.5}$$

where $\mathbf{H}_{sr} \triangleq [\mathbf{H}_{sr,1}^T, \mathbf{H}_{sr,2}^T, \cdots, \mathbf{H}_{sr,K}^T]^T$ is a $KN_r \times N_s$ channel matrix between the source node and all relay nodes, $\mathbf{H}_{rd} \triangleq [\mathbf{H}_{rd,1}, \mathbf{H}_{rd,2}, \cdots, \mathbf{H}_{rd,K}]$ is an $N_d \times KN_r$ channel matrix between all relay nodes and the destination node, $\mathbf{F} \triangleq \mathrm{bd}[\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_K]$ is the $KN_r \times KN_r$ block diagonal equivalent relay matrix, $\mathbf{v}_r \triangleq \left[\mathbf{v}_{r,1}^T, \mathbf{v}_{r,2}^T, \cdots, \mathbf{v}_{r,K}^T\right]^T$ is obtained by stacking the noise vectors at all the relays, $\bar{\mathbf{H}} \triangleq \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B}$ is the effective MIMO channel matrix of the source-relay-destination link, and $\bar{\mathbf{v}} \triangleq \mathbf{H}_{rd}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d$ is the equivalent noise vector. We assume that all noises are i.i.d. Gaussian noise with zero mean and unit variance. The diagram of the equivalent MIMO relay system with multiple data streams described by (3.5) is shown in Fig. 3.2. The transmission power consumed by each relay node (3.3) can be expressed as

$$\mathrm{E}\big[\mathrm{tr}(\mathbf{x}_{r,i}\mathbf{x}_{r,i}^H)\big] = \mathrm{tr}\big(\mathbf{F}_i\big[\mathbf{H}_{sr,i}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}\big]\mathbf{F}_i^H\big)\,, \qquad i = 1, \cdots, K. \tag{3.6}$$

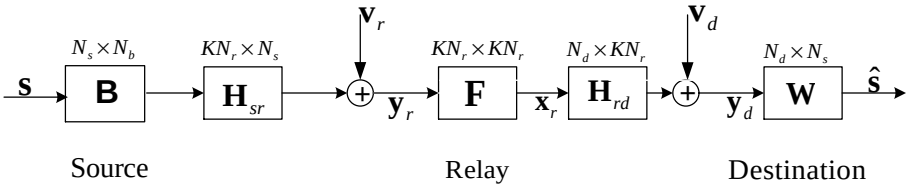

Figure 3.2: Block diagram of the equivalent MIMO relay system with multiple data streams.

By using a linear receiver, the estimated signal waveform vector at the destination node is given by $\hat{\mathbf{s}} = \mathbf{W}^H\mathbf{y}_d$, where $\mathbf{W}$ is an $N_d \times N_b$ weight matrix. The MSE of the

signal waveform estimation is given by

$$\begin{aligned}\text{MSE} &= \text{tr}\Big(\text{E}\Big[\big(\hat{\mathbf{s}}-\mathbf{s}\big)\big(\hat{\mathbf{s}}-\mathbf{s}\big)^H\Big]\Big)\\ &= \text{tr}\Big(\big(\mathbf{W}^H\bar{\mathbf{H}}-\mathbf{I}_{N_b}\big)\big(\mathbf{W}^H\bar{\mathbf{H}}-\mathbf{I}_{N_b}\big)^H+\mathbf{W}^H\mathbf{C}_{\bar{v}}\mathbf{W}\Big).\end{aligned} \tag{3.7}$$

The weight matrix $\mathbf{W}$ which minimizes (3.7) is the Wiener filter and can be written as

$$\mathbf{W}=(\bar{\mathbf{H}}\bar{\mathbf{H}}^H+\mathbf{C}_{\bar{v}})^{-1}\bar{\mathbf{H}}. \tag{3.8}$$

Substituting (3.8) back into (3.7), it can be seen that the MSE is a function of $\mathbf{F}$ and $\mathbf{B}$ and can be written as

$$\text{MSE}=\text{tr}\left(\left[\mathbf{I}_{N_b}+\bar{\mathbf{H}}^H{\mathbf{C}_{\bar{v}}}^{-1}\bar{\mathbf{H}}\right]^{-1}\right). \tag{3.9}$$

3.3 Joint Source and Relay Matrices Optimization

In this section, we address the joint source and relay matrices optimization problem for MIMO multi-relay systems with a linear MMSE receiver at the destination node. In particular, we show that optimal relay matrices have a general beamforming structure. Based on (3.6) and (3.9), the joint source and relay matrices optimization problem can be formulated as

$$\min_{\{\mathbf{F}_i\},\mathbf{B}} \quad \text{tr}\left(\left[\mathbf{I}_{N_b}+\bar{\mathbf{H}}^H{\mathbf{C}_{\bar{v}}}^{-1}\bar{\mathbf{H}}\right]^{-1}\right) \tag{3.10}$$

$$\text{s.t.} \quad \text{tr}\big(\mathbf{B}\mathbf{B}^H\big)\le P_s \tag{3.11}$$

$$\text{tr}\big(\mathbf{F}_i\big[\mathbf{H}_{sr,i}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr,i}^H+\mathbf{I}_{N_r}\big]\mathbf{F}_i^H\big)\le P_{r,i},\quad i=1,\cdots,K \tag{3.12}$$

where $\{\mathbf{F}_i\}\triangleq\{\mathbf{F}_i, i=1,\cdots,L\}$, (3.11) is the transmit power constraint at the source node, while (3.12) is the power constraint at each relay node. Here $P_s>0$ and $P_{r,i}>0$, $i=1,\cdots,K$, are the corresponding power budget. Obviously, to avoid any loss of transmission power in the relay system when a linear receiver is used, there should be $N_b\le\min(N_s,KN_r,N_d)$. The problem (3.10)-(3.12) is nonconvex and a globally optimal solution of $\mathbf{B}$ and $\{\mathbf{F}_i\}$ is difficult to obtain with a reasonable computational complexity. In the following, we develop an iterative algorithm to optimize $\mathbf{B}$ and $\{\mathbf{F}_i\}$. First we show the optimal structure of $\{\mathbf{F}_i\}$.

3.3.1 Optimal Structure of Relay Amplifying Matrices

For given source matrix $\mathbf{B}$ satisfying (3.11), the relay matrices $\{\mathbf{F}_i\}$ are optimized by solving the following problem

$$\min_{\{\mathbf{F}_i\}} \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_b} + \bar{\mathbf{H}}^H \mathbf{C}_{\bar{v}}{}^{-1} \bar{\mathbf{H}}\right]^{-1}\right) \tag{3.13}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{F}_i\big[\mathbf{H}_{sr,i}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}\big]\mathbf{F}_i^H\big) \le P_{r,i}, \qquad i = 1, \cdots, K. \tag{3.14}$$

Let us introduce the following singular value decompositions (SVDs)

$$\mathbf{H}_{sr,i}\mathbf{B} = \mathbf{U}_{s,i}\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H, \quad \mathbf{H}_{rd,i} = \mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{V}_{r,i}^H, \qquad i = 1, \cdots, K \tag{3.15}$$

where $\mathbf{\Lambda}_{s,i}$ and $\mathbf{\Lambda}_{r,i}$ are $R_{s,i} \times R_{s,i}$ and $R_{r,i} \times R_{r,i}$ diagonal matrices, respectively. Here $R_{s,i} \triangleq \mathrm{rank}(\mathbf{H}_{sr,i}\mathbf{B})$, $R_{r,i} \triangleq \mathrm{rank}(\mathbf{H}_{rd,i})$, $i = 1, \cdots, K$. Based on the definition of matrix rank, $R_{s,i} \le \min(N_r, N_b)$ and $R_{r,i} \le \min(N_r, N_d)$. The following theorem states the structure of the optimal $\{\mathbf{F}_i\}$.

THEOREM 3.1: By using the SVDs of (3.15), the optimal structure of $\mathbf{F}_i$ as the solution to the problem (3.13)-(3.14) is given by

$$\mathbf{F}_i = \mathbf{V}_{r,i}\mathbf{A}_i\mathbf{U}_{s,i}^H, \quad i = 1, \cdots, K \tag{3.16}$$

where $\mathbf{A}_i$ is an $R_{r,i} \times R_{s,i}$ matrix, $i = 1, \cdots, K$.

PROOF: See Appendix 3.A. □

The remaining task is to find the optimal $\mathbf{A}_i$, $i = 1, \cdots, K$. From (3.31) and (3.32), we can equivalently rewrite the optimization problem (3.13)-(3.14) as

$$\min_{\{\mathbf{A}_i\}} \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_b} + \sum_{i=1}^{K} \mathbf{V}_{s,i}\mathbf{\Lambda}_{s,i}\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H\left(\sum_{i=1}^{K}\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H + \mathbf{I}_{N_d}\right)^{-1} \times \sum_{i=1}^{K}\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H\right]^{-1}\right) \tag{3.17}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{A}_i(\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}})\mathbf{A}_i^H\big) \le P_{r,i}, \qquad i = 1, \cdots, K. \tag{3.18}$$

Both the problem (3.13)-(3.14) and the problem (3.17)-(3.18) have matrix optimization variables. However, in the former problem, the optimization variable $\mathbf{F}_i$ is an $N_r \times N_r$ matrix, while the dimension of $\mathbf{A}_i$ is $R_{r,i} \times R_{s,i}$, which may be smaller than that of $\mathbf{F}_i$. Thus, solving the problem (3.17)-(3.18) has a smaller computational complexity than solving the problem (3.13)-(3.14). In general, the problem (3.17)-(3.18) is nonconvex and a globally optimal solution is difficult to obtain with a reasonable computational

complexity. Fortunately, we can resort to numerical method, such as the projected gradient algorithm [107] to find (at least) a locally optimal solution of (3.17)-(3.18).

THEOREM 3.2: Let us define the objective function in (3.17) as $f(\mathbf{A}_i)$. Its gradient $\nabla f(\mathbf{A}_i)$ with respect to $\mathbf{A}_i$ can be calculated by using results on derivatives of matrices in [108] as

$$\nabla f(\mathbf{A}_i) = 2\big(\mathbf{R}_i^H \mathbf{M}_i^H (\mathbf{E}_i \mathbf{S}_i^H + \mathbf{D}_i^H) - \mathbf{R}_i^H \mathbf{G}_i^{-H} \mathbf{E}_i \mathbf{S}_i^H\big), \qquad i = 1, \cdots, K \tag{3.19}$$

where $\mathbf{M}_i, \mathbf{R}_i, \mathbf{S}_i, \mathbf{D}_i, \mathbf{E}_i$, and $\mathbf{G}_i$ are defined in Appendix 3.B.

PROOF: See Appendix 3.B. □

In each iteration of the PG algorithm, we first obtain $\tilde{\mathbf{A}}_i = \mathbf{A}_i - s_n \nabla f(\mathbf{A}_i)$ by moving $\mathbf{A}_i$ one step towards the negative gradient direction of $f(\mathbf{A}_i)$, where $s_n > 0$ is the step size. Since $\tilde{\mathbf{A}}_i$ might not satisfy the constraint (3.18), we need to project it onto the set given by (3.18). The projected matrix $\bar{\mathbf{A}}_i$ is obtained by minimizing the Frobenius norm of $\bar{\mathbf{A}}_i - \tilde{\mathbf{A}}_i$ (according to [107]) subjecting to (3.18), which can be formulated as the following optimization problem

$$\min_{\bar{\mathbf{A}}_i} \quad \mathrm{tr}\big((\bar{\mathbf{A}}_i - \tilde{\mathbf{A}}_i)(\bar{\mathbf{A}}_i - \tilde{\mathbf{A}}_i)^H\big) \tag{3.20}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\bar{\mathbf{A}}_i(\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}})\bar{\mathbf{A}}_i^H\big) \le P_{r,i}. \tag{3.21}$$

Obviously, if $\mathrm{tr}\Big(\tilde{\mathbf{A}}_i(\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}})\tilde{\mathbf{A}}_i^H\Big) \le P_{r,i}$, then $\bar{\mathbf{A}}_i = \tilde{\mathbf{A}}_i$. Otherwise, the solution to the problem (3.20)-(3.21) can be obtained by using the Lagrange multiplier method, and the solution is given by

$$\bar{\mathbf{A}}_i = \tilde{\mathbf{A}}_i\big[(\lambda+1)\mathbf{I}_{R_{s,i}} + \lambda\mathbf{\Lambda}_{s,i}^2\big]^{-1}$$

where $\lambda > 0$ is the solution to the nonlinear equation of

$$\mathrm{tr}\Big(\tilde{\mathbf{A}}_i\big[(\lambda+1)\mathbf{I}_{R_{s,i}} + \lambda\mathbf{\Lambda}_{s,i}^2\big]^{-1}(\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}})\big[(\lambda+1)\mathbf{I}_{R_{s,i}} + \lambda\mathbf{\Lambda}_{s,i}^2\big]^{-1}\tilde{\mathbf{A}}_i^H\Big) = P_{r,i}. \tag{3.22}$$

Equation (3.22) can be efficiently solved by the bisection method [107].

The procedure of the PG algorithm is listed in Table 3.1. δ_n and s_n are the step size parameters at the nth iteration, and ε is a positive constant close to 0. The step size parameters δ_n and s_n are determined by the Armijo rule [107], i.e., $s_n = s$ is a constant through all iterations, while at the nth iteration, δ_n is set to be γ^{m_n}. Here m_n is the minimal nonnegative integer that satisfies the following inequality $f(\mathbf{A}_i^{(n+1)}) - f(\mathbf{A}_i^{(n)}) \le \alpha\gamma^{m_n}\mathrm{tr}\Big((\nabla f(\mathbf{A}_i^{(n)}))^H(\bar{\mathbf{A}}_i^{(n)} - \mathbf{A}_i^{(n)})\Big)$, where α and γ are constants. According to [107], usually α is chosen close to 0, for example $\alpha \in [10^{-5}, 10^{-1}]$, while a proper choice of γ is normally from 0.1 to 0.5.

Table 3.1: Procedure of applying the projected gradient algorithm to solve the problem (3.17)-(3.18)

1. Initialize the algorithm at a feasible $\mathbf{A}_i^{(0)}$ for $i = 1, \cdots, K$; Set $n = 0$.

2. For $i = 1, \cdots, K$,

   Compute the gradient of (3.17) $\nabla f(\mathbf{A}_i^{(n)})$;

   Project $\tilde{\mathbf{A}}_i^{(n)} = \mathbf{A}_i^{(n)} - s_n \nabla f(\mathbf{A}_i^{(n)})$ to obtain $\bar{\mathbf{A}}_i^{(n)}$;

   Update $\mathbf{A}_i$ with $\mathbf{A}_i^{(n+1)} = \mathbf{A}_i^{(n)} + \delta_n(\bar{\mathbf{A}}_i^{(n)} - \mathbf{A}_i^{(n)})$.

3. If $\max \|\mathbf{A}_i^{(n+1)} - \mathbf{A}_i^{(n)}\| \leq \varepsilon$, then end.

   Otherwise, let $n := n + 1$ and go to step 2).

3.3.2 Optimal Source Precoding Matrix

With fixed $\{\mathbf{F}_i\}$, the source precoding matrix $\mathbf{B}$ is optimized by solving the following problem

$$\min_{\mathbf{B}} \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_b} + \mathbf{B}^H \mathbf{\Psi} \mathbf{B}\right]^{-1}\right) \tag{3.23}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{B}\mathbf{B}^H\big) \leq P_s, \tag{3.24}$$

$$\mathrm{tr}\big(\mathbf{F}_i \mathbf{H}_{sr,i} \mathbf{B}\mathbf{B}^H \mathbf{H}_{sr,i}^H \mathbf{F}_i^H\big) \leq \bar{P}_{r,i}, \qquad i = 1, \cdots, K \tag{3.25}$$

where $\mathbf{\Psi} \triangleq \mathbf{H}_{sr}^H \mathbf{F}^H \mathbf{H}_{rd}^H (\mathbf{H}_{rd} \mathbf{F}\mathbf{F}^H \mathbf{H}_{rd}^H + \mathbf{I}_{N_d})^{-1} \mathbf{H}_{rd} \mathbf{F} \mathbf{H}_{sr}$, and $\bar{P}_{r,i} \triangleq P_{r,i} - \mathrm{tr}(\mathbf{F}_i \mathbf{F}_i^H)$, $i = 1, \cdots, K$. Let us introduce $\mathbf{\Omega} \triangleq \mathbf{B}\mathbf{B}^H$, and a positive semi-definite (PSD) matrix $\mathbf{X}$ with $\mathbf{X} \succeq (\mathbf{I}_{N_s} + \mathbf{\Psi}^{\frac{1}{2}} \mathbf{\Omega} \mathbf{\Psi}^{\frac{1}{2}})^{-1}$, where $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is a PSD matrix. By using the Schur complement [106], the problem (3.23)-(3.25) can be equivalently converted to the following problem

$$\min_{\mathbf{X}, \mathbf{\Omega}} \quad \mathrm{tr}\,(\mathbf{X}) \tag{3.26}$$

$$\text{s.t.} \quad \begin{pmatrix} \mathbf{X} & \mathbf{I}_{N_s} \\ \mathbf{I}_{N_s} & \mathbf{I}_{N_s} + \mathbf{\Psi}^{\frac{1}{2}} \mathbf{\Omega} \mathbf{\Psi}^{\frac{1}{2}} \end{pmatrix} \succeq 0, \tag{3.27}$$

$$\mathrm{tr}\big(\mathbf{\Omega}\big) \leq P_s, \qquad \mathbf{\Omega} \succeq 0, \tag{3.28}$$

$$\mathrm{tr}\big(\mathbf{F}_i \mathbf{H}_{sr,i} \mathbf{\Omega} \mathbf{H}_{sr,i}^H \mathbf{F}_i^H\big) \leq \bar{P}_{r,i}, \qquad i = 1, \cdots, K. \tag{3.29}$$

The problem (3.26)-(3.29) is a convex semi-definite programming (SDP) problem which can be efficiently solved by the interior-point method [106]. Let us introduce the

eigenvalue decomposition (EVD) of $\mathbf{\Omega} = \mathbf{U}_\Omega \mathbf{\Lambda}_\Omega \mathbf{U}_\Omega^H$. Then from $\mathbf{\Omega} = \mathbf{B}\mathbf{B}^H$, we have $\mathbf{B} = \mathbf{U}_\Omega \mathbf{\Lambda}_\Omega^{\frac{1}{2}}$.

Table 3.2: Procedure of solving the problem (3.10)-(3.12).

1. Initialize the algorithm at a feasible $\mathbf{B}^{(0)}$ satisfying constraint (3.11); Set $m = 0$.
2. For fixed $\mathbf{B}^{(m)}$, obtain $\{\mathbf{F}_i\}^{(m)}$ by solving the problem (3.17)-(3.18) using the PG algorithm.
3. Update $\mathbf{B}^{(m+1)}$ by solving the problem (3.26)-(3.29) with known $\{\mathbf{F}_i\}^{(m)}$.
4. If $\|\mathbf{B}^{(m+1)} - \mathbf{B}^{(m)}\| \leq \varepsilon$, then end.
   Otherwise, let $m := m + 1$ and go to step 2).

Now the original joint source and relay optimization problem (3.10)-(3.12) can be solved by an iterative algorithm as shown in Table 3.2. This algorithm is first initialized at a random feasible $\mathbf{B}$ satisfying (3.11). At each iteration, we first update $\{\mathbf{F}_i\}$ with fixed $\mathbf{B}$ and then update $\mathbf{B}$ with fixed $\{\mathbf{F}_i\}$. Note that the conditional updates of each matrix may either decrease or maintain but cannot increase the objective function (3.10). Monotonic convergence of $\{\mathbf{F}_i\}$ and $\mathbf{B}$ towards (at least) a locally optimal solution follows directly from this observation.

3.4 Numerical Examples

In this section, we study the performance of the proposed the optimal source precoding matrix and relay amplifying matrices for two-hop non-regenerative MIMO multi-relay systems with multiple data streams when a linear MMSE receiver is used at the destination node. All simulations are conducted in a flat Rayleigh fading environment where the channel matrices have zero-mean entries with variances σ_s^2/N_s and $\sigma_r^2/(KN_r)$ for $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. The BPSK constellations are used to modulate the source symbols, and all noises are i.i.d. Gaussian with zero mean and unit variance. We define $\mathrm{SNR_s} = \sigma_s^2 P_s K N_r / N_s$ and $\mathrm{SNR_r} = \sigma_r^2 P_r N_d / (K N_r)$ as the SNR for the source-relay link ($\mathrm{SNR_s}$) and the relay-destination link ($\mathrm{SNR_r}$), respectively. We transmit 1000 randomly generated bits in each channel realization, and all simulation results are averaged

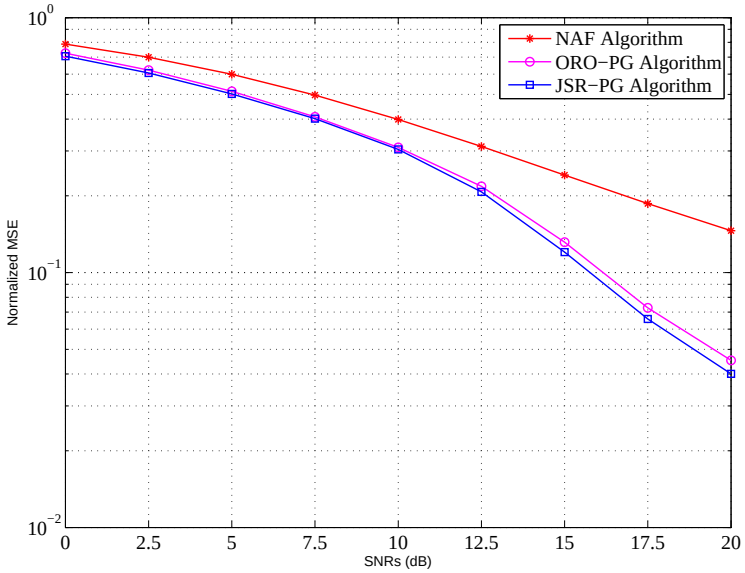

Figure 3.3: Example 3.1 Normalized MSE versus $\mathrm{SNR_s}$ with $K = 3$.

over 200 channel realizations. In all simulations, the linear MMSE receiver in (3.8) is employed at destination node for symbol detection.

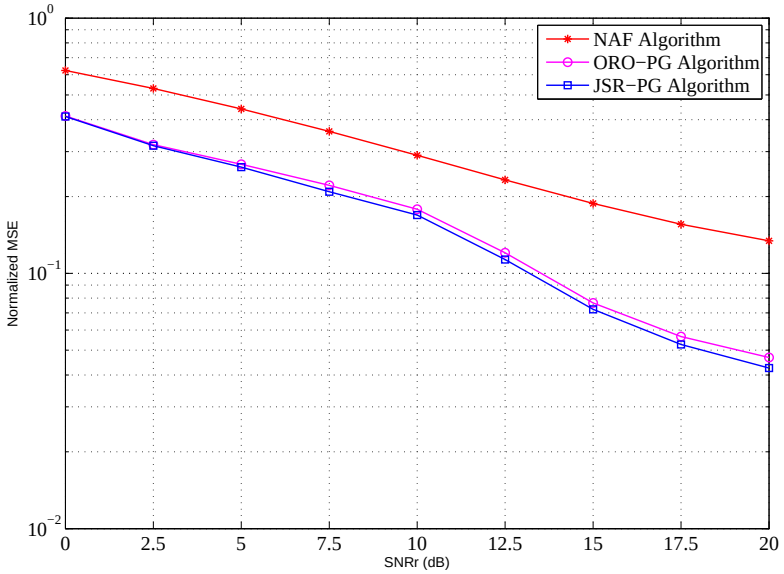

Figure 3.4: Example 3.1 Normalized MSE versus $\mathrm{SNR_r}$ with $K = 3$.

In the first example, a MIMO multi-relay communication system with $K = 3$ re-

lay nodes and $N_s = N_r = N_d = 3$ is simulated. We compare the normalized MSE performance of the proposed joint source and relay optimization algorithm using the projected gradient (JSR-PG) algorithm in Table 3.2, the optimal relay-only algorithm using the projected gradient (ORO-PG) algorithm in Table 3.1, and the naive amplify-and-forward (NAF) algorithm where both the source and relay matrices are scaled identity matrices. Fig. 3.3 shows the normalized MSE of all algorithms versus SNR_s for $\text{SNR}_\text{r} = 20\text{dB}$. While Fig. 3.4 demonstrates the normalized MSE of all algorithms versus SNR_r for SNR_s fixed at 20dB. It can be seen from Figs. 3.3 and 3.4 that the JSR-PG and ORO-PG algorithms have a better performance than the NAF algorithm over the whole SNR_s and SNR_r range. Moreover, the proposed JSR-PG algorithm yields the lowest MSE among all three algorithms.

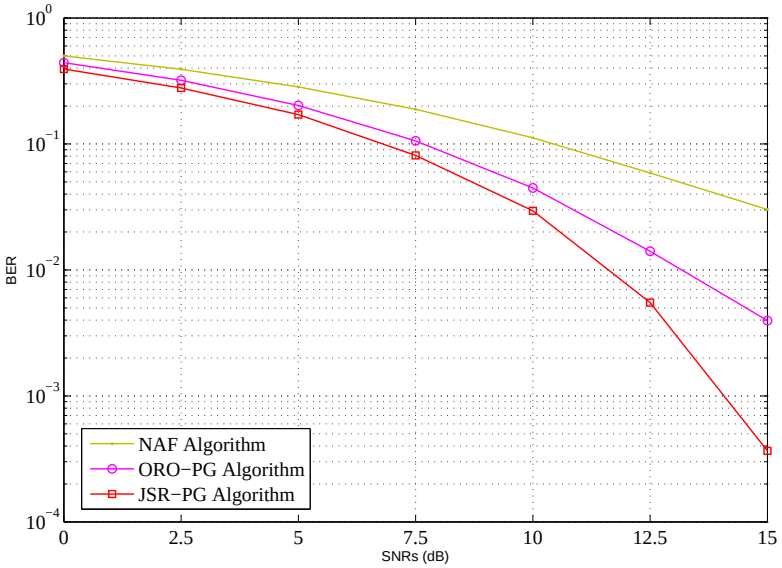

Figure 3.5: Example 3.2 BER versus SNR_s with $K = 3$.

In the second example, we compare the BER performance of the proposed JSR-PG algorithm in Table 3.2, the ORO-PG algorithm in Table 3.1, and the NAF algorithm. Fig. 3.5 displays the system BER versus SNR_s for a MIMO multi-relay communication system with $K = 3$ relay nodes, $N_s = N_r = N_d = 3$, and SNR_r fixed at 20dB. It can be seen from Fig. 3.5 that the proposed JSR-PG algorithm has a better BER performance than existing algorithms over the whole SNR_s range.

In the third example, we study the effect of the number of relay nodes to the system BER performance using the JSR-PG and ORO-PG algorithms. Fig. 3.6 displays the

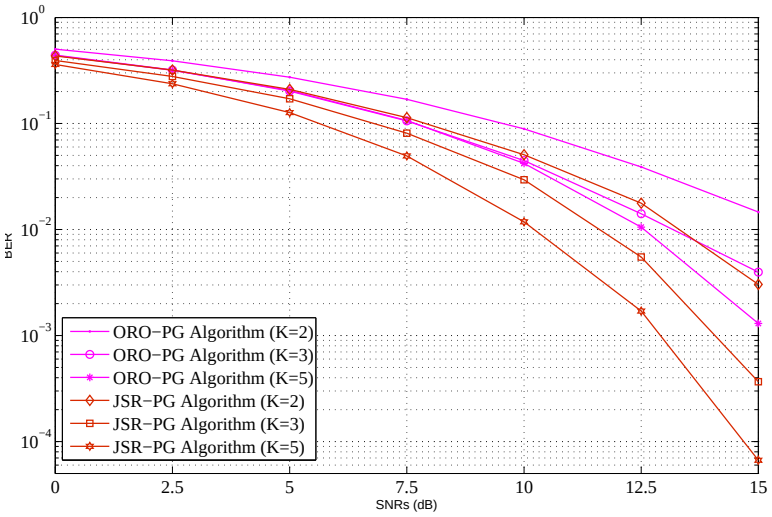

Figure 3.6: Example 3.3 BER versus $\mathrm{SNR_s}$ with different K.

system BER versus $\mathrm{SNR_s}$ with $K = 2$, 3, and 5 for $\mathrm{SNR_r}$ fixed at 20dB and $N_s = N_r = N_d = 3$. It can be seen that at BER $= 10^{-2}$, for both the ORO-PG algorithm and JSR-PG algorithm, we can achieve approximately 3dB gain by increasing from $K = 2$ to $K = 5$. It can also be seen that the performance gain of the JSR-PG algorithm over the ORO-PG algorithm increases with the increasing number of relay nodes.

3.5 Chapter Summary

In this chapter, we have derived the general structure of the optimal relay amplifying matrices for a linear non-regenerative MIMO relay system where each relay has multiple antennas and multiple data streams are transmitted from the source to the destination node. The proposed source and relay matrices minimize the MSE of the signal waveform estimation when a linear receiver is used at the destination node. Simulation results demonstrate the effectiveness of the proposed source and relay matrices design using the projected gradient (PG) approach.

3.A Proof of Theorem 3.1.

Without loss of generality, $\mathbf{F}_i$ can be written as

$$\mathbf{F}_i = \begin{bmatrix} \mathbf{V}_{r,i} & \mathbf{V}_{r,i}^{\perp} \end{bmatrix} \begin{bmatrix} \mathbf{A}_i & \mathbf{X}_i \\ \mathbf{Y}_i & \mathbf{Z}_i \end{bmatrix} \begin{bmatrix} \mathbf{U}_{s,i}^H \\ (\mathbf{U}_{s,i}^{\perp})^H \end{bmatrix}, \qquad i = 1, \cdots, K \tag{3.30}$$

where $\mathbf{V}_{r,i}^{\perp}(\mathbf{V}_{r,i}^{\perp})^H = \mathbf{I}_{N_r} - \mathbf{V}_{r,i}\mathbf{V}_{r,i}^H$, $\mathbf{U}_{s,i}^{\perp}(\mathbf{U}_{s,i}^{\perp})^H = \mathbf{I}_{N_r} - \mathbf{U}_{s,i}\mathbf{U}_{s,i}^H$, such that $\bar{\mathbf{V}}_{r,i} \triangleq [\mathbf{V}_{r,i}, \mathbf{V}_{r,i}^{\perp}]$ and $\bar{\mathbf{U}}_{s,i} \triangleq [\mathbf{U}_{s,i}, \mathbf{U}_{s,i}^{\perp}]$ are $N_r \times N_r$ unitary matrices. Matrices $\mathbf{A}_i, \mathbf{X}_i, \mathbf{Y}_i, \mathbf{Z}_i$ are arbitrary matrices with dimensions of $R_{r,i} \times R_{s,i}$, $R_{r,i} \times (N_r - R_{s,i})$, $(N_r - R_{r,i}) \times R_{s,i}$, $(N_r - R_{r,i}) \times (N_r - R_{s,i})$, respectively. Substituting (3.15) and (3.30) back into (3.13), we obtain that $\mathbf{H}_{rd,i}\mathbf{F}_i\mathbf{H}_{sr,i}\mathbf{B} = \mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H$ and $\mathbf{H}_{rd,i}\mathbf{F}_i{\mathbf{F}_i}^H\mathbf{H}_{rd,i}^H = \mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}(\mathbf{A}_i\mathbf{A}_i^H + \mathbf{X}_i\mathbf{X}_i^H)\mathbf{\Lambda}_{r,i}^H\mathbf{U}_{r,i}^H$. Thus we can rewrite (3.13) as

$$\begin{aligned} \text{MSE} = \text{tr}\Bigg(\Bigg[\mathbf{I}_{N_b} + \sum_{i=1}^{K} \mathbf{V}_{s,i}\mathbf{\Lambda}_{s,i}\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H \Bigg(\sum_{i=1}^{K} \mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}(\mathbf{A}_i\mathbf{A}_i^H + \mathbf{X}_i\mathbf{X}_i^H)\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H + \mathbf{I}_{N_d} \Bigg)^{-1} \\ \times \sum_{i=1}^{K} \mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H \Bigg]^{-1} \Bigg). \end{aligned} \tag{3.31}$$

It can be seen that (3.31) is minimized by $\mathbf{X}_i = \mathbf{0}_{R_{r,i} \times (N_r - R_{s,i})}$, $i = 1, \cdots, K$.

Substituting (3.15) and (3.30) back into the left-hand-side of the transmission power constraint (3.14), we have

$$\begin{aligned} &\text{tr}\left(\mathbf{F}_i[\mathbf{H}_{sr,i}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}]\mathbf{F}_i^H\right) \\ &= \text{tr}(\mathbf{A}_i(\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}})\mathbf{A}_i^H + \mathbf{Y}_i(\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}})\mathbf{Y}_i^H + \mathbf{X}_i\mathbf{X}_i^H + \mathbf{Z}_i\mathbf{Z}_i^H), \quad i = 1, \cdots, K. \end{aligned} \tag{3.32}$$

From (3.32), we find that $\mathbf{X}_i = \mathbf{0}_{R_{r,i} \times (N_r - R_{s,i})}$, $\mathbf{Y}_i = \mathbf{0}_{(N_r - R_{r,i}) \times R_{s,i}}$, and $\mathbf{Z}_i = \mathbf{0}_{(N_r - R_{r,i}) \times (N_r - R_{s,i})}$ minimize the power consumption at each relay node. Thus we have $\mathbf{F}_i = \mathbf{V}_{r,i}\mathbf{A}_i\mathbf{U}_{s,i}^H$, $i = 1, \cdots, K$.

3.B Proof of Theorem 3.2.

Let us define $\mathbf{Z}_i \triangleq \sum_{j=1, j \neq i}^{K} \mathbf{U}_{r,j}\mathbf{\Lambda}_{r,j}\mathbf{A}_j\mathbf{\Lambda}_{s,j}\mathbf{V}_{s,j}^H$ and $\mathbf{Y}_i \triangleq \sum_{j=1, j \neq i}^{K} \mathbf{U}_{r,j}\mathbf{\Lambda}_{r,j}\mathbf{A}_j\mathbf{A}_j^H\mathbf{\Lambda}_{r,j}\mathbf{U}_{r,j}^H + \mathbf{I}_{N_d}$. Then $f(\mathbf{A}_i)$ can be written as

$$\begin{aligned} f(\mathbf{A}_i) = \text{tr}\Big(\Big[\mathbf{I}_{N_b} + \left(\mathbf{Z}_i^H + \mathbf{V}_{s,i}\mathbf{\Lambda}_{s,i}\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H\right) \left(\mathbf{Y}_i + \mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H\right)^{-1} \\ \times \left(\mathbf{Z}_i + \mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H\right) \Big]^{-1} \Big). \end{aligned} \tag{3.33}$$

Applying $\left(\mathbf{I}_{N_b}+\mathbf{A}^H\mathbf{C}^{-1}\mathbf{A}\right)^{-1}=\mathbf{I}_{N_b}-\mathbf{A}^H(\mathbf{A}\mathbf{A}^H+\mathbf{C})^{-1}\mathbf{A}$, (3.33) can be written as

$$\begin{aligned}f(\mathbf{A}_i)=\mathrm{tr}\big(\mathbf{I}_{N_b}-(\mathbf{Z}_i^H+\mathbf{V}_{s,i}\mathbf{\Lambda}_{s,i}\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H)\left[(\mathbf{Z}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H)\right.\\ \times(\mathbf{Z}_i^H+\mathbf{V}_{s,i}\mathbf{\Lambda}_{s,i}\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H)+\mathbf{Y}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H\left.\right]^{-1}\\ (\mathbf{Z}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H)\big).\end{aligned}\tag{3.34}$$

Let us now define $\mathbf{E}_i\triangleq\mathbf{Z}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H$, $\mathbf{K}_i\triangleq\mathbf{Y}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H$, and $\mathbf{G}_i\triangleq\mathbf{E}_i\mathbf{E}_i^H+\mathbf{K}_i$. We can rewrite (3.34) as

$$f(\mathbf{A}_i)=\mathrm{tr}\big(\mathbf{I}_{N_b}-\mathbf{E}_i^H\mathbf{G}_i^{-1}\mathbf{E}_i\big)=\mathrm{tr}\big(\mathbf{I}_{N_b}-\mathbf{E}_i\mathbf{E}_i^H\mathbf{G}_i^{-1}\big).\tag{3.35}$$

The derivative of $f(\mathbf{A}_i)$ with respect to $\mathbf{A}_i$ is given by

$$\begin{aligned}\frac{\partial f(\mathbf{A}_i)}{\partial\mathbf{A}_i}&=-\frac{\partial}{\partial\mathbf{A}_i}\mathrm{tr}\big(\mathbf{E}_i\mathbf{E}_i^H\mathbf{G}_i^{-1}\big)\\ &=\frac{\partial}{\partial\mathbf{A}_i}\mathrm{tr}\big(\mathbf{G}_i^{-1}\mathbf{E}_i\mathbf{E}_i^H\mathbf{G}_i^{-1}\left((\mathbf{Z}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H)\mathbf{E}_i^H\right.\\ &\quad\left.+\mathbf{Y}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H\right)\big)-\frac{\partial}{\partial\mathbf{A}_i}\mathrm{tr}\big(\mathbf{E}_i^H\mathbf{G}_i^{-1}\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H\big).\end{aligned}\tag{3.36}$$

Defining $\mathbf{M}_i\triangleq\mathbf{G}_i^{-1}\mathbf{E}_i\mathbf{E}_i^H\mathbf{G}_i^{-1}$, $\mathbf{R}_i\triangleq\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}$, $\mathbf{S}_i\triangleq\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H$, and $\mathbf{D}_i\triangleq\mathbf{A}_i^H\mathbf{\Lambda}_{r,i}\mathbf{U}_{r,i}^H$, we can rewrite (3.36) as

$$\begin{aligned}\frac{\partial f(\mathbf{A}_i)}{\partial\mathbf{A}_i}&=\frac{\partial}{\partial\mathbf{A}_i}\mathrm{tr}\big(\mathbf{M}_i(\mathbf{Z}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H)\mathbf{E}_i^H\\ &\quad+\mathbf{M}_i(\mathbf{Y}_i+\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i}\mathbf{A}_i\mathbf{D}_i))-(\mathbf{E}_i^H\mathbf{G}_i^{-1}\mathbf{U}_{r,i}\mathbf{\Lambda}_{r,i})^T(\mathbf{\Lambda}_{s,i}\mathbf{V}_{s,i}^H)^T\\ &=\frac{\partial}{\partial\mathbf{A}_i}\mathrm{tr}\big(\mathbf{M}_i\mathbf{R}_i\mathbf{A}_i\mathbf{S}_i\mathbf{E}_i^H+\mathbf{M}_i\mathbf{R}_i\mathbf{A}_i\mathbf{D}_i\big)-(\mathbf{E}_i^H\mathbf{G}_i^{-1}\mathbf{R}_i)^T\mathbf{S}_i^T.\end{aligned}\tag{3.37}$$

Finally, the gradient of $f(\mathbf{A}_i)$ is given by

$$\begin{aligned}\nabla f(\mathbf{A}_i)&=2\left(\frac{\partial f(\mathbf{A}_i)}{\partial\mathbf{A}_i}\right)^*\\ &=2\big((\mathbf{M}_i\mathbf{R}_i)^T(\mathbf{S}_i\mathbf{E}_i^H)^T+(\mathbf{M}_i\mathbf{R}_i)^T\mathbf{D}_i^T-(\mathbf{E}_i^H\mathbf{G}_i^{-1}\mathbf{R}_i)^T\mathbf{S}_i^T\big)^*\\ &=2\big(\mathbf{R}_i^H\mathbf{M}_i^H(\mathbf{E}_i\mathbf{S}_i^H+\mathbf{D}_i^H)-\mathbf{R}_i^H\mathbf{G}_i^{-H}\mathbf{E}_i\mathbf{S}_i^H\big).\end{aligned}\tag{3.38}$$

Chapter 4

Simplified Source and Relay Matrices Design

To reduce the complexity of the algorithm in Chapters 3, we develop a simplified source and relay matrices design algorithm. We first relax the power constraint at each relay node to the power constraint at the output of the second-hop channel. We propose a jointly optimal source and relay beamforming algorithm which minimizes the MSE of signal waveform estimation at the destination node for the relaxed problem. Then we scale the relay matrices to satisfy the individual power constraint at each relay node. In Section 4.1, we show that both the optimal source precoding matrix and the optimal relay amplifying matrices have a beamforming structure. By using the optimal structure, a joint source and relay power loading algorithm is then developed to minimize the MSE of the signal waveform estimation. The simulation results in Section 4.2 demonstrate that the proposed joint source and relay beamforming algorithm has an improved performance-complexity tradeoff. The chapter is briefly summarized in Section 4.3.

4.1 MMSE Relay Design Algorithm

The communication process between the source and destination nodes for a parallel MIMO relay communication system with multiple data streams has been introduced in Chapter 3 in (3.1)-(3.5). Fig. 4.1 illustrates an equivalent two-hop MIMO relay system

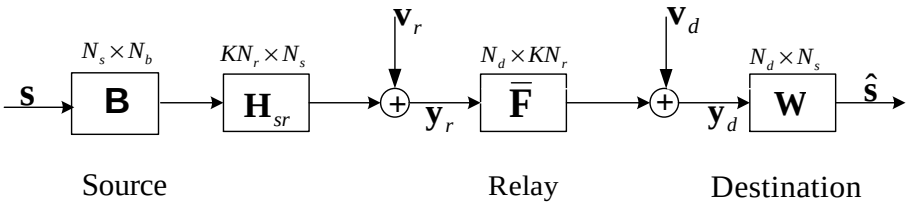

Figure 4.1: Block diagram of the equivalent MIMO relay system with multiple data streams.

with multiple data streams by introducing

$$\bar{\mathbf{F}} \triangleq \mathbf{H}_{rd}\mathbf{F}. \tag{4.1}$$

The received signal vector at the destination node can be equivalently written as

$$\mathbf{y}_d = \bar{\mathbf{F}}\mathbf{H}_{sr}\mathbf{B}\mathbf{s} + \bar{\mathbf{F}}\mathbf{v}_r + \mathbf{v}_d = \bar{\mathbf{H}}\mathbf{s} + \bar{\mathbf{v}}$$

where we define $\bar{\mathbf{H}} \triangleq \bar{\mathbf{F}}\mathbf{H}_{sr}\mathbf{B}$ as the effective MIMO channel matrix of the source-relay-destination link, and $\bar{\mathbf{v}}$ as the equivalent noise with $\bar{\mathbf{v}} \triangleq \bar{\mathbf{F}}\mathbf{v}_r + \mathbf{v}_d$. The transmission power consumed by each relay node (3.3) can be expressed in (3.6). Based on (3.6) and (3.7), the joint source and relay optimization problem with a linear MMSE receiver used at the destination node can be formulated as

$$\min_{\{\mathbf{F}_i\},\mathbf{B}} \quad \mathrm{tr}\Big(\big[\mathbf{I}_{N_b} + \bar{\mathbf{H}}^H\mathbf{C}_{\bar{v}}^{-1}\bar{\mathbf{H}}\big]^{-1}\Big) \tag{4.2}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{B}\mathbf{B}^H\big) \le P_s \tag{4.3}$$

$$\mathrm{tr}\big(\mathbf{F}_i\big[\mathbf{H}_{sr,i}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}\big]\mathbf{F}_i^H\big) \le P_{r,i}, \qquad i = 1, \cdots, K \tag{4.4}$$

where (4.3) is the transmit power constraint at the source node, while (4.4) is the power constraint at each relay node. Here $P_s > 0$ and $P_{r,i} > 0$, $i = 1, \cdots, K$, are the corresponding power budget. Obviously, to avoid any loss of transmission power in the relay system when a linear receiver is used, there should be $N_b \le \min(N_s, KN_r, N_d)$.

Due to the power constraint at each relay node (4.4), the source and relay matrices optimization problem (4.2)-(4.4) is much more challenging to solve when $K \ge 2$ compared with the case of $K = 1$. To overcome this difficulty, we relax the power constraints in (4.4) by considering the power of the signal at the output of $\mathbf{H}_{rd}$, which can be expressed as [73]

$$\mathrm{E}\big[\mathrm{tr}\big((\mathbf{H}_{rd}\mathbf{x}_r)(\mathbf{H}_{rd}\mathbf{x}_r)^H\big)\big] = \mathrm{tr}\big(\bar{\mathbf{F}}\big[\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \mathbf{I}_{KN_r}\big]\bar{\mathbf{F}}^H\big) \le P_r\mathrm{tr}(\mathbf{H}_{rd}\mathbf{H}_{rd}^H). \tag{4.5}$$

Here $P_r \triangleq \sum_{i=1}^{K} P_{r,i}$ is the total transmission power budget available to all K relay nodes. By using (4.5), the relaxed joint source and relay optimization problem can be written as

$$\min_{\bar{\mathbf{F}},\mathbf{B}} \quad \mathrm{tr}\Big(\big[\mathbf{I}_{N_b} + \bar{\mathbf{H}}^H \mathbf{C}_{\bar{v}}^{-1} \bar{\mathbf{H}}\big]^{-1}\Big) \tag{4.6}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{B}\mathbf{B}^H\big) \le P_s \tag{4.7}$$

$$\mathrm{tr}\big(\bar{\mathbf{F}}\big[\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \mathbf{I}_{KN_r}\big]\bar{\mathbf{F}}^H\big) \le P_x \tag{4.8}$$

where $P_x \triangleq P_r \mathrm{tr}(\mathbf{H}_{rd}\mathbf{H}_{rd}^H)$, and (4.8) is the power constraint at the output of $\mathbf{H}_{rd}$.

Let $\mathbf{H}_{sr} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{V}_s^H$ denote the singular value decomposition (SVD) of $\mathbf{H}_{sr}$, where the dimensions of $\mathbf{U}_s$, $\mathbf{\Lambda}_s$, $\mathbf{V}_s$ are $KN_r \times KN_r$, $KN_r \times N_s$, $N_s \times N_s$, respectively. We assume that the main diagonal elements of $\mathbf{\Lambda}_s$ are arranged in a decreasing order. By using Theorem 1 in [60], the optimal structure of $\bar{\mathbf{F}}$ and $\mathbf{B}$ as the solution to the problem (4.6)-(4.8) is given by

$$\bar{\mathbf{F}} = \mathbf{Q}\mathbf{\Lambda}_f\mathbf{U}_{s,1}^H, \quad \mathbf{B} = \mathbf{V}_{s,1}\mathbf{\Lambda}_b \tag{4.9}$$

where $\mathbf{Q}$ is any $N_d \times N_b$ semi-unitary matrix with $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_{N_b}$, $\mathbf{U}_{s,1}$ and $\mathbf{V}_{s,1}$ contain the leftmost N_b columns of $\mathbf{U}_s$ and $\mathbf{V}_s$, respectively, $\mathbf{\Lambda}_f$ and $\mathbf{\Lambda}_b$ are $N_b \times N_b$ diagonal matrices. From (4.9), we see that the optimal $\bar{\mathbf{F}}$ and $\mathbf{B}$ have a beamforming structure. In fact, they jointly diagonalize the source-relay-destination channel $\bar{\mathbf{H}}$. By using (4.9), the joint source-relay optimization problem (4.6)-(4.8) becomes

$$\min_{\mathbf{\Lambda}_f,\mathbf{\Lambda}_b} \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_b} + \big(\mathbf{\Lambda}_f\mathbf{\Lambda}_s\mathbf{\Lambda}_b\big)^2\Big[\mathbf{\Lambda}_f^2 + \mathbf{I}_{N_b}\Big]^{-1}\right]^{-1}\right) \tag{4.10}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{\Lambda}_b^2\big) \le P_s \tag{4.11}$$

$$\mathrm{tr}\left(\mathbf{\Lambda}_f^2\big[\big(\mathbf{\Lambda}_s\mathbf{\Lambda}_b\big)^2 + \mathbf{I}_{N_b}\big]\right) \le P_x. \tag{4.12}$$

Denoting $\lambda_{f,i}, \lambda_{s,i}, \lambda_{b,i}$, $i = 1, \cdots, N_b$, as the main diagonal elements of $\mathbf{\Lambda}_f$, $\mathbf{\Lambda}_s$, $\mathbf{\Lambda}_b$, respectively, the optimization problem (4.10)-(4.12) can be equivalently written as

$$\min_{\{\lambda_{f,i}\},\{\lambda_{b,i}\}} \quad \sum_{i=1}^{N_b}\left(1 + \frac{\big(\lambda_{f,i}\lambda_{s,i}\lambda_{b,i}\big)^2}{\lambda_{f,i}^2 + 1}\right)^{-1} \tag{4.13}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_b}\lambda_{b,i}^2 \le P_s \tag{4.14}$$

$$\sum_{i=1}^{N_b}\lambda_{f,i}^2\big[\big(\lambda_{s,i}\lambda_{b,i}\big)^2 + 1\big] \le P_x \tag{4.15}$$

$$\lambda_{b,i} \ge 0, \quad \lambda_{f,i} \ge 0, \quad i = 1, \cdots, N_b\,. \tag{4.16}$$

The problem (4.13)-(4.16) is nonconvex and a closed-form solution is intractable to obtain. In the following, we develop an iterative method to obtain a numerical solution of the optimal $\{\lambda_{f,i}\}$ and $\{\lambda_{b,i}\}$. Let us define

$$\begin{aligned} a_i &\triangleq \lambda_{s,i}^2, \qquad x_i \triangleq \lambda_{b,i}^2, \\ y_i &\triangleq \lambda_{f,i}^2\big[\big(\lambda_{s,i}\lambda_{b,i}\big)^2+1\big], \qquad i=1,\cdots,N_b. \end{aligned} \tag{4.17}$$

Then the optimization problem (4.13)-(4.16) can be equivalently rewritten as

$$\min_{\{x_i\},\{y_i\}} \quad \sum_{i=1}^{N_b}\left(1+\frac{\frac{a_i x_i y_i}{a_i x_i+1}}{1+\frac{y_i}{a_i x_i+1}}\right)^{-1} \tag{4.18}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_b} x_i \le P_s \tag{4.19}$$

$$\sum_{i=1}^{N_b} y_i \le P_x \tag{4.20}$$

$$x_i \ge 0, \quad y_i \ge 0, \quad i=1,\cdots,N_b\,. \tag{4.21}$$

For a fixed $\{y_i\}$ satisfying (4.20) and (4.21), the problem of optimizing $\{x_i\}$ can be written as

$$\min_{\{x_i\}} \quad \sum_{i=1}^{N_b}\frac{a_i x_i + y_i + 1}{a_i x_i y_i + a_i x_i + y_i + 1} \tag{4.22}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_b} x_i \le P_s \tag{4.23}$$

$$x_i \ge 0, \qquad i=1,\cdots,N_b. \tag{4.24}$$

The Lagrangian function associated with the problem (4.22)-(4.24) can be written as

$$\mathcal{L} = \sum_{i=1}^{N_b}\frac{a_i x_i + y_i + 1}{a_i x_i y_i + a_i x_i + y_i + 1} + \mu_1\Big(\sum_{i=1}^{N_b} x_i - P_s\Big) \tag{4.25}$$

where $\mu_1 \ge 0$ is the Lagrangian multiplier. Taking the derivative of (4.25) with respect to x_i equal to zero, we obtain

$$x_i = \frac{1}{a_i}\left[\sqrt{\frac{a_i y_i}{\mu_1(y_i+1)}} - 1\right]^{\dagger}, \quad i=1,\cdots,N_b$$

where μ_1 is the solution to the following nonlinear equation

$$\sum_{i=1}^{N_b}\frac{1}{a_i}\left[\sqrt{\frac{a_i y_i}{\mu_1(y_i+1)}} - 1\right]^{\dagger} = P_s.$$

In a similar fashion, for a fixed $\{x_i\}$ satisfying (4.19) and (4.21), we can update $\{y_i\}$ as

$$y_i = \left[\sqrt{\frac{a_i x_i}{\mu_2(a_i x_i + 1)}} - 1\right]^{\dagger}, \quad i = 1, \cdots, N_b$$

where $\mu_2 \geq 0$ is the solution to the following nonlinear equation

$$\sum_{i=1}^{N_b} \left[\sqrt{\frac{a_i x_i}{\mu_2(a_i x_i + 1)}} - 1\right]^{\dagger} = P_x.$$

Note that the conditional updates of $\{x_i\}$ and $\{y_i\}$ may either decrease or maintain but cannot increase the objective function in (4.18). Monotonic convergence of $\{x_i\}$ and $\{y_i\}$ follows directly from this observation. After the convergence of the alternating algorithm, $\lambda_{f,i}$ and $\lambda_{b,i}$ can be obtained from (4.17) as

$$\lambda_{f,i} = \sqrt{\frac{y_i}{\lambda_{s,i}^2 x_i + 1}}, \quad \lambda_{b,i} = \sqrt{x_i}, \qquad i = 1, \cdots, N_b.$$

By using (4.1) and the optimal structure of $\bar{\mathbf{F}}$ and $\mathbf{B}$ in (4.9), we have $\mathbf{H}_{rd,i}\mathbf{F}_i = \mathbf{Q}\mathbf{\Lambda}_f\mathbf{\Phi}_i$, where matrix $\mathbf{\Phi}_i$ contains the $(i-1)N_r + 1$ to iN_r columns of $\mathbf{U}_{s,1}^H$. Then we obtain

$$\mathbf{F}_i = \mathbf{H}_{rd,i}^{+}\mathbf{Q}\mathbf{\Lambda}_f\mathbf{\Phi}_i, \qquad i = 1, \cdots, K. \tag{4.26}$$

Finally, we scale $\mathbf{F}_i$ in (4.26) to satisfy the power constraint (4.4) at each relay node as

$$\tilde{\mathbf{F}}_i = \alpha_i \mathbf{F}_i, \qquad i = 1, \cdots, K \tag{4.27}$$

where the scaling factor α_i is given by

$$\alpha_i = \sqrt{\frac{P_{r,i}}{\mathrm{tr}(\mathbf{F}_i[\mathbf{H}_{sr,i}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}]\mathbf{F}_i^H)}}, \qquad i = 1, \cdots, K. \tag{4.28}$$

4.2 Numerical Examples

In this section, we study the performance of the proposed simplified joint source and relay beamforming algorithm for parallel MIMO relay communication systems with multiple parallel relay nodes. All simulations are conducted in a flat Rayleigh fading environment using the BPSK constellation, and the noises are i.i.d. Gaussian with zero mean and unit variance. The channel matrices have zero-mean entries with variances σ_s^2/N_s and $\sigma_r^2/(KN_r)$ for $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. We vary the SNR in the source-to-relay link ($\mathrm{SNR_s}$) while fixing the SNR in the relay-to-destination link ($\mathrm{SNR_r}$) to 20dB.

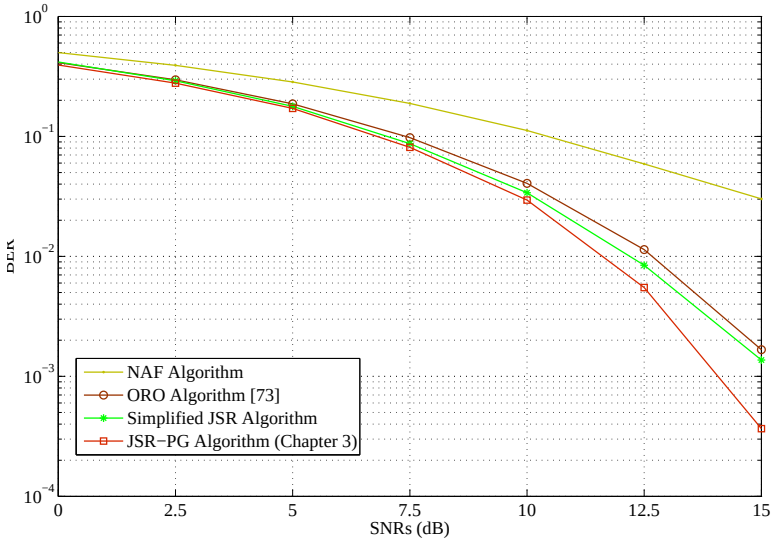

Figure 4.2: Example 4.1 BER versus SNR_s with $K = 3$.

We transmit 1000 randomly generated bits in each channel realization, and the BER results are averaged through 200 channel realizations. Here we set $N_b = N_s = N_r = N_d = 3$.

In the first example, we simulate a relay system with $K = 3$ and compare the BER performance of the following algorithms: (i) the proposed simplified joint source and relay scheme (Simplified JSR); (ii) joint source and relay optimization algorithm using the projected gradient (JSR-PG, Chapter 3); (iii) the NAF algorithm where both the source and relay matrices are scaled identity matrices satisfying power constraints (4.7) and (4.8); (iv) the optimal relay only (ORO) algorithm developed in [73] where the relay matrices are optimized based on the MMSE criterion, while the source precoding matrix is a scaled identity matrix. Fig. 4.2 shows the BER performance of four systems versus SNR_s. It can be seen from Fig. 4.2 that the NAF algorithm has the worst performance, since it does not exploit the channel knowledge available. The proposed algorithm has a higher BER than the algorithm in Chapter 3. However, the former algorithm has a much lower computational complexity than the latter one.

In the second example, we study the effect of the number of relays to the system BER performance using the proposed algorithm. Fig. 4.3 displays the system BER versus SNR_s with $K = 2$, 3, and 5. It can be seen that at BER $= 10^{-2}$, for proposed simplified algorithm, we can achieve approximately 4.5dB gain by increasing from $K = 2$ to $K = 5$.

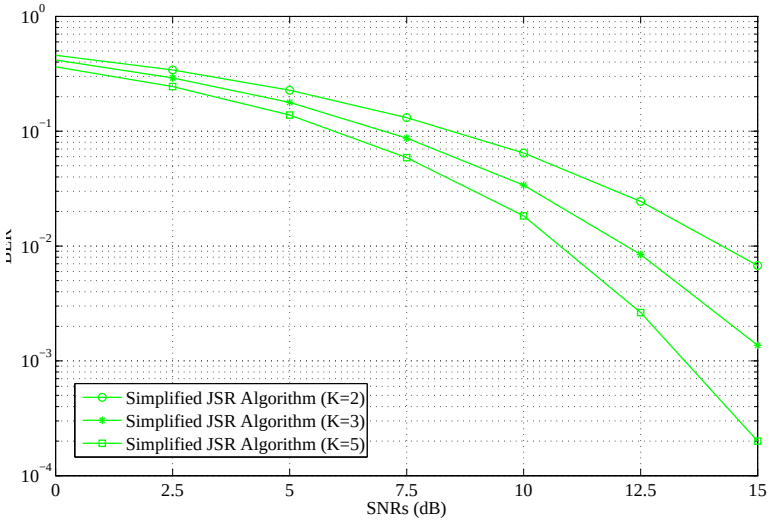

Figure 4.3: Example 4.2 BER versus $\mathrm{SNR_s}$ with varying K.

4.3 Chapter Summary

In this chapter, we have proposed a simplified source and relay matrices design algorithm for parallel MIMO relay communication systems with multiple parallel data streams. The proposed algorithm has an improved performance-complexity tradeoff.

Chapter 5

Parallel MIMO Relay Design Based on Nonlinear Receiver

In this chapter, we study the optimal structure of the source precoding matrix and the relay amplifying matrices for parallel MIMO relay communication systems with multiple data streams when a nonlinear DFE receiver is used at the destination node, and the MMSE criterion is used to detect the transmitted signal at each stream.

The rest of the chapter is organized as follows. Existing works are briefly summarized in Section 5.1 which is followed by performance comparison of the linear and the nonlinear receivers in Section 5.2. We show that the optimal source precoding matrix and the optimal relay amplifying matrices have a beamforming structure in Section 5.3. Using such optimal source and relay matrices and the MMSE-DFE receiver, a joint source and relay power loading algorithm is developed to minimize the MSE of the signal waveform estimation. In Section 5.4, numerical simulations are performed to demonstrate the performance of the proposed algorithm. The chapter is summarized in Section 5.5.

5.1 Overview of Known Techniques

It is well known that relay techniques are very efficient in enhancing the coverage and the energy efficiency of wireless communication systems [3, 4]. When nodes in the relay system are installed with multiple antennas, we call such system MIMO relay communication system. MIMO relay communication systems have attracted much research

interest and provided significant improvement in terms of both spectral efficiency and link reliability [12, 37, 38, 47, 54, 57–60, 73, 74, 89]. In [12, 38, 54, 57–60], the authors have studied MIMO relay systems with a single relay node at each hop. MIMO relay systems with multiple parallel relay nodes have been investigated in [73] and Chapters 2-4. In [73], the optimal relay amplifying matrices are developed to minimize the MSE of the signal waveform estimation. In Chapters 2-4, we investigated the jointly optimal structure of the source precoding matrix and the relay amplifying matrices when a linear MMSE receiver is used at the destination node.

In this chapter, we study MIMO parallel relay systems with a nonlinear DFE at the destination node. In particular, the MMSE criterion is used to estimate the transmitted signal at each stream. We call such receiver an MMSE-DFE receiver. We show that the optimal relay amplifying matrices have a beamforming structure. This result generalizes the optimal source and relay matrices design from a linear MMSE receiver case Chapter 4 to nonlinear MMSE-DFE receiver scenarios. Simulation results show that the proposed source and relay matrices together with the MMSE-DFE receiver yield a significant BER performance improvement compared with the linear MMSE based relay algorithm developed in Chapter 4.

5.2 Performance Comparison of Linear and Nonlinear Receivers

The basic MIMO relay system with SIC technique is illustrated in Fig. 1.1. In this section, we compare the performance of the MIMO relay systems with linear and nonlinear receivers through numerical simulations. In the simulations, the transmission signaling is in spatial multiplexing mode (i.e., the source transmits independent data streams from different antennas) with total transmit power uniformly distributed among the transmit antennas. We define the relay amplifying matrices as scaled identity matrices satisfying the power constraints at the relay nodes. Also, all simulations are conducted in a flat-fading Rayleigh environment using the BPSK constellation, and the noise variances are assumed to be the same for all antennas. We transmitted 10^3 randomly generated bits in each channel realization and the BER results are averaged through 200 channel realizations. We plot BER curves versus SNR.

In the first example, we simulate the system BER performance of ZF and MMSE receiver with and without SIC in MIMO relay channel with varying SNR in the source-

to-relay link (SNR_s) keeping the relay-to-destination link (SNR_r) at 20dB. Fig. 5.1 show the BER performance with $N_s = N_r = N_d = 2$, $K = 1$. It can been seen that at BER = 10^{-3}, we achieve 5 dB gain from MMSE to MMSE-SIC as well as from ZF to ZF-SIC.

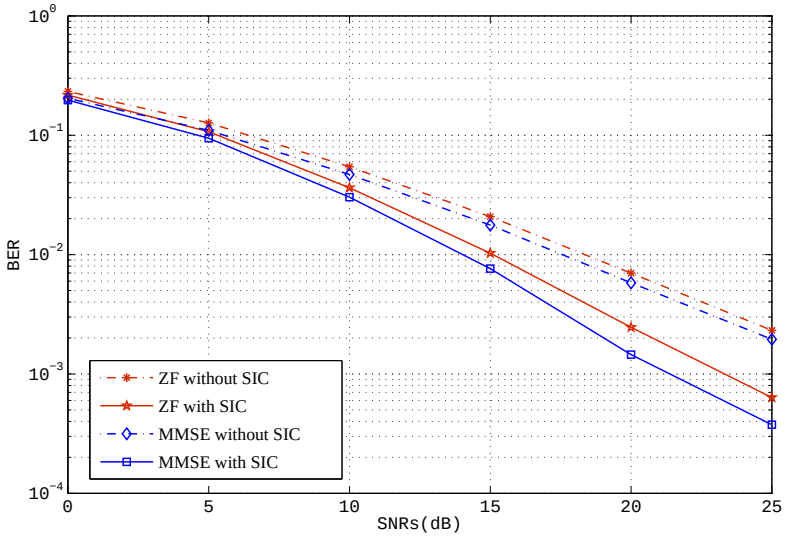

Figure 5.1: BER versus SNR_s. $N_s = N_r = N_d = 2$, $K = 1$ and $SNR_r = 20$dB for MIMO relay channel.

In the second example, we simulate the system BER performance of ZF and MMSE receiver with and without SIC in MIMO relay channel with varying SNR in the relay-to-destination link (SNR_r) keeping the source-to-relay link (SNR_s) at 20dB. Fig. 5.2 show the BER performance with $N_s = N_r = N_d = 2$, $K = 1$. Our results demonstrate that ZF-SIC and MMSE-SIC receiver algorithms have lower BER compared to the ZF and MMSE receiver algorithms.

5.3 Optimal Source and Relay Design with DFE

Fig. 1.1 illustrates a two-hop parallel MIMO relay communication system consisting of one source node, K parallel relay nodes, and one destination node. We assume that the source and destination nodes have N_s and N_d antennas, respectively, and each relay node has N_r antennas.

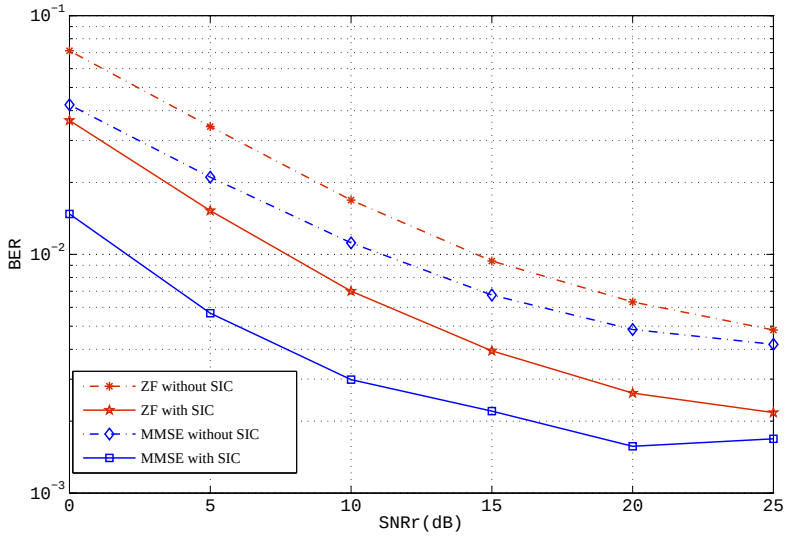

Figure 5.2: BER versus SNR_r. $N_s = N_r = N_d = 2$, $K = 1$ and $SNR_s = 20$dB for MIMO relay channel.

The communication process between the source and destination nodes for MIMO relay system has been introduced in Chapter 3 (3.1)-(3.6), and Chapter 4 (4.1).

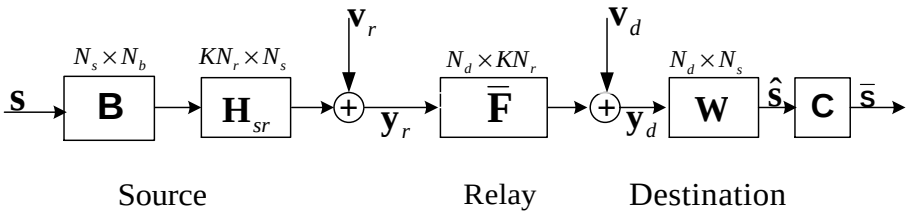

Figure 5.3: Block diagram of the equivalent MIMO relay system with SIC technique.

At the destination node, a nonlinear DFE receiver is used to detect the source symbols successively with the N_bth symbol detected first and the first symbol detected last. The equivalent MIMO relay system model is shown in Fig. 5.3. Assuming that there is no error propagation in the DFE receiver, the kth source symbol is estimated

as

$$\bar{s}_k = \bar{\mathbf{w}}_k^H \mathbf{y}_d - \sum_{l=k+1}^{N_b} c_{k,l} s_l, \quad k = 1, \cdots, N_b \tag{5.1}$$

where $\bar{\mathbf{w}}_k$ is the feed-forward vector for the kth symbol, and $c_{k,l}$, $l = k+1, \cdots, N_b$, are the feedback coefficients for the kth symbol. By introducing $\bar{\mathbf{W}} = [\bar{\mathbf{w}}_1, \bar{\mathbf{w}}_2, \cdots, \bar{\mathbf{w}}_{N_b}]$, $\bar{\mathbf{s}} = [\bar{s}_1, \bar{s}_2, \cdots, \bar{s}_{N_b}]^T$, and an $N_b \times N_b$ strictly upper-triangle matrix $\mathbf{C}$ with nonzero elements $c_{k,l}$, we can represent (5.1) in matrix form as

$$\bar{\mathbf{s}} = \bar{\mathbf{W}}^H \mathbf{y}_d - \mathbf{C}\mathbf{s} = (\bar{\mathbf{W}}^H \bar{\mathbf{H}} - \mathbf{C})\mathbf{s} + \bar{\mathbf{W}}^H \bar{\mathbf{v}} \tag{5.2}$$

where $\bar{\mathbf{W}}$ and $\mathbf{C}$ are the feed-forward matrix and the feedback matrix of the DFE receiver, respectively. To minimize error of the signal estimation in (5.2), we get

$$\mathbf{C} = \mathcal{U}[\bar{\mathbf{W}}^H \bar{\mathbf{H}}] \tag{5.3}$$

where $\mathcal{U}[\bar{\mathbf{W}}^H \bar{\mathbf{H}}]$ denotes the strictly upper-triangular part of $\bar{\mathbf{W}}^H \bar{\mathbf{H}}$. Substituting (5.3) back into (5.1), we obtain

$$\bar{s}_k = \bar{\mathbf{w}}_k^H \big([\bar{\mathbf{H}}]_{1:k} [\mathbf{s}]_{1:k} + \bar{\mathbf{v}} \big), \quad k = 1, \cdots, N_b \tag{5.4}$$

where $[\mathbf{a}]_{1:k}$ denotes a vector containing the first k elements of vector $\mathbf{a}$, and $[\mathbf{A}]_{1:k}$ stands for a matrix containing the first k columns of $\mathbf{A}$.

When the MMSE criterion is used to estimate each symbol, from (5.4) the feed-forward matrix $\bar{\mathbf{W}}$ is given as

$$\bar{\mathbf{w}}_k = \big([\bar{\mathbf{H}}]_{1:k} [\bar{\mathbf{H}}]_{1:k}^H + \mathbf{C}_{\bar{v}} \big)^{-1} \bar{\mathbf{h}}_k, \ k = 1, \cdots, N_b$$

where $\bar{\mathbf{h}}_k$ is the kth column of $\bar{\mathbf{H}}$. Let us introduce the following QR decomposition

$$\mathbf{G} \triangleq \begin{bmatrix} \mathbf{C}_{\bar{v}}^{-1/2} \bar{\mathbf{H}} \\ \mathbf{I}_{N_b} \end{bmatrix} = \mathbf{Q}\mathbf{R} = \begin{bmatrix} \bar{\mathbf{Q}} \\ \underline{\mathbf{Q}} \end{bmatrix} \mathbf{R} \tag{5.5}$$

where $\mathbf{R}$ is an $N_b \times N_b$ upper-triangular matrix with all positive diagonal elements, $\mathbf{Q}$ is an $(N_b + N_d) \times N_b$ semi-unitary matrix with $\mathbf{Q}^H \mathbf{Q} = \mathbf{I}_{N_b}$, $\bar{\mathbf{Q}}$ is a matrix containing the first N_d rows of $\mathbf{Q}$, and $\underline{\mathbf{Q}}$ contains the last N_b rows of $\mathbf{Q}$.

Using the QR decomposition (5.5), it has been shown in [89] that the feed-forward weight matrix $\bar{\mathbf{W}}$, the feedback matrix $\mathbf{C}$ and the MSE matrix $\mathbf{E} = \mathrm{E}\big[(\bar{\mathbf{s}} - \mathbf{s})(\bar{\mathbf{s}} - \mathbf{s})^H\big]$ can be written as

$$\bar{\mathbf{W}} = \mathbf{C}_{\bar{v}}^{-1/2} \bar{\mathbf{Q}} \mathbf{D}_R^{-1}, \quad \mathbf{C} = \mathbf{D}_R^{-1} \mathbf{R} - \mathbf{I}_{N_b}, \quad \mathbf{E} = \mathbf{D}_R^{-2} \tag{5.6}$$

where $\mathbf{D}_R$ is a matrix taking the diagonal elements of $\mathbf{R}$ as the main diagonal and zero elsewhere. Using (5.5) and (5.6), the joint source and relay optimization problem which minimizes the MSE of the signal waveform estimation can be formulated as

$$\min_{\bar{\mathbf{F}},\mathbf{B}} \quad \mathrm{tr}\big(\mathbf{D}_R^{-2}\big) \tag{5.7}$$

$$\text{s.t.} \quad \begin{bmatrix} \mathbf{C}_{\bar{v}}^{-1/2}\bar{\mathbf{H}} \\ \mathbf{I}_{N_b} \end{bmatrix} = \mathbf{QR} \tag{5.8}$$

$$\mathrm{tr}(\mathbf{B}\mathbf{B}^H) \le P_s \tag{5.9}$$

$$\mathrm{tr}\big(\bar{\mathbf{F}}\big[\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H+\mathbf{I}_{N_r}\big]\bar{\mathbf{F}}^H\big) \le P_x \tag{5.10}$$

where (5.9) is the transmit power constraint at the source node, while (5.10) is the power constraint at the output of $\mathbf{H}_{rd}$ (Chapter 4). Here $P_x > 0$ and $P_s > 0$ are the corresponding power budgets.

Let $\mathbf{H}_{sr} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{V}_s^H$ denote the singular value decomposition (SVD) of $\mathbf{H}_{sr}$, where the dimensions of $\mathbf{U}_s$, $\mathbf{\Lambda}_s$, $\mathbf{V}_s$ are $KN_r \times KN_r$, $KN_r \times N_s$, $N_s \times N_s$, respectively. We assume that the main diagonal elements of $\mathbf{\Lambda}_s$ are arranged in a decreasing order. We also introduce $M = \min(R_h, N_b)$, where $R_h \triangleq \min(\mathrm{rank}(\mathbf{H}_{sr}), \mathrm{rank}(\mathbf{H}_{rd}))$. Using the nonlinear MMSE-DFE receiver at the destination node of a parallel MIMO relay network, the optimal source precoding matrix and the relay amplifying matrices as the solution to the problem (5.7)-(5.10) are given by

$$\bar{\mathbf{F}} = \mathbf{V}\mathbf{\Lambda}_f\mathbf{U}_{s,1}^H, \quad \mathbf{B} = \mathbf{V}_{s,1}\mathbf{\Lambda}_b\mathbf{V}_r^H \tag{5.11}$$

where $\mathbf{\Lambda}_f$ and $\mathbf{\Lambda}_b$ are $M \times M$ diagonal matrices, $\mathbf{V}$ is any $N_d \times M$ semi-unitary matrix with $\mathbf{V}^H\mathbf{V} = \mathbf{I}_M$, $\mathbf{U}_{s,1}$ and $\mathbf{V}_{s,1}$ contain the leftmost M vectors of $\mathbf{U}_s$ and $\mathbf{V}_s$, respectively, and $\mathbf{V}_r$ is an $N_b \times M$ semi-unitary matrix ($\mathbf{V}_r^H\mathbf{V}_r = \mathbf{I}_M$) such that the QR decomposition in (5.8) holds. The proof of (5.11) is similar to the proof of Theorem 2 in [89].

From (5.11), we find that both $\bar{\mathbf{F}}$ and $\mathbf{B}$ have a beamforming structure. In particular, they jointly diagonalize the source-relay-destination channel matrix $\bar{\mathbf{H}}$ up to rotational matrices $\mathbf{V}$ and $\mathbf{V}_r$. It can be shown similar to [89] that the constraint (5.8) can be equivalently written as

$$\mathbf{d}[\mathbf{D}_R] \prec \boldsymbol{\sigma}_G \tag{5.12}$$

where $\prec$ stands for multiplicative majorization [109], $\boldsymbol{\sigma}_G$ is a column vector containing singular values of $\mathbf{G}$, and $\mathbf{d}[\mathbf{D}_R]$ is a column vector containing all diagonal elements of

$\mathbf{D}_R$. Let us denote $\lambda_{f,i}, \lambda_{s,i}, \lambda_{b,i}$, $i = 1, \cdots, M$, as the main diagonal elements of $\mathbf{\Lambda}_f$, $\mathbf{\Lambda}_s$, $\mathbf{\Lambda}_b$, respectively. Using (5.11) and (5.12), the optimization problem (5.7)-(5.10) can be equivalently rewritten as

$$\min_{\{\lambda_{f,i}\},\{\lambda_{b,i}\}} \quad \mathrm{tr}\big(\mathbf{D}_R^{-2}\big) \tag{5.13}$$

$$\text{s.t.} \quad \mathbf{d}\big[\mathbf{D}_R^2\big] \prec_{\mathrm{w}} \left[\left\{1 + \frac{\big(\lambda_{f,i}\lambda_{s,i}\lambda_{b,i}\big)^2}{\lambda_{f,i}^2 + 1}\right\}^T, \mathbf{1}_{N_b - M}\right]^T \tag{5.14}$$

$$\sum_{i=1}^{M} \lambda_{b,i}^2 \le P_s \tag{5.15}$$

$$\sum_{i=1}^{M} \lambda_{f,i}^2 \big[\big(\lambda_{s,i}\lambda_{b,i}\big)^2 + 1\big] \le P_x \tag{5.16}$$

$$\lambda_{b,i} \ge 0, \quad \lambda_{f,i} \ge 0, \qquad i = 1, \cdots, M \tag{5.17}$$

where $\prec_{\mathrm{w}}$ stands for weakly multiplicative submajorization [109], $\{a\}$ stands for an $M \times 1$ vector $[a_1, a_2, \cdots, a_M]^T$, $\mathbf{1}_{N_b - M}$ denotes a $1 \times (N_b - M)$ vector with all 1 elements.

The problem (5.13)-(5.17) is highly nonconvex and a closed-form solution is intractable to obtain. In the following, we develop an iterative method to obtain a numerical solution of the optimal $\{\lambda_{f,i}\}$ and $\{\lambda_{b,i}\}$. Let us define

$$\begin{aligned} &a_i \triangleq \lambda_{s,i}^2, \qquad x_i \triangleq \lambda_{b,i}^2, \\ &y_i \triangleq \lambda_{f,i}^2 \big[\big(\lambda_{s,i}\lambda_{b,i}\big)^2 + 1\big], \qquad i = 1, \cdots, M. \end{aligned} \tag{5.18}$$

Then using the definition of the operator $\prec_{\mathrm{w}}$ in [109], the optimization problem (5.13)-(5.17) can be equivalently written as

$$\min_{\{x_i\},\{y_i\}} \quad -\sum_{i=1}^{M} \log\left(1 + \frac{\frac{a_i x_i y_i}{a_i x_i + 1}}{1 + \frac{y_i}{a_i x_i + 1}}\right) \tag{5.19}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} x_i \le P_s \tag{5.20}$$

$$\sum_{i=1}^{M} y_i \le P_x \tag{5.21}$$

$$x_i \ge 0, \quad y_i \ge 0, \quad i = 1, \cdots, M\,. \tag{5.22}$$

For a $\{y_i\}$ fixed satisfying (5.21) and (5.22), the problem of optimizing $\{x_i\}$ can be

written as

$$\min_{\{x_i\}} \quad \sum_{i=1}^{M} \log\left(\frac{a_i x_i + y_i + 1}{a_i x_i y_i + a_i x_i + y_i + 1}\right) \tag{5.23}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} x_i \le P_s \tag{5.24}$$

$$x_i \ge 0, \qquad i = 1, \cdots, M. \tag{5.25}$$

The Lagrangian function associated with the problem (5.23)-(5.25) can be written as

$$\mathcal{L} = \sum_{i=1}^{M} \log\left(\frac{a_i x_i + y_i + 1}{a_i x_i y_i + a_i x_i + y_i + 1}\right) + \mu_1\Big(\sum_{i=1}^{M} x_i - P_s\Big) \tag{5.26}$$

where $\mu_1 \ge 0$ is the Lagrangian multiplier. Taking the derivative of (5.26) with respect to x_i equal to zero, we obtain

$$x_i = \frac{1}{2a_i}\left[\sqrt{{y_i}^2 + \frac{4a_i y_i}{\mu_1}} - y_i - 2\right]^{\dagger}, \quad i = 1, \cdots, M$$

where μ_1 is the solution to the following nonlinear equation

$$\sum_{i=1}^{M} \frac{1}{a_i}\left[\sqrt{{y_i}^2 + \frac{4a_i y_i}{\mu_1}} - y_i - 2\right]^{\dagger} = 2P_s.$$

In a similar fashion, for a $\{x_i\}$ fixed satisfying (5.20) and (5.22), we can update $\{y_i\}$ as

$$y_i = \frac{1}{2}\left[\sqrt{{a_i}^2{x_i}^2 + \frac{4a_i x_i}{\mu_2}} - a_i x_i - 2\right]^{\dagger}, \quad i = 1, \cdots, M$$

where $\mu_2 \ge 0$ is the solution to the following nonlinear equation

$$\sum_{i=1}^{M} \left[\sqrt{{a_i}^2{x_i}^2 + \frac{4a_i x_i}{\mu_2}} - a_i x_i - 2\right]^{\dagger} = 2P_x.$$

The iterative algorithm can be initialized at any random feasible $\mathbf{x}$ or $\mathbf{y}$. Since the conditional updates of $\{x_i\}$ and $\{y_i\}$ may either decrease or maintain but cannot increase the objective function in (5.19). Monotonic convergence of $\{x_i\}$ and $\{y_i\}$ to a locally optimum solution follows directly from this observation. After the convergence of the alternating algorithm, $\lambda_{f,i}$ and $\lambda_{b,i}$ can be obtained from (5.18) as

$$\lambda_{f,i} = \sqrt{\frac{y_i}{a_i x_i + 1}}, \quad \lambda_{b,i} = \sqrt{x_i}, \qquad i = 1, \cdots, M$$

and then the optimal structure of $\bar{\mathbf{F}}$ and $\mathbf{B}$ is given by (5.11). The rotation matrix $\mathbf{V}_r$ in (5.11) can be computed using the numerical method developed in [110]. From (5.11), we have $\mathbf{H}_{rd,i}\mathbf{F}_i = \mathbf{V}\mathbf{\Lambda}_f\mathbf{U}_{s,i}^H$, where matrix $\mathbf{U}_{s,i}^H$ contains the $(i-1)N_r+1$ to iN_r columns of $\mathbf{U}_s^H$. Then we obtain

$$\mathbf{F}_i = (\mathbf{H}_{rd,i})^+ \mathbf{V}\mathbf{\Lambda}_f\mathbf{U}_{s,i}^H, \qquad i = 1, \cdots, K. \tag{5.27}$$

Finally, we scale $\mathbf{F}_i$ in (5.27) to satisfy the power constraint (4.4) at each relay node as (4.27) and (4.28).

5.4 Numerical Examples

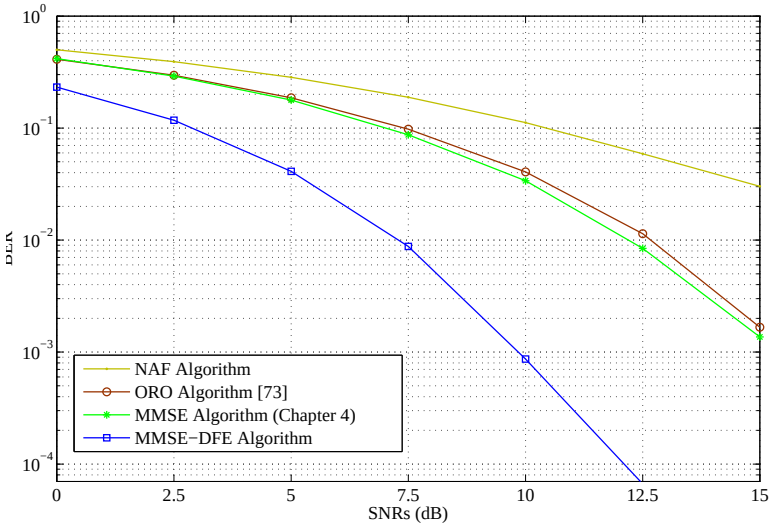

Figure 5.4: Example 5.1 BER versus $\mathrm{SNR_s}$ with $K = 3$.

In this section, we study the performance of the proposed optimal joint source and relay beamforming algorithm for parallel MIMO relay systems with MMSE-DFE receiver at the destination node. All simulations are conducted in a flat Rayleigh fading environment using the BPSK constellation, and the noises are i.i.d. Gaussian with zero mean and unit variance. The channel matrices have zero-mean entries with variances σ_s^2/N_s and $\sigma_r^2/(KN_r)$ for $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. We vary the SNR in the source-to-relay link ($\mathrm{SNR_s}$) while fixing the SNR in the relay-to-destination link ($\mathrm{SNR_r}$) to 20dB. We transmit 1000 randomly generated bits in each channel realization, and the BER

results are averaged through 200 channel realizations. Here we set $N_b = N_s = N_r = N_d = 3$.

In the first example, we simulate $K = 3$ and compare the BER performance of the following algorithms: (i) the proposed joint source and relay scheme with MMSE-DFE receiver; (ii) the joint source and relay algorithm for parallel MIMO relay systems where a linear MMSE receiver is applied at the destination node (Chapter 4); (iii) the NAF algorithm where both the source and relay matrices are scaled identity matrices; (iv) the ORO algorithm developed in [73] where the relay matrices are optimized based on the MMSE criterion, while the source precoding matrix is a scaled identity matrix. From Fig. 5.4, it can be seen that the NAF algorithm has the worst performance. The proposed algorithm outperforms the other three approaches.

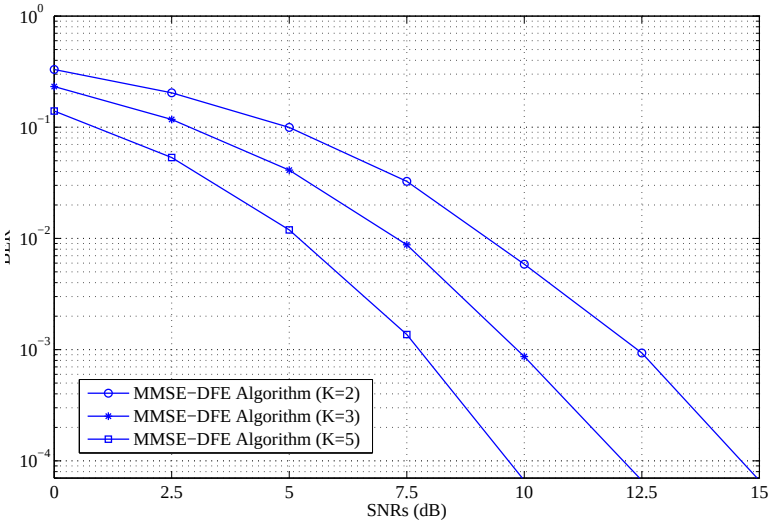

Figure 5.5: Example 5.2 BER versus SNR_s with varying K.

In the second example, we study the effect of the number of relays to the system BER performance using the proposed algorithm with MMSE-DFE receiver. Fig. 5.5 shows the BER performance with $K = 2$, 3, and 5. It can be seen that at BER $= 10^{-3}$, we achieve a 5dB gain by increasing from $K = 2$ to $K = 5$.

5.5 Chapter Summary

In this chapter, we have studied the optimal structure of the source precoding matrix and the relay amplifying matrices for parallel MIMO relay communication systems with

multiple parallel data streams when a nonlinear MMSE-DFE receiver is used at the destination node. The proposed source and relay matrices jointly diagonalize the source-relay-destination channel up to two rotation matrices and minimize the MSE of the signal waveform estimation. The proposed algorithm has an improved BER performance compared with existing techniques.

Chapter 6

Conclusions and Future Work

In wireless communications, MIMO relay communication systems have attracted much research interest and provided significant improvement in terms of both spectral efficiency and link reliability. In this thesis, we have developed several advanced algorithms for MIMO relay communication systems using parallel relays and successive interference cancelation technique.

6.1 Concluding Remarks

In Chapter 2, we studied the optimal transmit beamforming vector and the relay amplifying factors design problem for a parallel MIMO relay communication system with distributed single-antenna relay nodes when a single data stream is transmitted from the source to destination node. The proposed algorithm minimizes the MSE of the signal waveform estimation at the destination node. Simulation results demonstrate the performance of the proposed algorithm.

In Chapter 3, we have studied the general structure of the optimal source precoding matrix and relay amplifying matrices for a linear non-regenerative parallel MIMO relay communication system with multiple parallel data streams using the projected gradient approach. Simulation results demonstrate that the proposed algorithm has improved BER performance compared with the conventional techniques.

To reduce the complexity of the algorithm in the Chapter 3, we propose a simplified source and relay matrices design in Chapter 4 by first relaxing the power constraints at each relay node to the power constraint at the output of the second-hop channel. After solving the relaxed optimisation problem, the relay matrices are then scaled to

satisfy the individual power constraint at each relay node. Simulation results show a good performance-complexity tradeoff of the simplified algorithm.

Finally, in Chapter 5, we investigated the advantages of using detection algorithms in combination with SIC technique in MIMO relay communication systems and has derived the optimal structure of the source precoding matrix and the relay amplifying matrices for parallel MIMO relay communication systems with multiple parallel data streams when the nonlinear MMSE-DFE receiver is used at the destination node. Our results demonstrate that ZF-SIC and MMSE-SIC receiver algorithms outperform the ZF and MMSE receiver algorithms. The proposed optimal source precoding matrix and relay amplifying matrices together with the MMSE-DFE receiver algorithm has an improved BER performance compared with existing techniques.

6.2 Future Works

In this thesis, we have developed several advanced algorithms for MIMO relay communication systems using parallel relays and successive interference cancelation technique. However, there are still many possibilities for extending this dissertation work. We have proposed an iterative algorithm in Chapter 2 for distributed single-antenna relay nodes in a parallel MIMO relay communication system. Any possible closed-form solution to the problem can be an interesting future work since the complexity of iterative algorithms is comparatively higher than closed-form solutions.

We have extended the existing two-hop MIMO relay communication systems with single relay node schemes to two-hop MIMO relay communication systems with multiple parallel relay nodes schemes in Chapters 3-4 for linear receiver and in Chapter 5 for nonlinear receiver. However, the extension to multiple hops with parallel MIMO relay nodes still remains open as a challenging problem. Any closed form solution to the iterative algorithm proposed in Chapter 5 is also desirable.

Finally, we assumed that the channel state information (CSI) is fully known in all our algorithms. But in practice, CSI is not always perfectly known and has to be estimated. Therefore, optimal solutions robust to channel uncertainties for the problems we have solved will be of practical interest.

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