Western Australia School of Mines

Optimum Waste Dump Planning
Using Mixed Integer Programming (MIP)

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DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

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ABSTRACT

Mine planning is predominately ore production oriented, and the waste rock is only mined for the purpose of ore recovery. The strong interest in optimising ore production scheduling often leads to neglecting of the worthless waste rock and the rock dump. The reality, however, suggests an optimised waste dump plan can bring significant economic and environmental benefits to an operation. However, a review of the current practice reveals that no tool or methodology is available in the industry to create an optimum rock placement schedule to guide the planning of the waste rock dumps. Therefore, the aim of this research is to develop an engineering tool to generate the optimum rock placement schedule, and the corresponding dump schedule.

Through the modeling of waste rock dump planning and scheduling, a framework of integrated waste rock mining and dumping system is established. Based on this framework, a new mixed Integer Programming (MIP) model, known as location optimisation (OP) model, is created. It is theoretically possible to schedule the rock placement at a minimal haulage cost, in concurrent with acid mine drainage (AMD) prevention. Along with OP model, two variant MIP models are also produced. One minimises the opportunity cost by matching truck requirement with the budget, and the other minimises both haulage cost and opportunity cost, which are named truck balance (TB) and Combo models, respectively.

The OP model is tested by using a simplified data set, and the results are verified for cross-checking of the formulations. The output results prove that constraints that are designed for material segregation, non-acid forming (NAF) rock stockpiling and re-handling, and rock dump construction sequence are all correctly formulated and honoured.

The implementation of the three MIP models involves a synthetic data set. The solutions confirm that the developed MIP models are capable of automatically generating optimum rock placement schedules according to the defined objectives. The OP model schedule achieves a minimal cost of $119.2 million by focusing on minimising the haulage distance and the waste

rock re-handle. The TB model searches for a best-fitting schedule that has the minimal truck budget deviation, but much higher in cost of $122.8 million. The Combo model schedule considers both aspects, so the cost of $122.3 million is between OP and TB model schedules. To compare the potential saving in waste rock haulage, classic manual method is employed to schedule the waste rock. The estimated cost is $125.1 million, which is worse than that of by the MIP models, by up to $5.9 million or 4.95% compared to the OP model. This is already a significant cost saving, and it still has the potential to increase in larger scale mining operations.

The applicability of the MIP models is demonstrated via Tropicana Gold Project (TGP) case study. Three life of mine strategic dump schedules are generated by the three MIP models, along with relevant numerical data for each time period, such as the estimated haulage distance, the truck hour requirement, the truck productivity, and the growth medium (GM) material re-handle. These data will assist to better manage the truck fleet, and to achieve sustainable mining objective. The graphical results derived from the rock placement schedule provide direct visual assistance for better forecasting the long term outlook of operation, and it will allow the engineers to identify any misalignment between the planning and operation.

In addition, the MIP models are utilised for quick analysis of the proposed alternative rock dump design. The graphical results suggest that the alternative design will reduce the overall footprint, and the numerical results prove this observation. It is calculated that alternative dump design will reduce the growth medium (GM) material demand by 25%, lower the overall haulage distance by 9.7% and increase the truck productivity by 12.6%. Therefore the developed MIP models can provide numerical and graphical solutions to aid the decision-making for ‘what if’ scenarios.

In conclusion, three MIP models have been constructed, verified, and implemented in a real operation. It is proved that the models are capable of generating optimum rock placement schedules and corresponding dump schedule to assist with the optimum waste dump planning.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ADS	Advanced Destination Scheduler
AHS	Autonomous Haulage System
AMD	Acid Mine Drainage
AMPL	A Mathematical Programming Language
BCM	Bank Cubic Meter
GM	growth medium
ID	Identification
INAP	International Network for Acid Prevention
LCM	Loose Cubic Metre
LP	Linear Programming
MIP	Mixed Integer Programming
Moz	Million Ounces
MRD	main rock dump
MSP	marginal grade stockpile
NAF	Non-acid Forming
NAFSP	Non-acid forming (rock) stockpile
NPC	Net Present Cost
NPV	Net Present Value
PAF	Potential Acid Forming
RAM	Random-Access Memory
ROM	Run Of Mine
TSF	Tailing Storage Facility
TGP	Tropicana Gold Project

CHAPTER 1. INTRODUCTION

A mine waste rock dump does not have any intrinsic value; however, it is inevitable for most open pit mines. Due to the large volume of material handling and the associated haulage cost, a waste rock dump could be the most expensive structure to be built in an open pit mine. In addition, it has the potential to produce acid mine drainage (AMD), which threatens the natural environment even after mining has ceased. These economic and environmental issues are often studied as two individual problems when planning the waste rock dump, i.e. optimising one will sacrifice the other. Very little research has been conducted to explore a better integrated approach for the waste rock dump planning.

1.1 PROBLEM STATEMENT

A possible solution is an optimised rock placement schedule that selects the most appropriate dumping location. Such a schedule is required to minimise the overall haulage cost, in concurrent with AMD prevention by encapsulating the potential acid forming (PAF) rock. Furthermore, the marginal grade (or low grade) is to be separately stored for future process, should the commodity price increase in the future.

To achieve all these aims, the detailed dumping location for each mining block, or a rock placement schedule, must be decided once a mine production schedule is produced. The general rule is to selectively handle different type of rock, as illustrated in Figure 1-1. PAF rock is to be centralised in a designated location surrounded by non-acid forming (NAF) rock in the main rock dump. The NAF rock stockpile is optional if required. The precise volume of NAF rock to be stockpiled and the timing for it to be re-handled are dependent on the progression of the rock dump, which are to be determined along with the rock placement schedule.

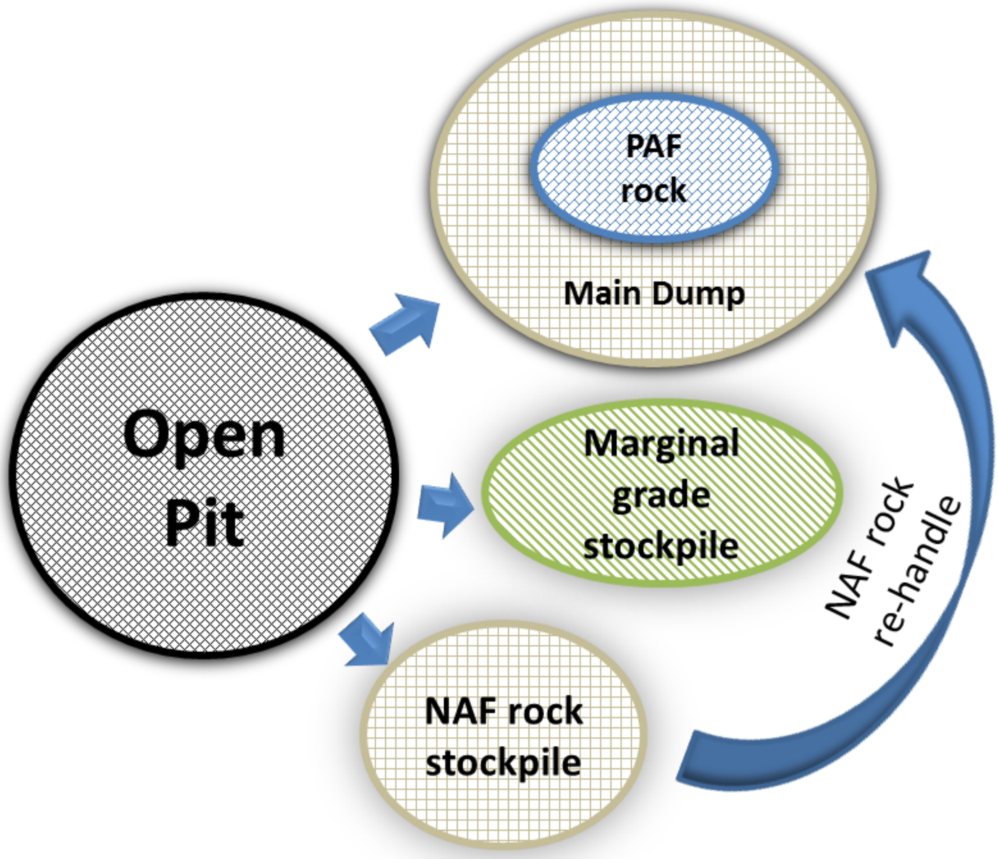


Figure 1-1 Ideal material flow chart in a typical open pit operation

The rock placement schedule needs to outline the volumetric rock movement from its original in-situ location in an open pit to the final resting location in a rock dump. To be practical, this schedule must consider the following:

1. It is fully integrated with the pre-defined mining schedule;
2. It selectively handles and segregates different material to different rock dumps or stockpiles;
3. It schedules the PAF rock to the designated location only;
4. It ensures complete encapsulation of the PAF rock by the NAF rock;
5. It determines the time and volume of NAF rock to be stockpiled and also the time and volume to be re-handled; and
6. It represents a logical rock dump construction sequence.

1.2 OBJECTIVES

The main objectives of the research are the following:

- To model the integrated mine waste rock mining and dumping system, which incorporates the existing mine production schedule;
- To create mathematical programming models that automatically generates the optimum rock placement schedule and corresponding dump schedule, under the condition of the environmental constraints;
- To generate multiple rock placement schedules to achieve different objectives, such as haulage cost minimisation, opportunity cost minimisation by matching equipment requirement with budget, or a balanced schedule in haulage cost and opportunity cost;
- To simultaneously optimise the rock placement schedule, NAF rock stockpile schedule, NAF rock re-handle schedule, and the rock dump progression by the proposed mathematical models; and
- To implement the developed methodology in a real case study.

1.3 SCOPE

The scope of the project is limited to optimising the rock placement, including, but not limited to the PAF rock, the NAF rock and the marginal grade (or low grade). It is aimed to schedule the mined rock in conjunction with a defined mine production schedule and, subsequently forming a detailed dump schedule for the life of mine. Although the mine production scheduling and mine waste rock dump design are the other two key components of the problem, they do not form part of this study.

1.4 SIGNIFICANCE AND RELEVANCE

The research project offers a new approach towards to the planning of mine waste rock dumps, and brings a number of benefits to the mining industry.

Firstly, the proposed mathematical models will generate detailed waste rock dumping schedules to suit different objectives, i.e. minimal haulage cost, or minimal opportunity cost, or a hybrid between the two. Such a schedule will significantly reduce the uncertainty in waste rock scheduling and rock dump planning, which provides a base for engineers to identify any flaws and to seek improvement towards sustainable mining.

Secondly, waste rock mining, dumping, and dump construction are modelled in an integrated system, which identifies the operational issues associated with scheduling waste rock dump(s).

Thirdly, operational policies, such as material segregation and PAF rock encapsulation, are formulated by mathematical equations. Therefore, the resulting rock placement schedule is imbedded with these considerations. This will eliminate human errors in producing waste rock dumping schedules, which may deviate from these operational policies.

Fourthly, the methodology utilises mathematical programming, which is a scientifically proved method for optimality. This ensures the resulting schedules are more accurate and better than any classical manual method.

Finally, the methodology itself is flexible for modifications. This will help to satisfy the increasing demand in optimising other operational problems related to the mine waste rock dump.

1.5 THESIS OVERVIEW

Chapter 1 presents the waste dump planning problem. It defines the objectives and scope of the study, then summarises the key original contributions of the research.

Chapter 2 provides an overview of current practice in waste rock dump planning and explains the methodology proposed for solving this problem.

Chapter 3 describes the main considerations in modelling the waste rock dump planning, from which the core problem for generating the optimum rock placement schedule is identified.

Chapter 4 explains the formulations of the three mathematical models, which focus on minimising haulage and re-handle cost, minimising the deviation from truck budget, and balancing the haulage cost with truck budget, respectively. A verification of the model is also included in this chapter.

Chapter 5 demonstrates the differences between the three developed models by solving the dump schedule problem using a synthetic data set. It compares the estimated haulage cost by the MIP models against to that of by the classical manual method. Furthermore, the real-world implementations are performed, using a greenfield mine site data.

Chapter 6 concludes all the findings of the research and states recommendations for future research.

CHAPTER 2. WASTE ROCK DUMP PLANNING AND PROPOSED RESEARCH METHODOLOGY

Traditional mine planning is ore oriented. The main objective is to maintain a constant ore production rate, in such a way that saleable product specifications are met and project net present value (NPV) is maximised. Hence, the waste rock mining, hauling, and dumping are not regarded as the most critical part of the production schedule. Consequently, the detailed spatial locations for placing the waste rock are not typically included in a mining schedule, which could potentially result in deviations between the actual operation and the mine planning. In this chapter, current practice in waste rock dump planning is discussed, from which, an existing problem is identified, together with a proposed methodology to address this issue.

2.1 BACKGROUND INFORMATION

2.1.1 Waste rock haulage cost

Material handling cost accounts for up to 50 percent of the operating costs in a typical open pit mine (Alarie and Gamache, 2002), among which a large proportion is for hauling waste rock to a waste rock dump (Wang and Butler, 2007; Williams *et al*, 2008; Russell, 2008; Sommerville and Heyes, 2009).

Taking a simple example, an open pit mine with a stripping ratio of five to one should expect the cost of hauling waste rock to be at least five times that of hauling ore. It can be assumed that ore is often sent to a fixed run of mine (ROM), then to the plant for processing. The haulage distance or cost from a pit exit to the ROM does not vary significantly over the time. In contrast, the bulk of waste rock volume accumulates at the rock dump, which will expand horizontally and then vertically, with extra lift. As a result, the travel distance increases, hence the haulage cost. Eventually, the cost for hauling waste rock will be greater than five times that of hauling ore. Therefore, any strategy to reduce the cost in the waste rock haulage and dumping could translate into a big saving.

Reducing the overall waste rock handling cost can be achieved by either minimising the haulage distance or maximising the productivity, to reduce the unit cost per unit volume or mass of waste rock handled. Available solutions include increasing the equipment size, improving the machinery reliability performance, providing better operating conditions, and implementing a dispatching system (Lizotte and Bonates, 1987). It is noted that all of the mentioned means are intended to increase the productivity, yet few studies have been able to conclude effective methods that reduce the actual haulage distance or the cost component. This circumstance is not only because of the lack of interest in waste rock, but also because of the ore-oriented production scheduling in the mining industry.

Since mining ore is the primary objective, the scheduling of waste rock haulage and dumping receives little attention. The basic criterion for planning the waste rock dumping, apart from considering the rock dump geotechnical stability and final designed shape, is to match the volumetric capacity of a rock dump with the waste rock volume mined out from an open pit. This approach often leaves the problem to the daily operation, i.e., where exactly within the rock dump should the dumping occur? Short-term planning engineers follow the availability principle and seek the shortest haul to minimise the short-term cost (Wang and Butler, 2007), as illustrated in Figure 2-1.

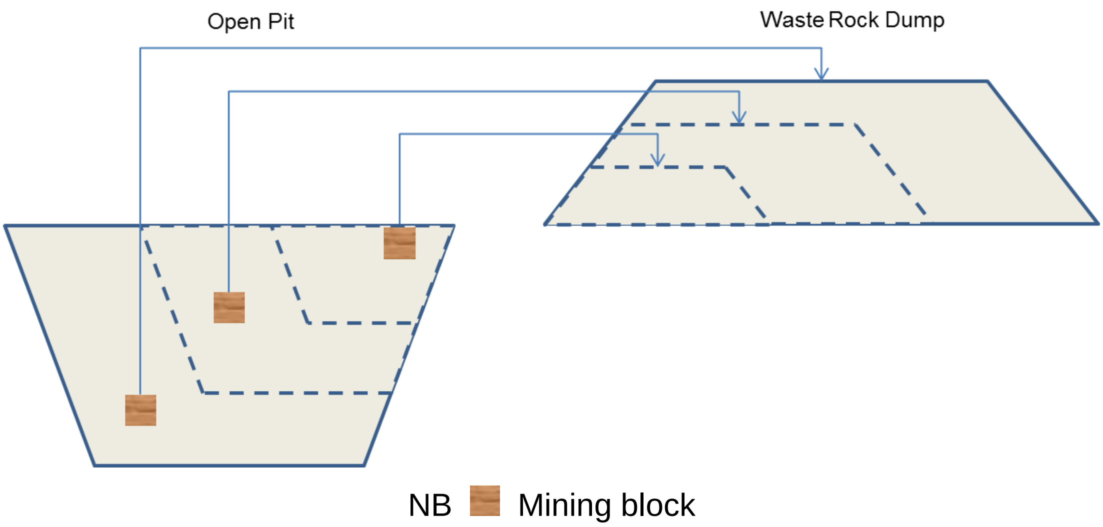


Figure 2-1 Dump progression with the shortest haul first strategy

This approach lacks of long term vision and could result in extra rehabilitation cost in the future. Sommerville and Heyes (2009) used a case study to demonstrate that this type of dump progression sequence requires a large amount of material re-handling for rehabilitation, which results in an 8% higher in cost compared to some other dump schedule.

There are also other ways to expand a waste rock dump, such as a centred progression followed by long-haul then short-haul dumping strategy, as illustrated in Figure 2-2 and Figure 2-3, respectively.

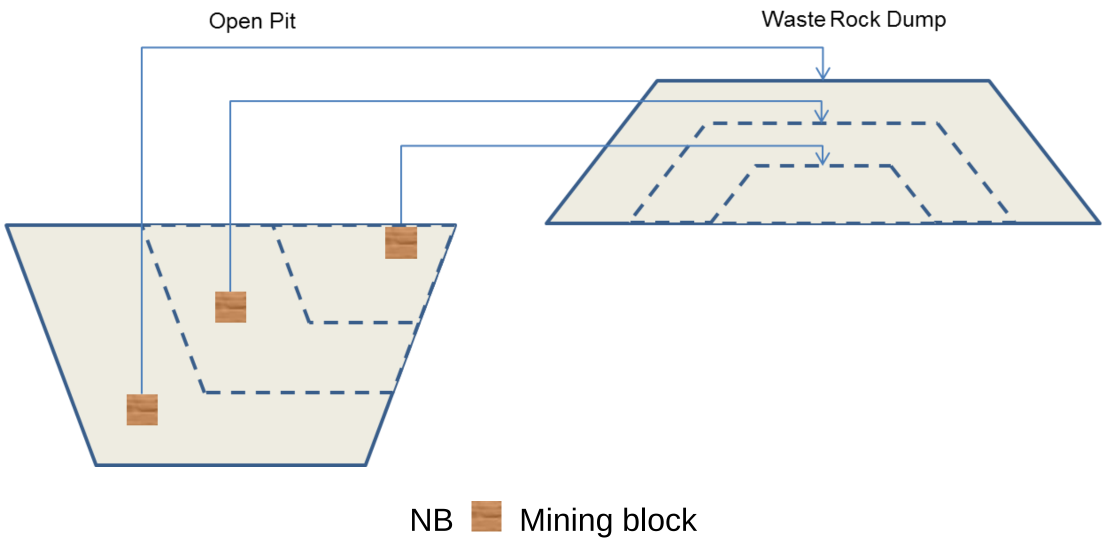


Figure 2-2 Dump progression with a centred haul strategy

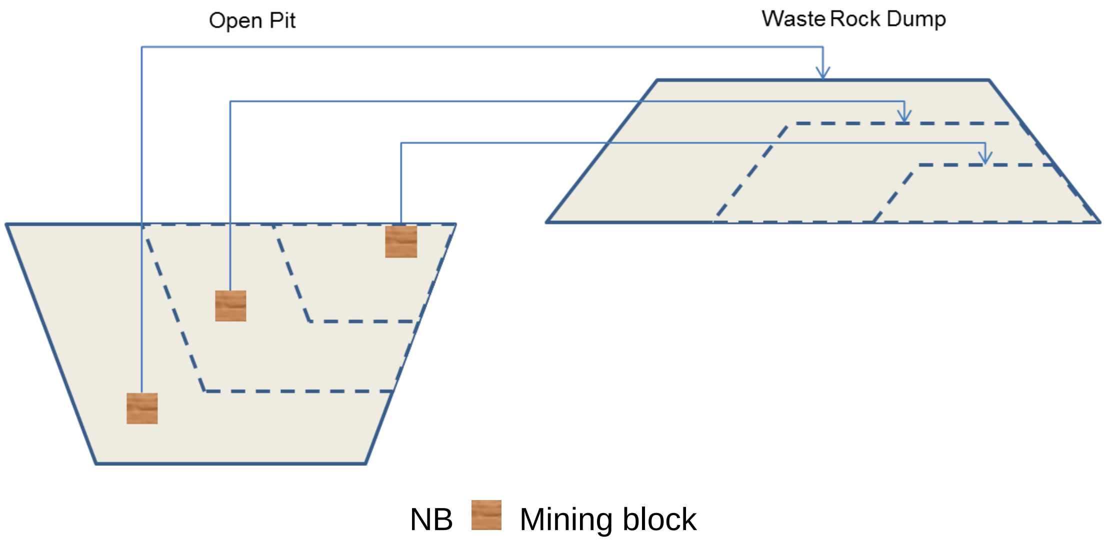


Figure 2-3 Dump progression with a long-haul then short-haul strategy

However, a quantitative analysis of the overall haulage cost under these dump progression sequences is very time consuming; nevertheless, there could be other better progression sequences available that are more economical than the three types listed above.

To seek the optimum solution, one must evaluate the haulage cost from each waste block to all possible dumping locations, as shown in Figure 2-4. The extent of the problem is proportional to the number of waste mining blocks and the possible dumping locations.

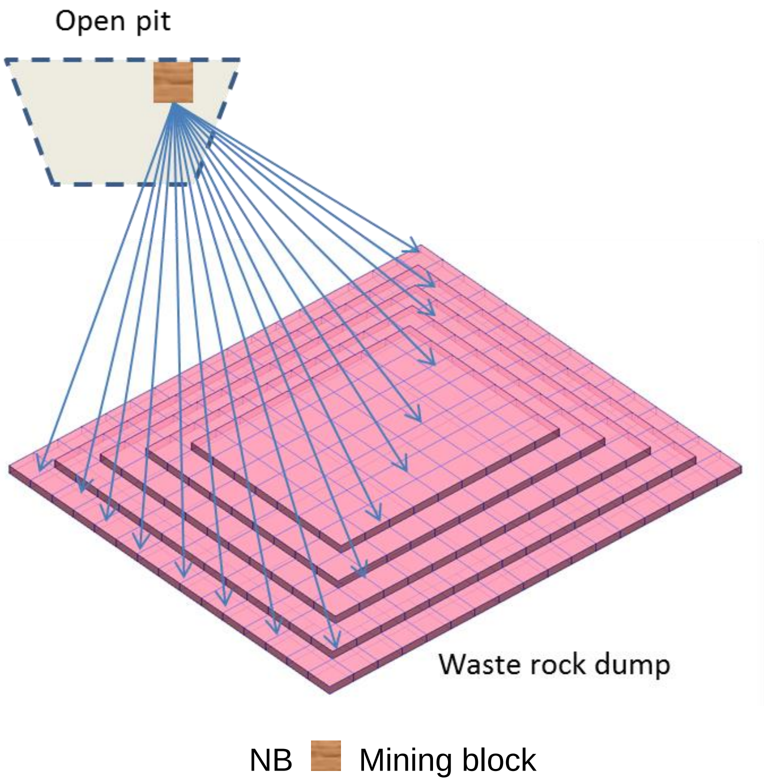


Figure 2-4 Evaluating all possible dumping locations for a mining block

In addition, one must also evaluate the timing cost for hauling waste rock to each individual dumping location, as shown in Figure 2-5. This is because of the time value of money.

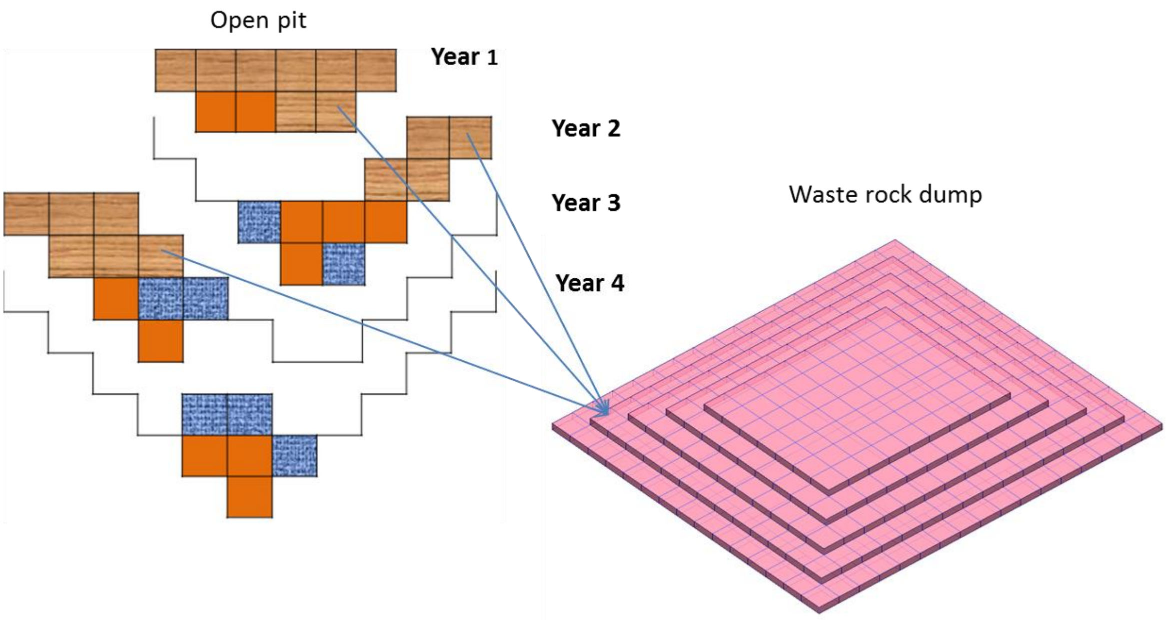


Figure 2-5 Evaluating haulage cost to one dumping location in different time periods

The possible dumping locations in the waste rock dump are often too numerous to analyse in a timely fashion, and the problem could become even more complicated if the waste rock is permitted to be stockpiled and then to be re-handled. A scientific method must be employed to determine the true optimum rock placement schedule and the corresponding dump progression sequence.

2.1.2 Acid mine drainage (AMD) and prevention

Acid mine drainage (AMD) is an acidic, heavily contaminated discharge from an operating or an abandoned mine site. This environmental problem can cause long-lasting threats to the natural environment and nearby communities. A mine waste rock dump is one of the potential pollution sources. The formation of AMD is described by Johnson (2003), which is often quoted as the following chemical equation (2.1).

$$4FeS_2 + 15O_2 + 14H_2O \rightarrow 4Fe(OH)_3 + 8SO_4^{2-} + 16H^+ \qquad \textbf{(2.1)}$$

In brief, it is due to the oxidation of pyrite and other sulphidic minerals, resulting from the exposure to both oxygen and water (INAP, 2013). The process is illustrated in Figure 2-6. Because much of Australian’s zinc, lead, silver, copper and nickel deposits are sulphide ores (Hore-Lacy, 1979), many mine sites have the potential risk of generating AMD.

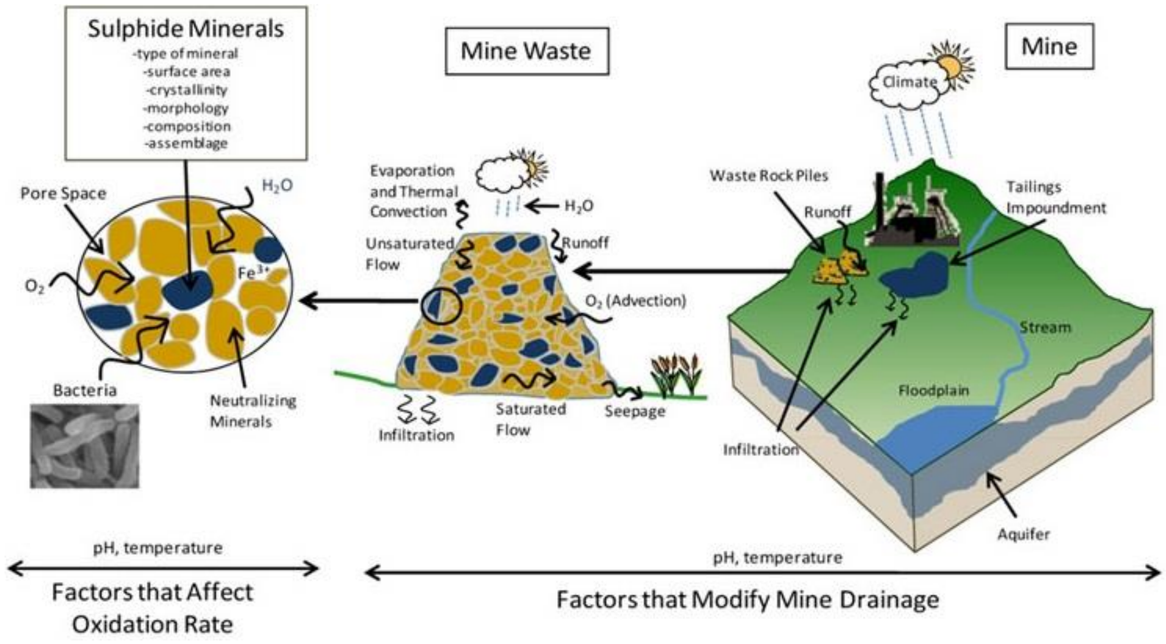


Figure 2-6 Illustration of AMD formation

(Source: INAP, 2013)

The treatment of such AMD comes at a high price, both environmentally and economically (Gazea, Adam and Kontopoulos, 1996; Harries, 1997; Kuyucak, 2002; Johnson and Hallberg, 2005). For example, a current operating AMD neutralisation plant in Japan has an estimated annual cost of approximately US$5 Million dollars. This plant became operational in 1982 (JOGMEC, 2012) and must remain operating because of the long-lasting characteristics of AMD.

Alternatively, AMD prevention by good mine planning could be a more preferred method (Johnson and Hallberg, 2005). Williams, Stolberg and Currey (2006) described the encapsulation of the PAF mineralised waste rock at Kidston Gold Mines in north Queensland, Australia, by weathered (oxidised) NAF rock. The simplified cross section of the final waste rock dump is illustrated in Figure 2-7. By this mean, contact between the PAF rock, air and water are controlled at a minimal level, thus reducing the potential for AMD occurrence.

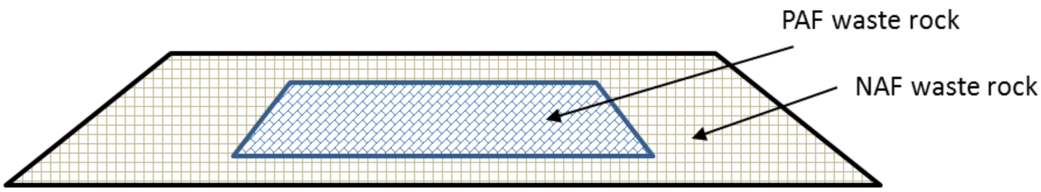


Figure 2-7 Simplified cross section view of a waste rock dump with PAF rock fully encapsulated in the centre of the rock dump

(After Williams, Stolberg and Currey, 2006)

However, it could be very difficult to achieve this final dump shape without the option of temporary stockpiling then re-handling the NAF rock. This is because the proportion of the PAF rock typically increases when mining progresses beyond the water table level. The NAF rock will need to be stockpiled in the early stages of mining; then, it is available for re-handling to encapsulate the PAF rock. The volume of the NAF rock to be stockpiled and the time to be re-handled for PAF rock encapsulation depends on the rock dump progression. Therefore, a detailed rock placement and dump schedule becomes the key to form an environmental sound waste rock dump.

2.2 Current Practice in Mining Industry

A waste rock dump is environmentally hazardous. For this reason, government agencies have established guidelines for mining companies to design, maintain and rehabilitate this structure, for example, ‘Guidelines For Waste Dump Design and Rehabilitation’ (Anon, 1992). The technical aspects, such as the dump design, stability of the waste dump, mine reclamation, and ground water and contamination can be found in other reference books, e.g., ‘Surface Mining’ (Bohnet and Kunze, 1990).

However, the descriptions are often too brief to aid the actual waste dump planning. The Federal Department of Environment Australia recognised this issue, and to promote sustainable mining, some leading practice examples have been compiled in ‘Best Practice Environmental Management in Mining: Landform Design for Rehabilitation’ (Anon, 1998).

On top of those guidelines, reference books, and best practice examples, many mining companies have developed their own policies in waste rock dumping and handling, because every mine site has its unique environment, location, size, topography, mineralisation and the lease boundary. Therefore, each mine waste rock dump design is tailored to best suit these external conditions. For example, Rio Tinto Iron Ore's Pilbara mine sites have a well-defined management plan for sulphides waste rock (Green, 2009), which describes the special design requirement and placement sequence for the PAF rock.

However, a well-designed waste rock dump is solely a graphical product if it is not constructed properly. The construction of the waste dump is based on the available waste material mined over the life of mine. It requires a detailed waste rock placement schedule or a dump schedule to guide an operation, in such a way that the engineering designed rock dump can be gradually formed.

2.2.1 Traditional manual rock dump scheduling

Different from extensively studied ore production scheduling, the rock dump scheduling has the lowest priority level. An engineer designed waste rock dump is often treated as a single point with a known volumetric capacity. The scheduling of the rock dump and the waste rock movement is normally conducted by the short term planning engineer, based on the availability principle during daily operation.

The tools used in most cases are excel spreadsheet for record keeping and mine design packages for dumping location capacity estimation. The preference in choosing the dumping location is subject to personal experience and judgment. Due to the absence of long term guidance, the manual method can be problematic. It could deviate from the long term plan, and ultimately fail to reach the original design objectives.

2.2.2 Software aided rock dump scheduling

There are many mine scheduling software available to create the optimised production schedule, yet only a few claim to be capable of producing or optimising the rock dump schedule.

2.2.2.1 Minemax™

Minemax (Minemax, 2013) is a software company that provides optimisation and scheduling solutions to the mining industry. Recently, the company has presented its 'Waste Dump Optimisation' technique at the 2013 SME Annual Meeting (Butler, George and Scott, 2013). The presentation is entitled 'Simultaneous Pit and Waste Dump Schedule Optimisation', and the authors stated that it is possible to jointly optimise the detailed waste scheduling together with a mine schedule.

In the example, the entire waste rock dump is divided into smaller dump blocks to better represent the different haulage distanced and, hence, trucking hours from a common dump entry point. The authors also took pit depth into consideration for the truck hour calculation. This approach is more accurate and realistic compared with manual practice, in which a rock dump is often regarded as a single point. The solution is an optimised mining schedule that satisfies the trucking hour budget under the pre-defined waste rock dump progression sequence.

However, due to the human intervention on the rock dump progression sequence, the claim of 'optimising detailed waste scheduling' is misleading. The dump progression is not an optimised result from the program, but instead, it is pre-defined. Furthermore, the software does not have options for an NAF rock stockpiling and re-handling. Without such options, it may fail to encapsulate the PAF rock in case of lacking NAF material scenarios.

In addition, the users are allowed to produce the production schedule by selecting one time period at a time, which is a function that is provided in the software to reduce the solution time. This approach will risk the overall optimality of the resulting schedule.

2.2.2.2 Whittle™

Whittle™ (GEOVIA, 2013) is a well-known open pit optimisation software package that was developed by Jeff Whittle (1988). It uses the Lerchs and Grossmann Algorithm (Lerchs and Grossmann, 1965) to determine the ultimate pit limit for a set of given block model. The alternative usage of this software in waste rock dump optimisation is noted by Dincer (2001). The basic technique is to treat the dump optimisation problem as a mirror image of the pit optimisation problem.

A dump cost model is created before optimisation is performed. Both the haulage cost and the area cost are assigned to each block in the modified block model. The resulting nested waste dump 'cones', as illustrated in Figure 2-8, describe the sequence of the dump progression and also outline the most economical final footprint. Because Whittle™ is readily available on the market, this exercise can be conducted by most mining companies, for which a dump cost model is pre-defined.

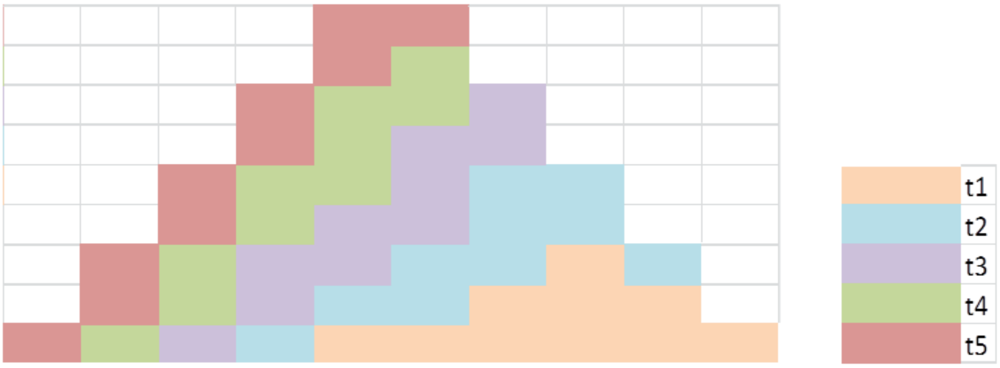


Figure 2-8 Cross sectional view of the nested waste rock dump 'cones' in relation to the time period

However, the dump cost model is limited to considering the haulage cost from a fixed point to a block. In case of the multiple pit exit points and dump entry points situation, it is required to assign the minimal haulage cost to a dump block. This limitation could lead to inaccurately solving the problems, particularly under the scenario of a mine site involving multiple pits/pit exit points, multiple dumps/ dump entry points.

The end result from the software is the optimised shape of a rock dump in fixed time interval, without any information about the waste rock that is mined and hauled. The lack of such information could result in a failure to reach the final shape, due to having a misalignment between the planning and the operation department.

2.2.2.3 XPAC Advanced Destination Scheduler (ADS)™

XPAC™ (RungePincockMinarco, 2013) is a powerful scheduling software package that is used by many mining companies. It is capable of generating mining schedules to meet target mining rates, crusher feeds and grades. The Advanced Destination Scheduler (ADS) is an add-on module for scheduling waste rock placement. Wang and Butler (2007) demonstrated the use of the ADS module for scheduling the waste rock dump at the BHP Billiton's Mt Whaleback operation. The main objective is to produce the most economical dump schedule that has the shortest travel time. After a long time of making preparations and scripting, the authors successfully obtained a dump schedule, with the corresponding dump progression in both numerical and graphical form. However, there are many flaws that prevent this schedule from being the optimum solution.

Firstly, it uses TALPAC™ script to calculate the travel time without the presence of a realistic haul route. This estimation requires many assumptions, such as an elevation conversion factor, gradient, corner radius, rolling resistance and speed limits. The calculated results might not correctly reflect the actual travel time; hence, the decision making of selecting the shortest haul path is questionable.

Secondly, the authors have considered segregation for different waste rock material, to centralise the PAF rock. However, without the enforcement of fully encapsulating PAF rock, it is still possible to expose the PAF rock at the end of mine life.

Thirdly, the shortest haul may not be the most economical method if waste rock dump rehabilitation cost is considered, due to large volume of material re-handle at the end of mine life (Sommerville and Heyes, 2009).

Fourthly, the schedule generated is feasible, but it is not optimal. The adjacent dependency rule has limited the dump progression direction, which could possibly violate the optimality of the system, resulting in a sub-optimum solution.

Fifthly, the haulage path selection satisfies the shortest haul in each discrete time period. The resulting solution is likely to schedule the waste to the nearest dump parcel in an early time period and further away as time progresses because the location of the shorter haul has been fully occupied in the earlier time periods. The local optimum for each time period does not equal the global optimum for the entire duration of the mine life.

Lastly, XPAC itself is not an optimisation tool (Youds, 2000); instead, it requires the user to vary the parameters manually, to achieve the project objective.

2.3 Research Methodology

Through the review of the available software, it is noted that none has the function in generating the optimum rock dump schedule. Human intervention is a must in deciding the dump progression sequence, so the scheduling result is not true optimum solution.

Mathematical modelling is a scientific method to obtain the true optimum solution under multiple criteria, yet it is flexible enough to perform modifications. It has been widely used in many industries to study and optimise the systems (Taha, 2007). For these reasons, it is proposed to use mathematical modelling to describe the waste rock mining and dumping system.

2.3.1 Mathematical modelling

The process of mathematical modelling is described by Kallrath (2004). It involves the following steps:

1. Construct a mathematical model that is based on a real-world problem;
2. Collect data for the problem generation;
3. Solve the problem and obtain the optimum solution;
4. Interpret the solution; and
5. Implement the solution to improve the system.

The modelling cycle could take a number of iterations to achieve the most accurate representation of the problem, as illustrated in Figure 2-9. Apart from these basic steps, choosing the most appropriate type of mathematical model is equally important. An efficient model would not only benefit the accuracy of the problem description, but also allow the user to obtain the optimum solution in a timely fashion.

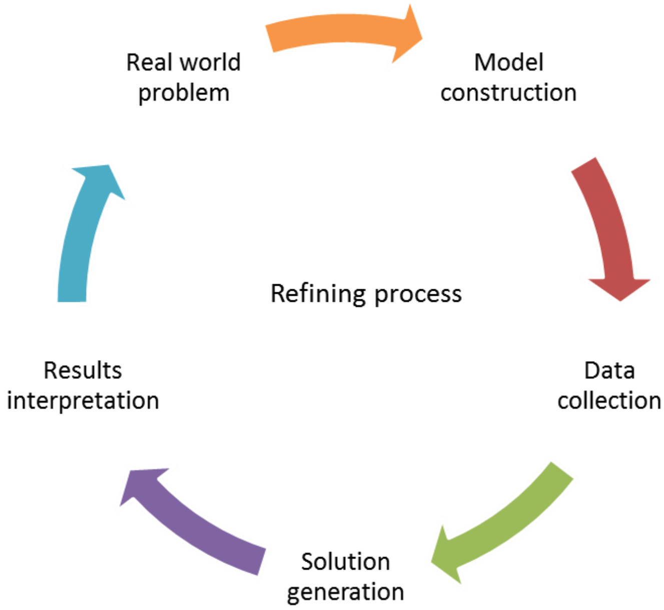


Figure 2-9 Typical modeling and problem solving cycle

(After Kallrath, 2004)

Among the many mathematical models, Linear Programming (LP) is the most common technique for solving optimisation problems. The generalised LP model consists of a linear objective function, with a number of linear constraints, and a set of non-negative restrictions, as shown in equations (2.2) to (2.4).

$$\text{Maximise (or Minimise) } Z = C_1X_1 + C_2X_2 + C_3X_3 \cdots + C_jX_i \tag{2.2}$$

$$\begin{Bmatrix} A_{11}X_1 + A_{12}X_2 + \cdots A_{1n}X_i \le B_1 \\ A_{21}X_1 + A_{22}X_2 + \cdots A_{2n}X_i \le B_2 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ A_{m1}X_1 + A_{m2}X_2 + \cdots A_{mn}X_i \le B_m \end{Bmatrix} \tag{2.3}$$

$$X_1, X_2 \cdots X_i \ge 0 \tag{2.4}$$

The objective Z is the value of interest, which is equal to a function of the decision variables X_i with the corresponding coefficients C_j. The Z value may stand for the cost or NPV, depending on the formulation. It provides a numerical indicator to compare the solutions. The limiting conditions of the problem are formulated in the constraint sets (2.3), and the constant A_{mn} and

B_m are derived from the problem. Moreover, the limitations of the variable X_i are regulated by the constraint (2.4).

When solving the LP problem, there are many possible solutions that exist to satisfy the constraints (2.3) and (2.4). However, there is only one set of solutions that allow the Z value to reach its maximum (or minimum) value. According to the preference of a maximisation or minimisation, such a solution set is known as the optimum solution, which is mathematically proved.

Mixed Integer Programming (MIP) is a restricted form of LP in which some variables must be integers while others are continuous. A binary variable is another special case of an integer variable. The value of the variable is zero or one, representing ‘no’ or ‘yes’ situation. With the introduction of a binary variable, MIP can specify a variety of logical conditions, which make the mathematical modelling more realistic and accurate in representing a real-word problem. Therefore, MIP will be used for this research.

2.3.2 Solving the optimisation problem

Upon successful construction of a mathematical model, a problem must be solved to determine the optimum solution. In the past, a graphical method has been used to solve simple LP problems, as shown in Figure 2-10.

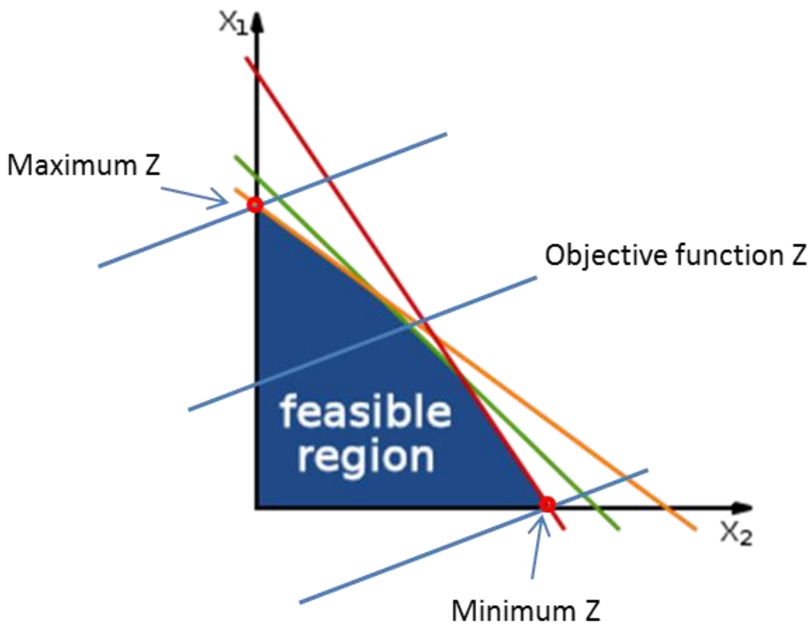


Figure 2-10 Illustration of the graphical method concept

The linear constraints outline the feasible region, where any points (X_1, X_2) within this region satisfy the condition. The Objective function Z is graphed and the value is calculated. An optimum solution set (X_1, X_2) can be located when Z value reaches the maximum or minimum, depending on the optimisation nature of the problem.

The graphical method becomes impractical when solving problems that have a large number of variables and constraints. In the late 1940's, the simplex algorithm was developed by Dantzig for solving more complicated linear programming problems (Fourer, Gay and Kernighan, 2002). This method provides a standard approach to any liner programming problems. It first converts a problem into standard form. Then the problem is reconstructed to a table form. The derivation of the optimum solution is via a series of row operations on the table. The detailed solving steps are discussed by Taha (2007).

With the advancement of computing technology, computerised row operation enables faster and accurate results generation. However, problems involving binary variables cannot be solved simplex method. It requires branch and bound technique to divide the problem into sub-problems before solving.

Branch and bound algorithm first solve the problem using LP relaxation method. If the solution contains correct integer value for the integer variable, then the optimal solution is obtained. Otherwise, an integer value will be assigned to either side of the non-integer value to create two new sub-problems, which are then solved by standard LP solution procedures. This process continues until all branches for a given 'level' have been evaluated and no better objective value can be calculated. Figure 2-11 illustrates the fundamental of branch and bound technique.

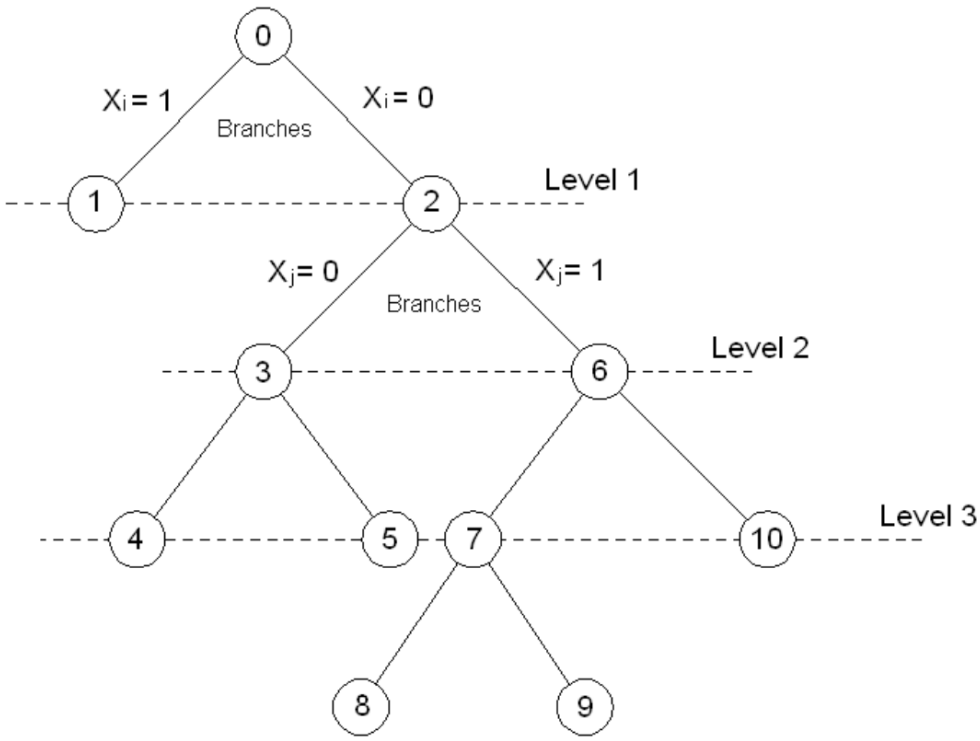


Figure 2-11 Illustration of branch and bond tree

(after Topal, 2003)

Modern optimisation engines, such as IBM ILOG CPLEX, have incorporated these algorithms to solve large-scale optimisation problems.

2.3.3 Available mathematical models in the mining industry

Mathematical modelling has been successfully applied in the mining industry since the 1960s. One of the most important achievements in open pit mining is the determination of the ultimate pit limit (Lerchs and Grossmann, 1965), which has been incorporated into many commercial software packages, such as Whittle™, Maptek Vulcan Chronos™ and CAE NPV Scheduler™ (formerly Datamine NPV scheduler™) (Newman *et al*, 2010). Other scholars have used mathematical modeling to optimise production scheduling (Gershon, 1983; Dagdelen, 1985; Smith, 1998; Caccetta and Hill, 2003; Ramazan and Dimitrakopoulos, 2004; Ramazan, 2006; Cullenbine, Wood and Newman, 2011; Kumral, 2012; Groeneveld, Topal and Leenders, 2012), and to solve operational equipment allocation problems (White and Olson, 1992; Topal and Ramazan, 2012).

The studies on underground mining started much later, because of fewer in number of mine sites and difficulty in modelling various mining methods. Despite these factors, there has been some mathematical models for optimising the underground mine design (Alford, 1995), mining sequencing (Trout, 1995), and integrated mine design, sequencing and machine allocation problems (Topal, 2003; 2008; Nehring and Topal, 2007; Little, Topal and Knights, 2011).

Mathematical modelling has been conducted extensively in open pit mining and to a lesser extent in underground mining; thus far, waste rock placement and dump scheduling has received very little attention. There is very limited literature on the existing models for waste rock placement optimisation and dump scheduling.

Ben-Awuah and Askari-Nasab (2011) built a framework to model the production scheduling of oil sands mining in Canada. Based on this framework, a mixed integer programming (MIP) model was constructed and solved by the optimisation engine CPLEX. The objective is to seek the production schedule that provides the maximum profit. At the same time, the authors set constraints to ensure that dyke material mining and dyke construction are concurrent with ore production, which involves dump schedule. The model was experimented on a synthetic data set, and the problem was solved, generating a smooth mining schedule to ensure the production of waste material for dyke construction. The implementation of Ben-Awuah and Askari-Nasab’s model in real-world oil sands operations was discussed by Ben-Awuah (2013). It is proven that the model is robust and reliable. However, the main focus of the model is still on the production scheduling; rather than on the waste rock placement. The model does have constraints for regulating the volume for the dyke material mining and dyke construction, yet it does not consider the actual dyke construction sequence or the option for re-handling of any waste material. Therefore the model may not be applicable in hard rock mine sites, especially under the scenario of rock dump with multi-lift design.

Williams *et al* (2008) noted the economic importance of a waste rock dump in an open pit mine. They attempted to use a mathematical programming model to optimise the system. The proposed model considers the volumetric movement of the waste rock from a mining block to a dumping location. This variable is a precise description of the waste rock movement and the subsequent dump schedule. Williams *et al* (2008) explained the haulage cost profile in horizontal and vertical direction, which are to be minimised by the proposed model, in such a way that the overall waste rock haulage or waste rock dump construction cost is minimised. However, no results verification was shown to prove the functionality of the proposed mathematical model.

Topal, Williams and Zhang (2009) revised the model, and attempted to minimise the overall waste rock haulage time. The model was tested by using actual field data, with a three staged waste blocks mining schedule and the corresponding dump designs. The implementation however, was in an idealised way. Firstly, the model seeks the shortest haul in each time period, but this optimisation is carried out in each discrete time period, which is a violation in achieving the global optimality. Secondly, the dump progression is pre-defined by the authors, rather than calculated by the mathematical model. This is another violation to reaching the true optimum solution. The choice of dumping location is limited by the staged dump progression design, which will prevent the model to seek the optimum dump progression path. Thirdly, the volume of NAF rock to stockpile and the timing of re-handle are both pre-defined by the users. No variable is stated in the model to represent this function; therefore no optimisation is performed in stockpiling or re-handling of NAF rock.

Among all the mentioned waste rock dump related models, none has the option to automatically calculating the NAF rock stockpiling and re-handling. This presents a risk in finding feasible solutions, especially under scenarios of insufficient NAF rock case. Therefore, an improvement is required.

2.4 SUMMARY

Scholars generally agree on the significance on the haulage cost involved in material handling, yet no effective solution is available to directly minimise such cost. This is because that there are too many options in scheduling the waste rock placement, making it impossible to search the optimum solution without the help of a scientific method. On the other hand, AMD issue can be possibly prevented via careful scheduling the placement of the PAF and the NAF rock, in such a way that fully encapsulation of the PAF rock by the NAF rock is achieved at the end of mine life. An integrated approach would consider both aspects when planning the waste rock dump, and produce the optimum rock placement and dump schedule.

Available guidelines, reference books, best practice examples and company policies endeavour to create a sustainable waste rock dump design, yet none will assist in generating a rock dump schedule to achieve the final rock dump design. The traditional manual scheduling, performed by short term planning engineer, is dependent on personal experience and judgment, which cannot be standardised in the industry. Modern software package are computerised manual scheduler, which requires a user to state the dump progression. Therefore, none has the capacity in achieving the true optimum dump schedule.

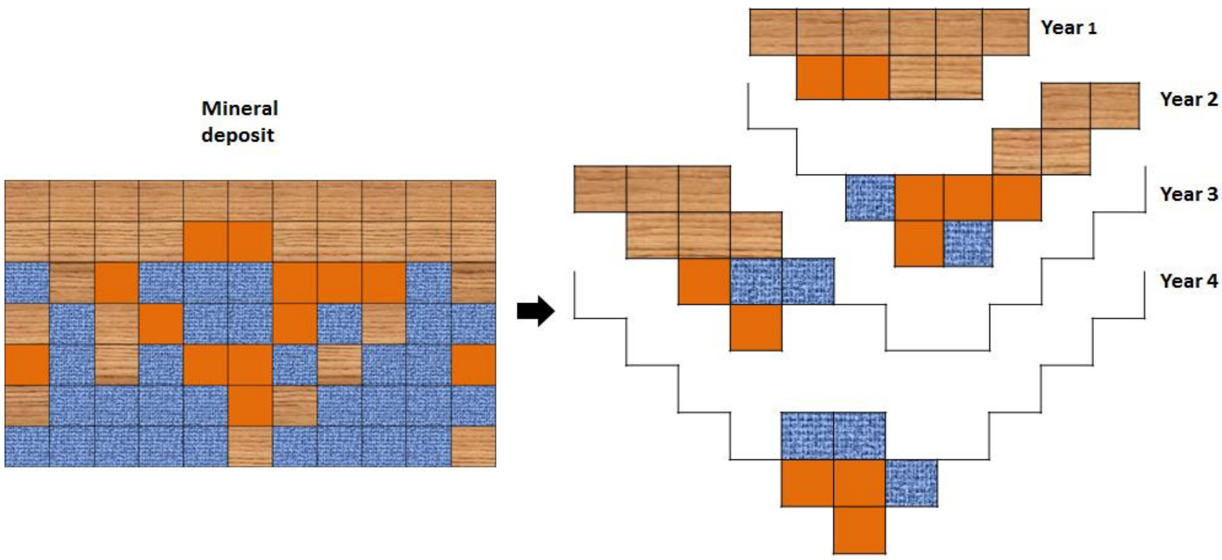
To resolve this complicated scheduling problem, mathematical modelling method is proposed. Available models have not considered NAF rock stockpile, which presents a risk in finding a feasible solution under insufficient NAF rock scenario. Therefore, further improvement is required.

CHAPTER 3. MAIN CONSIDERATIONS IN WASTE ROCK DUMP PLANNING AND SCHEDULING

The modelling of a real-world problem is essential to exploring the core problem in the system. This chapter describes a generic model for waste rock dump planning and notes the fundamentals of the optimisation problem within the system.

3.1 STRICT REMOVAL OF MINING BLOCKS ACCORDING TO THE MINING SCHEDULE

The most common objective of a mining schedule is to ensure the ore production, in such a way that the maximum NPV can be realised. As shown in Figure 3-1, assume that the simplified 2-D mineral deposit on the left will create the highest NPV under the outlined mining schedule on the right. Each mining block must be extracted from the open pit during the allocated time period.



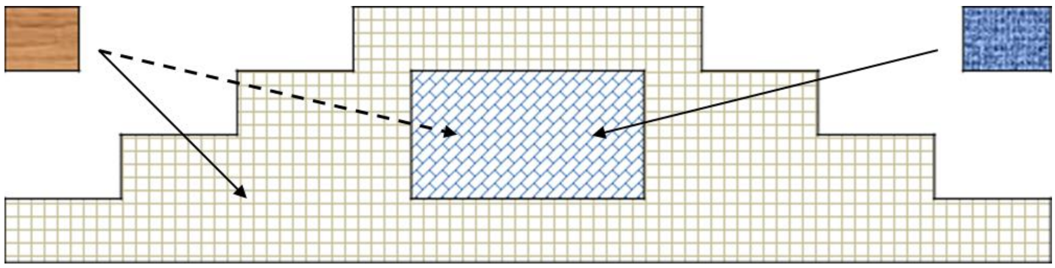
NB. NAF waste rock; PAF waste rock; Ore

Figure 3-1 Simplified 2-D mineral deposit and mining schedule

Once mined, the ore and waste rock are segregated to different destinations, usually the ROM for ore and the rock dump for waste. The actual dumping location for waste rock requires further attention, such as its chemical properties, i.e., NAF or PAF waste rock. Additional considerations could include the grade of the rock block, i.e., pure waste or marginal grade material (or low grade).

3.2 MATERIAL SEGREGATION BY SELECTIVE HANDLING

A generic 2-D waste rock dump cross section is shown in Figure 3-2, where NAF and PAF reserve areas are outlined. The PAF material is stored only in the centre of the rock dump, to minimise its contact with air and water, for AMD prevention. The NAF rock, however, is free to be transported anywhere within the rock dump, under the conditions of fully retained PAF rock within the reserved area. This selective handling of the different material will not add any complications to an operation, but instead, it will be an essential strategy for building an environmentally sound waste rock dump.



NB. NAF rock area; PAF rock reserve area

Figure 3-2 Allocation of NAF and PAF rock in a generic rock dump

In addition, the material segregation can also apply to the low grade material, if it is defined. This scenario will allow the possible recovery of low grade material, should there be a commodity price increase in the future. This approach is a strategy to increase the potential value of the rock dumps.

3.3 ENCAPSULATION OF THE PAF ROCK

AMD prevention may be realised by encapsulating PAF waste rock by NAF rock. Although the actual design can differ among each individual mine sites, the principle is illustrated in Figure 3-3. The key is not only to centralise PAF rock but also to provide a comprehensive NAF rock cover on the top. Such a strategy will effectively limit the amount of air and water that contact the PAF rock, minimising the AMD occurrence. In addition to the design, PAF rock cannot be located beneath the side slopes of a rock dump because of rainfall infiltration.

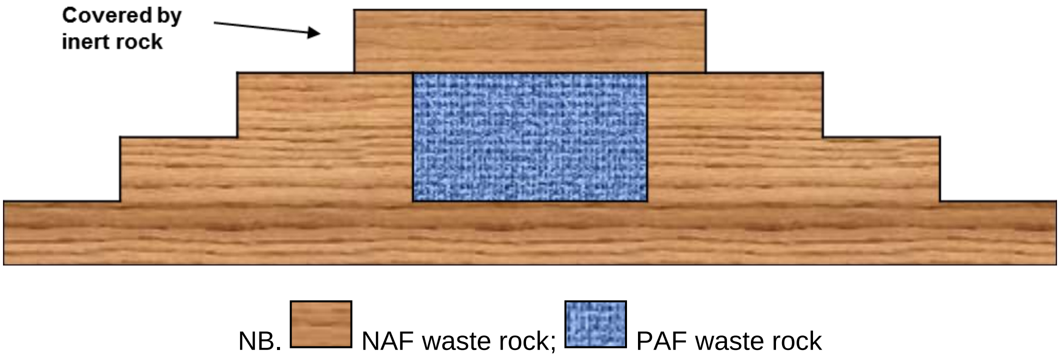


Figure 3-3 Illustration of a fully encapsulated 2-D waste rock dump

3.4 STOCKPILING OF INERT WASTE ROCK AND RE-HANDLING

Stockpiling of NAF rock is an important option. In general, the proportion of NAF waste rock becomes less and less as an open pit progresses below the water table, and the opposite situation occurs for the unoxidised sulphidic rock. Without a NAF rock stockpile, the amount of inert waste rock that is excavated in the later stages of mining might not be sufficient for fully encapsulating and covering the PAF rock.

For example, Figure 3-4 illustrates the scenario of NAF rock deficiency during the final year of operation. The solution is to stockpile certain amount of NAF rock in earlier stages of mining and then, to re-handle it for encapsulation purposes.

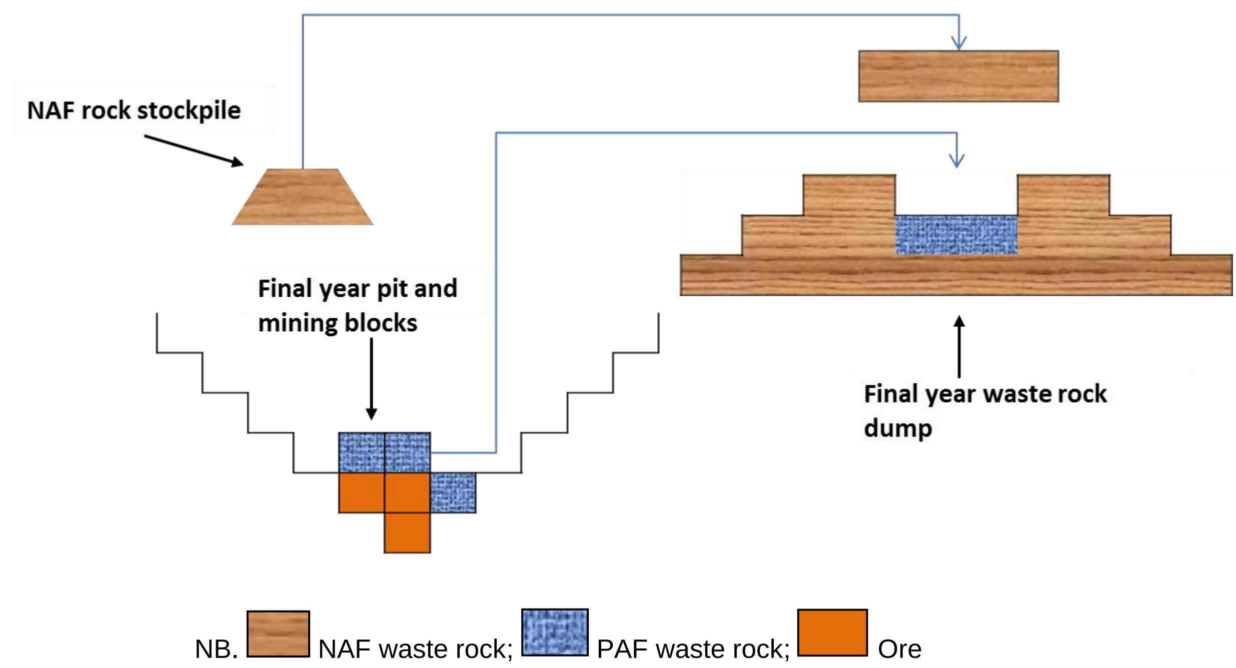


Figure 3-4 Year 4 mining blocks and waste rock dump

Re-handling waste rock incurs double-handling cost and must be minimised as much as possible. The precise amount of NAF rock to be stockpiled and to be re-handled can be determined along with a rock placement schedule.

3.5 Dump block modelling

In current practice, many of the mine scheduling software treats a rock dump as a single point or multiple points based on the number of lifts. The simplification improves the solution time, yet such a schedule often contains little information about the waste rock placement, i.e., the actual spatial dumping location is not available. The lack of detailed waste rock dumping information could potentially cause misalignment between the engineering design and the site operation, ultimately failing to achieve the long-term design objectives, such as the rock dump height, capacity, footprint, and slope angle.

In this thesis, it is proposed to further divide a rock dump into even smaller practical dump blocks, as shown in Figure 3-5, in such a way that each centroid of a dump block represents a unique dumping location with a nominal capacity.

The dump block model, however, is different from the conventional mining block model, i.e., a uniform block size is not enforced. The horizontal cuts are defined by the lift interval, and the vertical cuts are manipulated by the user.

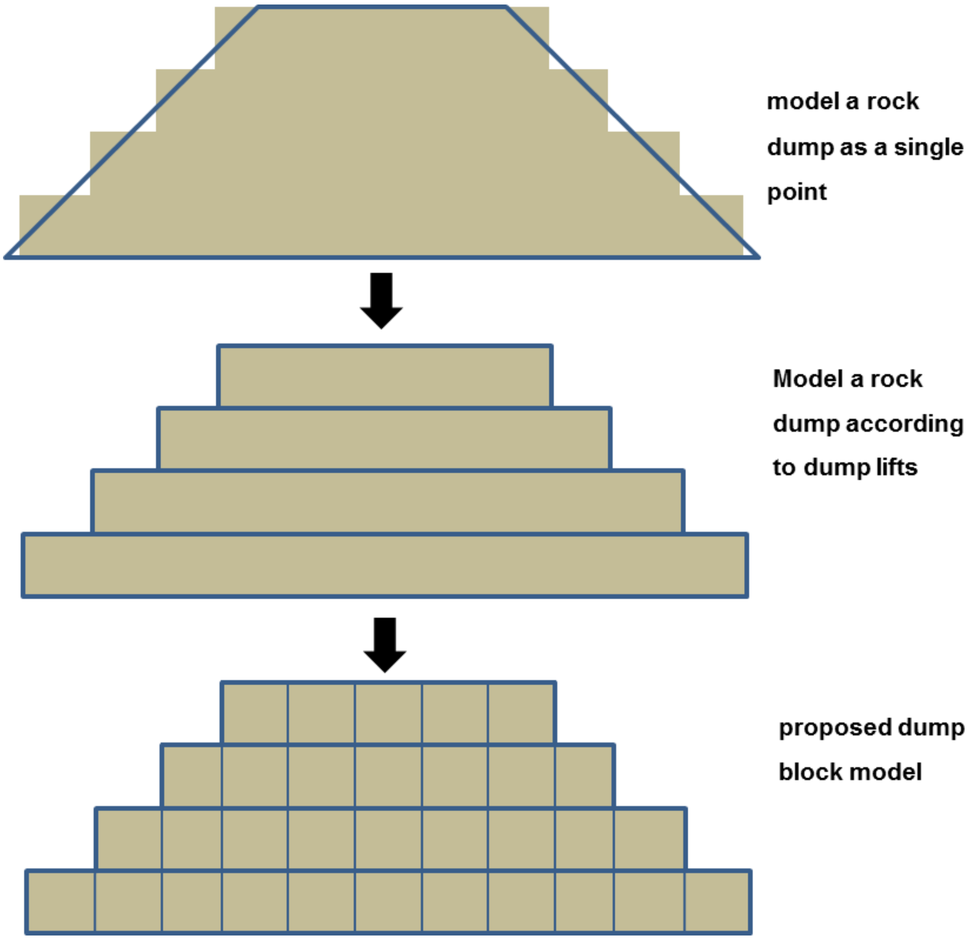


Figure 3-5 Modeling of dumping location -dump block model

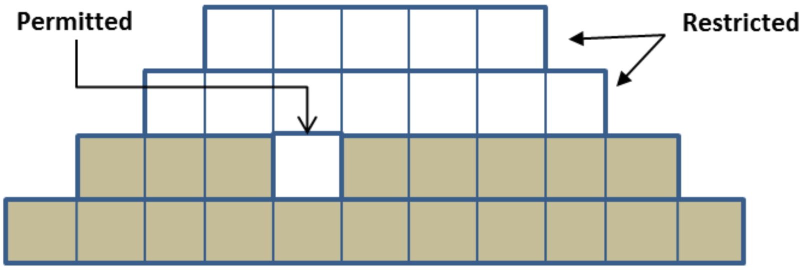
The purpose of the dump block model is to derive more spatial information about a rock dump, i.e., the detailed locations within a rock dump and the capacity at each individual location, in such way that a practical dump schedule can be realistically generated and utilised by mining operations. Furthermore, it allows direct estimation of haulage distance and haulage cost with much higher accuracy.

3.6 LOGICAL ROCK DUMP CONSTRUCTION SEQUENCE

To form a rock dump that has a multiple-lift configuration, some logical rules must be honoured, to allow the resulting rock placement schedule to be practical in the real world. Two dump construction sequences are considered, namely, lift-by-lift and multi-lift, as discussed in the following sections (3.6.1 and 3.6.2).

3.6.1 Lift-by-lift dump construction sequence

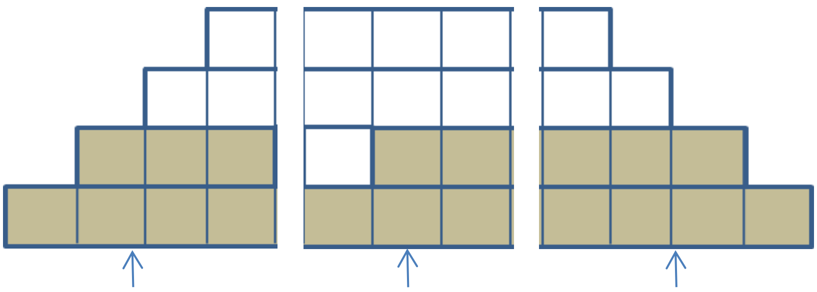
The lift-by-lift dump construction is controlled by the inter-lift dependency condition. This condition only allows waste rock dumping to occur at the lowest unfilled lift, i.e., dumping to a dump block in the upper lift is restricted if the previous lift is not fully filled. Figure 3-6 illustrates the condition, in which only one dump block in the second lift is permitted for dumping and all of the other dump blocks in any of the upper lifts are restricted.



NB. Filled block; Empty block

Figure 3-6 Lift-by-lift dump construction dependency condition

This simple construction sequence is not flexible with regards to the number of choices in the dump blocks. However, it can be improved by sub-dividing a rock dump, as shown in Figure 3-7. Each division is treated independently when applying this dependency condition, in such a way that the number of permitted dump blocks is increased.

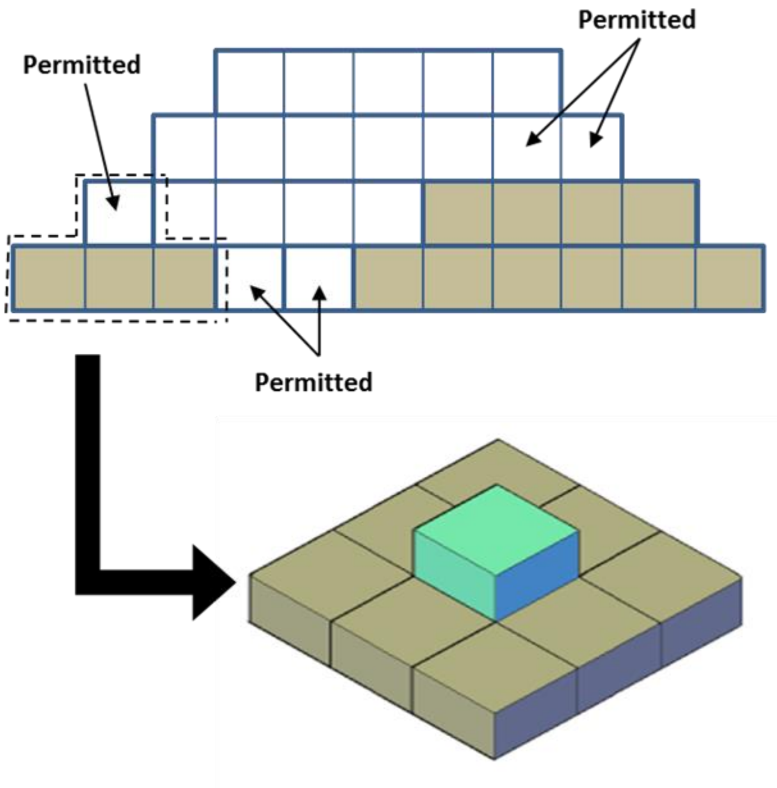


NB. Filled block; Empty block

Figure 3-7 Lift-by-lift dump construction dependency condition with dump division

3.6.2 Multi-lift dump construction sequence

Multi-lift dump construction is controlled by the inter-block dependency condition. As illustrated in Figure 3-8, this arrangement allows waste rock dumping to occur in multiple lifts.



NB. Filled block; Empty block; Permitted block

Figure 3-8 Multi-lift dumping construction dependency condition

This construction sequence is subject to the satisfaction of a stable slope condition, which is modelled by stacking one dump block on top of nine lower dump blocks, as shown in Figure 3-8. In the MIP model, the cumulative filling status of the nine precedence dump blocks in the lower lift must be monitored. Once the precedence blocks are fully filled, the upper lift block in the central position becomes available for waste rock dumping.

The multi-lift dump construction sequence will provide the most flexible solution to any rock dump schedule. However, the limitation is that each dump block must have nine blocks below its position, excluding the first lift. This arrangement could limit the applicability of the model.

3.7 FRAMEWORK OF THE INTEGRATED WASTE ROCK MINING AND DUMPING SYSTEM

Waste rock mining and the subsequent hauling and dumping are not discrete activities. Therefore, a dump schedule must be fully integrated with an optimised mining schedule, including not only the numerical value, such as the timing and volume for mining block removal, but also the graphical designs of the staged pits and rock dumps.

During the expansion period of an open pit, a series of staged pits will form; hence, it is possible to have the co-existence of temporary and permanent pit exit points. Such a fact must be accounted for when considering the possible haulage paths between pit exit point and dump entry point. As a result, the waste rock movement schedule must explicitly describe where the rock volume is from and to, which should include the following information:

- Time of the material movement;
- Mining block ID;
- Pit exit point;
- Rock dump entry point; and
- Spatial location of the dump block;

In addition to this primary ex-pit material transport, re-handled volume, if required, should also details its source location and the final resting position in the rock dump, which should include:

- Time;
- NAF rock stockpile (if multiple NAFSP exist); and
- Spatial location of the dump block.

To model this complicated system in a generic mine site, an integrated framework representing the waste rock mining and dumping is established, as shown in Figure 3-9.

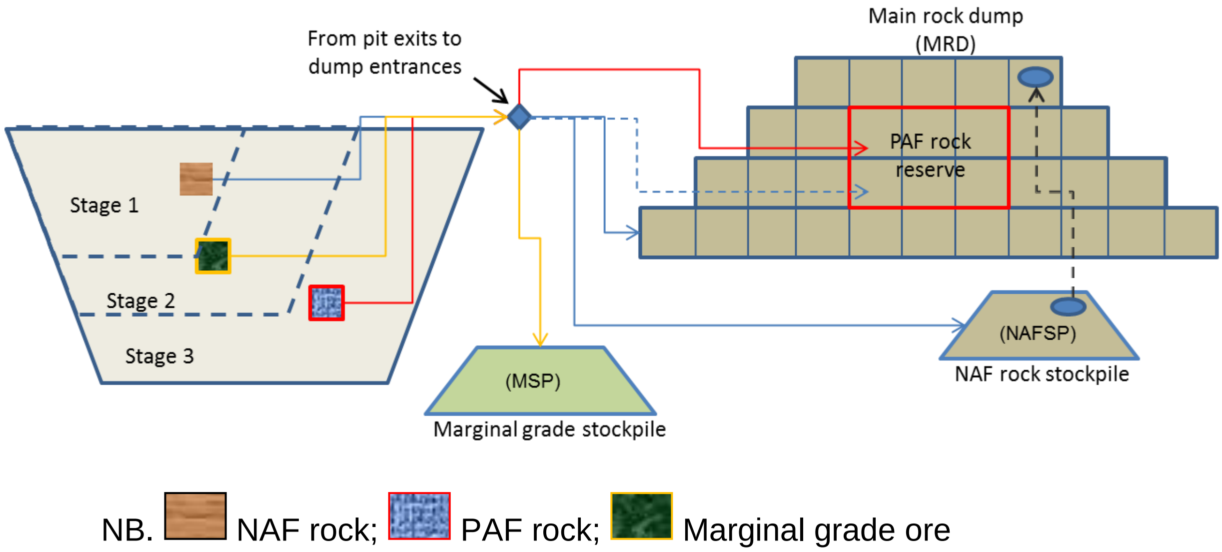


Figure 3-9 Integrated framework of mining and dumping system with material segregation rule applied in a generic mine site

It is noted that the primary waste rock transport is dependent on the mining schedule, available dumping location and the material type. The volume of NAF rock to be stockpiled and the timing to re-handle are dependent on the primary waste rock transport and the progression of the waste rock dump. These two unknowns are inter-dependent, and many quantitative solutions are potential candidates that may satisfy the balance of such material flow. To search the optimum solution, measuring criteria must be introduced, such as haulage distance for the material transport.

3.8 Equivalent Flat Distance Calculation

The rock volume from a mining block can be theoretically transported to any of the available dump blocks, without the constraints of material segregation; hence, all possible haulage paths and distances must be accounted for. The calculation of the haulage distance involves summing up three segments of the haul route, referring to in Figure 3-10, which are:

1. From a mining block centroid to a pit exit point;
2. From a pit exit point to a dump entry point; and
3. From a dump entry point to a dump block centroid.

This model is also suitable to be applied in the cases of multiple pits / pit exit points, and multiple dumps / dump entry points.

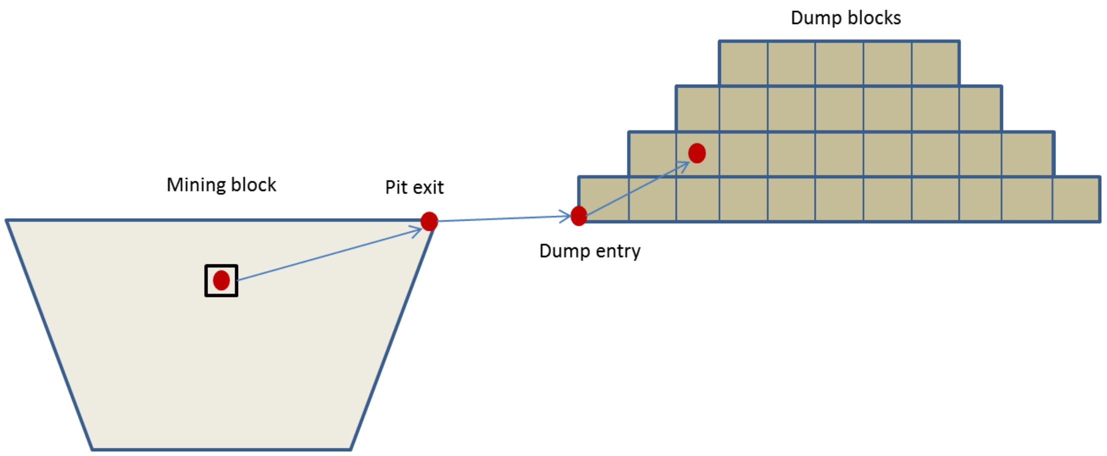


Figure 3-10 Haul route segments from a pit to a waste dump for a mining block

The direct distance between two points in 3-D space, i.e. from (X_1, Y_1, Z_1) to (X_2, Y_2, Z_2), must be converted into the equivalent flat-based distances if the points are located on different elevations. The basic principle of the conversion is to apply a road gradient to the elevation difference, in addition to the distance projection, as illustrated in Figure 3-11. The conversion to the equivalent flat distance reflects a slower truck speed on the (up/down) ramp and longer travelling time.

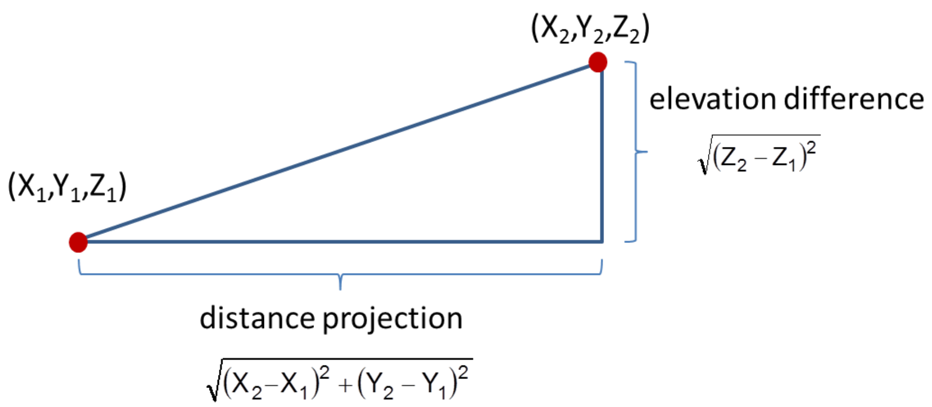


Figure 3-11 Equivalent flat distance calculation between two points in 3D space

The calculation of the equivalent flat distance can vary from site to site, depending on the calculation model established. Therefore, scale factors are applied to both the elevation difference and the distance projection proportions, as shown in equation (3.1). This arrangement allows flexibility for the user to create the most accurate model to the site-specific condition.

Equivalent flat distance (D)= $c_1 \times g \times \sqrt{(Z_2 - Z_1)^2} + c_2 \times \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$ **(3.1)**

where:

c_1 = scale factor for elevation difference;

c_2 = scale factor for distance projection; and

g = road gradient.

This equivalent flat distance model is capable of differentiating the haulage distance from each mining block to every dump block, which allows a full-scale evaluation of the preference in the dumping location(s) from each mining block.

3.9 SUMMARY

The modelling of the waste rock dump planning and scheduling provides better understanding of waste rock transport system. It outlines the important aspects that the material flow must obey, such as existing mining schedules, material segregation rules, PAF rock encapsulation and the option for NAF rock stockpiling.

The dump block model is proposed for this research, to provide a better representation of the actual rock dump. Based on this approach, two types of logical rock dump construction sequence are modelled, to allow flexibility in problem solving and, at the same time, ensure the practicality of the models. The waste rock mining and dumping problems are linked together by the integrated framework, which identifies that the volumetric material flow is the core problem in the system.

This material transportation system can be improved by optimising the two associated elements, i.e., the volume of material that is transported from the scheduled mining block to the dump block(s) and, the haulage distance of such a movement. These two elements will be formulated in mathematical equations for determining the optimum solution.

CHAPTER 4. MATHEMATICAL MODEL FORMULATION AND VERIFICATION

This chapter details the mathematical formulation of the three MIP models. The base model is developed to minimise the overall haulage distance and the required volume of re-handle. Variant one focuses on minimising the budget deviation between truck requirement and plan, and variant two is a hybrid model which minimises both haulage distance and budget deviation.

4.1 LOCATION OPTIMISATION (OP) MODEL

4.1.1 Indices

t = time periods;

p = available pits;

b = mining blocks;

p1 = staged pits;

e1 = staged pit exit points;

n = rock dumps, i.e., main rock dump (MRD), marginal grade stockpile (MSP), and non-acid forming rock stockpile (NAFSP);

e2 = rock dump entry points;

d = dump block location, containing information of the rock dump name, dump block easting, northing, and elevation; and

d’ = precedent dump blocks of the dump block d.

4.1.2 Sets

B_p^t = set of mining blocks located in pit p, to be removed during time period t;

E_{p1} = set of pit exit points located in staged pit p1;

E_n = set of dump entry points located in n rock dump;

M = set of dump blocks in MRD;

R = sub-set of M, representing the pre-defined PAF waste rock reserve in the centre of the MRD, which are permitted to receive both PAF waste rock and NAF waste rock;

F = set of dump blocks within MSP. This group of dump blocks are permitted to receive marginal grade material only;

P = set of dump blocks within NAFSP. This group of dump blocks are permitted to receive NAF waste rock only. The NAF waste rock stored is allowed to be re-handled;

MF = set of dump blocks in either set M or set F;

MP = set of dump blocks in either set M or set P;

MR = set of dump blocks located on the immediate top of the PAF reserve area. This group of dump blocks forms the cover of the PAF reserve area;

PS = set of precedence dump blocks under lift-by-lift construction sequence; and

PM = set of precedence dump blocks under multi-lift construction sequence.

4.1.3 Parameters

U_b = bulk volume (measured in bank cubic metres, BCM) of a mining block b;

G_b = grade of a mining block b (% or g/t, depends on ore type);

G_0 = cut-off grade to determine whether a block falls into marginal grade or waste (% or g/t, depends on ore type);

A_b = reactivity of a mining block b (no certain unit);

A_0 = cut-off reactivity value to determine whether a block is PAF or NAF (no certain unit);

C_d = volumetric capacity of a dump block d (m^3);

S = swell factor (%);

r = discount rate (%);

$D_{b,e1}$ = equivalent flat distance (m), from a mining block b to staged pit exit point e1;

$D_{e1,e2}$ = equivalent flat distance (m), from a staged pit exit point e1 to rock dump entry point e2;

$D_{e2,d}$ = equivalent flat distance (m), from rock dump entry point e2 to a dump block d; and

$D_{n,\,d}$ = equivalent flat distance (m), from rock dump n, i.e. n=NAFSP, to a dump block d.

4.1.4 Variables

$V^{t}_{b,e1,e2,d}$ = rock volume (BCM) hauled from a mining block b, via pit exit point e1 and rock dump entry point e2, to a dump block d, in time period t (linear variable);

$V^{t}_{n,d}$ = NAF waste rock (BCM) re-handled from NAFSP, to a dump block d in time period t (linear variable);

X^{t}_{d} = filling status (%) of a dump block d at the end of a time period t, ranging from 0 to 100% (linear variable); and

$$Y^{t}_{d} = \begin{Bmatrix} 1, \text{if dump block d is available for dumping in time period t;} \\ 0, \text{otherwise.} \end{Bmatrix}.$$

4.1.5 Objective function

The base model is called the OP model, to minimise the overall haulage distance and the volume of re-handling, hence the associated haulage cost over the life of mine. The volume of the material that is transported and the loaded travel distance are included in the cost estimation, by equation (4.1). The cost factor that is used is one cent per loaded flat meter per BCM handled, and time value of money is also considered by applying a discount rate in relation to the time period.

Minimise the net present haulage cost (NPC)

$$= \left\{ \begin{array}{l} \sum_{b}\sum_{e1}\sum_{e2}\sum_{d}\sum_{t}\left(D_{b,e1}+D_{e1,e2}+D_{e2,n,d}\right)\times V^{t}_{b,e1,e2,d}\times \text{cost factor} \Big/ (1+r)^{t} \\ + \\ \sum_{n=NAFSP}\sum_{d\in M}\sum_{t}\left(D_{n,d}\times V^{t}_{n,d}\right)\times \text{cost factor} \Big/ (1+r)^{t} \end{array} \right\} \quad (4.1)$$

4.1.6 Mining schedule and material segregation constraints

A production schedule is already determined by mine scheduling software; as a result, the waste rock placement must follow the defined schedule. Figure 4-1 represents that the entire volume of an NAF waste block must be fully removed from the open pit during a scheduled time period. The eligible dump blocks for receiving such a volume are specified, in such a way that the material excavated is guided to the appropriate location(s).

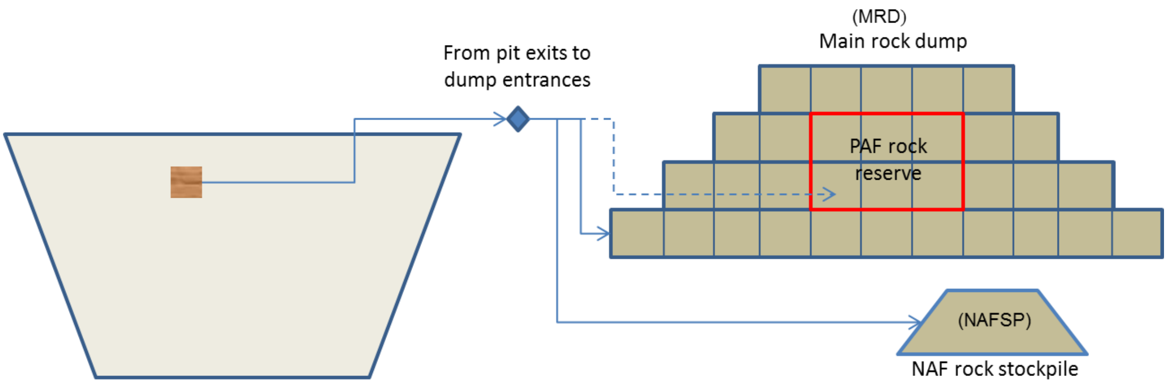


Figure 4-1 Illustration of mining schedule and material segregation constraint

Constraints (4.2) to (4.4) ensure the complete removal of a mining block during a scheduled time period, and they also provide guidance to send the material to the appropriate dump block(s) according to the material segregation rules.

$$\sum_{e1}\sum_{e2}\sum_{d} V^{t}_{b,e1,e2,d} = U_b \quad (4.2)$$

$$\forall t,b \,\middle|\, b\in B^{t}_{p};\ A_b < A_0;\ G_b < G_0;\ e1\in E_{p1};\ e2\in E_n;\ d\in MP$$

$$\sum_{e1}\sum_{e2}\sum_{d} V^{t}_{b,e1,e2,d} = U_b \quad (4.3)$$

$$\forall t,b \,\middle|\, b\in B^{t}_{p};\ A_b \geq A_0;\ e1\in E_{p1};\ e2\in E_n;\ d\in R$$

$$\sum_{e1}\sum_{e2}\sum_{d} V^{t}_{b,e1,e2,d} = U_b$$
$$\forall t,b \middle| b \in B^{t}_{p}\ ;\ G_b \geq G_0\ ;\ A_b < A_0;\ e1 \in E_{p1};\ e2 \in E_n;\ d \in F \tag{4.4}$$

4.1.7 Dump block capacity constraints

Each dump block has a nominal capacity. Therefore, the cumulative volume in a dump block should not exceed its maximum nominal capacity during any time period. This consideration is illustrated in Figure 4-2.

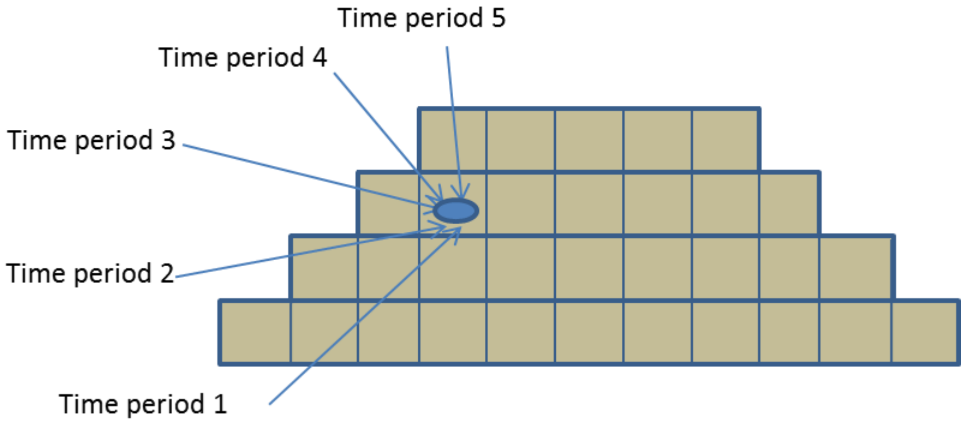


Figure 4-2 Illustration of dump block capacity constraint

As a result, the cumulative percentage of filling for each dump block in relation to its nominal capacity is monitored. This is applied to the dump blocks with one-way material transfer characteristics, i.e., only receiving material and no re-handling is sourced from it. The MRD and MSP satisfy the condition, and each dump block within the two rock dump will be monitored.

Constraints (4.5) and (4.6) monitor the percentage of filling for every dump block within MRD and MSP at the end of each time period. Constraint (4.7) ensures that the dump blocks located immediate above the PAF rock reserve are fully filled at the end of the mine life. Upon satisfaction of this constraint, the encapsulation of the PAF rock is achieved.

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{t} S \times V^{t}_{b,e1,e2,d} + \sum_{n=NAFSP}\sum_{t} S \times V^{t}_{n,d} = X^{t}_{d} \times C_d$$
$$\forall t,d \middle| d \in M\ ;\ b \in B^{t}_{p};\ e1 \in E_{p1};\ e2 \in E_n \tag{4.5}$$

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{t} S \times V^{t}_{b,e1,e2,d} = X^{t}_{d} \times C_d$$
$$\forall t,\ d \middle| d \in F;\ b \in B^{t}_{p};\ e1 \in E_{p1};\ e2 \in E_n \tag{4.6}$$

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{t} S \times V^{t}_{b,e1,e2,d} + \sum_{n=NAFSP}\sum_{t} S \times V^{t}_{n,d} = C_d \tag{4.7}$$

$$\forall d \,\middle|\, d \in MR;\ b \in B^{t}_{p};\ e1 \in E_{p1};\ e2 \in E_{n}$$

4.1.8 *Stockpile and re-handling material flow constraints*

Material re-handling is permitted to occur from the NAFSP to the MRD, as shown in Figure 4-3. The in-flow and out-flow are restricted under the nominal capacity and timing conditions.

The cumulative material within an NAFSP must be less than its nominal capacity. In terms of the timing, the in-flow in the current time period becomes available only from the next time period onwards. Therefore, the out-flow volume in a time period should be less than the cumulative remaining volume by the end of the previous time period. This rule results in a zero out-flow volume in the 1st time period.

The two conditions that were imposed on the NAF rock stockpile eliminate the possibility of using it as a by-pass, in such a way that only a necessary amount of rock volume will be transported to the NAFSP.

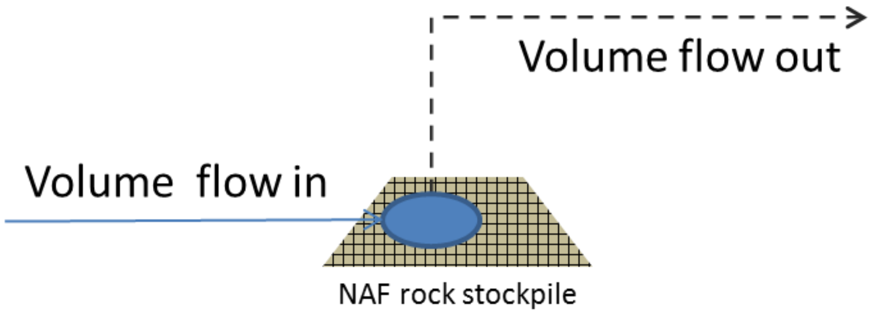


Figure 4-3 Illustration of the stockpile and re-handling flow constraints

Constraint (4.8) regulates the nominal capacity condition, which prevents material overflow situation. Constraints (4.9) and (4.10) specify the timing condition of the material flow into and out of the NAFSP, such a way that only the necessary amount of NAF rock will be stockpiled and re-handled.

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{d \in P}\sum_{t} S \times V^{t}_{b,e1,e2,d} - \sum_{n=NAFSP}\sum_{d \in M}\sum_{t} S \times V^{t}_{n,d} \leq \sum_{d \in P} C_d \tag{4.8}$$

$$\forall t \,\middle|\, b \in B^{t}_{p};\ e1 \in E_{p1};\ e2 \in E_{n}$$

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{d\in P}\sum_{t} S\times V^{t}_{b,e1,e2,d} - \sum_{n=NAFSP}\sum_{d\in M}\sum_{t+1} S\times V^{t}_{n,d} \geq 0 \qquad \text{(4.9)}$$
$$\forall t \;\big|\; b\in B^{t}_{p};\ e1\in E_{p1};\ e2\in E_{n}$$

$$\sum_{n=NWRS} V^{t}_{n,d} = 0 \qquad \forall\, t,d \,\big|\, t=1;\ d\in M \qquad \text{(4.10)}$$

4.1.9 Rock dump construction sequence constraints

Two types of rock dump construction sequences are proposed, which are described graphically in section 3.6. The mathematical control of the sequence is detailed below. They are limited to the rock dumps that have a one-way material flow condition, i.e., MRD and MSP.

4.1.9.1 Lift-by-lift dump construction dependence constraints

The lift-by-lift construction sequence in MRD and MSP are controlled by constraint set (4.11) and (4.12), and constraint set (4.13) and (4.14), respectively. A binary variable $Y^{t}_{n,k}$ is used to model the logical sequence. It first checks whether the cumulative volume in the lowest dump lift has reached its nominal capacity. This circumstance is enabled by constraints (4.11) and (4.13). If the nominal capacity is not reached, then the binary variable can only be equal to 0, and constraint (4.12) and (4.14) confirms that no dumping is permitted in the immediate upper lift. Only if the capacity of this lift is reached, can the binary variable be equal to 1, and constraint (4.12) and (4.14) will flag that the immediate upper lift is available for the waste rock dumping.

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{d'}\sum_{t} S\times V^{t}_{b,e1,e2,d'} + \sum_{n=NAFSP}\sum_{d'}\sum_{t} S\times V^{t}_{n,d'} - \sum_{d'} C_{d'}\times Y^{t}_{d} \geq 0 \qquad \text{(4.11)}$$
$$\forall t,d \;\big|\; d\in M;\ d'\in PS;\ b\in B^{t}_{p};\ e1\in E_{p1};\ e2\in E_{n};$$

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{t} S\times V^{t}_{b,e1,e2,d} + \sum_{n=NAFSP}\sum_{t} S\times V^{t}_{n,d} - C_{d}\times Y^{t}_{d} \leq 0 \qquad \text{(4.12)}$$
$$\forall t,d \;\big|\; d\in M;\ b\in B^{t}_{p};\ e1\in E_{p1};\ e2\in E_{n}$$

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{d'}\sum_{t} S \times V^{t}_{b,e1,e2,d'} - \sum_{d'} C_{d'} \times Y^{t}_{d} \geq 0 \quad \textbf{(4.13)}$$
$$\forall t,d \mid d \in F;\ d' \in PS;\ b \in B^{t}_{p};\ e1 \in E_{p1};\ e2 \in E_{n};$$

$$\sum_{b}\sum_{e1}\sum_{e2}\sum_{t} S \times V^{t}_{b,e1,e2,d} - C_{d} \times Y^{t}_{d} \leq 0 \quad \textbf{(4.14)}$$
$$\forall t,d \mid d \in F;\ b \in B^{t}_{p};\ e1 \in E_{p1};\ e2 \in E_{n}$$

4.1.9.2 Multi-lift dump construction dependence constrains

The multi-lift construction sequence is modelled by stacking one dump block on top of the nine lower dump blocks, as indicated in Figure 3-8. Mathematically, it is controlled by the binary variable Y^{t}_{d}.

According to constraint (4.15), the binary variable is equal to 0 if the precedence base blocks are not fully filled. It only turns into 1 when the base is full. The target dump block becomes available for receiving waste rock when Y^{t}_{d} turns into 1. This is controlled by constraints (4.16).

$$\sum_{d' \in PM} X^{t}_{d'} - 9 \times Y^{t}_{d} \geq 0 \quad \forall t,d \mid d \in MF \quad \textbf{(4.15)}$$

$$Y^{t}_{d} - X^{t}_{d} \geq 0 \quad \forall t,d \mid d \in MF \quad \textbf{(4.16)}$$

4.1.10 Non-negativity and Integrality constraints

Non-negativity and integrality of the variables are enforced by constraints (4.17) to (4.20) as appropriate.

$$V^{t}_{b,e1,e2,d} \geq 0 \quad \forall t,b,e1,e2,d \quad \textbf{(4.17)}$$

$$V^{t}_{n,d} \geq 0 \quad \forall t,\ n,\ d \mid n\text{=NWRS};\ d \in M \quad \textbf{(4.18)}$$

$$0 \leq X^{t}_{d} \leq 1 \quad \forall t,\ d \mid d \in MF \quad \textbf{(4.19)}$$

$$Y^{t}_{d}\ \text{binary} \quad \forall t \quad \textbf{(4.20)}$$

The AMPL coding of the OP models are attached in Appendix A.

4.2 TRUCK BALANCE (TB) MODEL

A waste rock schedule involves volumetric material movement, and the associated haulage distance. The product of the two equates to the required loaded haulage work, which is measured in BCMxm. Such work is conducted by haul trucks, which has a maximum work capacity each year. With a given budget for the number of available trucks, the required haulage work might not match the budget. This situation leads to the development of the first MIP model variant, which is called the TB model.

4.2.1 Additional parameters

TC = truck annual capacity, measured in BCM x m; and

TN^t = number of haul trucks available in time period t.

4.2.2 Additional variables

$\underline{U}^t$ = the haulage work capacity (BCMxm) under the requirements in time period *t*; and

$\overline{O}^t$ = the haulage work capacity (BCMxm) over the requirements in time period *t*.

4.2.3 TB model objective function

The TB model does not minimise the haulage distance or volume of re-handling, but it minimises the opportunity cost by matching the required truck capacity with the budget, hence minimising the over and under budget, as indicated in the objective function (4.21). At the same time, the time value of money is considered; hence a discounting effect is applied.

Minimise the opportunity cost

$$= \sum_t \left(\underline{U}^t + \overline{O}^t\right) \times \text{cost factor} \Big/ (1+r)^t \qquad \textbf{(4.21)}$$

4.2.4 TB model specific constraint

Constraint (4.22) is a specific constraint for the TB model. It enforces the required haulage work, from the rock placement selection, to equal the available truck work capacity plus (and minus) the amount of under (and over) budgeting. This constraint ensures that the resulting rock placement schedule mitigates the under or over budget situation, such that the available trucks are better utilised.

$$TC \times TN^t + \underline{U}^t - \overline{O}^t = \left\{ \begin{array}{l} \sum_{b}\sum_{e1}\sum_{e2}\sum_{d}\left(D_{b,e1} + D_{e1,e2} + D_{e2,d}\right) \times V^t_{b,e1,e2,d} \\ + \\ \sum_{n=NAFSP}\sum_{d\in M}\left(D_{n,d} \times V^t_{n,d}\right) \end{array} \right\} \quad \forall t \qquad \textbf{(4.22)}$$

Constraint (4.23) and (4.24) state the non-negativity condition for the newly introduced variables for TB model.

$$\underline{U}^t \geq 0 \quad \forall t \qquad \textbf{(4.23)}$$

$$\overline{O}^t \geq 0 \quad \forall t \qquad \textbf{(4.24)}$$

The AMPL coding of the TB models are attached in Appendix B.

4.3 OVERALL BALANCED (COMBO) MODEL

4.3.1 Combo model objective function

The second variant MIP model is called the Combo model; this model combines the OP and TB models. The objective function (4.25) aims to generate a balanced rock placement schedule that considers haulage distance, material re-handle and the budget deviation.

Minimise Overall Cost=

$$\left\{ \begin{array}{l} \sum_{b}\sum_{e1}\sum_{e2}\sum_{d}\sum_{t}\left(D_{b,e1} + D_{e1,e2} + D_{e2,d}\right) \times V^t_{b,e1,e2,d} \times \text{cost factor} \Big/ (1+r)^t \\ + \\ \sum_{n=NAFSP}\sum_{d\in M}\sum_{t}\left(D_{n,d} \times V^t_{n,d}\right) \times \text{cost factor} \Big/ (1+r)^t \\ + \\ \sum_{t}\left(\underline{U}^t + \overline{O}^t\right) \times \text{cost factor} \Big/ (1+r)^t \end{array} \right\} \qquad \textbf{(4.25)}$$

4.3.2 Combo model specific constraint

$$TC \times TN^t + \underline{U}^t - \overline{O}^t \geq \left\{ \begin{array}{l} \sum_b \sum_{e1} \sum_{e2} \sum_d \left(D_{b,e1} + D_{e1,e2} + D_{e2,d} \right) \times V^t_{b,e1,e2,d} \\ + \\ \sum_{n=NAFSP} \sum_{d \in M} D_{n,d} \times V^t_{n,d} \end{array} \right\} \quad \forall t \qquad \textbf{(4.26)}$$

From a budgeting perspective, the adverse impact by over-budgeting is less severe than under-budgeting, because a potential delay due to a truck shortage could lower the production rate. Therefore, the combo model specific constraint (4.26) gives some level of freedom, allowing deviation to occur, but ensures that the truck budget is greater than or equal to the required haulage target from the rock placement schedule.

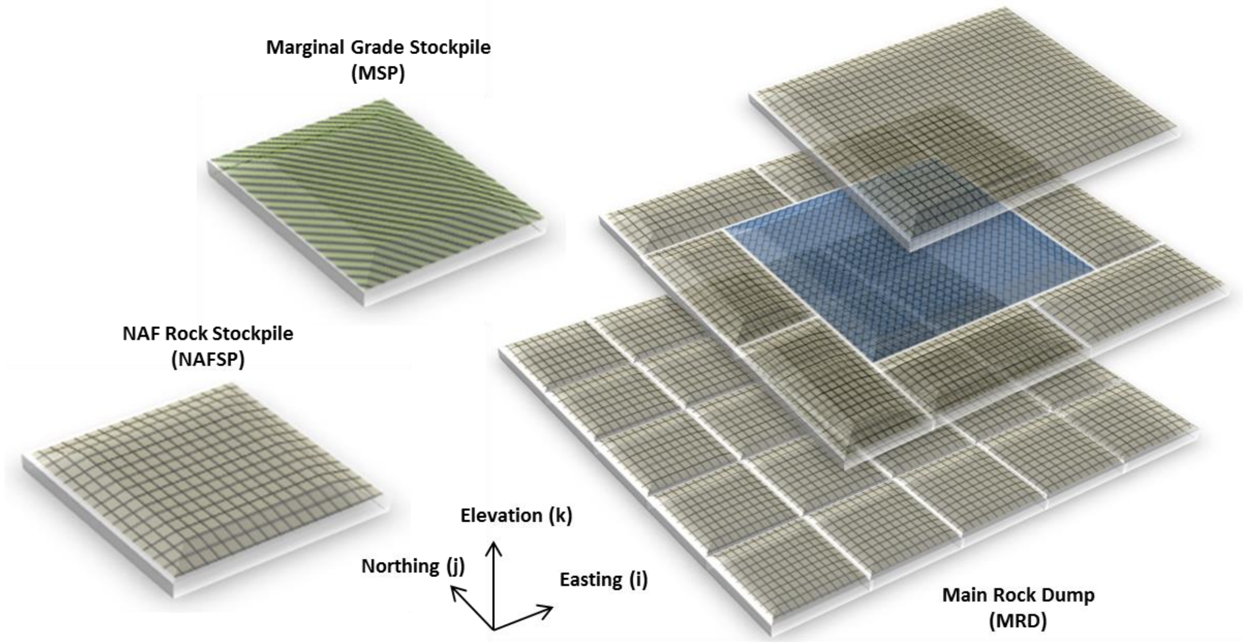
The AMPL coding of the Combo models are attached in Appendix C.

4.4 MIP MODEL VERIFICATION – FUNCTIONALITY TEST OF BASE MODEL

The models discussed above are programmed in AMPL code (Fourer, Gay and Kernighan, 2002). To examine the functionality of the MIP model, particularly the constraints, base OP model is tested by using a simplified data set. This section details the verification process, including the introduction of the input data, the solution interpretation and results verification.

4.4.1 Input data set

A conceptual design of main rock dump (MRD), marginal grade stockpile (MSP) and NAF rock stockpile (NAFSP) are shown in Figure 4-4. A total of 37 dump blocks are available for receiving rock volume from an open pit. The location and nominal capacity of each dump block are summarised in Table 4-1.



NB. NAF rock area; PAF rock area; Marginal grade area

Figure 4-4 Conceptual MRD, MSP and NAFSP

The shape and capacity of the dump blocks are solely designed for testing and verification purpose; thus it is noted that the PAF rock reserve in the centre of the MRD is larger than any other dump block to centralise the PAF rock storage.

Table 4-1 conceptual rock dump capacity in m^3

Rock Dump(n)	Elevation(k)	Northing(j)	Easting(i)				
			1	2	3	4	5
1 (MRD)	1	1					
		2					
		3			11,880		
		4					
		5					
	2	1					
		2		1,800	1,800	1,800	
		3		1,800	39,600	1,800	
		4		1,800	1,800	1,800	
		5					
	3	1	1,800	1,800	1,800	1,800	1,800
		2	1,800	1,800	1,800	1,800	1,800
		3	1,800	1,800	1,800	1,800	1,800
		4	1,800	1,800	1,800	1,800	1,800
		5	1,800	1,800	1,800	1,800	1,800
2 (MSP)	1	1	5,000				
3 (NAFSP)	1	1	10,000				

A total of 100 waste mining blocks are scheduled for removal evenly over five time periods. There are 30 PAF rock blocks, 4 marginal grade blocks and 66 NAF rock blocks. The mining schedule and material type breakdown can be seen in Table 4-2.

Table 4-2 Rock block removal schedule and material breakdown

Time Period	NAF block	PAF block	Marginal block	Sub-total
1	20	0	0	20
2	19	0	1	20
3	16	3	1	20
4	10	9	1	20
5	1	18	1	20
Overall	66	30	4	100

An ID number is assigned to each mining block, which ranges from 1 to 100. Table 4-3 details the ID in relation to the block material type. This information will be used for validating the destination in the automatically generated placement schedule, especially the destination that is selected for the PAF and marginal grade blocks.

Table 4-3 Mining block material type breakdown

Time Period	NAF block ID	PAF block ID	Marginal block ID
1	1-20	N/A	N/A
2	21-39	N/A	40
3	41-54	57-59	60
4	61-70	71-79	80
5	81	82-99	100

Each mining block contains 1000 BCM of material, and a net swell factor of 1.2 is assumed. Total of 100 waste blocks produce overall swelled volume of 120,000 LCM.

Pre-processing of the input data is required, to ensure a feasible schedule can be produced under the condition of material segregation rule. The initial volume comparison between the material that is mined out and the rock dump capacity is summarised in Table 4-4, which suggests that all three types of rock can be separately stored in the designed rock dumps. This result indicates that a potential solution does exist. However, infeasibility could still occur once the mining schedule is incorporated.

Table 4-4 Rock dump capacity check

Material type	Rock Dump	Dump Capacity (m^3)	Volume from pit (LCM)	Check
Marginal grade	MSP	5,000	4,800	Satisfy
PAF rock	PAF reserve	39,600	36,000	Satisfy
Inert rock	MRD	71,280		
	PAF reserve	3,600		
	ISP	10,000		
	Sub-total	84,880	79,200	Satisfy

4.4.2 Problem size and solution time

Two MIP problems are generated by the OP model with two types of dump construction sequence. Each problem involves more than 2,700 variables, as summarised in Table 4-5. These problems are not likely to be solvable by manual methods under any circumstances. However, the optimum solution is determined by the optimiser, which is run on a computer of 2.8GHz CUP and 12GB RAM, in less than one second.

Table 4-5 MIP problems generated and solution time

Model Name	Number of	OP lift-by-lift	OP multi-lift
Problem size	liner variable	2,762	2,761
	binary variable	12	144
	constraints	349	560
Solving process	simplex iterations	962	772
	branch and bound cut	0	0
Solution time (seconds)		0.06	0.03

4.4.3 Results verification

The results on each key linear variable were automatically written in to Microsoft Access, to form the optimum dump plan. Verification of the results include checking whether the material segregation, the NAF rock stockpiling and re-handling, and the dump lift construction sequence are correctly programmed.

4.4.3.1 PAF material segregation

The check point for material segregation is on the destination for the PAF block and marginal grade block. The designated dumping location for PAF block is the PAF rock reserve, and the corresponding spatial location {rock dump, elevation, northing, easting} is {1, 2, 3, 3}. The scheduling results are filtered to display only this dump block, as summarised in Table 4-6 and

Table 4-7 for the lift-by-lift and multi-lift dump construction sequence, respectively.

The entries in column b are the IDs of the rock blocks that are scheduled to the PAF rock reserve, among which, the bold entries match the IDs of the PAF blocks in Table 4-3. The volume of movement is 1000 BCM for each block, which suggests that the entire volume for every PAF block is directed to the PAF rock reserve.

Table 4-6 Filtered results of material flow into the PAF rock reserve under a lift-by-lift dump construction sequence

t	b	n	k	i	j	V (BCM)
3	51	1	2	3	3	1000
3	52	1	2	3	3	1000
3	53	1	2	3	3	1000
3	**57**	1	2	3	3	1000
3	**58**	1	2	3	3	1000
3	**59**	1	2	3	3	1000
4	**71**	1	2	3	3	1000
4	**72**	1	2	3	3	1000
4	**73**	1	2	3	3	1000
4	**74**	1	2	3	3	1000
4	**75**	1	2	3	3	1000
4	**76**	1	2	3	3	1000
4	**77**	1	2	3	3	1000
4	**78**	1	2	3	3	1000
4	**79**	1	2	3	3	1000
5	**82**	1	2	3	3	1000
5	**83**	1	2	3	3	1000
5	**84**	1	2	3	3	1000
5	**85**	1	2	3	3	1000
5	**86**	1	2	3	3	1000
5	**87**	1	2	3	3	1000
5	**88**	1	2	3	3	1000
5	**89**	1	2	3	3	1000
5	**90**	1	2	3	3	1000
5	**91**	1	2	3	3	1000
5	**92**	1	2	3	3	1000
5	**93**	1	2	3	3	1000
5	**94**	1	2	3	3	1000
5	**95**	1	2	3	3	1000
5	**96**	1	2	3	3	1000
5	**97**	1	2	3	3	1000
5	**98**	1	2	3	3	1000
5	**99**	1	2	3	3	1000

Table 4-7 Filtered results of the material flow into the PAF reserve under the multi-lift dump construction sequence

t	b	n	k	i	j	V (BCM)
3	49	1	2	3	3	500
3	50	1	2	3	3	1000
3	51	1	2	3	3	1000
3	52	1	2	3	3	500
3	**57**	1	2	3	3	1000
3	**58**	1	2	3	3	1000
3	**59**	1	2	3	3	1000
4	**71**	1	2	3	3	1000
4	**72**	1	2	3	3	1000
4	**73**	1	2	3	3	1000
4	**74**	1	2	3	3	1000
4	**75**	1	2	3	3	1000
4	**76**	1	2	3	3	1000
4	**77**	1	2	3	3	1000
4	**78**	1	2	3	3	1000
4	**79**	1	2	3	3	1000
5	**82**	1	2	3	3	1000
5	**83**	1	2	3	3	1000
5	**84**	1	2	3	3	1000
5	**85**	1	2	3	3	1000
5	**86**	1	2	3	3	1000
5	**87**	1	2	3	3	1000
5	**88**	1	2	3	3	1000
5	**89**	1	2	3	3	1000
5	**90**	1	2	3	3	1000
5	**91**	1	2	3	3	1000
5	**92**	1	2	3	3	1000
5	**93**	1	2	3	3	1000
5	**94**	1	2	3	3	1000
5	**95**	1	2	3	3	1000
5	**96**	1	2	3	3	1000
5	**97**	1	2	3	3	1000
5	**98**	1	2	3	3	1000
5	**99**	1	2	3	3	1000

At the same time, some NAF rock is also directed to this location, which indicates that the designed capacity of the PAF rock reserve is slightly higher than required. It also proves that the material segregation constraint for the NAF rock is correctly programmed.

4.4.3.2 Marginal grade material segregation

The destination for the marginal grade blocks is also verified. The given MSP location {rock dump, elevation, northing, easting} is {2, 1, 1, 1}, and the filtered results under both dump construction sequences display the same values, as shown in Table 4-8. This finding confirms that every marginal grade block is directed to the MSP, and no other mining block is to be scheduled to this dumping location. The overall required capacity for the MSP can be calculated, and the engineer will be able to tailor the design, in such a way that the footprint of the disturbed land is minimised.

Table 4-8 Filtered results of the material flow into the MSP

t	b	n	k	i	j	V (BCM)
2	40	2	1	1	1	1000
3	60	2	1	1	1	1000
4	80	2	1	1	1	1000
5	100	2	1	1	1	1000

4.4.3.3 NAF rock volume to be stockpiled and re-handled

The schedule results provide an insight to the amount of NAF rock to be stockpiled at NAFSP, which is located at {3, 1, 1, 1}. The filtered results to this dumping location are summarised in Table 4-9 and Table 4-10 under the two construction sequences. Some differences can be noticed in the volume from the mining blocks to the NAFSP, due to different rock dump construction sequences adopted. However, the total NAF rock volume scheduled to the NAFSP is the same, which is 12,500 BCM.

Table 4-9 Filtered results of the NAF rock to be stockpiled under the lift-by-lift dump construction sequence

t	b	n	k	i	j	V (BCM)
3	41	3	1	1	1	500
3	42	3	1	1	1	1000
3	43	3	1	1	1	1000
4	61	3	1	1	1	1000
4	62	3	1	1	1	1000
4	63	3	1	1	1	1000
4	64	3	1	1	1	1000
4	65	3	1	1	1	1000
4	66	3	1	1	1	1000
4	67	3	1	1	1	1000
4	68	3	1	1	1	1000
4	69	3	1	1	1	1000
4	70	3	1	1	1	1000

Table 4-10 Filtered results of the NAF rock to be stockpiled under the multi-lift dump construction sequence

t	b	n	k	i	j	V (BCM)
3	43	3	1	1	1	500
3	42	3	1	1	1	1000
3	41	3	1	1	1	1000
4	70	3	1	1	1	1000
4	69	3	1	1	1	1000
4	68	3	1	1	1	1000
4	67	3	1	1	1	1000
4	66	3	1	1	1	1000
4	65	3	1	1	1	1000
4	64	3	1	1	1	1000
4	63	3	1	1	1	1000
4	62	3	1	1	1	1000
4	61	3	1	1	1	1000

The required re-handling under the two construction sequences is the same, at 8,900 BCM, which will occur in the 5th time period. This re-handling volume is less than the total NAF rock stockpiled; thus the remaining NAF rock will be permanently stored in the NAFSP, or the MRD needs to be re-designed to accommodate the extra NAF rock.

4.4.3.4 Dump construction sequence

The dump construction sequence is cross-checked via the verification of the dump block filling status. Figure 4-5 displays the percentage of filling for each dump block within the MRD over the five time periods, under the lift-by-lift construction sequence. It is noted that during the 2nd time period, the dump block {1,2,2,2} in the second lift is utilised, which occurred after all of the dump blocks in the first lift are fully filled. Similarly, the dump block {1,3,3,3} in the top lift is used only after all of the dump blocks in the second lift are fully filled during the 5th time period. This verification shows that the lift-by-lift dump construction sequence has been achieved and that the mathematical formulation is correct.

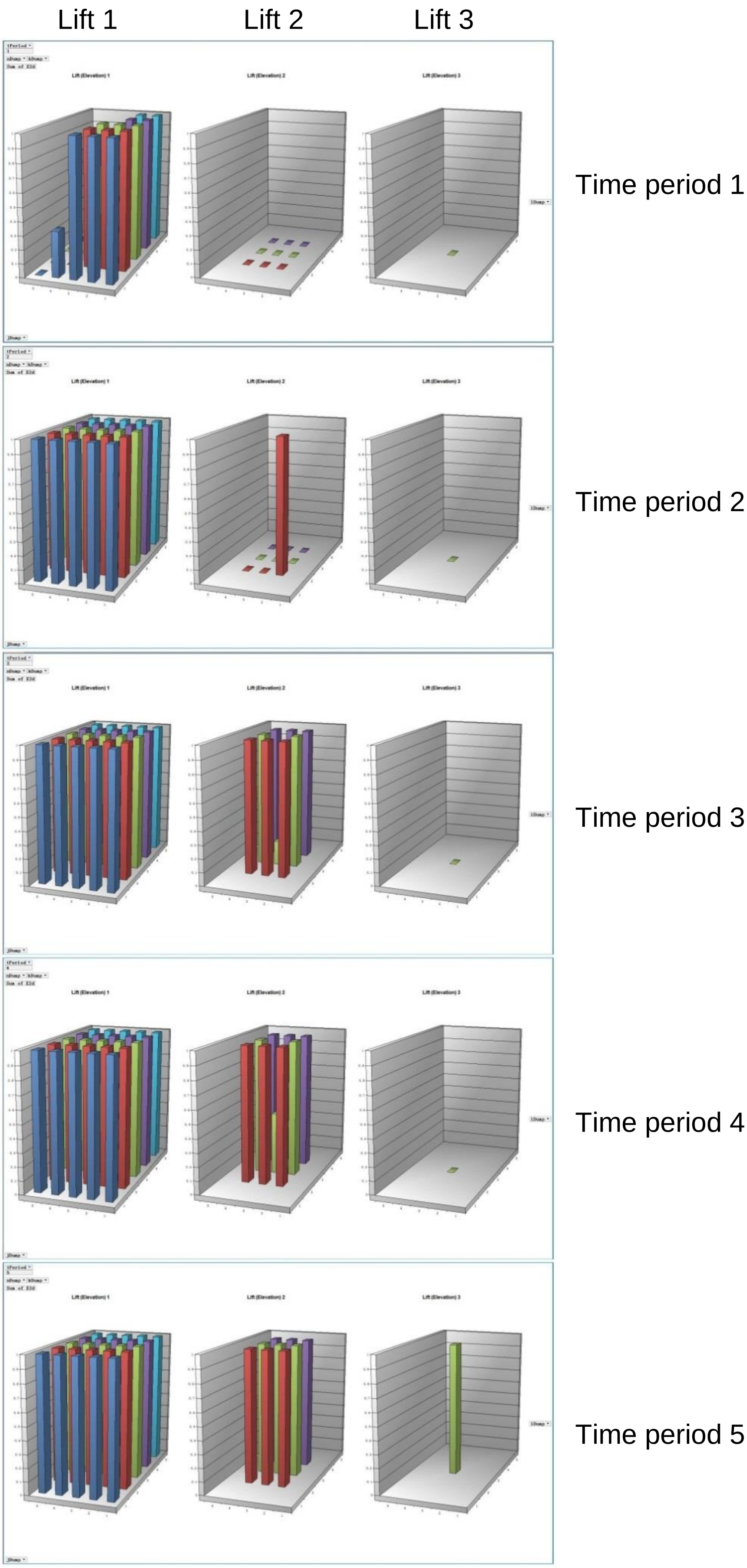


Figure 4-5 Progression of the dump block filling under the lift-by-lift dump construction sequence

The verification for the multi-lift dump construction sequence uses the same technique. The dump block filling status from the 1st time period to the 5th time period are presented in Figure 4-6.

In the 2nd time period, three dump blocks on the second lift are used, while the first lift is not fully filled; thus the filling status of the three blocks and their precedent blocks are investigated. The locations of the dump blocks are summarised in Table 4-11. It is found that all of the precedents blocks are filled to 100%, which means that those three dump blocks on the second lift have satisfied the dependency condition. Therefore, the multi-lift dump construction sequence is also correctly formulated.

Table 4-11 Investigation of the dump block and precedent blocks

	block location {n,k,i,j}								
Target block	1,2,2,2			1,2,2,3			1,2,3,2		
Precedent blocks	1,1,1,3	1,1,2,3	1,1,3,3	1,1,1,4	1,1,2,4	1,1,3,4	1,1,2,3	1,1,3,3	1,1,4,3
	1,1,1,2	1,1,2,2	1,1,3,2	1,1,1,3	1,1,2,3	1,1,3,3	1,1,2,2	1,1,3,2	1,1,4,2
	1,1,1,1	1,1,2,1	1,1,3,1	1,1,1,2	1,1,2,2	1,1,3,2	1,1,2,1	1,1,3,1	1,1,4,1

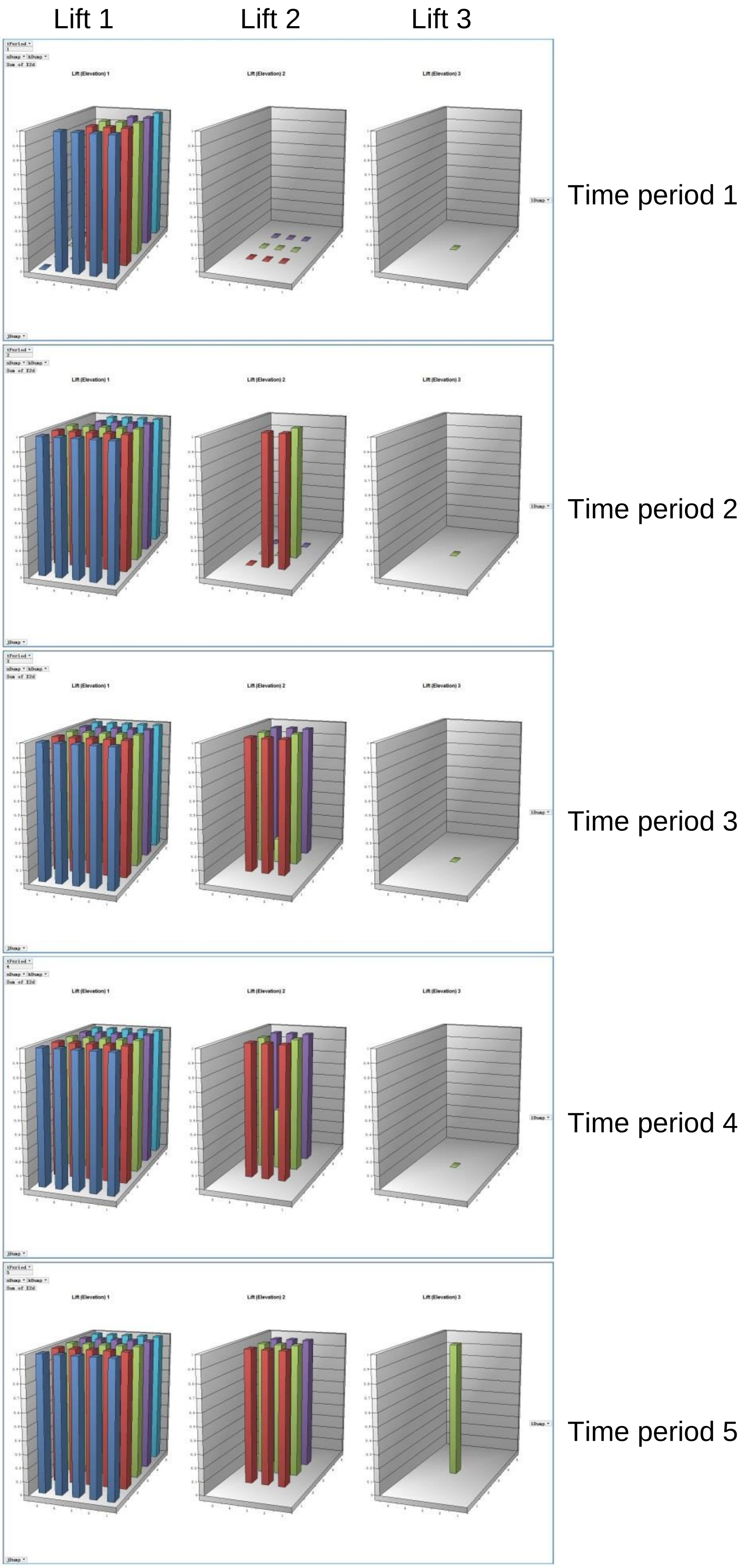


Figure 4-6 Progression of the dump block filling under the multi-lift dump construction sequence

4.5 SUMMARY

In summary, the OP model is developed to minimise the overall haulage distance and the required volume for re-handling. This model is formulated to achieve the material segregation condition while considering the pre-defined mining schedule. At the same time, each dump block in the MRD and the MSP is monitored in such way that its nominal capacity will not be exceeded in any time period. Material re-handling and the dump construction sequence are also described by mathematical equations, such that the logical flow of the material is controlled.

Moreover, the TB and Combo models are developed as a result of taking further thoughts about truck utilisation and budgeting. The purpose of the TB model is to seek the optimum rock placement schedule, with a minimal deviation between the required haulage work and truck budget, while the Combo model will search for the most balanced schedule, which minimises both the haulage cost and the deviation from truck budget.

The verification of the results from the base model shows that constraints to regulate material segregation, the NAF rock stockpile and re-handle, and the rock dump construction sequence are functioning properly.

The automatically generated rock placement schedule is optimised, containing valuable information of detailed volumetric flow for each mining block to the destination(s). It allows a quick analysis of the actual required capacity for the rock dumps so that a better designed can be developed. The timing and the volume for the NAF rock to stockpile and to re-handle are calculated by the MIP models, assisting the decision making in mine planning. Each dump block, excluding that of in NAFSP, is monitored, and the progression of the entire rock dump can be viewed in each planning time period, which is useful function for creating the staged dump plan and forecasting the future progression.

CHAPTER 5. MIP MODEL IMPLEMENTATION

Upon successful verification of the base model, especially the constraint sets, the MIP models developed are ready to be implemented. This section details the cross-comparisons of the three MIP models, and the implementation in the real world.

5.1 CASE STUDY ONE – MIP MODELS COMPARISONS

The differences between the base model and the two variants are to be analysed by examining the schedule solution to a synthetic mine site. This section compares the implementation results between the three proposed MIP models. In addition, manual scheduling was performed for comparing the estimated haulage cost.

5.1.1 Synthetic mine site layout

A synthetic mine site, as illustrated in Figure 5-1, is created by using the mine design packages Vulcan™ and Surpac™. The square-shaped rock dump is adopted to create regular dump blocks. An example is shown in Figure 5-2 to represent the dump blocks within the main rock dump.

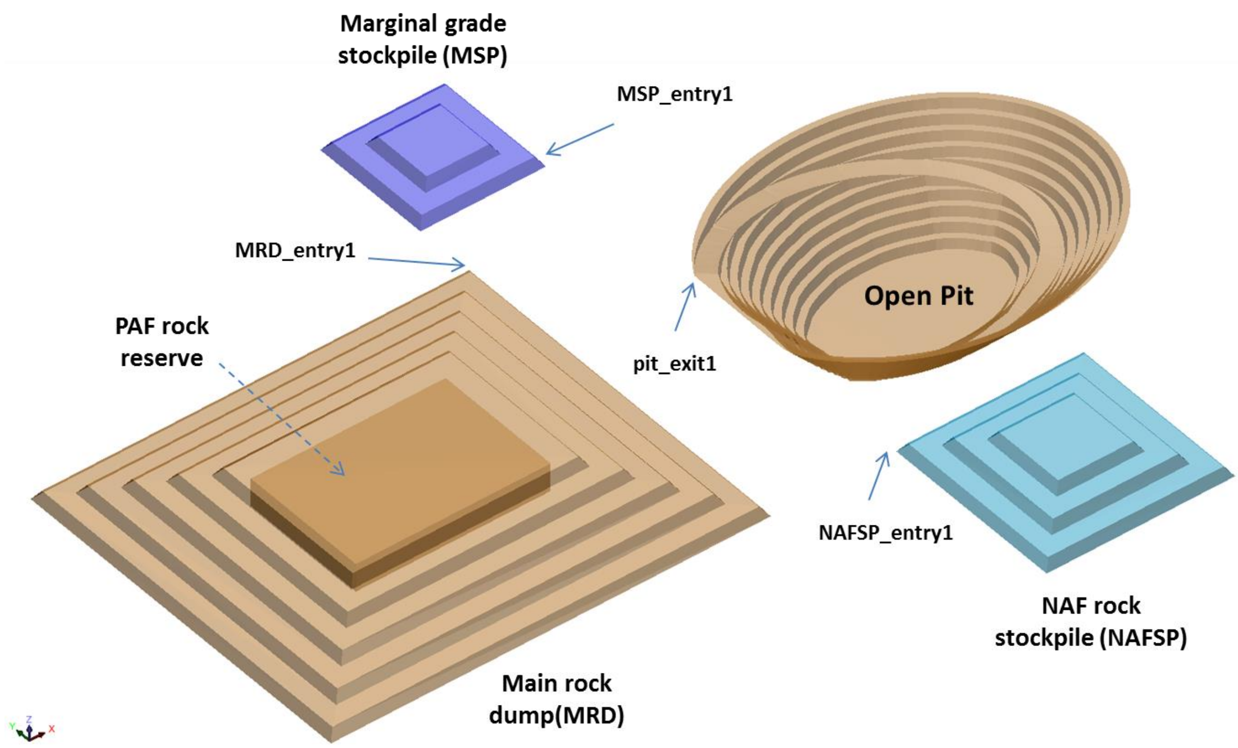


Figure 5-1 Synthetic mine site layout containing rock dumps and the pit

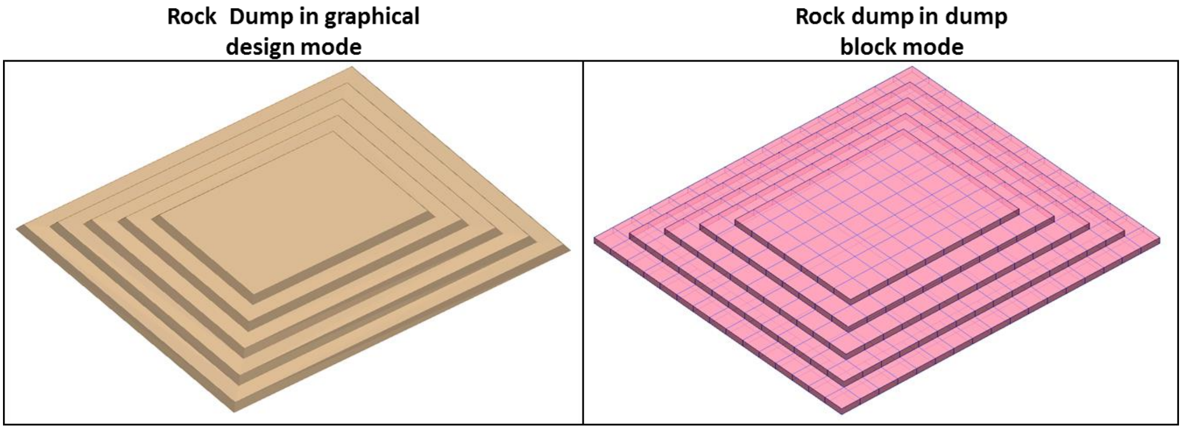


Figure 5-2 Dump blocks contained within the main rock dump (MRD)

The MRD is designed to the final landform shape, with all PAF rock to be fully encapsulated within the centre by the NAF rock. The design for MSP and NAFSP are based on the maximum allowable footprint, and the actual capacity required can be calculated from the rock placement schedule, which will be generated by the MIP models. Each rock dump is divided into a number of regular dump blocks, with the details summarised in Table 5-1. The total number of dump block is 969, and the overall capacity is 14.24 million m^3.

Table 5-1
Number of dump blocks in each rock dump

Dump name	Unit size	No. of dump blocks	Capacity (10^6 m^3)
MRD (PAF reserve*)	40x40x10	755 (160*)	12.08 (2.56*)
NAFSP	30x30x10	180	1.62
MSP	40x40x10	34	0.544
Total		969	14.24

*Contained within the MRD

5.1.2 Mining block removal schedule from an open pit and preliminary volume check

The open pit contains 2,547 regular waste rock blocks, with the block size measuring 20 m x 20 m x 10 m (length, width and height, respectively). These blocks are categorised into three types: the NAF rock, the PAF rock and the marginal grade ore, which must be segregated when choosing the dumping location(s). The removal schedule is displayed in Table 5-2, and the removal rate varies between 429 and 690, which reflects a deviation in the waste stripping.

Table 5-2
Number of mining blocks in the production schedule and the break down in terms of the material types

Time Period	Waste mining block	NAF	PAF	Marginal grade	Mixed PAF and marginal grade*
1	482	482	0	0	0
2	429	424	5	0	0
3	690	625	17	53	5
4	500	442	31	31	4
5	446	350	82	16	2
Total	2,547	2,323	135	100	11

*Mixed block containing both the PAF and marginal grade, which is treated as PAF.

A net swell factor of 1.25 is used in the preliminary check to compare the swelled volume from the pit and the dump capacity, as shown in Table 5-3. It can be observed from the table that each type of rock can be accommodated by the current design. It is noted that the PAF reserve is large enough to fully retain 675,000 thousand LCM PAF rock from the pit, with the balance of 1.885 million m^3 capacity for the NAF rock.

Table 5-3
Dump capacity check

Material type	Rock Dump	Dump Capacity (10^6 m^3)	Volume from pit (10^6 LCM)	Check
Marginal grade	MSP	0.54	0.445	Satisfy
PAF	PAF reserve	2.56	0.675	Satisfy
NAF rock	MRD	9.52		
	PAF reserve	1.885		
	NAFSP	1.62		
	Sub-total	13.025	11.615	Satisfy

5.1.3 Problem size and statistics of output results

Three MIP models are combined with the two pre-defined dump construction sequences, resulting in six MIP problems. Each MIP problem involves more than two million variables, as shown in Table 5-4 and Table 5-5. The scale of the problems are certainly beyond the capacity of humans to solve, but are readily solved by a PC with the specification of 2.8 GHz CPU with 24 GB RAM.

Table 5-4 MIP problem size and solution time by lift-by-lift dump construction sequence

Model Name	Number of	OP1	TB1	Combo1
Problem size	liner variable	2,282,510	2,282,520	2,282,520
	binary variable	35	31	31
	Constraints	7,422	7,415	7,415
Solving process	simplex iterations	400,464	40,265	532,358
	branch and bound cut	8	0	14
Solution time (minutes)		34	30	45

NB. Lift-by-lift dump construction is denoted by 1 in the results analysis section.

Table 5-5 MIP problem size and solution time by multi-lift dump construction sequence

Model Name	Number of	OP2	TB2	Combo2
Problem size	liner variable	2,286,039	2,478,583	2,478,583
	binary variable	3,927	3,945	3,945
	Constraints	18,906	19,069	19,069
Solving process	simplex iterations	31	81,443	42,346,106
	branch and bound cut	0	0	0
Solution time (minutes)		5	207	7,224

NB. Lift-by-lift dump construction is denoted by 2 in the results analysis section.

The results of the three key linear variables are exported to a database, forming a detailed rock placement schedule, including a rock volume movement schedule, a material re-handle schedule and a dump block filling status schedule. The sample for each schedule is shown in Table 5-6, Table 5-7 and Table 5-8, respectively. An example interpretation of the results is given using first row data from each table.

Table 5-6 Rock volume movement schedule

time	pit	block	pit exit	Dump	dump entry	elevation	easting	northing	V-move (BCM)
1	test_pit	478	exit_e1	MRD	MRD_e1	1955	68180	39440	800
1	test_pit	478	exit _e1	MRD	MRD _e1	1955	68220	39400	3200
1	test_pit	477	exit _e1	NAFSP	NAFSP_e1	1955	68515	39285	4000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
5	test_pit	2445	exit _e1	MSP	MSP_e1	1965	68440	40060	4000

Row 1 in Table 5-6 should be interpreted as follows: during time period **1**, **800**BCM material is hauled, from block **478** located in **test_pit** via **exit_e1** to **MRD** using dump entry **MRD_e1**, to a dump block, where the centroid is located at elevation **1955**, easting **68180**, and northing **39440**.

Table 5-7 Material re-handle schedule

Time	Dump	elevation	easting	northing	V-rehandle (LCM)
4	MRD	1975	68180	39400	12800
4	MRD	1975	68180	39440	12800
5	MRD	1985	67900	39440	12800
⋮	⋮	⋮	⋮	⋮	⋮
5	MRD	1985	67980	39440	12800

Row 1 in Table 5-7 should be interpreted as follows: during time period **4**, **12800** LCM of material from the NAFSP should be re-handled to a dump block within **MRD**, where dump block centroid is located at ‘elevation **1975**, easting **68180**, and northing **39400**.

Table 5-8 Dump block filling status

time	Dump	elevation	easting	northing	X-filling
1	MRD	1955	68180	39440	100%
⋮	⋮	⋮	⋮	⋮	⋮
4	MRD	1975	68180	39400	100%
⋮	⋮	⋮	⋮	⋮	⋮
5	MSP	1965	68440	40060	100%

Row 1 in Table 5-8 should be interpreted as follows: at the end of time period **1**, the dump block with centroid location of elevation **1955**, easting **68180**, and northing **39440** within **MRD** is filled to **100%** of its capacity.

5.1.4 Results Analysis

Six rock placement schedules are generated by solving the six MIP problems. To compare the different effects of each model, the loaded haulage work requirement, required material re-handle, and the estimated NPC are calculated.

5.1.4.1 Loaded haulage work requirement

Loaded haulage work links volumetric material movement and the associated haulage distance to form a key performance index. The resulting haulage work required under lift-by-lift dump construction and multi-lift dump construction sequences are presented in Figure 5-3 and Figure 5-4, respectively.

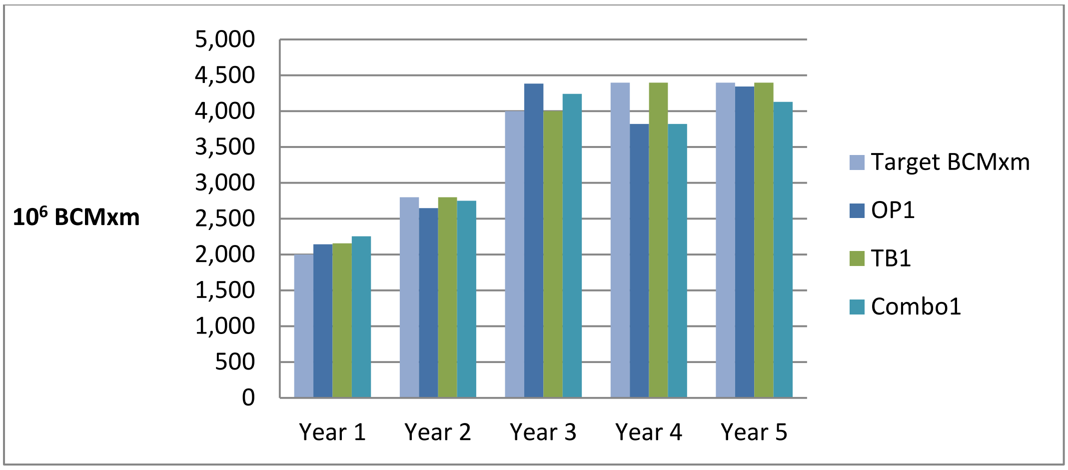


Figure 5-3 Yearly haulage work (BCMxm) requirement under lift-by-lift dump formation sequence

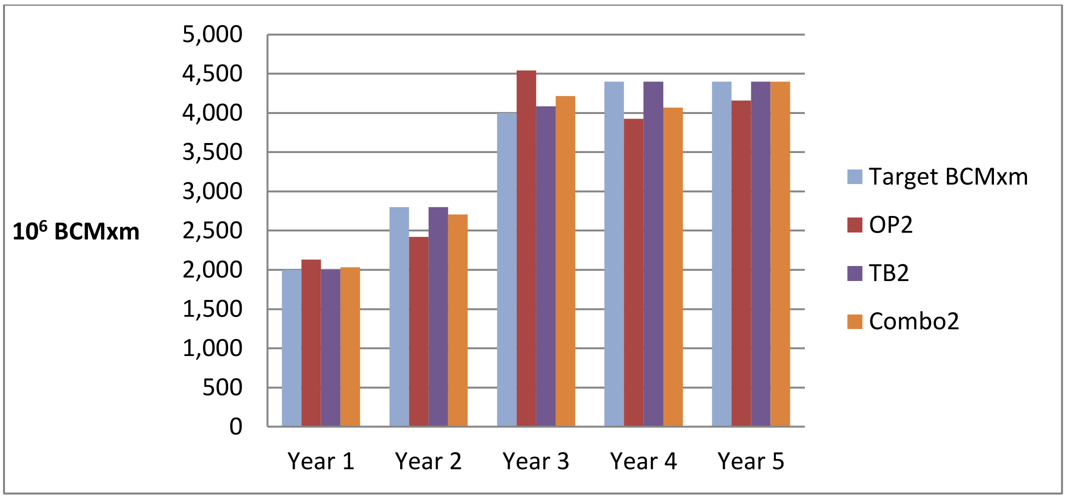


Figure 5-4 Yearly haulage work (BCMxm) requirement under multi-lift rock dump formation sequence

Regardless of the dump construction sequence, the OP model appears to perform the best in minimising the required haulage work but deviates from the target capacity. By contrast, the TB model yields the best match to the target capacity, and the Combo models' performance is between that of the OP and TB models.

Details of the total haulage work required by all six schedules are summarised in Table 5-9. It shows that the schedule generated by the TB models will result in less than 1% deviation from the target, which matches closely with the truck budget among the three models.

Table 5-9 Overall haulage work (BCMxm) required by all six schedules

BCM x m (million)	OP1	OP2	TB1	TB2	Combo1	Combo2	Target
Year 1	2,142	2,131	2,155	2,000	2,252	2,031	2,000
Year 2	2,649	2,421	2,800	2,800	2,749	2,706	2,800
Year 3	4,385	4,542	4,000	4,085	4,243	4,215	4,000
Year 4	3,824	3,925	4,400	4,400	3,823	4,069	4,400
Year 5	4,346	4,159	4,400	4,400	4,131	4,400	4,400
Total	17,345	17,178	17,755	17,685	17,197	17,421	17,600
Difference in %	-1.45%	-2.40%	0.88%	0.49%	-2.29%	-1.02%	

5.1.4.2 Volume of re-handle

The total re-handle volume determined by the MIP models is derived from the material re-handle schedule, as shown in Table 5-7. A comparison of the results is provided in Figure 5-5. It is observed that the TB models require double-handling the greatest amount of waste rock, as no minimisation is imposed on its objective function. The OP and Combo models determine to re-handle considerably less waste rock. An ideal solution, which requires no re-handle, is generated by the OP model under multi-lift dumping strategy.

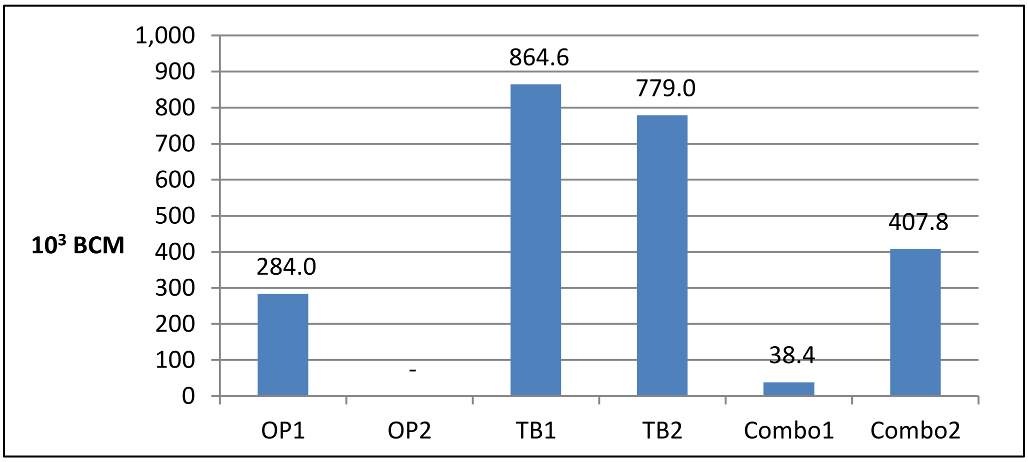


Figure 5-5 Comparison of waste rock re-handle by six rock placement schedules

5.1.4.3 Estimated haulage cost

The NPC for haulage is estimated using a cost factor of 1 cent per BCM per flat m hauled, with a discount rate of 12% applied annually. The NPC for each rock placement schedule is shown in Figure 5-6.

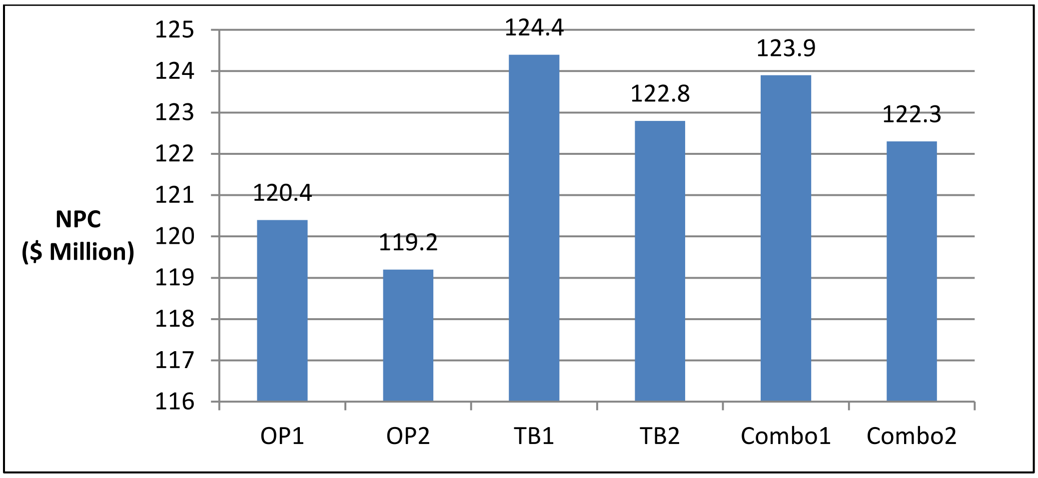


Figure 5-6 Estimated haulage cost of six rock placement schedules

Figure 5-6 indicates that generally, the multi-lift constructed rock dump is more economical than the lift-by-lift constructed ones, thus proving the flexibility and the potential cost savings gained by multi-lift dumping. Among the three models, the OP model results are the best for cost minimisation, which aligns with the designed objective.

5.1.5 Scheduling results by traditional method vs MIP models

In addition to this cross model comparison, classical manual method was employed to schedule the waste rock to the appropriate rock dumps, so that the overall haulage cost is estimated and is compared with that of by MIP models.

The fundamental technique adopted in manual method is trial and error, by matching the removal rock volume with the available dump capacity. Some simplifications was carried out to reduce the problem size to a manageable level, such as dividing the rock dumps into a number of points in according to the number of lifts and grouping the mining blocks located on the same bench.

Lift-by-lift construction sequence is utilised during the exercise, and Microsoft excel was used for record keeping. Without the assistance of computer program, it took the author three days to complete this schedule, which satisfies all the constraints as the mathematical model. A large proportion of the time spent is on the data pre-processing and results validation, which include:

- Calculating equivalent distance between the mining blocks to the dumping locations;
- Assigning waste rock volume to the appropriate rock dumps according to the rock types;
- Validating the swelled rock volume allocation with the in-situ rock volume from the mining schedule;
- Determining the remaining volumetric capacity in the rock dump; and
- Computing the required re-handle volume.

The differences in scheduling the waste rock using traditional manual method and MIP models are summarised in Table 5-10.

Table 5-10 Traditional waste rock scheduling vs MIP models

Comparison	Traditional Method	MIP models
Solving strategy	Trial and error	Branch and bound, and other scientific method
Problem scale	Over simplified	Full scale
Rock dump progression sequence	Human defined	Optimised by the models
Fast solution time	No	Yes
Accurate results	No	Yes

The main difficulty involved in the traditional method is to determine the re-handle volume, because it is a variable dependant on the schedule of the rock placement and dump progression. Meanwhile, the material segregation rule, in particular, the centralised PAF rock storage rule also brings some complication to the manual scheduling. Although a feasible schedule is generated, it is certain that a better solution would exist.

As the result, the estimated NPC for haulage is approximately $125.1 million, which is still $700,000 or 0.5% higher than the worst case generated by TB1 model, and $5.9 million or 4.9% more expensive than that OP2 model. The comparison of the results is illustrated in Figure 5-7.

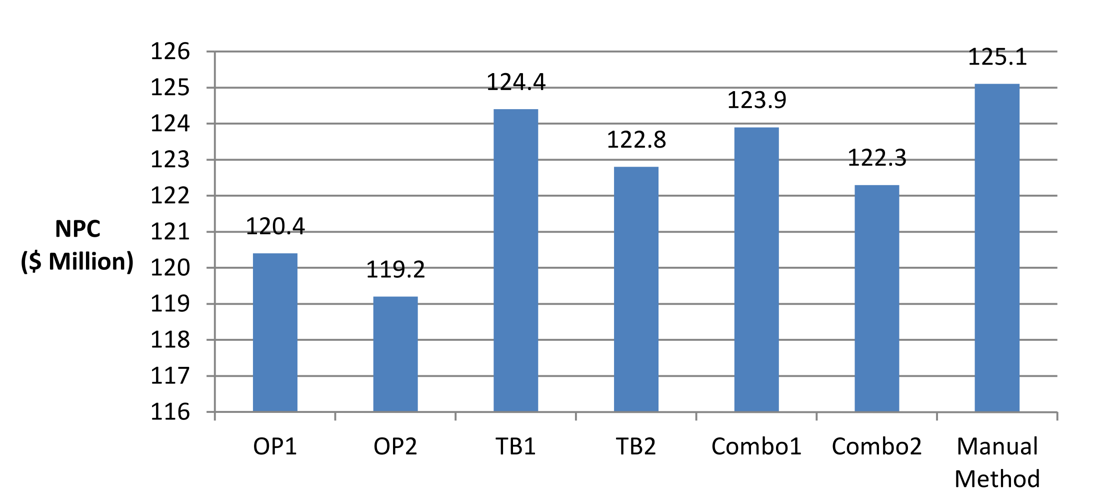


Figure 5-7 Manual schedule haulage cost vs MIP models

5.2 CASE STUDY ONE SUMMARY

The base MIP model and the two variants are implemented using a synthetic mine site data. Each problem contains more than two million variables, which are solved with IBM ILOG CPLEX, and rock placement schedules are automatically generated.

The analysis results indicates that the OP model schedules best at minimise the overall loaded haulage work, resulting in the minimum haulage cost of 119.2 to 120.4 million dollars and 0 to 284,000 BCM re-handle volume. The TB model schedules most closely adhere to the budget, but yield the highest haulage costs of 122.8 to 124.4 million dollars and 779,000 to 864,600 BCM re-handle volumes. The Combo model's schedules rank between the two extremes, at 122.3 to 123.9 million dollars and 38,400 to 407,800 BCM re-handle, because it considers both aspects in its objective function.

A manual schedule was conducted based on a simplified data set. The trial and error method took three days to generate a set of feasible schedule, which was very inefficient compared to some 5 minutes solution time by the OP models. The estimated overall haulage cost was $125.1 million, which is $700,000 higher than the worst case by TB1 model, and $5.9 million more expensive than that of by OP2 model.

The other finding is that the multi-lift dump construction sequence is more flexible than the lift-by-lift dump construction sequence, resulting in a lower estimated haulage cost across the three MIP models, and also reduces waste rock re-handle in OP and TB model. Combo model objective function consists of two components, i.e., haulage cost and opportunity cost, hence the re-handle volume under multi-lift construction sequence is higher, which increases the haulage distance, in order to reduce the opportunity cost. The overall cost is still lower than the lift-by-lift construction sequence.

A limitation of the multi-lift construction sequence is that it requires regular-shaped dump design to satisfy the dependency condition, which could be difficult to implement in reality.

5.3 Case study two – Strategic Dump Schedule in Tropicana Gold Project (TGP)

This section demonstrates the real world implementation of the developed MIP models. It includes an introduction to the mining project, model modification to site specific requirement, MIP problem solving, results generation and a comparative analysis.

5.3.1 Background information

Tropicana Gold Project (TGP) is located approximately 330 kilometres northeast of Kalgoorlie, as shown in Figure 5-8. It is a significant open pit operation in Western Australia that has an expected mine life of ten years, with possible extension for underground mining. The reserve estimation as of December 31, 2011 is 56.4 million tonnes of ore, grading at 2.16 g/t for 3.91 million oz. of contained gold (Tropicana_Joint_Venture, 2013).

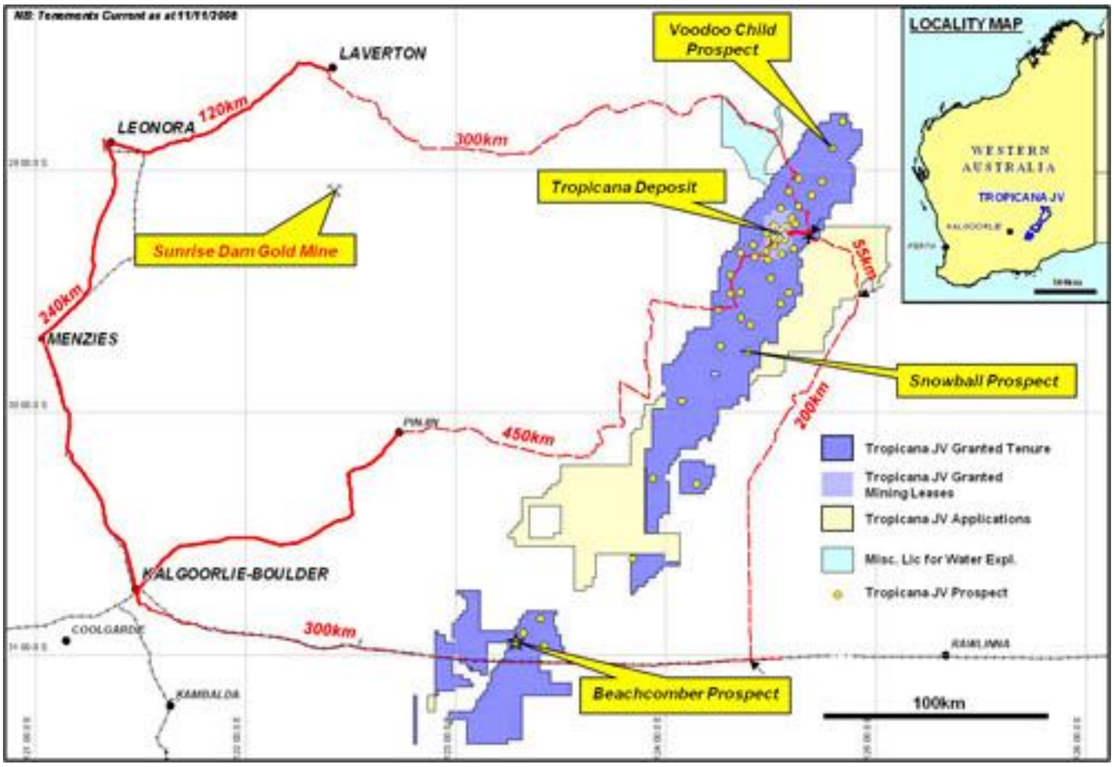


Figure 5-8 Tropicana project location

(Source: Tropicana_Joint_Venture, 2013)

As a large-scale open pit mine, invertible waste stripping and hauling are as important as ore mining and processing because great volume of material to be handled, hence the material handling cost. Under current volatile economic environment, cost reduction via optimising dump schedule would add significant value to the project.

5.3.2 Implementing MIP model in the current mine design

Three main rock dumps have been designed, namely LTA, LEA and LWE rock dumps, to accommodate the majority of waste rock. The tailing storage facility (TSF) and ROM-pad are to be built by the waste rock in the early stages of mining. Seven growth medium (GM) stockpiles will be used to store GM material for rehabilitation, covering the top of three main rock dumps by the end of mine life. The layout and landform design are illustrated in Figure 5-9.

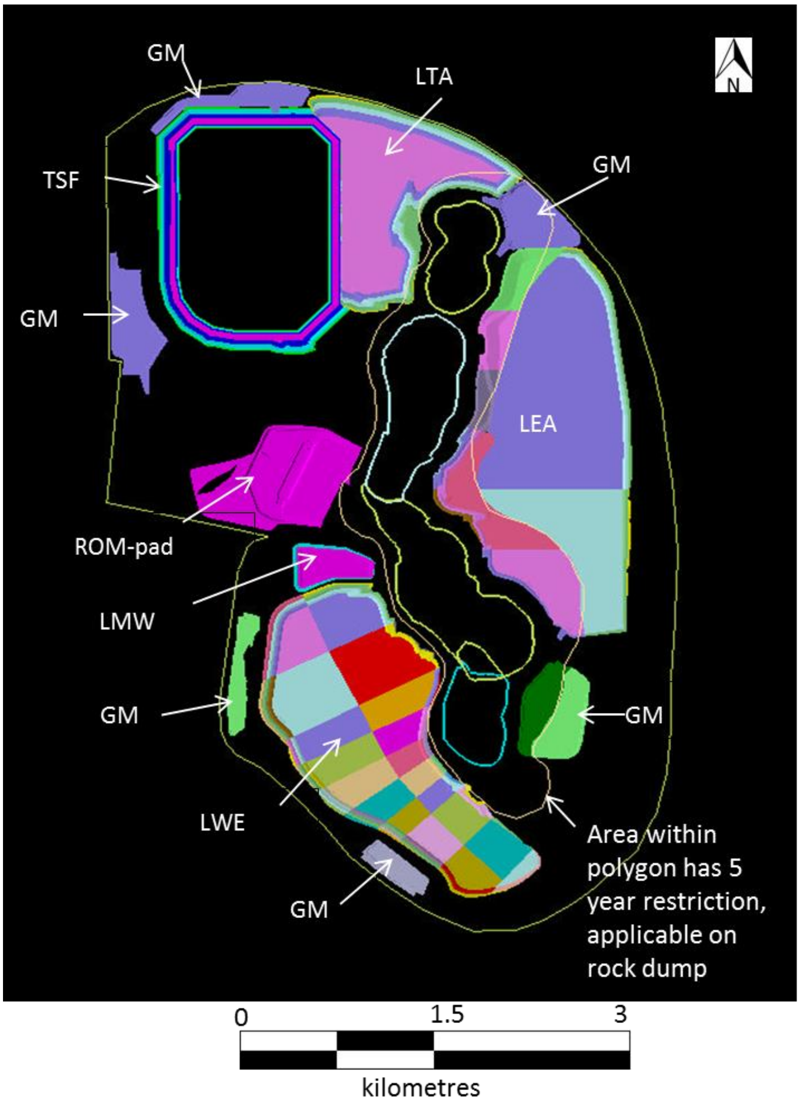


Figure 5-9 Mine site layout and overall landform design in Tropicana Gold Project

According to the given design, the total waste rock storage capacity is approximately 229.9 million loose cubic metre (LCM), as summarised in Table 5-11.

Table 5-11 Landform capacity for waste rock

Dump name	Capacity (m^3)
LTA	42,697,046
LEA	78,289,098
LWE	65,795,428
TSF	17,003,814
ROM pad	9,503,012
GM stockpile	16,587,882
Grand total	229,876,280

Apart from waste rock, low-grade stockpile (LMW) is also designed to separately store low-grade material (MW) for future processing. Ore is to be stockpiled on the top of ROM-pad, from where it is transported to a processing plant nearby with an assumed infinite capacity.

The main rock dumps are sub-divided into smaller divisions by vertical cuts, as described in section 3.6.1, then into dump blocks based on lift interval. The total number of possible dump blocks is 221, as summarised in Table 5-12.

Table 5-12 Summary of dump block in given design

Waste Dump	Number of division	Number of blocks
LTA	4	24
LEA	12	66
LWE	19	88
TSF	1	32
ROM_Pad	1	1
GM stockpile	7	7
LMW	1	2
Plant	1	1
Total	46	221

The production schedule is provided, and the yearly volumetric movement is presented in Figure 5-10. A preliminary check shows that the overall volume to be removed from the open pits is 234.8 million Loose Cubic Metre (LCM), comprising the following:

- 32.3 million LCM ore,
- 3.2 million LCM of MW, and
- 199.3 million LCM of waste rock.

This check indicates that all waste rock and MW material from the open pits can be fully contained within the current landform design.

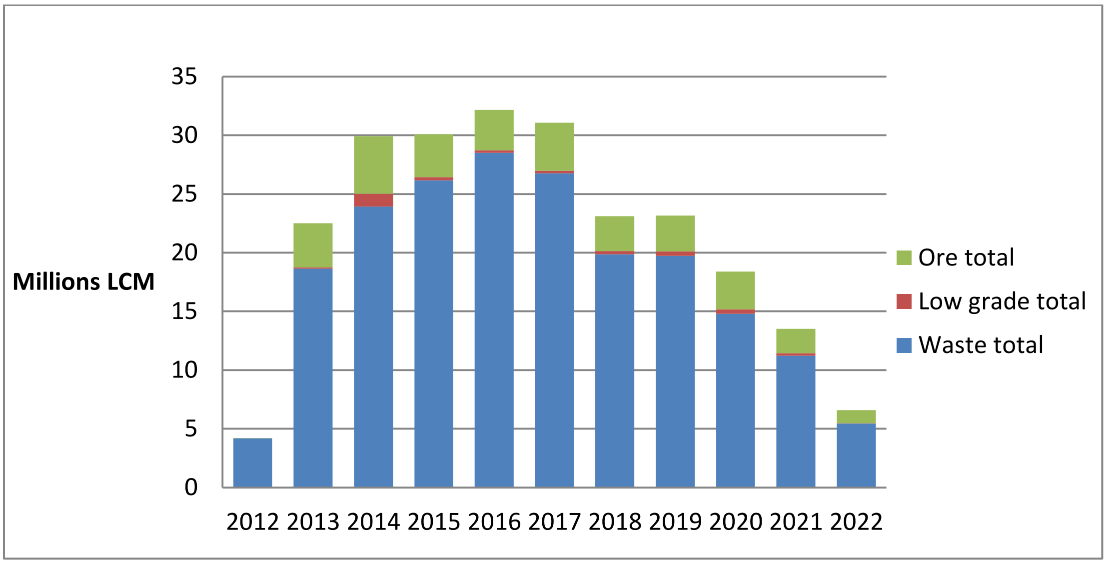


Figure 5-10 Yearly material movement schedule from operating pits

The provided block model is not a standard block model for scheduling software. The long-term planning engineer grouped some mining blocks based on their attributes and spatial location, thereby reducing the total number of mining blocks to 2,454, as shown in Table 5-13.

Table 5-13 Simplified schedule of mining blocks

Year	Waste	MW	Ore	Yearly sum
2012	14		11	25
2013	92	28	248	368
2014	105	37	278	420
2015	100	45	281	426
2016	30	20	109	159
2017	46	24	132	202
2018	41	21	144	206
2019	35	15	97	147
2020	26	17	116	159
2021	31	24	154	209
2022	18	18	97	133
Total blocks	538	249	1667	2,454

Given the number of dump blocks and mining blocks, it is estimated that 542,334 combinations are possible. All of which need to be evaluated by the MIP models before determining the optimum dumping strategy.

Additionally, the case study involves multiple pits, pit exits, rock dumps and dump entrances, which further complicates the problem. In case of the pit exit, for example, the design specifies 20 different exits, as summarised in

Table 5-14 and illustrated in Figure 5-11. Some exits are temporary, which will form and disappear as pits expand. That information must be read by the model so that the logic for pit expansion and the contained mining blocks are reflected in the solution output.

Table 5-14 Summary of pit exits

Pit(s)	Exit name	Number of exits
BS	J, O	2
TP	P, U,T,G,E,H,D,M	8
HA	L,R,B,I,A,C,Q,S,N,F	10
Total		20

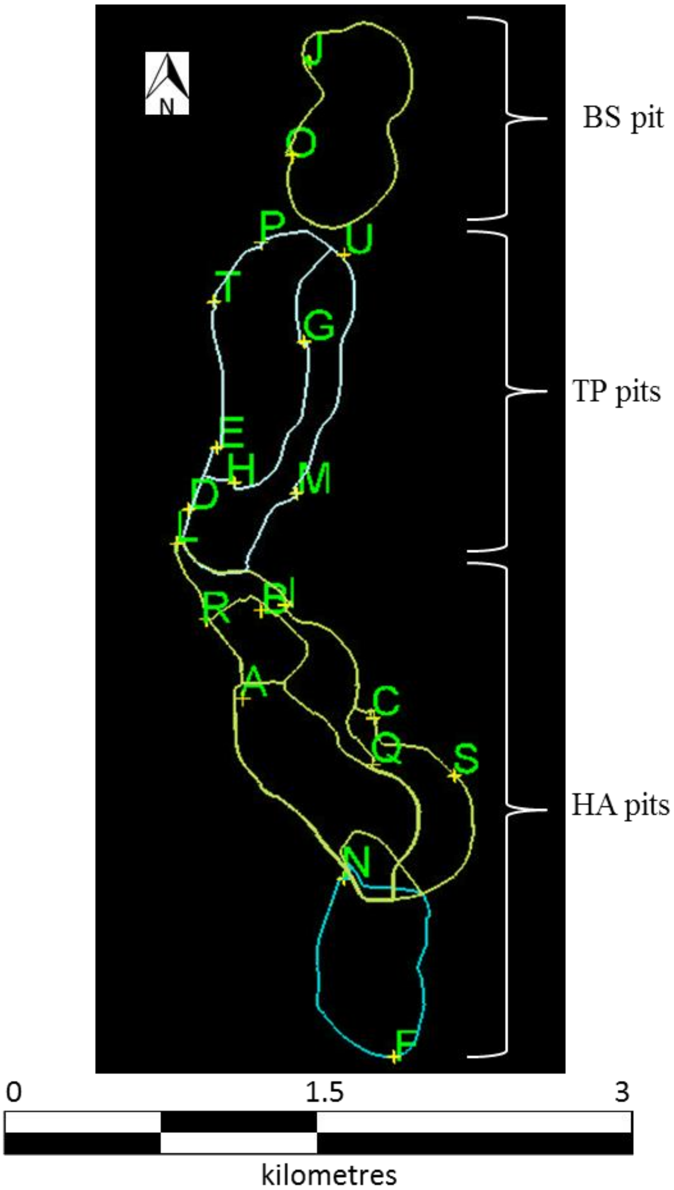


Figure 5-11 Temporary and permanent pit exits according to current design

5.3.2.1 Interpretation of site material flow

The generic model for waste rock dump planning and scheduling covers most aspects in the system, yet the individual mining project introduces special conditions. According to the provided information, ore movement is included, and waste rock is further categorised into Azone material, GM material and other waste rock. This variation requires minor modifications of the generic framework and the material flow, which are illustrated in Figure 5-12. The following modifications are required:

- ROM-pad is added as the dumping point for ore.
- PAF material is excluded.
- GM material is only allowed to be transported to the GM stockpile or to the top of the main rock dumps.
- Azone material is preferred to be sent to TSF to form an impermeable layer until the completion of TSF in 2015.

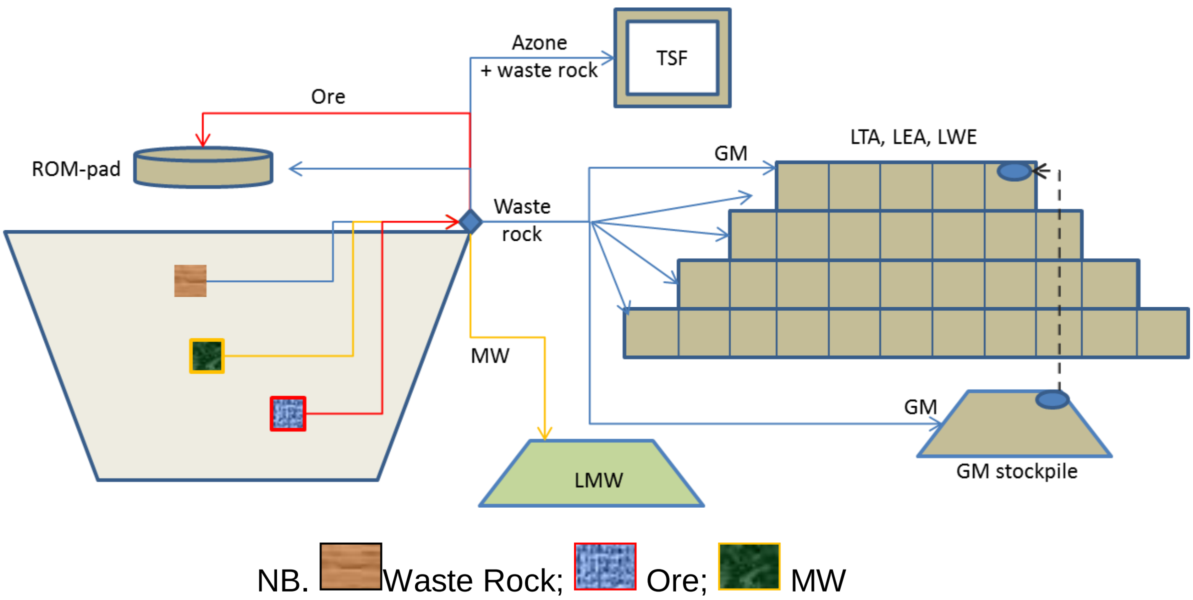


Figure 5-12 Modified mining and dumping framework for Tropicana project

5.3.2.2 Modification of MIP model to site specific condition

Apart from the material flow, site specific constraints are introduced to ensure the logic of the dumping schedule, which is required by the mine site:

- Restricting rock dumping to occur in part of the LEA dump, where it is located within the 5-year restriction zone, as indicated in Figure 5-9. The restriction will be lifted in 2017.
- Prioritising a two-staged ROM-pad construction, with stage one to be built by the end of 2013 and full completion in 2014.
- Prioritising a two-staged TSF construction, with stage one to be built by the end of 2013 and full completion in 2015.
- The TB model requires a pre-defined nominal truck capacity, which is not provided in the case study. The model must determine the optimum required loaded haulage work (LCMxm), such that the deviation between adjacent years is minimal, then the solutions are to be used as a guide for determining the truck budgeting.

These modifications are programmed using AMPL coding, which are specifically designed for the project, hence public disclosure is restricted.

5.3.3 MIP problem size and solution time

Each division of the rock dumps is treated as an individual structure, and a lift-by-lift dump construction sequence is applied. This implementation generates three MIP problems involving more than 640,000 variables each. The problem size, solving process and solution time are summarised in Table 5-15.

Table 5-15 Problem size and solution time

Model Name	Number of	OP	TB	Combo
Problem size	liner variable	643,063	643,085	643,085
	binary variable	1,719	1,719	1,719
	Constraints	12,537	12,547	12,547
Solving process	simplex iterations	335,094	51,550,326	8,184,228
	branch and bound cut	2,316	208	531
Solution time (minutes)		4.7	3557.4	244.3

Using the same computer as in case study one, the OP model required only five minutes to solve this scheduling problem. The most time-consuming problem, generated by TB model, was solved within two and half days.

5.3.4 Numerical results- comparisons between three schedules

The rock placement schedules are generated by the three MIP models, i.e., the OP, TB and Combo models. As requested by the data provider, for each time period, the total equivalent flat haulage distance, the truck hours required and truck productivity (LCM/km) are analysed. The MIP models also calculate the timing and the amount of GM material to be re-handled to cover the top of the three main rock dumps. The details of the findings are discussed below.

5.3.4.1 Overall return trip haulage distance

Table 5-16 summarises the overall return trip haulage distance, including the distance covered for the GM material re-handle. The estimated distances are presented in Figure 5-13. The OP model schedule specifies the least total distance to be covered, i.e.,14.06 million km (equivalent flat based distance), thus indicating that the minimal distance objective has been achieved.

Table 5-16 Yearly return trip haulage distance (thousand km)

Period	OP	TB	COMBO
2012	164	444	258
2013	1,239	1,188	1,372
2014	1,542	1,620	1,643
2015	1,663	1,836	1,976
2016	2,050	1,950	1,915
2017	1,693	1,676	1,876
2018	1,368	1,505	1,327
2019	1,505	1,363	1,418
2020	1,398	1,293	1,085
2021	870	874	791
2022	569	448	521
Sum	14,061	14,198	14,183

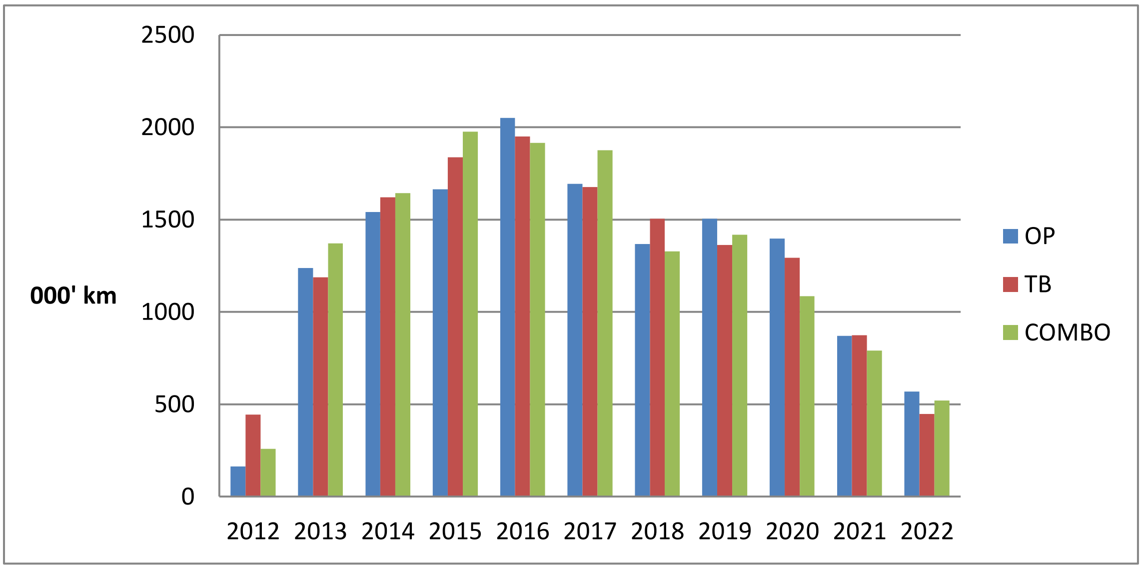


Figure 5-13 Overall haulage distance (thousand km) including re-handle by three options

5.3.4.2 Estimated truck hours requirement

The haulage distance is divided by the average truck travel speed of 40km/h, to facilitate the estimation of the required truck hours. This estimation follows the same trend as the haulage distance. These results are summarised in Table 5-17.

Table 5-17 Yearly truck hour requirement (hour)

Period	OP	TB	COMBO
2012	4,101	11,108	6,460
2013	30,963	29,698	34,291
2014	38,538	40,499	41,075
2015	41,587	45,911	49,395
2016	51,253	48,740	47,866
2017	42,323	41,889	46,903
2018	34,206	37,624	33,180
2019	37,630	34,076	35,461
2020	34,952	32,327	27,136
2021	21,761	21,862	19,787
2022	14,215	11,206	13,022
Sum	351,530	354,940	354,576

The deviation of required truck hours evident in Figure 5-14. Compared to the OP model schedule, the TB and Combo model schedules yield smaller deviations over the time. This output aligns with the objective function of the two models that additionally considers trucking deviations between two adjacent years.

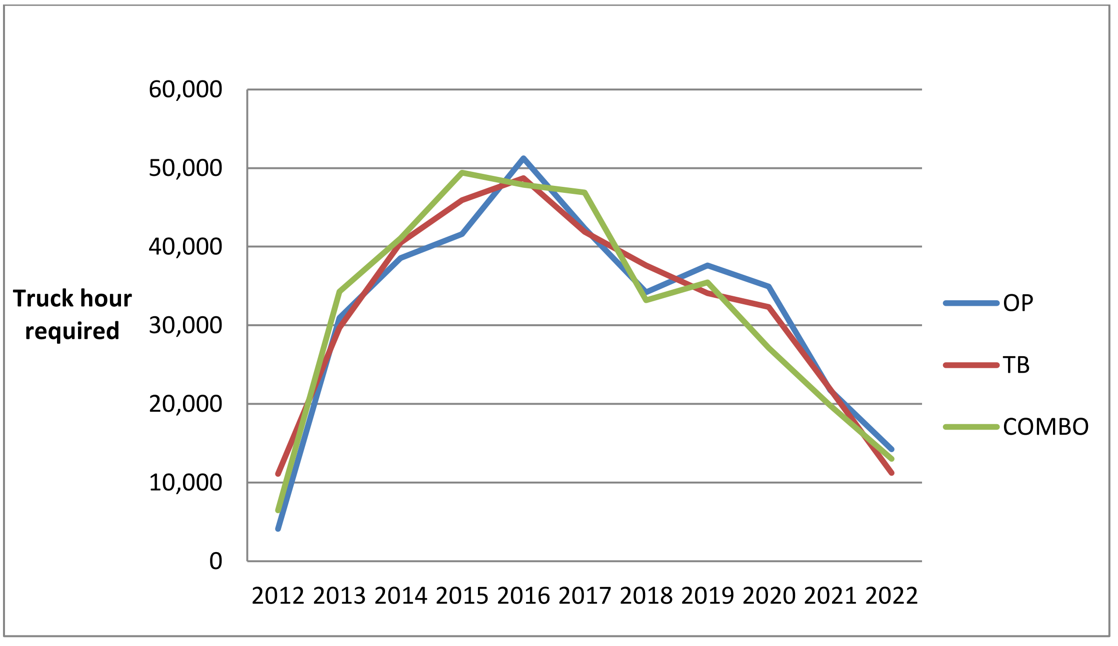


Figure 5-14 Estimation of truck hours required each year

5.3.4.3 Truck productivity per flat haulage distance (LCM/km)

The total rock volume handled each year is divided by the total return trip distance to calculate the truck productivity, measured in LCM/km. The calculated value is a performance index, which can be converted to tonnes/km if average density is applied. Generally, the higher the LCM/km value, the better efficiency the haulage system.

The yearly truck productivity yielded by the three schedules is summarised in Table 5-18, and displayed in Figure 5-15. Over the life of mine, the average productivity yielded by the OP, TB and Combo models are 17.90 LCM/km, 16.46 LCM/km and 17.03 LCM/km, respectively. These values suggest that the OP model most efficiently minimises haulage distance over the mine life. However, the TB and Combo model schedules yield a more balanced productivity over this time period.

Table 5-18 Yearly truck productivity in LCM/km

Period	OP	TB	COMBO
2012	25.73	9.50	16.34
2013	18.17	19.07	16.40
2014	19.41	18.54	18.21
2015	18.22	16.54	15.41
2016	15.76	16.56	16.80
2017	18.38	18.80	16.57
2018	16.90	15.63	17.42
2019	15.61	17.38	16.34
2020	14.23	14.57	17.48
2021	16.03	16.35	17.09
2022	18.49	18.07	19.23
Average	17.90	16.46	17.03

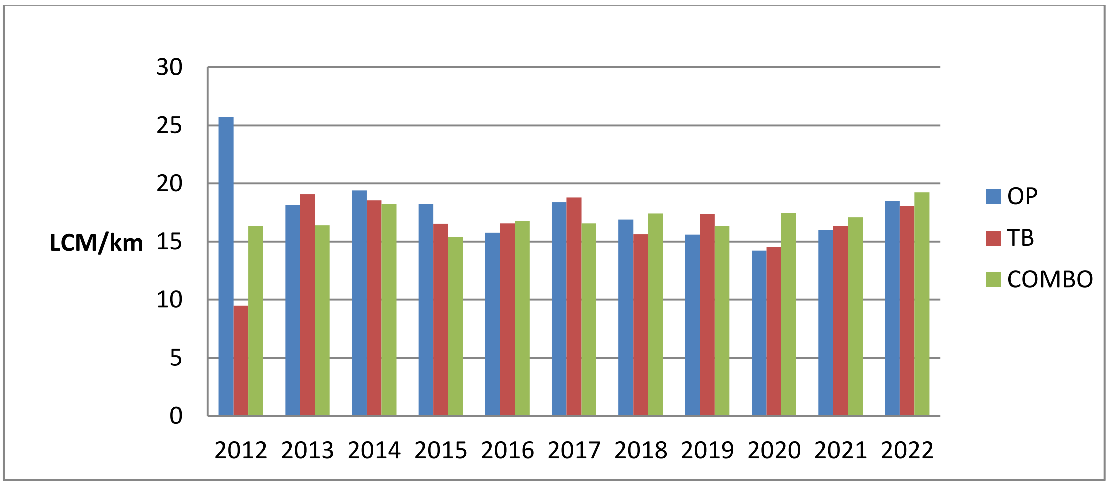


Figure 5-15 Yearly truck productivity (LCM/km) performance by three options

5.3.4.4 GM material re-handle

The production schedule is already determined, so the overall volume of movement from the open pits remains constant. The extra material movement is a result of material rehandle, i.e., GM material moved from a GM stockpile to the top of a rock dump. A summary of GM material re-handle calculated by each MIP model is provided in Table 5-19, and illustrated in Figure 5-16.

Table 5-19 Yearly re-handle schedule in LCM

Period	OP	TB	COMBO
2012			
2013		148,731	
2014		108,596	
2015	208,709	273,983	348,426
2016	154,736	135,072	
2017	41,143	424,985	
2018		398,392	
2019	324,941	516,879	
2020	1,497,434	441,258	584,235
2021	427,369	777,732	
2022	3,934,508	1,521,189	3,438,299
Total	6,588,840	4,746,819	4,370,960

The overall re-handle volume required by the TB and Combo model schedules are considerably less, by approximately 30% than that is required by the OP model schedule.

Figure 5-16 suggests that both the OP and Combo options prefer to rehab the rock dump at a later stage of mining, while the TB option allows this rehabilitation to occur in earlier years. The TB model is considered pro-active in terms of progressive rehabilitation. In fact, such a result is manipulated by its objective function, to minimise yearly deviation of truck budget.

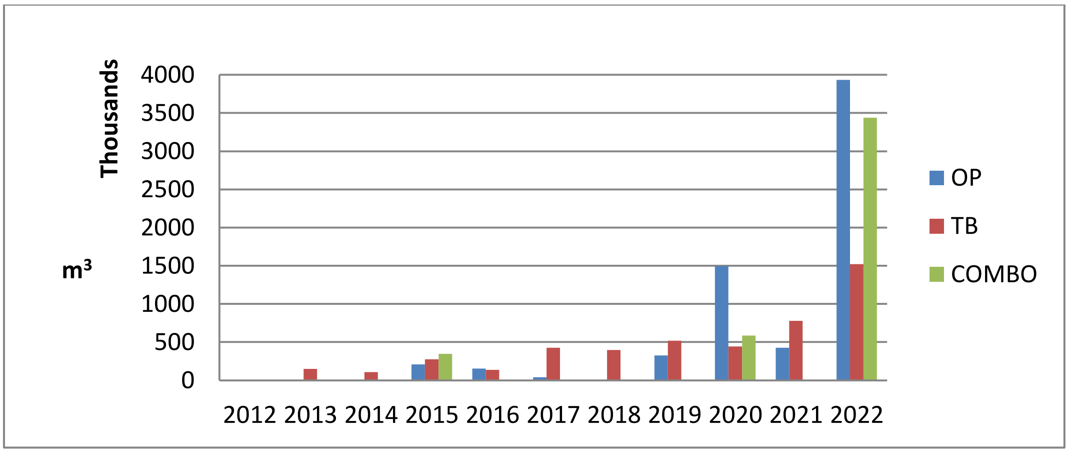


Figure 5-16 GM material re-handled (m^3) by three options

5.3.5 Graphical results-final footprint and landform progression

The dump block filling schedule enables predicting the final landform footprint and landform yearly progression, thus providing guidance for mine planning engineers in staged landform design. The final landform footprint prediction and yearly landform progression, generated by the OP model, are illustrated in Figure 5-17 and Figure 5-18, respectively.

Figure 5-17 Final footprint of landform predicted by OP model

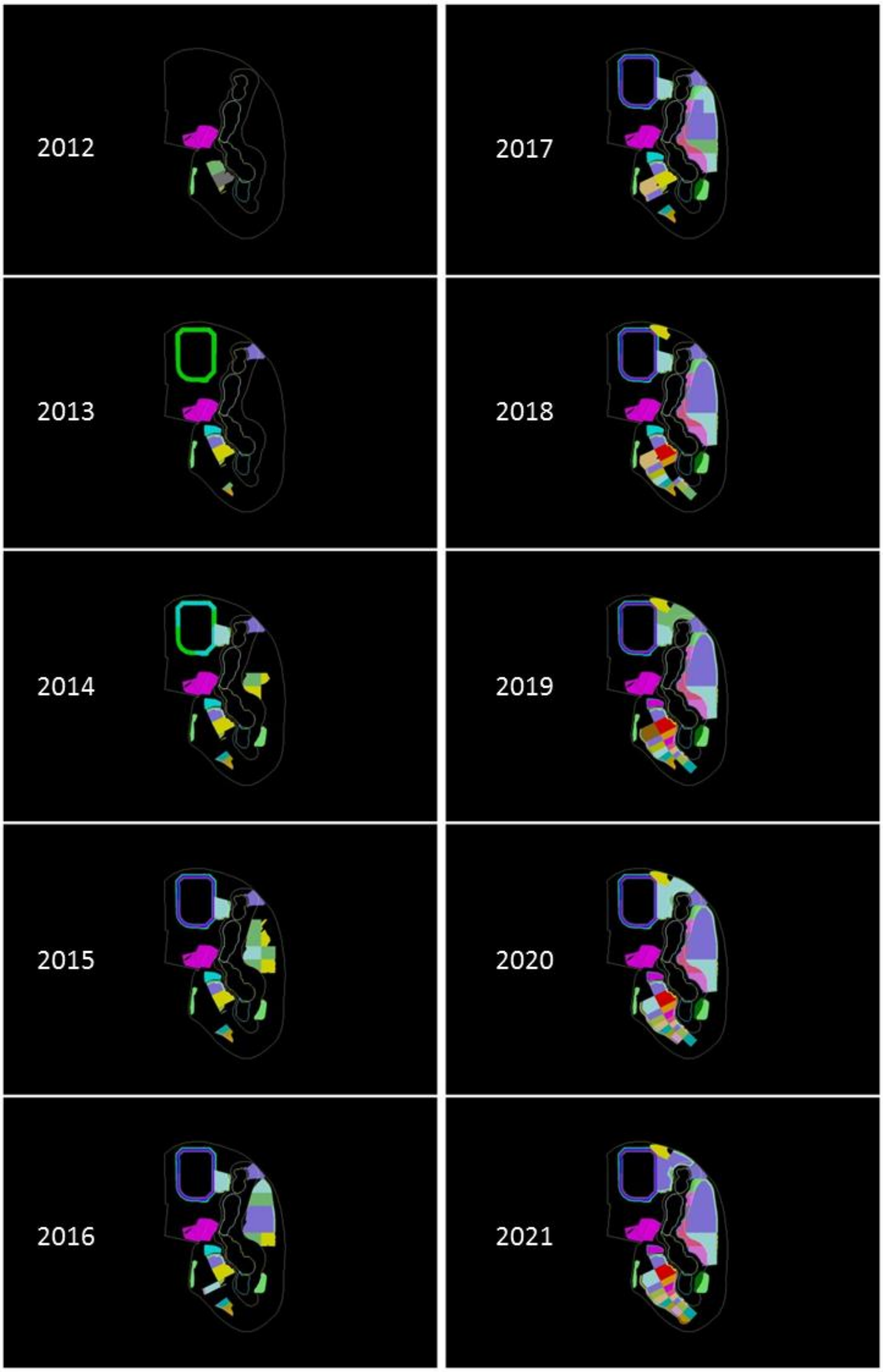


Figure 5-18 Landform progression according to the OP option dump schedule

The final landform footprint and progression as predicted by the TB model are illustrated in Figure 5-19 and Figure 5-20, respectively.

Figure 5-19 Landform footprint predicted by the TB model

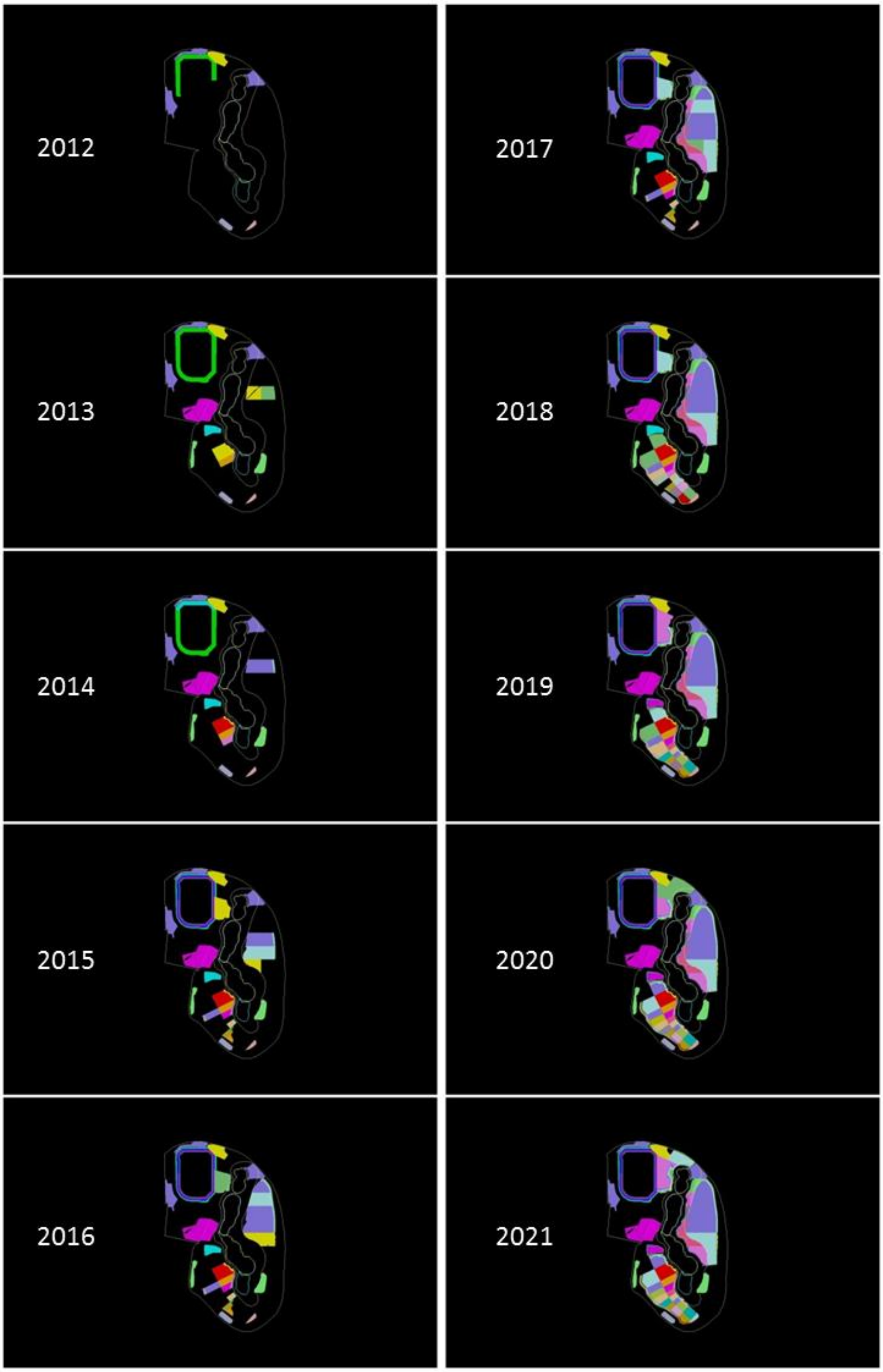


Figure 5-20 Landform progression predicted by the TB option dump schedule

The final landform footprint and progression as predicted by the Combo model are illustrated in Figure 5-21 and Figure 5-22, respectively.

Figure 5-21 Landform footprint predicted by the Combo model

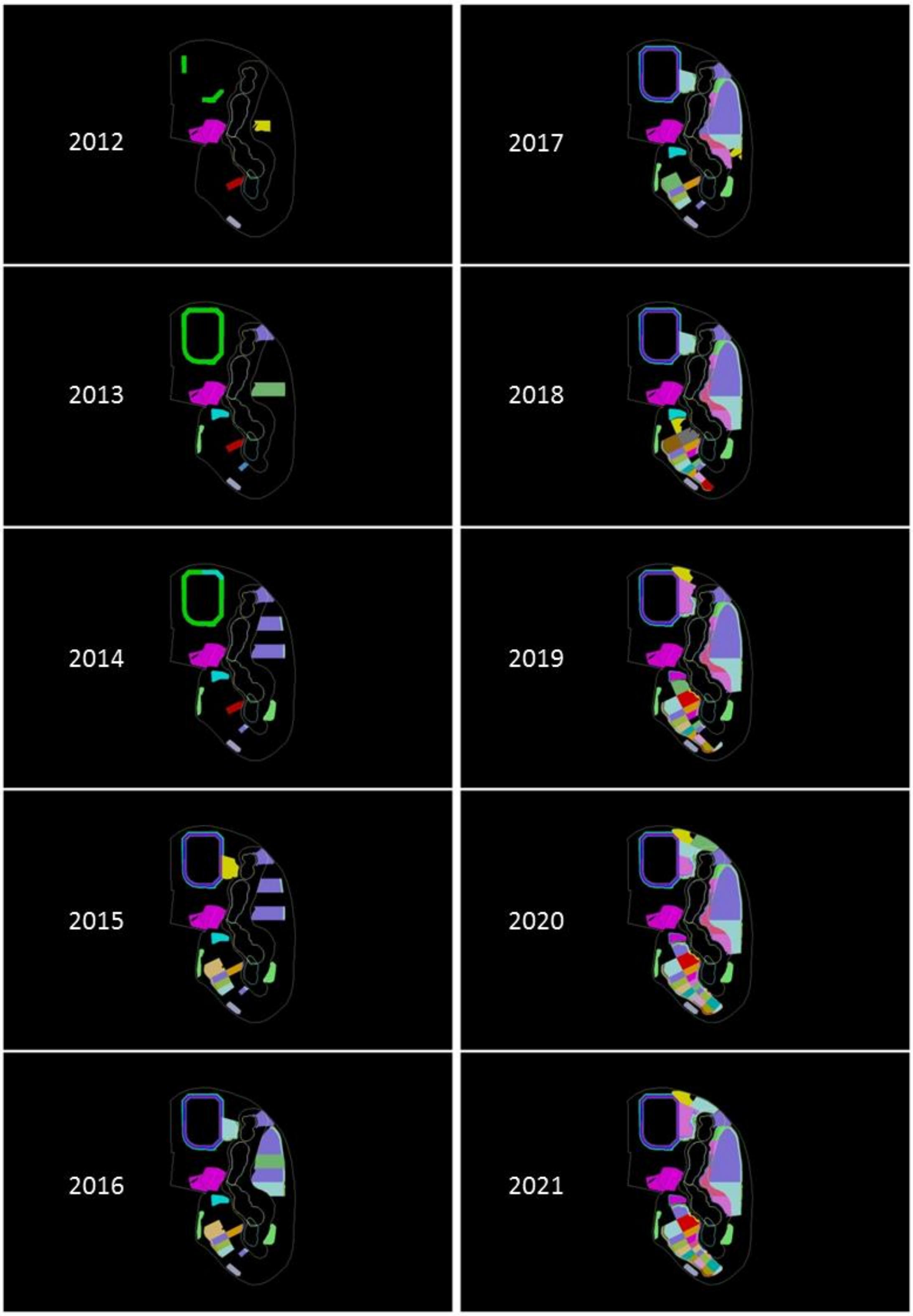


Figure 5-22 Landform progression according to the Combo option dump schedule

The differences in the landform progress are resulted from the different objective functions in each model. The user can select the model that satisfies the project objective.

5.4 Case study two extension – Tropicana Gold Project with an Alternative Rock Dump Design

A hypothesis in TGP is that both overall landform footprint and haulage cost will be reduced under the scenario of higher rock dump configuration. Therefore, an alternative scenario for the TGP is proposed. It allows three more lifts too be added to the current rock dump design, specifically, three lifts on LTA, LEA, and LWE, indicated in Figure 5-23. This design scenario has sufficient capacity to retain all the waste rock.

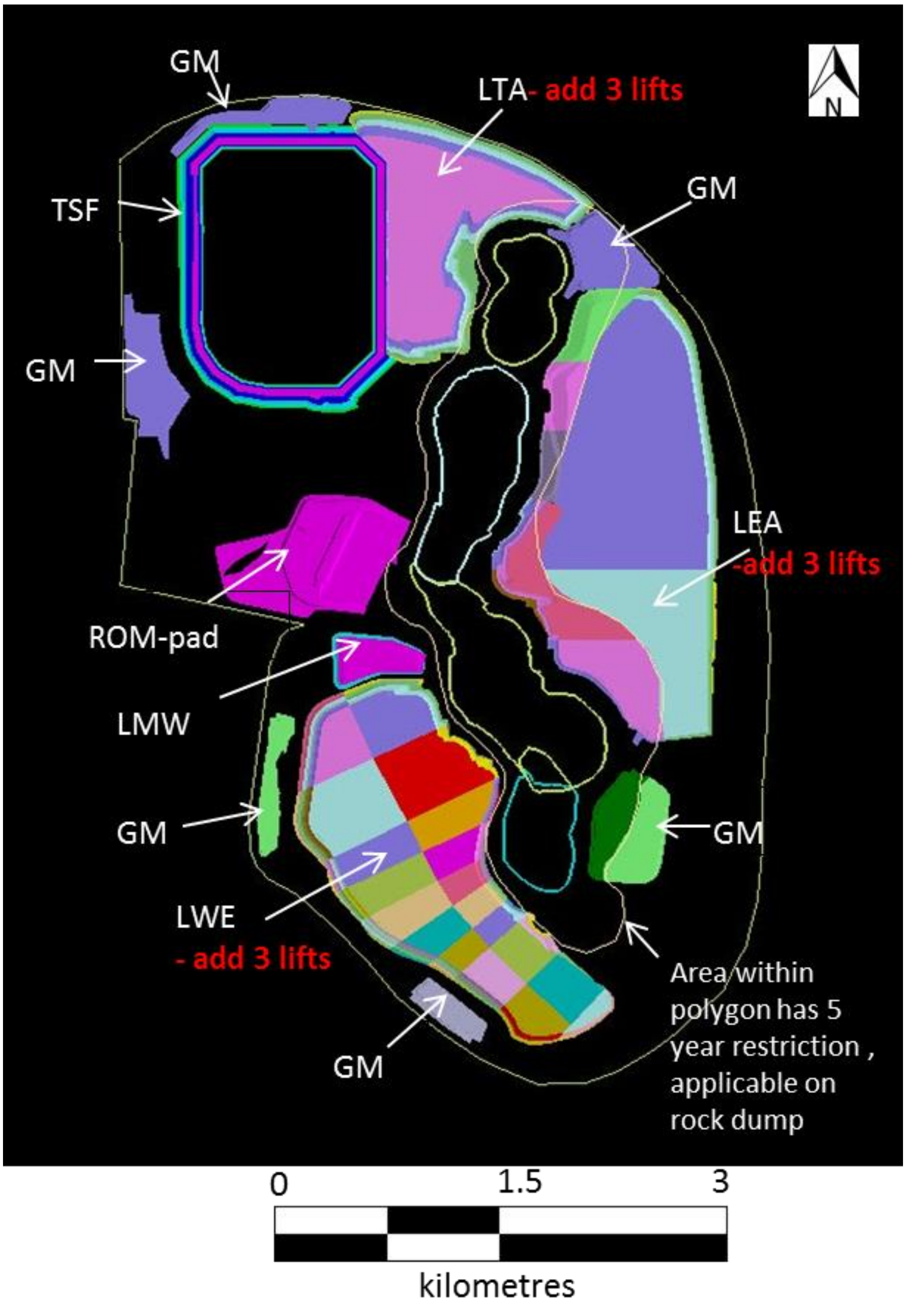


Figure 5-23 Alternative landform design

With the pre-defined rock dump divisions, the added lifts increase the number of possible dump blocks from 221 to 338, as detailed in Table 5-20. The TSF, ROM-pad, GM stockpile, LMW and Plant are unmodified.

Table 5-20 Number of dump blocks comparison

Waste Dump	Number of blocks		Additional blocks
	Current	Alternative	
LTA	24	36	12
LEA	66	114	48
LWE	88	145	57
TSF	32	32	
ROM_Pad	1	1	
GM stockpile	7	7	
LMW	2	2	
Plant	1	1	
Total	221	338	117

5.4.1 Study objective and model implementation

The change of input data is likely to result in a different output. The objective of this case study is to analyse the effect of the additional lifts on the decision making, in specifically to:

- Produce numerical proof to answer whether the added lifts are to be utilised, and if so, where are the preferred extra lift locations and how many lifts;
- Predict the final landform footprint based on this alternative design, and calculating the required GM material; and
- Compare the haulage distance, truck hour and truck productivity with the original design input.

The alternative dump design data is read by the OP and Combo models, and the number of variables is increased to 984,000, as indicated in Table 5-21. Both MIP problems are solved and results are automatically written into a database.

Table 5-21 Alternative dump design scenario problem size and solution time

Model Name	Number of	OP	Combo
Problem size	linear variable	982,762	982,784
	binary variable	2,778	2,778
	constraints	17,839	17,849
Solving process	simplex iterations	291,871	1,459,172
	branch and bound cut	1,038	146
Solution time (minutes)		4.5	109.2

5.4.2 Schedule results and analysis

The rock placement schedules generated by the OP and Combo models allow a clear view of the final landform. The analysis of two schedules indicates that the same final landform will be achieved, as displayed in Figure 5-24.

Figure 5-24 Footprint suggested by the OP and Combo models

The preferred location and number of additional lifts required are different in the OP and Combo model schedules, which are illustrated in Figure 5-25 and Figure 5-26, respectively.

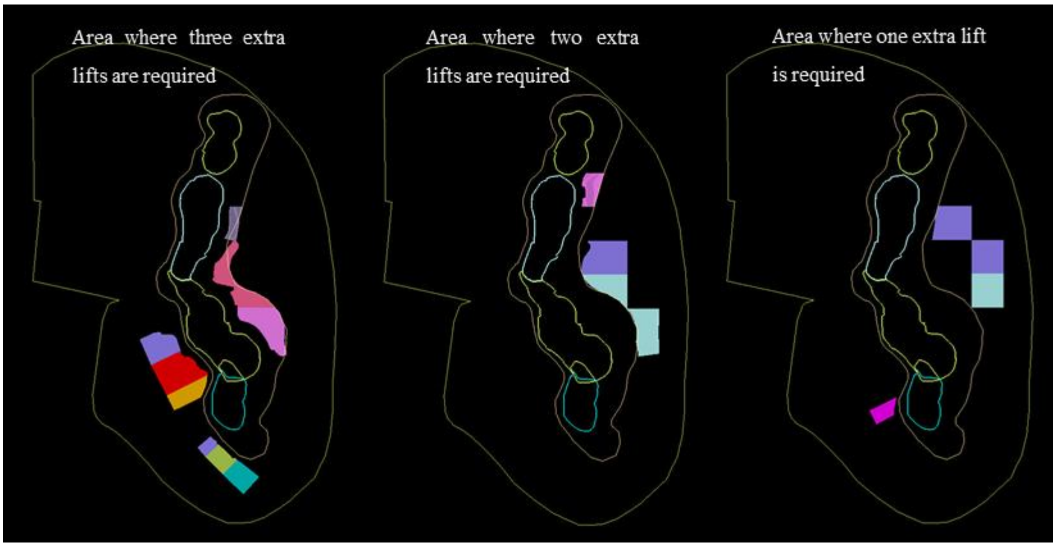


Figure 5-25 Preferred extra lifts location determined by OP model

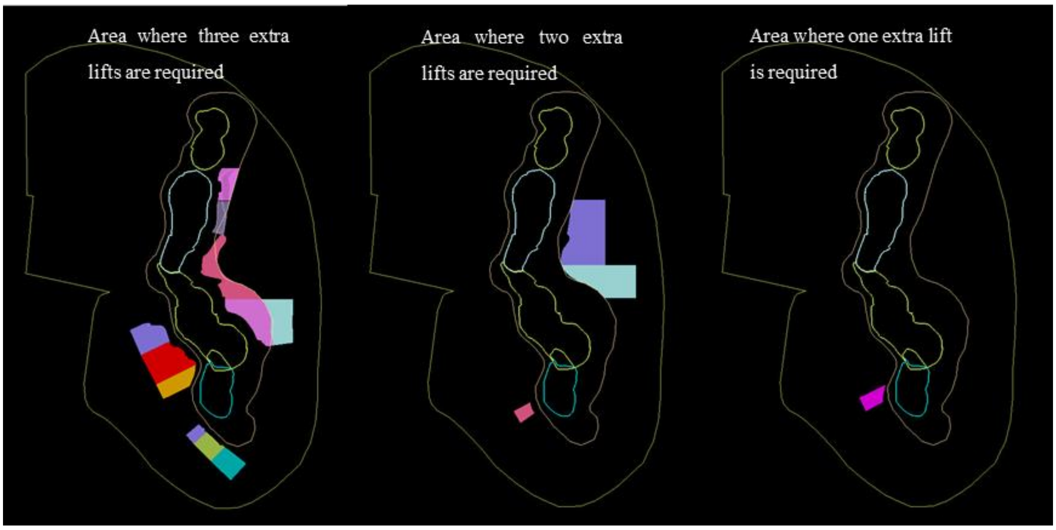


Figure 5-26 Preferred additional lifts location determined by the Combo model

It appears that additional lifts are utilised predominately in the area near the dump entrances, which is a reasonable decision. However, this numerical proof is required for the decision-making.

5.4.3 Results comparisons between original and proposed dump design

5.4.3.1 Comparison of GM material requirements

Because additional lifts will be utilised, less land will be disturbed. Such result influences the GM demand, as less GM material will be needed to cover the rock dumps. Table 5-22 summarises the GM requirement based on the final landform footprint, which indicates that the alternative design will reduce GM material demand by 1.95 million m^3, a 25% reduction.

Table 5-22 Comparison of GM material requirements

	GM material requirement (m^3)		GM available (m^3)
Landform name	Original design	Alternative design	
LEA Total	3,580,222	3,431,776	
LTA Total	1,451,059	1,167,513	
LWE Total	2,763,217	1,243,644	
Total	7,794,498	5,842,933	7,256,010

It is observed that significant variation in the final landform footprint is resulted by altering the design input. An additional comparison is conducted to examine the differences in haulage distance, estimated truck hours, and truck productivity.

The comparisons exclude volume and travel distance associated with the scheduled GM material re-handle. This is because:

1. The final landform shape is yet to be determined for the alternative design input, i.e., the MIP models will determine the final lift height for each dump division, to where the GM material is re-handled.
2. One constraint enforcing the re-handling of GM material from the GM stockpile has been dropped to allow a feasible solution under the alternative design input.

Therefore, the GM re-handle under the original design input are not accounted for to ensure the comparison is fair.

5.4.3.2 Return trip haulage distance

The yearly haulage distance based on the original and alternative design inputs are compared, summarised in Table 5-23, and presented in Figure -5-27.

Table 5-23 Return trip haulage distance in thousand km

	Original		Alternative	
Year	OP	Combo	OP	Combo
2012	164	258	164	161
2013	1,239	1,372	1,221	1,225
2014	1,542	1,643	1,632	1,508
2015	1,663	1,976	1,499	1,541
2016	2,050	1,915	1,924	1,864
2017	1,693	1,876	1,553	1,656
2018	1,368	1,327	1,223	1,247
2019	1,505	1,418	1,284	1,308
2020	1,398	1,085	1,182	1,106
2021	870	791	794	783
2022	569	521	415	411
Total	14,061	14,183	12,891	12,809

The analysis shows that the alternative design will reduce the overall haulage distance by 8.3% (from 14,061 thousand km to 12,891 thousand km) with the OP model schedule and 9.7% (from 14,183 thousand km to 12,809 thousand km) with the Combo model schedule. Therefore, the alternative design, which permits rock dump with more lifts, is more economical to construct as less haulage is required. However, this result is only valid for this case study, using the equivalent flat distance calculation model agreed by the data provider.

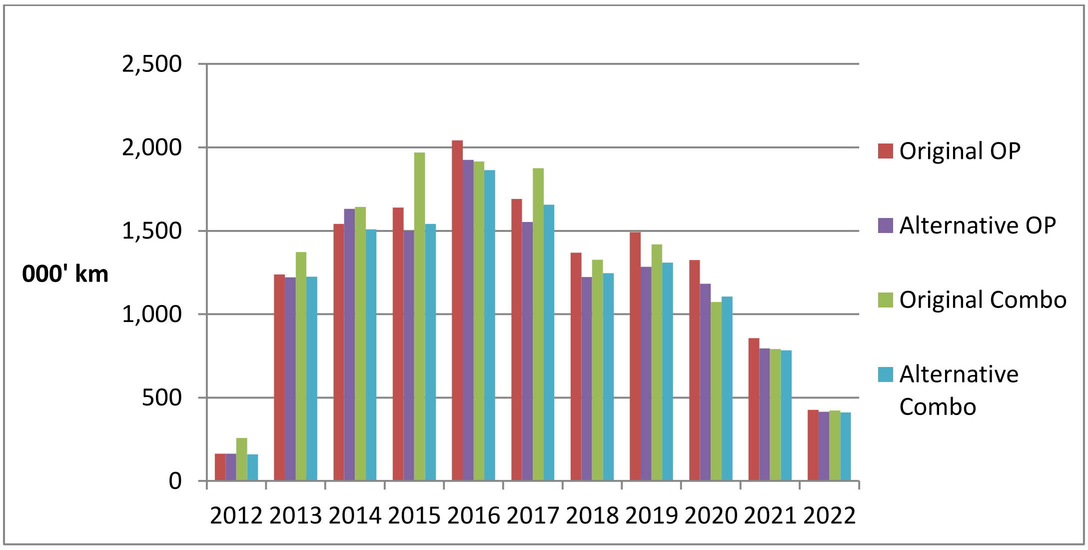


Figure -5-27 Yearly haulage distance comparison in thousand kilometre

5.4.3.3 Estimated truck hour requirement

The truck hour requirement is shown in Table 5-24 and presented in Figure 5-28. Truck hour is derived from dividing distance by 40km/h and trends identically to haulage distance.

Table 5-24 Yearly truck hour requirement (hour)

	Original		Alternative	
Year	OP	Combo	OP	Combo
2012	4,101	6,460	4,101	4,014
2013	30,963	34,291	30,534	30,614
2014	38,538	41,075	40,797	37,698
2015	40,997	49,253	37,481	38,521
2016	51,048	47,866	48,102	46,588
2017	42,294	46,903	38,817	41,410
2018	34,206	33,180	30,563	31,167
2019	37,278	35,461	32,106	32,711
2020	33,106	26,838	29,552	27,646
2021	21,428	19,787	19,850	19,583
2022	10,669	10,545	10,365	10,274
Total	344,628	351,659	322,267	320,226

The truck hours required by the alternative dump design are notably less than those by the original design, a comparison echoed in haulage distance comparison, which indicates 8.3% and 9.7% reduction in the alternative design, calculated by the OP and Combo models, respectively. Moreover, deviation in the truck hour requirement is less severe in the alternative dump design.

Figure 5-28 Yearly truck hour requirement

5.4.3.4 Truck productivity per flat haulage distance (LCM/km)

The yearly truck productivity is calculated, summarised in Table 5-25 and illustrated in Figure 5-29. The average improvement is 5.9% by the OP model schedule and 12.6% by the Combo model schedule.

Table 5-25 Yearly truck productivity (LCM/km) comparison

	Original		Alternative	
Year	OP	Combo	OP	Combo
2012	25.73	16.34	25.73	26.29
2013	18.17	16.40	18.42	18.37
2014	19.41	18.21	18.34	19.85
2015	18.36	15.28	20.08	19.54
2016	15.75	16.80	16.71	17.26
2017	18.37	16.57	20.02	18.76
2018	16.90	17.42	18.91	18.54
2019	15.54	16.34	18.05	17.71
2020	13.89	17.13	15.56	16.63
2021	15.78	17.09	17.03	17.26
2022	15.42	15.60	15.87	16.01
Average	17.57	16.65	18.61	18.75

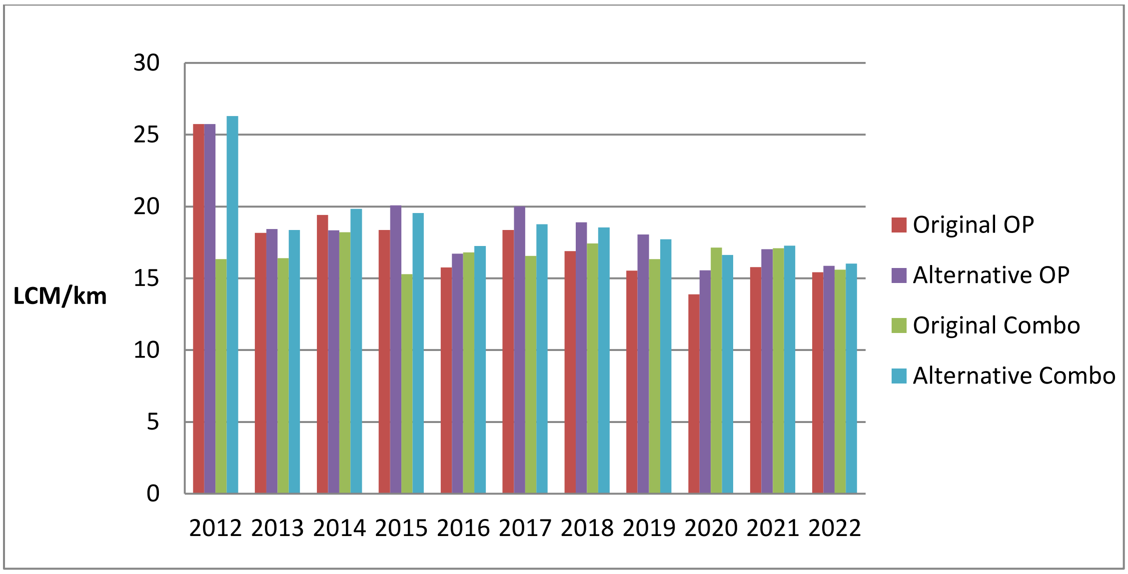


Figure 5-29 Yearly truck productivity (LCM/km)

5.5 CASE STUDY TWO SUMMARY

The three developed MIP models, i.e., a minimal overall haulage distance (OP) model, a minimal deviation in truck usage (TB) model, and a balanced haulage distance and truck usage (Combo) model, are implemented in Tropicana Gold Project, and three distinctive rock placement schedules and life of mine dumping strategies are automatically generated.

Under the current design, the OP model specifies the most efficient schedule for waste rock placement. It yields the lowest haulage distance, 14.06 million km, and the highest truck productivity, 17.90LCM/km on average. However, it predicts the largest GM re-handle volume among the three models, approximately 6.6 million LCM. The TB and Combo model schedules require greater truck haulage, yet their deviations in yearly haulage distance and truck hour requirement are smaller than those of by the OP model. The required GM material re-handle is also considerably less by the two models, an average of 30%, compared to that of by the OP model.

Under the alternative design, the north part of LTA rock dump, the south-west part of LWE rock dump and the north-east part of LEA rock dump will not be utilised, because of increased capacity in the three main rock dumps, thus reducing the final landform footprint and the GM material required to cover the waste rock dump. The alternative design will reduce GM material demand by 1.95 million LCM, a 25% reduction.

Furthermore, comparisons of haulage distance and truck productivity shows that the alternative design is more advantageous compared to the current design. Using the OP model, the overall haulage distance (including truck house required) will decrease 8.3% and truck productivity will increased 5.9%. Using the Combo model, the overall haulage distance will decrease 9.7% and truck productivity will increase 12.6%.

CHAPTER 6. CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

Waste rock mining, hauling and dumping are inevitable exercises in an open pit mine, in order to recover valuable ore. The haulage cost involved could be as high as 50% of the total operating cost, yet thus far, no scientific method has been developed to schedule the waste rock dump, to effectively reduce this cost. In addition, a waste rock dump is the potential source of acid mine drainage (AMD). Literatures show that it could be prevented via careful scheduling the waste rock dumping sequence, namely, encapsulating PAF rock by NAF rock. Despite of the fact, waste rock placement and rock dump scheduling are outside the scope of current practise in mine planning.

This research utilises mathematical modelling method to develop new MIP models for generating the optimum rock placement schedule, which will assist the planning of the optimum waste rock dumps.

Through the modelling process, an integrated waste rock mining and dumping system is established. Based on which, a base MIP model, called the OP model, is constructed and tested with a simplified data set. The results verification shows that all constraints are correctly formulated.

Case study one involves a synthetic data set, created using mine planning software. Upon solving the MIP problems, optimum rock placement schedules are automatically generating. Each schedule has a distinctive characteristic. The OP model schedule best minimises the overall loaded haulage work, thus resulting in lowest haulage cost of 119.2 million dollars, and zero re-handle volume. The TB model schedule yields the best match with the budget, but the highest haulage cost of 124.4 million dollars and re-handle volume of 864,600BCM. The Combo model considers both haulage cost and budget deviation; hence, its solution is between these two extremes, at cost of 122.3 million dollars, and rehandle volume of 38,400BCM. The model outputs include detailed rock volume movement schedule, material re-handle schedule and the dump block filling schedule.

Two types of dump construction sequences have been modelled in this research. The multi-lift dump construction sequence is more flexible than the lift-by-lift dump construction sequence, thus yielding a lower cost across the three MIP models and reduced waste rock re-handle across in OP and TB models. However, it requires a regular-shaped dump design to satisfy the dependency condition, which could be difficult to implement in reality.

Manual schedule was conducted using Microsoft Excel. The aim was to match the waste rock volume with the dump capacity. A feasible solution was generated after three days of trial and error, which is much longer than some 5 minutes solving time by OP model. The estimated cost was $125.1 million, up to $5.9 million or 4.9% worse than that of by OP model.

The real world application of the MIP models is demonstrated using the Tropicana Gold Project data and three life of mine strategic dump schedules are generated accordingly. OP model schedule resulted in the shortest overall haulage distance of 14.06 million km, least truck hour requirement of 351,530 hours, and highest truck productivity of 17.9LCM/km. These numerical data can be used by mine planning engineers for improving truck fleet management. The graphical results of the landform progression are determined from the optimum rock placement schedule. This visual information will improve the forecasting of the project and provide guidance to the operation.

The MIP models are utilised for quick assessment of alternative design scenarios. The final landform footprint of the proposed alternative design for the TGP is predicted. Consequently, the GM material requirement was accurately analysed, which is decreased by 25% compared with the original design. Other supporting data, such as the reduction in overall haulage distance by up to 9.7%, and incremental in estimated truck productivity by 12.6%, could potentially make the decision making process more straightforward.

In conclusion, the new methodology for generating optimum rock placement schedule has been developed, verified, and can be implemented in real operations. It will optimise the waste rock dump planning economically and environmentally.

6.2 RECOMMENDATIONS

First, future studies should focus on the applicability of the MIP models.

The modelling for waste rock dump planning and scheduling is based on an ex-pit rock dump situation. In reality, however, it may contain backfill, fully in-pit dump or erosion resistant rock dump design. This requires modification to current models to adapt the flexible design input.

In addition, variations in geological model and production scheduling may occur. Although neither forms part of the research, the models need to be capable of updating the changes in the given data, and generating new results.

Second area of improvement is the refining of mathematical models.

A weighting factor can be applied to the objective function in Combo model to emphasise the importance of a specific objective, and the output schedule would adjust this requirement automatically.

Another potential improvement is to introduce stochastic equivalent flat distance calculation. This stochastic parameter will realistically reflect the variations in operators' skill, and different road condition.

Current model requires a pre-defined production schedule, before optimising the waste rock placement, which is a post process to mine production schedule. Future study could consider optimising both mining and dumping in one model, in order to produce a well-integrated optimum schedule.

Lastly, a user-friendly interface need to be developed, then, a client can use this stand-alone software without the restriction of a specific mine scheduling package.

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APPENDIX A

OP MODEL WITH LIFT-BY-LIFT DUMP CONSTRUCTION SEQUENCE IN AMPL CODE

set BT dimen 3;
set tperiod = setof {(t,p,b) in BT} t;
set pit{t in tperiod}= setof {(t,p,b) in BT} p;
set block{t in tperiod, p in pit[t]}= setof {(t,p,b) in BT} b;
set PET dimen 3;
set pe {t in tperiod, p in pit[t]}=setof {(t,p,e1) in PET} e1;
set mDump dimen 4;
set fDump dimen 4;
set SP dimen 4;
set rDump within mDump;
set DUMP = mDump union fDump union SP;
set nDump= setof {(n,k,i,j) in DUMP} n;
set kDump{n in nDump}= setof {(n,k,i,j) in DUMP} k;
set iDump{n in nDump}= setof {(n,k,i,j) in DUMP} i;
set jDump{n in nDump}= setof {(n,k,i,j) in DUMP} j;
set DET dimen 3;
set de {t in tperiod, n in nDump}=setof {(t,n,e2) in DET} e2;
set mbDump:= mDump diff rDump;
set mfDump:= mDump union fDump;
set nofDump:= DUMP diff fDump;
set norDump:= DUMP diff rDump;
set rcDump dimen 4;
	Set declaration

param bcx{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcy{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcz{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bxsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bysize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bzsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param BV{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param Grade{t in tperiod, p in pit[t], b in block[t,p]};
param Acid{t in tperiod, p in pit[t], b in block[t,p]};
param GradeLB>0;
param AcidUB>0;
param S >=0;
param Dc{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcxsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcysize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dczsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param EDb2pe {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p]}>=0;
param EDpe2de {t in tperiod, p in pit[t], e1 in pe[t,p], n in nDump, e2 in de[t,n]}>=0;
param EDde2d {t in tperiod, n in nDump, e2 in de[t,n], k in
	Parameter declaration

kDump[n], i in iDump[n], j in jDump[n]}>=0;
param EDsp2d {t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}>=0;
param dl{n in nDump}>=0;
param dw{n in nDump}>=0;
param dh{n in nDump}>=0;
param Discount>=0;	

var V2d{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]} >=0, <=BV[t,p,b];
 	
var X2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump}>=0,<=1;
 	
var Vsp2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} >=0;
 	
var Bd{t in tperiod, n in nDump, k in kDump[n]: n<>'inert_rock_stockpile'} binary;
	Variable declaration

minimize Overall: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP} (EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])/((1+Discount)^t)*V2d[t,p,b,e1,n,e2,k,i,j] + sum {t in tperiod, n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} EDsp2d[t,n,k,i,j]/((1+Discount)^t)*Vsp2d[t,n,k,i,j];	Objective function

s.t. A{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]<GradeLB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in nofDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A1{t in tperiod, p in pit[t], b in block[t,p]: Acid[t,p,b]>=AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in rDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A2{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]>=GradeLB and Acid[t,p,b]<AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in Dump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
	Mining schedule and material segregation constraint sets

s.t. B0 {n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in rcDump}: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n]}S*V2d[t,p,b,e1,n,e2,k,i,j] +sum{t in tperiod}S*Vsp2d[t,n,k,i,j] = Dc[n,k,i,j]; s.t. B11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j] +sum{tt in tperiod: t-tt>=0}S*Vsp2d[tt,n,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j]; s.t. B13{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump }: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0 and Grade[tt,p,b]>=GradeLB and Acid[tt,p,b]<AcidUB}S*V2d[tt,p,b,e1,n,e2,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j];	Dump block capacity constraint sets

s.t. B2{t in tperiod}: 0<=sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], n in nDump, e2 in de[tt,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and t-tt>=0 and Acid[tt,p,b]<AcidUB and Grade[tt,p,b]<GradeLB}S*V2d[tt,p,b,e1,n,e2,k,i,j] - sum{ttt in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t-ttt>=-1 and (n,k,i,j) in mDump}S*Vsp2d[ttt,n,k,i,j] <= sum{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP}Dc[n,k,i,j]; s.t. B4: sum{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and Acid[t,p,b]<AcidUB and Grade[t,p,b]<GradeLB}V2d[t,p,b,e1,n,e2,k,i,j] - sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] >=0; s.t. B5: sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t=1 and (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] = 0;	Stockpile and re-handle material flow constraint sets

s.t. C10{t in tperiod, n in nDump, k in kDump[n]:n='main_dump'}:
sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump and t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j]
+sum{tt in tperiod, i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump and t-tt>=0 }S*Vsp2d[tt,n,k,i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k,i,j) in mDump}Dc[n,k,i,j]*Bd[t,n,k]>=0;
 	
s.t. C20{t in tperiod, n in nDump, k in kDump[n]: k+dh[n] in kDump[n] and n='main_dump'}:
sum{p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n], i in iDump[n], j in jDump[n]: (n,k+dh[n],i,j) in mDump}S*V2d[t,p,b,e1,n,e2,k+dh[n],i,j]
+sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in mDump}S*Vsp2d[t,n,k+dh[n],i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in mDump}Dc[n,k+dh[n],i,j]*Bd[t,n,k]<=0;
 	
s.t. C11{t in tperiod, n in nDump, k in kDump[n]:n='marginal_grade_stockpile'}:
sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump and t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k,i,j) in
	Lift-by-lift construction sequence constraint sets

fDump}Dc[n,k,i,j]*Bd[t,n,k]>=0;
 	
s.t. C21{t in tperiod, n in nDump, k in kDump[n]: k+dh[n] in kDump[n] and n='marginal_grade_stockpile'}:
 	
sum{p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n], i in iDump[n], j in jDump[n]: (n,k+dh[n],i,j) in fDump}S*V2d[t,p,b,e1,n,e2,k+dh[n],i,j]
 	
-
 	
sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in fDump}Dc[n,k+dh[n],i,j]*Bd[t,n,k]<=0;	

OP MODEL WITH MULTI-LIFT DUMP CONSTRUCTION SEQUENCE IN AMPL CODE

set BT dimen 3;
set tperiod = setof {(t,p,b) in BT} t;
set pit{t in tperiod}= setof {(t,p,b) in BT} p;
set block{t in tperiod, p in pit[t]}= setof {(t,p,b) in BT} b;
set PET dimen 3;
set pe {t in tperiod, p in pit[t]}=setof {(t,p,e1) in PET} e1;
set mDump dimen 4;
set fDump dimen 4;
set SP dimen 4;
set rDump within mDump;
set DUMP = mDump union fDump union SP;
set nDump= setof {(n,k,i,j) in DUMP} n;
set kDump{n in nDump}= setof {(n,k,i,j) in DUMP} k;
set iDump{n in nDump}= setof {(n,k,i,j) in DUMP} i;
set jDump{n in nDump}= setof {(n,k,i,j) in DUMP} j;
set DET dimen 3;
set de {t in tperiod, n in nDump}=setof {(t,n,e2) in DET} e2;
set mbDump:= mDump diff rDump;
set mfDump:= mDump union fDump;
set nofDump:= DUMP diff fDump;
set norDump:= DUMP diff rDump;
set rcDump dimen 4;
	Set declaration

param bcx{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcy{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcz{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bxsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bysize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bzsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param BV{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param Grade{t in tperiod, p in pit[t], b in block[t,p]};
param Acid{t in tperiod, p in pit[t], b in block[t,p]};
param GradeLB>0;
param AcidUB>0;
param S >=0;
param Dc{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcxsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcysize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dczsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param EDb2pe {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p]}>=0;
param EDpe2de {t in tperiod, p in pit[t], e1 in pe[t,p], n in nDump, e2 in de[t,n]}>=0;
param EDde2d {t in tperiod, n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]}>=0;
param EDsp2d {t in tperiod, n in nDump, k in kDump[n], i in
	Parameter declaration

iDump[n], j in jDump[n]: (n,k,i,j) in mDump}>=0;
 	
param dl{n in nDump}>=0;
 	
param dw{n in nDump}>=0;
 	
param dh{n in nDump}>=0;
 	
param Discount>=0;	

var V2d{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]} >=0, <=BV[t,p,b];
 	
var X2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump}>=0,<=1;
 	
var Vsp2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} >=0;
 	
var Bd{t in tperiod, n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump} binary;
 	
var Cd{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump
 	
and (i-dl[n]) in iDump[n] and (i+dl[n]) in iDump[n] and (j-dl[n]) in jDump[n] and (j+dl[n]) in jDump[n]};
	Variable declaration

minimize Overall: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP} (EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])/((1+Discount)^t)*V2d[t,p,b,e1,n,e2,k,i,j] + sum {t in tperiod, n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} EDsp2d[t,n,k,i,j]/((1+Discount)^t)*Vsp2d[t,n,k,i,j] ;	Objective function

s.t. A{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]<GradeLB}:
 	
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in nofDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A1{t in tperiod, p in pit[t], b in block[t,p]: Acid[t,p,b]>=AcidUB}:
 	
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in rDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A2{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]>=GradeLB and Acid[t,p,b]<AcidUB}:
 	
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in fDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
	Mining schedule and material segregation constraint sets

s.t. B0 {n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in rcDump}: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n]}S*V2d[t,p,b,e1,n,e2,k,i,j] +sum{t in tperiod}S*Vsp2d[t,n,k,i,j] = Dc[n,k,i,j]; s.t. B11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j] +sum{tt in tperiod: t-tt>=0}S*Vsp2d[tt,n,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j]; s.t. B13{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump }: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0 and Grade[tt,p,b]>=GradeLB and Acid[tt,p,b]<AcidUB}S*V2d[tt,p,b,e1,n,e2,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j];	Dump block capacity constraint sets

s.t. B2{t in tperiod}: 0<=sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], n in nDump, e2 in de[tt,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and t-tt>=0 and Acid[tt,p,b]<AcidUB and Grade[tt,p,b]<GradeLB}S*V2d[tt,p,b,e1,n,e2,k,i,j] - sum{ttt in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t-ttt>=-1 and (n,k,i,j) in mDump}S*Vsp2d[ttt,n,k,i,j] <= sum{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP}Dc[n,k,i,j]; s.t. B4: sum{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and Acid[t,p,b]<AcidUB and Grade[t,p,b]<GradeLB}V2d[t,p,b,e1,n,e2,k,i,j] - sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] >=0; s.t. B5: sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t=1 and (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] = 0;	Stockpile and re-handle material flow constraint sets

s.t. C10{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k-dh[n],i-dl[n],j-dl[n]) in mDump and (n,k-dh[n],i-dl[n],j) in mDump and (n,k-dh[n],i-dl[n],j+dl[n]) in mDump and (n,k-dh[n],i,j-dl[n]) in mDump and (n,k-dh[n],i,j) in mDump and (n,k-dh[n],i,j+dl[n]) in mDump and (n,k-dh[n],i+dl[n],j-dl[n]) in mDump and (n,k-dh[n],i+dl[n],j) in mDump and (n,k-dh[n],i+dl[n],j+dl[n]) in mDump }: Cd[t,n,k-dh[n],i,j]- 9*Bd[t,n,k,i,j]>=0; #C20= dumping sequence control2 s.t. C20{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: Bd[t,n,k,i,j] - X2d[t,n,k,i,j]>=0; #C11=dumping sequence control1 s.t. C11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k-dh[n],i-dl[n],j-dl[n]) in fDump and (n,k-dh[n],i-dl[n],j) in fDump and (n,k-dh[n],i-dl[n],j+dl[n]) in fDump and (n,k-dh[n],i,j-dl[n]) in fDump	Multi-lift construction sequence constraint sets

and (n,k-dh[n],i,j) in fDump and (n,k-dh[n],i,j+dl[n]) in fDump and (n,k-dh[n],i+dl[n],j-dl[n]) in fDump and (n,k-dh[n],i+dl[n],j) in fDump and (n,k-dh[n],i+dl[n],j+dl[n]) in fDump }: Cd[t,n,k-dh[n],i,j]- 9*Bd[t,n,k,i,j]>=0; #C21= dumping sequence control2 s.t. C21{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump}: Bd[t,n,k,i,j] - X2d[t,n,k,i,j]>=0; s.t. D10{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i-dl[n],j-dl[n]) in mDump and (n,k,i-dl[n],j) in mDump and (n,k,i-dl[n],j+dl[n]) in mDump and (n,k,i,j-dl[n]) in mDump and (n,k,i,j) in mDump and (n,k,i,j+dl[n]) in mDump and (n,k,i+dl[n],j-dl[n]) in mDump and (n,k,i+dl[n],j) in mDump and (n,k,i+dl[n],j+dl[n]) in mDump }: Cd[t,n,k,i,j]=sum{ii in iDump[n], jj in jDump[n]: abs(ii-i)<=dl[n]	

and abs(jj-j)<=dl[n] and (n,k,ii,jj) in mDump}X2d[t,n,k,ii,jj]; s.t. D11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i-dl[n],j-dl[n]) in fDump and (n,k,i-dl[n],j) in fDump and (n,k,i-dl[n],j+dl[n]) in fDump and (n,k,i,j-dl[n]) in fDump and (n,k,i,j) in fDump and (n,k,i,j+dl[n]) in fDump and (n,k,i+dl[n],j-dl[n]) in fDump and (n,k,i+dl[n],j) in fDump and (n,k,i+dl[n],j+dl[n]) in fDump }: Cd[t,n,k,i,j]=sum{ii in iDump[n], jj in jDump[n]: abs(ii-i)<=dl[n] and abs(jj-j)<=dl[n] and (n,k,ii,jj) in fDump}X2d[t,n,k,ii,jj];	

APPENDIX B

TB MODEL WITH LIFT-BY-LIFT DUMP CONSTRUCTION SEQUENCE IN AMPL CODE

set BT dimen 3;
set tperiod = setof {(t,p,b) in BT} t;
set pit{t in tperiod}= setof {(t,p,b) in BT} p;
set block{t in tperiod, p in pit[t]}= setof {(t,p,b) in BT} b;
set PET dimen 3;
set pe {t in tperiod, p in pit[t]}=setof {(t,p,e1) in PET} e1;
set mDump dimen 4;
set fDump dimen 4;
set SP dimen 4;
set rDump within mDump;
set DUMP = mDump union fDump union SP;
set nDump= setof {(n,k,i,j) in DUMP} n;
set kDump{n in nDump}= setof {(n,k,i,j) in DUMP} k;
set iDump{n in nDump}= setof {(n,k,i,j) in DUMP} i;
set jDump{n in nDump}= setof {(n,k,i,j) in DUMP} j;
set DET dimen 3;
set de {t in tperiod, n in nDump}=setof {(t,n,e2) in DET} e2;
set mbDump:= mDump diff rDump;
set mfDump:= mDump union fDump;
set nofDump:= DUMP diff fDump;
set norDump:= DUMP diff rDump;
set rcDump dimen 4;
	Set declaration

param bcx{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcy{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcz{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bxsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bysize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bzsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param BV{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param Grade{t in tperiod, p in pit[t], b in block[t,p]};
param Acid{t in tperiod, p in pit[t], b in block[t,p]};
param GradeLB>0;
param AcidUB>0;
param S >=0;
param Dc{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcxsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcysize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dczsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param EDb2pe {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p]}>=0;
param EDpe2de {t in tperiod, p in pit[t], e1 in pe[t,p], n in nDump, e2 in de[t,n]}>=0;
param EDde2d {t in tperiod, n in nDump, e2 in de[t,n], k in
	Parameter declaration

kDump[n], i in iDump[n], j in jDump[n]}>=0;
 	
param EDsp2d {t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}>=0;
 	
param dl{n in nDump}>=0;
 	
param dw{n in nDump}>=0;
 	
param dh{n in nDump}>=0;
 	
param Discount>=0;
 	
param Tc>=0;
 	
param Tn{t in tperiod}>=0, integer;	

var U{t in tperiod}>=0;
 	
var O{t in tperiod}>=0;
 	
var V2d{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]} >=0, <=BV[t,p,b];
 	
#according to mining schedule (t), vary waste block volume transported from pit exit to Dc location
 	
var X2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump}>=0,<=1;
 	
var Vsp2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} >=0;
 	
var Bd{t in tperiod, n in nDump, k in kDump[n]: n<>'inert_rock_stockpile'} binary;
	Variable declaration

minimize Truck_devi: sum {t in tperiod} U[t]/((1+Discount)^t) + sum {t in tperiod} O[t]/((1+Discount)^t) ;	Objective function

s.t. D1{t in tperiod}: Tc*Tn[t]+U[t]-O[t] = sum {p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP} (EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])*V2d[t,p,b,e1,n,e2,k,i,j] + sum {n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in mDump} (EDsp2d[t,n,k,i,j]*Vsp2d[t,n,k,i,j]) ;	TB model specific constraint

s.t. A{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]<GradeLB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in nofDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A1{t in tperiod, p in pit[t], b in block[t,p]: Acid[t,p,b]>=AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in rDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A2{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]>=GradeLB and Acid[t,p,b]<AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in Dump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
	Mining schedule and material segregation constraint sets

s.t. B0 {n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in rcDump}: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n]}S*V2d[t,p,b,e1,n,e2,k,i,j] +sum{t in tperiod}S*Vsp2d[t,n,k,i,j] = Dc[n,k,i,j]; s.t. B11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j] +sum{tt in tperiod: t-tt>=0}S*Vsp2d[tt,n,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j]; s.t. B13{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump }: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0 and Grade[tt,p,b]>=GradeLB and Acid[tt,p,b]<AcidUB}S*V2d[tt,p,b,e1,n,e2,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j];	Dump block capacity constraint sets

s.t. B2{t in tperiod}: 0<=sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], n in nDump, e2 in de[tt,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and t-tt>=0 and Acid[tt,p,b]<AcidUB and Grade[tt,p,b]<GradeLB}S*V2d[tt,p,b,e1,n,e2,k,i,j] - sum{ttt in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t-ttt>=-1 and (n,k,i,j) in mDump}S*Vsp2d[ttt,n,k,i,j] <= sum{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP}Dc[n,k,i,j]; s.t. B4: sum{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and Acid[t,p,b]<AcidUB and Grade[t,p,b]<GradeLB}V2d[t,p,b,e1,n,e2,k,i,j] - sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] >=0; s.t. B5: sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t=1 and (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] = 0;	Stockpile and re-handle material flow constraint sets

s.t. C10{t in tperiod, n in nDump, k in kDump[n]:n='main_dump'}:
sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump and t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j]
+sum{tt in tperiod, i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump and t-tt>=0 }S*Vsp2d[tt,n,k,i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k,i,j) in mDump}Dc[n,k,i,j]*Bd[t,n,k]>=0;
 	
s.t. C20{t in tperiod, n in nDump, k in kDump[n]: k+dh[n] in kDump[n] and n='main_dump'}:
sum{p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n], i in iDump[n], j in jDump[n]: (n,k+dh[n],i,j) in mDump}S*V2d[t,p,b,e1,n,e2,k+dh[n],i,j]
+sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in mDump}S*Vsp2d[t,n,k+dh[n],i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in mDump}Dc[n,k+dh[n],i,j]*Bd[t,n,k]<=0;
 	
s.t. C11{t in tperiod, n in nDump, k in kDump[n]:n='marginal_grade_stockpile'}:
sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump and t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k,i,j) in
	Lift-by-lift construction sequence constraint sets

fDump}Dc[n,k,i,j]*Bd[t,n,k]>=0;
 	
s.t. C21{t in tperiod, n in nDump, k in kDump[n]: k+dh[n] in kDump[n] and n='marginal_grade_stockpile'}:
 	
sum{p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n], i in iDump[n], j in jDump[n]: (n,k+dh[n],i,j) in fDump}S*V2d[t,p,b,e1,n,e2,k+dh[n],i,j]
 	
-
 	
sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in fDump}Dc[n,k+dh[n],i,j]*Bd[t,n,k]<=0;	

TB MODEL WITH MULTI-LIFT DUMP CONSTRUCTION SEQUENCE IN AMPL CODE

set BT dimen 3;
set tperiod = setof {(t,p,b) in BT} t;
set pit{t in tperiod}= setof {(t,p,b) in BT} p;
set block{t in tperiod, p in pit[t]}= setof {(t,p,b) in BT} b;
set PET dimen 3;
set pe {t in tperiod, p in pit[t]}=setof {(t,p,e1) in PET} e1;
set mDump dimen 4;
set fDump dimen 4;
set SP dimen 4;
set rDump within mDump;
set DUMP = mDump union fDump union SP;
set nDump= setof {(n,k,i,j) in DUMP} n;
set kDump{n in nDump}= setof {(n,k,i,j) in DUMP} k;
set iDump{n in nDump}= setof {(n,k,i,j) in DUMP} i;
set jDump{n in nDump}= setof {(n,k,i,j) in DUMP} j;
set DET dimen 3;
set de {t in tperiod, n in nDump}=setof {(t,n,e2) in DET} e2;
set mbDump:= mDump diff rDump;
set mfDump:= mDump union fDump;
set nofDump:= DUMP diff fDump;
set norDump:= DUMP diff rDump;
set rcDump dimen 4;
	Set declaration

param bcx{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcy{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcz{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bxsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bysize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bzsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param BV{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param Grade{t in tperiod, p in pit[t], b in block[t,p]};
param Acid{t in tperiod, p in pit[t], b in block[t,p]};
param GradeLB>0;
param AcidUB>0;
param S >=0;
param Dc{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcxsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcysize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dczsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param EDb2pe {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p]}>=0;
param EDpe2de {t in tperiod, p in pit[t], e1 in pe[t,p], n in nDump, e2 in de[t,n]}>=0;
param EDde2d {t in tperiod, n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]}>=0;
param EDsp2d {t in tperiod, n in nDump, k in kDump[n], i in
	Parameter declaration

iDump[n], j in jDump[n]: (n,k,i,j) in mDump}>=0;
 	
param dl{n in nDump}>=0;
 	
param dw{n in nDump}>=0;
 	
param dh{n in nDump}>=0;
 	
param Discount>=0;	

var U{t in tperiod}>=0;
 	
var O{t in tperiod}>=0;
 	
var V2d{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]} >=0, <=BV[t,p,b];
 	
var X2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump}>=0,<=1;
 	
var Vsp2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} >=0;
 	
var Bd{t in tperiod, n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump} binary;
 	
var Cd{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump and (i-dl[n]) in iDump[n] and (i+dl[n]) in iDump[n] and (j-dl[n]) in jDump[n] and (j+dl[n]) in jDump[n]};
	Variable declaration

minimize Truck_Devi: sum {t in tperiod}U[t]/((1+Discount)^t) + sum {t in tperiod}O[t]/((1+Discount)^t) ;	Objective function

s.t. D1{t in tperiod}: Tc*Tn[t]+U[t]-O[t] = sum {p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP} (EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])*V2d[t,p,b,e1,n,e2,k,i,j] + sum {n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} (EDsp2d[t,n,k,i,j]*Vsp2d[t,n,k,i,j]) ;	TB model specific constraint

s.t. A{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]<GradeLB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in nofDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A1{t in tperiod, p in pit[t], b in block[t,p]: Acid[t,p,b]>=AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in rDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A2{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]>=GradeLB and Acid[t,p,b]<AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in fDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
	Mining schedule and material segregation constraint sets

s.t. B0 {n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in rcDump}: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n]}S*V2d[t,p,b,e1,n,e2,k,i,j] +sum{t in tperiod}S*Vsp2d[t,n,k,i,j] = Dc[n,k,i,j]; s.t. B11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j] +sum{tt in tperiod: t-tt>=0}S*Vsp2d[tt,n,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j]; s.t. B13{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump }: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0 and Grade[tt,p,b]>=GradeLB and Acid[tt,p,b]<AcidUB}S*V2d[tt,p,b,e1,n,e2,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j];	Dump block capacity constraint sets

s.t. B2{t in tperiod}: 0<=sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], n in nDump, e2 in de[tt,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and t-tt>=0 and Acid[tt,p,b]<AcidUB and Grade[tt,p,b]<GradeLB}S*V2d[tt,p,b,e1,n,e2,k,i,j] - sum{ttt in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t-ttt>=-1 and (n,k,i,j) in mDump}S*Vsp2d[ttt,n,k,i,j] <= sum{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP}Dc[n,k,i,j]; s.t. B4: sum{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and Acid[t,p,b]<AcidUB and Grade[t,p,b]<GradeLB}V2d[t,p,b,e1,n,e2,k,i,j] - sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] >=0; s.t. B5: sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t=1 and (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] = 0;	Stockpile and re-handle material flow constraint sets

s.t. C10{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k-dh[n],i-dl[n],j-dl[n]) in mDump and (n,k-dh[n],i-dl[n],j) in mDump and (n,k-dh[n],i-dl[n],j+dl[n]) in mDump and (n,k-dh[n],i,j-dl[n]) in mDump and (n,k-dh[n],i,j) in mDump and (n,k-dh[n],i,j+dl[n]) in mDump and (n,k-dh[n],i+dl[n],j-dl[n]) in mDump and (n,k-dh[n],i+dl[n],j) in mDump and (n,k-dh[n],i+dl[n],j+dl[n]) in mDump }: Cd[t,n,k-dh[n],i,j]- 9*Bd[t,n,k,i,j]>=0; #C20= dumping sequence control2 s.t. C20{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: Bd[t,n,k,i,j] - X2d[t,n,k,i,j]>=0; #C11=dumping sequence control1 s.t. C11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k-dh[n],i-dl[n],j-dl[n]) in fDump and (n,k-dh[n],i-dl[n],j) in fDump and (n,k-dh[n],i-dl[n],j+dl[n]) in fDump and (n,k-dh[n],i,j-dl[n]) in fDump	Multi-lift construction sequence constraint sets

and (n,k-dh[n],i,j) in fDump and (n,k-dh[n],i,j+dl[n]) in fDump and (n,k-dh[n],i+dl[n],j-dl[n]) in fDump and (n,k-dh[n],i+dl[n],j) in fDump and (n,k-dh[n],i+dl[n],j+dl[n]) in fDump }: Cd[t,n,k-dh[n],i,j]- 9*Bd[t,n,k,i,j]>=0; #C21= dumping sequence control2 s.t. C21{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump}: Bd[t,n,k,i,j] - X2d[t,n,k,i,j]>=0; s.t. D10{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i-dl[n],j-dl[n]) in mDump and (n,k,i-dl[n],j) in mDump and (n,k,i-dl[n],j+dl[n]) in mDump and (n,k,i,j-dl[n]) in mDump and (n,k,i,j) in mDump and (n,k,i,j+dl[n]) in mDump and (n,k,i+dl[n],j-dl[n]) in mDump and (n,k,i+dl[n],j) in mDump and (n,k,i+dl[n],j+dl[n]) in mDump }: Cd[t,n,k,i,j]=sum{ii in iDump[n], jj in jDump[n]: abs(ii-i)<=dl[n]	

and abs(jj-j)<=dl[n] and (n,k,ii,jj) in mDump}X2d[t,n,k,ii,jj]; s.t. D11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i-dl[n],j-dl[n]) in fDump and (n,k,i-dl[n],j) in fDump and (n,k,i-dl[n],j+dl[n]) in fDump and (n,k,i,j-dl[n]) in fDump and (n,k,i,j) in fDump and (n,k,i,j+dl[n]) in fDump and (n,k,i+dl[n],j-dl[n]) in fDump and (n,k,i+dl[n],j) in fDump and (n,k,i+dl[n],j+dl[n]) in fDump }: Cd[t,n,k,i,j]=sum{ii in iDump[n], jj in jDump[n]: abs(ii-i)<=dl[n] and abs(jj-j)<=dl[n] and (n,k,ii,jj) in fDump}X2d[t,n,k,ii,jj];	

APPENDIX C

COMBO MODEL WITH LIFT-BY-LIFT DUMP CONSTRUCTION SEQUENCE IN AMPL CODE

set BT dimen 3;
set tperiod = setof {(t,p,b) in BT} t;
set pit{t in tperiod}= setof {(t,p,b) in BT} p;
set block{t in tperiod, p in pit[t]}= setof {(t,p,b) in BT} b;
set PET dimen 3;
set pe {t in tperiod, p in pit[t]}=setof {(t,p,e1) in PET} e1;
set mDump dimen 4;
set fDump dimen 4;
set SP dimen 4;
set rDump within mDump;
set DUMP = mDump union fDump union SP;
set nDump= setof {(n,k,i,j) in DUMP} n;
set kDump{n in nDump}= setof {(n,k,i,j) in DUMP} k;
set iDump{n in nDump}= setof {(n,k,i,j) in DUMP} i;
set jDump{n in nDump}= setof {(n,k,i,j) in DUMP} j;
set DET dimen 3;
set de {t in tperiod, n in nDump}=setof {(t,n,e2) in DET} e2;
set mbDump:= mDump diff rDump;
set mfDump:= mDump union fDump;
set nofDump:= DUMP diff fDump;
set norDump:= DUMP diff rDump;
set rcDump dimen 4;
	Set declaration

param bcx{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcy{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcz{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bxsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bysize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bzsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param BV{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param Grade{t in tperiod, p in pit[t], b in block[t,p]};
param Acid{t in tperiod, p in pit[t], b in block[t,p]};
param GradeLB>0;
param AcidUB>0;
param S >=0;
param Dc{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcxsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcysize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dczsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param EDb2pe {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p]}>=0;
param EDpe2de {t in tperiod, p in pit[t], e1 in pe[t,p], n in nDump, e2 in de[t,n]}>=0;
param EDde2d {t in tperiod, n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]}>=0;
	Parameter declaration

param EDsp2d {t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}>=0;
 	
param dl{n in nDump}>=0;
 	
param dw{n in nDump}>=0;
 	
param dh{n in nDump}>=0;
 	
param Discount>=0;
 	
param Tc>=0;
 	
param Tn{t in tperiod}>=0, integer;	

var U{t in tperiod}>=0;
 	
var O{t in tperiod}>=0;
 	
var V2d{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]} >=0, <=BV[t,p,b];
 	
var X2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump}>=0,<=1;
 	
var Vsp2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} >=0;
 	
var Bd{t in tperiod, n in nDump, k in kDump[n]: n<>'inert_rock_stockpile'} binary;
	Variable declaration

minimize Truck_Under: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP} (EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])/((1+Discount)^t)*V2d[t,p,b,e1,n,e2,k,i,j] + sum {t in tperiod, n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} EDsp2d[t,n,k,i,j]/((1+Discount)^t)*Vsp2d[t,n,k,i,j] + sum {t in tperiod} U[t]/((1+Discount)^t) + sum {t in tperiod} O[t]/((1+Discount)^t) ;	Objective function

s.t. D1{t in tperiod}: Tc*Tn[t]+U[t]-O[t] >= sum {p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP}(EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])*V2d[t,p,b,e1,n,e2,k,i,j] + sum {n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}(EDsp2d[t,n,k,i,j]*Vsp2d[t,n,k,i,j]);	TB model specific constraint

s.t. A{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]<GradeLB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in nofDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A1{t in tperiod, p in pit[t], b in block[t,p]: Acid[t,p,b]>=AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in rDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A2{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]>=GradeLB and Acid[t,p,b]<AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in Dump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
	Mining schedule and material segregation constraint sets

s.t. B0 {n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in rcDump}: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n]}S*V2d[t,p,b,e1,n,e2,k,i,j] +sum{t in tperiod}S*Vsp2d[t,n,k,i,j] = Dc[n,k,i,j]; s.t. B11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j] +sum{tt in tperiod: t-tt>=0}S*Vsp2d[tt,n,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j]; s.t. B13{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump }: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0 and Grade[tt,p,b]>=GradeLB and Acid[tt,p,b]<AcidUB}S*V2d[tt,p,b,e1,n,e2,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j];	Dump block capacity constraint sets

s.t. B2{t in tperiod}: 0<=sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], n in nDump, e2 in de[tt,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and t-tt>=0 and Acid[tt,p,b]<AcidUB and Grade[tt,p,b]<GradeLB}S*V2d[tt,p,b,e1,n,e2,k,i,j] - sum{ttt in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t-ttt>=-1 and (n,k,i,j) in mDump}S*Vsp2d[ttt,n,k,i,j] <= sum{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP}Dc[n,k,i,j]; s.t. B4: sum{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and Acid[t,p,b]<AcidUB and Grade[t,p,b]<GradeLB}V2d[t,p,b,e1,n,e2,k,i,j] - sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] >=0; s.t. B5: sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t=1 and (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] = 0;	Stockpile and re-handle material flow constraint sets

s.t. C10{t in tperiod, n in nDump, k in kDump[n]:n='main_dump'}:
sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump and t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j]
+sum{tt in tperiod, i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump and t-tt>=0 }S*Vsp2d[tt,n,k,i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k,i,j) in mDump}Dc[n,k,i,j]*Bd[t,n,k]>=0;
 	
s.t. C20{t in tperiod, n in nDump, k in kDump[n]: k+dh[n] in kDump[n] and n='main_dump'}:
sum{p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n], i in iDump[n], j in jDump[n]: (n,k+dh[n],i,j) in mDump}S*V2d[t,p,b,e1,n,e2,k+dh[n],i,j]
+sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in mDump}S*Vsp2d[t,n,k+dh[n],i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in mDump}Dc[n,k+dh[n],i,j]*Bd[t,n,k]<=0;
 	
s.t. C11{t in tperiod, n in nDump, k in kDump[n]:n='marginal_grade_stockpile'}:
sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump and t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j]
-
sum{i in iDump[n], j in jDump[n]:(n,k,i,j) in
	Lift-by-lift construction sequence constraint sets

fDump}Dc[n,k,i,j]*Bd[t,n,k]>=0;
 	
s.t. C21{t in tperiod, n in nDump, k in kDump[n]: k+dh[n] in kDump[n] and n='marginal_grade_stockpile'}:
 	
sum{p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n], i in iDump[n], j in jDump[n]: (n,k+dh[n],i,j) in fDump}S*V2d[t,p,b,e1,n,e2,k+dh[n],i,j]
 	
-
 	
sum{i in iDump[n], j in jDump[n]:(n,k+dh[n],i,j) in fDump}Dc[n,k+dh[n],i,j]*Bd[t,n,k]<=0;	

COMBO MODEL WITH MULTI-LIFT DUMP CONSTRUCTION SEQUENCE IN AMPL CODE

set BT dimen 3;
set tperiod = setof {(t,p,b) in BT} t;
set pit{t in tperiod}= setof {(t,p,b) in BT} p;
set block{t in tperiod, p in pit[t]}= setof {(t,p,b) in BT} b;
set PET dimen 3;
set pe {t in tperiod, p in pit[t]}=setof {(t,p,e1) in PET} e1;
set mDump dimen 4;
set fDump dimen 4;
set SP dimen 4;
set rDump within mDump;
set DUMP = mDump union fDump union SP;
set nDump= setof {(n,k,i,j) in DUMP} n;
set kDump{n in nDump}= setof {(n,k,i,j) in DUMP} k;
set iDump{n in nDump}= setof {(n,k,i,j) in DUMP} i;
set jDump{n in nDump}= setof {(n,k,i,j) in DUMP} j;
set DET dimen 3;
set de {t in tperiod, n in nDump}=setof {(t,n,e2) in DET} e2;
set mbDump:= mDump diff rDump;
set mfDump:= mDump union fDump;
set nofDump:= DUMP diff fDump;
set norDump:= DUMP diff rDump;
set rcDump dimen 4;
	Set declaration

param bcx{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcy{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bcz{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bxsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bysize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param bzsize{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param BV{t in tperiod, p in pit[t], b in block[t,p]}>=0;
param Grade{t in tperiod, p in pit[t], b in block[t,p]};
param Acid{t in tperiod, p in pit[t], b in block[t,p]};
param GradeLB>0;
param AcidUB>0;
param S >=0;
param Dc{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcxsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dcysize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param Dczsize{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in DUMP} default 0 ;
param EDb2pe {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p]}>=0;
param EDpe2de {t in tperiod, p in pit[t], e1 in pe[t,p], n in nDump, e2 in de[t,n]}>=0;
param EDde2d {t in tperiod, n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]}>=0;
param EDsp2d {t in tperiod, n in nDump, k in kDump[n], i in
	Parameter declaration

iDump[n], j in jDump[n]: (n,k,i,j) in mDump}>=0;
 	
param dl{n in nDump}>=0;
 	
param dw{n in nDump}>=0;
 	
param dh{n in nDump}>=0;
 	
param Discount>=0;	

var U{t in tperiod}>=0;
 	
var O{t in tperiod}>=0;
 	
var V2d{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]} >=0, <=BV[t,p,b];
 	
var X2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump}>=0,<=1;
 	
var Vsp2d{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} >=0;
 	
var Bd{t in tperiod, n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump} binary;
 	
var Cd{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mfDump and (i-dl[n]) in iDump[n] and (i+dl[n]) in iDump[n] and (j-dl[n]) in jDump[n] and (j+dl[n]) in jDump[n]};
	Variable declaration

minimize Truck_Under: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP} (EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])/((1+Discount)^t)*V2d[t,p,b,e1,n,e2,k,i,j] + sum {t in tperiod, n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} EDsp2d[t,n,k,i,j]/((1+Discount)^t)*Vsp2d[t,n,k,i,j] + sum {t in tperiod} U[t]/((1+Discount)^t) + sum {t in tperiod} O[t]/((1+Discount)^t) ;	Objective function

s.t. D1{t in tperiod}: Tc*Tn[t]+U[t]-O[t] >= sum {p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in DUMP} (EDb2pe[t,p,b,e1]+EDpe2de[t,p,e1,n,e2]+EDde2d[t,n,e2,k,i,j])*V2d[t,p,b,e1,n,e2,k,i,j] + sum {n in nDump,k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump} (EDsp2d[t,n,k,i,j]*Vsp2d[t,n,k,i,j]) ;	TB model specific constraint

s.t. A{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]<GradeLB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in nofDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A1{t in tperiod, p in pit[t], b in block[t,p]: Acid[t,p,b]>=AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in rDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
 	
s.t. A2{t in tperiod, p in pit[t], b in block[t,p]: Grade[t,p,b]>=GradeLB and Acid[t,p,b]<AcidUB}:
sum {e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]:(n,k,i,j) in fDump}V2d[t,p,b,e1,n,e2,k,i,j]= BV[t,p,b];
	Mining schedule and material segregation constraint sets

s.t. B0 {n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in rcDump}: sum {t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], e2 in de[t,n]}S*V2d[t,p,b,e1,n,e2,k,i,j] +sum{t in tperiod}S*Vsp2d[t,n,k,i,j] = Dc[n,k,i,j]; s.t. B11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0}S*V2d[tt,p,b,e1,n,e2,k,i,j] +sum{tt in tperiod: t-tt>=0}S*Vsp2d[tt,n,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j]; s.t. B13{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump }: sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], e2 in de[tt,n]: t-tt>=0 and Grade[tt,p,b]>=GradeLB and Acid[tt,p,b]<AcidUB}S*V2d[tt,p,b,e1,n,e2,k,i,j] =X2d[t,n,k,i,j]*Dc[n,k,i,j];	Dump block capacity constraint sets

s.t. B2{t in tperiod}: 0<=sum{tt in tperiod, p in pit[tt], b in block[tt,p], e1 in pe[tt,p], n in nDump, e2 in de[tt,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and t-tt>=0 and Acid[tt,p,b]<AcidUB and Grade[tt,p,b]<GradeLB}S*V2d[tt,p,b,e1,n,e2,k,i,j] - sum{ttt in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t-ttt>=-1 and (n,k,i,j) in mDump}S*Vsp2d[ttt,n,k,i,j] <= sum{n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP}Dc[n,k,i,j]; s.t. B4: sum{t in tperiod, p in pit[t], b in block[t,p], e1 in pe[t,p], n in nDump, e2 in de[t,n], k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in SP and Acid[t,p,b]<AcidUB and Grade[t,p,b]<GradeLB}V2d[t,p,b,e1,n,e2,k,i,j] - sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] >=0; s.t. B5: sum{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: t=1 and (n,k,i,j) in mDump}Vsp2d[t,n,k,i,j] = 0;	Stockpile and re-handle material flow constraint sets

s.t. C10{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k-dh[n],i-dl[n],j-dl[n]) in mDump and (n,k-dh[n],i-dl[n],j) in mDump and (n,k-dh[n],i-dl[n],j+dl[n]) in mDump and (n,k-dh[n],i,j-dl[n]) in mDump and (n,k-dh[n],i,j) in mDump and (n,k-dh[n],i,j+dl[n]) in mDump and (n,k-dh[n],i+dl[n],j-dl[n]) in mDump and (n,k-dh[n],i+dl[n],j) in mDump and (n,k-dh[n],i+dl[n],j+dl[n]) in mDump }: Cd[t,n,k-dh[n],i,j]- 9*Bd[t,n,k,i,j]>=0; #C20= dumping sequence control2 s.t. C20{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in mDump}: Bd[t,n,k,i,j] - X2d[t,n,k,i,j]>=0; #C11=dumping sequence control1 s.t. C11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k-dh[n],i-dl[n],j-dl[n]) in fDump and (n,k-dh[n],i-dl[n],j) in fDump and (n,k-dh[n],i-dl[n],j+dl[n]) in fDump and (n,k-dh[n],i,j-dl[n]) in fDump	Multi-lift construction sequence constraint sets

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and (n,k-dh[n],i,j) in fDump

and (n,k-dh[n],i,j+dl[n]) in fDump

and (n,k-dh[n],i+dl[n],j-dl[n]) in fDump

and (n,k-dh[n],i+dl[n],j) in fDump

and (n,k-dh[n],i+dl[n],j+dl[n]) in fDump

}:

Cd[t,n,k-dh[n],i,j]- 9*Bd[t,n,k,i,j]>=0;


#C21= dumping sequence control2

s.t. C21{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i,j) in fDump}:

Bd[t,n,k,i,j] - X2d[t,n,k,i,j]>=0;


s.t. D10{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]:

    (n,k,i-dl[n],j-dl[n]) in mDump

and (n,k,i-dl[n],j) in mDump

and (n,k,i-dl[n],j+dl[n]) in mDump

and (n,k,i,j-dl[n]) in mDump

and (n,k,i,j) in mDump

and (n,k,i,j+dl[n]) in mDump

and (n,k,i+dl[n],j-dl[n]) in mDump

and (n,k,i+dl[n],j) in mDump

and (n,k,i+dl[n],j+dl[n]) in mDump

}:

Cd[t,n,k,i,j]=sum{ii in iDump[n], jj in jDump[n]: abs(ii-i)<=dl[n]
```

and abs(jj-j)<=dl[n] and (n,k,ii,jj) in mDump}X2d[t,n,k,ii,jj]; s.t. D11{t in tperiod, n in nDump, k in kDump[n], i in iDump[n], j in jDump[n]: (n,k,i-dl[n],j-dl[n]) in fDump and (n,k,i-dl[n],j) in fDump and (n,k,i-dl[n],j+dl[n]) in fDump and (n,k,i,j-dl[n]) in fDump and (n,k,i,j) in fDump and (n,k,i,j+dl[n]) in fDump and (n,k,i+dl[n],j-dl[n]) in fDump and (n,k,i+dl[n],j) in fDump and (n,k,i+dl[n],j+dl[n]) in fDump }: Cd[t,n,k,i,j]=sum{ii in iDump[n], jj in jDump[n]: abs(ii-i)<=dl[n] and abs(jj-j)<=dl[n] and (n,k,ii,jj) in fDump}X2d[t,n,k,ii,jj];	