Science and Mathematics Education Centre

THE IMPACT OF LEARNERS' SPATIAL CAPACITY AND WORLD VIEWS ON THEIR SPATIAL CONCEPTUALISATION: A CASE STUDY

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ABSTRACT

This multi-sited case study aims to explore spatial capacity through pen-and-paper and hands-on activity tests, and explore world view perceptions of space in an attempt to show that spatial conceptualisation is a rich and complex blend of spatial capacity and world view. This study is oriented in a interpretive-naturalistic paradigm and characterised by multi-dimensional quantitative and qualitative methods. The research, set in five secondary schools in the Eastern Cape, was carried out with 32 Grade 11 learners and was designed around seven stages.

This study attempts to understand spatial conceptualisation by recognising that all learners have epistemological macrostructures (world views) that shape their perceptions of the world in general and of space in particular. The main contention of this study is that spatial conceptualisation cannot be understood in isolation, through studying achievements on traditional pen-and-paper tests only. A comprehensive understanding of an individual's spatial conceptualisation involves the recognition of hands-on skills and world views as well.

Spatial capacity, defined here in terms of spatial visualisation and orientation constructs, was explored through a pen-and-paper and a hands-on activity test. The results show only a weak to moderate correlation between the two tests, suggesting that performance in a traditional pen-and-paper test was not necessarily a good predictor for performance in a hands-on activity-based test.

The investigation of world views was underpinned by a logico-structuralist process centred in conversations around nine bi-polar themes. Through a process of content- and meta- analyses involving the participation of a validation team, world-view profiles were established.

In terms of the applied pen-and-paper test which explored spatial capacity, this study confirms males' dominance in all spatial tasks, particularly in three-dimensional problems. This was also found to be true for learners from the participating rural school and for those from disadvantaged socio-economic backgrounds. In the hands-on activity test, however, the study revealed no observable gender difference in favour of the males, except for items that were characterised by the spatial orientation construct.

Both the participating rural and township schools performed poorly in items characterized by the spatial visualisation and orientation construct compared to the other participating schools. Although participants from the rural and township schools found it difficult to articulate their world views in depth, the world-view perspectives of space of this sample reveal rich and complex profiles that are similar across all the schools. Despite leaning towards a Newtonian division of absolute and relative space and containing strong religious elements, this sample generally views space as mysterious, infinite and somewhat obscure. It often refers to space in Kantian ideas and related space in terms of subjective feelings. Females in particular, refer to their own 'space bubble', for example.

Out of the world-view profile analysis, a meta-analysis was conducted which explored thinking skills in terms of capacity to abstract, to be insightful, deal with complex issues, engage critically, and be imaginative. This reveals that for this case, females were rated on a higher level than their male counterparts for their capacity to abstract and be complex (the capacity to identify related parts and to deal with composites), whereas males rated higher for showing insight, being imaginative, and being critical. Although there appear to be high correlations between the various tests, meta-levels and school performance for some of the participants, the same cannot be said for the sample as a whole. The world-view aspect of this study reveals a rich, often complex, understanding of space, strengthening the notion that world views are integral to a learner's cognition process.

This thesis is dedicated to

JEAN,

DOMINIQUE

and

SARAH

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CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION TO THE STUDY

It is a common perception that spatial understanding is fundamental to the understanding and "appreciation of our inherently geometric world" (NCTM, 1989) in general and to mathematical cognition in particular. Battista *et al.* (1982) recognised this by suggesting that spatial thinking is an important aspect of mathematical performance. Much research has focused on the relationship between spatial understanding and mathematics achievement (Bishop, 1980). Educators and policy-makers the world over have intuitively recognised the role of spatial thinking and most mathematics curricula incorporate aspects of developing spatial thinking, usually in the form of formal geometry.

There is, however, little consensus on the concept of space, even while most western mathematics curricula are firmly rooted in a Euclidean paradigm which sees space as a measurable and rational system. Euclidean space is seen as ordered, where shapes are measurable and positioned in a definite manner (Euclid, 1956). There is a sense of geometry – "the branch of Mathematics concerned with the properties and relations of points, lines, surfaces and solids; the relative arrangement of objects or parts" (*Concise Oxford Dictionary*, 1995). Newtonian space is consistent with Euclidean space in terms of its sense of order and tangibility. It affirms the "reality of space" (Gardner, 1999) and maintains that space exists independently of the subject's awareness. Kantian space, on the other hand, suggests that "space (and time) are not features of absolute reality but only forms of sensibility, elements of our subjective cognitive constitution, and that everything that has spatial properties – all the objects of our experience – are mere appearances as opposed to things in themselves" (Gardner, 1999). This implies that space should not be conceptualised in terms of objective features only. In Kant's concept of space, the space is independent of its content. This means that our subjective understandings and experiences form an integral part of ours overall perception of space. Kant suggests that the order we find in nature is the order that exists in our minds, an order which is embedded in or reflects our own structure of mind (Stumpf, 1994; Want and Klimowski, 1996).

There is a tendency in current thinking to embrace a broader view of geometry. The post-1994 curriculum for Mathematics in South Africa for example, sees space and shape within a context of social experiences. One of the specific outcomes of the mathematics curriculum suggests that learners need to be able to "describe and represent experiences with shape, space, time and motion, using all available senses" (South Africa, 1997). This apparent shift is consistent with a global epistemological paradigm move towards recognising that cognition is an active and complex process of social interaction.

The core epistemological principles that underpin this shift can be summarised as follows:

- Knowledge is actively constructed by the learner, not passively received from the environment (Matthews, 1992);
- The function of cognition is adaptive (von Glasersfeld, 1990, cited by Taylor, Fraser, and Fisher, 1993). This process organises our experimental world; it does not discover an independent, pre-existing world outside the mind of the learner (Matthews, 1992);
- The cognition process is not personal and insular, but one that relies on social interaction in general and, according to Vygotskian thought, on language in particular (Vygotsky, 1986).

The notion that cognition is an active process and not a passive intake of information expresses a clear contrast with traditional didactic and expository teaching based on the metaphor of "teaching as transmission of knowledge" (Taylor *et al.*, 1993:3). In his earlier writing, von Glasersfeld (1988), often labelled as a proponent of 'radical constructivism', makes the observation that in traditional theories of knowledge the activity of 'knowing' is taken as a matter of course, "an activity that requires no justification and functions as an initial constituent" (von Glasersfeld, 1988:208). The learner is seen as a passive recipient of knowledge with very little say in his/her cognitive destiny. The "knowing subject" is conceived of as 'pure' entity "unimpeded by biological and psychological conditions" (von Glasersfeld, 1988:208).

More recent researchers (Driver and Oldham, 1986; Kuiper, 1991; Ernest, 1994) widely accept that learners develop understanding, ideas and beliefs about the natural world outside the 'formal' learning environment, and as Driver and Oldham (1986:105) suggest, "long before they are formally taught". Further, it is asserted that learners develop a sense about the world, albeit a 'wrong' sense sometimes, and move towards a cognitive perspective based on their past experiences. Driver and Oldham (1986) therefore maintain that an individual's knowledge is not considered a set of discrete 'bits' but a series of structures, and learning involves the development and change of such structures. In my view, learning occurs over a continuum and develops on the basis of continued reflection and evaluation within our own experiential context - it is a cyclical process of continuous modification and adaptation, and I therefore concur with Taylor *et al.*'s. (1993) deduction that the learner's new understandings can be formed on the basis of his/her own prior knowledge and experience. The process of making sense of our experience, the genesis of the individual's knowledge, can be a result of the individual's "purposeful and subjective interpretation of his/her experience of the physical and social world" (Taylor *et al.,* 1993:4).

Ernest (1992) emphasises the 'social' dimension in the cognition process and suggests that 'social constructivism' regards the individual subjects and the realm of the social as "indissolubly interconnected". It is the social reality that creates or constrains the shared experience underlying physical or social understanding. This development in constructivist thought is strengthened by the work of Vygotsky, who sees language as a key component in the cognition process. When considering the formation of concepts, Vygotsky (1986:107) suggests that "real concepts are impossible without words, and thinking in concepts does not exist beyond verbal thinking. That is why the central moment in concept formation, and its generative cause, is a specific use of words as functional tools." The above assertion appears dogmatic in its implication that the cognition process cannot be a personal and insular one. This in my view needs to be questioned, as it assumes that *all* cognition is based upon social interaction.

The implications that constructivism has for spatial conceptualisation and world view are important, but Lerman (1992a) warns against merely replacing one rhetoric with another. Although the debate on constructivism has succeeded in questioning and replacing the transmission metaphor with one that sees students constructing their own knowledge, the practical implications for the teacher need careful consideration. Lerman (1992a) laments the emergence of terminology such as 'the constructivist teacher' and 'the constructivist classroom', and in-service courses that convince teachers to become 'constructivist' when the implications of 'constructivism' are not clearly understood. Although a supporter of constructivist theory, Solomon (1994) identifies with the dilemma that understanding the nature of 'constructivist teaching' is still difficult.

I identify with Lerman's (1992a) reservations and the dangers of replacing one dogma with another. As I see it, constructivism has much to offer education in general, and the understanding of cognitive processes in particular. It provides a useful framework for understanding spatial conceptualisation within a social context. Social constructivism recognises that past experiences, presuppositions and perceptions are central to the cognition process. It also reinforces the notion that untutored beliefs (within a socio-cultural context) influence the construction of meaning. Cobern (1991:7) refers to these presuppositions about the world as epistemological macrostructures and he asserts that "each person can be seen as having a fundamental, epistemological macrostructure which forms the basis for his or her view of reality. The common term is world view". I therefore argue that an individual's world view is fundamental to his or her conceptualisation of space and shape.

World view theory has its roots in anthropology, and Kearney (1984:10) suggests that there are three basic problems in the study of world view:

- What are the necessary and therefore universal types of images and assumptions which are part of any world view, and what are the specific contents of these universals?
- What relationship do these images and assumptions have with the world they represent?
- What influence does this world view have on behaviour?

There has been growing interest in world-view theory in science education research, and Cobern (1993a, 1993b, 1994, 1997) in particular has embraced the notion of world view in arguing the importance of fundamental beliefs with respect to learning science. This research draws from the experiences of Cobern in the context of learning mathematics with special reference to spatial conceptualisation. The enabling framework which facilitates the construction of world-view profiles in this research, referred to as concept maps by Cobern (1993) and Slay (2000), is based on what Kearney (1984) terms logico-structuralism. According to logico-structuralism, world views are constructed on the basis of the following universal categories: Self, The Other (Non-Self), Classification, Relationship, Causality, Space, and Time (Cobern, 1993; Kearney 1984). This study focuses on the space category. In order to establish individual profiles of the notion and conceptualisations of space, I made use of Cobern's framework which uses "vector pairs" such as naturalism/religion, chaos/order, mystery/knowledge, function/purpose, mundane/special (Cobern, 1993) to establish concept maps of what individuals think of nature. I have included additional vector pairs such as mathematical/non-mathematical, measurable/random, plane/multi-dimensional in order to set up profiles for individual world views on space.

The central assertion of this study is that spatial conceptualisation is a complex blend of spatial skills (such as spatial visualisation and spatial orientation as defined in 2.2.1 on page 19) and world-view perceptions of space. In my experience, traditional spatial conceptualisation research in mathematics education has exclusively focused on the former dimension (by using pen-and-paper tests) and ignored the notion of world view.

In order to illustrate this complex blend, my research strategy for this study therefore made use of a multi-faceted approach which included exploring spatial skills by utilising not only a pen-and-paper test, but also a hands-on activity test, and by engaging in the world-view profiles of the participants.

This study involved adolescent learners (Grade 11) from various classroom settings. By virtue of its subjective, implicit and personal processes, world view is culturally dependent (Cobern, 1991 and Kearney, 1984) and this study makes use of the rich cultural diversity in the South African educational landscape in its attempt to set up world-view profiles about the nature of space.

1.2 OBJECTIVES OF THE STUDY

The research question examined by this study is:

> Can understanding of a world-view-theory approach contribute towards the exploration of an individual's spatial conceptualisation?

To answer the question this study pursues and is structured around the following objectives:

1. to explore spatial capacity (spatial visualization and orientation skills) using both pen-and-paper test items and hands-on activities;
2. to explore spatial conceptualization by engaging in world view profiles of space using a logico-structural approach;
3. to investigate consistencies and relationships between spatial capacity, world views and aspects of mathematics performance of Grade 11 learners;
4. to show that spatial conceptualisation is a rich and complex blend of spatial capacity and world view.

The implication this study has for teaching and learning is considered in the conclusion of this dissertation.

1.3 METHODOLOGY

This study is grounded in an interpretivist-naturalistic paradigm as it is concerned with the individual and is aimed at understanding the subjective world of the individual (Schwandt, 1994; Lincoln and Guba, 1985; Denzin and Lincoln, 1994; Cohen and Manion, 1994; Cohen, Manion and Morrison, 2000) and makes use of multiple sources and types of data (Le Compte, Millroy and Preissle, 1993) to ensure credibility (Lincoln and Guba, 1985). This research comprised a multi-sited case study (Stake, 2000) involving a group of 32 Grade 11 learners from five different secondary schools in the Eastern Cape, a province of South Africa, from a diverse range of cultural and socio-economic backgrounds. In order to answer the research question and meet the objectives, several techniques for data collection were employed. They included the use of questionnaires, administering a pen-and-paper and a hands-on activity test and engaging in conversations as discussed in detail in chapter three. A logico-structuralist approach was used in the exploration of word views. (Kearney, 1984). Both quantitative methods (in the form of fundamental descriptive statistics) and qualitative methods (in the form of interview analyses) were used for analysis.

1.4 SIGNIFICANCE OF THE STUDY

The significance of this study is twofold:

- that it is research about how pupils think *in* space **and** *about* space;
- that it is unique and a new kind of research in the South African context.

Consistent with Cobern's (1996) view, I believe that it is of great interest (particularly to educators) how people see the world. In many instances the very point of teaching and learning is about how students change their perceptions of the world. In my experience of visiting many mathematics classrooms in South Africa over the years, the essence of living in a world has all but disappeared at the cost of a very technical and content-driven approach to mathematics. A glance through the many traditional textbooks used in South African schools would confirm this. Although in a phase of curriculum transformation, the current geometry curriculum (for Grade 11) for example, is dominated by a Euclidean approach, which in practice is generally characterised by regurgitating proofs and doing 'riders'. A pedagogical strategy to facilitate a broader understanding of space is noticeably absent. Assessment strategies are dominated by regular summative pen-and-paper tests and end-of-year (semester) examinations. A learner's capacity and conceptualisation (spatial capacity and conceptualisation in this case) are thus traditionally reflected as a quantitative mark on a score sheet. The point of this study is to emphasise and show that spatial capacity and conceptualisation is far more complex than that.

This study breaks new ground in its attempt to integrate the exploration of spatial conceptualisation as seen in pen-and-paper tests **and** world-view profiles. This study reveals that spatial conceptualisation is a rich blend of skills and competencies manifested in pen-and-paper and hands-on activity tests, and world view perceptions of space. Further, this study adds to the research literature in mathematics education in terms of its use of a logico-structuralist approach in investigating world views of space.

This study is of interest to mathematics educators on most levels of practice. It is of particular significance to those who hold the constructivist view that prior knowledge and individual epistemological macrostructures are cornerstones of cognition. It highlights the importance of meta-knowledge and suggests that curriculum developers should take cognisance of the notion that spatial conceptualisation involves a blend of spatial capacity and world views of space.

1.5 LIMITATIONS OF THE STUDY

This study does not claim to tell the whole story of spatial conceptualisation and world view; rather it attempts to provide a rich description of the complexity of spatial conceptualisation as it exists at a particular time, for a specific group of people within a particular context. As with all case studies, caution should be exercised in extrapolating and generalising from its findings. The best I can hope for is that the theme of this work will resonate with other mathematics educators and inspire other researchers to investigate further. Limitations of the study in general and methodological and procedural limitations in particular are discussed in more detail in the last chapter.

1.6 THESIS OVERVIEW

Chapter two attempts to contextualise the study within the literature and provides the theoretical fundamentals which have informed this project. Firstly, a review of key terminology and definitions is provided. It is noted that inconsistencies exist in the research literature in the use of terminology; this chapter defines the key concepts that frame this study. Spatial conceptualisation is seen as a blend of spatial capacity (spatial visualisation and orientation skills) and world-view perceptions of space. Secondly, spatial capacity is reviewed in terms of gender and culture differences and cognitive development. Thirdly, the notions of world view and space are contextualised within the literature. The chapter ends with a discussion of the epistemological underpinnings of this study.

Chapter three outlines the research process employed. It brings to attention the paradigm within which this research is located, the design of the research process, and the techniques employed. It also discusses the data-analysis processes and highlights the issue of validation, while c**hapter four** introduces, describes and analyses the pen-and-paper and the hands-on activity tests that were used to investigate spatial capacity. It provides an item-by-item discussion for both tests and explores gender, individual and school differences in performance.

Chapter five provides an in-depth content analysis of the interviews and deals with *what* the participants said about space while **chapter six** deals with *how* the participants thought about space. It analyses the interviews on a meta-level and engages with the resulting individual world view profiles. The chapter ends by exploring linkages between spatial capacity, world views and performance in school mathematics.

Finally, **chapter seven** provides a synopsis and summary of the findings, documenting teaching and leaning strategies that capitalise on the relationships between spatial capacity, world view and mathematics achievement, articulating the limitations of the study and its research methodology, offering some recommendations, and reflecting upon the entire research process.

CHAPTER TWO

THEORY INFORMING THE STUDY

2.1 INTRODUCTION

This chapter deals with the literature pertinent to this study. As I seek to show that spatial conceptualisation is a complex blend of spatial capacity (as defined below) and perceptions of space (world view of space), this literature review is divided into three sections. First the notion of spatial capacity is reviewed. This includes a brief review of the inconsistent terminology used in the literature and a synopsis of some of the research undertaken with specific reference to mathematics performance and gender difference. This chapter thus aims initially to clarify some of the terminology and then formulate a workable framework for a theoretical base upon which this study can rest. The development of spatial capacity in the context of cognitive development will also be analysed. Secondly, the theoretical underpinnings of world-view theory will be discussed with special reference to the logico-structural approach; and thirdly, the concept of space will be analysed in its psychological and philosophical contexts.

2.2 SPATIAL CAPACITY

2.2.1 A review of key terminology and definitions

The notion that spatial capacity and the ability to operate in a spatial context are fundamental to learning in general and to the learning of mathematics in particular is not new. Nickson (2000:49) observes that "children's earliest mathematical experience is spatial in nature as they explore the space around them by moving within it and discovering their relation to it". Historically, research into spatial ability is rooted in mechanical aptitude and "practical ability" (McGee, 1979). Tsakalos (1993) makes the observation that research into spatial capacity dates back at least 50 years and she cites work done by Hamly and Thurstone in the 1930s. Piaget's work, for example, resulting in the interesting assertion that a "child's order of development in geometry seems to reverse the order of historical development" (Piaget, 1953:74), which inspired much follow-up research, dates to the early fifties. There has been a plethora of research literature since then on spatial conceptualisation and its impact and importance on the cognitive development of spatial capacity. Much of the literature, however, is in agreement that there is little consensus in the terminology and definitions used. Equally, it appears that there is little consensus in the theoretical frameworks underpinning spatial development. As Battista *et al.* (1982:332) note, "the role that spatial thinking plays in mathematical performance has not been adequately described. It is still not known how important spatial ability is for learning various topics in mathematics, nor are the spatial components of mathematical thinking understood." Eliot and Hauptman (1981) note that spatial ability is referred to by researchers in such a variety of ways that it is often difficult to be precise about the exact meaning of the term. It is noted, for example, that some scholars view spatial ability in terms of individual differences in performance of a set of specific operations on spatial tasks, whereas others see it in terms of differences in the processing of any non-linguistic information. Bishop (1980) also laments the inconsistencies in terminology and methodology of research in the general discourse of spatial ability. He emphasises the notion that there is very little conclusive and clear evidence about the extent of the relationships between spatial ability and mathematical ability. He says that the findings "differ from one study to another" (Bishop, 1980:258). He was particularly critical of the factor-analysis research, which makes very little or no reference to individuals, to their approaches to solving a particular problem, and/or to their classroom situation. This research emphasises the complexity of spatial conceptualisation by not only analysing spatial capacity through pen-and-paper and hands-on activity tests but also by exploring individual world views on space.

It is not the intention here to provide a chronologically correct and historically precise and all-encompassing inventory of terminology used in past research into spatial capacity and spatial conceptualisation. The following terms appear most frequently in the literature; I will engage with each one of them and analyse them on an individual basis in my ultimate endeavor to synthesize the various concepts into a workable framework of definitions for this study:

- spatial ability
- spatial visualization and orientation
- spatial factor
- visual imagery
- spatial perception
- spatial conceptualisation
- spatial reasoning

Spatial ability

Spatial ability is often used as an all-encompassing and generic concept involved in any mental operation or problem-solving situation which is remotely spatial in nature. It may or may not be that the task requires specific spatial thinking. Lean and Clements (1981: 267) refer to spatial ability as "the ability to formulate mental images and to manipulate these images in the mind". Gardner (1983:8) refers to spatial ability as one of the "human intelligences – a relatively autonomous human intellectual competence".

Spatial visualisation and orientation

Based on research done by Michael *et al.* (1957), McGee (1979:889) identifies the existence of at least two spatial "factors" which make up an individual's spatial ability. These are *spatial visualisation* and *spatial orientation.*

Spatial visualisation

One of the earliest definitions developed by Guilford and Lacey as cited by Michael *et al.* (1957:188) describes spatial visualisation as "an ability to imagine:

the rotation of depicted objects,

the folding or unfolding of flat patterns, the relative changes of position of objects in space,

the motion of machinery".

It is further suggested that this factor is best assessed in tests that present a stimulus pictorially and in which some manipulation or transformation to another visual arrangement is involved (Michael *et al.*, 1957).

Thurston cited by Michael *et al.* (1957:188) describes spatial visualisation as an "ability to visualize a configuration in which there is movement or displacement among the internal parts of the configuration." French, also cited by Michael *et al.* (1957:188) concurs with the dynamic dimension of spatial visualisation when he suggests that it is "an ability to comprehend imaginary movements in three-dimensional space or the ability to manipulate objects in the imagination." It is interesting to note that French specifies the three–dimensionality of the space involved, whereas Guilford and Lacey appear to incorporate three- and two-dimensional space. Michael *et al.* (1957:188) cite research done by Ekstrom, French and Harman who assert that spatial visualisation is "an ability to manipulate or transform the image of spatial patterns into other arrangements."

The underlying ability in spatial visualisation appears to be connected to movement, transformation and manipulation. It is dynamic and involves motion. This is supported by McGee's definition (1979:893) that spatial visualisation is the "ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object." The theoretical assumptions of this study will remain consistent with the above definition, but extend the concept of stimulus objects to include physically presented stimuli and not restrict the concept to pictorially presented stimulus objects only.

In their study on the use of spatial visualisation in mathematics by girls and boys, Fennema and Tartre (1985:184) concur with McGee's observation (1979) that spatial visualisation is "logically related to mathematics". The quantitative work of Schonberger cited by Fennema and Tartre (1985:184), has shown that mathematics achievement and spatial visualisation seem to correlate in the range of 0,3 to 0,6. It is noted, however, that the mathematical tasks under observation here were overtly spatial in nature. Fennema and Tartre's study (1985) thus incorporated a broader spectrum of mathematics tasks and their conclusion suggests that students who had a high spatial visualisation skill factor "solved no more mathematical problems than students who had a low spatial visualization skill [factor]". Research done by Eisenberg and McGinty (1977), however, suggests that students with a high spatial visualisation test score generally achieved correspondingly high calculus scores. This conclusion is consistent with McGee's assertion (1979:899) that spatial visualisation is important for success in college mathematics, "especially geometry and algebra"

The present study further explores this notion by analysing the relationship between spatial visualisation factors of the participants and mathematics performance in tasks that contain specific spatial concepts **and** also in tasks that are devoid of spatial ideas (such as general school performance in algebra for example). These concepts are covered in chapter five.

Spatial orientation

Thurstone, cited by McGee (1979), suggests that spatial orientation is the ability to recognise the identity of an object when it is seen from different angles. This also involves the ability to visualise a rigid configuration when the object is moved into different positions. French, also cited by McGee (1979), sees spatial orientation as the ability to perceive spatial patterns accurately and to compare them with each other.

An important feature of spatial orientation is the ability to make sense of spatial orientations of objects relative to different positions of itself or of other objects. McGee (1979:897) suggests that spatial orientation involves the "comprehension of the arrangement of elements within a visual stimulus pattern, the aptitude to remain unconfused by the changing orientations in which a spatial configuration may be presented, and the ability to determine spatial orientation with respect to one's body".

Apart from exploring spatial orientation explicitly through a pen-and-paper test and through hands-on activities, this study also explores correlations between spatial orientation factors and mathematics performance. See chapter five.

Spatial factor

Researchers consider that a spatial factor describes the mental manipulation of shapes and is quite distinct from factors such as verbal ability (McGee, 1979). I argue for a wider definition and suggest (in chapter three) that a spatial factor not only describes the mental but also includes the physical manipulation of shapes. Further, Eliot and Hauptman (1981:55) cite French, who in 1951 suggested that in addition to spatial visualization and spatial orientation "factors" there was enough evidence to support the existence of a *spatial* factor which is the "ability to perceive spatial patterns accurately and to compare them with each other".

Visual imagery

Lean and Clements (1981:268), citing Hebb, take visual imagery to mean "imagery which occurs as a picture in the mind's eye". Suwersono's (1982) research isolated this concept and focused on the extent to which a person prefers to use visual methods when solving a mathematical problem. These three researchers built their work on Krutetskii's (1976) interesting assertion that many children use visual imagery when thinking about topics which do not appear to require visual thinking. In terms of solving mathematical problems and engaging in processing mathematical information, he divides individuals into three categories:

a) the analytic type, who prefers verbal-logical modes of thinking;
b) the geometric type, who prefers visual-pictorial modes of thinking and;
c) the harmonic type, who uses both modes.

Spatial perception

The way an individual relates to space and has come to terms with the relationships between objects is fundamental to spatial perception. Spatial perception is central to the process of cognition across all disciplines. Recent research in the field of geography for example, suggests that development in spatial perception, conceptualisation and knowledge of the environment is essential to the process of learning (and teaching) in that field (Wilmot, 1998; van Harmelen and Boltt, 1995; Spencer *et al.*, 1989). According to Stern and Robinson (1994), perception in general is "the gathering of information through our senses and the organizing of that information in order to create meaning". It is subjective and relative and depends on the way in which individuals structure it at different times. As individuals make sense of their experiences, perceptions change constantly (Boardman, 1983; Stern and Robinson, 1994). Wilmot (1998) thus suggests that spatial perception in particular relates to how an individual perceives space. The extent of an individual's spatial perception depends on spatial perceptual skills that comprise (amongst others) the ability to:

- recognise objects in the environment;
- orientate ourselves in the world;
- orientate objects in relation to other ourselves and/or objects;
- transfer three-dimensional space into two-dimensional forms;
- achieve perceptual constancy (recognition that real objects are constant in shape, size and colour, but that they may appear distorted when viewed from different perspectives);
- recognise depth and distance/proximity;
- identify perceptions of elevations including vertical/aerial, oblique and horizontal/normal views;
- identify and understand relationships of location/position, scale and size.

(van Harmelen and Boltt, 1995)

Spatial conceptualisation

Spatial perception relates to how an individual 'sees' space whereas, according to some researchers, spatial conceptualisation specifically refers to how an individual 'understands' these perceptions (Yelon and Weinstein, 1977; Grove *et al.,* 1989, Wickens, 1992; Atkinson *et al.,* 1993, Golledge *et al.*, 1995). Following Wilmot, 1998:28) spatial conceptual skills include the ability to:

- categorise and reduce the complexity of the environment;
- organise, categorise, structure/order and interpret/make sense of our perceptions of what objects are, where they are, and how and why they are such;
- identify, describe, analyse, explain, and justify objects and relationships of both a concrete and abstract nature; and
- acquire, organise, store, recall and decode information obtained about the relative location and attributes of objects and phenomena in the external environment, by means of a 'cognitive map'.

Spatial reasoning

The term spatial reasoning often occurs in the literature in conjunction with discussions on spatial ability. Clements and Battista (1992:420) describe spatial reasoning as consisting of "the set of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed and manipulated". Nickson (2000) suggests that, with time, as individuals develop a spatial awareness as a result of experience and perceptions, this awareness eventually develops into spatial reasoning.

A summary of fundamental terminology and concepts underpinning this study

For the sake of ensuring clarity and seeking consistency in the use of terminology from the literature (Bishop, 1980), this research project is shaped around the following fundamental concepts:

Spatial capacity: the all-encompassing concept which embraces spatial visualisation, orientation, perception and ability. In this research spatial capacity refers in particular to that aspect of spatial conceptualisation which is measurable in a pen-and-paper and/or hands-on activity test (this is dealt with in detail in chapter four);

Spatial visualisation: the ability to manipulate mentally, rotate, twist, or invert a pictorially or physically presented stimulus object. The underlying ability in *spatial visualisation* appears to be connected to movement, transformation and manipulation. It is dynamic and involves motion (see the definition on page 13);

Spatial orientation: the ability to recognise the identity of an object when it is seen from different angles. It is the ability to make sense of spatial orientations of objects relative to different positions of itself or of other objects (see the definition on page 15);

Spatial conceptualisation: the fundamental concept that ultimately incorporates spatial capacity and perceptions of space as discussed in chapter five.

2.2.2 Spatial capacity and mathematics performance

Clements and Battista (1992:442) assert that spatial thinking is essential to scientific thought; "it is used to represent and manipulate information in learning and problem-solving" and there seems to be general consensus amongst scholars that there exists a relationship between mathematical performance and spatial capacity. The extent of that relationship, however, appears to be a contentious issue. Smith (1964), for example, suggests that spatial ability is positively related to 'high-level mathematical conceptualisation' but has little to do with the acquisition of 'low-level concepts and skills'. What is meant by high-level and low-level conceptualisation is not articulated, but one assumes that the latter is associated with fundamental concepts such as simple calculations and working with simple two-dimensional figures. Guay and McDaniel (1977) define low-level spatial abilities as requiring the vizualisation of two-dimensional configuration but no mental transformations; and high-level spatial abilities as requiring visualisations of three-dimensional configuration with mental manipulations. In contradiction to Smith (1964), Guay and McDaniel (1977) argue that a positive relationship exists between spatial capacity and mathematical performance for both low-and high-level conceptualisations. This is supported by Sherman (cited in Lean and Clements, 1981), who emphasises that the spatial ability factor is in fact one of the main factors that affect mathematical performance. Fennema and Tartre (1985:203), however, question this very strongly and suggest that much more research is required before one can "safely conclude that an emphasis on spatial visualisation skills will improve mathematical thinking". They question the idea that spatial visualization skills are highly important in the learning of mathematics and indeed disagree with the notion that the development of such skills should become a major goal of mathematics education. It is interesting to note, however, that in earlier research, Fennema and Sherman (1977 and 1978) openly supported the view that there is a positive correlation to be found between spatial ability and mathematics achievement. Although there appear to be contradictions and inconsistencies in the research literature, these inconsistencies need, to a large extent, be seen in terms of inaccurate definitions of terms or vague and ambiguous terminology. Clements and Battista (1992) remind us that there exists a natural positive relationship between spatial capacity and mathematical performance involving obvious spatial and visual dimensions. They cite Soviet researchers such as Yakimanskaya, who emphasize the importance of spatial thinking in geometry, for example. In chapter five of my study, links are drawn between spatial capacity and mathematical performance in school.

2.2.3 Spatial capacity differences in gender and culture

Gender

In suggesting that research literature often assumes that gender is an issue in mathematics, Ernest (1998:2) identifies two aspects of the "so-called gender and mathematics problem". He notes that

- There are significant gender differences in participation in mathematics ;
- There are significant gender differences in performance of mathematical tasks, examinations and activities. Females perform significantly worse, on average, than their male counterparts.

Participation

Leder (1992) makes the observation that more American males than females take the more advanced high school mathematics courses and that more males enroll for elective mathematics courses. Citing Dekkers, Leder (1992) finds that a similar situation prevailed in Australia in the mid-eighties, where at least twice as many males studied mathematically specialised courses than females. Ernest (1992) concurs with this and suggests that these imbalances are particularly widespread in the developed world. A reverse trend, however is, identifiable in some Latin and Latin-American countries as well as South-East Asia, Scandinavia and the Carribean. Ernest (1992) therefore cautions against making global generalisation based on American and British experiences.

Performance

Although it is claimed that performance and participation go hand in hand, much more research seems to focus on the performance (sometimes referred to as attainment) aspect of the 'problem'. In investigating relative performance, Shuard's (1986:23) research, for example, confirmed the notion that although "little difference between the mathematical attainment of boys and girls existed in the primary stages of their lives" the boys soon 'overtook' the girls "at those aspects of mathematics which are fruitful for future mathematical development and insight". Much of the research seems to support the relative under-performance in mathematics by girls. Nickson (2000:51) refers particularly to research that suggests that girls have a "less strongly developed spatial ability compared with boys". This has particular relevance to this research (see 4.4.2 on page 174) and I will thus isolate some of the main contentions found in the literature.

Building on Fennema and Carpenter's (1981) work which indicates that there is little difference between males and females in overall mathematics achievement at ages 9 and 13, but that at age 17, however, males' performance was higher, Battista's (1990) research into the role that spatial thinking plays in mathematical problem-solving, concluded that males and females differed in spatial visualisation. Males scored significantly higher than females on spatial visualisation, geometry achievement and geometric problem-solving. However, there was no gender difference in logical reasoning. Other research on this theme has suggested that boys are better at more complex items requiring abstract thought, problem-solving and conceptualisation. Girls are better at simple, repetitive tasks requiring low-level skills such rule-following (Walkerdine, 1998 citing the work of Shuard). These findings are consistent with the popular hypothesis that the outcomes in the processes requiring spatial and geometric thinking favour males. This point is explored in detail with the sample used for this study in section 5.5.8 on page 271.

In trying to provide an answer to this apparent discrepancy between the sexes, there have been numerous attempts to identify differences in strategies amongst females and males. Clements and Battista (1992), for example, cite Tartre who reports that males prefer non-verbal modes of thinking and females the verbal mode. Further, Clements and Battista (1992) found that females use less effective concrete strategies than their male counterparts and that males use more efficient abstract strategies on spatial tasks.

Many possible explanations have been put forward to explain these gender discrepancies. Leder (1992) identifies the following variables as having possible effects on gender differences:

- biological variables
- environmental variables
    - school
    - teachers
    - peer group
    - parents
    - wider society
- cognitive variables
- psychological variables.

More recently, Ernest (1998) and Walkerdine (1998) have adopted a more critical stance than the traditional approach to the interpretation of gender research. Walkerdine (1998:20) divides the traditional approaches into two arguments:

- the "nature" argument, which focuses on differences in terms of ability and brain lateralisation;
- the "nurture" argument, which tends to blame personality and socialisation processes.

She argues that, either way, most of the studies "work from the premis that girls are worse at mathematics and then attempt to explain this by global generalization". This is problematic as it simply perpetuates a self-fulfilling prophecy. She suggests that, instead, research should focus on the similarities rather than the differences – this might provide more useful insight. The main point of her argument, however, is more fundamental. As most of the traditional gender research was embedded in the quantitative tradition, the interpretation of the data was based on statistical analyses. The significant gender differences that the researchers talk about was therefore rooted in statistical differences. This, according to Walkerdine (1998), is dangerous as it can lead to gross misinterpretations and distortions. She therefore argues that the results need to be viewed critically and with circumspection. By virtue of the 'negative' generalisations and assumptions that females are lacking, a deficit model research approach was adopted, and "deficit models tend to blame the victim" (Walkerdine, 1998:28).

Ernest (1998) also views the interpretations of past research data in a critical light. He links the various interpretations to the fundamental philosophical assumptions that underpin each of the researchers' paradigms. He refers to five "interest groups" – each of which has a unique view of gender and mathematics. These views, or philosophical underpinnings as I would call them, inform the way a researcher not only gathers data, but also interprets the data. His five interest groups and their views on gender and mathematics are:

The industrial trainers: They subscribe to a fixed view of knowledge and society. It is the fixed biological differences that make males better than females. Feminism and attempts at gender equality are viewed as undesirable political interventions in the natural state of affairs.

The technological pragmatists: They have a pragmatic, utilitarian and absolutist approach to mathematics. They believe that both males and females should benefit society equally, even if females are inferior.

The old humanists: They are committed to the preservation of the purity of mathematics. They see mathematics as inherited and primarily as a male domain – but women need to be encouraged and progress as far as their nature allows.

The progressive educator: They subscribe to an individualistic ideology within a traditional liberal education framework. Mathematics is very much located within the individual, as is the gender issue. The solution lies in providing support.

The public educator: Their view of knowledge is a social constructivist one with a strong commitment to reform in social justice. They see gender inequity as a result of sexism and stereotyping (Ernest, 1998).

The above model may appear very firmly compartmentalised with little space for overlap. Ernest (1998:8) does not claim a "logical" link between the groups, however, nor does he entertain the idea that they are mutually exclusive. His groups are meant as an "illuminating conceptual tool" in an effort to make sense of the plethora of often contradictory research findings on gender equity.

Although not central to this study, the issue of gender equity features quite prominently in this research. Gender differences are teased out in the analysis of spatial capacity in a pen-and-paper test and a hands-on activity test (chapter four). Gender differences are also analysed on a meta-level (see chapter six)

Culture

Building on his earlier work in which he reported that native Africans of all nationalities were poorly developed perceptually when compared to Europeans, Mitchelmore (1980) claims that students from Jamaica were about three years behind their counterparts in

Ohio, who in turn were three years behind their counterparts in England in spatial thinking. Graham's (1988:119) work in Australia was inspired by the concern that Aboriginal children are "failing to learn mathematics effectively in schools".

The type of results and conclusions illustrated above is characteristic of research done from a western premise and which investigates performance and attainment in mathematics in terms of western and eurocentric assumptions – these are not very helpful in understanding how various cultures solve mathematical problems and develop cognitively. Bishop (1988) advocates an approach which is sensitive to cultural preservation and development and which acknowledges the notion that mathematics is not value-free. He views mathematics as a cultural product derived from humans engaging in:

- counting
- locating
- measuring
- designing
- playing
- explaining.

Although my research analyses spatial capacity across diverse cultural perspectives, the aim is not to do a cross-cultural study. Culture is viewed in broad terms. According to Erez and Early (1993:42), the most general view of culture is "that it is a set of characteristics common to a particular group of people". The sample of this study consisted of five classes from five different secondary schools (see to section 3.3 on page 49). Inherent in each of these classes (groups) is their own classroom culture and this study recognised that each mathematics classroom and each school had its own culture. I agree with Bruner(1996:4) that "learning and thinking are always situated in a cultural setting" – in this case the cultural setting was the school and the classroom. The analyses of data (chapters four, five and six) thus often took place within the five different participating schools. It is mostly within the type of classroom culture mentioned earlier that this investigation was embedded. The focus was not on ethnicity and race.

2.2.4 Cognitive development of spatial capacity

Gardner (1985) makes the observation that although spatial capacity (he uses the term spatial intelligence) has long been recognised as central to learning, little research has been conducted on the cognitive development of this capacitiy. In my experience and from my readings, this situation has not changed in 20 years.

The influential studies conducted by Piaget and Inhelder (1967) and van Hiele (1986) in particular are reviewed only briefly as it is not within the scope of this thesis to provide an in-depth analysis of the two theories. Although cognitive development is not the focus of this study, it is relevant, being strongly linked to mathematics performance and the development of an epistemological macroctructure as discussed in the next section. A very brief overview of the classical research done on cognitive development in spatial capacity therefore seems appropriate.

Piaget and Inhelder

The developmental model proposed by Piaget and Inhelder (1967) suggests that the evolution of spatial relations in an individual proceeds at two different levels:

- at the perceptual level;
- at the level of thought or imagination.

Clements and Battista's (1992:422) interpretation suggests that individuals construct spatial capacity through "progressive organization" of their motor and internalised actions resulting in operational systems – it is a build-up from "prior active manipulation" of their environment. Further, this progressive organisation follows a definite logical (rather than historical) order. Initially it starts with topological relations and progresses to a Euclidean understanding. The former focuses on how various figures relate to themselves, whereas the latter emphasises the way figures relate to one another. According to Clements and Battista (1992) this polarisation received much criticism because the differentiation in classifying figures as topological or Euclidean is nebulous at the best of times. They themselves argue against a mutually exclusive classification and suggest that every figure possesses both characteristics to an equivalent degree.

The van Hiele levels of geometric thinking

Clements and Battista (1992) provide a useful and critical overview of van Hiele's theory and it is not my intention to repeat their observations. The theory developed by van Hiele in 1957 suggests that individuals progress through five levels in sequential order in their development of geometric thinking (Mayberry, 1983). Van Hiele's thinking was greatly influenced by his own difficulties with geometry and the inspiration of the renowned Dutch mathematics educator Freudenthal, who pointed him in the direction of the Gestalt movement in psychology (van Hiele, 1986). It is there that van Hiele came across the work of Piaget, who himself subscribed to the idea of levels of development. Van Hiele emphasises the notion that one of the main differences between his theory and that of Piaget lies in the fact that Piaget's theory is one of development and his is one of learning.

It is asserted by van Hiele, cited in Clements and Battista (1992), that "students progress through levels of thought in geometry from a Gestalt-like visual level through increasingly sophisticated levels of description, analysis, abstraction, and proof". The theory is underpinned by the following assumptions:

- learning is a discontinuous process;
- learning occurs in discrete and different levels of thinking;
- levels are sequential and hierarchical;
- to move from one level to another an individual must have mastered much of the learning at the lower level;
- concepts that are implicitly learnt at a lower level become explicitly understood at a higher level;
- each level has its own particular language with its own linguistic symbols and its own system of relations connecting these symbols (Nickson 2000, Clements and Battiste 1992).

Some of these assumptions have been subjected to critical attention and become focuses of research. Wilson (1990), for example, examined the claim that the levels are discrete and hierarchical. In his re-analysis of an investigation into the learning sequence in geometry based on the work of van Hiele, Wilson (1990) citing Usiskin concludes that the levels are not always ordered in the way that van Hiele predicted. Further, he suggests that through interaction with skills that are outside of the hierarchy, the van Hiele hierarchy can indeed be obscured.

The van Hiele levels can be described as follows (Burger and Shaughnessy 1986, Clements and Battista 1992, Nickson 2000):

Level 1 Visualisation: Students identify and recognise geometric configurations as wholes and according to appearance without regard to properties of their components.

Level 2 Analysis: Students are able to reason about shapes and objects by their properties.

Level 3 Abstraction: Students can form abstract definitions. They can classify figures hierarchically and can distinguish between the necessity and sufficiency of a set of properties in determining a concept.

Level 4 Deduction: Students can establish theorems within an axiomatic system. They are able to reason within the context of a mathematical system and its definitions, axioms and theorems.

Level 5 Rigour: Students can reason formally about mathematical systems. They can compare systems based on different axioms and appreciate the role of logic in deductive systems.

The progression from one level to the next depends on the mastery of the previous level. This, according to van Hiele (1986), is not necessarily a function of an individual's genetic makeup; instead it relates directly to the influence of the teacher/learning process. The interaction process between teacher and learner is thus central to the van Hiele theory. Van Hiele (1976, 1986:176) proposes five phases in the "learning process":

Phase 1 Information: The learner is acquainted with the context of the field of study and the content domain (Clements and Battista, 1992).

Phase 2 Bound orientation: The learner becomes acquainted with the content principles.

Phase 3 Explicitation: The learner begins to elaborate on his/her intuitive knowledge. The teacher's role is to bring the content to an explicit level of awareness (Clements and Battista,1992).

Phase 4 Free orientation: The learners are able to orientate themselves within the concepts and relations previously elaborated.

Phase 5 Integration: Learners integrate their knowledge into a coherent network that can be described and applied (Clements and Battista, 1992).

Naturally, the van Hiele assertions have been at the centre of much research and have generated mixed reactions. It is not my intention to review this research in detail, but just highlight some of the central questions that have arisen:

- Do the five levels represent a hierarchy (Mayberry, 1983; Wilson, 1990)?
- Are students at different levels for different concepts (Mayberry, 1983)?
- Are the van Hiele levels discrete (Burger and Shaughnessy 1986, Fuys *et al.*, 1988; Wilson, 1990; Clements and Battista, 1992)?
- To what extent does van Hiele's theory relate to mathematics achievement (Senk, 1989)?

Although much of the research has been critical, other investigations have been naïve leading to gross overgeneralisations. A report on Van Niekerk's (1998:67) research, for example, can be construed as claiming that she has "confirmed the theory of van Hiele for the South African situation". In her subject didactical analysis, within her methodological framework, she may have shown that her participants conform to the five van Hiele levels, but then to claim national consistency is questionable.

In concluding this section, the key theoretical issues as outlined in 2.2 that are specifically followed up in this study in order to achieve part of the objectives articulated on pages six and seven are:

- the importance of spatial ability for learning various topics in mathematics;
- relationships between spatial ability and mathematical ability;
- relationships between spatial visualisation and orientation factors and mathematical performance;
- gender differences and equity in spatial capacity, and mathematical performance;
- relationships between spatial capacity and spatial thinking .

2.3 WORLD VIEW

As Funk (2001) suggests, the meaning of the term world view seems self-evident. The German translation *Weltanschauung* implies a perception of the world, "a conception of the world" or a "particular philosophy of life" (*Concise Oxford Dictionary,* 1995). The 19th century German philosopher Wilhelm Dilthey was especially interested in world views. "He was fascinated by the familiar, yet vexing, fact that world views vary so widely and conflict so sharply even when they are based on the reasoned arguments of philosophers" (Rickman, 1979:47). According to Dilthey (Rickman, 1976:141) "the formation of world views is determined by the will to stabilize the conception of the world, the evaluation of life and the guidance of the will". World views are dynamic and develop under varied conditions. Dilthey (Rickman, 1976:139) asserts that:

> …climate, race and nationality, determined by history, and the development of states, the temporal delimitation into epochs and ages in which nations cooperate,

> combine to produce the special conditions which influence the rise of differences in world views.

Dilthey's views need to be seen in context with his own epistemological assumptions. He assumes that "we know the world through our feelings and strivings as well as through our sense impressions and thinking" (Dilthey cited in Rickman, 1976:15). The epistemological underpinning of my study has elements of Dilthey's assumption and is rooted in the idea that an individual's cognition is a complex process which is informed by world views and presuppositions.

Not unlike Dilthey, Funk (2001:4) suggests that there are many interrelated elements that make up a world view:

- ***epistemology*** **-** beliefs about the nature and sources of knowledge;
- ***metaphysics*** - beliefs about the ultimate nature of reality;
- ***cosmology*** - beliefs about the origins and nature of the universe, life, and especially man;
- ***teleology*** **-** beliefs about the meaning and purpose of the universe, its inanimate elements, and its inhabitants;
- ***theology*** **-** beliefs about the existence and nature of God;
- ***anthropology*** - beliefs about the nature and purpose of man in general and, oneself in particular;
- ***axiology*** - beliefs about the nature of value, what is good and bad, what is right and wrong.

Funk (2001) makes an interesting observation with which I strongly identify. He suggests that an individual's world view may not always be explicit. Few people take time to thoroughly think out, much less articulate, their world view. When exploring and discussing their world views (dealt with in chapters four and five) the participants of this research project intimated that they had never really thought and talked about their presuppositions and conceptions of the world around them.

Holmes' work on Dilthey, as cited in Cobern (1991), suggests that our world view, or *Weltanschauung* initially rests on a more fundamental implicit world picture, or *Weltbild*, which develops in the context of the world in which we live, or *Lebensbild*. This concept of world view has influenced current world view theorists and has occupied an important place in anthropology, but according to Kearney (1984) no comprehensive model had been formulated prior to his logico-structural model which will be discussed later. He refers to world view as a "culturally organized macrothought: those dynamically interrelated basic cognitive assumptions of a people that determine much of their behaviour and decision making" (Kearney, 1984:1). Cobern (1991:19), a science-education researcher who has embraced the notion of world-view theory in science-education research, takes Kearney's definition a step further and asserts that "a world view inclines one to a particular way of thinking". In Cobern's work (1991), which rests heavily on Kearney's (1984) theoretical framework, world-view research focuses on students' presupposition about their world, that is, their epistemological macrostructures. One of the central themes of this research is Cobern's (1991:20) observation that "knowing more about students' world views should help researchers come to a better understanding of conceptual change by providing a more complete understanding of conceptual structure. It should enable educators to better understand students' attitudes and achievement in general".

Kearney's (1984:2) unique theoretical world-view framework is based on the assumption that a world view is a "dynamic, more or less internally consistent system which demonstrates logical and structural regularities". He suggests that the structural composition of a world view consists of seven universals, or cognitive categories (Cobern, 1991): Self, The Other (Non-Self), Relationship, Classification, Causality, Space and Time. Kearney refers to his model as logico-structural integration (Kearney, 1984) because he asserts that the world-view categories are filled with "logically consistent presuppositions about reality" (Cobern, 1991:39).

As Slay (2000) notes in reviewing these universals, they can serve as an effective framework for analysis of a world view. Although there is a recognition that the universals above need to be seen holistically, my research specifically focuses on the category space and, in conjunction with exploring issues of spatial capacity, investigates presuppositions about space held by 32 Grade 11 students. It is my assertion that spatial conceptualisation is a complex blend of spatial capacity and world view of space. I will dwell on the space universal in depth, but in order to provide a wider context and overview I will briefly touch on the other six universals:

Self and *Non-Self*: According to Kearney (1984) the first requirement for a world view is the presence of a Self. An individual's primary reference point is himself or herself (Cobern, 1991). However, the Self cannot be isolated from the Non-Self , which Kearney (1984) refers to as the Other, as the Self is either distinct from his/her surroundings or has a definite relationship with the surroundings. Cobern (1991) refers to the Non-Self as everything in the universe except the Self. It is difficult to conceive of individuals "floating freely suspended in some environment ([the NonSelf]) without having to respond to it in at least some minimal way" (Kearney, 1984:68)

Relationship: This refers specifically to the relationship between the Self and the NonSelf. Kearney (1984:73) notes that there are "various forms that this interaction may take and in turn be so cognized by the individual. The particular manner in which individuals perceive their relationship vis-à-vis the NonSelf is in effect a stance towards the world."

Classification: This universal relates to the notion of class. According to Kearney (1984), depending on one's philosophical predispositions, this notion is either an innate capacity of the mind or in some other way acquired from the environment. Cobern (1991) uses the analogy of a situation in the science classroom where the science teacher and the students could conceivably use various classification categories for classifying a certain scientific phenomenon, depending on their attitudes and predisposition to that particular phenomenon.

Causality: Kearney (1984) argues that the formation of a world view is based on the dialectical forces in one's life, especially in the formative childhood years. The notion of cause and effect is inherent in this universal. Kearney embraced this notion with particular reference to Piagetian tenets which suggest that the nature of cause and effect changes for a child with growth and experience (Cobern, 1991).

Time: Kearney (1984:94) asserts that "time is a more complex and abstract concept [than space]". He justifies this by arguing that perceptions of space are dependent upon immediately sensed information such as object and body location. Time, on the other hand, is a perception that is "not so directly tied to objects" (Kearney, 1984:94). Although my research does not consider the time universal it shows that young students have very rich, often very complex and abstract perceptions of space. The assumption that perceptions of space are always object-related is, according to evidence produced in this project, questionable. The assertion that time is a more complex and abstract concept than space therefore needs further explanation. Cobern's interpretation (1991) of the time universal suggests that there are three basic time orientations, namely, past, present and future. These can vary in depth and range. For example the future can be a few hours, a few days or far more. Kearney (1984:96) cites some interesting research done in 1961 by Kluckhohn and Strodtbeck which attempts to show how perception of time contributes to different world views in different communities. They believe, for example, that Hispanic people are more present-oriented whereas Anglos are more future-orientated. They also suggested that the Anglos often have a false stereotype of Latin Americans as unreliable and oblivious of time. The Anglos are often irritated by failure of people to keep appointments and be punctual. These examples are manifestations of different time world views.

World view in the context of this research

The key theoretical issues as outlined in 2.3 that are particularly relevant to this study and specifically frame this research in order to achieve part of the objectives articulated in section 1.2 on page six and seven are:

- the systematic application of the logico-structural model as proposed by Kearney (1984) and applied by Cobern (1991) and Slay (2000) to explore world views of space;

- the creation and exploration of world-view profiles to illustrate students' presupposition about space.

2.4 SPACE

As articulated in objective 2 on page six, this study attempts to explore spatial conceptualization by engaging in world-view profiles of space (refer to chapters five and six). The notion of space is therefore central to this research and of particular relevance. Space means many things to many people and is key to this study to deconstruct some of the understandings of the concept of space. Kearney's (1984) space universal therefore deserves more extensive attention than the other universals.

In Kearney's (1984) logico-structural model, space is the seventh universal although he links this very closely to time. He theorises that as things are located in space, they are also inextricably located in time. For the purpose and scope of this project, however, space and time have been segregated. When looking at the world-view aspect of space, as wide a definition for space as possible is sought. In Kearney's work (1984:92) he observed that space is used to refer "to many different concepts, ranging from an easily measurable geographical space to more metaphorical usages such as psychological, life and mathematical space". In his anthropological dealing with world-view aspects of space, Kearney (1984) was mainly concerned with the relationship between the environmental space of a people and their images of it. In my attempt to ground this research in the literature, my readings led me along a fascinating path of interesting interpretations of space. This journey facilitated transient visits to Gestalt psychology, ancient Greek philosophy, mathematics and modern western philosophy.

2.4.1 Absolute and relative space

Among theorists in Gestalt psychology, Koffka (1935) distinguished between geographical environment (absolute space) and the behavioural environment (relative space). He held that the geographical environment is 'stimulus providing' and that the behavioural environment depends upon the geographical environment and the organism itself. This means that the "results of an animal's behaviour depend not only upon his behavioural but also on his geographical environment" (Koffka, 1935:31).

2.4.2 Mathematical space, physical space and psychological space

Downs and Stea (1973) cite Lewin (another Gestalt psychologist), who stressed the relationship of and distinctions among mathematical space, physical space and psychological life space. These concepts of space resemble those of Koffka, who collapsed mathematical and physical space into absolute space and referred to Lewin's psychological space as relative space. In Greek philosophy Plato and Aristotle explored the notion of space in terms of its physical characteristics and boundaries. Plato viewed space as a receptacle or vessel for objects (Caygill, 1995), whereas Aristotle suggested that space is "the boundary of the containing body at which it is in contact with the contained" (Aristotle cited in McKeon, 1941:31). One of the most influential geometers who speculated with mathematical space, Euclid, acquired his early education in Plato's Academy (Hollingdale, 1994) and was taught by geometers from that school of thought (Euclid, 1956). In his work, *The Elements*, regarded as one of the most influential texts of mathematics (Hollingdale, 1994), Euclid employed a precise, innovative, rigorous and logical methodology (using assumptions, postulates and theorems) to describe and prove geometrical concepts (Mlodinow, 2001), which at the time informed the frames of reference for spatial understanding.

Descartes, in his quest to describe the universe in terms of definable co-ordinates or positions, leaned towards the Platonic position, with the identification of space with "extension in length, breadth, and depth" (Descartes cited in Caygill, 1995:368). In his communication with Sir Henry More, the seventeenth-century English philosopher and theologian, on 5 February 1649 Descartes, although acknowledging the existence of a 'imaginary space' wrote:

> …people commonly think [of space] as being sometimes full and sometimes empty, or something as real and at other times as imaginary. But in space – no matter how imaginary and empty it may be – everyone easily imagines different parts of a determinate size and shape, and in their imagination they can move some parts into the place of others but they cannot in any way conceive of two parts that simultaneously interpenetrate in one and the same place because it implies a contradiction for this to happen unless some part of space is removed. Since I think that such properties, which are so real, can exist only in real bodies, I dared to claim that there cannot be a space which is completely empty, and that every extended entity is a real body. (Descartes, 1998:169)

Building on the work of Galileo, Descartes was one of the first advocates of geometrical physics. Sorrell (1987) suggests that Descartes' physics was constructed out of mathematical facts about material things, from facts about size, shape, composition and speed. Descartes claimed that "he was in the habit of turning all problems into geometry" (Davis and Hersch, 1986). Descartes asserted that mathematics (geometry in particular), which was based on direct observation, is fundamental to understanding the universe. The result of this reductionist approach was a very heavily structured and grid-like outlook on space. Indeed, Descartes is credited as the founder of the Cartesian plane, the foundation for co-ordinate geometry, a system which facilitates the solving of algebraic problems through applying geometric principles. In his book *La Geometrie* he suggests that "any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction" (Descartes, 1925:2)

Newton distinguished between absolute space and relative space. In his view, absolute space is "without relation to anything external, remains always similar and immovable" (Newton cited in Caygill, 1995:368) – it is the space of God. Relative space on the other hand "is some movable dimension or measure of absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable space" (Newton cited in Caygill (1995:368). According to Gardner (1999) Newton's view is of space as an absolutely real, self-subsistent 'container' which would exist even if no physical objects were contained within it.

Leibniz refutes both Descartes' and Newton's views that space is in some sense substantial. He argues that space is relative (Caygill, 1995). Leibniz suggests that the universe is made up of monads, which have no shape or size. A monad is a point, not a mathematical or a physical point but a metaphysically existent point. Whereas Descartes is arguing for a rational and material space, Leibniz clearly proposes a space consisting of non-corporeal forms. Leibniz, however, says that there must be some relation between all the monads which make up the universe, some explanation for their orderly actions which Leibniz refers to as pre-established harmony (Stumpf, 1994). Leibnizian view is of space as a logical construction out of relations between objects (Gardner, 1999).

It can be conceived that in Newton's absolutist model of space, "the universe could shift its position in space and could have been created at a different time from that at which it actually came into existence" (Gardner, 1999:71). Leibniz's relational view of space, however, "grants the plain possibility of empty space and empty time".

Locke avoids the material non-material debate by arguing that space is a simple concept based entirely on our senses of sight and touch. Space therefore is "simple idea which is modified into measures of distance and into figures" (Caygill, 1995:368).

Kant's view on space changed and evolved over time. He initially identified with the Leibnizian definition of space "as the objective relation of substances" (Caygill, 1995:369). With the emergence of a revised definition of metaphysics, which was no longer equated with the science of substantial forces but was identified as the science of the limits of human cognition, Kant turned to a more subjective understanding of space. He asserted that space is an intuition (Gardner, 1999). More specifically, Kant speaks of space as *a priori* intuition which means that it is not derived from experience. This is to say that a perception of space is pure and cannot be derived from either sensibility or understanding. "Space cannot be derived from outer experience" (Caygill, 1995:372).

2.4.3 Personal space

Personal space refers to “an area with invisible boundaries surrounding a person’s body into which intruders may not come” (Sommer, 1969:26). It is not spherical in shape, according to Sommer (1969), as some people are able to tolerate closer presence of strangers at their sides than directly in front. Further, the extent of this space changes from individual to individual as some people like to be close enough for warmth and comradeship whereas others like to be far enough to avoid space invasion. The invasion of personal space is an intrusion into a person’s self-boundaries. These may be physical or they may be psychological. I may feel uncomfortable by an individual’s physical closeness; or I may be feel uneasy at an individual’s intrusion into my relationship space with another person, say. When exploring issues of space with my sample (see 5.3.12 on page 204) the aspect of personal space surfaced on numerous occasions.

2.4.4 Space in the context of this research

One of the central aims of this research was to construct individual world view profiles of space of the participating pupils. As I sought as wide a perception of space as possible I consciously tried not to let the above definitions and understandings dictate my interview approach (see chapter three). I therefore steered clear of providing a single dogmatic definition of space. As it happened, it was fascinating to experience how many of the above ideas emerged throughout the conversations (see chapters four and five). Many of the subjects took a Platonic stance and saw space as something empty into which objects could be placed. Others identified with the personal dimension of space and articulated how they treasured their own space bubble. Others again adopted a more Kantian perspective and equated space with their feelings – as something that feels good and beautiful. Many saw space merely in its physical and planetary (cosmic) form while others took the Newtonian and Cartesian stance and saw space as an ordered, absolute and definable system. The different ideas and positions on space were not mutually exclusive – there was a lot of overlap and diffusion of different ideas.

The extent to which the participants' world view of space relate specifically to the above theoretical notions are discussed in chapter five on page 202.

2.5 EPISTEMOLOGY

The underpinnings of the theoretical constructs of this study are framed by the constructivist paradigm, "a theory of knowing rather than a theory of knowledge" (von Glasersfeld, 1993:24), in general, and social constructivism in particular. The fundamental tenet of this study is consistent with Bruner's claim (1996:19) that "reality is made, not found". This study was influenced by the key constructivist notion that learners are actively involved in their cognition process and are not just passive recipients of information. Von Glasersfeld, (1988), who is often labelled as a proponent of 'radical constructivism' in the literature, suggests that constructivism is in contrast to the traditional epistemological view which regarded cognition as a response to transmissive teaching. In his view, "knowledge is the conceptual means to make sense of experience" (von Glasersfeld, 1990:27) rather than a 'representation' of something that is supposed to lie beyond it. This implies that "the concepts and relations in terms of which we perceive and conceive the experimential world we live in are necessarily generated by ourselves" (von Glasersfeld, 1990:28). Numerous researchers (Ernest, 1991a; Goldin, 1990; and Lerman, 1992b) have criticised von Glasersfeld's apparent emphasis on the individual construction of meaning at the expense of underlining the importance of social interaction in an individual's cognitive development. Von Glasersfeld (1995:12) disputes this by arguing that, consistent with Piagetian thought, "the 'others' with whom social interaction takes place are part of the environment, no more but also no less than any of the relatively 'permanent' objects the child constructs within the range of his or her lived experience".

Proponents of 'social constructivism' view cognition as a social construct, meaning that learning is a process based on interaction with peers, teachers and others (Cobb, Wood and Yackel, 1990). Referring specifically to mathematical knowledge as a social construct, Ernest (1991:43) views the following as fundamental:

- Linguistic knowledge forms the basis for mathematical knowledge – language is fundamental to social construction;
- Interpersonal social process are key to cognition;
- Mathematical knowledge can be either objective or subjective knowledge – social constructivism "links them in a creative cycle".

Ernest's views above are strengthened by the work of Vygotsky, who sees language as a key component in the cognition process. When considering the formation of concepts, Vygotsky (1986:107) suggests that "real concepts are impossible without words, and thinking in concepts does not exist beyond verbal thinking. That is why the central moment in concept formation, and its generative cause, is a specific use of words as functional tools". Ernest (1994b) himself, however, cautions against conflating Vygotskian and Piagetian forms of social constructivism. He suggests that the latter is based on the cognitive aspects of the psychology of mathematics education located within a specific and complex research methodology and theoretical underpinning, whereas the former focuses on the acquisition of semiotic skills, the learning of written and spoken language, the role of the teacher, and the overall social context.

It needs to be noted that although much of the research around constructivist theory pertains to content such as electrolysis, energy and momentum in science and trigonometry and algebra in mathematics, scant attention is given to meta-knowledge – "views of the nature and range of science [and mathematics], as well as scientific [mathematical] ways of thinking" (Duit, 1995:283). Duit's (1995:283) claim that constructivism "highlights the importance of meta-knowledge" is very pertinent and central to this study. From a science-education perspective he argues that "students' views about science are not only issues in their own right, but also have a significant influence on the acquisition of science". I would argue that a parallel situation for mathematics education also holds. This study focuses specifically on meta-knowledge by analysing world view of space within the broader theme of spatial conceptualisations. Duit (1995:283) quite rightly observes that in science education there is "only little research available in this field" – this experience, in my view, can be directly transferred to mathematics education.

As Ireland (2000) notes, the learners' prior knowledge is an essential cornerstone of constructivism. It refers to learning experiences that learners bring to the classroom and knowledge which the learners may have acquired outside the classroom situation. The Australian National Statement for Mathematics, for example, recognises the importance of prior knowledge when it articulates that "learning experiences which teachers provide should build on strengths with which students come to the classroom and broaden their horizons and the range of contexts in which they can function" (AEC, 1990:9). Regrettably the South African Curriculum Statement Grades R-9 (Statements for Grades 10-12 are in the process of being constructed) does not explicitly recognise the importance of prior knowledge or the constructivist notion that all learners bring unique learning experiences to the classroom and that these experiences need to be harnessed by the teacher. The preamble of the South African Constitution, however, states inter alia that the aims of the Constitution are to "heal the divisions of the past and establish a society based on democratic values, social justice and fundamental human rights" (South Africa, 2002:1). Reference to the past implies that recognising earlier experiences will form a crucial element in the development of a new society in South Africa. Indeed the Norms and Standards for Educators refers explicitly to "recognition of prior learning" (South Africa, 2000:8). It suggests that recognition of prior learning "means the comparison of the previous learning and experience of a learner howsoever obtained against the learning outcomes required for a specific qualifications, and the acceptance for purposes of qualifications of that which meets the requirements" (Republic of South Africa. 2000:8)

For the purpose of this study, the notions of world view (the epistemological macro-structure of an individual) and prior knowledge are very closely related. Constructivism is not without its problems and pitfalls. Lerman (1992a), for example, cautions against replacing one rhetoric with another. He suggests that labels such as 'the constructivist teacher' and 'the constructivist classroom' are often bandied about when the implications of constructivism are not clearly understood. I have often come across teachers who, in their repertoire of techniques, have used 'constructivist tools' yet have failed to embrace the fundamental paradigm shift implied by constructivism. I therefore agree only partially with Ireland's (2000:72) assertion that "working within a constructivist paradigm … implies adopting a style or process that facilitates the students in their construction of knowledge". Embracing constructivism has deeper implications than adopting teaching styles and techniques: it requires of one a fundamental shift in how one views the learner and the cognition process. I agree with Duit's (1995) observation that unfortunately the notions of constructivism are often adopted in a superficial manner and viewed as somewhat 'fashionable'.

Although it is beyond the scope of this literature review to engage on a philosophical level about mathematical knowledge, in the context of social constructivism I wish to point out that Ernest (1991b:60) warns against the problem of "relativism of mathematical knowledge". He argues that if mathematical truth rests on social conventions, as advocated from a social constructivist perspective, then it is both "arbitrary and relative". In other words, it can be construed that mathematical knowledge "only holds relative to a particular culture at a particular period".

Related to Ernest's concern above, Duit (1995) notes that it can be construed that constructivism can lead to "solipsism" - the view that the self is all that exists or can be known (*The Concise Oxford Dictionary*, 1995). "The idea that individuals actively construct their knowledge on the basis of already existing conceptions, is in danger of leading to solipsism" (Duit, 1995:273). I argue, however, that social constructivism explicitly favours learning within a social context that will prevent "idiosyncratic" (Duit, 1995:274) and egoistic constructions.

Implications for this study

The epistemological underpinnings articulated above are not only at the heart of this study, but also reflect part of my own world view. This interconnection has had a direct influence on the choice of research methodology as discussed in the following chapter, and the analysis and interpretation of the data.

2.6 CONCLUSION

This chapter has described the multiple theoretical perspectives which inform this study. The widespread use of various terminologies in the research literature was noted and an attempt was made to synthesis these. Workable definitions that would underpin the theoretical framework of this study were articulated. This included definitions for:

- spatial visualisation
- spatial orientation
- spatial capacity
- spatial conceptualisation
- world view

The concept of space was briefly discussed in its philosophical and psychological context. Specific references and linkages were made as to how and where the underpinning theory will inform particular aspects of the research. The underlining epistemology was dominated by references to social constructivism and specific reference was made to the inter-relatedness between prior knowledge, a cornerstone of constructivism, and world-view. This study attempts to break new ground and extend the body of literature in mathematics education in two main areas:

- It adopts a broader understanding of spatial conceptualisation by incorporating individuals' world-view perspectives;
- it attempts to apply the logico-structuralist approach of Kearney's (1984) world-view model to explore world views.

The following chapter will further contextualise this research within its research paradigm and focus on the research orientation, design and process that framed this study.

CHAPTER THREE

THE METHODOLOGY

3.1 INTRODUCTION

This chapter describes and explains the process of research followed in this study. After a brief overview of the study, its methodology is articulated in terms of the following structure:

- orientation
- design
- process

Although this structure may give the impression of being linear (Southwood, 2000), this was not the intention of the effect: the entire research process was a complex and dynamic exercise, entangled with interrelationships, spanning three years of interaction with many interesting people.

The essence of this research is to capture the complexities and richness of the processes that contribute to the overall spatial conceptualisation of individual learners. The research suggests that traditional pen-and-paper tests do not necessarily reflect an individual's spatial conceptualisation accurately. There is much more to spatial conceptualisation than only spatial skills. Every individual has an underlying world view of space that is often not manifested in his/her spatial skills and capacities; our world view of space is integral to our overall spatial conceptualisation. The research therefore explores the notion of spatial conceptualisation not only through a pen-and-paper test, but also with the aid of an innovative hands-on-activity test and through the establishment of individual world view profiles. The two tests, the Adapted Monash Space Test (AMST) and the Hands-On Activity Test (HAT), that were central in the exploration of the spatial visualisation and orientation constructs are described in detail towards the end of this chapter.

This research takes the form of a multi-site case study (Burgess *et al.,* 1994; Yin, 1994) and involves 32 learners in the age group 16-19 years, and their mathematics teachers in five secondary schools in the Eastern Cape, South Africa.

As suggested in section 1.5 on page 9, generalist claims are not being made although it is expected that some of the interpretations and results will be useful in other contexts and will perhaps lead to further investigations.

3.2 ORIENTATION

Although it is difficult to pigeonhole this research within one methodological category due to its multidisciplinary and multifaceted nature, the paradigm within which this research is located is the interpretivist-naturalistic paradigm (Schwandt, 1994; Lincoln and Guba, 1985; Denzin and Lincoln, 1994; Cohen *et al.,* 2000). The methodological approach of this work is consistent with Cohen *et al.*'s. (2000:23) notion that interpretivist research " …begins with individuals and set[s] out to understand their interpretations of the world around them." Much of the research process is characterised by dialogical interaction with the participants in order to "uncover what people believe" (Cantrell, 1993:84).

Key features within the interpretivist paradigm that characterises this study can be listed as follows:

- Interpretation is key to understanding any situation;
- Understanding a situation is based on the perceptions and points of view of those who live in it;
- This research thus started with individuals and sought to understand their interpretation of their world (Schäfer, 1999);
- The study can be described as small-scale research;
- It focuses on interpreting the specific;
- It seeks to understand the meanings rather than the causes;

- The research relies on micro-concepts: individual perspectives and personal constructs (Cohen *et al.*, 2000);
- Although an enabling and strategic framework guided the research, it is seen as a process that emerged and grew;
- The methodology is multidimensional and multifaceted;
- The data gathered used methods of interviews, questionnaires, pen-and-paper tasks and hands-on activity tasks.

Give the above, it would be expected that this study be characterised by the qualitative tradition that "focuses on the in-depth, the detail, the process and the context " (Lemmer, 1992:294). The multifaceted nature and the complexity of the data coupled with a multidimensional design, and the desire to explore links between different sets of data has, however, inspired me to use both qualitative and quantitative methods. The motivation behind integrating quantitative and qualitative data was not to validate the different data sets. They give "data on different things, so the relationship between them cannot be one of confirmation or contradiction" (Mason, 1994:109), although they may contain similar or discrete themes. The purpose was to yield a meaningful and rich narrative that accurately reflects the situation being studied.

Researchers drawn to the 'Oppositional Diversity' Thesis (Walker and Evers, 1988) regard the integration of qualitative and quantitative methods as incompatible because the mehods are epistemologically underpinned by different paradigms that are "incommensurable in that neither educational research nor any form of inquiry can provide a rational method for judging between them." The quantitative tradition is located within a positivist philosophy of science whereas the qualitative tradition is rooted within a post-positivist paradigm.

From the perspective of a post-positivist paradigm of research, this study has a strong interpretive orientation focusing on "gaining insight and understanding" (Southwood, 2000). It also has a naturalistic approach to the world in that it attempts, in the main, to make sense of, or interpret, phenomena in their natural settings (Denzin and Lincoln, 2000).

It can, however, be argued that much of the study is also based on quantitative principles which are characteristic of a more positivist orientation. Where quantitative methods (in the form of descriptive statistics) were applied, it was in the broadest context of obtaining baseline information as starting points for interpretation rather than for articulating generalisations. Quantitative data were also used to establish initial tentative links and connections between different sets of information that were later analysed and interpreted in depth.

In support of an integrative approach, Schulman (1985) not only suggests that qualitative and quantitative traditions complement each other but that any social science field committing itself to a single paradigmatic view can be counterproductive. Mason (1994:106) asserts that an integrative approach is useful when:

- following up similar themes in different data sets "as a way of linking the data";
- addressing a particular topic from a variety of angles.

Mason (1994:107) warns, however, that the necessary technical competencies need to be developed in order to "deal with data that have different logical principles". Further she emphasises that to integrate quantitative and qualitative data analysis "involves developing mechanisms" to ensure that meaningful, sensible and appropriately limited questions are asked of the data set. I took encouragement from Hammersley (1992:172) who argues strongly that there is no necessary relationship between adopting a particular position – "many combinations are quite reasonable". He emphasises that these positions ought to depend on the purposes and circumstances of the research "rather than being derived from methodological or philosophical commitments".

3.3 DESIGN

The research design can be described as a multi-sited case study (Sowden and Keeves, 1988; Stenhouse, 1988) focusing on 32 high school learners from five secondary schools in the Eastern Cape, South Africa. The sampling process was consistent with Cantrell's (1993:90) qualitative approach of using a "small, information-rich sample, selected purposefully to allow the researcher to focus in depth on issues important to the study". According to Stenhouse (1988:50), evaluative and ethnographic case studies usually form single cases, but in the case of "educational case studies" multi-sited approaches are increasingly being used.

In particular, this inquiry may be called an instrumental case study (Stake, 2000:437) as it examines a (multi-sited) case to "provide insight into an issue", namely spatial conceptualisation and world view. Although the case is studied and analysed in detail, its context scrutinised and its activities detailed, its role is secondary to the prime purpose of understanding spatial conceptualisation and world view.

3.3.1 Sample

The sample consisted of 32 learners in the age group 16-19 years old from five secondary schools situated in the Eastern Cape and readily accessible from Grahamstown.

As this in-depth case study focused particularly on spatial conceptualisation amongst South African learners, I selected schools across the cultural spectrum of the South African educational landscape. The cultural spectrum represented in this sample not only pertained to ethnicity but also classroom culture. I was therefore also interested in included single-sex schools. The prime criteria for selection were:

- geographical accessibility;
- willingness to participate;
- cultural representivity.

S1 is a single-sex secondary state school for girls (Grade 8 – Grade 12), often referred to as the sister school of S4. It has a learner population of 400 girls and is characteristic of a typical 'middle-class' (see page 54) school situated in a former white suburb. It has a very attractive campus with beautifully crafted buildings. Prior to the 1994 democratisation process of the South African education system, S1 was a whites-only school. It has a boarding establishment and attracts learners from all population groups throughout the Eastern Province. The school has a good infrastructure and is relatively well resourced (compared to its counterparts in the townships) with a full complement of appropriately qualified staff. It is strongly supported by a financially viable parent body - to the extent that it can employ its own 'governing body' staff in areas where the state does not provide sufficient personnel. In order to make up for the short fall in state subsidy and maintain an average class size of 30-40, it charges school fees of R3640 per learner per annum.

S2 is a largely 'middle class' state co-educational secondary school located in a small coastal town of the Eastern Cape. During the apartheid era it was the only secondary school that serviced the white community of that town, but currently draws learners from all population groups of the area. It accommodates 335 learners and has an infrastructure comparable to S1. As with all other state schools it is dependent on a skeletal state subsidy. Additional funding for extra staff and resources are acquired by annual school fees, which amount to R2950 per learner per annum, and self-organised fundraising events. It has a relatively strong local support base.

S3 is also a co-educational state secondary school situated in the 'township' of the same town as S1 and S4. Despite being housed in buildings no different from S2 it is in a poor state of repair, inadequately resourced and bursting at its seams in terms of accommodating the 800 learners who attend it. The school operates with minimal resources and at times has to cope with classes of over 70 learners. As in the apartheid years, it currently draws its learners only from the black population group. It relies on a support base characterised by apathy, minimal involvement, high unemployment and poverty. As with the other schools, it draws its skeletal state subsidy which at best covers the teachers' salaries and minimal terrain maintenance. Other sources of income are derived from erratically paid school fees (R960 per learner per annum) and donations from aid agencies and other welfare organisations.

S4 is a single-sex state secondary school that accommodates 310 boys from Grade 8 to Grade 12. It is a largely 'middle-class' state school situated in a former white suburb with an ethos and history similar to S1. The school has a well-maintained and attractive campus with an excellent infrastructure. It also has a boarding establishment and hence draws its learners from all over the Eastern Cape and further afield. It attracts learners from all the population groups. It is a well-resourced school supported by a financially viable parent body. In order to boost the state subsidy and enable the school to employ more teachers it charges school fees of R4160 per boy per annum.

S5 is rural school situated in the remote Winterberg mountains of the Eastern Cape. It forms part of a newly established community trust which was developed to uplift local farm communities. The trust offers a range of upliftment programmes such as literacy courses and other self-help schemes. The heart of the trust is a resource centre that boasts facilities including a library, a computer centre, and a school that caters for learners from Grade 1 to Grade 10. The school draws its learners mainly from families of local farm labourers.. The school population at present stands at 220 learners, of which 65 are in the secondary school (Grade 8-Grade 10). The learners are bussed to and from the school every day. The school is well resourced with the learners being able to use the other facilities of the resource centre. As with the other schools, the minimal state subsidy covers the salaries of the teachers. Extra funding is acquired from the private sector by tapping into corporate social development programmes, and from school fees which amount to R50 per learner per annum.

The initial intention was to involve six Grade 9 learners from each of the above schools, but as the research design developed and evolved it became apparent that the participants would require the capacity to articulate complex concepts regarding their own world views. They needed to have developed clear personal perceptions and ideas, and I felt that the involvement of older adolescents would be more suitable. As I required the commitments of the participants for the duration of an entire year and as the Grade 12 learners would be preoccupied with preparating for their matriculation examinations, I settled for at least six learners in the age group 16-19 years who were in either Grade 10 or Grade 11. The selection procedure is explained in 3.3.2 on page 58 and the distribution of the sample across the schools is illustrated in Table 3.1. The table also indicates the age (in years and months at the end of 2001), the gender, and the home-language of each of the participants.

The same sample was used for the entire duration of the study. With the exception of one pupil (P21), the commitment of the participants was excellent. P21's attendance was erratic due to commitments at school and it was difficult to pin him down in his free time. This did not prove to be problematic for the research as there were already six participants from that particular school.

Table 3.1 School, age, gender and language distribution of sample.

PUPIL	SCHOOL	AGE December 2001	GENDER	FIRST LANGUAGE
P1	S1	16.10	Female	Xhosa
P2	S1	18.2	Female	Xhosa
P3	S1	17.6	Female	Xhosa
P4	S1	16.11	Female	English
P5	S1	17.12	Female	English
P6	S1	16.8	Female	Xhosa
P7	S1	17.4	Female	English
P8	S2	17.3	Female	English
P9	S2	16.10	Female	English
P10	S2	17.8	Female	English
P11	S2	18	Male	English
P12	S2	17	Female	English
P13	S2	17.4	Female	English
P14	S3	17.1	Male	Xhosa
P15	S3	17.1	Female	Xhosa
P16	S3	17.6	Male	Xhosa
P17	S3	16	Female	Xhosa
P18	S3	17.1	Female	Xhosa
P19	S3	16.9	Female	Xhosa
P20	S4	17.1	Male	Xhosa
P21	S4	16.6	Male	Xhosa
P22	S4	16.11	Male	English
P23	S4	17.4	Male	English
P24	S4	17.7	Male	Sotho
P25	S4	17.7	Male	English
P26	S4	18.3	Male	English
P27	S5	17.3	Female	Xhosa
P28	S5	18.10	Female	Xhosa
P29	S5	14.8	Female	Xhosa
P30	S5	17.11	Male	Xhosa
P31	S5	17.5	Female	Xhosa
P32	S5	17.3	Female	Xhosa

As can be expected in the South African context, the socio-economic backgrounds of the participators varied greatly. As mentioned in the profiles of the participating schools, S1, S2 and S4 draw on a middle class clientele whereas S3 and S5 draw on a very poor community. To obtain a sense of the participants' socio-economic background I used an adapted questionnaire (see 3.4.1) which was similar to the one used by the Third International Mathematics and Science Study (TIMSS) (Howie, 2002). In order to establish a crude socio-economic profile of the sample, the participants were asked to respond to the following indicators:

- how many people lived in their house;
- whether, in their homes, they had access to:
    - a calculator
    - a computer
    - a television
    - a dictionary
    - an encyclopedia
    - electricity
    - running water;
- whether they had their own desks;
- the number of books (other than magazines) present in their homes.

The rationale behind these criteria is based on largely western middle-class conditions (those associated with families from professional and business workers), which suggest by and large that an adolescent would come from a family of approximately four to five and have access to a calculator, computer, television, own desk, dictionary, encyclopedia and some other books. His or her home would have running water and be connected to a permanent electricity supply.

Table 3.2 provides a summary of the responses. In terms of indicating access to a calculator, computer, TV, own desk, dictionary, encyclopedia, electricity and running water, a '1' indicates access and a '0' no access.

As is evident, S3 and S5 deviated sharply from the other schools for most of the indicators:

People living at home: For S1, S2 and S4 the average number fluctuated between four and five, whereas for S3 and S5 the average was 7,5 and 5,7 respectively.

Calculator: All the participants except for P14 and P32 in S3 and S5 had access to a calculator at home.

Computer: None of the pupils in S3 and S5 had a computer at home, whereas for the rest of the sample, between 70% and 80% owned a computer.

TV: Interestingly enough, everyone except P27 from S5 had access to a television set.

Own desk: Once again, only pupils from S3 and S5 did not have access to their own desk at home. Only one pupil from S3 had her own desk.

Dictionary: Although all the participants from S1, S2 and S4 had a dictionary at home, very few from S5 owned such a resource.

Encyclopedia: None of the participants in S5 had access to these reference books, whereas only two from S3 indicated that they were in possession of such books.

Electricity: This effective indicator of socio-economic background shows that whereas 100% of the pupils from S1, S2 and S4 had electricity at home, only 70% of those from S3 and S5 did.

Running water: A similar scenario existed in the case of running water. Surprisingly only 30% of the township pupils had running water, compared with 80% of the rural pupils.

Books: All the participants from S5 indicated that they had less than 10 books in their homes.

Table 3.2 Socio-economic profile of sample

Name	People in home	Calculator	Computer	TV	Own desk	Dictionary	Encyclop.	Electricity	Running water	Books
P1 S1	5	1	1	1	1	1	1	1	1	26-100
P2 S1	9	1	0	1	1	1	1	1	1	26-100
P3 S1	5	1	0	1	1	1	0	1	1	26-100
P4 S1	4	1	1	1	1	1	0	1	1	>200
P5 S1	4	1	1	1	1	1	0	1	1	26-100
P6 S1	6	1	1	1	1	1	1	1	1	11-25
P7 S1	2	1	1	1	1	1	1	1	1	26-100
Average	5.0	1.0	0.7	1.0	1.0	1.0	0.6	1.0	1.0	
P8 S2	4	1	1	1	1	1	1	1	1	101-200
P9 S2	7	1	1	1	1	1	1	1	1	101-200
P10 S2	3	1	1	1	1	1	1	1	1	101-200
P11 S2	3	1	1	1	1	1	1	1	1	101-200
P12 S2	4	1	0	1	0	1	1	1	1	26-100
P13 S2	5	1	1	1	1	1	1	1	1	>200
Average	4.3	1.0	0.8	1.0	0.8	1.0	1.0	1.0	1.0	
P14 S3	5	0	0	1	0	0	0	1	0	<10
P15 S3	8	1	0	1	0	1	0	0	0	<10
P16 S3	17	1	0	1	0	1	0	1	1	11-25
P17 S3	6	1	0	1	0	1	1	1	0	101-200
P18 S3	4	1	0	1	1	1	1	1	1	26-100
P19 S3	5	1	0	1	0	1	0	0	0	<10
Average	7.5	0.8	0.0	1.0	0.2	0.8	0.3	0.7	0.3	
P20 S4	2	1	0	1	1	1	0	1	1	101-200
P21 S4										
P22 S4	4	1	1	1	1	1	1	1	1	101-200
P23 S4	6	1	1	1	1	1	1	1	1	26-100
P24 S4	5	1	0	1	1	1	1	1	1	26-100
P25 S4	4	1	1	1	1	1	1	1	1	26-100
P26 S4	5	1	1	1	1	1	0	1	1	101-200
Average	4.3	1.0	0.7	1.0	1.0	1.0	0.7	1.0	1.0	
P27 S5	10	1	0	0	1	0	0	0	1	<10
P28 S5	5	1	0	1	1	0	0	0	1	<10
P29 S5	4	1	0	1	0	1	0	1	1	<10
P30 S5	6	1	0	1	1	1	0	1	1	<10
P31 S5	5	1	0	1	0	0	0	1	0	<10
P32 S5	4	0	0	1	0	0	0	1	1	<10
Average	5.7	0.8	0.0	0.8	0.5	0.3	0.0	0.7	0.8	

3.3.2 Structure

Due to the potential complexity of a multidimensional approach to the data collection process a fairly tight design structure was initially adopted. The purpose was not to stifle an emergent process (Cantrell, 1993) but rather to provide for an enabling framework in which to collect appropriate data. The design comprises of seven stages and a flexible approach was adopted to allow for of an emerging process. This flexibility proved particularly valuable in the final phases of the data analyses where the development and identification of meta-themes grew out of an emergent process of analyses (see 5.4.1 on page 228). Each stage played an important part in informing the next stage. In terms of Cantrell's (1993:89) design continuum which ranges from 'prestuctured' to 'emergent', this research leans towards the prestructured with elements of an emergent design. Miles and Huberman (1984:27) caution against an extremist position: "one can make a case for tight, prestructured qualitative designs and for loose, emergent ones". I took note of their warning that "the looser the initial design, the less selective the collection of data; everything looks important at the outset to someone waiting for the key construct or regularities to emerge … and that wait can be a long one … the researcher, awash with data, will need months to sort it out".

Stage 1 Personal acquaintance

This stage consisted of establishing a personal relationship with the participating learners and their mathematics teachers. This initially required two meetings with the five mathematics teachers from each school and myself. The issues of spatial conceptualisation and world-view as outlined in chapter two were discussed and explored with the intention of motivating and inspiring the teachers to become involved. They all showed immediate interest and pledged their support and commitment. This was very important to me as I would not rely only on the infrastructure of their school but also their organisational assistance in setting up meetings with the participating learners – not always an easy task considering the extensive extramural programme some of the learners were involved in. When seeking permission from the principals of the five schools and the parents I made it very clear that I would not use any of the learners' school time and that the research would be conducted 'after hours'. As the teachers were well acquainted with the pupils who would be involved in this research, I saw their role in the selection process of the learners as very important. As this research is underpinned by an interpretivist-naturalistic paradigm and constructivist epistemology, and I had no intention of generalising, the notion of randomness was not an issue. I needed participants who:

- wanted to participate;
- were reasonably articulate in the English language;
- were reflective and able to talk about their world view;
- took mathematics as a subject (their performance was not a criterion).

After our two meetings, as explained above, the selection process was left to the teachers. The initial interest shown by the learners was sufficient and in the end there was no need to shortlist any of the learners and engage in a further 'selection round'. To ensure familiarity and security, the whole data-gathering process took place in the schools over the entire year of 2001. The first contact session was spent in getting to know each other and ensuring familiarity with the intentions of the research. At first, a number of learners expressed anxiety and concern, but as they realised and appreciated their roles in this project they became very excited. This was reflected in their commited attendance and participation throughout the year.

Stage 2 Personal profiling

In order to construct individual profiles and contribute towards creating narratives it was important to obtain personal details such as socio-economic and home background information. Further, it was necessary to establish an initial and 'baseline' overview of the participants' understanding and perception of space. This stage offered also the participants an initial opportunity to express themselves and reflect on their experiences and perceptions. This stage was used to 'break the ice' and provide an indication to the participants of what lay ahead.

Stage 3 World- view profiles

This stage consisted of one-to-one conversations between the individual learners and myself. These took place at the individual schools, were based on a set of themes (see page 77) and built on what the learners had articulated in Stage 2. The average duration of each conversation was one hour.

Stage 4 Spatial visualisation and spatial orientation

Characteristic of this stage was the exploration of the participants' spatial visualisation and spatial orientation capacities through the Adapted Monash Space Test (AMST), the Mathematical Processing Questionnaire (MPQ) and the Hands-On Activity Test (HAT) (see 3.5 on page 85). This stage commenced with the HAT, which took one afternoon per school to complete. This was followed up by the pen-and-pencil AMST, which took each learner two hours to complete. The MPQ was administered in conjunction with the AMST.

Stage 5 Data analysis and exploring theories of consistencies

This initially involved a quasi-quantitative analysis of the AMST and the HAT, establishing patterns and trends for a more detailed interpretative analysis process. In conjunction with this, the conversations were transcribed and analysed on two levels:

- Establishing themes and trends for creating individual space world-view profiles;
- With the aid of the profiles established in level 1, establishing meta-profiles through a process of meta-analysis (see 5.4 on page 226).

Stage 6 School performance

This stage involved gathering data from learners regarding their performance in the mathematics classroom throughout the year. Appendix 2 gives the questionnaire that was used.

Stage 7 The narrative

The resultant narrative was an attempt to combine the data from Stage 4, Stage 5 and Stage 6 into a rich discussion and exploration of spatial conceptualisation. This study made use of extensive descriptive narrative to enable the reader to "vicariously experience" (Stake, 2000:439) the case and "draw conclusions (which may differ from those of the researcher)". Stake (2000:439) observed that case study researchers face "strategic decisions in regard to how much and how long the complexities of the case should be studied". He warns that "damage occurs when the commitment to generalize or to theorize runs so strong that the researcher's attention is drawn away from the features important for understanding the case itself". I have therefore relied on thick descriptions (Stake, 2000) in exploring the objectives of this study.

From the above, the research design may appear to be very neat and linear in approach. In reality, however, the individual stages overlapped significantly and the activities characteristic of the stages often happened simultaneously.

3.4 PROCESS

3.4.1 Collection of data

Questionnaires

A questionnaire (Appendix 1) containing both "closed" and "open-ended" questions (Cohen *et al.*, 2000:248) was initially used during Stage 2 in order to obtain specific information about the participants:

- initial ideas about space;
- personal details;
- socio-economic background.

As I wanted a 100% completion and return rate, the individual learners were required to complete the questionnaire during one of my early contact sessions. Further reasons for insisting that the participants complete the questionnaire during the contact session was to avoid having to make use of an unreliable postal service. Also, I did not wish to add an administrative burden on already over-committed teachers.

A second questionnaire conducted during Phase 6 of the project focused on the school performance of the participants. This questionnaire (see Appendix 2) was filled in by the mathematics teachers concerned and provided quantitative data on the learners' mathematics achievement throughout the year as well as qualitative information regarding the teachers' perception of the individual learners. Although I expected a 100% return rate (only 5 teachers were involved), I struggled to receive them all back in good time. One of the teachers, for example, took six months to complete the questionnaire.

Interviews

Constructing world view profiles of the participants in terms of how they perceived space in the broadest sense required a research tool that facilitated a situation where the participants could articulate detailed and rich descriptions (Weiss, 1994). It also required a situation where I as a researcher could "gather descriptive data in the subject's own words and … access the unobservable", which would enable me "to develop insights into how the participants interpret and make meaning of the world" (Cantrell, 1993:96). The semi-structured interview seemed most appropriate as it is a "two-person conversation initiated by the interviewer for the specific purpose of obtaining research-relevant information, and focused by him [her] on content specified by research objectives of systematic description, prediction and explanation" (Cohen *et al.,* 2000:269). Interviews "seek the words of the people we are studying, the richer the better, so that we can understand their situations with increasing clarity" (Ely *et al.,* 1991:58).

Recorded interviews formed the central data-gathering method for Stage 3. The approach was a semi-structured one where the participants were encouraged to talk "freely and emotionally" (Cohen *et al.*, 2000:270) in order to encourage "candour, richness, depth, authenticity and honesty". The interviews were shaped, however, by a framework of themes that were inspired by the work of Cobern (1991), Kearney (1984) and Slay (2000) on establishing word-view profiles (see 3.4.4 on page 77). This structure (Fontana and Frey, 2000) remained more or less consistent for all the 32 interviews.

Using interviews and dialogues as central data-gathering strategies suited the overall style and epistemological spirit of this research. In many ways this thesis centres on people's stories – the verbal articulation of experiences, perceptions and feelings. The Vygotskyan school of thought would view stories as a microcosm of people's consciousness (Vygotsky, 1986).

In order to alleviate initial fears and soften the tension and anxiety of being recorded, I commenced each interview with reflecting on how each student responded on the item in the questionnaire that asked:

> **5. What do you feel like when you think about SPACE?**
>
> Complete the sentence:
>
> When I think of space I ____
>
> See Appendix 2

Despite giving me permission to record our conversations, the participants were at first too nervous to talk freely in the presence of a tape-recorder, but their self-consciousness diminished as the conversations progressed. I assured them of the confidentiality of the research, their anonymity, and their editorial right to study and alter their transcripts if they so wished. I felt that the interviews went surprisingly smoothly – they talked freely, expressed themselves spontaneously and generally appeared very comfortable to talk about their perceptions on space. Many commented after the conversations that they "*never had to talk and think about these things*". They clearly were challenged by the topic, and P10 suggested that she would *"instigate a family discussion of the topic once I get home"*.

Although I recognised the advantages of transcribing the conversations myself, I had time constraints and therefore employed a transcriber to document each of the conversations. The time I saved I used to analyse the transcripts.

It is important to point out that although English was the prime medium of communication, it was not the first language of all participants (see Table 3.1). In the case of S3, the participants were happy that I asked questions and provoked conversation in English but agreed to respond in Xhosa if they could not manage the English. In those instances the transcripts were written out in Xhosa and translated into English for analysis. In the case of S5, the conversations were a mixture of English and Xhosa. A translator (who subsequently also transcribed the interviews) was present throughout the conversations and translated into the vernacular when necessary. At S1 and S4 there was no need to translate as all the participants were fluent in English. The same applied for S2 as none of the participants there had English as a second language. Southwood (2000) alluded to the notion that the mediation role of a translator could negatively affect the relationship between the researcher and the participants. To avoid such a situation and to ensure accessibility, an experienced translator was used who was familiar to the participants – someone they could relate to and be comfortable with.

At the end of each interview the participants were invited to reflect on what they had said by choosing a picture from a portfolio of 11 artworks. Most of the pictures were prints created by the artist M.C. Escher (see Appendix 3). The rationale behind this exercise was to offer the participants an opportunity to illustrate, reinforce or enrich what they had said during the interviews by using works of art. They could choose three pieces from a portfolio of 11 pictures. They were then asked to articulate why and in what way the chosen pictures resonated with what they had said. The works could either contradict or complement what they had said.

Seven of the pictures were lithographs and woodcuts by M.C. Escher and were chosen because of his wonderful talent of challenging our spatial assumptions. Escher was a master in dealing with tensions "inherent in any flat representation of a spatial situation.

In many of his prints he causes the spatial to emerge from the flat surface. In others he makes a conscious attempt to nip in the bud any spatial suggestion that he may have brought about" (Ernst, 1985). The other pictures were 'Mankind breaking through the Clouds of Heavan and Recognition of New Spheres' by C. Flammarion, the 'Last Supper' by Salvador Dali, the 'Emblemata Nova de Secretiis Naturae Chymica' by Michael Maier, and a NASA satellite photograph of earth taken whilst orbiting the moon.

Pen-and-Paper Space Test and Hands-On Activity Test

Stage 4 of the research centred on the completion of two tests. The first was of an adaptation of the Monash Space Test (Wattanahawa, 1977), a traditional pen-and-paper test devised by researchers from Monash University in 1977 for investigations into spatial performance (see Appendix 4). As the original test was not fully consistent with my intentions I altered some items to suit the needs of this research (see page section 3.5.2 on page 86 and chapter four for more details). The Adapted Monash Space Test (AMST) retained 13 of the original tasks and added another 12 that suited this research. The specific aim of the AMST was to test capacity in spatial visualisation and spatial orientation (see pages 86 and 87) and it was felt that the original test did not adequately cater for that aim. To ensure a reasonable spread between spatial visualisation and orientation items, some of the original items were therefore discarded and others added. Further, to ensure that the test was pegged at the appropriate level (for 16-19 year olds) suitable items had to be included. This involved discarding some of the original items and replacing them with others. This was done in consultation with my supervisors. A detailed discussion regarding validity and robustness follows in section 3.4.3 on page 69. As this was not a time trial, the participants were under no time pressure to complete the test. The pilot revealed that two hours was ample time for themt. I piloted this test with 10 pupils of similar ages to those in the sample in order to ensure:

- that the language was accessible;
- there were no errors in the items;
- that the difficulty level of the AMST was appropriate;
- that the time allocated was suitable.

In conjunction with the AMST I made use of an adapted processing questionnaire (AMPQ) which attempted to ascertain how the participants solved the individual tasks and what strategies they adopted (see Appendix 5). This test was inspired by Suwersono (1982), who devised a Mathematical Processing Test in his research into the extent to which students used visual imagery in their mathematical thinking. The AMPQ was administered immediately after the participants had completed their AMST.

The second test, the HAT (see section 3.5.3 on page 107 and chapter four), presented eight 'hands-on' activities involving spatial visualisation and orientation skills (see Appendix 6). A number of the activities were used by Wilmot (1998) in her research on graphicacy and Vye (2001) in her work on spatial skills. The activities were piloted many times with teachers and learners in numerous contexts by other researchers and myself. I had thus accumulated sufficient experience in the management and administration of the activities to administer this test to my sample without another pilot. The activities were set up in a physical circle and the participants moved from one activity spot to the next. As timing was crucial in some of the tasks, I made use of an assistant to help with time-keeping.

So as not to disrupt the flow of this discussion, a detailed description of the two tests are postponed to 3.5 towards the end of this chapter.

3.4.2 Analysis

> To analyse is to find some way or ways to tease out what we consider to be essential meaning in the raw data; to reduce and reorganize and combine so that the readers share the researcher's findings in the most economical, interesting fashion. The product of analysis is a creation that speaks to the heart of what was learnt.
>
> (Ely *et al.*, 1991:140)

I took much encouragement from the above quote as it implies that the analysis process can be a highly innovative and unique process – one that does not necessarily rely on accepted norms and standards. Validity and reliability checks cannot be ignored, however, and a detailed discussion follows in section 3.4.3 on page 69. My initial intentions were naively based on the traditional linear approach of first collecting the data and then analyzing it. In reality, however, both happened simultaneously, with analysis going on all the time. I can identify with Ely *et al.*'s (1991) assertion that analysis is an integral part of data-gathering. Southwood (2000:62) makes a useful distinction between informal (inductive) and formal (deductive) types of analysis, suggesting that the former occurs throughout the process of research whereas the latter occurs after the data has been collected and "is more specific in technique".

The analysis of the raw data of this project can be broadly divided into two categories:

The one category of analysis focused on the data received from Stage 4 of the research process, the 'performance' on the AMST and the HAT. The initial analysis in the AMST performance was quantitative. The participants either correctly or incorrectly answered the tasks. Rudimentary statistical comparison and correlation techniques were applied. These results were used to identify trends and patterns that were then dealt with interpretatively. Performance on the HAT was assessed in the same way. Cross-correlation techniques were applied to once again ascertain rudimentary trends between the two tests before engaging in a more qualitative analysis process. I found this process very useful as it created 'accessible hooks' for me to hang my raw data on very early in the analysis stage.

The other category of analysis concerned data received from Stage 3 of the research process, the interviews. Miles and Huberman (1984:21) suggest that qualitative analysis consists of three "concurrent flows of activity". This resonated with my research strategy and fitted the research process. The first 'flow' is that of *data reduction* where raw data is put into a form that is manageable and conducive to interpretation. The overarching reduction process in this research was the transcribing of the conversations. Themes were then clustered and world-view profiles (in the form of concept maps) were constructed (see section 3.4.4 on page 77). This was consistent with the model adopted by world-view studies conducted by Cobern (1991) and Slay (2000). The concept maps facilitated in-depth analysis and provided an effective tool for teasing out initial consistencies and inconsistencies. The concept maps were effective and useful in establishing meaningful overviews of the individual world views of the participants. They gave an interesting glimpse into the participants' perception of space and illustrated the richness and complexities of it. The extent to which this perception related to their spatial capacity in terms of spatial visualisation and orientation (see page 18 for definitions) was, however, nigh impossible to establish. A deeper level of analysis was required. Through interaction with my supervisors and a team of peers, a process and model of 'meta-analysis' was conceived (see 5.4 on page 228). Three meta-themes crystallised out of the initial analysis. I entitled them: extent of abstraction, insight, and complexity.

As these were very obscure and abstract themes to analyse and tease out, a team of three expert researchers was convened which would not only verify my analysis but also contribute to the understanding of those themes. The team consisted of:

1. a researcher who had just completed her PhD research in teacher professional development. Her work was firmly embedded in the interpretive paradigm and her research involved the interpretive framing of conversations with teachers through a process of consensual validation with teachers;

2. a co-researcher who is an expert in the field of mathematics education currently also doing PhD research through the same institution as myself. His data analysis was on the interpretation of videos of teachers in action;

3. a colleague, teaching in the same department, also in the process of completing his PhD thesis. His research involved textual analyses of integral theories of education, exploring the history of western educational paradigms.

Each member of the team initially analysed one transcript firstly to confirm the existence of the three meta-themes and secondly to identify any others. At our second meeting each team member shared his/her analysis, and a process of negotiating consensus then ensued. The meanings and criteria of the meta-themes were discussed and it was felt that a fourth meta-theme, that of critical engagement, should be added. Each team member then went away with five transcripts that I had selected. I ensured that the five transcripts were representative of the sample in terms of gender, school and race. The group met two weeks later. The main topic of discussion revolved around the issue of 'extent'. Could we quantify the extent of insight, for example? Did we want to quantify it? We agreed that it was neither helpful nor appropriate to attach an absolute quantity to each of the meta-themes. It was suggested instead that we should rather position the participants on a continuum and in that way build individual profiles. These profiles could then be used to inform the narrative and contribute to the bigger picture of each individual's broader spatial conceptual profile. Further, an additional meta-theme was identified – that of imagination. This arose out of the difficulties that we had in determining the extent of an individual's capacity to abstract. It was felt that many of the responses were characterised by rich and abundant imagination rather than abstractions. The final set of meta-themes which thus informed the meta-analysis consisted of:

- the extent of abstraction;
- the extent of insight;
- the extent of complexity;
- the extent of critical engagement.
- the extent of imagination

(The applications and analyses of these concepts are discussed in more detail in 5.4.2 on page 227).

Through discussion, reasonable consensus amongst the team members was reached and the individual meta-profiles coincided remarkably well. The process of consensual validation continued with another five transcripts. The team felt that in the previous round however, their assessment and analysis was affected by the names of the candidates that appeared on the transcripts. For the next round I therefore omitted the names and any other evidence that would suggest who the participant was or what school he/she came from. I randomly chose five transcripts. Further, it was felt that if the resultant profiles of the three team members coincided with my profiles there was no need to meet again. I would then continue the analysis on my own.

The second 'flow' of analysis is that of *data display*, "an organized assembly of information that permits conclusion drawing and action taking" (Miles and Huberman, 1984:21). In my research this took the form of a narrative text that sought to 'display' the complex and rich dimensions of spatial conceptualisation of 32 mostly Grade 11 learners in the Eastern Cape, South Africa.

The third 'flow' of analysis is the *conclusion-drawing and verification*, which is characterised by a process of reflection and evaluation on the "plausibility, sturdiness, confirmability and validity" (Miles and Huberman, 1984:21) of the research.

It needs to be emphasised that the above model needs to be viewed holistically. Data-reduction, for example "is not something separate from analysis, it is part of analysis" (Miles and Huberman, 1984:22). Analysis is a continuous and iterative process that is "ongoing and intertwined" (Ely *et al.,* 1991:86).

3.4.3 Validation

> Validity is not seen as part of a final product control process or verification, but rather a continuous process of credibility, growth and understanding.
>
> (Schäfer, 1999:26)

The issue of validity is complex, controversial and multifaceted and deserves attention. Cohen *et al.* (2000:105) view validity as a key issue and regard it as a "requirement" for both quantitative and qualitative research. Traditional and conventional conceptions of validity "have been based on positivist standards of objectivity and neutrality" (Southwood, 2000:66). In the traditional paradigm the validity issue was specifically with reference to the data and the appropriate use of them to make inferences, conduct analyses, test hypotheses and draw conclusions, whereas in qualitative research the emphasis of validity is more specifically on the process of research (Southwood, 2000). As this research has an integrative design, the issue of validity becomes very complex and I would like to deal with it as holistically as possible. There exists an abundance of critical literature that questions qualitative methodologies and accuses them of being 'soft' options in terms of their validation processes and lack of generalisability. Conversely much has been written in defence of the qualitative approach, justifying its position. I do not want to enter into this type of debate. Rather, a detailed analysis of some specific aspects of validity relating to this study is deemed more meaningful in the context of this work:

Internal validity

Internal validity, conventionally, pertains to the credibility of inferences that experimental treatments cause effects under certain well-defined conditions (Eisenhart and Howe, 1992). Cohen *et al.* (2000:107) suggest that it seeks to demonstrate that the explanation of an event, issue or set of data can be sustained by the data; in other words, "the findings must accurately describe the phenomena being researched". Although this applies to both qualitative and quantitative research, Cohen *et al.* (2000) assert that the researcher's discussion of validity needs to be located within the research paradigm that is being used.

The AMST used in Stage 4 of this research has its origin entrenched in a traditional quantitative paradigm. The original sample was relatively large (2346 students) and the research sought to generalise sex differences in performance of spatial tasks (Wattanawaha, 1977). The validity of the original test was argued in statistical terms only, based on the Rasch analysis of a trial test. Thus, although my research is mainly located within an interpretivist paradigm, the validity of the AMST is based on a positivist position. It is recognised that this can be viewed as a direct contradiction of Cohen's assertion above, but a flexible approach is needed here as the results of the

AMST were used to inspire an in-depth and interpretive analysis. Further, the AMPQ was used in conjunction with the AMST to facilitate an interpretative analysis. Although I agree with Cohen *et al.*'s (2000) assertion in principle, researchers need to guard against the self-limitation of simply committing to one paradigm for the sake of pigeon-holing their research. I would therefore argue for the notion that paradigms are not necessarily mutually exclusive, and concur with Hammersley (1992:159) who challenges "the widely held idea that there are [only] two methodological paradigms in social research: the quantitative and the qualitative". Social research is characterised by diversity including a huge variety of strategies, techniques and approaches. Hammersley (1992) therefore suggests that it is counterproductive to encapsulate this diversity within two paradigms. He thus advocates a multi-paradigm approach.

For maximising credibility, I found the framework of Guba and Lincoln (1989:239) useful and committed myself to the following techniques:

a) prolonged engagement
b) peer debriefing
c) progressive subjectivity
d) member checks.

a) Prolonged engagement

The process of data collection and analysis spanned two years and the contact sessions with the participants were spread over an entire year to ensure "substantial involvement" (Guba and Lincoln, 1989:237). The sustained contact also facilitated the building of rapport and trust. Although I did not immerse myself to the extent an ethnographer would, an atmosphere of warmth and mutual understanding was established.

b) Peer debriefing

Numerous peers have been involved in the research process in one way or another and have explicitly and implicitly contributed to the validation of the research. Guba and

Lincoln (1989:237) and Cohen *et al.* (2000:108) recommend a process of engaging with a "disinterested peer" in order to discuss findings, conclusions and tentative analyses extensively. The notion 'disinterested' implies a certain distance between the peer and the researcher and an absence of any vested interest. This type of interaction took place throughout the research process in formal situations and discussions, as with a paper which I presented at an international conference on the work in progress (Schäfer, 2001) and PhD seminars that I conducted, or in informal situations in different contexts such as tea room, corridor, classroom and workshop discussions.

Other peer validation, in the form of critical friends (Southwood, 2000), was fundamental to the validation process of the research. This process of consensual validation, described in 3.4.2, proved invaluable in the analysis of the interviews. Le Compte *et al.* (1992:748) emphasise that the consensual validation process rests not only on critical friends but also on "the agreement of competent others that the researchers account is valid". This does not imply that the goal is to achieve uniformity in the criticism, but rather that the views are valid, meaningful and useful. Although one of the particular roles of the consensual validation process in my research was to verify my interpretations of meta-themes, the overarching role was to validate the analysis process.

Alongside consensual validation as outlined above, my supervisors for this study played an integral role in the validating the research process. I was fortunate to interact with two supervisors who at times offered different critical perspectives to issues but in the process enriched both the research process and my engagement with the project. All the tests and questionnaires were scrutinised by the chief supervisor. Apart from yielding expert advice, this process added credibility to the research tools and contributed greatly to their validity.

Another forum which proved helpful in providing insight, constructive criticism and encouragement was a support group consisting of fellow PhD students. This group met on a regular basis and served as a 'bouncing board' to discuss initial findings, share joys and frustrations, and exchange useful ideas. Ely *et al.* (1991:99) acknowledges the role of support groups: they can "consider each member's emerging findings, suggest alternative explanations and act as auditors of the research process".

c) *Progressive subjectivity*

Guba and Lincoln (1989) observe that nobody engages in an inquiry with a blank mind. The "involvement of self" (Le Compte *et al.,* 1992:717), the notion of subjectivity, is unavoidable and it is becoming increasingly acceptable among qualitative researchers that subjectivity, as integral to research, no longer be viewed with the invariably negative consequences for the outcome of the inquiry. Nevertheless, Guba and Lincoln (1989:238) warn that the inquirer's subjective interpretation and construction cannot be "given privilege over that of anyone else", hence the suggestion that any construction should be a joint one. They therefore recommend that a process of 'progressive subjectivity', a continuous process of debriefing and reflecting with other parties, be embarked upon in order to critically evaluate original constructs. As a validation technique this process resonated with me. The issue is not whether the constructs are subjective or not, but whether they have been "sufficiently opened to the light of criticism" (Phillips, 1993). The process of consensual validation discussed in b) above can be viewed in terms of progressive subjectivity.

d) Member checks

This is a process whereby the participants in an inquiry have access to the data (and the interpretations thereof) and are given the opportunity to verify them. According to Guba and Lincoln (1989:239), this is the "most crucial technique for establishing credibility". This technique was applied particularly during Stage 2 and Stage 3 of this research. Once the interviews were transcribed and individual profiles were established, the participants were invited to respond by adding or deleting anything that they felt was inaccurate or misleading. The reaction was mixed, many expressed amusement at what they had said, and very few altered their transcripts. 'Member checking' is also referred to as face or participant validation by some researchers (Cantrell, 1993).

Intuition

Although the meta-criteria for our meta-analysis were defined and there was general consensus on those definitions (see 5.4.2 on page 227), much of the interpretation of the interviews in terms of the meta-themes rested on intuition. Intuition, in Polanyi's view (cited in Grene, 1969:118), "is a skill, rooted in our natural sensibility to hidden patterns and developed to effectiveness by a process of learning … it is not more mysterious than perception – but not less mysterious either". Polanyi argues for subjective involvement in scientific thought. He speaks of "personal judgement[s] which cannot be replaced by the operation of reasoning" (Grene, 1969:105). This is consistent with the Kantian view which asserts that into all acts of judgements there enters a personal decision which cannot be accounted for by any rules (Gardner, 1999).

Writing on decision-making, Jankowicz (2001) observes that many situations require people to draw on their experiences to make decisions based on intuition and subjective judgement. He recognises the importance of intuition, which may not be "based on explicit, propositionally statable principles" (Jankowicz, 2001:61).

The point here is that, particularly in the meta-analysis of the interviews as explained in 3.4.2 on page 67, intuition and subjective involvement based on the experience of the assembled team played an important and integral role in interpreting of the transcripts and the eventual construction of the meta-stars.

External validity

External validity centres on the issue of generalisability and from a positivist's perspective refers to the extent to which research results can be "generalised, or transferred to other settings" (Punch,1998). According to Schofield (1993:91) researchers in the quantitative tradition view generalisations as an important feature and thus "devote considerable thought to the question of how generalisability … can be enhanced". The notion of transferring research results to another setting is very problematic in qualitative research as, by its very nature, qualitative research often seeks in-depth understanding of a particular setting. Many researchers in the qualitative tradition therefore reject generalisability or give it very low priority (Schofield, 1993). Guba and Lincoln (1985:316), for example, assert that the establishment of transferability in naturalistic research is impossible because the naturalist researcher can only "set out working hyphotheses together with a description of the time and context in which they were found to hold". In this respect external validity therefore requires thick description (Cohen *et al.*, 2000) to enable someone interested in making a transfer to reach a conclusion is able to do so (Guba and Lincoln, 1985).

Le Compte *et al.* (1992:660) make the point that another useful criterion for external validity is that of worth: "valid studies must be worthwhile". This implies that the study needs to be contextualised within current research that in turn is 'worthwhile'. It needs to be open to scrutiny by peers and "accessible to the general educational community" (Le Compte *et al.*, 1992:660). From the outset of this research project I was determined not to work in isolation and, as far as I could, to involve my professional community. As this project began to take on a life of its own and evolve into a major part of my life it seemed natural to integrate it into my interactions with my community. I used many of the findings and ideas in my own teaching, I presented papers at conferences and the project was a source of useful information for discussions and debates. After completion of this thesis I intend to publish from it in journals and periodicals. As a result of using this project in my masters courses numerous students have based their research on similar themes and issues.

Construct validity

Cohen *et al.* (2000) identify this type of validity as important in research that seeks to establish and explore constructs. They argue that validity needs to be assured that a researcher's particular construction of a particular issue is in agreement with other constructions of the same particular issue. This has particular relevance in this research which explores issues around spatial visualisation and orientation. It was important that the interpretation of these constructs was consistent with the interpretation of other researchers in order to contextualise appropriately within the literature and other research. Construct validity relates closely to external validity in the sense that for research to be worthwhile and add value to a particular discourse, there needs to be agreement, to some degree at least, between the researched constructs. "Demonstrating construct validity means not only confirming the construction with that given in relevant literature, but looking for counter examples which might falsify [a researcher's] construction" (Cohen *et al.*, 2000:110). Eisenhart and Howe (1992) further suggest that construct validity implies that the themes that a qualitative researcher might be using need to be meaningful to the participants themselves. For my interview process this had special relevance. Although I had a particular framework of themes in mind I needed to be sensitive to letting other themes emerge that were meaningful to the participants. This was achieved by responding to their cues and by being open and flexible enough in spontaneous conversation.

Triangulation

Stake (2000:443) makes the observation that no case study researcher is unconcerned about "the clarity and validity of their own communications" and hence would employ various procedures to maximise validity. The process of using "two or more methods of data collection in the study of some aspect of human behaviour" (Cohen *et al.*, 2000:112) as a credibility check is often referred to as triangulation. The multi-dimensional approach to data collecting in this research can be viewed as triangulation because numerous data-collecting techniques were employed to gather data around the same issue.

Increasingly, however, the notion of triangulation in qualitative research is being re-examined. Guba and Lincoln (1989:240), for example, reject triangulation as a credibility check because "triangulation itself carries too positivistic an implication". They question the assumption that there exist unchanging phenomena so that triangulation can logically be a check. Lynch (1996) makes the observation that triangulation, in itself, will not guarantee validity. He cites Pearsol, who claims that credibility is defined in terms of the researcher, not in terms of the methodological technique of triangulation. I concur strongly with Lynch (1996:62) and support the position that triangulation should be a tool for providing richer and clearer information rather than "give some sort of privileged status, just because multiple data sources were used."

Crystallisation

Richardson (2000:923) highlights a unique aspect of research and its validation thereof, which is often neglected. She asserts that the very process of writing is a method of inquiry. Texts often validate themselves. " By writing … we discover new aspects of our topic and our relationship to it. Form and content are inseparable … it offers an additional – or alternative – research practice". As opposed to a process of triangulation (which she views as rigid, fixed and two-dimensional) she proposes a process of crystallisation which is more dynamic and creative. She advocates that in the act of writing, or creating a narrative researchers reflect and think about their findings. "Writing is not just a mopping-up activity at the end of a research project – it is a method of discovery and analysis" (Richardson, 2000). I can strongly identify with this, as often during writing this narrative my analysis of issues and data made new sense. The process of committing our thoughts to words and text forces us increasingly to clarify ideas and perceptions – hence the metaphor of crystallisation.

In summary, validity remains complex; and whilst grappling with the nuances of this issue, sight must not be lost of the notion that the main purpose of validity is to establish what counts as evidence (Lynch, 1996). What is important for researchers in any paradigm is to ensure that their research is trustworthy and communicated to others as clearly and non-defensively as possible (Ely *et al.*, 1991).

3.4.4 Interview schedule, concept maps and other tools for analysis

As the interview process in Stage 3 was methodologically central to this project and produced fundamental data, I feel that it is important to provide a detailed description and analysis of the process. The overarching objective in engaging the participants in conversations was to establish a sense of their world view on space. As discussed in 2.3 on page 30, world view refers to the "epistemological macrostructure" (Cobern, 1991:7) of a person. A world view encapsulates a person's beliefs, perceptions and presuppositions, and forms the basis of his or her view of reality (Kearney, 1984). The notion of a world view is very complex and involves a number of components (see page 33). This research focused on only one of these components, namely the one that centres on a person's perception of space. The interview was used in this research as a technique to tease out the participants' own view of space. As I felt that the concept of space could be very abstract and possibly inaccessible for many of them and consequently a very difficult topic for conversation, I modeled my strategy on Cobern's (1993) and Slay's (2000) world-view research in which they made use of a framework of themes around which conversations and interviews could be structured. I was reluctant to over-structure my interviews and so stifle spontaneous conversation and insight. Nevertheless, I thought that some enabling framework which remained consistent for all the 32 interviews would facilitate the eventual analysis more effectively than having to analyse 32 totally open-ended conversations. In retrospect I was pleased that I had a frame of reference for analysing the conversations and building meaningful concept maps (see page 206).

The framework originally devised by Cobern (1993:935) relied on a set of "bipolar descriptive codes". These bipolar codes (also sometimes called vector pairs), representing the opposing ends of a continuum, were used to explore students' conceptualisation of and beliefs about nature. The set of bipolar codes were: naturalism and religion, chaos and order, mystery and knowledge, function and purpose, mundane and special, and science and no science.

In addition to a few extra vector pairs, I made use of Cobern's (1993) model above (see Table 3.3), and for each of the pairs I devised some guiding questions that would frame the structure of the interviews (see Appendix 7).

Table 3.3 Framework for interviewing using bipolar codes

WORLD VIEW OF SPACE
Naturalism - Religion
Chaos - Order
Mystery - Knowledge
Function - Purpose
Mundane - Special
Mathematical – Non-mathematical (Cartesian – Non-Cartesian)
Finite - Infinite
Tangible – Non-Tangible
Internal - External

Citing Jones, Cobern (1993:939) suggests that "bipolar coding involves selecting two related codes that together distinguish a range of beliefs with respect to one presupposition". For example, as described below, people could believe space to be orderly, chaotic, or something in between the two extremes. The nine descriptive pairs of bipolar codes, as used in this study, are discussed below. (A full analysis of the conversations together with extensive quotes and narratives is provided in chapter five).

Naturalism and religion

Naturalism, as used in this study, refers to the belief that "material or physical causation provides the basis" (Cobern, 1993:939) for understanding space. It fundamentally rules out theistic involvement in space. Religion, however, is the opposite – it asserts the involvement of the supernatural in space.

Chaos and order

Chaos implies that space is changeable, random and unpredictable, whereas order implies the opposite – space is predictable, based on rules and principles.

Mystery and knowledge

This vector pair describes "the extent to which one believes [space] to be fathomable" (Cobern, 1993:942). Those who find space mysterious are clearly more impressed with what is not known than what is, whereas those who find space knowable have significant material understanding of space or events in space (Cobern, 1993).

Function and purpose

A function explanation of space refers to a "teleonmic or a structure/function explanation of space whereas purpose explanations refer to transcendent purpose or cosmic teleology" (Cobern, 1993:944).

Mundane and special

This vector pair seeks to deconstruct whether space is perceived as something "beyond the ordinary" (Cobern 1993:946) or something mundane and prosaic.

Mathematical and non-mathematical

As this study is situated in a mathematical context there was an interest in the extent to which mathematics informed a participant's view of space.

Finite and infinite

This vector pairs seeks to describe the extent to which participants view space as an infinite concept (infinitely large or infinitely small) or whether they see space in terms of defined parameters.

Tangible and non-tangible

A 'tangible' explanation of space is one where space is perceived as something that one can touch and see, whereas a 'non-tangible' perception of space is characterised by mystery and abstractions.

Internal and external

This vector pair seeks to explore the extent to which space is perceived as a phenomena 'out there' or something which is internalized and part of the self.

The interview transcripts then formed the basis for the construction of concept maps. The transcripts and concept maps were reviewed and edited by the participants (member-checked, see 3.4.3 on page 73) before they were finalised and analysed. The concept maps proved very useful for the initial 'content' analysis and the final 'meta' analysis (see chapter five). They served as effective route maps of where the conversations went and provided a basis for creating final narrative. Figure 3.1 is an example of such a concept map.

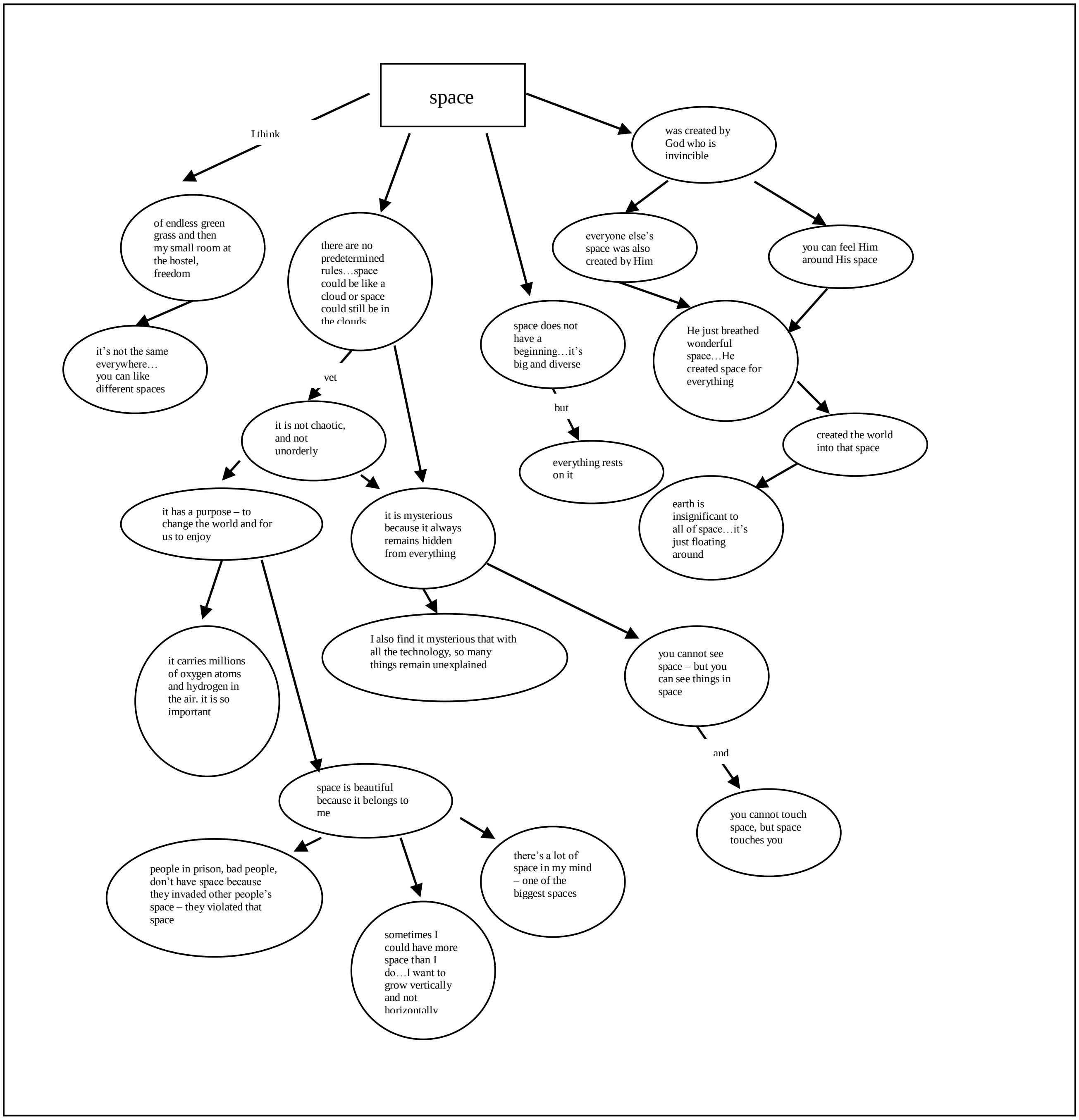


Figure 3.1. P6's concept map of her conceptualisation of space

The tool used for the final meta-analysis discussed in 3.4.2 is illustrated in Figure 3.2. It consists of a ‘meta-star’ whereby each arm of the star represents each participant’s emphasis on the five meta-criteria: abstraction, imagination, critical, insight and complexity. Details regarding the development, construction, interpretation and use of the meta-star are provided in chapter five on page 231. The idea of a metastar originated during discussions with my consensual validation team and its final shape was based on the work of Ken Wilber, who used numerous such techniques to illustrate human consciousness and nature of development (Wilber, 2000).

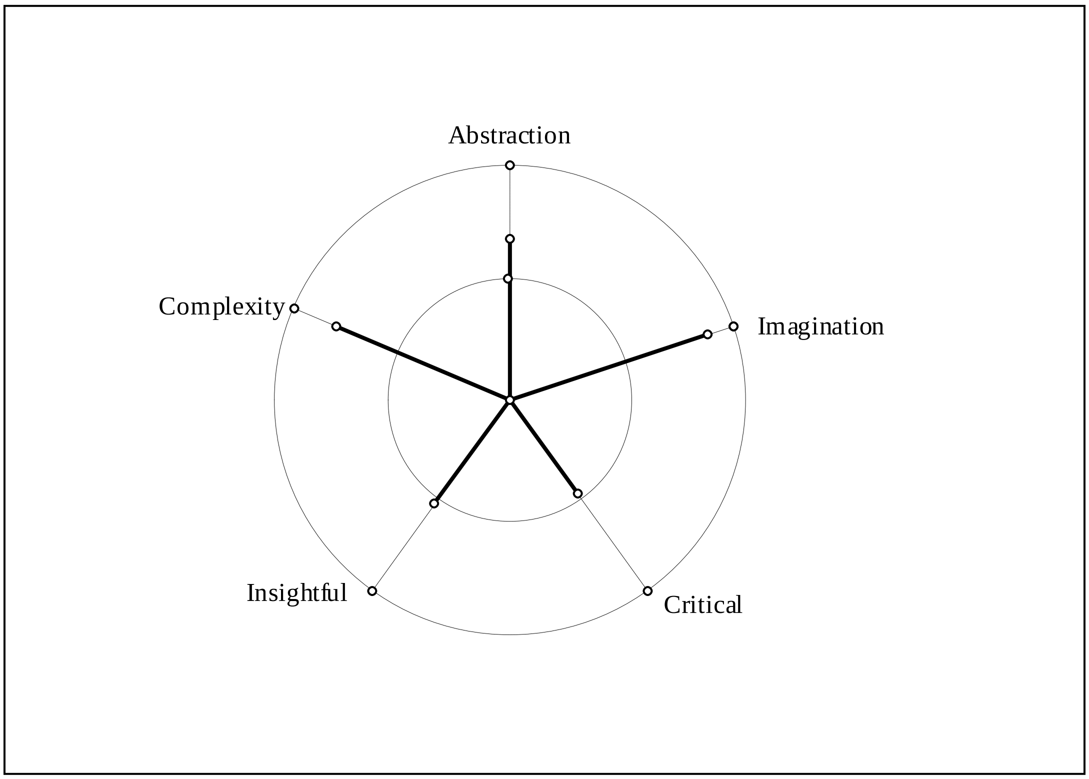


Figure 3.2 P6’s meta-star

Although the five meta-criteria are analysed in finer detail in 5.4.2 on page 229 where they are incorporated in the narrative, a brief discussion of them as they are used for the purpose of this study follows below:

Abstraction

This criterion gauges the extent to which the participants can talk about space in terms of non-tangible and non-concrete concepts. The *Concise Oxford Dictionary* (1995) defines abstract as "to do with or existing in thought rather than matter, or in theory rather than in practice".

Imagination

This criterion shows the extent to which participants can form mental images of space. The *Concise Oxford Dictionary* refers to imagination as "the ability of the mind to be creative and resourceful".

Critical

This criterion refers to an individual's ability to be discerning and being able to critically reflect and in particular refers to a person's capacity for "making censorious comments or judgments" (*Concise Oxford Dictionary*, 1995).

Insightful

When talking about his/her perception of space, participants' ability to articulate their understanding about hidden truths is assessed by this criterion.

Complexity

This refers to the participants' ability to articulate ideas in terms of related and composite parts.

3.5 DESCRIPTION OF THE AMST AND THE HAT

3.5.1 Introduction

This section deals with the two tests that were used in order to explore and investigate spatial visualisation and orientation. These are the Adapted Monash Space Test (AMST) and the Hands-On Activity Test (HAT). I use the term 'test' with hesitancy as they are not used only in the traditional quantitative sense of measuring and assigning a numeric value to the participants' spatial capacity, but also form part of a bigger picture of spatial conceptualisation. The test results paint only part of the spatial picture of the participants. They are integral to the research however, and hence require description. The spatial visualisation and orientation constructs, as discussed in the previous chapter, are central to the tests and each item of the tests is described in terms of the two constructs which dominate the particular item. It is recognised that there is overlap of the constructs – they are not necessarily mutually exclusive.

3.5.2 Pen-and-paper spatial test (AMST)

The selection of a suitable pen-and-paper test to explore spatial ability was not an easy task for the following reasons:

- There exist a plethora of spatial tests each with their own agendas and validity criteria (Wattanawaha, 1977);
- It was also noted that "all well known existing tests purporting to measure spatial ability do little more than measure a tiny aspect of that ability" (Wattanawaha, 1977:59). In my experience this applies now as it did in the seventies;
- I needed a reputable test that was valid and whose results were readily accessible. Ideally I was looking for a research project that was well documented;
- Many of the existing spatial pen-and-paper tests are restricted to being administered only by qualified psychologists;
- I required a test that reflected a world view compatible with the spirit of this research.

During one of my residency periods at Curtin University I came across the Monash Space Test in Wattanawaha's (1977) thesis. The instrument had undergone a rigorous validity process (albeit in a positivist paradigm) and most of the individual test items were suited to what I was looking for, namely, testing spatial visualisation and orientation constructs. As discussed in 3.4.3, the validity issue was important, albeit not in terms of generating widespread generalisations, and I sought test items which were reliable. Further, Wattanahawa presented a very useful and extensive analysis of each item that served as a basis for my own initial analysis. Wattanahawa (1977:61) also claimed that her test "reflected Piagetian spatial ideas". This caught my attention, as this was the only test that I could locate that came close to the epistemological spirit of my work as intimated in chapter two.

To serve the purposes of this project I modified the original Monash Space Test by retaining 13 of the original 24 items and adding an additional 12 items. (Refer to 3.4.1 for more details.) Further, I wanted to include some open-ended items of an exploratory nature that could be linked up with the participants' conversations and their initial ideas of space articulated in the questionnaire. The first three tasks of the AMST were thus of the open-ended type.

The other tasks needed to contain a mixture of items that could be classified as 'spatial visualisation tasks' and 'spatial orientation tasks'. The reader will recall from page 14 that spatial visualisation refers to:

> the ability to mentally manipulate, rotate, twist, or invert a pictorially or physically presented stimulus object. The underlying ability in *spatial visualisation* appears to be connected to movement, transformation and manipulation. It is dynamic and involves motion.

and spatial orientation (page 15) refers to:

> the ability to recognise the identity of an object when it is seen from different angles. It is the ability to make sense of spatial orientations of objects relative to different positions of itself or of other objects.

These definitions are important, as they are carried across to the HAT test and incorporated in the analysis of that part of the research. In consultation with my supervisors and in my considered opinion 14 tasks can be characterised by their emphasis on spatial visualisation while nine tasks focus on spatial orientation. It is recognised that some problems can be categorised as containing both spatial visualisation and orientation constructs. For this research, however, I have categorised them according to the construct which in my view and experience, dominates.

The model that underpinned the original Monash Space Test was based on the DIPT classification system. This system is based on the assumption that the most important general characteristics of spatial tasks are:

- the Dimension of thinking required by the task;
- the degree of Internalisation required ;
- the manner in which the task requires an answer to be Presented;
- the Thought process required by the task.

(Wattanahawa, 1977)

For a more detailed explanation of the DIPT classification model see Appendix 8.

I initially incorporated the DIPT model in my research project, but as the research unfolded I focused more and more on spatial visualisation and orientation. As many of the DIPT criteria fall within the definitions of spatial visualisation and orientation (as used for the purpose of this study and articulated above), I dispensed with the structure of the DIPT model and exclusively situated my research around the notions of the spatial visualisation and orientation constructs.

Below follows an item-by-item discussion of each task of the AMST. To save space and facilitate easier reading, I have reduced the illustrations and the lay-out of the items. The original version of the test as it was presented to the participants appears in Appendix 4. I discuss each task under the following headings:

- rationale
- where applicable, spatial visualisation or spatial orientation.

Task 1

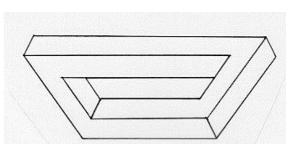
We have talked a lot about space and you have described the concept of space in many different ways. Below, SKETCH your image of space:

Rationale

Throughout my interaction with the participants, the concept of ‘space’ was emphasised and this test item was another opportunity to express their perceptions using a different medium.

Task 2

Describe as accurately as possible what you see in the diagram below:

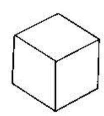


Rationale

The diagram describes an optical illusion. Although the 'object' is feasible as a two-dimensional drawing it is impossible to conceive of it as a three-dimensional object. The notion of two-dimensionality and three-dimensionality is superficially explored in the interviews and also in the HAT and this item serves as provoking the participants sense of two- and three- dimensions.

Task 3

Describe as accurately as possible what you see in the diagram below

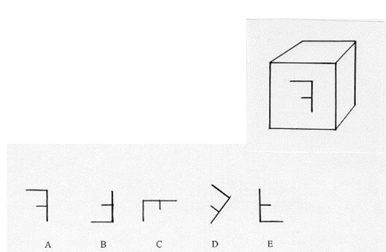


Rationale

The diagram can be viewed either in three dimensions as a cube (an isomorphic projection of a cube), or a hexagon in two dimensions with an inscribed Y-shape. As in the previous task, this item serves to explore the participants' sense of two- and three-dimensions, albeit with a diagram that is more familiar to them and which they would, in all likelihood, have come across in their mathematics classes.

Task 4

Which of the signs A, B, C, D or E can be printed with the stamp in the figure alongside?



Rationale

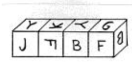
This item appears on the original Monash Test as Question 16 (Wattanahawa, 1977:229) and can be regarded as a perspective problem. It requires the candidate to imagine what the resultant figure will look like once the stamp has been printed. It relies on the ability to transfer a three-dimensional object (the stamp) to a two-dimensional print.

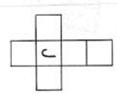
Spatial visualisation

This is a classic spatial visualisation problem which relies on manipulating, rotating and reflecting the image on the cube. It also requires pictorial inversion.

Task 5

In the figure below every cube is lettered EXACTLY ALIKE. Complete the net of ONE of the cubes below to show how the letters are arranged on the six faces.





Rationale

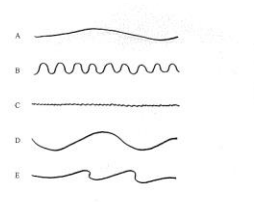
This task is characterised by undergoing the reverse process of the previous item. The starting point is a three-dimensional object which is to be unfolded to produce a two-dimensional net.

Spatial visualisation

This task relies heavily on complex manipulation and rotation. The unfolding of a cube into a two-dimensional net requires transformation skills from three-dimensions to two-dimensions.

Task 6

Which of the paths A, B, C, D, or E is the LONGEST?



Rationale

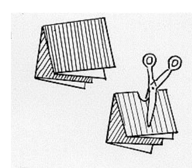
This task appears as Question 2 in the original Monash Space Test (Wattanahawa, 1977). It is a very straightforward task which requires imagining the stretched path.

Spatial visualisation

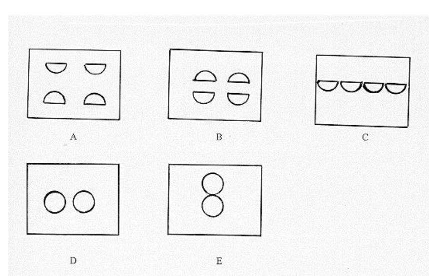
It represents a spatial visualisation task because it is dynamic and involves motion – the stretching out of the paths.

Task 7

Suppose a rectangular piece of paper is folded TWICE as shown alongside. Suppose you cut HALF A CIRCLE out of the folded paper.



If you then opened out the piece of paper, which of A, B, C, D or E alongside would it look like?



Rationale

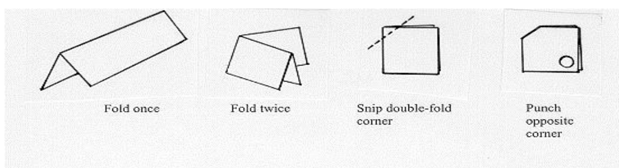
This task appears as Question 3 in the original Monash Space Test (Wattanahawa, 1977) and once again involves a combination of two- and three-dimensional skills.

Spatial visualisation

The task requires imagining what a cut-out figure, once unfolded, would look like. It asks for the mental manipulation of a mental picture to produce a 'new' picture.

Task 8

Sketch what a SQUARE sheet of paper will look like when it is opened up after the following sequence of FOLDS, SNIPS and PUNCHES.



Rationale

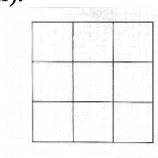
This is similar to Task 7 except that there is no choice of correct solutions to choose from. The participants need to draw their solutions so the notion of solving the problem by elimination on their part is minimised.

Spatial visualisation

As with Task 7, this problem requires the mental manipulation of a picture to produce a 'new' picture.

Task 9

Shade SIX of the nine squares of a three-by-three grid so NO THREE shaded squares are in a STRAIGHT line (row, column or diagonal).



Rationale

This task requires the participants to arrange six squares according to a rule and relies totally on a two-dimensional perspective.

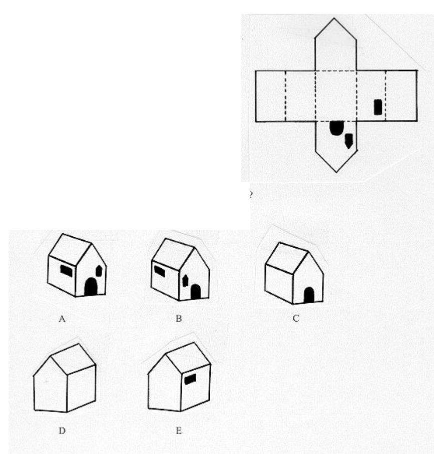
Spatial orientation

This is a static problem in that it does not involve the rotation of transforming of a shape. It seeks for the ability to make sense of spatial orientations of objects (squares) to different positions of itself or of others objects (the grid of squares).

Task 10

The figure alongside shows the plan (net) of a house which has only one door and two windows.

Which of A, B, C, D, or E could be the house?



Rationale

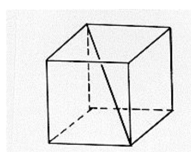
This task appears as Question 5 in the original Monash Space Test (Wattanahawa, 1977). This task combines two- and three-dimensional perceptions.

Spatial visualisation

Apart from having to transform a mental image from three-dimensions to two-dimensions, this problem involves a change in mental perspectives, in that the front of the house has to be seen from the other end.

Task 11

A picture of a MAIN AXIS of a cube is shown. A main axis of a cube goes from a corner to the far opposite corner of a cube.
ALTOGETHER, how many different main axes does a cube have?



Rationale

This task appears as Question 6 in the original Monash Space Test (Wattanahawa, 1977). Unlike the previous tasks which were either only two-dimensional, or consisted of a combination of two- and three-dimensions, this task is situated only in a three-dimensional perspective.

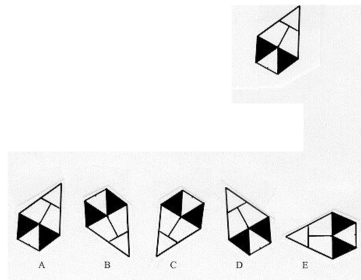
Spatial orientation

This problem requires an ability to identify the four main axes of the cube. The cube remains static and the correct positioning of the main axes requires an ability to make sense of the spatial orientations of the axes relative to the vertices of the cube and themselves.

Task 12

Suppose you see the picture alongside on a window.

You then looked at the picture from the other side of the window. Which of the figures A, B, C, D or E would you see?



Rationale

This task appears as Question 10 in the original Monash Space Test (Wattanahawa, 1977). This task mainly requires the manipulation of shapes in two-dimensions, although it can be argued that having to imagine what the shape looks like from the other side of the window involves a three-dimensional perspective.

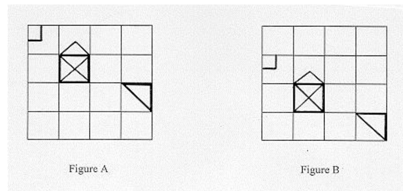
Spatial visualisation

This is a classic spatial visualisation problem as it involves a combination of rotation, reflection and transformation techniques to solve.

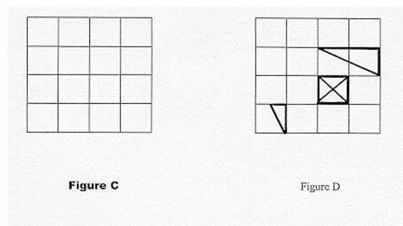
Task 13

Study Figure A and Figure B alongside.

Think of a RULE by which Figure B is obtained from Figure A.



Now complete Figure C so that Figure D is obtained from Figure C.



Rationale

This task appears as Question 11 in the original Monash Space Test (Wattanahawa, 1977) and serves as a lead-in example for Task 14. It relies entirely on a two-dimensional perspective.

Spatial visualisation

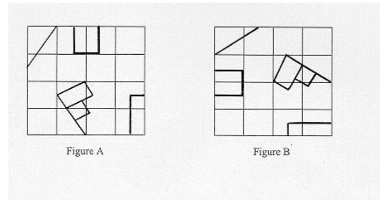
This task requires simple linear translation skills to solve.

Spatial orientation

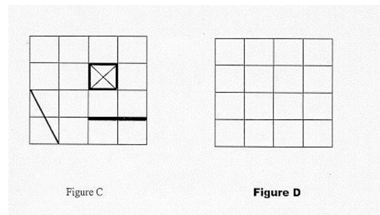
The definition for spatial orientation used in this study suggests that the task could fall into this category because it requires a sense of relative positions.

Task 14

Study Figure A and Figure B below. Think of a Rule by which Figure B is obtained from Figure A.



Now complete Figure D below so that Figure D is obtained from Figure C by the same Rule as above.



Rationale

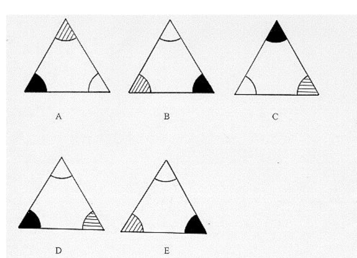
This task is similar to Question 11 in the original Monash Space Test (Wattanahawa, 1977). Although limited to a two-dimensional perspective, it is more complex than Task 13 and requires more insight.

Spatial visualisation

This problem requires a complex combination of rotation and reflection to solve.

Task 15

Find the ‘odd-one out’.



Rationale

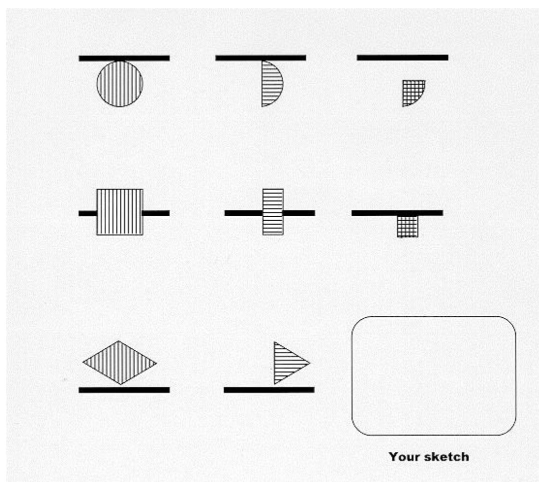
This is a typical 'IQ-test type' question which would be familiar to many of the participants. It is restricted to two-dimensions.

Spatial visualisation

This problem is best solved by rotation of the triangles about its centre. The manipulation required is restricted to the plane, i.e., no 'flipping-over' is required.

Task 16

Sketch the figure missing in the lower right corner of this three-by-three pattern.



Rationale

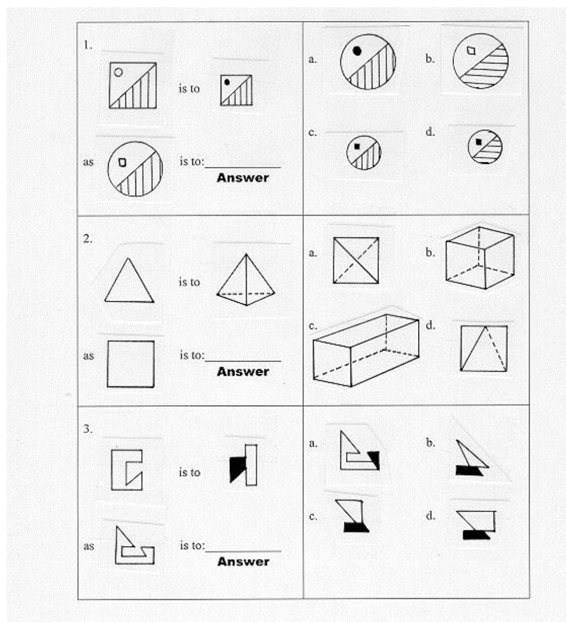
This task involves identifying the sequential metamorphosis of the original shape in the left-hand column. The shapes in the second column are obtained by halving the original shape along the vertical axis of symmetry and removing the left-hand half. The third column is obtained by once again halving the shapes in the second column along the horizontal axis of the shapes and removing the top half. It is situated purely in two-dimensional space.

Spatial orientation

The objects involved remain static – there is no manipulation of objects required to solve this problem. Hence it can be classified as a spatial orientation task.

Task 17

Which of the designs in the right hand column complete the statements in the left hand column?



Rationale

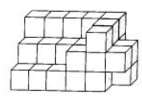
This is a multidimensional task which consists of two-dimensional and three-dimensional problems.

Spatial orientation

In terms of the static nature of the shapes (except perhaps for 3 where a linear translation can be used) this task can be classified as a spatial orientation task. The successful solving of this problem relies on the recognition of objects and patterns and their relative position to one another.

Task 18

How many blocks (unit squares) are there in this stack**?**



Rationale

This task is similar to Question 14 in the original Monash Space Test (Wattanahawa, 1977) and involves a three-dimensional block counting task.

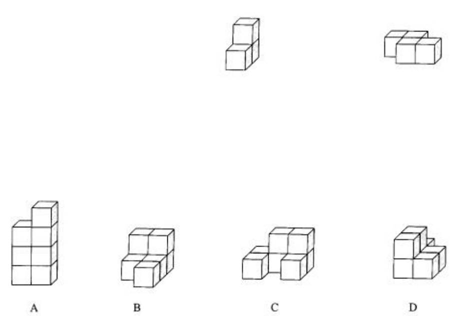
Spatial orientation

This specifically involves the identification of an object from different angles. It requires the ability to imagine different orientations of the stack and then counting the cubes.

Task 19

The two objects alongside can be placed together to form each of the shapes below, except ONE.

Which one?



Rationale

This task involves three-dimensional manipulations and is similar to the HAT task 4 (see page 111).

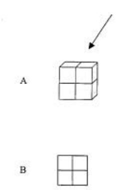
Spatial visualisation

This task requires an ability to manipulate the two given shapes by rotating and inverting them and placing them together in order to produce another shape. The problem can of course also be solved by working backwards i.e. identifying the two given shapes by analysing the composite shapes. Either way it requires mental manipulation of the two given shapes.

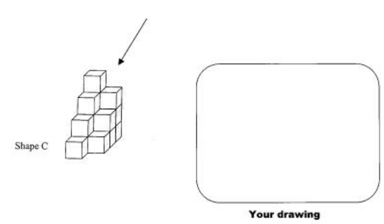
Task 20

Suppose you looked at shape A so that your eyes were looking along the arrow.
(You are looking at the shape from behind)

The shape you would see looks like B alongside.



Suppose this time you looked at the shape C alongside so that your eyes were looking along the arrow. You are looking at shape C from behind. In the space provided draw the shape that C would look like to you.



Rationale

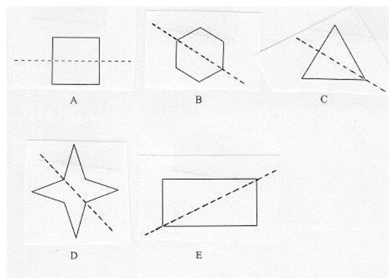
This task appears as Question 17 in the original Monash Space Test (Wattanahawa, 1977) requires a sense of three-dimensional and two-dimensional perspective. As in tasks 8 and 16, the subject is required to draw the solution.

Spatial orientation

This task requires the subject to recognise and imagine an object as seen from a different angle. The subject needs to position the objects differently in order to identify their new façade.

Task 21

Which of A, B, C, D, or E CANNOT be folded along the dotted line such that one half fits exactly overt the other half?



Rationale

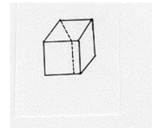
This task appears as Question 18 in the original Monash Space Test (Wattanahawa, 1977) and involves the transformation of a two-dimensional flat shape into a three-dimensional object by folding. This task is not unlike Task 7 although here there is more emphasis on oblique reflections. It differs from Task 8 in that the subject does not have to draw the answer.

Spatial visualisation

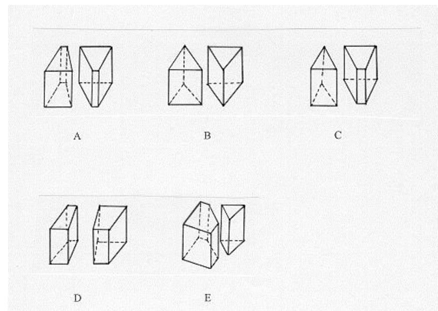
Once again there is manipulation involved – a combination of folding and reflecting.

Task 22

Suppose the cube alongside is cut into two sections along the dotted lines shown.



Which of A, B, C, D, or E shows the two sections which could be obtained.



Rationale

This task appears as Question 20 in the original Monash Space Test (Wattanahawa, 1977) and involves three-dimensional orientations.

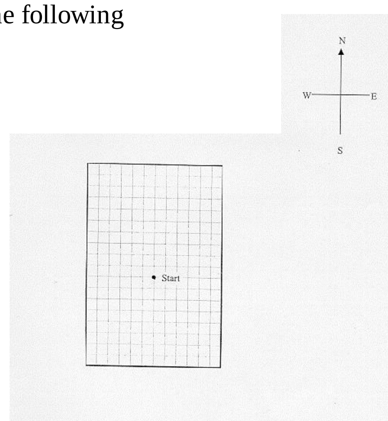
Spatial orientation

This task requires the ability to recognise and identify consistencies between the union of the two shapes and the original cube. It requires identifying the spatial orientation of shapes in relation to another shape.

Task 23

On the grid alongside, draw YOUR path from the following instruction:

From the start, face WEST and go 1 block.
Then turn SOUTH and go 4 blocks.
Then turn EAST and go 3 blocks.
Then turn LEFT and go 2 blocks.



Rationale

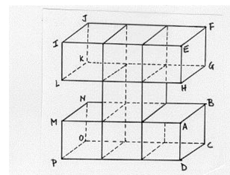
This task is very similar to Question 24 in the original Monash Space Test (Wattanahawa, 1977) and suggests a two-dimensional orientation. It requires the candidates to move along a map in the plane according to specific directional instructions.

Spatial orientation

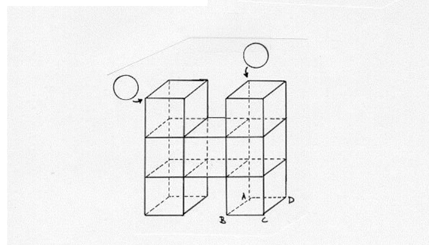
There is no manipulation of shapes involved. The problem relies purely on spatial orientation and making sense of different positions.

Task 24

If the shape in the figure alongside was placed in the position shown below, which would be the letters for the corners indicated by the arrows.



Write the correct letters in the circles.



Rationale

This task appears as Question 22 in the original Monash Space Test (Wattanahawa, 1977) and involves a set of complex rotational manipulations in three-dimensional space.

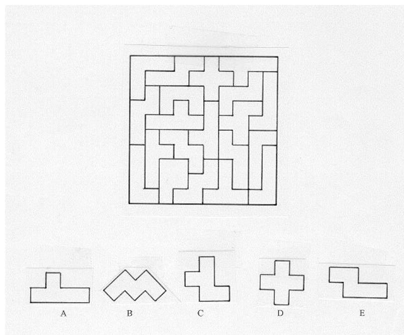
Spatial visualisation

This task involves the rotation and spatial inversion (twisting) of the original shape.

Task 25

Which of the five pieces shown below cannot be found in the jigsaw puzzle?

Do not flip over any pieces.



Rationale

The elimination of flipping any of the pieces situates this problem in two-dimensions.

Spatial visualisation

Although there are spatial orientation skills involved in this task (recognition of the identity of shapes and spatial orientation), it relies heavily on the rotating and translating of the pieces to fit the original matrix.

3.5.3 Hands-On Activity test (HAT)

The purpose of the HAT, which consisted of eight tasks, was to explore and investigate spatial visualisation and orientation skills in a context of a series of practical tasks as opposed to a pen-and-paper test situation. I was particularly interested to investigate the extent of the connections that existed between spatial visualisation and orientation ‘performance’ in a pen-an-paper situation and a hands-on-activity context.

I was once again faced with the task of selecting or adapting an appropriate test which had credibility and validity. Unlike the situation with the pen-and-paper tests, I could not trace anything suitable in the literature I read. The main criterion for the selection of items was that the tasks had to rely on spatial visualisation and orientation skills as defined earlier. I had worked with a group of researchers who investigated the role of graphicacy in many different contexts (Boltt and van Harmelen, 1995; Wilmot, 1998). Graphicacy for these researchers was defined as “the communication of spatial information that cannot be conveyed adequately through verbal or numerical means alone” (Balchin, 1985:8). One of the research tools used by Wilmot (1998:70) was a “diagnostic activity” to “identify spatial perceptual and spatial conceptual skills”. This activity consisted of eight hands-on activities and included tasks such as puzzle-building and drawing. On closer analysis of the individual tasks in terms of spatial visualisation and orientation, I found that most of them fitted my criteria very snugly and I was able to use Wilmot’s activities as a solid foundation for my test.

Below follows an item-by-item discussion of each task of the HAT. Once again I have reduced the illustrations and the layout of each item. Where appropriate I have included a photograph of the instrument used for easier reference for the reader. The original version of the test as it was presented to the participants appears in Appendix 6. I discuss each item under the following headings:

- rationale
- where applicable, spatial visualisation or orientation.

Once again in some of the activities the spatial visualisation and orientation constructs overlapped. As in the case of the AMST, I have isolated the dominant construct for each activity.

Activity 1

Shape sorter

- Each yellow shape fits through ONE of the openings in the ball.
- Your task is to place the shapes through the openings into the ball AS QUICKLY AS POSSIBLE.
- You can have two attempts.
- Both attempts will be timed.
- You will be asked to reflect on your strategy in solving this problem.

Rationale

This activity focuses on skills required to place a three-dimensional shape into a matching two-dimensional hole in the shape-sorter ball.

Each participants was allowed two attempts and the times of both attempts were recorded.

Spatial visualisation

The solving of this problem relies heavily on the ability to rotate and match the shapes with the hole in the ball. The location of the correct matching hole, which involves rotation and recognition, is also fundamental.

Activity 2

Puzzle

- Your tasks is to complete this puzzle AS QUICKLY AS POSSIBLE.
- You will be asked to reflect on your strategy in solving this problem.
- You will be timed.

Rationale

This activity consists of a classic puzzle problem where each participant is required to assemble a 24-piece puzzle in as quick a time as possible. They are supplied with the final picture they are working towards. This activity is essentially restricted to two-dimensional space in the sense that the pieces did not have to be flipped over.

Each participant was allowed two attempts and the times of both attempts were recorded.

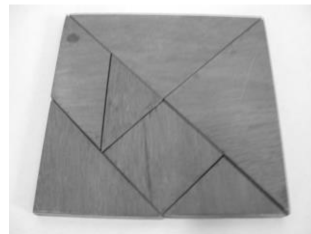
Spatial visualisation

This problem required recognition of shapes and the ability to match and combine them to create a bigger picture. It could also involve colour and pattern recognition in terms of what appeared on each shape and the combination of these to create a bigger picture. It is a dynamic exercise which involves transformation techniques such as rotation and translation.

Activity 3

Tangram

- Assemble the seven pieces into a perfect square.
- None of the pieces may overlap and there may be no gaps in between the pieces.
- You will be timed.
- You will be asked to reflect on your strategy in solving this problem.



Rationale

This activity consists of the classic Chinese tangram. The participants are required to construct a perfect square using all the seven pieces provided. Unlike the previous task they are not supplied with a picture of the assembled square. Once again this task is restricted to two-dimensions.

Each participant was allowed two attempts and the times of both attempts were recorded.

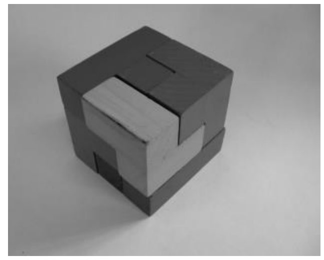
Spatial visualisation

Once again, this is can be characterised as a dynamic task which requires fundamental rotational and translational skills. The individual pieces need to be carefully manipulated and combined to form a new shape.

Activity 4

Soma cube

- Assemble the seven pieces into a CUBE.
- You will be timed.
- You will be asked to reflect on your strategy in solving this problem.



Rationale

This activity consists of assembling all seven pieces provided into a cube. The participants are not supplied with a picture of the assembled cube. This activity is very similar to the previous task except that it relies exclusively on three-dimensional perspectives.

Each participant was allowed two attempts and the times of both attempts were recorded.

Spatial visualisation

The skills required to solve this problem are very similar to those in the previous activities except that the additional dimension now asks for rotation in three dimensions.

Activity 5a

Front view

- The two objects in front of you must not be moved.
- Draw the objects on the table as you see them in front of you as accurately as possible.

Rationale

This activity requires the participants to draw a 'still-life' of two objects without moving the objects and without changing positions to obtain a different perspective. This activity was designed to establish how the participants perceive and understand the relative positions of the two objects. It also requires the ability to show reversibility of three-dimensional objects to a two-dimensional drawing.

Spatial orientation

This activity relies specifically on spatial orientation skills in that the participant needs to be able to identify the relative positions of the two objects and make sense of their orientation.

Activity 5b

Obscured reverse view

Without moving from your present position, draw the same two objects as they would appear to you if you were sitting on the opposite side of the table.

Rationale

In this activity the participants have to be able to imagine what the two objects would look like from a view not directly observable from where they are sitting.

Spatial orientation

The central spatial orientation skill required in this activity is the ability to recognise and identify objects when seen from a different angle (in this case the reverse angle). It also asks for the ability to imagine the relative orientations of the objects when viewed from a reverse angle.

Activity 6

Aerial view

- The picture shows a number of objects which were photographed from the side.
- Draw an aerial view of the same objects, i.e., draw them as they would appear from the top.

Rationale

This is a task similar to the previous one where the subjects had to imagine what the objects would look like from an aerial perspective. The situation is made more complex, however, by the fact that the participant first has to create a mental picture from the photograph and then form the aerial image.

Spatial orientation

This activity is a classic spatial-orientation task where the participant's ability to identify and recognise objects as seen from a different angle is challenged. The orientation and the relative positions of the objects as they appear on the photograph and as they appear from above are fundamental.

Activity 7

Route map

- Draw an accurate map of the route that you would take from your school to the Standard Bank building in the centre of town.
- Provide as much detail as you think is necessary for a stranger to follow your map.

Rationale

The scene that the participants have to imagine is a concrete image – a bird's eye view of the route from their school to the Standard Bank building in the centre of town. It could be drawn using three-dimensional perspective or as a plane map in two dimensions.

Visual orientation

The task involves imagining a complex combination of shapes and objects in relation to each other and the entire town. It requires an astute awareness of the identity of objects as seen from different angles and orientation.

Activity 8

School hall

- Imagine that you are standing in the middle of your empty school hall facing the stage.
- There are stairs leading up to the stage on either side.
- Just in front of the stage are two long rectangular tables and just to the right of you is a round table.
- Just behind you is a trapezoidal table (in the shape of a trapezium).
- Draw the above scene.

Rationale

This task is similar to the previous activity except that it relies on an imaginative situation which needs to be mentally constructed. The participants have first to construct the scene, then imagine it and then draw it, whereas in the previous task the scene was there already.

Spatial orientation

This activity relies heavily on the ability to conceptualise a situation mentally from a position totally removed from the self. The relative positions of the objects in the school hall and the hall itself require complex orientation skills because the entire scene is not a concrete one. One cannot physically see it – it unfolds in the minds of the participants.

3.6 CONCLUSION

This chapter has described the research project in terms of its orientation, design and process. It explained that this research:

- used qualitative and quantitative methods and is predominantly located within an interpretivist paradigm;

- is a multi-sited case study;
- used numerous techniques to ensure validity.

It has also described the instruments used to explore the notions of spatial visualisation and orientation in a traditional pen-and-paper situation and a hands-on activity-based context. For the purpose of this study, the spatial visualisation and orientation constructs are used to describe an individual's spatial capacity.

A reflection on the effectiveness and appropriateness of the methodology chosen appears in the final chapter on page 291 of this thesis.

The following chapter provides an analysis of the AMST and the HAT before constructing world-view profiles.

CHAPTER FOUR

DATA NARRATIVE 1: THE AMST AND THE HAT

4.1 INTRODUCTION

This chapter's intention is primarily to address the first objective of this study (see page six) and will hence engage with the data obtained from:

- the AMST
- the HAT.

An item-by-item analysis and discussion is provided for each test. So as not to interrupt the flow of the narrative, the graphic and quantitative data of the results of each item for the AMST are presented in Appendix 9. Details of the participants' performance in the HAT are given in Appendix 10. The discussion in this chapter should be read in conjunction with the description details of each item of both tests in chapter three and appendices 9 and 10.

4.2 THE AMST

For each item of the AMST the following descriptive information is provided in Appendix 9:

- the acceptable response;
- a diagram of each item to remind the reader what each item was about;
- where appropriate, a list of other responses given by the participants;
- the maximum mark obtainable;
- the internal difficulty factor (see explanation below);
- classification in terms of spatial visualisation and orientation (shaded accordingly);

- classification in terms of two dimension and three dimension perspective (shaded accordingly);
- a table indicating the performance of the participating schools and gender performance differences;
- a bar graph illustrating the performance of each participant.

Internal difficulty factor

The internal difficulty factor is a useful indicator as to how difficult each item was perceived by the participants. It is simply a rank based on the performance of the entire sample. For example, the internal difficulty factor of 1 for task 5 suggests that this was the most difficult item because only 22% of the sample managed the correct answer. Refer to Table 4.1 on page 120 for an overview of the difficulty factors for each item and the performance for the entire sample.

In the narrative below I specifically refer to:

- gender performance differences;
- differences in performance within the five participating schools;
- the findings in Wattanahawa's study, where appropriate.

4.2.1 Item-by-item analysis

For details of the requirements of each task refer to chapter three, or consult the original format of the test in Appendix 9. As task 1, task 2 and task 3 were very open-ended tasks, an analysis of them is provided at the end of this section.

Refer to Table 4.1 for a detailed overview of the performances of each participant on each item (except for tasks 1-3) of the AMST.

Table 4.1 Summary of performance of entire sample of all tasks of the AMST

NAME	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21	T22	T23	T24	T25	Tot	Percentage	Rank
Maximum				1	5	1	1	2	1	1	1	1	3	3	1	1	3	1	1	1	1	1	1	2	1	34	100	
Acceptable response				B		B	D			D	4	D			D		CBD	37	C		E	C		JM	C			
P1 S1				1	1	1	AB	2	1	BE	3	1	3	0	1	1	2	1	1	1	C	B	0	1	1	19	55.9	12
P2 S1				1	0	C	1	0	1	0	1	A	3	0	D	0	3	36	1	0	1	1	1	IP	0	15	44.1	19
P3 S1				A	0	1	1	0	1	A	1	1	1	0	0	1	1	78	1	0	C	0	1	0	B	10	29.4	27
P4 S1				1	2	1	1	2	1	A	1	1	3	3	1	1	2	1	1	1	1	1	0	2	1	28	82.4	1
P5 S1				C	0	1	E	0	0	A	1	E	3	1	B	1	3	21	B	0	B	1	0	KO	E	11	32.4	25
P6 S1				E	1	1	E	2	1	A	2	1	3	3	1	0	1	44	1	0	B	E	1	FJ	A	16	47.1	16
P7 S1				1	0	1	E	2	1	B	1	1	3	0	C	0	2	1	1	1	1	B	0	2	B	18	52.9	15
P8 S2				A	5	1	B	2	1	B	1	A	0	3	A	1	3	1	1	0	D	D	0	JO	1	21	61.8	9
P9 S2				1	0	1	1	2	1	E	12	A	0	0	C	0	2	1	D	0	1	E	1	2	BE	13	38.2	21
P10 S2				1	1	1	1	2	1	1	1	A	3	1	1	1	3	1	1	1	1	1	1	1	1	26	76.5	4
P11 S2 male				A	3	1	E	2	1	B	1	1	0	2	1	0	2	1	1	1	1	1	1	EB	1	21	61.8	9
P12 S2				C	0	D	E	2	0	none	1	A	0	0	all	0	3	1	1	1	1	E	0	LP	1	11	32.4	25
P13 S2				1	2	1	1	2	1	1	1	1	3	1	1	1	3	1	1	0	D	1	1	2	1	27	79.4	3
P14 S3 male				A	0	1	1	2	1	B	5	1	3	0	A	0	3	21	A	0	0	1	0	2	E	15	44.1	19
P15 S3				1	0	C	A	0	0	B	1	1	3	0	C	1	2	45	1	0	0	B	1	1	1	13	38.2	21
P16 S3 male				BD	1	1	1	1	1	1	10	A	3	1	C	1	2	41	1	0	1	B	0	KO	1	16	47.1	16
P17 S3				1	1	1	C	1	1	E	1	1	3	1	C	0	0	33	1	0	C	E	0	0	1	13	38.2	21
P18 S3				A	0	1	1	0	0	B	8	B	3	1	A	0	1	35	1	0	1	1	1	2	all	13	38.2	21
P19 S3				1	0	1	1	1	1	B	0	A	3	0	C	1	2	20	1	0	1	1	0	2	all	16	47.1	16
P20 S4 male				1	0	1	B	1	1	B	1	A	3	2	1	1	1	42	1	0	1	E	1	2	1	19	55.9	12
P21 S4 male				E	2	1	1	2	1	1	1	1	3	1	1	0	3	43	1	1	1	1	0	1	1	24	70.6	5
P22 S4 male				all	2	1	1	1	1	B	1	1	3	1	A	0	2	1	1	0	1	1	1	2	1	22	64.7	8
P23 S4 male				1	4	1	B	1	1	1	1	A	3	1	B	1	3	1	1	0	1	B	0	2	0	23	67.6	6
P24 S4 male				1	0	1	B	1	1	1	1	A	3	1	A	1	3	1	1	1	1	1	1	2	1	23	67.6	6
P25 S4 male				1	3	1	1	2	1	1	1	1	3	1	1	1	3	1	B	1	1	1	1	2	B	28	82.4	1
P26 S4 male				1	0	1	B	1	1	1	1	A	3	0	A	1	3	1	1	1	1	1	0	2	1	21	61.8	9
P27 S5				A	0	1	C	0	0	AB	10	ABC	3	0	D	0	BAA	40	A	0	D	D	0	FH	B	5	14.7	30
P28 S5				AD	0	1	AC	0	0	AB	2	1	0	0	AC	0	BAA	116	1	0	B	1	0	LD	B	4	11.8	31
P29 S5				A	2	1	C	0	0	B	8	1	0	0	B	0	1	70	0	0	1	A	0	0	E	6	17.6	29
P30 S5 male				1	5	1	B	2	0	B	1	1	3	1	A	0	2	24	A	0	1	A	0	1	B	19	55.9	12
P31 S5				A	0	1	C	0	0	A	6	1	0	0	C	0	1	40	D	0	C	D	0	IC	1	4	11.8	31
P32 S5				A	0	1	A	0	0	A	6	A	3	0	B	1	2	22	B	0	1	1	0	KO	1	10	29.4	27
Total				16	35	29	13	36	22	8	20	17	73	25	9	16	64	14	23	10	20	16	13	31	17		48.7	
% of sample correct				50	22	91	41	56	69	25	63	53	76	26	28	50	67	44	72	31	63	50	41	48	53			
Difficulty factor				10	1	22	6	15	19	2	16	13	21	3	4	10	18	8	20	5	16	10	6	9	13			

Table 4.1 provides the following information:

- Each of the 25 AMST tasks is labelled T1, T2 up to T25;
- The maximum scored attainable is given for each task;
- Where appropriate, the acceptable response is given for each task. For some tasks, such as T8, the participants were required to provide a drawing, hence the blank space;
- The participants are referred to as P1 to P32 and the corresponding school (S1 to S5) is indicated;
- Where male, the gender is indicated, otherwise the participant is female;

- For each task the correct response for each participant is recorded by the score obtained. In the case of an incorrect answer, the participant's response is recorded and shaded where appropriate, or a zero given;
- For each task, the total of the participants' correct answers are tallied;
- For each task, the percentage of correct answers for the sample is recorded;
- The difficulty factor is given for each task;
- The total score attained for each participant is given;
- The percentage score is given;
- The rank for each participant is given.

Task 4

According to the internal difficulty classification, this task ranked tenth and as was the case in the original Wattanahawa study, no strong gender difference occurred. On average, the males performed marginally better than the females.

Except for S5, the other schools achieved similar results. It is interesting to note that A was incorrectly chosen as the correct answer by 11 participants. There could be numerous explanations for this:

- failure to recognise that a reflection is fundamental to this problem;
- failure to recognise that the stamp represented a three-dimensional object;
- failure to transfer from three dimensions to two dimensions.

Task 5

This was the most difficult task and reflected a big gender difference in performance. This task relied heavily on three-dimensional perception and the gender results strongly support research which suggests that males perform more successfully in three-dimensional problems (see page 19).

The gender difference was clearly illustrated by the relatively poor performance of the all-girl school S1. It is interesting to note however, that S3 also performed poorly in this task.

Task 6

This was the easiest problem, as can be seen from the internal difficulty factor and chart 5.3: only three participants, P2, P12 and P15, did not provide the correct answer. It was correctly answered by 91% of the sample. Although all the males achieved full marks, the gender difference is marginal, which supports the notion that two-dimensional problems favour gender equality.

Further, the school performance across the schools was constant.

The above observations were consistent with Wattanahawa's results.

Task 7

This task was the sixth most difficult and was completed correctly by only 41% of the sample. The gender difference only marginally favours the males but there is a noticeable non-performance of S5. None of the participants there managed to solve this problem.

The answer B was given as the next most frequent response which points to a misinterpretation of the unfolding of the sheet of paper. There is a recognition that the folded paper consisted of 'four sheets', hence the choice of four half circles, but the correct transformation to a two-dimensional image was not made.

Task 8

This task was very similar to the previous one. Except for the actual cuts the number and configuration of folds is exactly the same. It is therefore quite surprising that the participants found this task much easier than the previous one – it was ranked fifteenth on the difficulty scale.

Further, it is interesting to note that the gender difference in performance is more pronounced in favour of the males in this problem than in the previous task.

Once again S5 performed very poorly compared to the other schools.

Task 9

This purely two-dimensional problem was characterised by a surprisingly big gender difference in performance. Nearly all the males managed full marks whereas just over one half of the females received full marks. As mentioned before, in terms of statistical analysis the sample in this study was very small and the statistical manipulation of the data generated therefore needs to be treated with circumspection. Yet it is interesting to note that in this case a two-dimensional problem did not favour the girls. This result reinforces the notion that gender generalisations need to be dealt with critically.

According to the difficulty factor, this task was regarded as a fairly easy problem.

Further, S5 struggled to make any sense of the problem. This could be due to language – although the problem was explained to the class and it appeared at the time that they understood what the issue of the problem was.

Task 10

This task was rated the second most difficult task. This concurs with Wattanahawa's difficulty rating. It is notable that S1, the all-girl school, performed very poorly as did the township school S3 and the rural school S5. The general poor performance by the girls is consistent with Wattanahawa's findings.

A pattern that seems to be emerging is that the participants from the rural school struggled particularly with problems that were dominated by three-dimensional perspectives.

Task 11

This was a fairly easy task, which could have been worked on on the diagram itself. The male dominance in performance was once again very noticeable, as was the failure of the rural school.

Task 12

As with the two-dimensional emphasis of task 4 and task 6, this task was characterised by relative equity in terms of gender difference. This is consistent with Wattanahawa's findings on this item. It is also interesting to note that the rural school coped very well with this item.

The other answer provided was that of A (frequency of 13). This suggests a misconception similar to the one on task 4 - the notion of reflection was ignored entirely.

Task 13

This task was registered as a very easy one and this was reflected in the relatively good performance throughout the sample (see Table 4.1). This item served as a lead-in to the next task, which in Wattanahwa's research was perceived as very difficult. I thought that this task would set the scene and scaffold the process for the following task.

I was surprised at the extent to which the males dominated this task as it was a purely two-dimensional problem involving a simple translation. This seems to go against the assertion that performance in two-dimensional problem favours neither males or females.

The relatively good performance of the rural school supports the conjecture made in the previous comment on page 123 which implies that two-dimensional problems seem to be more accessible to those participants than three-dimensional tasks.

Task 14

As in the Wattanahawa study this task proved to be very difficult. Surprisingly, in this task the females performed better in relation to their male counterparts than in the previous task. This would support the view that performance in two-dimensional tasks between males and females is relatively equitable.

It is interesting to note that although the participants in the rural school coped well in the previous task they found this one very difficult. This is not surprising, as it involves a combination of reflection and rotation.

Task 15

This straightforward two-dimensional task proved to be surprisingly difficult. It registered as the fourth most difficult task. The correct strategy involved a simple rotation in the same plane. As expected, there was equity between the girls and boys, but S3 and S5 did very poorly.

The overall performance as illustrated in Table 4.1 shows that just over one-third of the sample managed this problem. The other third of the sample chose A as the correct answer. The reasons for this are not clear as most of those who chose A indicated in their AMPQ that they used the correct strategy. P26 however argued that he chose A because:

I counted the stripes in the stripy corner – they all have 6 except A, which has 5.

Task 16

Half of the sample answered this correctly and once again a relatively equitable performance between boys and girls was achieved. As in the previous task, this is not surprising, as this problem was situated entirely in two-dimensional space.

Once again S5 found this a difficult task to perform. The recognition of pattern and shape is quite complex, yet, with the exception of P32, none of the participants even recognised that the truncated triangle should appear above the black solid line.

Task 17

This task was not regarded as very difficult and two-thirds of the sample managed to solve it correctly. Despite the dominance of two-dimensional space (except in question 2), the boys performed better than the girls.

Despite the more intricate nature of the problem, S5 managed much better than in the previous two tasks.

Task 18

This three-dimensional task was recorded as the eigth most difficult task, mainly due to the fact that both S3 and S5 could not manage the problem at all. In their AMPQ four participants of S3 and S5 indicated that they counted the cubes in the vertical columns and then added them up, whereas one said that he counted the cubes in the horizontal rows. One participant admitted to guessing a number and the rest suggested that they imagined the stack in their minds and then counted.

There was no consistency in the alternative answers given, except that two participants (P5 and P14) gave the answer as 21, which is the number of cubes that are visible in the diagramme.

On the whole the males performed better than the females.

Task 19

This was perceived as a very easy problem and across the board (except for S5) the participants managed very well.

It is interesting to note that this problem showed the most equity between the boys and girls in terms of performance, of all the problems of the AMST. This surprised me as I would have expected most equity in a two-dimensional problem. It could be argued that this three-dimensional problem was very easy and therefore accessible to everybody irrespective of gender.

Task 20

Characteristic of this task once again was the failure of S3 and S5 to perform at all. There are similarities to the previous two tasks in that the participants needed to be able to form mental images of the stacks from different angles. It is interesting to note that both those schools obviously found it very difficult to operate in three-dimensional space. The diagram that most of the participants offered consisted of an arrangements of cubes as seen from the front, i.e., the left column higher than the right column. It could be argued that the question was misread or that the instructions were not clear enough in emphasising that the view required was one from behind, although one would have expected the pilot to have revealed this.

The observation that the boys performed much better than the girls is consistent with the Wattanahawa study.

Task 21

Although this task involved two-dimensional flat shapes, the act of folding suggested a transfer into three-dimensional space. The transferral to three-dimensional space may be the reason why the boys outperformed the girls.

Except for S4, which achieved a full score, the difference in performance between the schools appeared to be marginal.

Task 22

As in Wattanhawa's study, the overall male dominance in this three-dimensional task was self-evident.

Only one-third of the rural school S5 participants managed to solve this task. This appears to be consistent with other three-dimensional tasks, where participants of the rural school seemed to struggle.

Task 23

This task was surprisingly difficult and less than half the sample managed to draw the correct map.

Once again none of the participants in S5 managed the entire task. This could be due to language difficulties although all of them managed at least half of the task. Interestingly, all of them started off correctly by going one block west. Three of them correctly headed south for four blocks and east for three. However, none of them then turned left.

The male dominance in this task is very marginal.

Task 24

This was a difficult three-dimensional task with males doing better than females. The all-boys school S4 scored nearly full marks whereas the all-girls school S1 struggled to achieve half marks. This concurs with Wattanahawa's findings.

Once again the rural school could not make any sense of this problem. The township school, however managed relatively well, outscoring both S1 and S2.

Task 25

In the final task, which was purely two-dimensional, gender equity would have been expected, yet the males once again showed their dominance.

It is interesting to note that the next popular answer was B (frequency 6). This may be because B at first appears very obscure. It needs careful rotation to make it coincide with its counterpart in the body of the puzzle.

4.2.2 Overall trends

The overall trends are discussed under the following headings:

- gender differences
- school differences
- individual differences.

It is recognised that statistically the findings below need to be viewed with circumspection as the sample was very small. They were, however useful as starting points for discussing particular differences and identifying trends peculiar to this case.

Gender differences

From the analysis in Chart 4.1 and Table 4.2, it is evident that for each item the males dominated the females in performance. The extent of dominance varied however. Wattanahawa's research suggested that difference in performance diminished for two-dimensional problems.

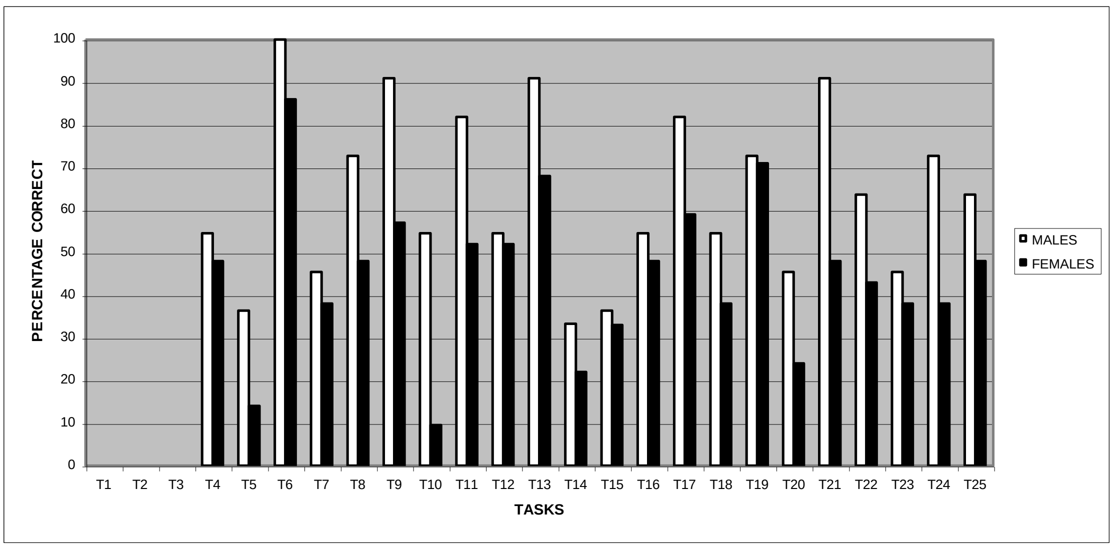


Chart 4.1 Item-by-item performance according to gender

This is illustrated in Table 4.2, which indicates the mean performance for males and females for the two-dimensional and three-dimensional tasks. I have only included those tasks which are exclusively two- or three-dimensional. The smaller mean difference for the two-dimensional tasks confirms the notion that generally, in two-dimensional problems the girls are not as heavily outstripped by the males as in three-dimensional problems. Statistical test revealed a significant difference at the 5% level of significance: girls three-dimensional compared to girls two-dimensional gave t=8,42 and pv=2,776 and girls three-dimensional compared to males three-dimensional gave t=2,4 and pv=2,2.

Table 4.2 Average performance of males and females on specifically two- and three-dimensional tasks

	Two-dimensions		Three-dimensions						
Task	Boys	Girls	Boys	Girls	S1	S2	S3	S4	S5
T6 vis	100	86			57.14	50.00	50.00	71.43	16.67
T9 orien	91	57			85.71	83.33	66.67	100.00	0.00
T11 orien			82	52	71.43	83.33	33.33	100.00	16.67
T13 vis	91	68			71.43	83.33	33.33	100.00	16.67
T15 vis	36	33			57.14	50.00	0.00	42.86	16.67
T16 orien	54	48			57.14	50.00	50.00	71.43	16.67
T18 orien			54	38	57.14	50.00	50.00	71.43	16.67
T19 vis			73	71	85.71	83.33	83.33	85.71	16.67
T20 orien			45	24	42.86	50.00	0.00	57.14	0.00
T22 orien			64	43	42.86	50.00	50.00	71.43	33.33
T24 vis			73	38	35.71	50.00	58.33	92.86	8.33
T25 vis	64	48			28.57	83.33	50.00	71.43	33.33
Average	72.7	56.7	65.2	44.3					
Mean Diff.	16.0		20.9						
Average two-dimension					59.52	66.67	41.67	76.19	16.67
Average three-dimension					55.95	61.11	45.83	79.76	15.28

In terms of the spatial visualisation and orientation constructs, Table 4.3 illustrates that, although the males dominated in each task, there is a slightly higher mean between the difference of performance of the males and females in the spatial orientation problems than in the spatial visualisation ones. This is consistent with the observation that females are not as heavily dominated by the males in two-dimensional tasks as three-dimensional ones, as the spatial orientation constructs are characterised by three-dimensional tasks.

Table 4.3 Difference in mean performance between males and females

NAME		T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21	T22	T23	T24	T25		Ave
BOYS		54.5	36.4	100.0	45.5	72.7	90.9	54.5	81.8	54.5	90.9	33.3	36.4	54.5	81.8	54.5	72.7	45.5	90.9	63.6	45.5	72.7	63.6		61.8
GIRLS		48	14	86	38	48	57	9.5	52	52	68	22	33	48	59	38	71	24	48	43	38	38	48		41.9
Difference		6.5	22.4	14.0	7.5	24.7	33.9	45.0	29.8	2.5	22.9	11.3	3.4	6.5	22.8	16.5	1.7	21.5	42.9	20.6	7.5	34.7	15.6		19.9
Spatial visualisation		6.5	22.4	14.0	7.5	24.7		45		2.55	22.9	11	3.4				1.7		43			35	16		18.2
Spatial orientation							33.9		29.8						23	16.5		21.5		21	7.45				21.8

School differences

As intimated in the comments of the item-by-item discussion, the rural school S5 consistently fared worse than any other school in 18 out of the 22 tasks, as can clearly be seen from Chart 4.2 and Table 4.4. It was only in tasks 6, 12, 13 and 21 that S5 did better than one or more of the other schools. These four tasks were mainly two-dimensional in nature and all of them were dominated by the spatial visualisation construct. By inference this would suggest that the rural school learners in this sample found it difficult coping with:

- three-dimensional problems;
- problems that are characterised by the spatial orientation construct.

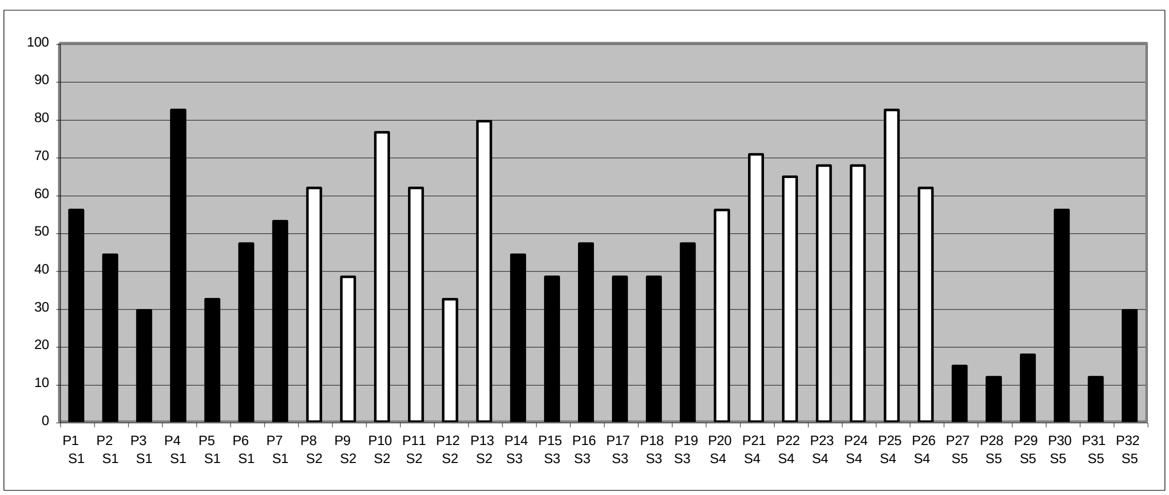


Chart 4.2 Individual performances clustered in schools

Table 4.4 Average school performance for each task

	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21	T22	T23	T24	T25	Average
S1	57.14	11.43	85.71	42.86	57.14	85.71	0.00	71.43	71.43	90.48	33.33	57.14	57.14	66.67	42.86	85.71	42.86	42.86	42.86	42.86	35.71	28.57	52.36
S2	50.00	36.67	83.33	50.00	100.00	83.33	33.33	83.33	33.33	33.33	38.89	50.00	50.00	88.89	100.00	83.33	50.00	66.67	50.00	66.67	50.00	83.33	62.02
S3	50.00	6.67	83.33	66.67	41.67	66.67	16.67	33.33	50.00	100.00	16.67	0.00	50.00	55.56	0.00	83.33	0.00	50.00	50.00	33.33	58.33	50.00	43.74
S4	71.43	31.43	100.00	42.86	64.29	100.00	71.43	100.00	42.86	100.00	33.33	42.86	71.43	85.71	71.43	85.71	57.14	100.00	71.43	57.14	92.86	71.43	71.13
S5	16.67	23.33	100.00	0.00	16.67	0.00	0.00	16.67	66.67	50.00	5.56	16.67	16.67	33.33	0.00	16.67	0.00	50.00	33.33	0.00	8.33	33.33	22.90

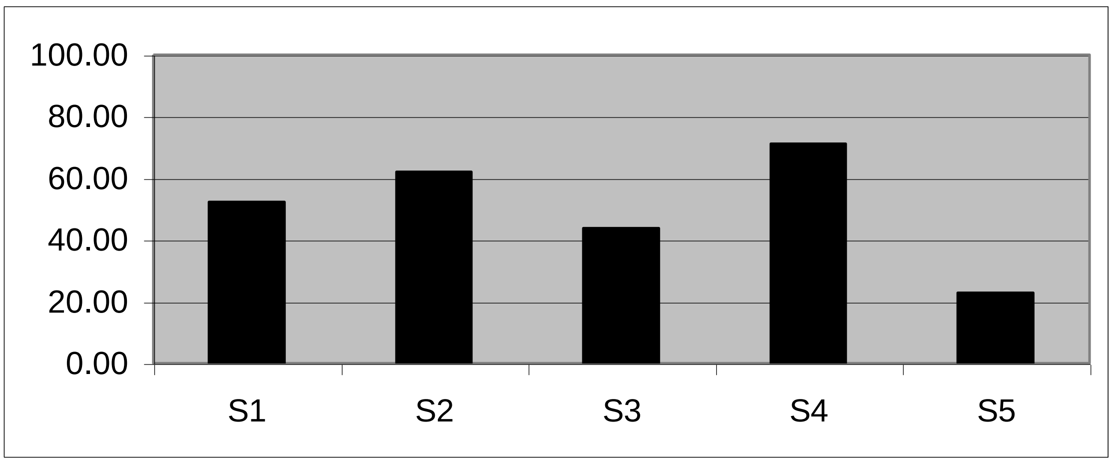


Chart 4.3 Average school performance

In terms of the global mean performance of 48,7% (Table 4.1 on page 129), the township school S3 performed below that. Its relative position as compared with the other participating schools is illustrated in chart 4.3 above. The tasks which S3 found particularly problematic were task 5, task 8, task 10, task 11, task 14, task 15, task 18, task 20 and task 23 (all the ones in which S3 on average scored below the global mean). Except for task 15 and task 23, all the above tasks are characterised by being entirely or partly three-dimensional in nature. Interestingly enough, with the exception of task 11, task 18 and task 23, they were all classified as spatial visualisation problems. So in summary, the township school learners of this sample found it difficult in coping with:

- three-dimensional problems;
- problems characterised by the spatial visualisation construct.

In terms of the male dominance in performance of this sample, it is not surprising that on average the all-boys school S4 performed better than any of the other schools.

Individual differences

As mentioned before, on average the males outperformed the females in every task, but on closer analysis this could be misleading. On an individual basis it is interesting to note for example, that the top five scores were dominated by females. The top score of 82,4% was shared by a females and a male (P25 and P4) and the next two positions were occupied by females (P13 and P10). By the same token, however, the bottom five positions were also dominated by females (P31, P28, P27, P29 and P3). In fact the lowest position that any male achieved was nineteenth place (P14). This could explain why, on the whole, the statistics indicate that the males outperformed the females.

From a socio-economic perspective, the statistics would indicate that those from a poor socio-economic background did not fare as well as their counterparts from a more privileged environment. This study confirmed other research, as mentioned on page 25, which points to a high correlation between poor performance in general spatial pen-and-paper tests and poor socio-economic background. It was therefore not surprising that the general performance of the participants of S3 and S5 in the AMST was compromised. Their relative poor performance is further illustrated by their low average rank of 19 and 26,7 respectively.

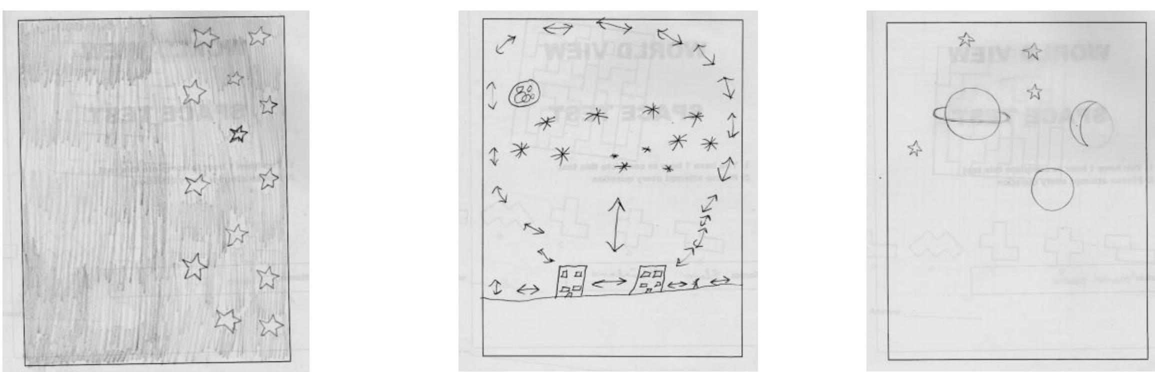
Once again we need to guard against overlooking individual anomalies which go unnoticed in mean tendency analyses. P30 from S5 is a case in point. Although this participant had most of the characteristics of his peers in S5, he was ranked twelfth in the entire sample. In his overall performance he achieved 55,9%, which was above the general average of 48,7% and the average of 22,9% of his school.

4.2.3 Analysis of open-ended tasks 1-3

Task 1

This task simply asked the participants to sketch their image of space. The rationale behind it was to explore the extent of consistency between the participants' written definitions of space as articulated in the questionnaire (see page 60), and their visual representations.

Those whose written definition described space in terms of the universe, stars and planets generally drew an accurate sketch of what they had said. Figure 4.1 shows three such sketches.

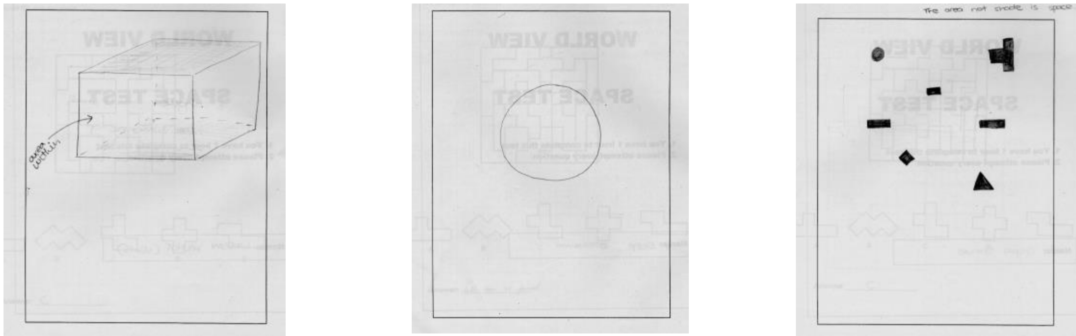


P3 P25 P10

Figure 4.1 Examples of sketches which show space in terms of its planetary definition

Other sketches did not appear to show any consistencies between what was said and what was drawn. In Figure 4.2, for example, the sketch by P4 shows space as confined and having clear boundaries, whereas her written definition suggests space as: *open expanses, a place where movement is not restricted and things seem uncluttered.*

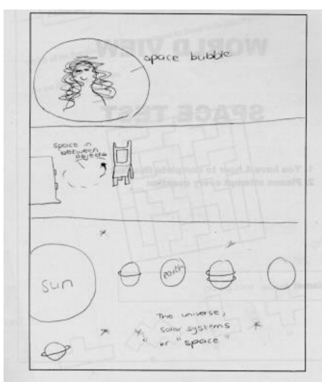
The sketch of P19 is very consistent with what she had written: *space is a big hole which is round.* The same applies for P13 who wrote that space *is just air between things, but it can keep things apart.*



P4 P19 P13

Figure 4.2 Further examples of sketches

Figure 4.3 provides an example of a sketch by P12 who attempted to depict her multitude of ideas about space. She referred to space as her 'space bubble', the space between objects and the universe.



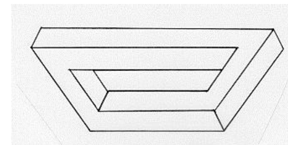
P12

Figure 4.3 Sketch showing multitude of perceptions

Many of the respondents simply left the page blank.

Task 2

This task asked the participants to describe as accurately as possible what they saw in the diagram. The responses varied a great deal and some participants were more articulate than others. In the analysis it was important to recognise that what was written by the individual participants was not necessarily fully representative of what was seen.



Reminder

The comments below therefore relate only to their articulation. Table 4.5 provides a summary of the main ideas with which the participants associated the diagram.

Only P13, P21 and P22 explicitly wrote that the figure was an impossible object and cannot be constructed:

> *A three dimensional drawing – it is impossible to build.* [P22; AMST]
>
> *A trapezium shape, drawn so that the sides are twisted – to physically make such a shape would be impossible.* [P13; AMST]

Just over one-third of the sample, however, suggested that there was a 'twist' in the shape:

> *I see a three-dimensional shape with the middle missing – I also see the shape as if it was twisted.* [P12; AMST]
>
> *It looks like the block has been twisted.* [P11; AMST]
>
> *I see a bending shape. It's bending on the top right corner and there's emptiness all around it.* [P25; AMST]

Ten of the respondents wrote that it was a three-dimensional shape whereas only one explicitly said it was two-dimensional.

Table 4.5 Main ideas that the diagram in task 2 was associated with

NAME			A	B	C	Trap.	Rect.	Hex.	Twist.	Lin.	Cub.	Holl.	Ideas
P1	S1				1		1		1			1	4
P2	S1				1						1	1	3
P3	S1									1			1
P4	S1						1		1			1	3
P5	S1				1								1
P6	S1				1	1		1				1	4
P7	S1			1		1	1						3
P8	S2				1		1		1				3
P9	S2								1				1
P10	S2				1				1				2
P11	S2	male							1	1		1	3
P12	S2				1				1			1	3
P13	S2		1			1			1				3
P14	S3	male						1					1
P15	S3											1	1
P16	S3	male				1	1						2
P17	S3											1	1
P18	S3											1	1
P19	S3										1		1
P20	S4	male					1					1	2
P21	S4	male	1									1	2
P22	S4	male	1		1				1				3
P23	S4	male			1								1
P24	S4	male			1		1		1		1		4
P25	S4	male							1			1	2
P26	S4	male										1	1
P27	S5						1						1
P28	S5										1		1
P29	S5										1	1	2
P30	S5	male				1	1						2
P31	S5						1						1
P32	S5						1					1	2
Total			3	1	10	5	11	2	11	2	5	15	
A - explicitly mentions the illusion							Hex. - Hexagon						
B - two-dimensional object							Twist. - twisting evident						
C - three-dimensional object							Lin. - lots of lines						
Trap. - Trapezium							Cub. - Cube						
Rect. - Rectangle							Holl. - Hollow middle						

Of the standard geometric shapes that the diagram was associated with, the rectangle dominated, followed by the trapezium, hexagon and the cube. Two respondents suggested that the diagram consisted of many lines:

> *I see lines drawn from the inside all along until a shape was formed.* [P3; AMST]

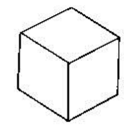
The dominant feature which was identified (48% of the sample) was the hollow centre of the shape:

> *It has an open space in the middle, so that you could stick your hand through it.* [P1; AMST]
>
> *I see a block that has space and air particles in it.* [P15; AMST]
>
> *I see a sketch of a block which has a hole inside.* [P17; AMST]

As intimated above, it was not possible to make general comments and recognise general trends as we cannot be sure of what was not said!

Task 3

As in task 2, the participants were asked to write down as accurately as possible what they saw.
As mentioned elsewhere, I was interested to see who identified this diagram as a three-dimensional cube, and who recognised it as a two-dimensional hexagon with a y-shape inside.



Reminder

As in the previous activity we need to recognise that what was written by an individual is not necessarily fully representative of what s/he actually saw. The comments below therefore are based entirely on what was articulated.

Table 4.6 provides a summary of the responses.

Table 4.6 Summary of responses to task 3

NAME	Cube	Hex Y	Hex	Square
P1 S1	1			
P2 S1	1			
P3 S1	1			
P4 S1	1			
P5 S1	1			
P6 S1	1			1
P7 S1	1			1
P8 S2	1			
P9 S2	1			
P10 S2	1			
P11 S2 male	1			
P12 S2	1			
P13 S2	1			
P14 S3 male				1
P15 S3	1			1
P16 S3 male	1			
P17 S3	1			
P18 S3				1
P19 S3	1			
P20 S4 male	1			
P21 S4 male	1		1	
P22 S4 male	1			
P23 S4 male	1			
P24 S4 male	1			
P25 S4 male	1			
P26 S4 male	1			
P27 S5	1			1
P28 S5	1			1
P29 S5	1			1
P30 S5 male	1			1
P31 S5	1			1
P32 S5	1			1
Total	30	0	1	11

Quite clearly, the majority (94%) associated the diagram with a cube, with only one participant (P21) suggesting it could also be a hexagon. No one suggested that it could be a hexagon with an internal y. P14 simply said it was a four-sided figure – a square.

Very evident was the lack of terminology, particularly for pupils in S5. It was clear to me that they described a cube, but used the term ‘square’:

> *This is a square and I see three sides, but if I take this block I see many sides. This square has six sides.* [P32; AMST]
>
> *In this diagram I can see three sides and they are equal. This diagram is a square.* [P27; AMST]

In terms of gender differences the results above were a little surprising as I was expecting the females to identify more fully with the two-dimensional option. As mentioned in 4.2.2 on page 129, the results in the AMST concur moderately with the research literature, which suggests that on the whole females do not achieve as well as males in three-dimensional problem-solving situations. This does not however necessarily imply that females do not recognise three-dimensional objects, as the result of this task demonstrated.

4.2.4 Conclusions relating to the AMST

This section provides an item-by-item discussion of the AMST by looking specifically at gender-performance differences, school-performance differences, individual differences, and comparisons with Wattanahawa’s study.

The most evident trends observable were:

- The males performed better than the females in every task (see section 4.2.2. on page 128);
- The gender difference was more pronounced in three-dimensional problems (see section 4.2.2 on page 129);
- The rural school in particular performed poorly relative to the other participating schools. This was particularly pronounced in three-dimensional problems (see section 4.2.2 on page 131). This concurs with research mentioned in section 2.2.2 on page 25.

The following section provides an item-by-item discussion of the HAT.

4.3 THE HAT

Activities 1, 2, 3 and 4 of the HAT were all puzzle-type activities characterised by the spatial visualisation construct. They will be focused on first. Activities 5, 6, 7 and 8 were activities that involved sketching and drawing. They were characterised by the spatial orientation construct and will be focused on second.

A summary of the participants' performance is provided in Appendix 10.

4.3.1 Item-by-item discussion of the spatial visualisation activities

Activity 1

Two dominant strategies were used to solve this problem:

a) The one that was used most consistently consisted of selecting any one shape at random with the one hand. The other hand was used to hold the ball, which was rotated rapidly until the appropriate hole appeared into which the shape could then be placed;

b) The other strategy consisted of first identifying a hole in the ball and then selecting the appropriate shape that fitted the selected hole.

Strategy a) proved to be more time-efficient.

Chart 4.4 illustrates that although the boys were quicker initially than the girls, there is very little difference between the two in the average minimum time.

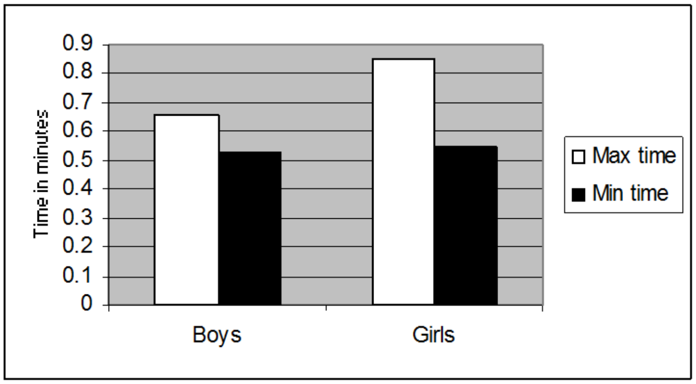


Chart 4.4 Gender difference in time for activity 1.

From Table 4.7 it can be seen that there is very little difference in the average time achieved by the two single-sex schools S1 and S4. The average times achieved by S1 and S4 were lower that those attained by the township school S3 and the rural school S5. Statistical testing revealed a significance difference at the 10% level (t = 1,39 and pv = 1,36). Interestingly enough there was also very little difference in time between S3 and S5. Statistical testing revealed only a significance difference at the 10% level for S3 and S5 (t = 0,125 and pv = 2,28). It could be suggested that one of the reasons why the participants of the poorer schools were slower was due to their possible lack of childhood experience with these type of apparatus. From the questionnaire it transpires however, that all the participants except for P14 had had experience with the shape-sorter in their childhood.

Table 4.7 Summary of times attained in spatial visualisation activities of the HAT

NAME	Activity1			Activity2		A1+A2		Activity 3	Activity 4
	Max time	Min time	Rank	Time	Rank		Rank	Time	Time
P1 S1	0.63	0.53	19	1.83	3	2.36	5	>5	>5
P2 S1	0.53	0.48	10	2.5	10	2.98	10	4.92	>5
P3 S1	1	0.5	11	5	26	5.5	26	>5	>5
P4 S1	0.75	0.42	4	1.92	5	2.34	4	0.67	2.5
P5 S1	0.75	0.6	23	4.17	24	4.77	24	>5	>5
P6 S1	0.58	0.58	20	2.97	16	3.55	17	>5	>5
P7 S1	0.52	0.5	11	2.17	9	2.67	9	>5	>5
	0.68	0.52	14	2.94	13.3	3.45	13.57		
P8 S2	0.58	0.45	6	2	7	2.45	6	>5	1.67
P9 S2	0.58	0.42	4	3.08	19	3.5	16	>5	>5
P10 S2	0.58	0.33	2	0.25	1	0.58	1	>5	>5
P11 S2 male	0.58	0.5	11	2.83	15	3.33	15	>5	>5
P12 S2	0.55	0.5	11	2	7	2.5	7	>5	>5
P13 S2	0.5	0.47	9	1	2	1.47	2	>5	2
	0.56	0.45	7.17	1.86	8.5	2.31	7.83		
P14 S3 male	0.58	0.45	6	6.5	31	6.95	31	>5	>5
P15 S3	1.17	0.87	30	5.42	29	6.29	29	>5	>5
P16 S3 male	0.6	0.6	23	5.08	28	5.68	28	>5	>5
P17 S3	2.25	0.67	25	3.17	20	3.84	19	>5	>5
P18 S3	0.92	0.5	11	2.5	10	3	11	>5	>5
P19 S3	0.75	0.67	25	4.58	25	5.25	25	>5	>5
	1.05	0.63	20	4.54	23.8	5.17	23.83		
P20 S4 male	0.67	0.45	6	4.07	22	4.52	22	>5	>5
P21 S4 male	dna	dna		dna		dna		dna	dna
P22 S4 male	0.5	0.28	1	1.92	5	2.2	3	>5	1.92
P23 S4 male	0.83	0.73	28	3.45	21	4.18	21	>5	>5
P24 S4 male	0.58	0.5	11	2.5	10	3	11	>5	>5
P25 S4 male	0.92	0.58	20	2.67	13	3.25	14	>5	4.17
P26 S4 male	0.6	0.52	18	4.08	23	4.6	23	>5	>5
	0.68	0.51	14	3.12	15.7	3.63	15.67		
P27 S5	1	0.5	11	5	26	5.5	26	>5	>5
P28 S5	0.92	0.33	2	2.67	13	3	11	>5	>5
P29 S5	1.5	0.58	20	6.08	30	6.66	30	>5	>5
P30 S5 male	0.72	0.67	27	3.03	18	3.7	18	>5	>5
P31 S5	0.92	0.88	31	3	17	3.88	20	>5	>5
P32 S5	0.9	0.78	29	1.83	3	2.61	8	>5	>5
	0.99	0.62	20	3.60	17.8	4.23	18.83		
Average	0.78	0.54		3.19		3.73		2.80	2.45

Activity 2

Relatively few of the participants (39%) used the photograph of the completed puzzle as a guide to putting the individual pieces together. The individual pieces obviously provided enough information in themselves to fit them all together.

Numerous strategies were adopted to solve this problem (see Table 4.8):

a) Initially all the pieces with straight edges were collected and put together to form the ‘frame’ of the puzzle. Then all the remaining pieces in the middle were fitted together;

b) Any straight edge was constructed (left, right, bottom or top) and the remaining pieces built around that edge;

c) The corners were identified first and the remaining pieces built around the corners;

d) The individual pieces were fitted together using colour patterns of the different shapes as the main criterion for construction;

e) The pieces were fitted together using the shapes of the individual pieces as the main criterion for construction;

f) The pieces were fitted together using the characters (Mickey Mouse etc) as the main criterion for construction;

g) Any two pieces were found as starting points and then the puzzle was constructed from there;

h) A random approach with no strategy evident.

Table 4.8 Summary of strategies followed in activity 2

NAME	Strategies								
	a	b	c	d	e	f	g	h	Used picture
P1 S1									no
P2 S1		1					1		no
P3 S1				1					no
P4 S1			1	1	1				no
P5 S1			1			1			yes
P6 S1				1					no
P7 S1	1			1					no
	1	1	2	4	1	1	1	0	
P8 S2	1			1					yes
P9 S2		1							yes
P10 S2		1							yes
P11 S2 male		1							no
P12 S2	1								no
P13 S2		1	1						yes
	2	4	1	1	0	0	0	0	
P14 S3 male					1	1			yes
P15 S3			1						no
P16 S3 male						1			yes
P17 S3		1							no
P18 S3						1			yes
P19 S3			1						
	0	1	2	0	1	3	0	0	
P20 S4 male			1	1					no
P21 S4 male									
P22 S4 male	1								no
P23 S4 male						f			yes
P24 S4 male			1						yes
P25 S4 male		1							yes
P26 S4 male								1	no
	1	1	2	1	0	0	0	1	
P27 S5								1	no
P28 S5								1	no
P29 S5			1						no
P30 S5 male				1					no
P31 S5			1						yes
P32 S5			1						no
	0	0	3	1	0	0	0	2	
Total	4	7	10	7	2	4	1	3	

Chart 4.5a illustrates that strategy (c) (selecting the corners first and building from there) was the most popular of the strategies. Building from any of the four edges (b), or matching appropriate colour patterns (d) were the next two popular strategies followed.

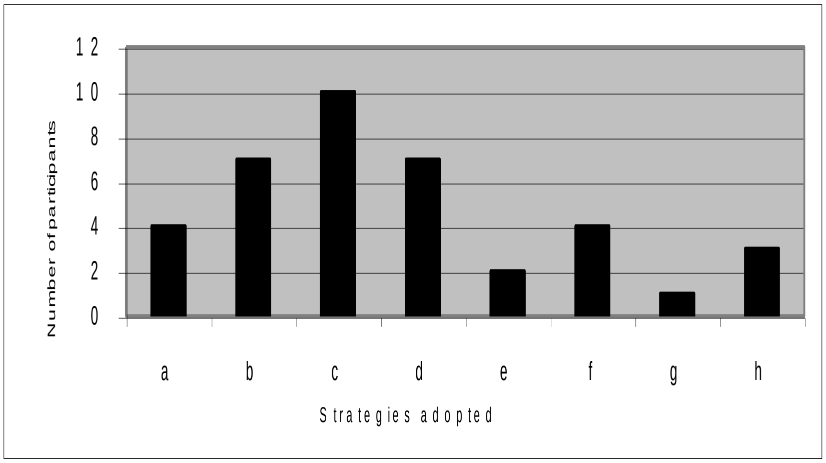


Chart 4.5a Strategies adopted for activity 2

Chart 4.5b illustrates that, on average, the boys were marginally quicker than the girls.

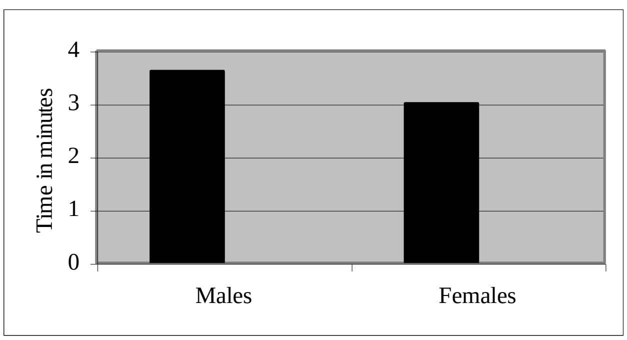


Chart 4.5b Gender difference for activity 2

Table 4.7 on page 144 reveals once again that the participants from the rural school S5 and the township school S3 were slower in completing the puzzle than their counterparts from S1, S2 and S4. Participants particularly from S5 recorded very slow times. On average they were nearly a whole minute slower than those from the rural school and more than 2,5 minutes slower than the quickest school S2. As in the previous activity it could be argued that the possible lack of childhood experience in playing with puzzles of this kind could have contributed to such discrepancies. From the reflections it transpired that of the students from the rural school only one member (P32) did not have childhood experience in playing with puzzles, whereas half of the students from the township school did not have the opportunity to play with puzzles of this kind when they were small.

Activity 3

Numerous strategies were adopted in participants' attempts to construct the square out of the seven pieces:

a) The outline of a square was constructed with some of the pieces and the remaining pieces were used to 'fill' in the 'inside' of the square;
b) The two big triangles were put together to form a square and the remaining pieces were placed around them in an attempt to form a bigger square;
c) There was random selection of pieces and trial-and-error;
d) The 'half' pieces were put together to form lots of little squares and then an attempt was made to put all the little squares together to form a big square;
e) People attempted to fit the five smaller pieces on top of the two big triangles;
f) Half a square was constructed using the two big triangles and then the remaining five pieces were fitted together to form the other half of the square.

Some of the participants' reflections:

Strategy a)

> *Tried to look for the corners and then fill in the middle.* [P11;R]

Strategy b)

> *My first plan was to construct with the two big blocks…*[P25;R]
>
> *I tried to put the two big ones together, and sort out the little ones without success.* [P20;R]
>
> *Positioned the two bigger triangles and arranged the smaller pieces around it.* [P4;R]

Strategy c)

> *It will bug me until I get it right – I felt I pretty much tried everything!* [P8;R]

> *It was so difficult to put this together – I have never meet something difficult like this – maybe I will try next time.* [P30;R]

> *Pick and match is the best way to describe it – it's like choosing a name for a pet, you've got so many choices you don't know which one to pick.* [P24;R]

Strategy d)

> *I first tried to put all the half pieces together…*[P23;R]

Strategy e)

> *I tried to build two bigger triangles out of the smaller ones….by putting them on top of the bigger triangles.* [P13;R]

Strategy f)

> *I put the two big triangles on the outside – fitted the rest next them.* [P2;R]

Table 4.9 and Chart 4.6 shows that strategy (c) (random, trial-and-error) was dominant, followed by strategy (b) (placing the two big triangles first).

It is significant to note that strategy (c) was the one mostly chosen by S3 and S5. Although I did not specifically ask whether the participants had ever played with a puzzle of this nature it is evident from their reflections that they found this activity very difficult.

> *This one is very difficult – even if you have to think a lot. First, one should be shown first so that you can make it. I think it's nice to work your brain.* [P14;R]

> *It was difficult to make the square – but I couldn't do it. I did my best, but it was difficult using all the seven pieces of different shape.* [P18;R]

> *This was one of the most difficult activities that I've done in this maths session. I couldn't figure it out – and I think even if the time was big I wouldn't be able to finish it.* [P16;R]

> *It was so difficult to put this together. I have never met something difficult like this. Maybe I will try next time.* [P30;R]

In terms of gender differences there are not enough data to make any meaningful observations, although it is interesting to note that the only two participants who managed to construct the required square were two girls (P2 and P4) from S1.

Table 4.9 Summary of strategies followed in activity 3

NAME	Strategies					
	a	b	c	d	e	f
P1 S1			1			
P2 S1						1
P3 S1		1				
P4 S1		1				
P5 S1			1			
P6 S1		1				
P7 S1		1				
	0	4	2	0	0	1
P8 S2			1			
P9 S2		1				
P10 S2		1				
P11 S2 male	1					
P12 S2				1		
P13 S2					1	
	1	2	1	1	1	0
P14 S3 male			1			
P15 S3			1			
P16 S3 male			1			
P17 S3			1			
P18 S3			1			
P19 S3			1			
	0	0	6	0	0	0
P20 S4 male		1				
P21 S4 male						
P22 S4 male					1	
P23 S4 male				1		
P24 S4 male			1			
P25 S4 male		1				
P26 S4 male		1				
	0	3	1	1	1	0
P27 S5				1		
P28 S5			1			
P29 S5			1			
P30 S5 male			1			
P31 S5		1				
P32 S5			1			
	0	1	4	1	0	0
Total	1	10	14	3	2	1

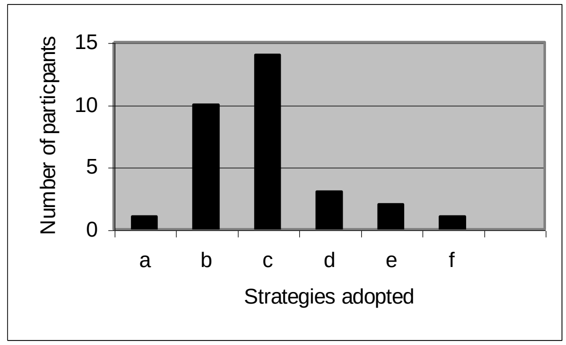


Chart 4.6 Strategies adopted for activity 3

Activity 4

The strategies were not quite as manifold as in the tangram of the previous activity:

a) Construct a square base and move up from there;

b) Random, trial-and-error;

c) Start with the obvious corner piece and moved from there;

d) Move up one level at a time;

e) Start with the flat shapes and then build with the irregular shapes.

Some of the reflections:

Strategy a)

> *I found the soma cube activity also difficult. I attempted to use the bigger blocks that could be laid down to form a square base…*[P10;R]

Strategy b)

> *I can't explain how I do this game because it was too difficult.*[P31;R]
> *I tried to fiddle around with it, but did not get it.*[P7;R]

Strategy c)

> *I knew the one shape had to be on the corner and I took it from there.* [P22;R]

Strategy d)

> *I tried moving up one level at a time…*[P24;R]

Strategy e)

> *I tried to figure out how many shapes had a flat shape and settled them down – then I made the more irregular shapes settle at the top…*[P5;R]

From Table 4.10 and Chart 4.7 it is evident that the trial-and-error strategy was the most popular approach. Even those who managed to construct a cube within the time constraints used strategies which at best can be described as trial-and-error.

Table 4.10 Summary of strategies followed in activity 4

NAME	Strategies				
	a	b	c	d	e
P1 S1		1			
P2 S1		1			
P3 S1		1			
P4 S1		1			
P5 S1				1	
P6 S1					1
P7 S1		1			
	0	5	0	1	1
P8 S2		1			
P9 S2				1	
P10 S2	1			1	
P11 S2 male	1				
P12 S2				1	
P13 S2		1			
	2	2	0	3	0
P14 S3 male		1			
P15 S3		1			
P16 S3 male		1			
P17 S3		1			
P18 S3		1			
P19 S3		1			
	0	6	0	0	0
P20 S4 male	1	1			
P21 S4 male					
P22 S4 male	1		1		
P23 S4 male		1			
P24 S4 male		1		1	
P25 S4 male	1				
P26 S4 male		1			
	3	4	1	1	0
P27 S5		1			
P28 S5		1			
P29 S5		1			
P30 S5 male		1			
P31 S5		1			
P32 S5		1			
	0	6	0	0	0
Total	5	23	1	5	1

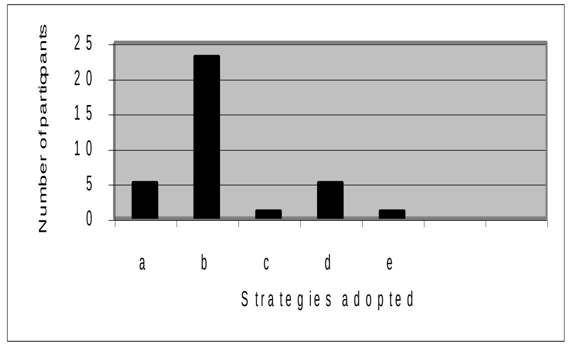


Chart 4.7 Strategies adopted for activity 4

Once again many of the participants of S3 and S5 commented explicitly on the difficulty of the task:

> *It is difficult to do a cube. But I enjoyed it and I must plan and think about it – but I am not doing it.* [P32;R]
>
> *The cube was very difficult but I try to do the cube. It was giving me a hard time.* [P27;R]
>
> *I failed to do it – it is difficult to me…I never do that* [P19;R]
>
> *This was difficult to make with all the seven pieces. I tried my best.* [P18;R]

In terms of gender differences, the data are not substantial enough for me to make meaningful comment. On a superficial level it can be seen that two boys and three females managed to construct the cube successfully. P4, a girl from S1, was the only pupil who managed both the tangram and the soma cube in the allotted time.

4.3.1.1 Overall trends

As in 4.2.2, the overall trends are described under the following headings:

- Gender differences
- School differences
- Individual differences.

Gender differences

As intimated in the discussion of each activity above, the statistics available do not reveal meaningful gender differences. Whilst the participants were engaged in the activities I kept field notes of my observations of each pupil. The primary aim was to verify their reflections but also to show obvious trends and inconsistencies. I found no observable gender differences. As all the activities were situated in three-dimensional space (with the exception of perhaps activities 2 and 3 which occur in the flat plane) it could have been expected that the girls performed worse than the boys. This did not occur.

School differences

In all of the activities above there was strong evidence that the township (S3) and the rural school (S5) did not manage as easily as their other counterparts. Generally they were much slower in completing the puzzle in activities 1 and 2 and from their reflections it transpired that they found the tangram and the soma cube exceedingly difficult. Hence they mostly adopted a strategy-free approach and relied on trial-and-error techniques. Although most of the pupils of S3 and S5 indicated that they had had experience with the shape-sorter and the jig-saw puzzle in their childhood, most of them had never played with a tangram or a soma-cube-type puzzle. This does not explain the time discrepancy for activities 1 and 2, but sheds some light on why their approach to activities 3 and 4 was haphazard and arbitrary. From my experience in working with rural school teachers (Schäfer, 1999) and supervising students in township schools, I was always struck by the general lack of resources and apparatus available to learners in those classrooms. On only rare occasions have I come across classroom environments in those schools which facilitated play and problem-solving with apparatus used in the HAT. The lack of widespread experience with apparatus of this kind could explain the discrepancy observed above.

Individual differences

In the timed activities 1 and 2, the two top cumulative times were achieved by P10 and P13, both from S2. The next two best times were scored by P22 from S4 and P4 S1. In terms of relating activities 1 and 2, I initially expected that those who managed to solve activity 1 swiftly would also solve 2 reasonably fast. This is not the case, however, as the rank correlation coefficient between activities 1 and 2 is only 0,37. This poor correlation suggests that there is no close relationship between the time it takes to finish them. This is clearly illustrated by the performance of P14, for example, who managed the sixth best time in activity 1 but came thirty-first for activity 2. A similar scenario occurred for P32 who came twenty-ninth for activity 1, but came a swift third for activity 2.

4.3.2 Item-by-item discussion of the spatial orientation activities

For each of the four spatial orientation activities, which all involved sketching (for details refer to Appendix 10), a maximum of three marks was achievable. The following rubric applied:

Table 4.11 Rubric for assessing spatial orientation activities

Activity	Marks					
	1		2		3	
5a	Correct position of tin and box		Correct details of perspective		Attention to detail	
5b	Correct position of tin and box		Correct details of perspective		Attention to detail	
6	Correct relative position of objects		Correct aerial/top view		Attention to detail	
7	Accurate map (relative position of landmarks)		Correct relative perspective		Attention to detail	
8	Accurate drawing (according to instructions)		Correct relative location and perspective of objects		Attention to detail	

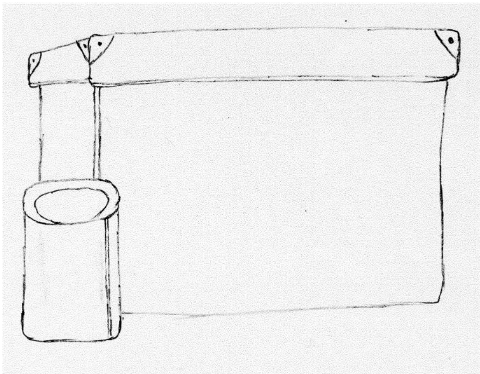
Activity 5a

On the whole, the drawings were accurate with 61% of the sample achieving full marks. See Table 4.12 for details and page 112 for the original view. Those participants who achieved only two marks mostly struggled to illustrate the three-dimensionality of the scene, whereas those who managed to only score one mark could not draw the relative position of the tin and the box accurately.

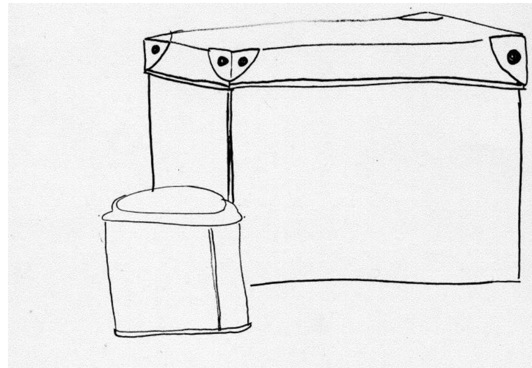
Some sketches of those who scored the full three marks are illustrated in Figure 4.1

Table 4.12 Summary of times attained for spatial orientation activities of the HAT

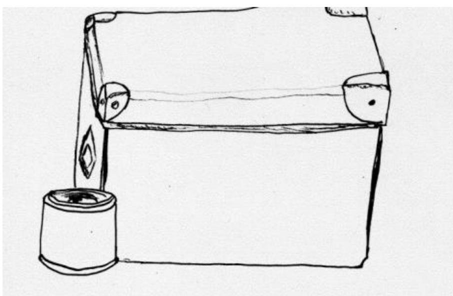
NAME	Activity 5a	Activity 5b	Activity 6	Activity 7	Activity 8	Total		Rank
Max	3	3	3	3	3	15	100	
P1 S1	3	3	3	3	3	15	100.0	1
P2 S1	3	3	1	2	2	11	73.3	12
P3 S1	3	2	2	1	3	11	73.3	12
P4 S1	3	2	3	2	3	13	86.7	7
P5 S1	3	1	1	1	0	6	40.0	28
P6 S1	3	2	0	3	3	11	73.3	12
P7 S1	3	2	1	2	2	10	66.7	18
	3.00	2.14	1.57	2.00	2.29	11.00	73.33	
P8 S2	2	2	2	2	3	11	73.3	12
P9 S2	2	0	2	2	3	9	60.0	20
P10 S2	3	2	2	2	2	11	73.3	12
P11 S2 male	3	3	3	2	3	14	93.3	3
P12 S2	3	2	3	2	3	13	86.7	7
P13 S2	2	2	3	3	3	13	86.7	7
	2.50	1.83	2.50	2.17	2.83	11.83	78.89	
P14 S3 male	3	2	1	3	3	12	80.0	10
P15 S3	1	0	1	0	2	4	26.7	29
P16 S3 male	2	2	1	1	2	8	53.3	24
P17 S3	2	2	0	2	2	8	53.3	24
P18 S3	2	1	1	0	3	7	46.7	27
P19 S3	1	1	3	2	1	8	53.3	24
	1.83	1.33	1.17	1.33	2.17	7.83	52.22	
P20 S4 male	2	2	2	2	1	9	60.0	20
P21 S4 male								
P22 S4 male	3	3	3	2	3	14	93.3	3
P23 S4 male	3	3	3	3	3	15	100.0	1
P24 S4 male	3	2	3	3	3	14	93.3	3
P25 S4 male	3	2	2	2	2	11	73.3	12
P26 S4 male	3	3	3	2	3	14	93.3	4
	2.83	2.50	2.67	2.33	2.50	12.83	85.56	
P27 S5	3	2	0	2	2	9	60.0	20
P28 S5	3	2	2	1	2	10	66.7	18
P29 S5	1	0	1	1	1	4	26.7	29
P30 S5 male	3	3	2	2	2	12	80.0	10
P31 S5	0	0	0	1	2	3	20.0	31
P32 S5	2	2	1	2	2	9	60.0	20
	2.00	1.50	1.00	1.50	1.83	7.83	52.22	
Average	2.45	1.87	1.78	1.87	2.32		69.05	
Difficulty	5	2	1	2	4			



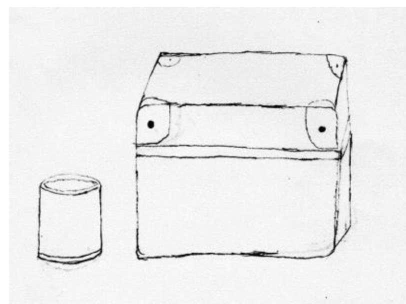
P1



P2



P23

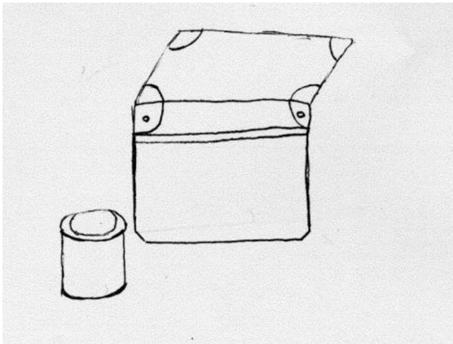


P30

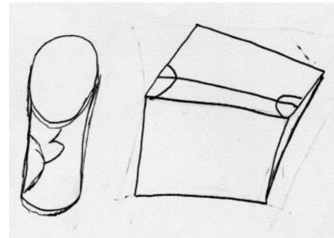
Figure 4.4 Examples of sketches by participants who scored full marks

In all the four cases in Figure 4.4 the three-dimensionality is clearly discernable as is the position of the tin relative to the box. The details on the corners of the box are true to the original.

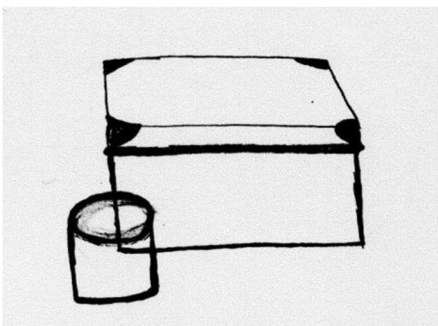
Figure 4.5 provides some examples of those participants who achieved only two marks.



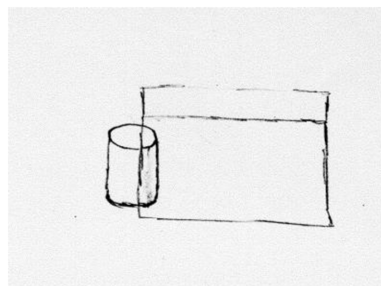
P8



P16



P18

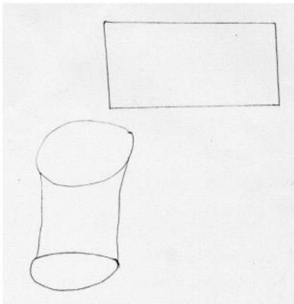


P20

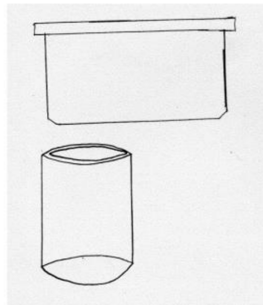
Figure 4.5 Examples of sketches by participants who scored 2 marks

Characteristic of the sketches of P18 and P20 in Figure 4.2 is the absence of convincingly drawing the third dimension. P20 particularly gives a very 'flat' impression. The relative position of the tin in P18 is not convincing and the details in P16 are poor. P8 does not provide for the third dimension as the side of the box is incomplete.

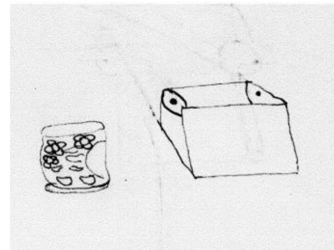
Figure 4.6 provides examples of sketches which neither illustrate the third dimension nor provide sufficient detail to represent the original.



P15

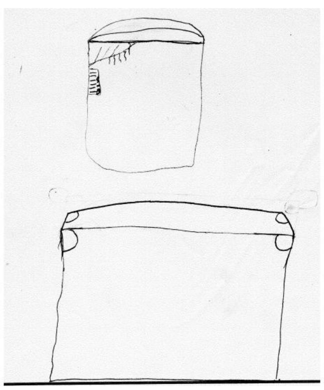


P19



P31

Figure 4.6 Examples of sketches by participants who scored 1 mark



P31

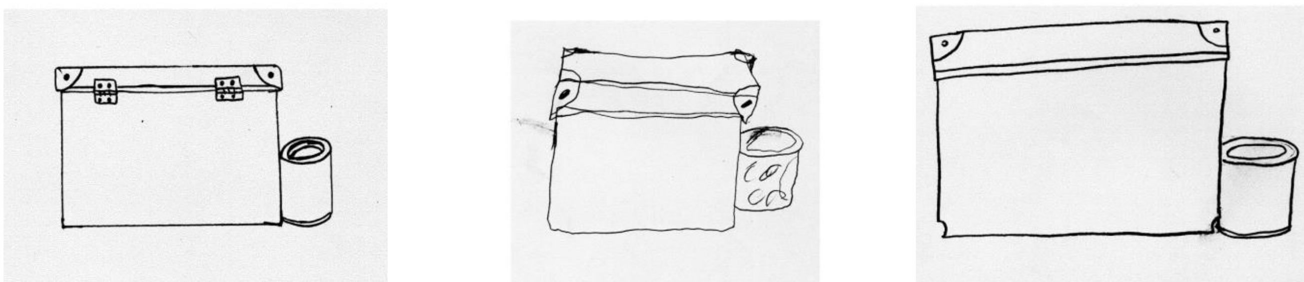
Figure 4.7 Sketch which achieved no marks

Figure 4.7 above shows a sketch which lacks perspective, falsely places the tin relative to the box, and shows little detail.

Activity 5b

In this activity only 22,6% of the sample managed to score full marks while 45% of the sample drew the tin on the wrong side of the box (see page 113 for the original view).

Figure 4.8 shows three sketches which scored full marks;

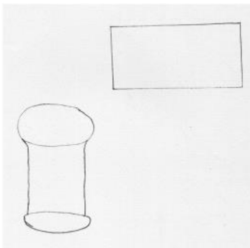


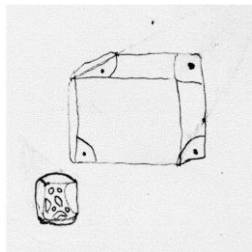
P26 P25 P22

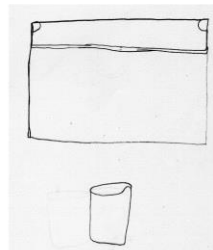
Figure 4.8 Examples of sketches by participants who scored 3 marks

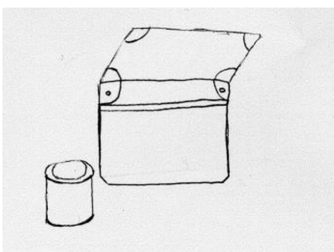
In order to score full marks the correct position of the tin was fundamental. It is interesting to note that P26 in Figure 4.8 even anticipated the hinges located at the back of the box!

The correct orientation of the tin relative to the box was difficult to imagine for many of the participants. Some examples are given in Figure 4.9.


P15


P29


P31


P5

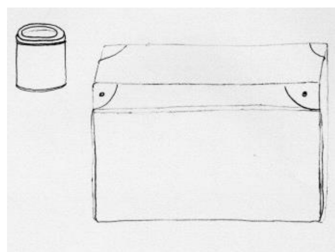

P12

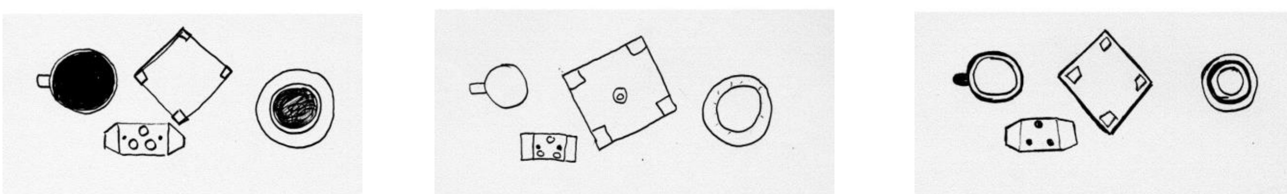
Figure 4.9 Examples of sketches by participants who scored 2 or less marks

From Figure 4.9 it is self-evident that the tin is situated on the incorrect side and in most cases in the wrong position. P12 at least recognised that that the tin will be set slightly back relative to the box. P31, as in the previous activity, misplaced the tin entirely and P15 was not able to illustrate the third dimension effectively.

Activity 6

Just under one-third of the sample (32%) managed to score full marks whereas 54,8% of the sample struggled to imagine the objects correctly as viewed from above. Of the sample, 35% could not position the objects correctly relative to each other (refer to page 114 for the original view).

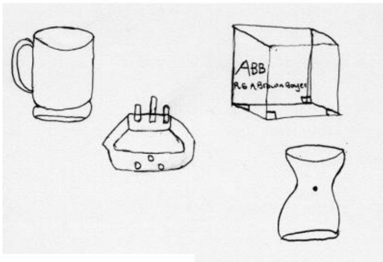
Figure 4.10 gives three sketches by participants who fulfilled all the criteria to obtain three marks.



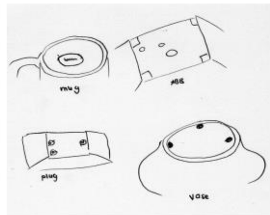
P4 P11 P13

Figure 4.10 Examples of sketches by participants who scored 3 marks

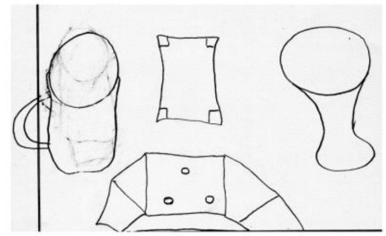
Characteristic of those who did not manage the sketch entirely successfully were those who could not imagine the correct orientation of the objects relative to each other as illustrated in Figure 4.11 by P6 and P31. Others could not imagine what the objects would look like from the top, as shown by P7 and P27.



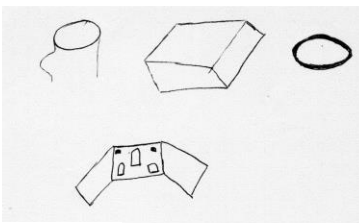
P27



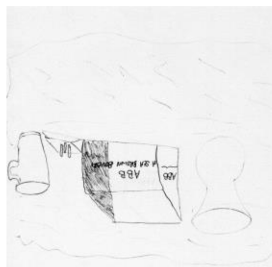
P6



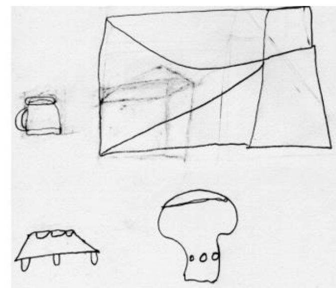
P16



P7



P17



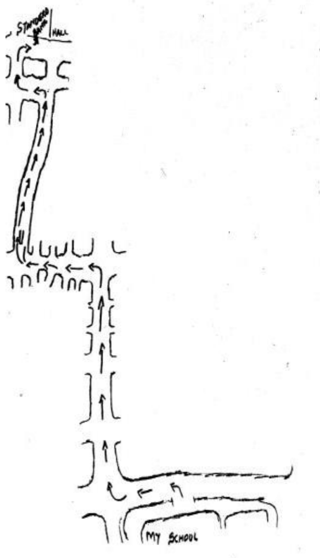
P31

Figure 4.11 Examples of sketches by participants who scored 2 or less marks

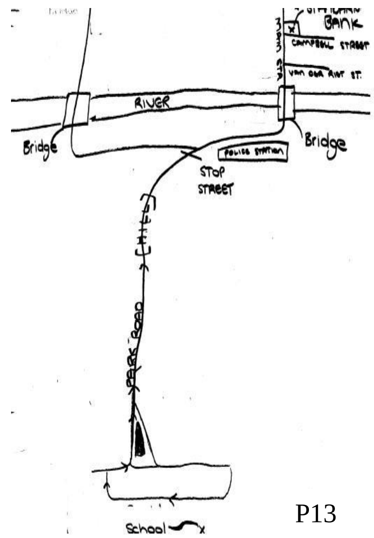
It is interesting to note that P17 drew his objects upside down suggesting a total lack of understanding of what the objects look like from above. This is also true of P31, who interestingly enough also struggled with activities 5a and 5b as illustrated in Figure 4.3 and 4.6.

Activity 7

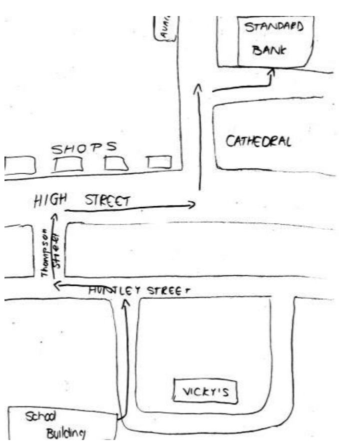
Only 19% of the sample managed to accumulate the three marks. Figure 4.12 provides four examples of those who scored the full marks.



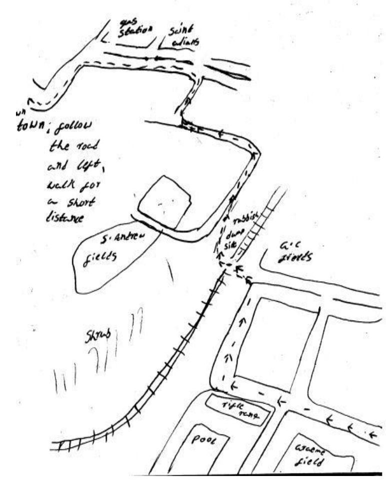
P14



P13



P1



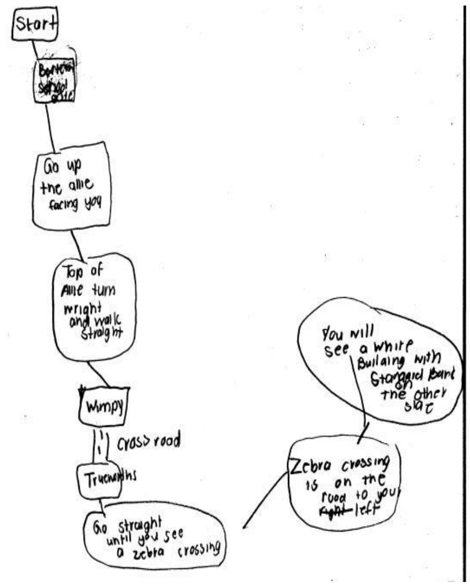
P23

Figure 4.12 Examples of route maps which scored full marks

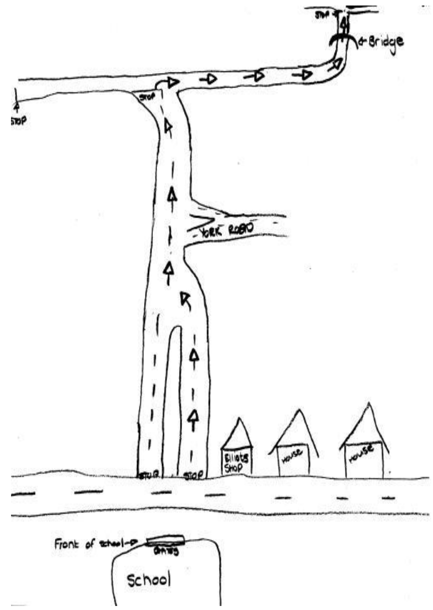
P14 drew a very accurate map of the route and indicated correctly all the streets that one has to cross to get to the bank. The same applies with P13 but in addition she chose to provide all the street names and indicate strategic features such as river and bridge. P1 is also accurate with numerous details and names. P23's map is interesting in that he provided a bird's eye view of the route very successfully. His map is at a slight angle and provides for an interesting perspective.

Figure 4.13 illustrates maps which lack accuracy, scale and detail.

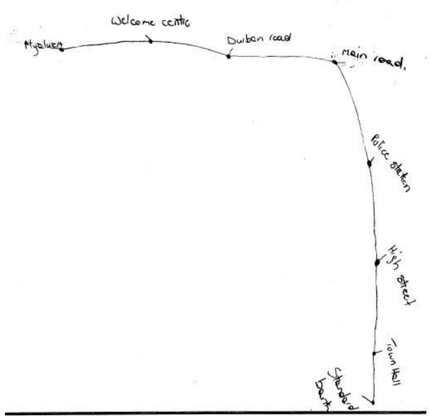
P5 drew her map devoid of any spatial context, in the form of a flow chart that gives strategic instructions as to how to get to the bank. P15's map points to strategic buildings and landmarks but is incorrect in terms of direction. Although the map of P8 is correct in terms of orientation, it is distorted and inaccurate in terms of scale. P26 accompanied his skeletal sketch map, which is not accurate, with a written description of how to get to the bank. The map of P18 does not make any sense.



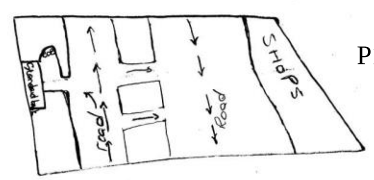
P5



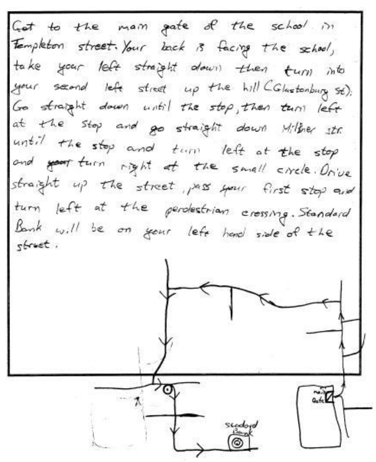
P8



P15



P18



P26

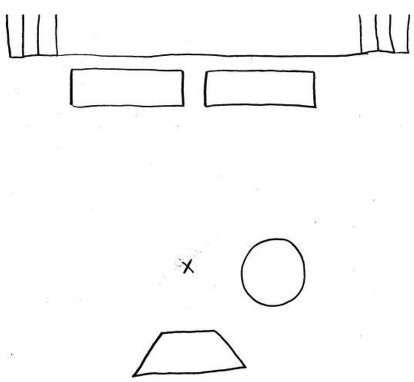
Figure 4.13 Illustrations of route maps which lack accuracy, detail and scale

Activity 8

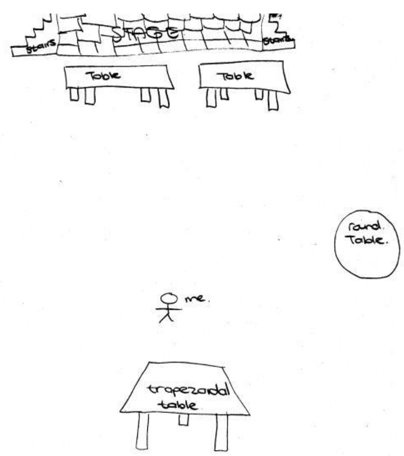
Just over one half of the sample (58%) achieved full marks whereas the rest lacked either detail or misplaced one or more of the object relative to the others.

Figure 4.14 provides two examples of scenes which scored full marks.

P22 strikes one with its simplicity yet all the required objects are placed in the correct relative position. P9 attempted to add some depth into her drawing and drew the scene from an angle – not vertically above as P22.



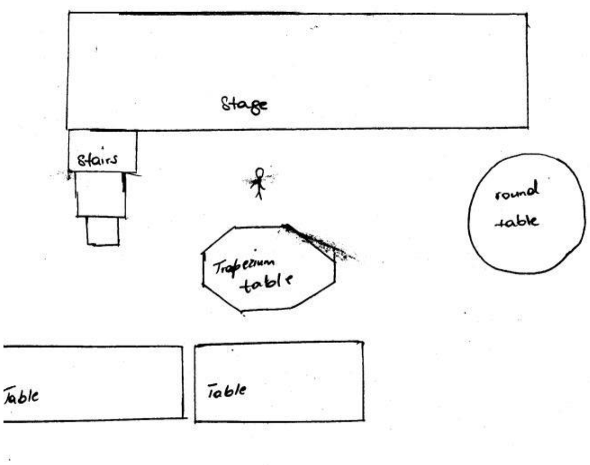
P22

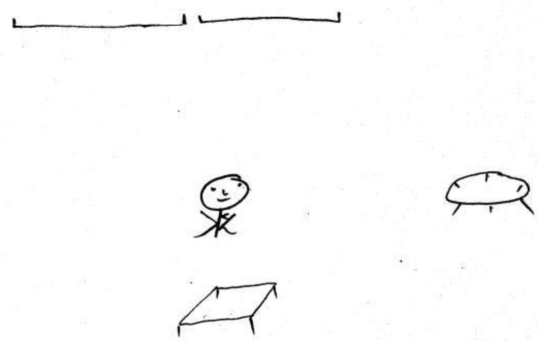


P9

Figure 4.14 Examples of sketches that scored full marks

Figure 4.15 shows examples of sketches which lack one or more of the criteria above. The most evident difficulty was situating the objects in the correct position relative to the person who is located in the middle of the hall.



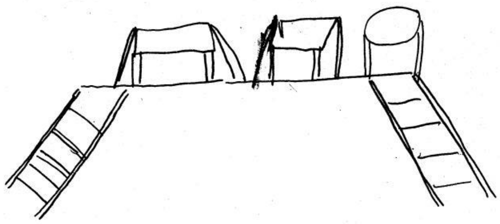


P19 P20

Figure 4.15 Examples of sketches which displayed incorrect location of objects.

P19 failed to recognise the correct location of the rectangular tables. They were supposed to be situated in front of her and in front of the stage. Further, the second set of stairs is missing. P20's diagram is confusing as the person is meant to be facing the stage – he is clearly looking in the opposite direction. The placing of the furniture is correct however.

Figure 4.16 provides an example of a sketch where any sense of orientation, both in terms of locating the pieces furniture relative to each other, and to the person herself, is absent.



P5

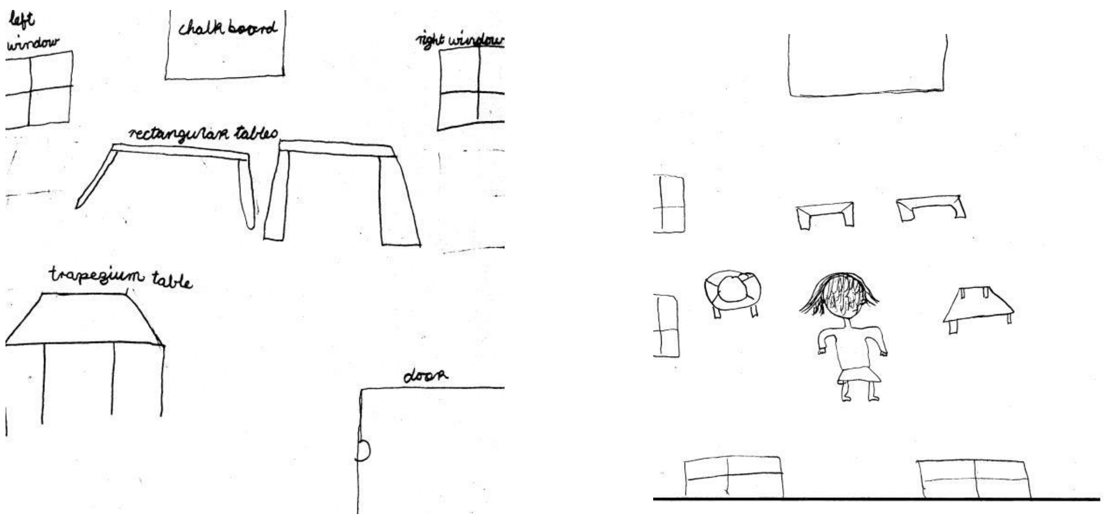
Figure 4.16 Example of sketch which lacks any sense of orientation

Figure 4.17 provides two examples of the participants from S5. This school does not have a school hall so the instructions for drawing the imaginary scene were altered. The instructions read as follows:

- Imagine that you are standing in the middle of an empty classroom facing the chalkboard;
- There is a window on either side of the chalkboard;
- Just in front of the chalkboard are two long rectangular tables and just to the right of you is a round table;
- Just behind you is a trapezoidal table (in the shape of a trapezium);
- Draw the above scene.

The difference between this scene and the one above is that the stage was replaced by the chalkboard and the stairs leading up to the stage were replaced by windows on either side of the board.

None of the participants of S5 sketched the scene according to the instructions. P31and P28 in Figure 4.17 were the most accurate. Both had problems with placing the round table in its correct location – P31 omitted it completely, whereas P28 has it on the wrong side. P31's own position is not clear and P28 has the chalkboard and the windows on opposite ends of the room.



P31 P28

Figure 4.17 Two sketches for activity 8 from participants of S5.

4.3.2.1 Overall trends

As previously, the overall trends are described under the following headings:

- Gender differences
- School differences
- Individual differences.

Gender differences

Overall, the males in this sample scored an average of 82% and the females 62%, which would indicate a substantial difference. Chart 4.8 illustrates the gender differences for each item of the orientation activities.

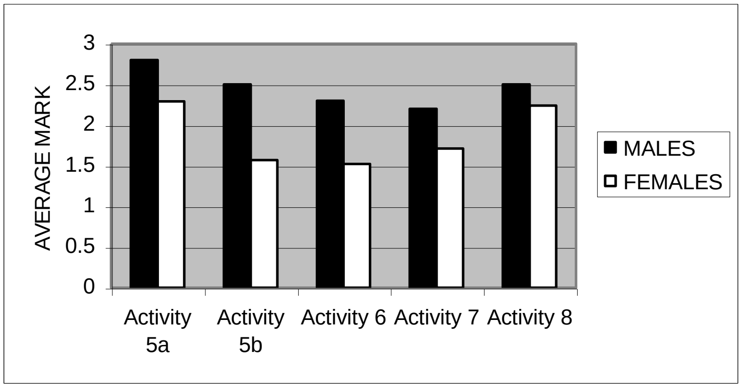


Chart 4.8 Gender differences for HAT orientation activities

As all the activities involved three-dimensional problem-solving, this result is consistent with those of the previous chapter, which suggested a male dominance in three-dimensional tasks.

School differences

It would appear that once again participants from S3 and S5 encountered more problems in completing the above tasks than their counterparts from S1, S2 and S4. Chart 4.9 illustrates the differences in school performance. It could be argued that the pupils from S3 and S5 were at a disadvantage due to the relatively complex and intricate language that was used to articulate the instructions of each activity. But as mentioned before, all measures were taken to ensure that all participants clearly understood what was required. The interpreter in S5 was briefed and she translated each instruction into the mother tongue. For S3, the instructions for each activity were carefully explained.

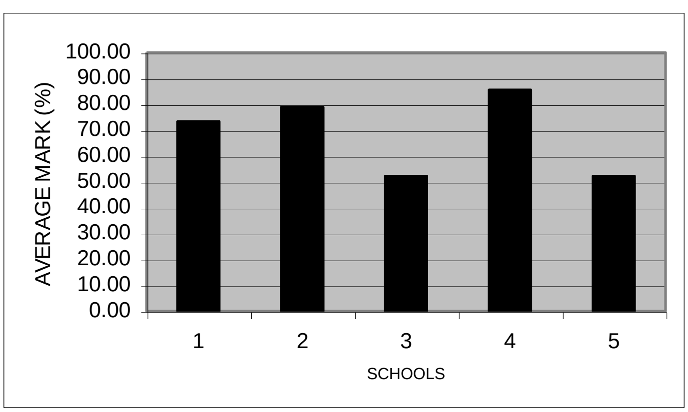


Chart 4.9 Difference in school performance

Individual differences

On an individual basis, P1 and P23 achieved full marks for all the activities (see Table 4.11 on page 155 for details) and P11, P22, P24 and P26 scored an average of 93,3%. Not surprisingly they were all pupils from either S1, S2 or S4. P14 and P30 from S3 and S5 respectively, were both placed tenth overall with an average of 80%. They stood out vis-à-vis their counterparts in their schools and out-performed them by a large margin.

4.3.3 Conclusions relating to the HAT

This section provided an item-by-item discussion of the HAT by focusing on various solution strategies that were adopted by the participants. Specific attention was given to gender, school and individual differences.

The most evident trends observable were:

- In the spatial visualisation activities 1 and 2 the gender difference was very marginal in favour of the boys (see section 4.3.1.1 on page 153);
- In the spatial visualisation activities 3 and 4 there were no gender differences apparent (see 4.3.1.1 on page 153);
- In the spatial orientation activities 5-8 the gender difference was heavily in favour of the boys (see page 4.3.2.1 on page 170);
- For both the spatial visualisation and orientation constructs, the township school and the rural school performed poorly relative to the other schools (see pages 154 and 171).

The following section analyses and discusses performances across the AMST and the HAT to explore relationships and connections between the two tests.

4.4 RELATIONSHIP BETWEEN THE AMST AND THE HAT

Correlational techniques are often used to explore the relationship between two sets of data. Cohen *et al.* (2000:199) suggest that correlational research is “mainly concerned with achieving a fuller understanding” of a phenomenon. It is the intention of this section to explore the extent to which the two tests (AMST and HAT) are related to each other. The AMST is a test that teases out the spatial visualisation and orientation constructs by using two- and three-dimensional problems in a traditional pen-and-paper mode. The HAT has similar intentions except that the mode is no longer pen-and-paper but hands-on activities. In the context of this research, the general question that now arises is:

- Is a pen-and-paper test such as the AMST a useful and reliable tool to provide insight into an individual’s conception of space?

In order to answer this question, the following specific questions are addressd:

- Generally, do those who scored high in the pen-and-paper test also score high in the hands-on activity test?
- Conversely, do those who perform poorly in the AMST also score poorly in the HAT?
- What is the correlation of performance in the AMST and the HAT within gender groups?

To answer these questions, the following set of rank correlation investigations were performed:

- AMST visualisation items and HAT visualisation items;
- AMST orientation items and HAT orientation items;
- AMST visualisation items by males and HAT visualisation items by males;
- AMST orientation items by males and HAT orientation items by males;
- AMST visualisation items by females and HAT visualisation items by females;

- AMST orientation items by females and HAT orientation items by females.

It is recognised that correlational studies have their limitations. Firstly, it must be emphasized that correlations do not imply a causal relationship. In this instance this means that performance in the AMST need not have any causal effect on performance in the HAT. Secondly, due to the relatively small sample size of this project, any trends need to be seen in the context of this case.

4.4.1 Rank correlations

Table 4.13 provides a matrix of rank correlations for the above framework:

Table 4.13 Rank correlation coefficient matrix for the AMST and the HAT

	Rank correlation
AMST visualisation items and HAT visualisation items	0,42
AMST orientation items and HAT orientation items	0,67
AMST and HAT visualisation items by males	0,58
AMST and HAT visualisation items by females	0,61
AMST and HAT orientation items by males	0,66
AMST and HAT orientation items by females	0,60

4.4.2 Discussion

Reference is made to Table 4.14 which isolates the performances of the participants in the spatial visualisation items and Table 4.15, which isolates the performance in spatial orientation items. Consult Table 4.1 on page 120 for the global overview of performances in all the AMST items.

AMST and HAT visualisation construct

The rank correlation on the spatial visualisation items of the AMST and the HAT is 0,42 which, according to Cohen *et al.* (2000:202), suggests that only “crude group predictions may be possible”. This means that participants who scored well on the AMST visualisation items did not necessarily score well on the HAT visualisation items. P4, for example, came first in the AMST (see Table 4.13 and Table 4.7) and fourth in the HAT. This would suggest a fairly high correlation. If we take P25, however, we see that he came second in the AMST and only fourteenth in the HAT; and a similar scenario applies for P23 who came fifth in the AMST but only twenty-first in the HAT.

If we look at the individual school performances, though, we notice that in terms of rank S3 and S5 have very few in the top ten of the AMST (see Table 4.14). The exception is P30, who came fifth in the AMST (but only eighteenth in the HAT). A similar scenario applies for the HAT (see Table 4.7), where very few participants of S3 and S5 scored in the top ten. The exception is P32, who came eighth in the HAT.

Table 4.14 Performance and rank on spatial visualisation items of the AMST

NAME		T1	T2	T3	T4	T5	T6	T7	T8	T9			T12	T13	T14	T15				T19		T21			T24	T25		Percentage	Rank
Maximum					1	5	1	1	2		1		1	3	3	1				1		1			2	1	24	70.588235	
Acceptable response					B		B	D			D		D			D				C		E			JM	C			
P1	S1				1	1	1	AB	2		BE		1	3	0	1				1		C			1	1	13	54.2	11
P2	S1				1	0	C	1	0		0		A	3	0	D				1		1			IP	0	8	33.3	22
P3	S1				A	0	1	1	0		A		1	1	0	0				1		C			0	B	5	20.8	26
P4	S1				1	2	1	1	2		A		1	3	3	1				1		1			2	1	20	83.3	1
P5	S1				C	0	1	E	0		A		E	3	1	B				B		B			KO	E	5	20.8	26
P6	S1				E	1	1	E	2		A		1	3	3	1				1		B			FJ	A	13	54.2	11
P7	S1				1	0	1	E	2		B		1	3	0	C				1		1			2	B	12	50.0	15
P8	S2				A	5	1	B	2		B		A	0	3	A				1		D			1	1	14	58.3	9
P9	S2				1	0	1	1	2		E		A	0	0	C				D		1			2	BE	8	33.3	22
P10	S2				1	1	1	1	2		1		A	3	1	1				1		1			1	1	16	66.7	5
P11	S2				A	3	1	E	2		B		1	0	2	1				1		1			EB	1	13	54.2	11
P12	S2				C	0	D	E	2		none		A	0	0	all				1		1			LP	1	5	20.8	26
P13	S2				1	2	1	1	2		1		1	3	1	1				1		D			2	1	18	75.0	2
P14	S3				A	0	1	1	2		B		1	3	0	A				A		0			2	E	10	41.7	20
P15	S3				1	0	C	A	0		B		1	3	0	C				1		0			1	1	8	33.3	22
P16	S3				BD	1	1	1	1		1		A	3	1	C				1		1			KO	1	12	50.0	15
P17	S3				1	1	1	C	1		E		1	3	1	C				1		C			0	1	11	45.8	18
P18	S3				A	0	1	1	0		B		B	3	1	A				1		1			2	all	10	41.7	20
P19	S3				1	0	1	1	1		B		A	3	0	C				1		1			2	all	11	45.8	18
P20	S4				1	0	1	B	1		B		A	3	2	1				1		1			2	1	14	58.3	9
P21	S4				E	2	1	1	2		1		1	3	1	1				1		1			1	1	17	70.8	4
P22	S4				all	2	1	1	1		B		1	3	1	A				1		1			2	1	15	62.5	8
P23	S4				1	4	1	B	1		1		A	3	1	B				1		1			2	0	16	66.7	5
P24	S4				1	0	1	B	1		1		A	3	1	A				1		1			2	1	13	54.2	11
P25	S4				1	3	1	1	2		1		1	3	1	1				B		1			2	B	18	75.0	2
P26	S4				1	0	1	B	1		1		A	3	0	A				1		1			2	1	12	50.0	15
P27	S5				A	0	1	C	0		AB		ABC	3	0	1				A		D			FH	B	5	20.8	26
P28	S5				AD	0	1	AC	0		AB		1	0	0	AC				1		B			LD	B	3	12.5	31
P29	S5				A	2	1	C	0		B		1	0	0	B				0		1			0	E	5	20.8	26
P30	S5				1	5	1	B	2		B		1	3	1	A				A		1			1	B	16	66.7	5
P31	S5				A	0	1	C	0		A		1	0	0	C				D		C			IC	1	3	12.5	31
P32	S5				A	0	1	A	0		A		A	3	0	B				B		1			KO	1	6	25.0	25
Total					16	35	29	13	36		8		17	73	25	10				23		20			32	17			
Average																												46.2	

AMST and HAT orientation construct

The rank correlation on the spatial orientation items of the AMST and the HAT is 0,67 which "make it possible for group predictions that are accurate enough for most purposes" (Cohen *et al.*, 2000:202). For our purposes it means that on the whole, participants will be placed in similar positions for both tests on the orientation tasks. P4 is a good example – she was positioned sixth and seventh in the AMST and the HAT respectively (see Table 4.15 and Table 4.2). P24 also shows a good correlation – he was placed first in the AMST and third in the HAT. In terms of the lower rankings, P29 for example was ranked twenty-ninth on both the AMST and the HAT.

The relatively high correlation for the orientation construct is manifested in school performance. None of the participants in S3 and S5 scored higher than seventeenth position in both the AMST and the HAT with the exception of P30 who came tenth in the HAT. See Table 4.15 and Table 4.2 for details.

Table 4.15 Performance and rank on spatial orientation items of the AMST

NAME	T1	T2	T3						T9		T11					T16	T17	T18		T20		T22	T23				Percentage	Rank
Maximum									1		1					1	3	1		1		1	1			10	29.411765	
Acceptable response											4						CBD	37				C						
P1 S1									1		3					1	2	1		1		B	0			6	60.0	13
P2 S1									1		1					0	3	36		0		1	1			7	70.0	8
P3 S1									1		1					1	1	78		0		0	1			5	50.0	17
P4 S1									1		1					1	2	1		1		1	0			8	80.0	6
P5 S1									0		1					1	3	21		0		1	0			6	60.0	13
P6 S1									1		2					0	1	44		0		E	1			3	30.0	25
P7 S1									1		1					0	2	1		1		B	0			6	60.0	13
P8 S2									1		1					1	3	1		0		D	0			7	70.0	8
P9 S2									1		12					0	2	1		0		E	1			5	50.0	17
P10 S2									1		1					1	3	1		1		1	1			10	100.0	1
P11 S2									1		1					0	2	1		1		1	1			8	80.0	6
P12 S2									0		1					0	3	1		1		E	0			6	60.0	13
P13 S2									1		1					1	3	1		0		1	1			9	90.0	4
P14 S3									1		5					0	3	21		0		1	0			5	50.0	17
P15 S3									0		1					1	2	45		0		B	1			5	50.0	17
P16 S3									1		10					1	2	41		0		B	0			4	40.0	23
P17 S3									1		1					0	0	33		0		E	0			2	20.0	28
P18 S3									0		8					0	1	35		0		1	1			3	30.0	25
P19 S3									1		0					1	2	20		0		1	0			5	50.0	17
P20 S4									1		1					1	1	42		0		E	1			5	50.0	17
P21 S4									1		1					0	3	43		1		1	0			7	70.0	8
P22 S4									1		1					0	2	1		0		1	1			7	70.0	8
P23 S4									1		1					1	3	1		0		B	0			7	70.0	8
P24 S4									1		1					1	3	1		1		1	1			10	100.0	1
P25 S4									1		1					1	3	1		1		1	1			10	100.0	1
P26 S4									1		1					1	3	1		1		1	0			9	90.0	4
P27 S5									0		10					0	BAA	40		0		D	0			0	0.0	32
P28 S5									0		2					0	BAA	116		0		1	0			1	10.0	29
P29 S5									0		8					0	1	70		0		A	0			1	10.0	29
P30 S5									0		1					0	2	24		0		A	0			3	30.0	25
P31 S5									0		6					0	1	40		0		D	0			1	10.0	29
P32 S5									0		6					1	2	22		0		1	0			4	40.0	23
Total									22		20					16	64	14		10		16	13					
Average																											54.7	

Gender and the visualisation construct

As far as the visualisation construct is concerned, the correlation for the males is 0,58 and for the females 0,61 which once again suggests that only crude predictions can be made.

Gender and the orientation construct

There is a reasonably high correlation for the orientation items. For the male group the rank correlation coefficient is 0,66 which, according to Cohen *et al.* (2000), suggests that group predictions can be made with high enough accuracy for most purposes. For the female group, the correlation coefficient is slightly lower at 0,60, which falls into the correlation range where only crude group predictions may be possible (Cohen, *et al.* 2000).

Focusing on schools, if we do a rank correlation on S5, for example, we get an even lower correlation of –0,036 on the AMST visualisation items and a correlation of 0,39 on the orientation items. This would suggest that the pen-and-paper test is a relatively poor predictor, in terms of the spatial visualisation and orientation constructs, of what pupils in this sample would achieve in the HAT.

Gender difference in perspective

It would appear that the gender-difference results and findings of this study are consistent with those documented in the international literature (Caplan and Caplan, 1994; Brannon, 1996; Caplan, Crawford, Hyde and Richardson, 1997). It needs to be highlighted, however, that the results of this study are to be viewed *within the context of the definitions* that frame *this* case study (see to 2.2.1 on page 18). Gender research (particularly involving 'spatial ability') remains controversial, partly due to the difficulties involved in defining the terms associated with spatial ability (Brannon, 1996). Caplan *et al.* (1997:108) observe that although "narrative reviews have generally come to the conclusion that there are gender-related differences in spatial ability, favoring males, … there is little agreement on the definition of the term spatial ability".

4.4.3 Conclusion relating to the correlations

On the whole, the rank correlation coefficients are of such a magnitude that at best only crude and moderate predictions can be made, suggesting that performance in the AMST is not necessarily a very good predictor for performance in the HAT. In particular:

- For the correlations between the visualisation items of the AMST and the HAT the following holds:

  - generally a weak correlation;
  - a weak to moderate correlation in the case of males;
  - a moderate correlation in the case of females.

- For the correlation between the orientation items between the AMST and the HAT the following holds:
  - generally a moderate correlation;
  - a moderate correlation in the case of males;
  - a moderate correlation in the case of females.

This is significant for this case study as it raises the question as to the role of pen-and-paper testing. Can a pen-and-paper test such as the AMST be used as a useful and reliable tool to provide insight into an individual's conception of space? If the answer is yes, then we would expect more reliable rank correlation coefficients between the AMST and the HAT, indicating a closer relationship between pen-and-paper tasks and actual hands-on activities.

4.5 CHAPTER CONCLUSION

This chapter first engaged with the results of a pen-and-paper test (the AMST) by providing an item-by-item analysis and interpretation of the sample's responses. In general it found that:

- The males in this sample performed better than the females in every task;
- The gender difference was more pronounced in three-dimensional problems;
- The rural school in particular performed poorly relative to the other participating schools of this sample. This was particularly pronounced in three-dimensional problems.

Secondly this chapter engaged with the Hands-On Activity test (the HAT) by analySing the various solution strategies. The most evident trends observable were:

- In the spatial visualisation activities the gender difference was very marginal in favour of the males for the first two activities. For the other two activities there were no gender differences apparent;
- In the spatial orientation activities the gender difference was heavily in favour of the males;
- For both the spatial visualisation and orientation constructs, the township school and the rural school performed poorly relative to the other schools.

Thirdly, this chapter investigated the relationship between the two tests and found that for the correlation between the two there was:

- generally a weak correlation;
- a weak to moderate correlation in the case of males;
- a moderate correlation in the case of females

for the spatial visualisation items.

For the correlation between the orientation items between the AMST and the HAT the following was found:

- generally a moderate correlation;
- a moderate correlation in the case of males;
- a moderate correlation in the case of females.

In order to broaden our understanding of spatial conceptualisation and achieve the second objective of this study that deals with our world views (see page 30), the next chapter delves deeper into the participants' conception of space, and explores their perceptions and world view of space and their capacity to talk about their visions and images of space.

CHAPTER FIVE

DATA NARRATIVE 2:
WORLD VIEW OF SPACE

5.1 INTRODUCTION

Throughout this project it has been suggested that there is more to a person's spatial conceptualisation profile than a mere score in a pen-and-paper test. In addition to the AMST, the previous chapter also dealt with the HAT, which was an attempt at exploring spatial conceptualisation issues using a hands-on activity approach, and we saw that there was not necessarily a high correlation between performance in a pen-and-paper test and a hands-on activity test. This chapter takes this a step further and explores the pupils' broader perceptions of space using a world-view theory approach (see 3.4.4 on page 77). In view of Cobern's (1991) contention that a student's world view should help researchers (and teachers) come to a better understanding of conceptual change (see 2.3 on page 30), I believe that an individual's spatial conceptualisation profile is not complete without also taking into account his/her spatial world view. Throughout this research I was impressed with how many participants possessed a very complex, definite and in some cases sophisticated perception of space. Yet this aspect of space, which in my view, is fundamental to the global component of an individual's spatial conceptualisation profile, is potentially lost in pen-and-paper tests. It is therefore the aim of this chapter to analyse the various world views that the participants possess about space, and thus provide an enriched and more complete picture of their conceptualisation of the spatial concept. World-view profiles were established through a process of content- and meta-analyses of interviews.

5.2 ANALYSIS OF INTERVIEWS

As indicated in chapter three, world-view profiles were established through a process of conducting interviews and conversations with the same 34 participants who took part in the AMST and the HAT. These recorded conversations were structured around Cobern's (1993) bipolar codes (also referred to as vector pairs) as illustrated in Table 3.1 on page 79. This framework provided a useful structure around which meaningful conversations could take place. It also provided for a useful framework to subsequently analyse these conversations, which, at times touched on very abstract and complex issues.

The bipolar codes were:

- naturalism – religion
- chaos – order
- mystery – knowledge
- function – purpose
- mundane – special
- mathematical – non-mathematical (Cartesian – non-Cartesian)
- finite – infinite
- tangible – non-tangible
- internal – external.

(See Chapter 3.4.4 for more details.)

The analysis of the interviews occurred on two levels:

a) a content level;

b) a meta-level.

a) The content level

The content analysis focused exclusively on how the participants responded to the bipolar codes and on what the participants said and described. On the basis of these articulations and the framework above, concept maps were drawn that facilitated pattern and trend identification. (An example of a concept map was given on page 82.) As explained in chapter three on page 78, the conversations were guided by the bipolar codes (vector pairs). Each code was analysed separately and the narrative of each bipolar code commences with a table that illustrates the cumulative position of the sample on the bipolar continuum (Refer to Table 5.1).

Table 5.1 Example of bipolar continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Naturalism	2	7		2	4	15	4	Religion

Each participant was asked at the conclusion of each interview to reflect on how they responded to each bipolar theme and commit to a position on the table. Table 5.1 for example, shows the position of the sample in the naturalism – religion vector pair. Fifteen members of the sample positioned themselves as ‘strongly convinced’ in terms of identifying with the religious elements of space, whereas only two members expressed strong convictions that space was a naturalistic concept.

As I wish to let the voices of the participants come through and be heard, I let the conversations speak for themselves and hence reproduce detailed extracts of the transcripts. All the quotes are coded and can be traced to the original transcripts housed at the Education Department, Rhodes University, Grahamstown in South Africa. Each participant was given a number and the source of the quote is also given. For example, [P26;I] means that the quote originates from P26 (participant 26) and the source is found in I (the interview).

b) The meta-level

The meta-level analysis used five criteria for analysing the conversations. The criteria were established through a process of consensual validation as explained in section 3.4.2 on page 67, and focused on the extent of the participants’:

- capacity to abstract
- use of imagination
- capacity to be critical
- insightfulness

- ability to articulate complex ideas.

This chapter is dedicated to the content level of the conversations and the next chapter to the meta-levels.

5.3 CONTENT LEVEL ANALYSIS

The analysis below commences with a brief overview of the participants' initial images of space as articulated in the questionnaire (see 3.4.1 on page 60). These initial images provided a convenient point of reference for the eventual interviews. The analysis then focuses on each of the nine vector pairs:

- naturalism – religion
- chaos – order
- mystery – knowledge
- function – purpose
- mundane – special
- mathematical – non-mathematical
- finite – infinite
- tangible – non-tangible
- internal – external

5.3.1 Initial image of space

In the questionnaire referred to in section 3.4.1 on page 60, one of the questions asked the participants to complete the sentence:

> When I think of space I …

Predictably many answered this in terms of the popular image that space is about the universe, planets and spaceships, a concept out there removed from the person:

> *... think of air moving around us and gases like weather, including geography, airplanes, spaceships – all stuff like that.* [P14; Q]

> *... think of stars and planets. The space that air fills also comes to mind.* [P24; Q]

> *... think of all the aliens and stars and everything else up there.* [P2; Q]

Others, identified with the expanse and the vastness of space:

> *... have an imagination of a big hole which is around me.* [P15; Q]

> *... think of a huge area to move around in.* [P1; Q]

> *... think about having a lot of room.* [P10; Q]

Many expressed their sense of space in terms of freedom and openness:

> *... think of freedom and openness and movement is possible and just open.* [P25; Q]

> *... think of freedom and anything that comes to mind.* [P3; Q]

> *... think of open expanses, a place where movement is not restricted and things are uncluttered.* [P4; Q]

Others identified their own personal role in space:

> *...I feel small and unimportant, because there is so much space in the universe.* [P9; Q]

> *...I enjoy it. I like having a lot of space around me.* [P12; Q]

> *...feel free because I imagine this big open place and I can do what I want with it.* [P8; Q]

> *...I would love to go there – I find it interesting and isolated.* [P5; Q]

A number of them recognised space as their own personal space:

> *... think of space as an area where you do something.* [P27; Q]

> *... I feel great because I can see anything.* [P30; Q]

> *... I love my space – it's peaceful and relaxing to my mind – I think of total silence.* [P1; Q]

> *... think about being by myself.* [P11; Q]

> *... at home I have my own space where I do my homework and hang out.* [P7; Q]

5.3.2 Naturalism – Religion

Here the conversation revolved primarily around the question of whether space was created or whether it evolved. Religion asserts the involvement of the supernatural in space whereas naturalism implies the material or physical causation (Cobern, 1993). The notion of whether space had always been there was also touched upon.

Table 5.2 Distribution of sample on Naturalism – Religion continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Naturalistic	2	7		2	4	15	4	Religious

Table 5.2 suggests that nearly half the sample (15) is strongly convinced about the existence of theistic involvement in space. In terms of the divine creation of space there were two dominant views:

a) God created space and everything in it from the start;

b) Space had always been there and God created things, such as the earth, into that space.

Evidence to illustrate a):

> *… I think He created all of it. Ja, I scheme He created that space there and how we can't build anything there because it's His space…*[P11;I]
>
> *... Space was always there – it's nature and nature was created by God.* [P15; I]

Evidence to illustrate b):

> *... I've always visualised it as lots of space and then He created things in that space. I never really visualized that He created the space as well.* [P12; I]
>
> *... In my view, OK. In the beginning there was nothing, so there was probably only space. And then He created things to fill in that space.* [P13; I]
>
> *... I guess space was there right from the beginning ... space has always been there ... and then God started creating.* [P24; I]
>
> *... Space was always there. The thing that was created, is the thing that occupies the space.* [P16; I]
>
> *... I think He made it* [space] *before He created the world so He could see how empty everything was and how He would need to fill it.* [P6; I]

> *... Yes, first is the space then the earth ... He created the space, the earth and the others...*[P30; I]

Another view took a composite approach and suggested that initially God created space and evolution then took over in the development of all the other elements of the universe:

> *… they say God first created space and then things evolved.* [P20; I]

The question of evolution came up repeatedly and much to my surprise the dominant view (90% of the participants) placed creation ahead of the process of evolution. The majority of the participants were resolute in the opinion that God created the universe and everything in it:

> *... I don't know what to believe in evolution – it sounds pretty interesting, but I don't know all the facts about it so I can't really say ... if I believe in it...although it sounds quite possible. It's weird, I mean ... I don't thinks we were monkeys at that stage.* [P9; I]
>
> *... I'm not too much bothered by evolution, and the big bang theory, Sir.* [P25; I]
>
> *... In Biology, we did a section on Darwin's theory. I don't know, it's interesting what other people think, but it's not what I think.* [P22; I]

Only one participant emphatically expressed doubt about the evolution/creation debate:

> *... but the scientists, it seems like they're talking about big bangs and expanding of the universe and stuff like that. My sense – actually I don't know what to believe. I really don't ... 'cause I wasen't there to experience it and I wasen't there to see it happen.* [P20; I]

Only isolated participants saw space as a naturalistic concept that can be explained scientifically:

> *... I think space is scientific ... like a mix of gases and oxygen with no starting place and ending place.* [P14; I]

5.3.3 Chaos – Order

The central issue of exploration here was the extent to which space was perceived as an ordered system which has elements of predictability, or as something chaotic and disorderly. Chaos would imply that space has elements of changeability, randomness and unpredictability. Order implies that space is predictable, based on rules and principles.

Table 5.3 Distribution of sample on Chaos – Order continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Chaos	1	7	5	1	6	14	2	Order

It is interesting to note that many who were convinced of the divine creation also saw space as something quite orderly and predictable. A number of them however were undecided and placed themselves on both sides of the polarity simultaneously. Nearly half the sample (14) were strongly convinced that space was orderly (see Table 5.3).

Evidence of orderly perception:

> … *it is orderly, because it was always there – no matter what.* [P30; I]

> *… everything there is in its good place – therefore it is orderly.* [P28; I]

> *… It's ordered, 'cause I think it's peaceful – if it's chaotic then it wouldn't be* [peaceful][P5; I]

> *… well, I think there is order in space. Because you can imagine the planets – they're arranged in order. So if there was no order, they would be mixed up.* [P16; I]

> *… God says there's a plan for your life, Sir. Everything is done for a reason. That's right, in Christian life – whatever you, there's a plan.* [P22; I]

> *… It's ordered because it's specifically designed.* [P25; I]

> *… it's orderly because it has always been there.* [P12; I]

Evidence of chaotic perception:

> *… I think space just happens – I mean – I don't know. If there were a principle, what would it be? I think it just happens.* [P3; I]

> *… It's very disorganised – disorderly, because you can't predict it. It's like changing every single day.* [P7; I]

> *… I don't think it's orderly, because of the things happening to earth – like earthquakes and stuff like that.* [P20; I]

A few responded indifferently, saying that it had elements of disorder and order:

> **…** *I think it's very unorderly, things happen in space and people can't explain it – I suppose ontheother hand it is quite orderly, because everything orbits around each other…*[P4; I]

> *… I can't say it's ordered, because it's sort of like there –all over. But sometimes it can be ordered if it's in a certain confinement or whatever – but it's generally just out there.* [P24; I]

> *... Well, it depends on how you see space. Whether you use it to reveal order, you know, or in a chaotic fashion. Let's take for example a playing field – Somerset field. We can decide to put a whole bunch of guards over there – it's orderly. Or someone might come along and say 'lets have a small match of rugby" – that's chaotic in a way. Or decide for something even more dangerous, more hectic, like bringing a war into that small space. Now that's even more chaotic. So it depends on how a person sees space. He can either use it to help him or not help him. So it depends, it's two different things.* [P23; I]

Some differentiated between space 'out there' and the space 'here, around us':

> *... I think it's quite ordered, because like – out there the meteorites and stuff like that are flying into each other so it's quite chaotic. But over here everything's – I don't know –there's some sort of order.* [P9; I]

5.3.4 Mystery – Knowledge

This vector pair explored the extent to which the perception of space is based on facts and knowledge. It seeks to describe "the extent to which one believes [space] to be fathomable'' (Cobern, 1993:942) or not. Questions asked to provoke discussion were:

- Do you think that we can ever understand what space is, or is it something mysterious?
- Do you think that space is something certain, something that can be defined or explained?

Table 5.4 Distribution of sample on Mystery – Knowledge continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Mystery	8	12	2		1	9	1	Knowledge

Two-thirds of the sample identified with the notion that space is something mysterious and obscure. The other third was convinced that it is based on facts and is explainable with rational ideas (see Table 5.4).

Not surprisingly, many of those who perceived of space as chaotic also regarded it as mysterious.

Those who were convinced that space is a mysterious concept:

> *... it is quite mysterious ... because you can't really examine it because you can't physically see it or touch it.* [P13; I]

> *... it is mysterious because there are things that people cannot understand like where does space come from.* [P17; I]
>
> *... Yes mysterious – people talk about if there are aliens on mars and everything, and I mean I wouldn't really know if it's true, but ja, to go up there and find out all the mysteries would be interesting.* [P2; I]
>
> *... It's something very mysterious...like the mystery of a new star being born, the mystery of what's happening under the earth. Nobody quite knows exactly, although there are many theories, but it could be fact or it could be theory. That's still quite mysterious.* [P7; I]
>
> *... I find it mysterious because with all the technology and everything we're still unsure of what atoms actually are – how space carries them.* [P6; I]
>
> *... Its mysterious because no one can control space.* [P26; I]

Some were not quite sure and adopted an ambivalent stance:

> *... Well, it's partly based on facts – but it is mysterious how it's just there – it's a bit of both.* [P12; I]
>
> *... I don't think space is very mysterious – although there are mysteries like where God is, like why there is no gravity there, things like that...*[P11; I]
>
> *... Well it's mysterious in a way because we don't know what's happening on Pluto, but it's based on facts in that we know how to get to the moon.* [P22; I]

Those who thought that space was based on facts:

> *... Space is factual – because like air it is always there.* [P14; I]
>
> *... Space is based on fact and knowledge because we have to know about space and we have to learn to know about space.* [P16; I]
>
> *... It is based on facts because it has a purpose for us.* [P18; I]
>
> *... It is not mysterious, because it is there, if I cannot...* [be in]*... space I will die.* [P28; I]
>
> *... I know space very well – I can tell them that space is the reason why we are surviving...*[P29; I]

Some suggested that our immediate space – the space around us - had a stronger factual base than the space 'out there' which is mysterious:

> *... Well the space out there I reckon is pretty mysterious because you can't really know what there is. But our space here – all the molecules and stuff around – they work together.* [P9; I]

5.3.5 Function – Purpose

The aim initially was to determine whether space had an underlying function; and did it have a mode of action to fulfil its purpose? Cobern (1993) referred the function aspect of this vector pair to structure/function (teleonmic), and purpose to transcendent purposes or cosmic teleology.

In conversation with the participants, however, I saw that they found it difficult to differentiate the two. The terms function and purpose were used synonymously. The positioning of the sample on the function – purpose continuum therefore became meaningless.

The views on the function of space were varied.

Some saw its purpose in a cynical light:

> *... it's there – some have benefited from it – like ...*[Neil] *... Armstrong by being the first man on the moon – America making mega money ... things like that.* [P20; I]

Others had a clear sense of the purpose of space:

> *... to provide life.* [P24; I]
>
> *... to give us room around us.*[P26; I]
>
> *... it contains all the planets – the earth and the other planets...*[P30; I]
>
> *... Space connects everything – it has a certain power – it puts things into place.* [P6; I]
>
> *... It is very important for people to have their own space - everyone needs to kind of unwind...*[P1; I]
>
> *... to fill up the emptiness between things.* [P4; I]
>
> *... to give a person a sense of being, because if you don't have space you feel so crowded and suffocated that you don't know what peace and tranquility is...to just enjoy being yourself and often just to make contact with yourself.* [P7; I]
>
> *... The purpose of space I think in my own view, is to protect the earth, but not only the earth but all the planets, because I heard something about a big stone that's coming slowly towards the earth that's going to crash into the earth maybe in a hundred year's time. But I think the purpose of space is to slow the speed of the meteorites – they say it's a meteorite. If there was no space it would just quickly come. Because the space makes it go s-l-o-w-l-y.* [P16; I]

> *…I think to hold matter.* [P10; I]

For those who differentiated space:

> … [the space here] – *it facilitates us people to live in. The other space the sun is 100%. It will scorch and burn you. And there is like no shade – it will disintegrate you.* [P23; I]

5.3.6 Mundane – Special

The aim with this vector pair was to explore the extent to which space was perceived as special, as something beyond the ordinary or something mundane and prosaic (Cobern, 1993).

Just above half the sample were convinced (to various degrees) that space was something special (see Table 5.5).

Table 5.5 Distribution of sample on Mundane – Special continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Mundane	1	8	5		3	11	4	Special

The concept special was often identified with beauty or lack of it:

> *… I think space is something beautiful and people should actually cherish their space…also space out there, all these gases and things you know, all those dust particles, I think it should look beautiful…*[P1; I]

> *… It is beautiful – just all the vastness that there's nothing hindering it, just free to do as it pleases.* [P4; I].

> *… it's beautiful because there are lots of different colours.* [P2; I]

> *… I don't think it's beautiful 'cause there isn't anything there…*[P9; I]

… or with freedom and vastness:

> *… I mean space is equal to freedom I think. And the people in prison don't have space – they have to give up space, because it's just like so important, it's a good thing and bad people can't have it …*

> *… I think it's special, because without space there would be nowhere to go.* [P17; I]

Others perceived its unknown elements as special:

> *… it is special 'cause it's so unknown, and no one knows much about it. People don't think about it in their everyday lives – they just take it for granted.* [P10; I]

… while others had other reasons:

> *… I think it's something quite special because without it, it would not work – because everything would be stuck together…*[P13; I]

> *… it's special because without space I can't read or write my homework in my room…*[P29; I]

> *… it's special since someone created it. 'Cause everything has got a purpose. So space has t o have something.* [P24; I]

For those with a differentiated perception of space:

> *… it's quite special, because you're constantly having things created, like stars being born…my own space is also special – your inner feelings and if you like need to go out and just reflect on your life you get to know who you are better.* [P7; I].

Many perceived it as something which is just there:

> *… It's just there – nothing special.* [P3; I]

> *… It's just something out there, which is around me.* [P18; I]

> *… It's not something special because there's space everywhere, so I don't think it's special.* [P19; I]

5.3.7 Mathematical – Non-mathematical

This vector pair sought to discover the extent to which the pupils' perception of space was informed by mathematical elements. For example, the notion of two-dimensions and three-dimensions was spoken about. The concept of location was also explored: where do they locate themselves in space?

Table 5.6 Distribution of sample on Mathematical – Non-mathematical continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Mathematical		13	7	4	3	6	1	Non-math.

Most of the sample (59%) were content to suggest that they were slightly to strongly convinced that space had a mathematical explanation to it see Table 5.6). This was not evident from their conversations, however. It was apparent that the majority of the participants were unable to articulate the difference between two and three dimensions, for example; nor were they able to elaborate on their mathematical perception of space. In terms of locating themselves (or the earth) in space, most of them positioned themselves relative to the tangible solar system, whereas very few expressed their position in more philosophical terms.

In terms of location:

> *… I see earth in a faraway corner of space…*[P5; I]
>
> *… Well it's … I'm not sure … I think it's the third rock from the sun…* [P1; I]
>
> *… We're sort of in a line with a whole bunch of planets with moons around.* [P4; I]
>
> *… On the bottom of Africa.* [P2; I]
>
> *… I am somewhere in space – on the ground.* [P3; I]
>
> *… Earth is part of the planets and I'm in the southern hemisphere somewhere…*[P18; I]
>
> *… Space is up there and earth is down here … and me I am just on earth – in Grahamstown, under the equator.* [P15; I]
>
> *… they say space is nowhere – so I cannot locate the position of the earth in space. I know where the earth is, but the thing is I know that earth is in the space, but I cannot locate the position of earth in space. But I believe that earth is in space.* [P16; I]
>
> *… Where do I locate earth? The third planet from the sun. I mean it's out there with all the rest, but it's near the sun. In outer space, but very small.* [P9; I]

In terms of the more philosophical:

> *… I know the world is round and everything, but still it's like South Africa's at the bottom and I place myself as a little space just contracting – you know and a bigger breathing space that wants to get bigger in my own space consuming my own space. I want to grow vertically and not horizontally – I don't want to disturb any other people's space. But I feel I could have more space than I do.*[P6; I]
>
> *… Earth is where He [God] wants it to be. That's how I see it, Sir, it must be somewhere He put it, and where He enjoyed it. Maybe halfway between like His home – whatever, His like heaven or whatever, and then He made…quite hard…I don't know, that's a problem…I'd say it's half way between hell and heaven.* [P11; I]
>
> *…The earth is in the centre of everything.* [P8; I]
>
> *… Unimportant – because out there, there are so many things – I mean even a star is more important than a human in space. Then you've got all your planets and …ja we look at the stars and we don't look at ourselves at night. Well we do, but not really. Everyone else is more interested in aliens and everything else that is out there…*[P9; I]

... I think earth in the space that I can see is in a way the centre. I could say it's small and generalise the whole solar system and everything, but in my way, space is centred around me, in earth. Earth is o big that the space that I occupy – the region and everything – I quite enjoy. I feel free around, because it's sort of like confinement or whatever. [P24; I]

Many expressed their location by emphasising their insignificance:

... Well I see myself – I don't think I'm anything really on earth, I'm just one of those minor thing son earth. [P1; I]

... Ja, you're some minute but you almost assume insignificance compared to the bigger picture and ja...like this little dot in the middle of this huge expanse. [P4; I]

... as a tiny spot in a huge universe – there are so many planets that we haven't discovered, that how can we say we're the only planet with human life? But we are significant – I mean if we ever did make contact with another human race we could like benefit each other. So basically we're just as significant as any other race in the universe if there were any. [P7; I]

... I think earth is just insignificant to all of space. It's just floating around...and I place myself;f on the foot of the earth – I mean Africa at the tip. [P6; I]

... Earth is just this small small scale to this space. It's just vast and huge I think it's very small. Probably won't even see it. [P10; I]

In terms of two- and three-dimensions:

... Two-dimensional is that you're not getting the full picture, you're only seeing what was taken form one angle, so you don't get the whole feel of the object – three dimensional you get like a better idea of the whole object and how...where its place is I suppose – where it fits together – you can see the whole way around. [P4; I]

Two dimensions is where we are – where we actually are – I sort of understand it, I don't quite understand it, but sort of. Three-dimensions it's like three different sides to a story – or to a shape. Say for instance you've got a square, you can make so you can see different sides of it. So you can see a top, side and bottom at the same time. [P4; I]

... Two-dimensional space? – maybe you can see space as two different things – I don't know. Three-dimensions is like a cube. Because it's like you can see only three, you know three angles of it. [P2; I]

... Two dimensions? – I think it takes up the least space...I mean it doesn't have any inside. Ja, it's a sheet of paper sitting there and consuming as little space as possible. Three dimensional is more powerful and it's ... it's big things more around us ... ja ... and it's really solid. [P6; I]

... Three dimensions – no, I don't know. [P15; I]

... Two dimensions – no I don't think I've ever heard of it. And three dimensions? No I've never heard of it. [P16; I]

... Three – dimensions – you see three sides of it at once. I suppose two-dimensions you'll see only two sides of it. [P11; I]

... Two-dimensions is two times the amount – and three-dimensions is three times the amount. [P10; I]

... Three-dimensions is when you look through like those movies and you look through those glasses and check movies and things like stand out. [P9; I]

... Two-dimensional is when you can only see the outline of. [P12; I]

... Two-dimensions is like flat and you don't see through it. But three-dimensions you see the shapes and stuff through it...I don't know much about two-d and three-d and all that. [P22; I]

... Two-dimensions – most animations are two-d cartoons. [P25; I]

5.3.8 Finite – Infinite

This vector pair was used to explore the magnitude of space. The conversation revolved around the following questions:

- Where do you think space starts?
- Where do you think space ends?
- How big do you think space is?
- How small do you think space is?

Table 5.7 Distribution of sample on Finite – Infinite continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Finite		1	1	1	1	16	13	Infinite

There seemed to be consensus that space was infinite (see Table 5.7) and that just as there is no beginning, there is no end:

... There is no starting point because it goes right around and comes back. It's always there. It's like infinity – you can't get to the end of infinity. [P11; I]

... It doesn't start, it doesn't end. It's got no beginning and no end. [P10; I]

... I don't think it has a space where it starts and ends, it's just there. [P8; I]

... It doesn't have an end because it doesn't have a start [P9; I]

... It should be humungous, because if you think about it, what's on the other side of space? I can't even ... I don't know ... so it should be quite big ... ja. [P1; I]

... Gee whiz ... that's quite a difficult question ... I don't really think it has a beginning, it's just ... I don't know ... it's just expanse after expanse after expanse – there's just nothing stopping it so there's no beginning to it. [P4; I]

> *... You're asking me - I have no clue. I think that space is just kind of there – it has no beginning, it has no end.* [P7; I]

> *... I think space is something which is formed in a circle, which has no end. We cannot see the end of it – it's like a circle – you turn around and around, all around but you cannot find its starting point or its ending point.* [P16; I]

> *... I don't think there is anybody who knows where space starts – I don't think it has an end.* [P30; I]

Some, however were convinced that the space around earth is limited:

> *... But on earth there must be an end to it. On earth it could be small or big, but up there I think it is huge.* [P11; I]

Only a few perceived space as having boundaries:

> *... Yes it ends – because there couldn't be that big without an end.* [P5; I]

> *... Yes, maybe space does have an end. If you choose to crush something, then it doesn't occupy space anymore. If ... um ... he said if you crumple something up and throw it, that space that it had is used up, it's gone. Ja, space can end ... exactly.* [P6; I]

> *... Maybe it ends somewhere, but us as human beings haven't seen the end, so we just assume it goes on ... it is big, too big. But it can be very small – yes microscopic – ja end it gets to an end. But if the small space gets to an end, the big one must as well. It's too big and it's too small, I don't know.* [P24; I]

5.3.9 Tangible – Non-tangible

This vector pair facilitated an exploration into how tangible space was perceived. The questions that underpinned the discussion focused on whether one could feel and/or see space.

Table 5.8 Distribution of sample on Tangible – Non-tangible continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Tangible	1	4	4	1	1	17	4	Non-tangible

The majority (69%) of the sample was convinced that space was something non-tangible (see Table 5.8). This is possibly linked with the result that reflected that the majority identified with the mysterious nature of space.

Those who perceived space as visible:

... Well you can't see it in the air, but in the open field you'll have space. I don't know...you can sit there but you can't see space in like air – you know what I mean. I s'pose you could – I mean when you go up into outer space you can see it like goes forever, so there must be a thing, but you can't like – if you're sitting there you can't see space on the other side there – it depends where you're situated. [P11; I]

... Ja, I see space – like that ...um ... to some extent, but ... um ... you can't like feel it. [P8; I]

... Ja, I can look out into the sky and you'll probably see space, unless there's a bird flying around or something, you'll still see space. [P9; I]

... Um ... you can see that there is a space but you can't touch the space that's there because space means that there is nothing there. [P13; I]

... I suppose you can see space. You can see that there is nothing there. [P12; I]

... Yes, I can see it. Ja well, you can see it when you're like the only person in like some huge desert, yes that's space – you can see it. [P1; I]

... It's more that we see the objects in space and you know that there's space between it but you can't actually see it. [P4; I]

... I can see space, but I cannot see air. [P14; I]

... I can see that there's space between us, but I cannot touch it. [P17; I]

... I can see the space, but I cannot touch it. [P29; I]

... Ja. You know, now I found out that space can be an English literal and figurative meaning, you know. The two different meanings of space. As in space, the literal one is the space all around me, filling up the air, and then there might be the space that sort of like is inside or the space that's a block, you know, something that occupies that space or in the space that conscience and all that – that occupies something as well, so that has to be somewhere. [P24; I]

... My space is the one I'm looking into right now. [P26; I]

For those who perceived space as invisible:

... No, you can't see or touch it. It's just there. [P10; I]

... I don't think you can see space. [P5; I]

... Not really. Some parts of it, I could say. I mean, if I stand somewhere and take a telescope, I'll be able to see that part, but not all of it. I'll see stars and small things, that OK. [P2; I]

... Not really, whatever doesn't have something is space. [P3; I]

... You can only see things in space, you can't see space. [P6; I]

... No, I don't think you can see space, you can feel it [P16; I]

... No. Unless they put like one of those...no you can't. If they spray that stuff like smoke where you can see lasers through it, then in a way the laser will be picking up all the particles which is in the space. Like sometimes behind a fire you see a shimmer? There I suppose

you're seeing the space but you're seeing a reflection of something. It's like a laser as well – it's picking up all the particles. [P22; I]

... You know that you are in space, but you can't see it. [P23; I]

Those who thought that you cannot touch space:

... No – I don't think like up there. I suppose you can touch like air particles up there in space, but I don't think down here. [P11; I]

... You can't touch it because it's not there. [P13; I]

... You know it's there, but you can't see it or touch it. [P3; I]

... You can't really touch space, but space is always touching you. [P6; I]

... I can see space, but I can't touch it. [P18; I]

... Space is something that is untouchable. [P19; I]

... It is not something that is touchable...[P30; I]

... You can touch the things that occupy the space but I'm not sure you can touch the space. [P24; I]

... You cannot touch it because it's like swimming in water. You can go through it and you know you're in some substance. So it's the same in space – you know you are there but you cannot feel it – yeah. [P23; I]

Some thought that you can touch space:

... Well, there's space around you, so you're probably touching it without even knowing it. But, I don't think you can physically see yourself touching it like you can touch a plug. [P9; I]

... Ja, we're always touching it. [P22; I]

... I have to be touching it if it's the space right here! [P26; I]

Others were of the opinion that although you cannot touch space you can feel it:

... Well, it's like well space is not condensed into something you can touch or feel. I suppose you could touch it, but you can't feel it because it's not dense enough to feel. It's like all loose. [P4; I]

... You can feel space when you are out in outer space, because as I said your weight changes when you are out in space. You cannot see space but you can feel like something going on...no you cannot touch space...[P16; I]

5.3.10 Internal – External

This vector pair was used to explore the extent to which space could be perceived as a phenomena 'out there' or something that exists within us – something which is internalized and part of us.

Table 5.9 Distribution of sample on Internal – External continuum

	Totally convinced	Strongly convinced	Slightly convinced	No strong Feelings	Slightly convinced	Strongly convinced	Totally convinced	
Internal	3	8	11	2		6	2	External

The questions that were used to provoke discussion revolved around the notion of whether we had space inside us in general, and whether there was space in our minds in particular.

Just over two-thirds of the sample (see Table 5.9) were convinced that space could be internalised and that we had space within us.

Some of those who thought that we had space within us would not go beyond the notion of gaps and 'real' space:

> *... There's probably space like all your organs and stuff are not totally cramped up together. There's space that keeps them apart, blood, veins and stuff.* [P10; I]
>
> *... Inside me – yes, 'cause the stomach's got like gaseous – what's the word – your stomach's full of acids and stuff and that's like open space unless you've really been eating. And your arteries have a little bit of space, because they have to open and close even though there's blood there.* [P9; I]
>
> *... only very very small spaces...*[P12; I]
>
> *... My lungs have some gases inside. So I think there's space in my lungs and in my mouth. So I have space inside me.* [p14; I]
>
> *... Yes, the air that I breathe.* [P2; I]
>
> *... Yes I can feel that there is a space inside of me but I cannot touch that space and I cannot see it because I cannot see inside me.* [P29; I]

Others had a more philosophical approach and recognised the existence of space in our minds, possibly manifested in our dreams and thoughts:

… Our mind … that's not really a space, but a slot where we could slot in more education, or you know, things like that, learning more and slot in. I don't think it's really an empty space where you can actually put something in … something actually like a filing system … like there is something there, but just gets filled up. Just like this little chip there and you put everything onto it. So I think the brain is a chip and you just fill in information into it…[P11; I]

… I think there is – there must be. There's got to be space for you to learn new things. [P8; I]

… I think your dreams have a lot to do with space. When you're dreaming, you dream of all places and times and stuff, gaps between your dreams – all over where your brain takes you. Just a lot of space and stuff. [P10; I]

… Physically in my brain – no. In my mind, it's probably a different kind of space. [P13; I]

… Ja, between your brains, so that things that I will remember from the past or the future. [P18; I]

… Yes, I mean it, I might feel quite tired now, nervous and everything, but I think there's a lot of space in my mind – that's one of the biggest spaces that there can be, but it's…what's so wonderful about it – and so powerful. [P6; I]

… that's a different kind of space, that you've got space to expand your mind – space where you can grow – but it's not really the same kind of space – I suppose it could be because there's no definition of space – could be the same but …um…there's not..I don't say there's like hollows in your head, but you can expand for it the same way you can fill space. [P4; I]

… Yip – there's space in everything. [P22; I]

… It's a space where thoughts, everything is generated, because there has to be something happening inside to take it out – everything – dreams – thinking has to have space. [P24; I]

… My entire body is made up of space – in my head – otherwise if it didn't my mind would not function. [P21; I]

Some denied the notion of space within us totally:

… No, I don't think so – I can't fill it. [P15; I]

… No, I can't imagine what is happening in my mind. So I don't think there's space in my mind. [19; I]

… Inside me I think my body is being occupied, so I don't think there's space in there. [P16; I]

… My head is full of brains and those kinds of nerves – no space there. [P17; I]

… No, because I think my body is liquid inside. [P20; I]

5.3.11 Summary of general image of space

If one had to draw a picture of the general world view of space of the participants, then this picture would reflect space as a complex, multifaceted and rich concept that:

- has a strong religious element. The general belief is that space has always been there and that God created things, such as the earth and the planets to occupy this space;
- is relatively orderly because it is underpinned by a design and it represents something peaceful;
- is nevertheless mysterious and somewhat obscure. Space is viewed as bipolar in that we have space immediately around earth which is based on facts and principles. The other space is the one 'out there' which is mysterious and unclear;
- has purpose and function, namely to provide and sustain life;
- is special and beautiful. For some however, it is perceived as mundane and 'just there';
- has mathematical characteristics in terms of locating earth and oneself. It has direction and is overpowering to the extent that it makes us insignificant and very small;
- it has no beginning and no end – it is infinitely big;
- is something that we cannot touch, yet feel. It is something that is visible in the sense that we can see that there is nothing there;
- we can internalize in the sense that we must have space inside us to store dreams and thoughts.

Reflecting back to what the literature said about space in chapter two, the Newtonian division of absolute and relative space came through strongly in the perceptions of this sample. Absolute space, which Newton refers to as the space of God (see page 37), was identified with by many of the participants. When considering the naturalism – religion vector pair, for example, the evidence shows a strong leaning towards a godly role in space (see 5.3.2 on page 186). The strong conviction by the sample that space is orderly (see 5.3.3 on page 187) further strengthens a Newtonian perspective of space which argues for a space which always remains similar and immoveable (refer to page 37). The participants' general view, that the function of space is to provide and sustain life for those in it, implies a Newtonian absolutist perspective which advocates that space is a self-subsistent 'container' which would exist even if no physical objects were contained in it (see page 37).

In terms of its magnitude and extent, space was viewed as infinite by most of the participants (see 5.3.8 on page 196). This can be viewed as counter to the Platonic position which asserts that space has length, depth and breadth and is a receptacle or vessel for objects (see page 36). Although Descartes leant towards the definability of space, he recognised the existence of an imaginary space, suggesting that space could be regarded as infinite. In terms of the Cartesian reductionist and grid-like outlook on space, it is something which is measurable and orderly. The perceptions of this sample however refuted this notion to a degree. Although orderly, most of the participants viewed space as something mysterious and obscure (see 5.3.4 on page 187). This is consistent with their often subjective understanding of space. Most of the sample expressed a view suggesting that space is intangible – something which you cannot touch, but feel. As P16 said: *you cannot see space, but you can feel it* (refer to 5.3.9 on page 197). This Kantian view of space suggests that space is an intuition – it is *a priori*, which means that it is not derived from experience (see page 38). Just over half the sample referred to space in Kantian terms and related their perspectives in terms of their feelings and subjectivity.

The richness of the participants' world view of space was impressive and it would be naïve to suggest that the various ideas and positions on space fit neatly into particular categories as perhaps suggested above. There was a lot of overlap and diffusion of ideas, and many participants adopted a multitude of positions. This added to the complexity, depth and wealth of their *Weltanschauung* of space.

5.3.12 Gender differences

Although no gender differences were detectable in the conversations, one particular theme was emphasised frequently by the female pupils, namely that of 'personal space'. This theme was not anticipated and emerged spontaneously as the conversations took their course. Many of the females, particularly from S1 and S2, commented on their awareness of their own space 'bubble':

> *... each person, each planet has a different space around it ... um ... space between one person and another – could be a humungous border where you could have millions of people around you and not mind about it Then you could have another person and if they have millions of people around they would go crazy...If you got your space, at least you know what like – to just enjoy being yourself and often just to make contact with yourself. It helps you to reflect on who you are and what you want in life...*[P7; I]
>
> *... when you're in a room, you feel claustrophobic and there's like not a lot of space around you – you feel enclosed, captured kind of thing.* [P4; I]
>
> *... I think that every person has their space sometimes, just to be alone and think.* [P1; I]
>
> *... space is beautiful because it belongs to me ... bad people always want to invade other people's spaces and spaces are very private things.*[P6; I]
>
> *... It tells what you are comfortable with and what you feel uncomfortable when they come too close to you, and that would be your space bubble that they're invading.* [P12, I]
>
> *... I feel relaxed and myself…*[in my space]*...like personally, like for me, something that's for me. I just think that 'cause it's my space, nobody else will really know. I will know myself, so I'm not like – when I'm in my space, nobody else can be. Because it's me – it's the only place that's for me, and me alone, that nobody else can be there, whether they're trying to be there or feel that maybe they don'yt understand or whatever, but say, how can I say now ... um ... when it's my space, nobody else can be there.* [P8; I]

Only one male mentioned the notion of personal space explicitly:

> *…and you're quiet by yourself – you get into a space...like you've got troubles or something you can think about those troubles or you know, because you're quiet and you're by yourself I your space. Ja well, I love going like to the beach and just sitting there and listening to the waves or where ever...if there's quietness I like sitting and thinking – I just enjoy it ... it clears up my mind.* [P11; I]

The reasons for the female pupils isolating this particular theme could be manifold. In discussion with colleagues and critical friends at work the following explanations were suggested:

- Female adolescents are more aware and protective of their personal space than their male counterparts;
- Some females of that age are more reflective than their male counterparts, and identify their personal space as a place to reflect and think;
- In this period of violence, insecurity and widespread sexual abuse in South Africa, females are acutely aware of their vulnerability and their personal space.

In explaining gender inequalities, Millett (1970) asserts that females tend to develop a passive temperament and a sense of inferiority as a result of the prevailing patriarchal ideology of our society and have taken men's dominance for granted. This has resulted in females being more introverted and hence more aware of their personal space, whereas males tend towards a more worldwide outlook. Other evidence suggests that females are perceived to be "closer to nature" (Haralambos and Holborn, 1995:596) and their psychological make-up is concerned with "childcare and primary socialization. They develop more personal and intimate relationships with others" and themselves. This could be a further reason why the females in this sample tended to isolate the importance of personal space. Sinclair, Sherman and Wood (1989:160) suggest that when children enter adolescence they have to a large extent internalised society's "view of sex-appropriate attitudes and behaviour". The male role is generally seen as assertive and independent, whilst the female role is perceived to be introspective and private; hence the emphasis on personal space.

As mentioned in 2.4.3 on page 38, personal space was referred to as an "area of invisible boundaries surrounding a person's body into which intruders may not come" (Sommer, 1969:26). This definition affirms the feelings of the female members of this sample in particular. Sommer (1969) suggests that the 'space bubble' is not spherical in shape, as some people are able to tolerate closer presence of strangers at their sides rather than directly in front, and the extent changes from individual to individual. The varying degree of emphasis of their personal space by the participants supports Sommer's suggestion.

5.3.13 School differences

The similarity of the world views of space across the spectrum of the five schools impressed me. There was no one school with a particularly strong and unique response to the various vector-pair themes. This was a little surprising as I had expected the pupils from the rural school, for example, to perhaps provide a more 'rural' perspective (whatever that is) than their urban counterparts. I anticipated the use of rural metaphors and descriptors. I expected them to articulate a world view which perhaps reflected a more 'ethnic and traditional african' perspective which included indigenous stories and anecdotes.

There was, however, a noticeable range in the extent and depth of articulation and discussion. Although a translator was present for the pupils from S5, the responses were short and in many instances superficial and merely skimmed the surface of the issue. This is clearly illustrated by comparing the extent of the concept maps representing the world-view profiles of the six pupils from S5 (see Figures 5.1, 5.2 and 5.3) and some others in the sample. The concept map approach was modelled after Cobern's (1993) and Slay's (2000) world-view work around perceptions of nature where they made use of concept maps to provide a visual overview of their participants' view of nature. The fundamental framework which determined the shape of the concept map was based on the nine vector-pair themes which framed the interviews. After the construction of the concept map each participant was asked to verify the authenticity of his or her concept map. This was done in conjunction with the verification of each of the interview transcripts (see member checks in 3.4.3 on page 73).

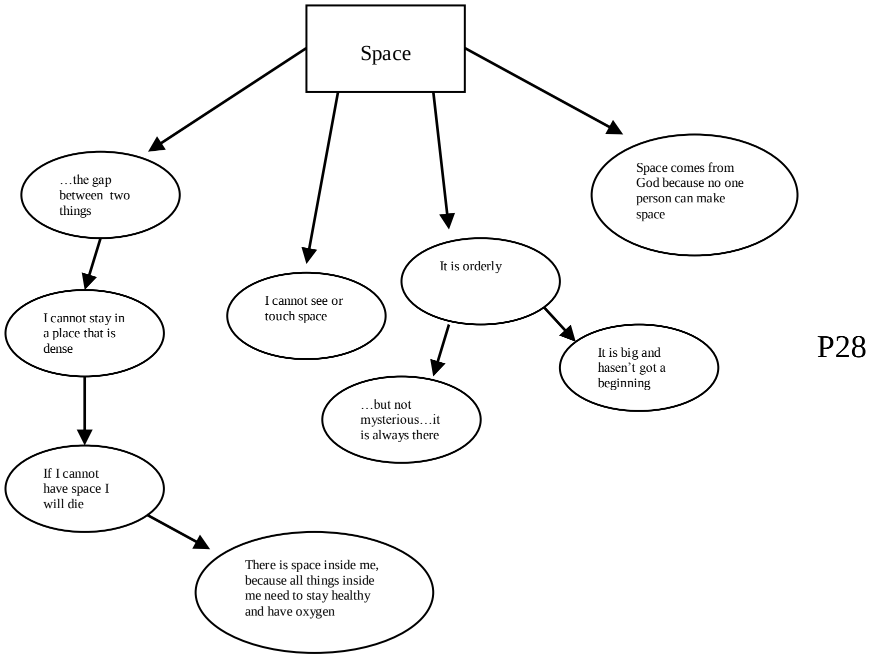
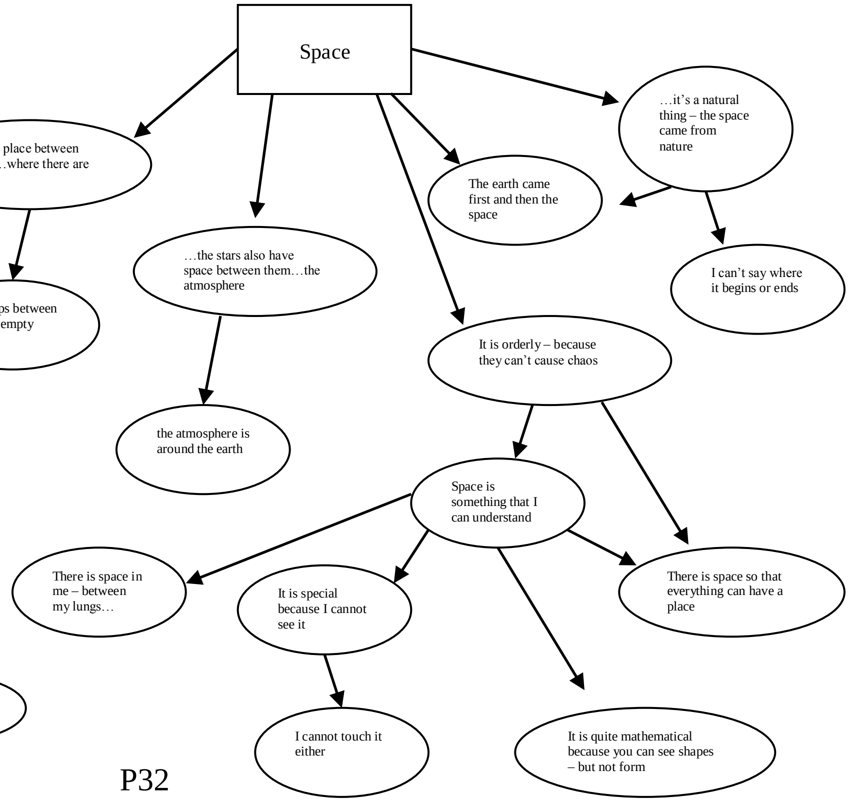
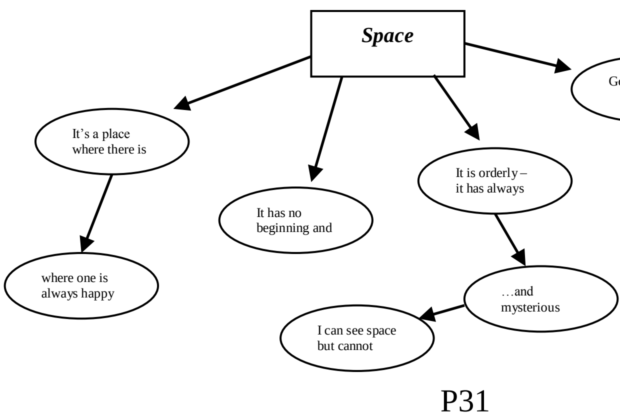


Figure 5.1 World-view profiles of pupils in S5





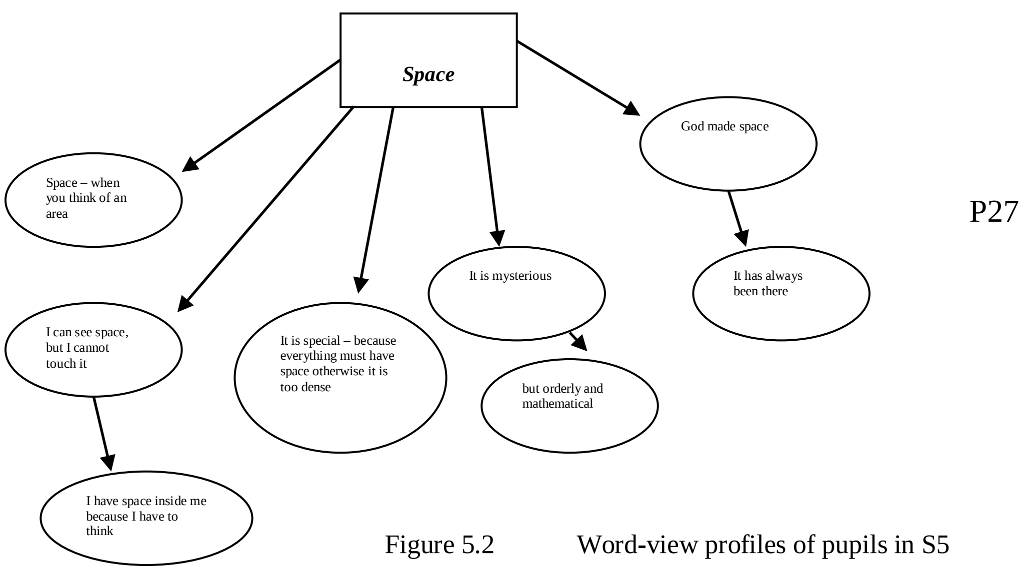
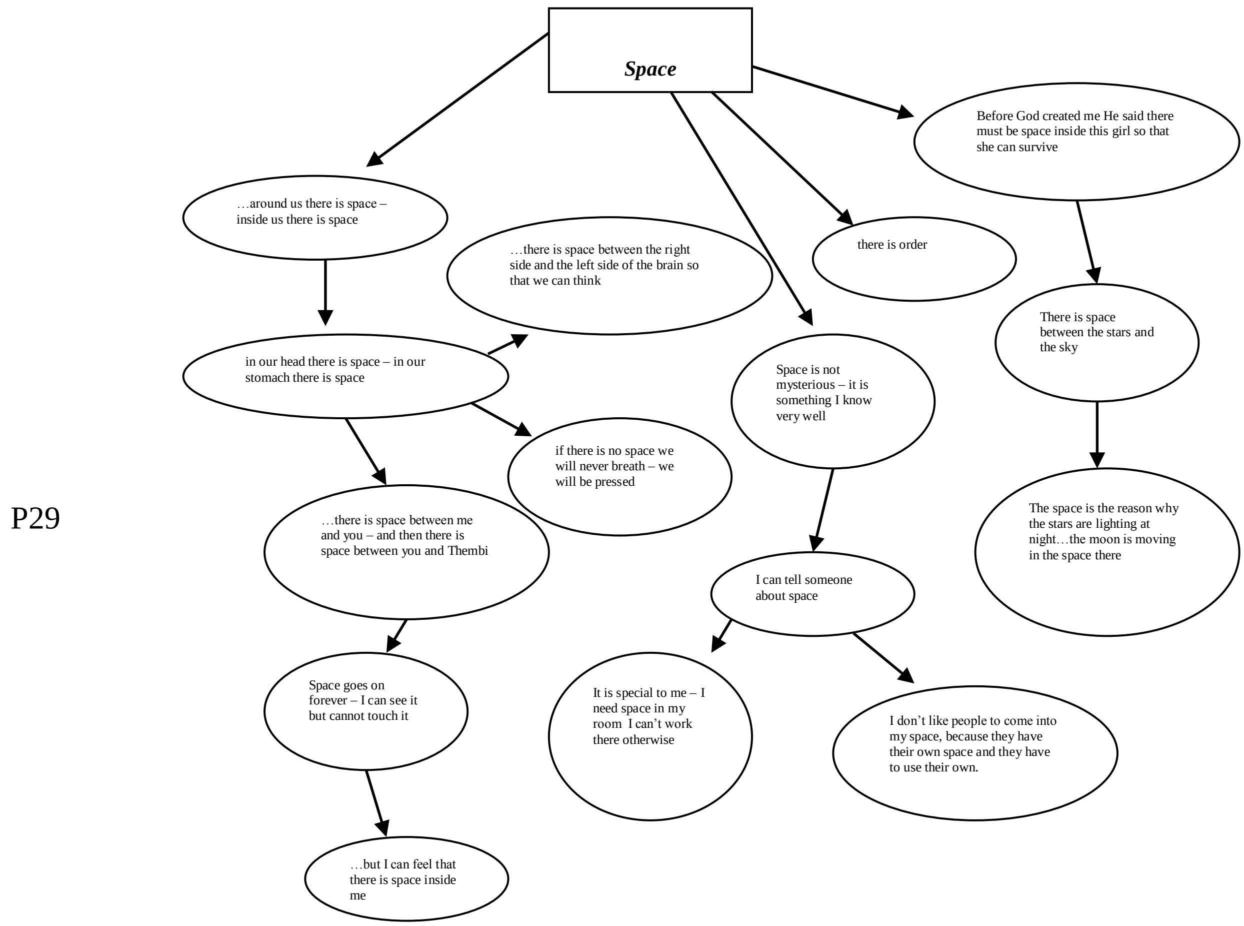


Figure 5.2 Word-view profiles of pupils in S5



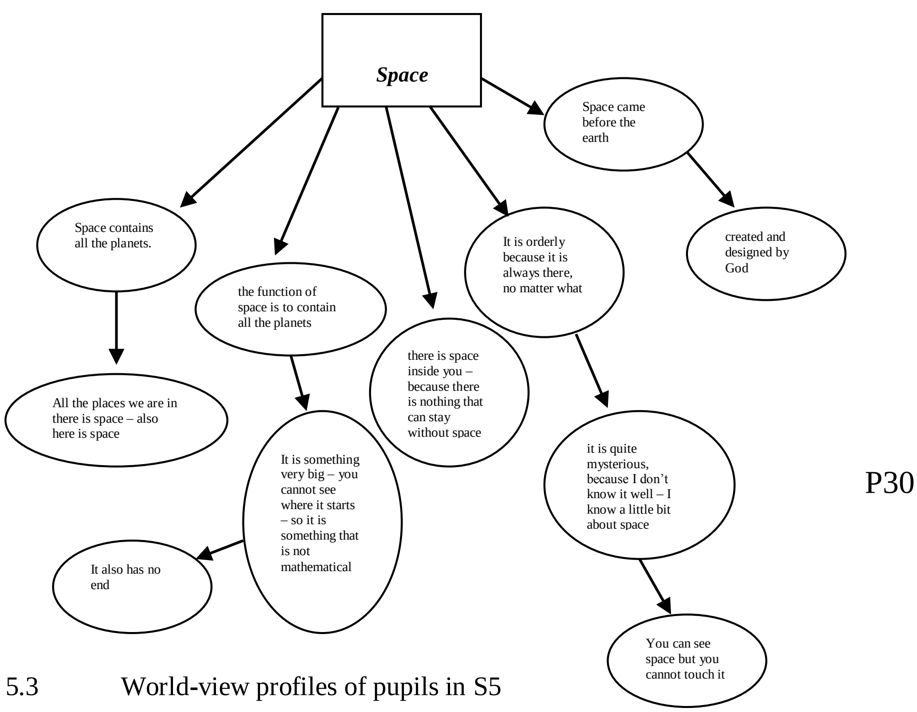


Figure 5.3 World-view profiles of pupils in S5

Figures 5.4 and 5.5 provide two examples of world-view concept maps of other members of the sample from S1 and S4. They are clearly more elaborate and sophisticated than the other six from S3 and S5.

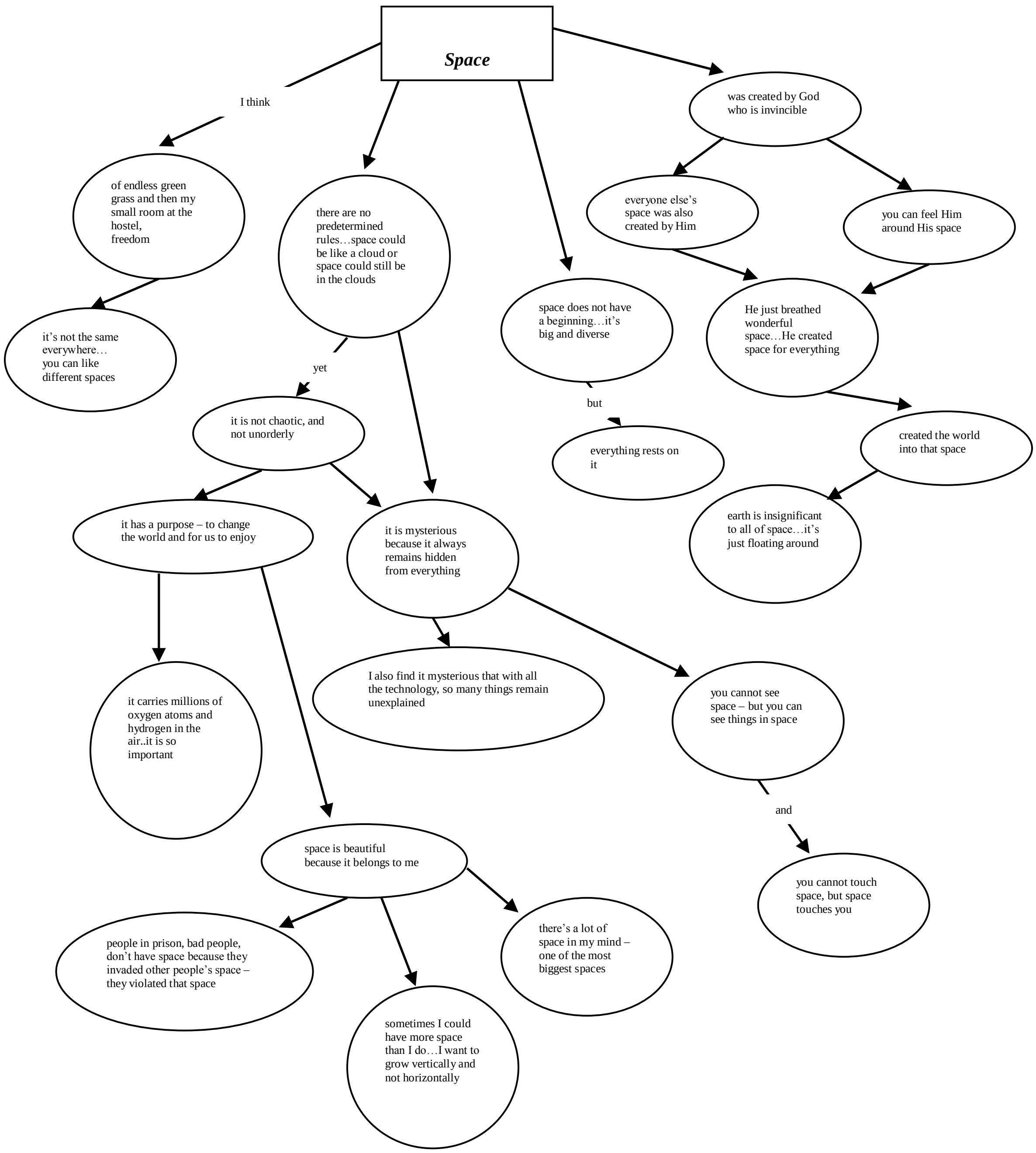


Figure 5.4 World-view profile of P6 from S1

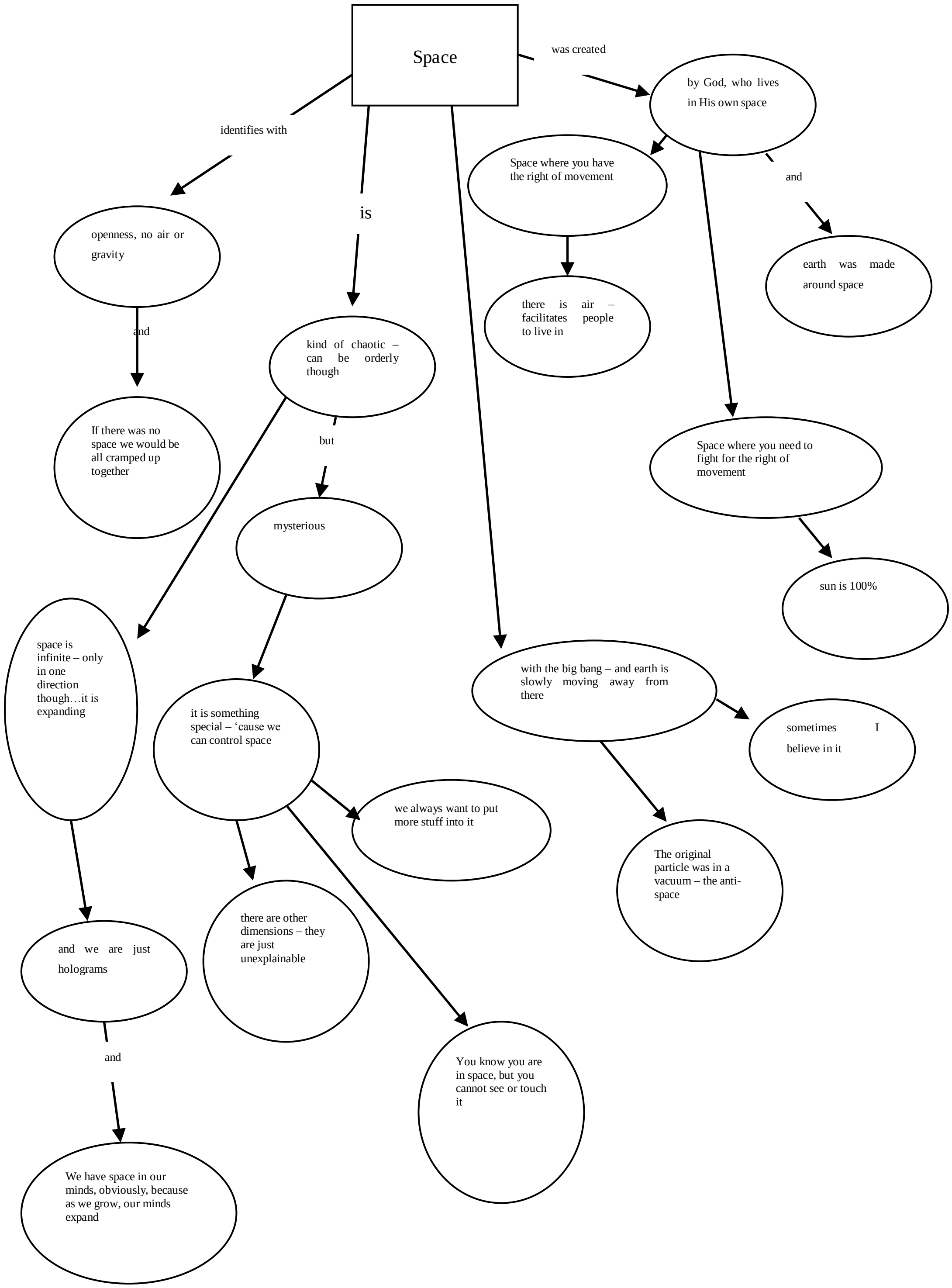


Figure 5.5 World-view profile of P23 from S4

The reasons for this difference between S3/S5 and S1/S4 could be manifold:

- Despite my attempts not to disadvantage second-language speakers by putting contingencies in place such as employing the use of an interpreter and by encouraging the use of the vernacular, pupils from S3 and S5 may still have been prejudiced. Actually, the assumption that the use of an interpreter solves the language issue is naïve. For example, one needs to take into consideration the hidden roles that the translator may play – how familiar or unfamiliar is that person to the participants? Are there power relations between the translator and the participants that may affect the interview dynamics, about which the researcher is unaware?
- From the socio-economic profiles in section 3.2 on page 54 it is apparent that most pupils from S3 and S5 come from deprived backgrounds in terms of access to information such as literature and the internet. Many of the parents of those pupils themselves have had limited access to knowledge, and therefore are possibly unable to facilitate stimulating and inspiring conversation;
- The schools (S3 and S5) themselves lack the resources and infrastructure to stimulate and challenge their pupils beyond the content of their limited stock of textbooks.

It is interesting to note that the discrepancies above appear to be directly linked to socio-economic factors as opposed to cultural factors. The evidence shows that the concept maps in Figures 5.4 and 5.5, illustrating a more elaborate and sophisticated world view of space than those in Figures 5.1, 5.2 and 5.3, refer to P6 and P23 from S1 and S4 respectively, both pupils from cultural backgrounds similar to those from S3 and S5, but from different socio-economic settings (see Tables 3.1 on page 53 and 3.2 on page 56). P6 is of Xhosa origin but has access to all the facilities characteristic of a middle-class environment (see 3.3.1 on page 49). Although of Ghanaian origin, P23's situation is the same.

5.3.14 The use of artworks to illustrate world views of space

As mentioned in 3.4.1 on page 63, the participants were asked (at the end of their interview) to chose three pictures from a portfolio of 11 selected artworks (see Appendix 3 to complement and illustrate further their image of space. It was suggested to them that they were to choose three pieces in all which either reinforced or contradicted what they had said during the interview. The reason behind this exercise was twofold:

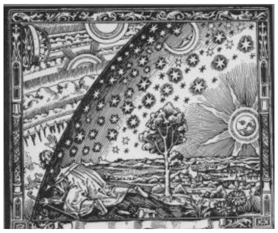
- to enable the participants to illustrate and support their articulated world view of space through another medium and to complement formulation of what was a very complex concept;
- to enhance the initial views and images of space as articulated by the participants and thereby enrich the narrative.

Their responses were recorded.

The selection of the portfolio was based on my own fascination with art in general, and in particular with that of M.C. Escher, the artist responsible for most of the 11 selected pictures. Escher's art is renowned for its exploration of spatial perspective and dimensions. His prints reflect the tension inherent in any flat representation of a spatial situation (Ernst, 1985). Through his drawings of visual deception and optical illusions he challenges our own perceptions of space, dimension and perspective.

The driving criterion for the selection of the 11 pictures was that, from my perspective, they could all be used to illustrate aspects of the nine vector pairs that framed the world-view profiles of the participants. It was hoped that the portfolio would assist participants in articulating their world view.

Picture 1



This work by C. Flammarion entitled 'Mankind breaking through the clouds of Heaven and Recognition of New Spheres' is a woodcut done in 1888 showing the astronomer reaching for truth. "He is depicted as breaking through the shell of appearances to arrive at an understanding of the fundamental mechanism that lies behind appearances" (Davis and Hersh, 1991:69). This work could thus be used to possibly illustrate the divide between space 'down here' and space 'out there'.

Six participants selected this picture.

P25 used it to emphasis his point about the differentiation of space:

> … *it shows the barrier between the two spaces – this one* [pointing to where the astronomer has come from] *looks like more compressed, more full and this one* [pointing to where the astronomer is going to] *looks like somewhat emptier, more spaceful. He's just discovering the other space of something.*

P17 also saw two different forms of space:

> *… it is a picture of earth. I think it shows the things on earth – the trees and the houses…this [pointing to where the astronomer is going to] is outer space … there are things around it, the stars.*

P18 saw space:

> *… between the house. You can plant crops so that there will be food in that space.*

P22 saw the divide as follows:

> *This is like the sun and the stars and everything. That is also part of the earth and what was made* [pointing to the surface of the earth] … *and like there was space* [pointing to the universe] … *it could represent the changing from day to night.*

Picture 2

This lithograph, 'Belvedere', created by Escher in 1958, depicts a royal palace built on top of a prison (Bool *et al.*, 1982). The structure on top of the prison is unusual in that it could not exist in our three-dimensional reality, no more than the cube-like structure that the seated man is holding. This work in particular challenges our traditionally held views of three-dimensional perspective.

In this work (an optical illusion) Escher used the two dimensional plane surface to 'legitimately' create a three-dimensional structure which cannot exist in reality. This work was therefore selected to possibly explore issues of two and three dimensions.

Only three participants selected this picture. None of the three explicitly spoke about the illusion although they all emphasised the strangeness of the people and the building.

P7 used it to illustrate that one can be a prisoner of one's own space:

> *... it shows like ... in your own homes that you can be prisoners..*

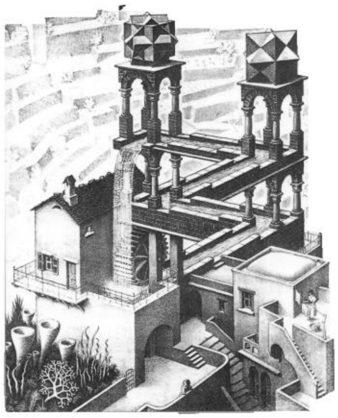
She continues:

> *... and then, on top, shows this jester, and this man is like climbing up. Now he could be going to see the woman or he could be going up to see the view. And then it's also like a different time altogether because of the way that they dress, and it just shows how, over the years our lives have actually changed. It's a strange house – I've never seen a house like that. An open-view type of thing.*

P6 said:

> *Space was used creatively – it's a palace and it does not have any closed doors...*

Picture 3

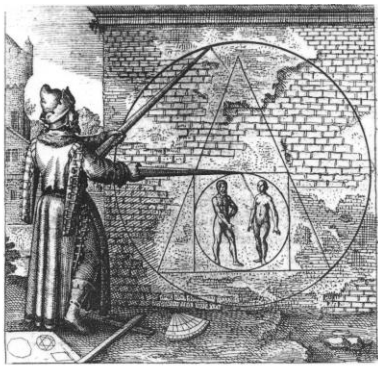


This Escher lithograph, 'Waterfall' completed in 1961, is based on the Penrose's triangle which once again is an object that cannot be constructed in 'real' three-dimensional space. I included this picture for those who wished to illustrate their understanding of two- and three-dimensional perspective.

P13, one of only two participants who chose this picture, used it to reinforce her mystical perception of space:

> *... It doesn't really make sense because these things are still like there's only – there isn't a third dimension. These things are joining at the back there ... and it's strange because there's these different houses here and there's a waterfall over here. And also the water's flowing upwards. And it's a very strange picture because there's seaweed things here and this woman doing her washing here ... um ... I don't know what it is about – the strange shapes at the top here too. The space in this one doesn't really fit together because these things should be further apart, because these should be under the sea and this should be on land somewhere. And also houses that style because they should just be far apart. And I don't know where these things fit together – colours and things. It doesn't work very well. Maybe because it doesn't make sense. Because all these things in the same picture doesn't make sense. Well I said that space was mystical and all that and this picture is quite a mystery – in fact it doesn't make sense. So in a way – it could almost be true, 'cause nobody really understands space, so this could happen, we don't know.*

Picture 4



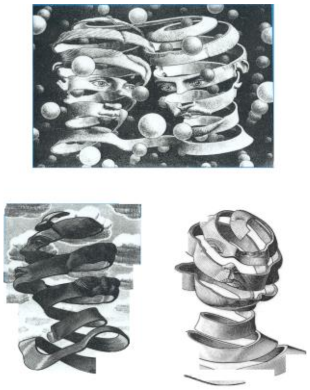
To me, this work from *Emblemata Nova de Secretii Naturae Chymica,* by Michael Maier, completed in 1618, explores the notion of universal rationality and measurability. The figure on the left, possibly God, is depicted as a powerful geometer calculating ratios and proportions. Humankind is depicted as mere figures that fit into a measurable system.

I chose this work to possibly assist people exploring the notion that space is mathematically underpinned. Alternatively it could also be used to illustrate the existence of a divine entity in the creation of space.

Of the three participants who selected this work, P22 used it to reinforce his conviction that God ensures that '*everything fits together*' :

> *... I s'pose this is like basically saying that every person, everything has to be in this circle to make it work properly, so there's a reason for everything. That's why they made you know and man and that's why they made ... a female, like there's a purpose for everything. If there wasn't them, then something would go wrong.*
>
> *Just like another object, like a human, like may they're trying to show that man is in the small thing which affects the square which affects the triangle, which affects the whole circle. ... um ... (pause) ... That could be God showing you how everything fits together, once you're in heaven, He's standing next to you and showing you how it is. Because you know none of us actually have the mind capacity to understand any of this.*
>
> *So we try and put it into words but we can't really do that, because none of us really understand, we only know like when there's a ...*
>
> *Ja. Well we think it fits together. It could be totally different. We try and relate it to what we know. But our minds aren't big enough to understand the concepts ...*

Picture 5



This picture consists of three Escher lithographs entitled 'Bond of Union', 'Rind', and 'Study for Rind', completed in 1956, 1955 and 1954 respectively. Escher's fascination with the plasticity of the earth's crust, which gave him a "vision of a gaseous and/or liquid sphere surrounded by a thin crust of solid material which is still subjected to all sorts of transforming influences" (Bool, *et al.*, 1982: 82) inspired the works. These works of art therefore could challenge us to look within us and question what lies at the core of our minds. They could be used to explore one's view of internal space.

This picture was indeed a popular choice with 12 pupils choosing it to reinforce their notions of internal space:

P23 used it to support his metaphor of us being like holograms:

> *... Picture 5 represents us as holograms. This represents what we'd look like in a way, not whilst we see each other now, but what I think of us as seeing ourselves. You can see part of yourself, but within you there's like space. That's what's in my mind, this is what I think of, ja.*

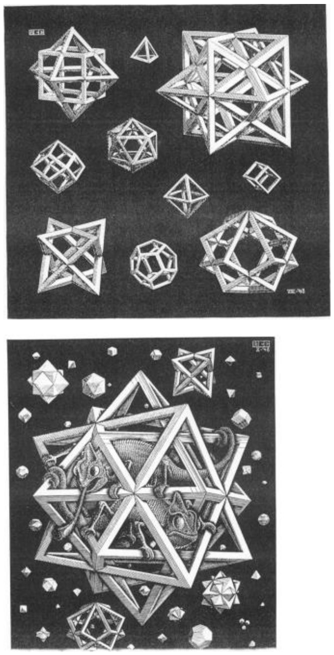
P24 also recognised the notion of internal space:

> *... Picture number 5 is the space in us. When I saw it, that's the first thing I thought of. Um ... space that enables us to think, make decisions, do right or wrong, everything. Sort of like now, art is made in such a way that it makes us see that there is a space within us, but in human beings you don't actually think like that – I didn't think like that first. But now that I see that there is this space and he's turned it into a literal sort of thing, but there is a space that's in us for thinking, making decisions and all that, so that's what attracted me to this picture.*

A similar interpretation was offered by P10:

> *... Picture 5 to me I take these to be humans and this is all the space around them, but there's actually space everywhere. It's even in them. The space is just everywhere. The same over here – everything is just – there's no solid thing – there's just space.*
> *I said there was slightly space internally, but more around you, with these particles which I take to be space around you, all around, and the same with this picture. There's just space around, but I didn't see it really in that view – of it being actually in everywhere. I didn't see it to be here. I thought the object was closed and it was all around but this is just space everywhere.*

Picture 6



This picture consists of two woodcuts, 'Study for Stars', and 'Stars', created in 1948, by Escher. In both prints Escher used mathematical solids and structures to represent components of 'little universes' (Ernst, 1985).

Both prints suggest that the universe is made up of symmetrical, mathematical and well organised structures. In 'Stars', humankind, represented by the chameleons, is entrapped in its own world wanting to break loose and be liberated.

These pictures were selected to possibly illustrate mathematical perceptions of space – its order and its predictability

It was the third-favourite choice and selected by 12 pupils, mostly as an appropriate representation of space in general.

P21 chose it for its depiction of emptiness and voids:

> *... You can see that there are gaps in between and they have holes, meaning that there is space in between and it creates the – the objects look more real. I think that this picture is well created in terms of the voids in them. That's what I feel about this picture ... The reason why I chose it is because I can see the gaps in between the things.*

P16 identified the solids with the stars:

> *... Picture Number 6 is also like space – I think maybe it's stars on it.*
> *And you can actually see the stars are also floating because there's no gravity there. If there were a gravity, they would certainly go towards the centre of that. So we can see that there's stars and space, and I don't know about these chameleon – lizards. About them, I don't know... I would say, maybe if you ... maybe they will live there forever, because there's nothing that can take them out.*

P10 suggested that it illustrates the omnipresence of space:

> *... With this picture, these could be ... particles – all sorts of particles – there's dust – dust particles, even condensation for rain everywhere. There's space around them, there's space everywhere. And I just took these to be the smallest little particles, like atoms for instance. There's just space even in atoms.*

> *Yes, there's space everywhere – even inside the smallest particle of matter.*
> *Not in the sense of – because the atom is matter so that can have space, but to me I didn't see space as being everywhere even …*

P23 used this picture to describe his two types of space:

> *… It describes the two different types of spaces. You have the two chameleons in a certain space, although it's much too small, this is the type of space you might live in. And that outer space is what is holding that space in*
> *And these other spaces there, I see them as like the planets, in which you can travel, but then you need to cross this space in order to get to that space over there.*
> *us. They represent us in this space. It's kind of small for them in a way. Depends on where they've been put …*

Picture 7

In this woodcut done in 1947, entitled 'Other World', Escher experimented with the relativity of vanishing points. In this image up, down, left, right, front and back can be substituted arbitrarily depending on which window we look through (Bool, *et al.*, 1982).

I chose this work for two reasons. Firstly it can be used to highlight and explore the often paradoxical situation of a three-dimensional image on a two-dimensional plane. Secondly it mischievously illustrates space in terms of comets, aliens, planets and craters.

This work was the second favourite and was chosen by 14 pupils to mainly illustrate that image of space which corresponded to my second reason above, but also to highlight the complexity of space in terms of perspective:

P28's justification for selecting this picture:

> *… Outside the earth … up there in the space. I see these things and stars …*
> *… I don't know this building. Yes, but I don't know what these things are …*
> *… nothing's funny-* in response to my question whether there she noticed anything odd about the building.

P9's response:

> *... I was saying space is mysterious, full of aliens and how we are unimportant. Even though these things are here, they're still very small compared to the shooting star and the other planets and stuff that are around space ... very funny looking birds. Aliens out in space.*
> *I don't know if there really are aliens out there, but I was just saying how we are unimportant because everyone's always saying how aliens being out in space, not humans, so they always used to describe aliens if you watch movies and stuff, as these funny heads and noses and ears and ...*

P10 recognised the relativity of the vanishing point:

> *... And in this last picture, this shows the outer space to me here in the background and stuff, and it just shows even with the stars and the planets and all these things, there's space everywhere. Between every star, between every planet ...*
> *This building – you can't actually see where the photo's coming from because you look at it from this way, that could be the top, but if you look from another angle, that could be the side.*
> *Because this thing's upside down but here it's the right way around. So it's actually just telling me that there's just space everywhere, no matter what angle you look at it from. You can be looking at it from this angle, there's space, from this angle ... you can't get away from it – there's just space everywhere.*

P12 also commented on the interchangeable planes:

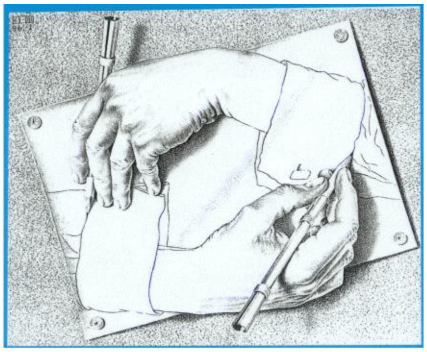
> *... Then I chose this one because it sort of confused me because it was like space inside other space.*
> *There's all this space outside here like you see there, well there seem to be stars and other things going on, and then there's a building type of thing and then there's space inside that building. So it sort of caught my attention because it was space inside space – that's why I chose that one. ... if it's like something almost touching this building or if it's something that has been put against the building or I'm not really sure ... or if it's something that's blocking something else off. And um, ja, and um, I don't know why these would be here – these structures whatever they are, because I don't really know – it just seems to be some sort of planet with craters or whatever, and I don't know how these tie in with the space/planet type thing. And that sort of struck me as strange. Yes – these birds are facing upwards and that's horizontal and that's sort of like looking up at me and that's sort of like looking that way, and that as well, and then that's going the opposite direction to that one. So that's strange. And this also looks ... the way they've done the outside here – the border, looks like this could carry on for a very long time. I'm not sure if it's like infinite or whatever. It looks like it carries on. Is it? It's a strange picture and I think maybe that's why it draws people's attention.*

P6 engaged extensively with the perspective issue:

> *... I like this picture because it's like looking ... you've got you at the bottom and you're looking up. And then figure at the top – it's excellent it shows you all the different dimensions, all the different ways. Like looking at this bird in three different ways, like there and there and looking also at the heart, it's like a mirror image of different – different kinds of figments with spaces behind them, but you can still see that they are a figment of the same thing.*
> *Um ... differences of space could be like so much squares*
> *Yes – it's too many squares in line, and it just a doesn't seem um ... like we're just ticking off them – I mean the background is like completely different.*
> *Maybe it's a picture of three-dimensions. It/s like a three-dimensional person trying to fit another dimension into two-dimensions, trying to incorporate those ...but it's quite difficult, it's almost as if it were three-dimensional ...*

Um... ja well, two-dimensional is something you can see and you can see that it's two-dimensional but only part is not three-dimensional, otherwise this is being used like a three-dimensional ...

Picture 8



This famous Escher lithograph, 'Drawing Hands', done in 1948, is all about visual deception on numerous levels. Firstly we are persuaded that we are looking at a three-dimensional world, whereas the drawing is merely two-dimensional (Ernst 1985). The conflict between the plane and three-dimensional plane is expressed very effectively. For either of the three-dimensional drawing hands, the other drawn hand is flat (Bool *et al.*, 1982*)* Secondly, we are enticed into a self-recursive situation where the hand that is being drawn is in turn drawing the original hand.

Hence, this work could be used to explore the issue of infinity and dimension deception.

This work resonated only with three participants.

P13 identified strongly with the dimension deception and the recursive illustration:

... I like this picture 'cause ... I don't know ... the hands are drawing themselves and it doesn't make any sense because one of them would have had to start ... (haha)
Um ... and ja, one of them would have had to be the first but they seem to be drawing at the same time. Um ... and also the hands are coming out of the picture. They should be two-dimensional when they're becoming three- dimensional.

P8 used this picture to counter her view that space is a liberating concept:

... OK, this one sort of, how can I say – this is more opposite to what I feel because when I look at this it makes me think that you're tied down and you just can't let go.
Because I think in space that that's when it's you and you feel free, so ja, I think that's the opposite. You can do what you want but here it seems like they can't let go what they're doing ...
I don't know, it just seems like the two hands and the way they're like this, makes

> me imagine somebody trying to pull away but can't. Like you can't let go and be free.

Picture 9

This photograph shows the 'rising' earth taken from a spacecraft orbiting the moon. The moon's surface is visible in the foreground.

I selected this photograph to be part of the portfolio as it represents a familiar view of our universe that should be recognisable to school pupils. However, many did not recognise earth and insisted it was another planet.

This picture was chosen by the majority of the sample. Twenty-four pupils selected this photograph to reinforce and complement the classic image of space – namely, that of our galaxy and universe.

P16 felt this picture supported his perceptions of space very strongly:

> *... Yes. Now number 9. This thing proves to me when you are out of the earth – when you are away from the earth, you can actually see what is earth. You can see what type, what shape earth is. The view I can observe when I am far from earth. So this picture number 9 supported me on that. Because you can actually see what is earth.*
> *I think the earth – this because I can see there are no houses. Because if you are far from earth you can actually see there is a little bit of something there. But here I think you are on another planet or somewhere on the moon when you are here. But when you are on the moon you see the earth as a smaller thing than the moon, but when you are on the earth you see the moon is a smaller thing than the earth*
> *I think maybe it's a freak of the eye, or maybe it's your brain. I don't know. But that's what's happening – when you are on the moon you see the earth smaller than the moon, but on the earth you see the moon smaller than the earth. But the moon is actually smaller than the earth.*
> *Ja, it is. So I think picture number 9 supports that you can see a clear view of the earth when you are not one earth.*

P21, P26, B28 and P27 had similar explanations:

> *... First of all, this is space*
> *The gap between the earth and the moon. It's another way of showing that space doesn't end because it carries on.*
>
> *... There's space in between right there ... the earth and this moon or planet, and that's the kind of space that I'm thinking of*
>
> *... there is space between the moon and the soil.*
>
> *...space between the moon and the earth.*

P13 chose this picture to support the scientific view of space:

> *... This one is also quite scientific in fact because it shows the two planets. It shows from a view of another planet. And it makes the space look very small when it's actually very big and also it shows how space can make things look very small – and the earth looks very small and so does the space. I don't know – also it shows the outer space and that space is all empty.*

Picture 10

In this woodcut, 'Smaller and Smaller', done in 1956, Escher expresses the notion of infinity using one of his popular motifs, the tessellated lizard. The lizards are arranged in a ever-decreasing pattern and the work gives the impression of an infinitely converging sequence.

The idea of infinity was central when I selected this work.

In my experience, when considering the concept of infinity one often thinks of it only in terms of its vastness and enormity, but seldom in terms of its minuteness. In this work Escher captures both essences of infinity.

P24 identified with Escher's exploration of infinity:

> *... All shapes and sizes go to infinity. Big to small ... graded down. All shapes, everything, spaces, different shapes, different sizes, different types .... Just cannot find – us as human beings don't know where it ends, so this artist goes to a place where you cannot see properly, just going down, going down and then sort of like multiplicates, gets bigger, magnifies and goes down. So this is also the way that I see space 'cause it's all around and it's sort of like to infinity as I said. Um you can't exactly get the end or the beginning or ... you can get the middle though. It's just there, but no one knows where the end is.*
> *Forever.*
> *... When I wrote that, OK space is out there, and as we were talking, the questions, sort of like, "Wait a minute" – there might be space here. Space might be different shapes, space might be there and the co-ordinates as you said. Then I thought Ja, you can use maths to find space and then space can be found, you know so it opened a lot of different ways of seeing space for me ... interesting ...*

P9 selected it for the same reason:

> *... I said space is pretty like infinite – you never see the end. Here you see the – it looks like the beginning but ... uh, it just looks like the beginning because you're taking it from a certain like point of view. But it goes on and on, you can still see further in this like ... what can I say ... into the shape you go you'll still see the same figures, so space never ends.*

Both P17 and P12 felt it illustrated the notion that space can be very cluttered:

> *... I think there's no space. Because the lizards sit there. They can't go anywhere. There is another lizard there and there's no space for them to move. So that's why I think space provides us to move.*
>
> *... this one because it sort of was very cluttered. If you know what I mean – what I spoke about earlier. And um it sort of seems to have an end here on this side and you can't see the end of the middle. And I'm sure if you took a magnifying glass and looked at it, it would look as if it were carrying on and on and on. And so it looked as if it was infinite going down. That one contradicted the fact that I like space and that.*
> *It has a nice maybe design, but it's very, very cluttered.*

Picture 11

This painting by Salvador Dali, 'The Last Supper', depicts Christ at the table with his disciples. The meal takes place in a room that is framed by pentagonal shapes which form a geodesic-type dome.

I selected this work because it explicitly refers to a familiar God and alludes to spatial ideas in numerous ways. The geodesic dome could have spatial relevance, as could Christ's resurrected body in the background. Further, the view onto an open landscape could have spatial implications.

This artwork was chosen by 10 participants. They used it mainly to illustrate the role of God in their perceptions of space.

P25 suggests:

> *... Picture number 11 is more the religious type of space. It's more probably like God represented He's ... floating ... ja. Probably meant to be Jesus*
> *Ja ... the Apostles and ... ja ...and the space all around them.*
> *Probably the light*

P18 also identifies with the religious interpretation:

> *... I think these guys are praying, and God created the earth and planets and they are grateful for what He did.*

P6, who usually commented in depth about her religious conviction did not see any religious relevance in this work:

> *... I felt that these were the most limiting ones ...*
> *Limiting picture.*
> *Um, well, it's limited, it stops. These people or things are so limited and their world is just – the space around them is just the same – just continuous space. In space it's meant to be different, and meant to be no shapes and sizes and everything seems so uniform. And space is like, it ... it seems as though up to where this beam outstretched it seems as though that's the only important space and I think that there's no space that's more important than the other and this space is very limited ... and yes ... it's just too limited and ... ja, you just think*

> *that it's the most important of everything in that world and it's not true at all. If this beam wanted to show how important things are, then it would have to have very big arms to consume everything. But it just seems as if it's like a special thing – space – although space can put people in place, this totally limits my idea of space … even if it's a beautiful scene and everything, it's very limited …*

5.4 CONCLUSION

The interviews and conversations that formed the basis of exploring world-view profiles of space were structured around a framework of nine vector pairs or bipolar codes. These codes also ultimately formed the themes around which the analysis took place.

This chapter was dedicated to the content analysis of the interviews and focused exclusively on what was said (as opposed to how it was said). Each vector pair was analysed separately and it was my intention to let the interviews speak for themselves. Hence, original quotes from the transcripts were provided. They were coded for easy tracking of the original. The responses reflected pupils' ideas of how space was perceived in many different contexts. Some responded very eloquently and went into complex detail about their perceptions and opinions. Others, however, responded quite superficially.

The participants also made use of a portfolio of 11 selected artworks, mainly by M.C. Escher to complement, enhance and illustrate their ideas and perceptions. This alternative medium proved to be an effective and useful way of assisting in the interpretation of their original ideas.

The next step in the analysis focuses deeper into what was said and looks beyond the content of the conversations. This 'second order' analysis is referred to as a meta-level analysis.

CHAPTER SIX

DATA NARRATIVE 2
META LEVEL ANALYSIS

6.1 INTRODUCTION

As mentioned in chapter three (3.4.4 on page 77) and section 5.3.15 above, a second level of analysis was performed on the interviews. In order to establish as comprehensive a picture as possible of the sample's conceptualisation of space, I considered it necessary to establish some sense of *how* the participants thought of space, and not merely confine the study to *what* they said about space. This chapter therefore investigates beyond the content level (previous chapter) and explores some of the higher-order thinking processes that the participants were involved in – hence the reference to the term meta-level analysis. The participants' dialogue evolved into a vehicle through which a meta-analysis of higher-order thinking was conducted. The world-view profiles of space, as illustrated in the previous chapter, thus emerged into important tools to explore meta-level thinking.

The chapter concludes with a discussion of the participants' mathematics achievement in relation to their world views and performance in the AMST and HAT.

6.2 DEVELOPMENT OF THE META-LEVEL ANALYSIS

As briefly outlined in 3.4.2 on page 67, the methodological model which facilitated this meta-analysis was conceived and developed through a process of consensual validation with a team of three experienced researchers. The task of the team was twofold:

a) to assist in the identification of themes that would frame the analysis;

b) to validate the analysis.

The validation role of the team was very important for this aspect of the research process in particular, as an analysis on a meta-level is inevitably very subjective. The validation of the analysis was therefore crucial for the general validity of this project.

6.3 THE IDENTIFICATION OF INITIAL META-THEMES

As the content-level analysis unfolded (see page 184) and the richness of what was said became evident, I became aware of the use of three central thinking skills used by the participants:

a) the capacity to abstract;
b) the capacity of insight;
c) the capacity to deal with complex concepts and issues.

I felt that these could form a useful base for the construction of a framework of criteria for a comprehensive meta-level analysis. The team members agreed to work initially through one transcript each, firstly to verify the existence or otherwise of these three meta-skills, and secondly comment on the feasibility and appropriateness of using them to inform a possible meta-analysis.

Once this was completed, the team met, and in discussion it was felt that an additional two criteria should be considered. These were:

a) the capacity to critically engage and
b) the capacity to be imaginative.

The next step was to agree on workable definitions of the five criteria. All the team members would then adhere to these when working through the next set of transcripts. A number of dictionaries were consulted and consensus was reached that the following definitions underpinned the five criteria:

> The capacity to explain in theory rather than practice, in intangible rather than concrete terms. The capacity to denoting a quality or condition or intangible thing, rather than a concrete object. (*Concise Oxford Dictionary*, 1995)

The capacity to **abstract**

> The capacity of understanding hidden truths. (*Concise Oxford Dictionary*, 1995)
> A view into anything, enlightenment. (*Chambers's Twentieth Century Dictionary*, 1972)

The capacity of **insight**

> The capacity to identify related parts, to deal with composites. (*Concise Oxford Dictionary*, 1995)
> The ability to see more than one, or many parts: not simple but intricate. (*Chambers's Twentieth Century Dictionary*, 1972)

The capacity to deal with **complex** concepts and issues

The capacity to **critically** engage

The capacity to finding fault, being able to censure. Able to express criticism. (*Concise Oxford Dictionary*, 1995) Discerning and reflective.

Having or showing in a high degree the faculty of imagination. Being able to form a mental image or concept. (*Concise Oxford Dictionary*, 1995) Being able to conjecture; to think vainly or falsely. (*Chambers's Twentieth Century Dictionary*, 1972)

Being **imaginative**

The team members and I then took five transcripts to analyse according to the five meta-criteria agreed upon above. A further brief was to suggest ways to document or illustrate each participant's 'meta-profile'.

6.4 THE DEVELOPMENT OF THE META-STAR

It was felt inappropriate to attach a quantity for each participant 'performance' in terms of the criteria. Instead, each participant was positioned on a continuum in each of the meta-criteria. Thus the notion of a 'meta-profile' developed. The initial idea of a meta-profile consisted of a grid of five vector pairs (similar to the world-view bipolar codes) on which each participant could be positioned – see Figure 6.1. This, however, involved the articulation of the antitheses of the respective meta-criteria. This proved to be cumbersome and in effect doubled the number of criteria in our framework. The idea was abandoned in favour of a model which had a zero starting point for each criteria.

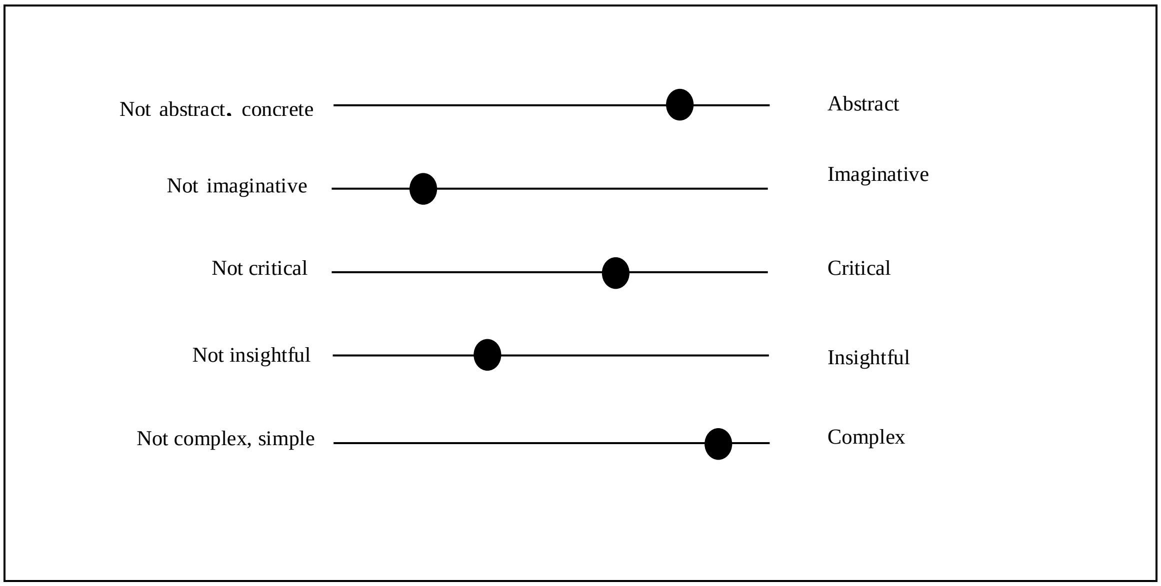


Figure 6.1 Initial idea of the continuum model consisting of five vector pairs

Hence the idea of a meta-star grew, where the centre of the star represents a 'zero level' for each of the criteria. The criteria are represented as the arms of the star. See Figure 6.2 for an example of a meta-star for P6. The meta-criteria need to be seen in relation to each other. The positions of the end-points of the bold radii were determined on the basis of intuition and personal judgement. Further details follow in section 6.7 on page 267. P6's star therefore, would suggest a high level of imagination, complexity and abstraction. Her ideas are however not very insightful and critical.

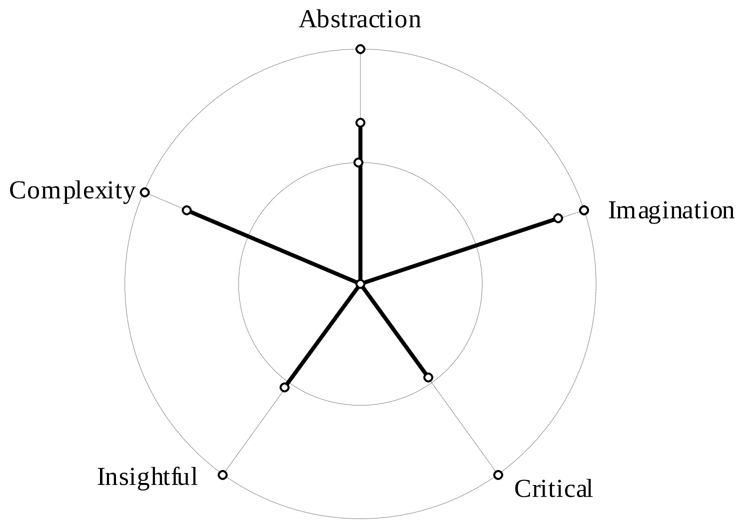


Figure 6.2 P6's meta-star

6.5 ANALYSIS STRATEGY

The five school clusters formed the broad framework around which the meta-analysis was conducted. The motive for analyzing in terms of themes was mainly one of consistency. The analysis of the ASMT and the HAT in chapter four was also done in terms of performance of each of the five participating schools. The design gave a good overview from which general trends could then be isolated and pursued. The five meta-themes which framed the narrative were analysed in the same order as they were defined in section 6.3 on page 230 and 231 namely: *abstraction, insight, complexity, critical engagement* and *imagination.* In an attempt once again to let the voices of the participants come through as strongly and authentically as possible, extensive use was made of their own words and articulations.

6.6 META-ANALYSIS BY SCHOOLS

Meta-profiles of S1

See Figures 6.3a and 6.3b for the individual meta-stars for S1.

Abstraction

Three of the participants (P4, P6 and P7) articulated their ideas beyond the tangible and made use of abstract ideas. P4 spoke of space in terms of freedom and openness:

> *… like in a room, like this is where space is – it's a sense of freedom, openness. The more space there is the more freedom you have. When you feel claustrophobic and there's like not a lot of space around you, you feel enclosed and captured kind of thing.*

She made use of picture 6 as an abstract illustration of our planets:

> *... you know like, this could be like our planets. This one's got a creature inside it, we could be such a creature...we could be either that, or this could be just in space like all the little micro-organisms and all the things that are floating around in space could be us. Could be the bigger picture or the smaller picture. It was like...it first gave me a feeling of space and how everything sort of grips together.*

Picture 10 was also used by her as an abstract representation of reality:

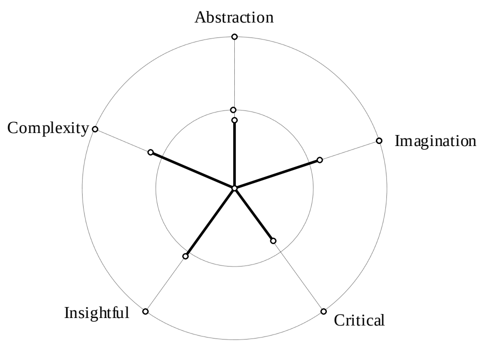
> *...This is also what I said about how we could be the small thing in the centre of this big picture, or we could be here in the universe, in this muddle in the whole universe. And the ants, you know, we could be like the smallest little thing in the centre... The universe and the galaxies and everything could be close to us...*

P6 also used the image of freedom to explain her idea of space:

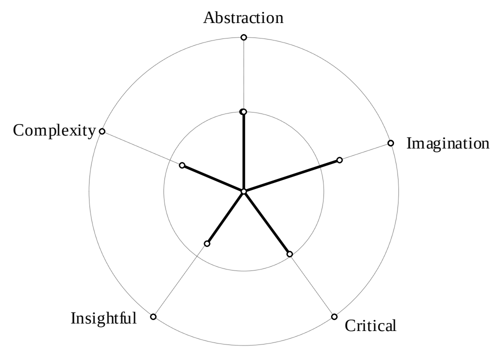
> *... I see it as something beautiful because it belongs to me, it's mine, it's a breath of fresh air, and yet you know, people in prisons and everything ... I mean space is equal to freedom I think. And freedom is basically equal to what you will, what you want to do. And the people in prisons don't have space, they have given up space, because it's just like so important, it's a good thing and bad people can't have it.*

She did not see space only in its concrete form but abstracted it in terms of personal space:

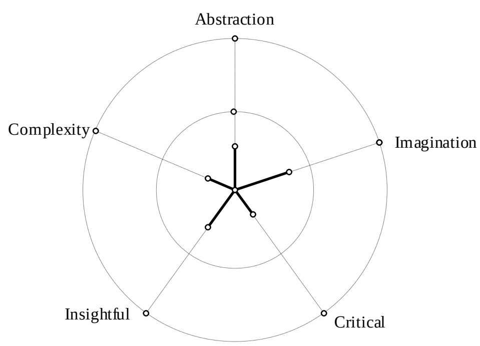
> *... bad people always want to invade other people's spaces and spaces are very private things.*
>
> *... I mean a person should be allowed to decide what ... how – I mean in outside spaces like in the outside world, like outside prison, a person should be able to decide what they want with their space and how their space should be good for them. But criminals violate that space...*



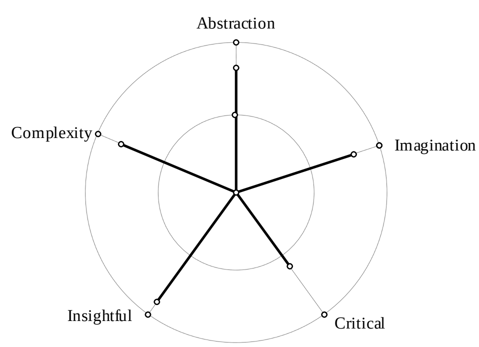
P1



P2

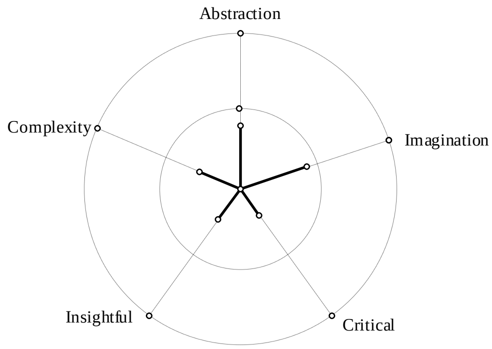


P3

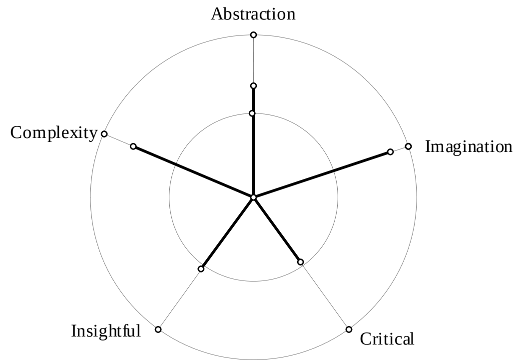


P4

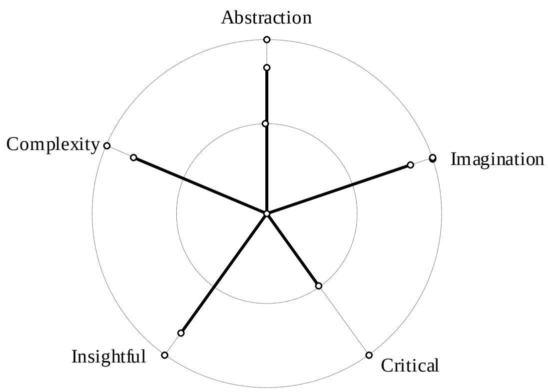
Figure 6.3a Meta-stars of S1



P5



P6



P7

Figure 6.3b Meta-stars of S1 continued

P6 continued with the theme of abstract space:

> *… I want to grow vertically not horizontally – I don't want to disturb any other people's space. But I feel I could have more space than I do.*

P7 also made use of abstractions to illustrate her images of space. They were similar to those of her peers in her class:

> *… Um … let me think … um … I think the function of space is to like um give a person a sense of being, because if you don't have space you feel so crowded and suffocated that you don't know what peace and tranquility is, and if you've got your space, at least you know a bit of*

> *what peace is, and you know what like … to just enjoy being yourself and often just to make contact with yourself*.

She believed that picture 7 was a good abstract representation of space:

> *… Picture 7 represents something that's vast and there's no two ones that are actually exactly the same because it's got like different planets, different galaxies, different – you name it, it's got it. And it's got like different types of animals and like horns and that to it. Because why should we be the exact same? Why should everybody be the exact same? And just because … if you look at different types of races then, why should they be in human form?*

Insight

The capacity of insight involves the understanding of hidden truths. This implies an informed or an enlightened view on something, according to the definition on page 230.

Apart from P4 and P7, and a bit from P1 and P6 there was little evidence of insight and understanding. Those who displayed insight mostly articulated it in the context of describing the planetary system and the workings of the universe.

When talking about the orderliness of space P4 explained:

> *… I s'pose on the one hand it is quite orderly, because everything orbits around each other and everything's very… um … like the planets they all orbit around one another.*

When P4 identified different types of space she displayed evidence of some interesting insights:

> *… I think that's a different kind of space, that you've got space to expand your mind – space where you can grow … but it's not really the same kind of space … I s'pose it could be because there's no definition of space … could be the same but um there's not … I don't say there's like hollows in your head, but you can expand for it the same way you can fill space.*

P7 recognised the formation of 'new stars and galaxies':

> *… You've got more planets, which have been formed. You've got more galaxies which are being formed every single day, every single year, and then, that's … it. It's just kind of grown.*

She also had some insight into the duality of science and religion:

> *… I think more … I think that what I'm trying to do is put both science and God into one category and … they've got so many scientific facts and they've also got so many religious facts and somewhere along the line you've got to come to a common factor.*

Complexity

Once again P4, P6 and P7 articulated their perceptions with intricate and complex ideas.

P4 showed evidence of being able to see things in many parts. She recognized, for example that space can be conceived of as something very small, and very big:

> *... I could actually see that because it's not so many years ago that people thought that this is it,*[something small and finite] *but now people are starting to open their minds to thinking that there's more to it. And like an ant's view is thinking that the world is what we might think the um whole of space is and that space could be thought of as something much bigger.*

She used picture 10 to illustrate the above:

> *... This is also what I said about how we could be the small thing in the centre of this big picture, or we could be here in the universe, in this muddle... in the whole universe. And the ants, you know, we could be like the smallest little thing in the centre ...*

P6 had complex religious views in that she saw God's role as multidimensional:

> *... God is invisible, but at the same time you can feel Him if you come around His space. It's like if you pray and when you go to worship or something – 'cause I'm a Christian – it's like you actually feel that you're not in just a classroom or you're not in just a room. You're actually around someone else's space and you begin to feel different 'cause you know it's someone else's space and it makes you think that all those spaces should be created because if you can just enter a special space it means there are lots of them, and it's a common ... well Christians believe that all space was created by God and also it feels like all space was created by God, 'cause space ... um ... some people feel that if you're in a dark alley, and maybe you feel lots of space but you're not happy there, different spaces feed different needs and we all know that at least one person has to have created different moods and different feelings. Spaces are like ... spaces are like ...um a realistic ... spaces I think are like a realistic way of showing different places, how you feel in different places.*
>
> *... God is like a pillar of your life. He is a great ... He is like a great father. He conceives all things and everything was made from Him. Nothing happens that does not pass through Him. He is the mediator in everything and at the same time He is powerful. He is not just peaceful, but powerful as well. He is not just something, but He's everything.*

As mentioned above, in P7's opinion there is a complex relationship between religion and science:

> *... I think more ... I think that what I'm trying to do is put both science and God into one category and ... they've got so many scientific facts and they've also got so many religious facts and somewhere along the line you've got to come to a common factor.*

In her interpretation of picture 2, P7 had some complex ideas:

> *... It actually shows like ... in your own homes and that you can even be prisoners. You can even be a prisoner or you can be a free person, because at the bottom it's got this person that's like locked up.*

... And then, on the top, it shows this jester, and this man is like climbing up. Now he could be going to see the woman or he could be going up to see the view. And then it's also like a different time altogether because of the way that they dress, and it just shows how, over the years our lives have actually changed

...[It's a] *strange house. I've never seen a house like that. An open-view type of thing –Like, where are these people going to sleep? Also, it's in the middle of nowhere, because it's like on mountaintops and how are they meant to get food and that, unless they just had to grow it on the mountain, which would be a very interesting task for them. Just looking at it, you can like feel peace, and that – if you look at the house and you don't look at the background.*

Critical engagement

In terms of their critical engagement there was little evidence of reflective and discerning ability. There was also little evidence of critical expression or finding fault. Most of the girls had a profoundly religious conviction, which in their deliberations was often used uncritically.

P5's conversation, for example consisted of very short answers, usually in the form of 'yes' and 'no'. She rarely embellished her thoughts and seldom went beyond the superficial. Here is an extract of our conversation when we explored the issue of creation and evolution of space:

Do you think that space was created or do you think that space evolved?
I think it evolved.
You think it evolved. OK. Why do you think that?
I don't know. I just can't picture it being created.
Yes, that's interesting. Why can't you picture it? What makes you think that?
Because ... I don't think ... created y God, yes, but created by ... I don't know what creation do you mean? By God? ... or ?...
Well…
Yeah, I think it was first created by God.
OK. So it was created do you think?
Yes.
By God?
Yes.

Another extract when we talked about the size of space:

Do you think space is infinite?
No
Explain …
The earth?
No, space.
I don't know.
How big is space?

Quite huge.
Quite huge?
Quite huge.
Do you think space can be small?
... No I don't think it can be small. No it can't be small.
Can you see space?
Yeah ... I don't know – I don't think you can see space.
OK, can you touch it?
No, I don't think so.

In terms of the same issues, P4, who was a little more critical than P5, had this to say:

Now do you personally think that space was created?
I think that everything was created, but I think that in the beginning there was probably a large expanse of nothingness, but then there has to be something in space to make the space, I don't know 'cause something's got to hold it all together.
Absolutely. But now, looking at the concept of space rather than what's in it, that original space, do you think has that been there always, was someone behind it, or has it always been there, do you think?
I've got mixed feelings about it, because I think that it could have been, I think I'm more to the side of thinking that it just always was there.
OK, so would you be more on the side that space has evolved?
Ja ... and changed ...and adapted.
And changed … what do you understand by "changed"?
Like ... there was a space and then, ja, it was created into and adapted for people to survive in it, and having things inside that space for people to survive.
OK, and by whom was it changed?
I think by God.
By God?
Yes.
And what do you understand by 'God'?
That he's greater than what our minds can actually um think and anticipate or whatever and He creates more and He's got more um ability than anything can ever know ... um ...

And, talking about the size of space:

OK. Where do you think space starts?
Gee whiz ... that's quite a difficult question ...I don't really think it has a beginning, it's just ... I don't know ... it's just expanse after expanse after expanse – there's just nothing stopping it so there's no beginning to it.
So are you saying that space is expanding – is it getting bigger, do you think?

I don't know, 'cause we only see a small part of the whole thing. We don't even know how much further it can go... it could be forever
OK – a similar question – where do you think space ends?
That's like my biggest – I've always wondered that because I always think that there must be an end somewhere but perhaps if we had the power or the ability to go further than what we have been able to go to then we would be able to find the end, but what would be holding the end together ...
Yes, good question, good question. Now just deep down, what is your gut feeling – do you think there is an ending to space?
I've always thought there must be, 'cause my logic thinks that there has to be somehow or other .
Uh … OK. I think that you've answered this one already – how big do you think space is?
I said it's infinite.
But you'd like to think that there is an ending?
Ja, I'd like to think that there's is an ending.

OK. Can you conceive of space being very minute?
I could actually see that because it's not so many years ago that people thought that this is it, but now people are starting to open their minds to thinking that there's more to it. And like an ant's view is thinking that the world is what we might think the um whole of space is and what space could think could be something much bigger.
OK, my following question is how small do you think space can get then?
Ja ... it's probably about the same but in reverse.

Imagination

What the participants lacked in critical engagement was generally made up by articulating rich and imaginative perceptions. P6 and P7 particularly showed a high degree of imagination. They were able to talk metaphorically and use mental images to illustrate their ideas.

When P6 spoke about the creation of space she described her scenario quite graphically:

> *I think that He created it at a particular time. When ... I think maybe He made it before He created the world so He could see how empty everything was and how He would need to fill it. Maybe God was up in heaven and everyone was singing and everything was nice and everything, but then He felt the need for more people and for make ... and everything in heaven was perfect and so He saw a need to create people which could actually depend on Him and He would like ... people not so perfect but who can see the light, people who can really glorify His name, starting from that evil and everything and to holy people and not just holy people. He really like felt the need for different space and to like expand. So that when H created the world everything was like claustrophobic, everything was like dark and no air, no atmosphere and He just made a world and He just um .. I don't know ... He just took an empty space and created ... breathed into it to see to clear and to see how it would feel and that's how space was created.*

She also spoke imaginatively about her perceptions of two- and three-dimensionality:

OK. What do you understand by two-dimensional space?
Two-dimensional space ... I think it takes up the least space ...
Takes up the least space. What do you mean by that?
I mean it doesn't have anything inside. What you see is what you get and there is nothing much to investigate in two-dimensional space. Ja, it's sheet of paper sitting there and consuming as little space as possible and you can easily compress it and easily just ignore that space, you can put something on top of it, you can put something next to it, you can even squash it, you can even crumple it up ...
Interesting – and three-dimensional space?
It's more powerful, and it's ... it's big things more around us. If you throw a piece of paper even the gravity can control it, even the atmosphere, not even gravity can control it. It's very weak and if you put it on a table it can just fall off. Whereas if you put like a tree on a table and it's two-dimensional, even if it's got nothing inside if you blew it, it would change its position, and ja, it's really solid, it wants to move solidly. You never see something three-dimensional – a triangle or something standing in one corner. It always wants to occupy its own space.
Mm. Interesting. So do you think there are other dimensions? We've talked about three dimensions and two dimensions …

> *Ja – maybe there are other dimensions. We are the dimensions, like um … although I don't believe … although I choose I try not to go into supernatural or anything, I definitely believe that there are other dimensions, like things can come in other forms than the ones we just know. Three-dimensional things and two-dimensional things you think you see everything but yet there are some dimensions where um … things can change form, like these re-incarnations and everything. Things can come in different forms, things can be alive and yet not see what kind of dimension they had before that.*
> **Mmm …**
> *… two-dimensional and three dimensional, there are things that are alive and are breathing and … have lots in them, have lots of content in them which you can't see everything, so you wouldn't call that two-dimensional and you wouldn't call it three dimensional …*
> **So you think that there are other dimensions?**
> *Yes there are.*

P7's imaginative faculty was revealed when she described her meaning for picture 2:

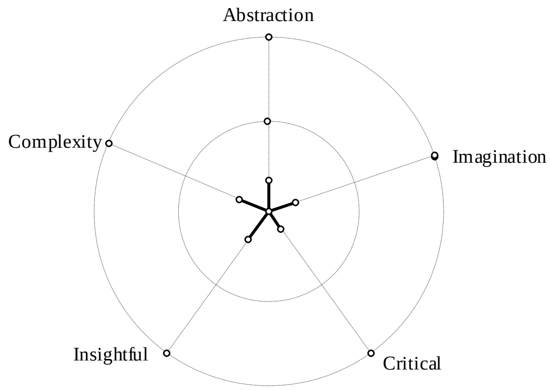
> *It actually shows like … in your own homes and that you can even be prisoners. You can even be a prisoner or you can be a free person, because at the bottom it's got this person that's like locked up.*
> *And then, on the top, shows this jester, and this man is like climbing up. Now he could be going to see the woman or he could be going up to see the view. And then it's also like a different time altogether because of the way that they dress, and it just shows how, over the years our lives have actually changed.*
> *Strange house.*
> *I've never seen a house like that. An open-view type of thing –*
> *Like, where are these people going to sleep?*
> *Also, it's in the middle of nowhere, because it's like on mountaintops and how are they meant to get food and that, unless they just had to grow it on the mountain, which would be a very interesting task for them.*
> *Just looking at it, you can like feel peace, and that – if you look at the house and you don't look at the background.*

Meta-profiles of S2

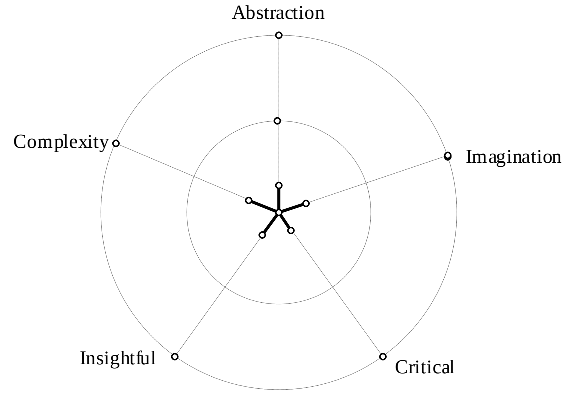
See Figure 6.4 for the individual meta-stars for S2.

Abstraction

Apart from P8, all the other candidates made use of theoretical arguments to illustrate their perceptions as opposed to using only practical and tangible ideas.



P27



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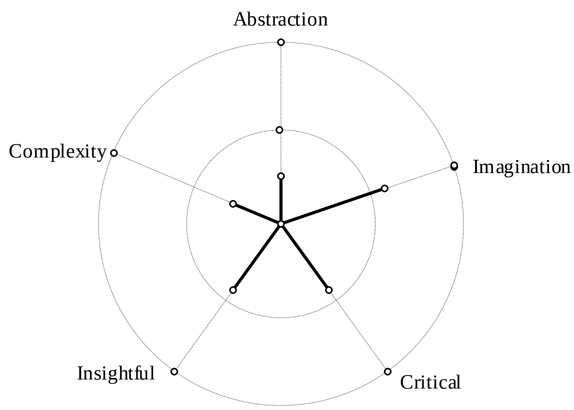

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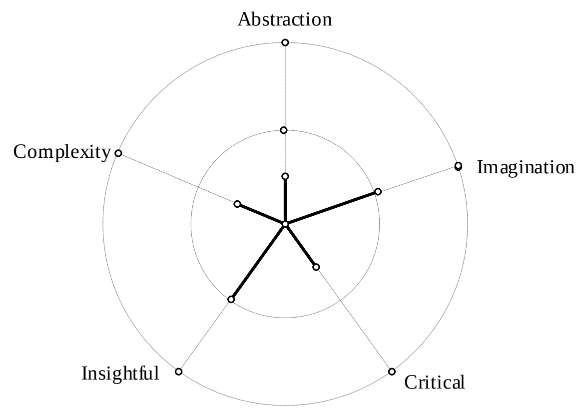

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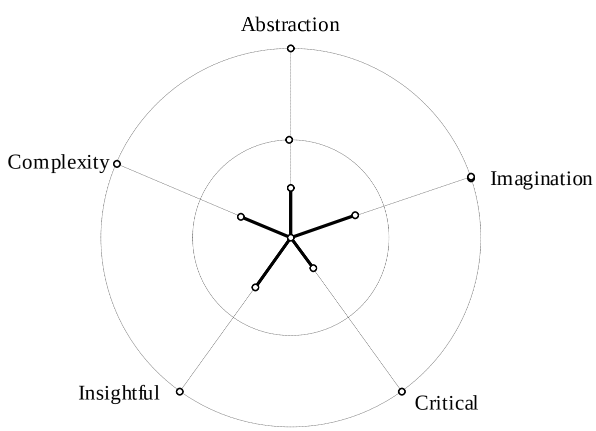

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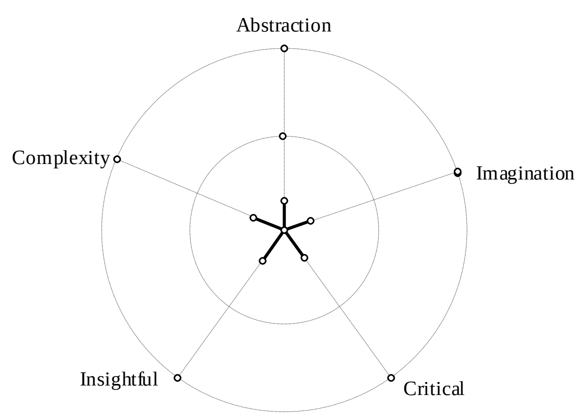


P32

Figure 6.4 Meta-stars of S5

In P12's attempt to describe how to create a sense of space when decorating a room, for example she made use of her capacity to abstract:

> *Well, … for instance in Home Economics we've learnt about interior designing and then how you used space and then they define space and stuff like that. So it's partly based on facts, but it is mysterious how it's just there … it's a bit of both.*
> *Well you get space and you get shapes and you get form. Now form is three-dimensional things that you find in the space, and shapes are two-dimensional things – they're just outlines, and then space they define as everything in between. And that's how they use space to decorate places, maybe not using space but using it as an element of decorating. Like you want to create a tranquil aura, then you'll have a lot of space and make I very spacious without being cluttered up and full of things. Because that would be taking the tranquillity out of it if it was all chaotic … ja … so … we learnt about space in interior design.*

When P9 used picture 7 to illustrate her understanding of the mysteriousness of space, she made use of abstract images and ideas:

> *Picture 7: I was saying space is mysterious, full of aliens and how we are unimportant. Even though these things are here, they're still very small compared to the shooting star and the other planets and stuff that are around space … very funny looking birds.*
> **And what do they represent?**
> *Aliens out in space*
> **Aliens in outer space …**
> *I don't know if there really are aliens out there, but I was just saying how we are unimportant because everyone's always saying how aliens being out in space, not humans, so they always used to describe aliens if you watch movies and stuff, as these funny heads and noses and ears and …*

When talking about the tangibility of space (in itself an abstract notion) P13 articulated her ideas quite abstractly:

> *Well, you can't* [see or touch it]. *Well you can see that there is space, but you can't see the actual space. Or maybe … (haha) … um you can see that there is a space but you can't touch the space that's there because space means that there's nothing there. There are those two objects there and there's a space between them. So you can't touch it because it's not there.*

When speaking about how he values his personal space, P11 referred to the tranquillity of the ocean waves:

> *Ja, well I love going like to the beach and just sitting there and listening to the waves or wherever … if there's just quietness I like sitting and thinking … I just enjoy it … it clears up my mind …*

P10 used picture 5 to help her theorise on the notion of internal space:

> *Picture 5 to me I take these to be humans and this is all the space around them, but there's actually space everywhere. It's even in them. The space is just everywhere. The same over here – everything is just – there's no solid thing – there's just space. I said there was slightly space internally, but more around you, with these particles which I take to be space around you, all around, and the same with this picture. There's just space around, but I didn't see it really in that view – of it being actually in everywhere. I didn't see it to be here. I thought the object was closed and it was all around but this is just space everywhere.*

Insight

On the whole, participants of this school displayed more insight than the previous school. They made more use of truths such a scientific facts than their counterparts in S1.

P13, for example, often made use of her good general knowledge to illustrate and embellish her ideas:

> [Talking about her own definition of space]: *I don't really know. Well if you look at like small things, then I s'pose it's gravity that keeps them apart really. Ja, that's probably the main thing. I don't really know … Space is where there's nothing ….*
>
> [She distinguishes between different spaces]: *Um … maybe space like um … what it's filled with, makes it different. Because if you think about space on earth and there's like air there. Once you're out of the atmosphere there's no air – there's still space. There's probably all the same kind of space, just different things that fill it.*

She had some interesting insights into the Esher pictures. She was one of the few who identified the optical illusions that were prevalent in some of them:

> *And it's strange because there's these different houses here and there's a waterfall over here. And also that the water's flowing upwards. And it's a very strange picture because there's seaweed things here and this woman doing her washing over here (haha) um … I don't know what it is about it – the strange shapes at the top here too. The space in this one doesn't really fit together because these things should be further apart, because these should be under the sea and this should be on the land somewhere. And also houses that style because they should just be far apart. And I don't know where these things fit together – colours and things. It doesn't work very well.*
> *Maybe because it doesn't make sense. Because all these things in the same picture doesn't make sense. And all these things in the background.*

Her views on the order in space was based on scientific observations:

> *Well the space around … the space around like our solar system is quite orderly in the way that it's – everything goes in order or they – that's what scientists say anyway – that everything goes in order in their orbits and everything around the sun. I s'pose in that sense it's quite orderly. If you had to take space in small quantities then it would be quite chaotic.*

P9 expressed similar insights when she differentiated between outer space and space within the atmosphere:

> *No, because if it doesn't have a start, how can it have an ending because it all like gels together in a way. We're talking about space here and all the molecules move around and they don't stop, so you can't just like say this is the beginning because everything changes. Ja, 'cause it's two different kinds of space. Like here we've got oxygen, they don't really have oxygen up there.*

P12 referred to the earth's position in space in terms of locality relative to the moon:

> *Well, it's close to the moon – closer to the moon than anything else, and um ... it's surrounded by planets, so it's sort of in the middle. Well not in the middle but I mean it's in the middle of a variety of planets.*

Complexity

Most of the participants used intricate descriptions and attempted to see things as related parts rather than just one simple whole.

P12 saw the earth as the central to the universe in a philosophical sense, yet realising that in terms of planetary sense it is of course positioned not in the centre:

> *Ja. Well I don't think it's in the centre of the universe, but it's in the centre in the sense that it's got space and stars all around it, so it's in the middle of something – not necessarily the whole universe.*

P9 attempted to articulate her dual sense of space:

> *OK, well if it's space around us as human beings, then it'll be like an area that's not occupied by any objects. But if it's space – the universe, then there's lots of meteorites and stuff like that flying through the air – that's how I see it.*
> **So you distinguish quite clearly?**
> *Ja, like around us on earth, would be the one that's got the objects and ... no objects in its way. But if it's in the universe, then it'll probably be like stars and – space would be the stars and satellites flying around.*
> **And where is that division between space here and space out there?**
> *Space here and space out there?*
> **Ja, where is that division?**
> *Um ... I don't know how to explain that one. 'Cause space here doesn't really have ... how do I say ... space is the air around us, and the rest is like outside our atmosphere, if that makes sense. So like space would be where the stars are and where our planets are, but on the planet itself would be the space I'm thinking of with no objects. Does that make sense?*

P11 also divided his space into parts:

> *Space is some area I said to you – like an area that's open you can put something down there in that space. OK, then you get the other space where there's no gravity, and then this one's got gravity. It's like there's two different spaces – that's what I can understand.*

P10 had a similar view, although she emphasised the notion that the two spaces made one:

> *Well I think that all the space in this universe is just one space. But it's just different densities and stuff. Like it's all one, but it's just different densities – like outer space there's no air but here there is air. But it's all one, I think.*

P13's views on space were intricate:

> *Um … well it's probably always been there because , ja, if something's there then there's no space. Like if there's … space is where there's nothing. So it couldn't really have been created. Because if something was created, then there wouldn't be space.*

Critical engagement

As with the previous school the participants were not deeply critical. They generally appeared very comfortable in rationalising concepts within a Christian world view and not questioning their assumptions or finding fault with their perceptions.

When looking at the issue of infinity, P13 was discerning and critical about the stereotypic belief that space was infinite:

> *Well I don't know, um I also think it's infinite because that's what I've heard all my life, but there isn't really proof that it's infinite, there isn't proof that it's not infinite, so um it's not really something anyone can say, because there could be an end and we just can't see it. I don't know.*

P10 was open to the idea that there could be more than one God:

> *Well, I don't know. Maybe our God of the earth – maybe He also created the other planets, but it's just not written or anything or there's just no life on those other places or whatever.*

The lack of critical reflection, and blind acceptance of a particular belief, are illustrated by P11's unconditional acceptance that God is in control:

> *I think it's orderly, because He's got control of it, I scheme. Because I mean He can predict what happens, He know when like it says in the Bible, He knows when we're going to die, so I scheme He organized all that up there anyway. So if He wants a meteorite to go that way, it's going to go that way. It's the way I see it, so it's orderly, Sir.*

It can be argued, however that beliefs, religious or otherwise, may be based on faith, in which case my comments may be overcritical.

Imagination

Once again the participants were generally imaginative in their descriptions, though often conjecturing with false information and misconceptions. P11 was quite imaginative when he used the computer as a metaphor to illustrate his perception of internal space:

> **And what about space in your mind?**
> *Um … that might be where we could fill up. That's not really a space, but a slot where we could slot – a computer slot where we could slot in more education, or you know, things like*

> *that, learning more and slot in. I don't think it's really an empty space where you can actually put something in … something actually like a filing system …*
> **Like your dreams, I mean where do they go?**
> *That must go to one like filing cabinet under 'd' or something.(haha). It's sad, like you clip it in, but there's a space to be filled like not really a space you can put something, but like an image, not like a physical …*
> **So the analogy is like a hard drive ..**
> *Ja …*
> **Like in a computer, where does it go?**
> *Like there is something there, but just gets filled up. Just this little chip there and you put everything onto it. So I thin the brain's a chip and you just fill in information into it, and it can't fill ours because we only use like one eighth of it or something. So I mean if we could fill it then we would know everything. That's how I see it.*

P12 also described her views about internal space imaginatively:

> *I don't know, … it's kind of like a box where you store everything but in your mind you can't store things because you can't put them in your head. You kind of just memorise them, while with a box you can physically put them there to keep it as a memory in your head. Probably special events or whatever that is something important, or maybe not even import, it could be something very little but it could be something special to you – your brain will store it there maybe for future use, and maybe just because you want it there.*

In her description about the mysteriousness of space, P9 used some interesting images:

> *Well the space out there I reckon is pretty mysterious, because you can't" really know what" there. I mean there could be anything there. We could be flying out there, but we think actually I don't know … it doesn't make sense. We could have our desks flying out there or whatever … but our space here – all the air molecules and stuff are around, so they like attract each other or whatever and it makes them pretty … work together. It makes them work together I reckon.*

The way she explained the relative insignificance of humans within the bigger context of space demonstrates the use of her imaginative faculties:

> **You said that 'it makes me feel small and unimportant'**
> *Oh. Because out there, there so many more things – I mean even a star is more important than a human out in space. Then you've got all your planets and … ja, we look at the stars and we don't look at ourselves at night. Well we do, but not really. Everyone else is more interested in like the aliens and everything else that's out there, not really a human being floating around space.*

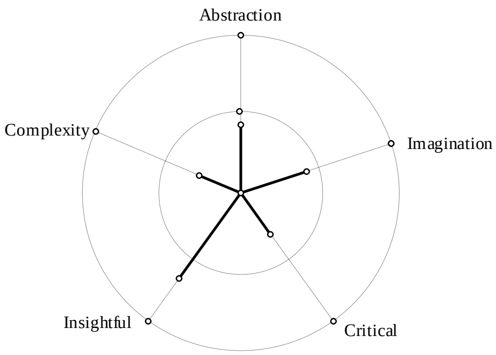
Meta-profiles of S3

Refer to Figure 6.5 for the individual meta-stars for S3.

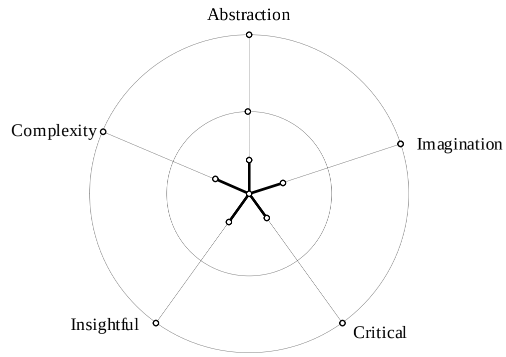
In terms of the five meta-themes, only P14 and P16 conversed beyond the superficial level. The others found it very difficult to explain their perceptions and there seemed to be resistance on their part to embellish their thoughts and views. My impression was of a certain withdrawal the moment I 'dug a little deeper' in an attempt to provoke a richer response.

The reasons for this could be manifold:

- The language issue has been referred to before (see page 212). Although the participants were encouraged to respond in their mother tongue, they felt it very difficult to express their perceptions effectively;
- Many of them commented that they never had to engage in a conversation of this nature before. The philosophical and theoretical dynamics may have proved threatening. The consequential withdrawal and inability to articulate their views effectively would therefore be only natural;
- As most of the participants came from severely disadvantaged socio-economic backgrounds (see page 54), the opportunity to engage in a discourse of this nature, be it in conversation with family and peers or by reading texts, was very limited;
- The nature of the school itself may have been such that topics of this genre were not discussed and engaged with. The calibre of the teaching that took place may not have encouraged engagement at the meta-level.



P14



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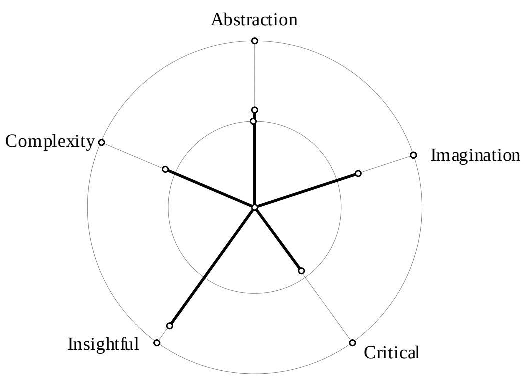

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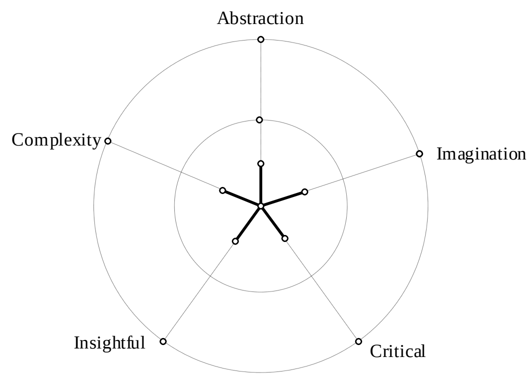

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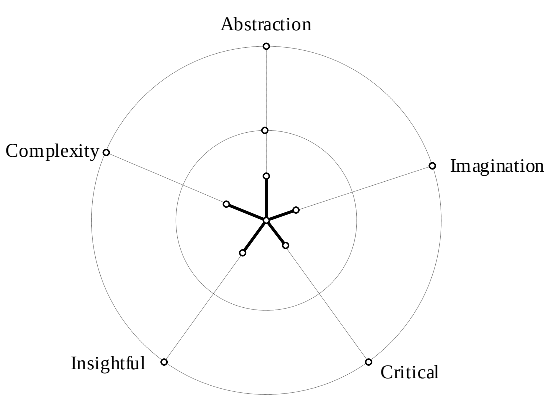

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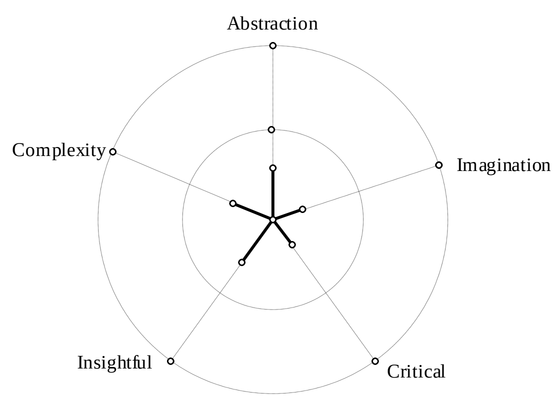


P19

Figure 6.5 Meta-stars of S3

Abstraction

There was little evidence of abstract thought. The conversations seldom went beyond the concrete to the intangible.

In talking about picture 7 (which in my opinion could be viewed very abstractly) P14's interpretation did not go beyond the concrete objects:

> *Picture 7. Here I see stars and here are situated stars.*
> **Where are the stars – show me –**
> *There are stars ... So I see stars here and they are on outer space too. It's what I said. Outer space and the black one is the space.*
> **And this building here?**
> *This building also has space too because it contains little birds here and horns. And horns have space too inside ...*
> **Do you notice anything strange about this building?**
> *This building – it's long.*
> **What do you mean by that?**
> *It's tall. And it has many windows and it is near to the stars.*
> **Why do you say it's long?**
> *Because of its shape.*
> **Because of its shape. Am I looking out of the building or into the building?**
> *You. Now. I am inside of it.*
> **You are inside of it, looking out. Where's the top and the bottom of this building?**
> *The top of it is this bird here.*

P16 referred to numerous intangible images to explain his theories. For example, he alluded to the notion that one needs to step away from earth to realise what earth is all about:

> *... when you get out in space you can actually see the whole earth. But not the whole earth, but the side that is facing. But I also think it's to learn about where you belong, to go to the space and look back where you came from and see what type of place I come from, because you cannot know earth when you are on earth. You have to be away from earth and then you can get a clear view of what is earth and what shape earth is – those kinds of things.*

Insight

P16 was insightful in his perceptions of planetary space:

> *Well, I think there is order in space. Because you can imagine the planets – they're arranged in order. So if there was no order, they would be mixed up, not maybe in their places, so maybe the earth would not be in the third place from the sun. If it wasn't in order, it would be in order, then the next day it would be in the fifth place. So I think there is order in space.*
>
> *OK , the purpose of space I think in my own view, is to protect the earth, but not only the earth but all the planets, because I heard something about a big stone that's coming slowly towards the earth that's going to crash into the earth maybe in a hundred year's time. But I think the purpose of space is to slow down the speed of the meteorite – they say it's a meteorite. If there was no space – if there was air like this, it would just quickly come.*

> *Because the space makes it go s-l-o-w-l-y. So I think the purpose of space is to protect all the planets and also to divide the planets – divided them so that these planets can have air just like ours. I don't know about the other planets – if they do have air , so I think the purpose of the space is to protect the planets.*
> *The other thing I forgot to mention about space – I think it's where you learn that you're not always the same person. OK you are the same person but your weight's no always the same. Because when you are in outer space you weight stays less than here on earth. So I think it's something that where you are learning something – where you can compare your weight on earth and in outer space. I think it's a purpose to learn about science.*
> *It means that there's a lack of gravity in space, because your weight is not your mass. Because your mass is the same here and in space. You weight is the pull of the earth on you, so there's no force that pulls you down when you have the contact with the surface of the moon. There's no gravity that can pull you. Most people confuse weight and mass, like in boxing where you say he weighs 65 kgs, I don't think that's weight, that's nonsense. That mass and weight is the pull of the earth on you. On everything that's on earth.*
> *I think when you move above the ozone layer that's where you can actually get the space. You can actually look back on the earth and see the air. Then you are above the ozone of the earth.*

P18 had some insight into the planetary make-up of the universe:

> *The universe is something that is made up by planets.*
> *Earth is part of the universe.*

Complexity

In terms of our definition, only P16 showed any capacity to identify related parts and deal with composite concepts:

He, like some of his counterparts in the other schools, divided space into two components, namely, the space immediately surrounding earth and outer space:

> *There is space if you look outside a window because you don't see it. But on the other hand I would say that there's no space when you look outside because space is where there's no air. So when you look outside the window, you see that the space that is outside is occupied by air, so basically there is no space outside the window. But if you look with a naked eye you can see there's space but if you can use science equipment you can see there's no space. The space that I think of space is there above the ozone where there's no gravity, where you float. The place where you float, where you cannot walk properly, the real walk, I think that's the space. I don't say it's wrong what I said before that outside the window there is space, but on another point I would say it's a little bit wrong , because there's no space when you look outside.*

Critical engagement

Except for P16, there was no critical engagement evident in any of the conversations. For example, in the extract below he implied that he recognised that his opinion was one of many opinions. This recognition suggests that he was reflective and discerning:

> *I think space cannot know where it starts, but I just imagine it starts maybe from the sun, close to the sun, towards the planets, all the end of the planets. That's why I think it starts close to the sun. It occupies the whole place where the planets are. So I think it starts from the sun.*
> *I think space is something which is formed in a circle, which has no end. We cannot see the end of it – it's like a circle - you turn around and around, all around, but you cannot find it's starting point or its ending point.*
> *It is my imagination*

Imagination

As was the case with the other four meta-themes, not much use was made of mental images, analogies and metaphors. There was very little evidence of conjecturing and theorising.

P16 made some interesting (yet obscure) references to a lift when he talked about the being able to touch space:

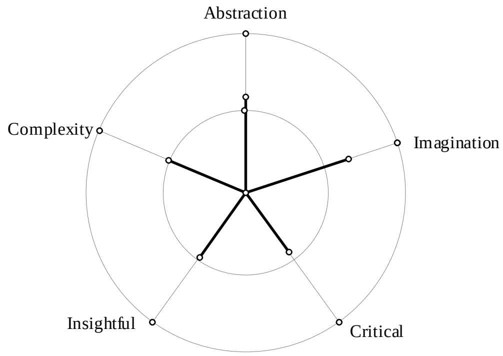
> **Can you touch it?**
> *Touch space – no you cannot touch space. Just like in outer space it's like a long lift, because when you're on a lift from one floor to another, there's something coming up at you and some people get headaches and I think that's the thing that's happening – when those guys – astronauts – when they enter the space maybe they have something like that – maybe they're not feeling well, maybe have those headaches …*

Also, in his analysis of picture 5 he makes use of images and analogies:

> *Number 5: Well I think that number 5 supports me in the thing that I've said that when you I think picture number 5 is a picture about space this big picture because they are floating – these balloons. They do not float like this on earth, so this proves that there is a lack of gravity here. And you can see the people there …*
> **What about the people there?**
> *Well I think these people – basically what he's trying to say in this picture, is that change of feeling that we have …*
> **That change of feeling …**
> *Their faces have been cut – it's just to prove that there is the change of feeling in the outer space.*
> **OK, what change of feeling?**
> *What I've said, when you're in outer space you can feel it's maybe something as you enter the space – those headaches and those …*

Meta-profiles of S4

Refer to Figure 6.6 for the individual meta-stars for S4.



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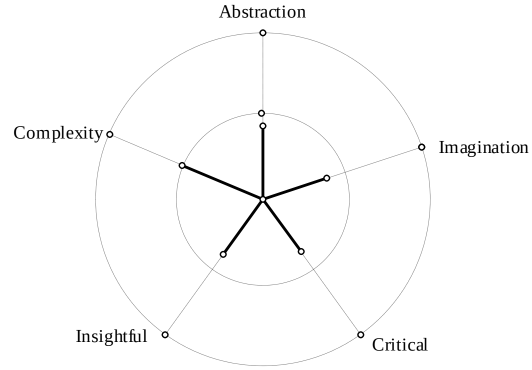

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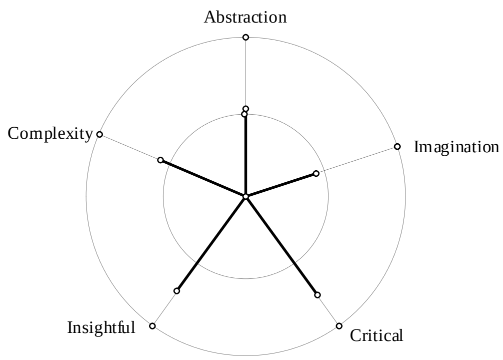

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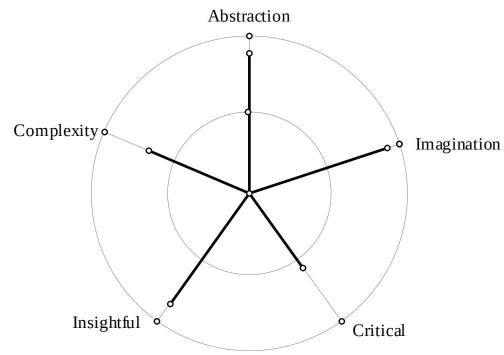

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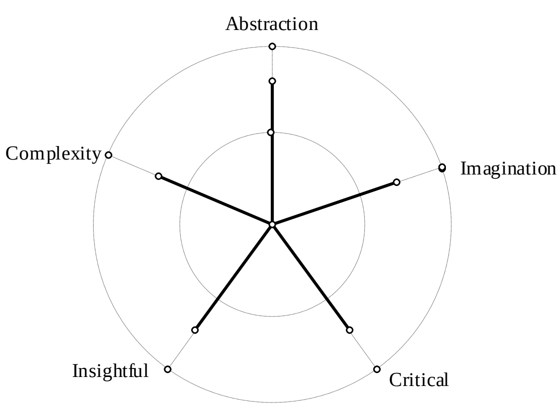

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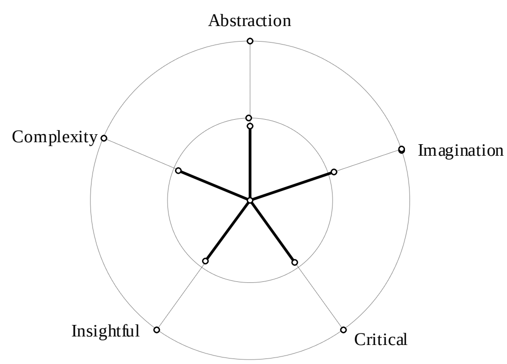

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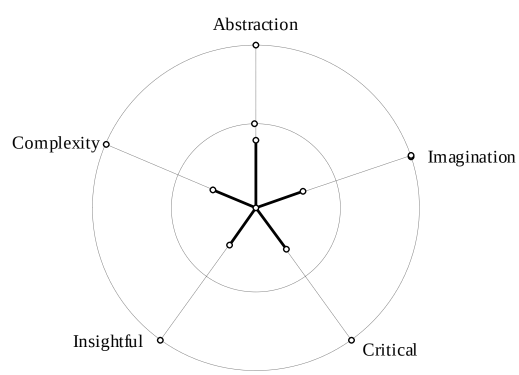


Figure 6.6 Meta-stars of S4

P26

The meta-stars of the participants of this school were diverse and varied. My conversations with P23 and P24 in particular were very interesting. They both made use of fascinating images and had intriguing insights into issues relating to their perceptions of space.

Abstraction

Of all the participants in the entire sample, P23 were able to theorise and reinforce his views with ideas that went beyond the concrete. His views on the role played by God, for example were quite different from that of the others. His theory makes for interesting reading:

> *Um ... I think it was made by God, 'cause if He was to make people to live, He would need people to live in a sort of place – area, so He would have to make space. 'Cause if there was no space, we would all be cramped up together. Won't be able to move anywhere... one mass, and you wouldn't be able to move. Like if you want to move to another place, it means you would have to take a whole lot of people together with you.*
> *So you wouldn't feel so comfortable doing that. So to give a better chance to us, He said let me give a bit of space to each one of you. And so that's how I think the earth was made around that.*
> **And how does evolution fit into this?**
> *Evolution like as in you will make humankind and stuff?*
> **Well I don't know – is space not part of evolution, do you think?**
> *Well space has been around for some time, and God – I believe that God is living in space, like in His own type of space. So I think evolution here on earth – He could decide whether or not to put space in it. So I think it does in a way have to do with evolution.*
> *There was a vacuum – I think it was just a vacuum – nothing there. Before he could make human life He first had to do evolution, and then decide whether or not to put in space for us to move around, 'cause H could decide I don't want these people to move around this way, that way, I want them together, close, packed up tight. So He might have not put in space, but I think He could have decided, "Let me bring in the free will of movement", so He put in space for that.*
> *I think He needed to do the evolution first, then after that decide whether or not to put in space.*
> **Interesting. Now – vacuum and space – is that the same thing?**
> *Vacuum is like you're just plummeting. Or you can just float. In vacuum, you're in one spot constantly, you cannot move anywhere 'cause there are forces pushing and pulling in all directions. They're pushing in one direction, and the other one coming from a different direction so you have no right of movement. You're just stuck over there, you can't move your hands or go anywhere else.*
> **Interesting. So you don't think vacuum is a type of space?**
> *No.*

In terms of the magnitude of space, P23 was able to articulate his ideas abstractly:

> **Does it have an end?**

Just like numbers – even though they start from 1 or 2, they can go on to eternity. I believe space continues within us. There's a whole lot of space we haven't explored. We only know of the two types of space, but for all I know there could be a hundred thousand – a billion thousand types of spaces around us. We still need to find them out and follow them, so I believe it's still going on.
You believe it's still going on. All right, tremendous. Do you think space can be something minute, something small?
Well, in a way, you can't. You can't whatever. Small quantities of space you are able to reduce it into a small area, but you cannot bring it, let's say you want to try bringing earth into a small minute space area, the forces within it, within us, would rebel against that and want to be free, would want to expand to the space which they are, or were originally in. So it would be a kind of a difficult thing unless you have more of a – a much more powerful anti-space which would be able to hold down that space you want and keep it there.
OK. You are happy that space is something that's infinite then, or not?
I'm happy with infinity.
But only in one direction?
No. Like I said, that small particle there – it would have more been circular ...
So it's infinite outwards.
Yes.
Not inwards?
No, not inwards
Does space rest on anything do you think?
No, I don't think it does rest on anything. Um ... I can't exactly say how it is, because we don't know how it is. We still have much to learn about space in a way. We still don't know much about it, so we can't decide whether or not it's lying on something or not. Or is it just drifting or floating over some sort of source. 'Cause there might be some sort of source that is making it float like that. I also wanted to say like the space outside this space – is it the one that you're talking about – like that space if there was something on the ground like holding it, if there wasn't ground it would fall back down ...

When we considered the abstract notion of multiple dimensions, P23 had some definite ideas:

Is there anything like 4-D or 5-D or 6-D? Are there other dimensions?
Um ... if there were 5-D, 4-D, I wouldn't know exactly what they were. More dimensions – I think there would be more dimensions. We still need to see whatever it looks, because so far we're only able to find 2-D where you only see two sides, and 3-D where you see the whole 180 degrees, next 360. Now 4-D would be something much more unexplainable – like someone would have to explain ...
But do you think that in space there are other dimensions?
Other dimensions, yes. 'Cause as I was saying about different types of space, then there will be different types of dimensions.

When he spoke about his internal space he used images beyond just the organic and the concrete:

You have space in your mind obviously, because as you grow up, your mind expands, yes. So obviously there is some space in there. So the space accumulates as you grow. As you grow the space within you widens to accommodate the more records of what you perceive around you, to be placed in there. So there is space in there ...

P24 was also very eloquent in his descriptions of space using abstract ideas and images:

OK … How I explain it is that space can never be equal to something, from what I know so far. You can never say space equals like sitting down and able to move your hand around and everything, like space is all over. It can be different shapes, different form, different types and different things can be in the space – different gases, different liquids, different everything. So that's why it's mysterious – no one actually knows you know, the format of space.

He conjectured that there was a literal space and a figurative space, the former being the space around you, and the latter pertaining to your conscience:

Ja. You know, now I found out that space can be an English literal and figurative meaning, you know. The two different meaning of space. As in space, the literal one is the space all around me, filling up the air, and then there might be the space that sort of like is inside or the space that's a block, you know, something that occupies that space or in the space that conscience and all that – that occupies something as well, so that has to be somewhere in …
Tell me a little bit about the space inside of you.
It's a space where thoughts, everything is generated, because there has to be something happening inside to take it out – everything – dreams … thinking has to have space.
Wonderful. Absolutely.
Thinking … can't just happen
Where does all your knowledge go for example?
Exactly. It is stored. It gets into the brain, but there has to be some sort of space that can't be actually seen or it's inside of you.

Interestingly his choice of pictures were all very abstract in nature and his interpretations of them were fascinating:

Picture 6, 5, and 10. Right, talk to me about picture 6 first of all.
It's sort of like relevant to what I think of space, 'cause the different sort of like types, shapes and sizes around, and it looks like it's just bothering around the air. Sort of like – there, 'cause I see space as a big area that different things can fit into. And sort of like all these shapes and sizes are in the same sort of space and the two things in there sort of like confined by something in the space and ja, it's like different spaces and you talk about co-ordinates and everything, so you want to find that top over there in space in a particular place, you can use that. That's what I think of space – it's not ordinary, it's different things, different places …
Great. And that creature in the middle there?
OK … as in how … must I explain?
What does that creature in the middle mean to you?
Me. It's sort of like confined in – it's sort of like locked into something so … I've got two ways I think it might be … in two ways it's confined in a sort of like literal area into its space and cannot go out, or it's sort of like inside its mentality or inside its conscience that's confined – it's spatial thinking and being able to make decisions …
Beautiful. Well explained. Picture number 5.
Picture number 5 is the space in us. When I saw it, that's the first thing I thought of. Um … space that enables us to think, make decisions, do right or wrong, everything. Sort of like now, art is made in such a way that it makes us see that there is a space within us, but in human beings you don't actually think like that – I didn't think like that first. But now that I see that there is this space and he's turned it into a literal sort of thing, but there is a space that's in us for thinking, making decisions and all that, so that's what attracted me to this picture.
Picture number 10?
All shapes and sizes go to infinity. Big to small … graded down. All shapes, everything, spaces, different shapes, different sizes, different types …. Just cannot find – us as human

> *beings don't know where it ends, so this artist goes to a place where you cannot see properly, just going down, going down and then sort of like multiplicates, gets bigger, magnifies and goes down. So this is also the way that I see space 'cause it's all around and it's sort of like to infinity as I said. Um you can't exactly get the end or the beginning or … you can get the middle though. It's just there, but no one knows where the end is.*
> **Beautiful. And if you go in further and further you would go in forever…**
> *Forever.*
> **Right, [P24], I really enjoyed listening to you, I really did …**
> *… When I wrote that, OK space is out there, and as we were talking, the questions, sort of like, "Wait a minute" – there might be space here. Space might be different shapes, space might be there and the co-ordinates as you said. Then I thought Ja, you can use maths to find space and then space can be found, you know so it opened a lot of different ways of seeing space for me … interesting …*

Insight

Some of the boys, particularly P22, P23 and P24 had some enlightened ideas and showed remarkable insight:

P22 often referred to what he learnt:

> *Ja, all the planets revolve around the sun, everything's in a system. If this planet over there – people say it was out by like three millimetres it's unordered. It's actually getting like that, every year it's a little bit more out, I s'pose. And one day, in millions of years, we're going to spin out of our … pattern.*

He liked to contextualise his perceptions in scientific knowledge:

> **Do you see space as something quite mysterious, or is it based on facts?**
> *What we know is based on facts, but I mean, out of our solar system or whatever, I think there's millions more out there, we just can't see that far. Like all those planets they know, that's like in our … like solar system, and there's another one there and another one there. That's all we know.*

P23 shared his insights on the big bang theory:

> *Seeing that most of the people are talking about the 'big bang', might as well go to that side. I would think we're not that far from the small particle from which earth and the rest of the other galaxies were from. Like the big bang in a way moved in, like exploded next in that position, so we are moving away from quite far, we're moving slowly …*
> *Away from the source from where space originated from. 'Cause that's where it is. Originally came from that small atom or particle. The formation of space did evolve from there. So like … I think I've said … ja.*
> **So do you believe in the 'big bang' theory?**
> *I must admit that sometimes I do believe in it, and sometimes I have some – I like argue with what they say. So it's half way through.*

He also had insight into how the atmosphere works:

So what does it do, this space here? What is it actually there for – what does it actually facilitate?
Well it facilitates us people to live in it. Because like, in this space there is air – you can breathe in it; there is like – sunlight is able to come in. The sunlight is more reduced in the way that it doesn't harm you, yeah. And like, there is shade, like the trees and the buildings and all that stuff. But in the other space the sun is 100% - will scorch and burn you or whatever. And there is like no shade unless you put something in front of you and that's you shade. But even that won't do, so in time it will disintegrate in a way. So – that's what I think.

He was one of the few in the entire sample who were able to articulate to some extent the difference between two dimensions and three dimensions:

You've heard of the concept of two-dimensional space. What is it? How do you understand two-dimensions?
I see 2-D as like the forward and the back – that's all you can see. Not the sides, 'cause on the sides is just nothing. You can see the forward like, you look at the back, OK, you just see blackness on its sides, then the back, a house, that's all. So that's what I perceive of 2-D.
And 3-D?
3-D is like a whole sort of like portion – you can see everything around you. Up, down, sides – you're able to see everything which is around you, yeah.

Complexity

Like many of his counterparts in the sample, P23 divided his space into two types, albeit into two entirely different types:

I think I find space mysterious, because you might never know – we already know we have two different types of space. The space where you have the right of movement and the space where you need to fight for the right of movement. Then you can find another type of space, you never know, where those two types are combined, where you have the right of movement on one side and the next moment you don't have the right of movement. Or there could be another sort – type of movement – I believe it's more a mystery, yeah.
Let's concentrate on those two spaces – the one where you have the right of movement, and the other one where you don't have the right of movement. Is there a clear distinction between the two? Is there a boundary?
Yes, there is a boundary. Like you can sense in a way that you have right of movement, 'cause you are moving around. Then as you try and move into the other type of space, it comes, OK if you were in a capsule of some sort and you still had the right of movement in that small capsule, and it suddenly breaks, and when it breaks you are in this other type of space, you have no time whatsoever, it just comes on at you quickly, you won't be able to move. You need some sort of whatever – propulsion. You don't have it, so now that space now has control of you. It can do whatever it wants to you, it can make you get the way or make you come back.

P24 saw space as a complex concept that was at the same time remote, but also right next to you:

You said that space is somewhere out there or right next to you. Can you explain that?
OK. You can say that space is sort of out of the way – will never be what your eyes can see. It's out there but it can quite well be right next to you because there's space right here. Or

right next to you and someone sits here and that's sort of like a shape or space that is there, or you can just have a round ball with a hollow with a space there. Then in the same way that the earth is big, there's space there, and in the same way in the galaxy, the space, earth occupies a small space. Then this Milky Way – bigger space you know is sort of like ...
The space around you – unpack that a little bit for me.
OK. When I'm out there I had in mind, when you're packing your stuff in a drawer, OK, you pack it so tightly there's maybe no enough space for air, not enough space to put more stuff in, and just say different shapes and sizes so there's not enough space for them all to come together, so now when you're just sitting there the space around you it's sort of like freedom in a way. You know when you're walking down the street it's like when you can wave your hands, there's nothing to sort of like confine you I guess ...

Critical engagement

P22 was critical, even cynical at times. He would question concepts and events that others would take for granted. I was taken by surprise for example when he questioned the landing on the moon in 1969:

I'm not too convinced that they landed on the moon anyway.
Are you not?
Not really.
Why not?
I don't know – I think they faked it. I mean they could have landed on it by now, but not when they did.
Really?
I think so.
That's interesting. Do you think it's a conspiracy?
When it did happen I think so.
Why?
I don't know. I've just got this feeling ... I can't base it on anything 'cause I don't have any knowledge about that.

In another instance he was quite cynical about where the world was heading:

All the planets revolve around the sun, everything's in a system. If this planet over there – people say it was out by like three millimetres it's unordered. It's actually getting like that, every year it's a little bit more out, I s'pose. And one day, in millions of years, we're going to spin out of our ... pattern.
Out of our order?
That's what they say, but the world will probably end before that, Sir.
Do you think so?
It's going to end soon, I think.
Soon. Why do you say that?
Well it says in the Bible it's going to be near this time sometime. I reckon. While I'm living, while we're living.
Do you think so?
Mmm. I think so.
What makes you think that?
I don't know. There's so much – this earth is going to die before that time – there's so much rubbish happening ...
Do you think more rubbish happens now than before?
Ja. I'm saying like litter, ozone layer and that.

Tell me about the ozone layer –
Well it's got a big hole in it, from all the pollution and stuff.

Imagination

Many of the boys in this school made use of rich and fascinating imagery to reinforce their perceptions and visions. They made use of complex mental images to come up with interesting (and sometimes obscure) conjectures about aspects of space.

P23 in particular had a very strong sense of imagination. When exploring the notion of orderliness he used images of a rugby field:

…Right. Tell me, [P23], is space something orderly or is it something chaotic?
Well it depends on how you use space. Whether you use it to reveal order, you know, or in a chaotic fashion. Let's take for example a playing field – Somerset Field. We can decide to put a whole bunch of guardings or a park over there – it's orderly. Or someone might come along and say, "Let's have a small match of rugby" – "That's chaotic in a way. Or decide to go for something even more dangerous, more hectic, like bringing a war into that small space. Now that's even more chaotic. So it depends on how a person sees space. He can either use it to help him or not to help him. So it depends, it's two different things. It has to be the way you think about it.
And that space out there – is that orderly or is that chaotic?
I think it's more chaotic. You have to first fight against it in order to be able to control the movement, so it's kind of chaotic.

He likened humans to holograms:

I think of myself more or less as a hologram.
As a hologram?
Yes
What's that?
You think you're here and you know exactly you're here, 'cause with this moving of the earth which I think is happening – the earth moving into different positions – if you stayed as a solid particle, as a solid state, you would just be by pushed by all this movement, so I think of myself as a holograms moving together, because a hologram you won't feel like particles as such, and you'll be able to move together with this space in order to live in this space. But in a solid state you won't be able to withstand the space.

He took this image further when he discussed his choice of picture 5 and picture 11:

Picture 5 represents us as holograms. This represents what we'd look like in a way, not whilst we see each other now, but what I think of us as seeing ourselves. You can see part of yourself, but within you there's like space. That's what's in my mind, this is what I think of, ja.

Picture number 11 shows a group of people in a certain building, and with then a sort of human being outside as the hologram. I didn't exactly perceive this as the sort of hologram which you see floating around. I believe it is a human, yes, but within this space, movement in space you are a hologram, living as a hologram in space. So that's why … I don't go exactly with this one here …

Which one?
This picture over here where there's like ...The building is there but it's off the hologram. You could say the building's not a hologram.
Oh I see. OK. But we are like holograms?
Ja. We are holograms.
And that figure there with those outstretched arms?
Ja.
That represents the hologram for you.
Ja, it's an actual hologram living in the space. You are not exactly holograms in the space, like living in it, like moving around. Holograms are sort of like invisible – you have that sort of like the building in a way, but then as you move you do turn into a hologram.

When we spoke about his internal space he once again used the hologram idea:

Can you see space?
Space. You cannot exactly see space. But you know that you are in space. You know you are in space but you cannot see it.
Can you touch it?
You cannot touch it because it's like swimming in water. You can go through it and you know you're in some substance. So the same is in space – you know you are there but you cannot feel it, yeah.
Do you think that there's space inside you? Do you have space inside you?
No. It's like when I said I was a hologram you wouldn't see any space. But if, OK, if you were to be in another type of space where you were in this position constantly, for a prominent reason, then you would know you had some space in you.
OK. Do you have space in your mind?
You have space in you mind obviously, because as you grow up, your mind expands, yes. So obviously there is some space in there. So the space accumulates as you grow. As you grow the space within you widens to accommodate the more records of what you perceive around you, to be placed in there. So there is space in there ...

P24 was also quite graphic in his explanations:

… Uh-huh. What do you think is the function of space? Why is it there? What does it do?
Provide life.
Provide life.?
In a way.
In what way?
Give – this is to the space on earth, the whole, sort of like. Without having space, if everything was in a sort of like house, sort of thing, with no space, confined, we wouldn't have the water cycle because that needs to go round and everything, wouldn't have rain obviously, wouldn't be able to breathe – no gases, no anything. It'd be sort of like too ... it's like now the earth is becoming too crowed, so now this thing, issue that space might be running out and all the pollution also there and everything... So in the general format, space is ...
OK. Wonderful. Provides life … good one. You've explained what you perceive of space – I quite like that – but you differentiated in your original definition in that questionnaire – you said that space is somewhere out there or right next to you. Can you explain that difference?
OK. You can say that space is sort of out of the way – will never be what your eyes can see. It's out there but it can quite well be right next to you because there's space right here. Or right next to you and someone sits here and that's sort of like a shape or space that is there, or you can just have a round ball with a hollow with a space there. Then in the same way that the earth is big, there's space there, and in the same way in the galaxy, the space, earth occupies a small space. Then this Milky Way – bigger space you know is sort of like ...
OK. Interesting. When you said it's like being alone and having no one to confine you – the space around you – unpack that a little bit for me –

> *OK. When I'm out there I had in mind, when you're packing your stuff in a drawer, OK, you pack it so tightly there's maybe no enough space for air, not enough space to put more stuff in, and just say different shapes and sizes so there's not enough space for them all to come together, so now when you're just sitting there the space around you it's sort of like freedom in a way. You know when you're walking down the street it's like when you can wave your hands, there's nothing to sort of like confine you I guess …*

In contrast, P26 for example, responded blandly and superficially:

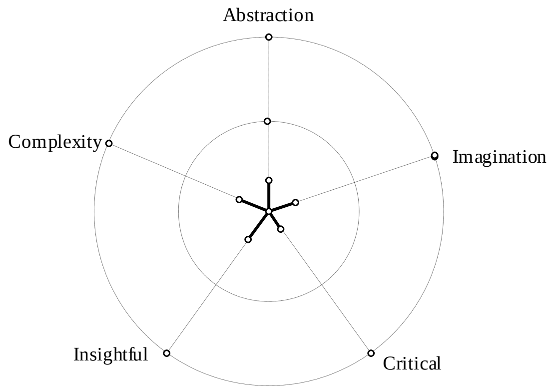
> **So, [you said] God created the earth?**
> *Uh … yes … ja.*
> **You're happy with that? Was there space when that earth was created.**
> *Shoo … ja.*
> **So space was always there.**
> *Ja.*
> **Do you think that space is the same everywhere?**
> *Yes.*
> **Why do you say that?**
> *There is only … are you saying like space here and space in other places?*
> **Ja**
> *Ja, it has to be the same.*
> **Good. Do you think that space is something chaotic or is it something quite ordered?**
> *Chaotic*
> **Why do you say that?**
> *No one can order space at all.*
> **Why do you say that?**
> *Shoo … um … why do I say that … OK I don't know.*

Meta-profiles of S5

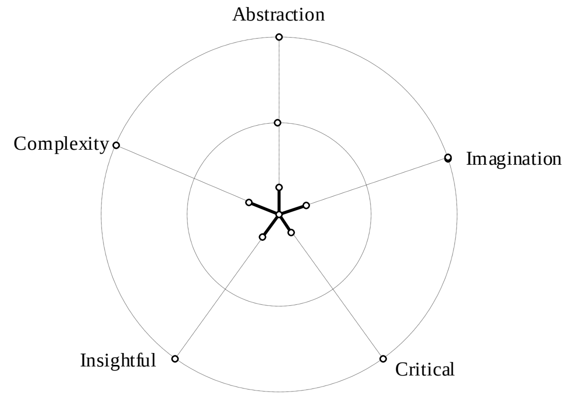
See Figure 6.7 for the individual meta-stars for S5.

The meta-stars of S5 are similar to those of S3, generally suggesting a lack of abstraction, critical engagement and complexity. The reasons for this could be attributed to similar factors articulated on page 249.

Three participants, however, stood out above the rest: P29, P30 and P31. They articulated their thoughts with certain insights and imagination.



P27



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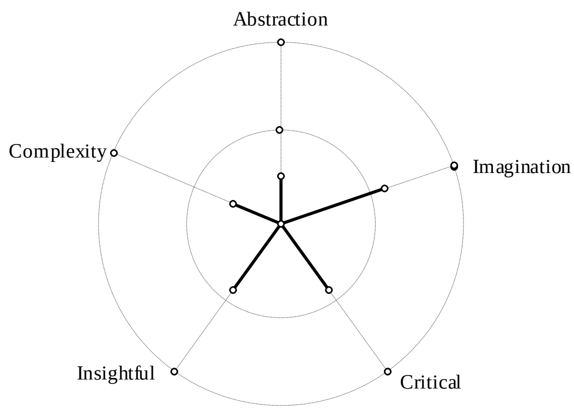
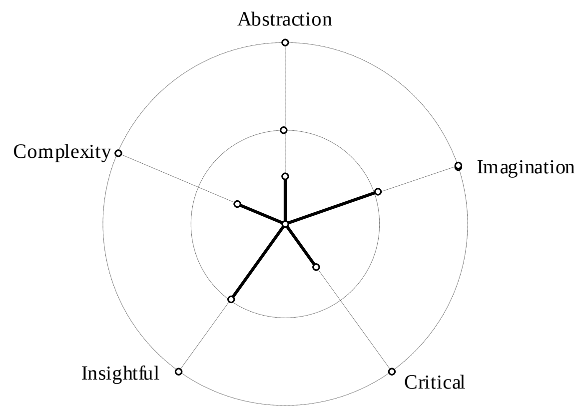
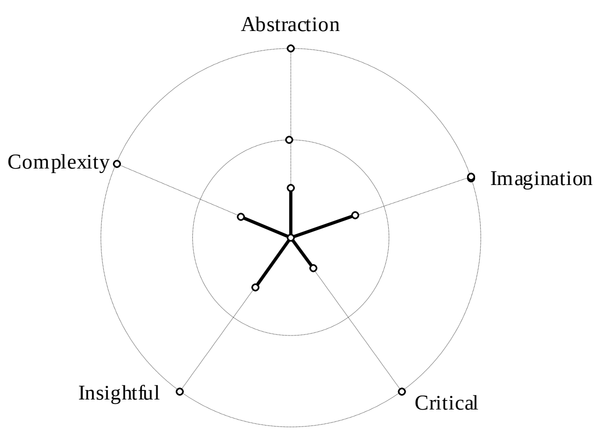


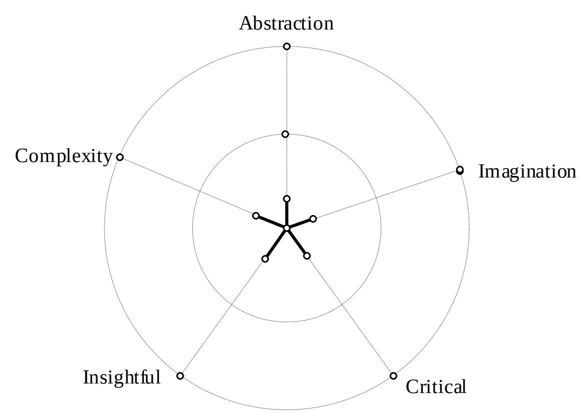
P29



P30



P31



P32

Figure 6.7 Meta-stars of S5

Abstraction

Very little abstract thought was evident in the conversations. The transcripts suggest that the participants did not articulate their ideas beyond the concrete. The capacity to denote a quality, condition or intangible description to their perceptions was absent.

Even in the case where participants selected a picture from the Escher collection which was abstract in nature, they used it superficially. Below an extract from the conversation with P28:

> **Why did you select picture 6?**
> *… I see the space between these things.*
> **What are these things?**
> *These are shapes.*
> **What do they represent?**
> *They represent space.*
> **And picture 5?**
> *… they have space between … he nose and the eyes …*
> **What does it represent?**
> *… it tells me about my body…*
> **What about your body?**
> *In my body there is space.*

And one with P27 who selected picture 7:

> **Tell me about picture 7. Why did you take picture 7?**
> *Because in number 7 there is space In this thing which is not closed.*
> **What is this [pointing to the building]… what does it represent to you?**
> *… these are the stars her e…hm… is not the church this one?*

Insight

P30 was the most articulate and insightful:

> **Tell me a little bit more about your idea of space.**
> **…***the space contains all the planets and everywhere in this place there is also space. All the places that we are in there is also space. Here there is space. If we haven't got space we will never survive.*
> **Why not?**
> *Because the earth that we stay in would never exist.*
> **What is the function of space – what does it do?**
> *…it contains all the planets and everything is in space, the earth and the other planets were in the space. So that is the function of space.*

Also P29 had a few insights which she shared:

> **You said there is space around us. Describe that space to me.**

> *When I am sitting here there is space between me and you and there is space between you and Thembi. So in this space I can say there is air moving around us … we are also breathing because there is a space between us. If there is not a space we will never breathe.*
> **Why not?**
> *Because if … maybe someone something like a…around your nose and mouth and tied it up you can't breathe because there is no space. And I can also say there is space in our head because in our right side … our … our brain has got two types…two sides. One side is the right side, the other is the left side, we use them … I'm not sure if you use the right side in order to draw … and the left side to think carefully. There is a space between those sides so that air can move around our brain so that you can think and the oxygen is also moving inside of our bodies.*

She had this to say about the planetary space:

> *Yes, there is space between the stars and the sky. I think if … maybe if those stars were pressing against each other I don't think … maybe there will be dim light, when they are dense and there is no space … like clouds in the sky. If there is a space like there is a space between the clouds and the sky, the clouds are white and the sky is blue. When they meet … when all the clouds meet, it becomes dark because there is no space for the sky to light so it becomes dark …*

She also referred to her own personal space:

> *I don't like people in my space.*
> **You don't like that?**
> *No, because they have their own spaces and they have to use their own space and I have mine too.*
> *Even when I am at home I can't … I can't read or learn or write my homework when my room is dirty and when there is no space in my room.*
> *In my room there is space between my bed and my sister's bed …*

It was not surprising that she selected picture 5:

> *Here is the space between this…and the space between…so that this person can breathe.*
>
> **So that this person can breathe?**
>
> *Yes and there is also space between these three things here…and between this man there is a space so that they can both breathe and also in their heads.*
> **What do you think those people represent – who are they?**
> *… maybe a … as I have said … there is a space even here in my brain there is space here in my brain and here in my head. Maybe here in my body there is a space … something like that. Because there is space in the body of a person so all these things they told me about the space that is in the person.*

Complexity

In terms of our definition suggesting that complexity implies the capacity to identify related parts and deal with composites, the participants of this school tended to confine their perceptions to a simplistic and rudimentary level.

Critical engagement

P29 was the only participant who showed any signs of discernment or critical thought. She would often start her deliberations with 'I think' or 'maybe', suggesting that she was open to other views and open for her opinion to be challenged. She did not blindly launch into a dogmatic and subjective account of her convictions.

Also P30 at times showed signs of looking at things critically. For example, while believing in God, he was critical and showed openness to other interpretations:

> **How did space come about?**
> *... according to the Bible's beliefs .... maybe it's designed by God. But we don't know – maybe it's designed by God.*

Imagination

P29 and P30 argued their case with imagination. They illustrated their ideas with interesting mental images.

In P29's account of her personal space, she made use of images to reinforce her insights. P30's use of imagination when he described his internal space is interesting:

> *Yes, it is possible that you can have space inside you. You must have space because there is nothing that can stay without a space. The brain sometimes sleeps – it's the way you give it a peace of mind and have a relax. It's when the brain relaxes which makes you to dream because now immediately when you wake up the dream is finished. The brain cannot work when you are dreaming. It is not that it is gone, it is still there, but it is not working.*

He used his imagination to describe the significance of picture 11:

> *Right here there are people, these people bend their heads forward ... however maybe they are praying these people. But here is the big person ... there is a big person there with open hands. Maybe that person is God. Maybe he is in space because here is the earth and these people are here down on earth. This big person is there at the top ...*

6.7 SYNTHESIS OF FINDINGS

For the purpose of synthesizing the data and identifying trends, each criterion of the meta-stars was subdivided into three levels: low, medium and high. The meta-star for P16 (see Figure 6.5 on page 250) for example, was classified as follows:

Abstraction:	medium
Imagination:	medium
Critical engagement:	medium
Insightful:	high
Complexity:	medium

It is recognised that for many cases the division between the levels was diffuse and not well defined. The following position of the individual arms of the star served as a rough guideline:

Close to the centre of the star:	low level
In the vicinity of the middle circle:	medium level
Close to the perimeter of the outer circle:	high level

The above classification is also used in the next section of this chapter when the meta-stars are analysed within the school context in an attempt to tease out relationships between the meta-criteria and performance in school mathematics.

Abstraction

In terms of the above classification, seven participants (21,8%) articulated their ideas using high levels of abstractions:

- three were from S1 (all females);
- two from S4 (both males);
- two from S2 (both females).

Most of the participants (40,6%) were classified as using medium levels of abstraction while 37,5% of the sample used low levels of abstraction. The latter were mostly from S3 and S5 (see Table 6.8 on page 281 and Table 6.10 on page 282).

Insight

Eleven participants (34,4%) used high levels of insight in their deliberations:

- five were from S2 (four females, one male);

- three were from S4 (all males);
- two were from S1 (all females);
- one was from S3 (male).

25% of the sample were classified as using medium levels of insight whereas 41% showed low levels of insight. The latter was once again dominated by S5 and S3.

Complexity

Seven participants (21,8%) were highly complex in their articulations:

- three were from S1 (all females);
- three were from S2 (all females);
- one was from S4 (male).

Of the sample, 31,3% were classified as medium in the complexity of their conversation whereas the majority (46,8%) used a low level of complexity in their articulation. Once again the latter was characteristic of S5 and S3.

Critical engagement

Only two participants (both males from S4) showed a high level of critical engagement. The rest of the sample did not engage very critically: 43,7% on a medium level and 50% on a low level.

Imagination

Ten participants (31,2%) were very imaginative in their descriptions:

- Four from S1 (all females);
- Four from S2 (three females and one male);
- Two form S4 (all males).

Ten participants were not so imaginative, and 37,5% engaged with a low level of imagination.

6.8 SYNTHESIS ON A SCHOOL LEVEL

S1

Of the five participating schools, S1 engaged at high levels of abstraction, imagination and complexity. Few participants engaged with high insight except for both P4 and P7.

None of the participants involved themselves in a high level of critical engagement.

S2

S2 engaged at high levels of imagination, insight and complexity. Few participants engaged at a high level of abstraction, and none engaged critically.

P9 stands out as highly rated in abstraction, imagination, insight and complexity.

S3

Apart from P16, who was highly rated in insight, all the participants in S3 engaged at a low or medium level for all the criteria.

S4

In general, S4 demonstrated a high level of insight, and two participants (P22 and P24) engaged critically. None of them scored highly in terms of the complexity criterion. The contribution of P24 is noteworthy as he was highly rated in all of the meta-criteria.

S5

Participants of S5 mostly engaged at a low level for all five meta-criteria. P29 and P30 can be singled out as having participated at a higher level than their counterparts.

A graphic overview of the above synthesis appears in Tables 6.6, 6.7, 6.8, 6.9 and 6.10 of section 6.11.1 on page 276 which explores linkages between meta-profiles and mathematics performance.

6.9 GENDER DIFFERENCES

Abstraction

As Table 6.1 illustrates, nearly one quarter (23,5%) of the female sample engaged at a high abstract level, whereas only 18,2% of the male population operated on a similar level. However, more than half the females (52,4%) did not engage abstractly at all, whereas 0,9% of the males used only concrete ideas.

Table 6.1 Percentage score of males and females on abstraction criteria

ABSTRACTION			
	Low	Medium	High
Males (%)	0,9	72,7	18,2
Females(%)	52,4	23,8	23,8

Insight

Table 6.2 shows that more than half of the females (52,4%) had low levels if insight whereas 45,4% of the males displayed high levels of insight.

Table 6.2 Percentage score of males and females on insight criteria

INSIGHT			
	Low	Medium	High
Males (%)	18,2	36,4	45,4
Females(%)	52,4	19	28,6

Complexity

Evidence from Table 6.3 shows that more than one quarter of the females (28,6%) were highly complex in their perceptions of ideas, whereas only 18,2% displayed the same level of capacity. However more than one half of the females (57%) engaged at a very simple level whereas only 18,25 of the males recorded a low level of complexity.

Table 6.3 Percentage score of males and females on complexity criteria

COMPLEXITY			
	Low	Medium	High
Males (%)	18,2	63,6	18,2
Females(%)	57	14,2	28,6

Critical engagement

As mentioned above there was little evidence of critical engagement. Only 18,2 % of the males participated on a high level of critical engagement, whereas no females engaged at that level. Table 6.4 indicates that most of the females (57%) and 36,4% of the males were not critical at all.

Table 6.4 Percentage score of males and females on critical engagement criteria

CRITICAL ENGAGEMENT			
	Low	Medium	High
Males (%)	36,4	45,4	18,2
Females(%)	57	42,8	0

Imagination

Table 6.5 suggests that the bulk of the males (63,6%) recorded a medium level use of imagination and 52,4% of the females a low level of imagination.

Table 6.5 Percentage score of males and females on imagination criteria

IMAGINATION			
	Low	Medium	High
Males (%)	0,9	63,6	27,3
Females(%)	52,4	28,6	19

6.10 CONCLUSION OF META -ANALYSIS

Participants' comments (section 5.3) formed the vehicle for a meta-level analysis. The five meta-criteria which framed the analysis were arrived at by consensual validation. They were:

- capacity to abstract;
- capacity of insight;
- capacity to deal with complex concepts;
- capacity to critically engage;
- capacity to be imaginative.

They formed the underlying framework upon which individual meta-stars were constructed. The individual meta-stars were used to create a meta-profile for each participant. They were clustered in schools for reasons of consistency with the AMST and the HAT analyses. To synthesise the various meta-profiles, the meta-stars were then subdivided into three levels: low, medium and high.

As the meta-stars and the three-level classification demonstrate, the meta-profiles of the sample were diverse and varied. General trends can be identified:

- Both S3 and S5 engaged on a low to medium level on all of the meta-criteria;
- In general, the pupils participated on a low level of critical engagement;
- Particularly in S1, S2 and S4 pupils engaged with insight and imagination;
- Females were rated on a high level for their capacity to abstract and be complex;
- Males were rated at a high level for their capacity to be critical, show insight and be imaginative.

The question that now arises is the extent to which these meta-profiles relate to school performance. The next section completes the empirical investigation of this project and focuses particularly on the connection between the meta-profiles of the participants, their performance in the AMST and the HAT, and their performance in mathematics during their Grade 11 year.

6.11 PERFORMANCE IN MATHEMATICS

The final component (Stage 6) of the empirical research (see 3.3.2 on page 59) of this project focused on the participants' performance in mathematics in 2001, the year that coincided with the data collection and my engagement with the pupils. The rationale behind this aspect of the project was:

- to contextualise the research within the classroom situation;
- to explore links between the HAT, the AMST, the world-view profiles, and school performance.

Towards the end of 2001, the mathematics teacher responsible for each of the study's school groups completed a questionnaire (see Appendix 2*)* which sought information on each of the participants' performance in the mathematics classroom during that year. As the assessment strategies of all the five schools consisted only of written summative tests and examinations, the teachers were requested to provide the following specific data:

- marks achieved for all the tests written during the year;
- topics of the tests to facilitate easy classification into 'geometry' and 'algebra';
- marks achieved for the examinations.

Further, the teachers were asked to provide qualitative comments on each participant in terms of their:

- general mathematics performance;
- performance in the geometry class;
- social skills and interaction in the class;
- general world view.

In the analysis, the tests written throughout the year were sorted into an algebra and a geometry category. The marks were then averaged out. A year-mark for algebra and a year-mark for geometry were thus produced. The marks for the examinations (which were written in their discrete algebra and geometry categories) were likewise averaged out.

To facilitate analysis, a table (see Tables 6.6, 6.7, 6.8, 6.9 and 6.10) for each school was then produced in such a way that for each participant the following information was displayed:

- the synthesised meta-stars in terms of the three-level classification (low l, medium m, high h)as discussed in section 6.7 on page 267
- rank performance in the HAT for the spatial visualisation and orientation constructs;
- rank (R) and score performance (S) expressed as a percentage in the AMST for the spatial visualisation and orientation constructs;
- the average score performance expressed as a percentage in the AMST for the participants in the school;
- school performance (expressed as a percentage) in terms of year and examination mark achieved for algebra and geometry.

First, each school was analysed individually. Those participants whose profile showed interesting trends, consistencies and inconsistencies were isolated and briefly discussed. Comments made by their mathematics teacher were integrated into the discussion.

The analysis then specifically focused on spatial capacity, as discussed in chapter two on page 19, and mathematics performance under the following headings:

- relationships between spatial visualisation and orientation factors, and mathematical performance;
- gender differences and equity in spatial capacity, and mathematical performance;
- relationship between spatial ability and mathematical ability.

A summary of the findings was then compiled.

6.11.1 World view, AMST, HAT, mathematics performance by schools

Analysis of S1

Table 6.6 displays a global overview of the performance of S1.

Table 6.6 Meta-profile, HAT, AMST and school performance for S1.

NAME	WORLD-VIEW INTERVIEWS (META-LEVEL)															HAT		AMST				SCHOOL PERFORMANCE			
	Abstraction			Imagination			Critical			Insightful			Complexity			Spatial orientation	Spatial visualisation	Spatial orientation		Spatial visualisation		Algebra		Geometry	
	l	m	h	l	m	h	l	m	h	l	m	h	l	m	h	Rank	Rank	R	S 54,7	R	S 46,2	year	exam	year	exam
P1		*			*			*			*			*		1	5	13	60	11	54,2	37,5	32	60	53
P2		*				*		*			*			*		12	10	8	70	22	33,3	59,6	73	67	63
P3	*			*			*			*			*			12	26	17	50	26	20,8	40,16	56	51	51
P4			*			*		*				*			*	7	4	6	80	1	83,3	57,8	63	66,6	63
P5	*			*			*			*			*			28	24	13	60	26	20,8	11,8	14	42	36
P6			*			*		*			*				*	12	17	25	30	11	54,2	57,3	48	40,3	58
P7			*			*		*				*			*	18	9	13	60	15	50	50,3	55	19,4	36

P4 and P7 were singled out previously in section 6.6 on page 234 as having high-level thinking skills in terms of being able to abstract, be imaginative and, critical, and having insight and the capacity to think in complex terms. P4's mathematics performance was also relatively high (63% in the algebra examinations and 63% in the geometry examinations).

In response to her general mathematics performance her teacher had this to say:

> *Likes Maths – algebra better. Trig weak. Geom OK. She works hard. Exponents weak. Geometry is not strong – new problems are difficult for her. Did not get exam theorem. She enjoys Maths and asks a lot of questions. She takes time to understand some questions. Always does better than I expect in tests and exams. At times leads the class – usually in algebra. She is neat – has problems with applications and fine detail. Interpretations of graphs weak.* [P4; TQ]

In her performance in the HAT and the AMST she also performed relatively well, achieving high ranks in both.

The same cannot be said of P7, however, who ranked highly in the meta-levels but achieved poorly in the geometry examination (36%). Her teacher's response:

> *My perception:*
> *Weak – especially trig and geom.. Does not get concepts easily. Works slowly. Tries, but sometimes ineffectively. Gets basics, but application and harder examples are impossible. Graphs 3/40 – very weak. Many errors*
> *Pupils' perception:*
> *Same as other subjects. Enjoys maths – but not geometry.*
>
> *Proofs wrong – almost no marks in Trig and Geometry exam for Geometry. Non starter. Extremely weak. Application impossible. Can't identify concepts on diagram.* [P7; TQ]

Her rankings in the HAT and the AMST were also relatively low.

P2's profile is interesting in that she appeared to be a good performer in mathematics compared to her peers, but apart from her imaginative capacity she was positioned on the medium level for the other meta-criteria. Her teacher's response to her mathematics capacity:

> *My perception:*
> *Works neatly - good logic flow. Graphs also good. Trig not as strong as in June. Good understanding of concepts. Perceptive, natural interest and ability. Asks few questions – gets concepts fairly easily. Excellent concentration – observer.*
> *Pupils perception:*
> *Highest subject mark. Prefers algebra. Trig OK Geom weaker. Enjoys maths more now than in the past.*
>
> *Geometry weaker than algebra. Good understanding of basics – good notation. Finds it difficult to express herself in geometric terms at times. Application a bit of a problem – but still.* [P2;TQ]

Her ranking in the HAT and the AMST appeared generally to be in the top third.

P5 is another interesting candidate in that her teacher singled her out as an exceptionally weak student of mathematics:

> *My perception:*
> *Extremely weak – insists on taking maths against our recommendations – parental pressure. Stronger at the end of the year, but will need close to a miracle in order to pass.*
> *Pupils' perception:*
> *Enjoys maths – especially geometry. Battles with concepts – mental block. Indicates that she enjoys mechanical things. Writes a fair amount, gets no marks or too many errors. Basic weak. Most of marks in algebra were for graphs. Roots/exponents no marks.*
> *History best subject.*
>
> *Theorems incorrect in exams. Only got grade 9 concepts. Trig accounted for most of marks in paper 2. Asks questions doesn't help. Perceptual problems. Application impossible.*
>
> *Pupil: Always weaker than I expected. I feel I understand.* [P5; TQ]

This is manifested in her very poor examination results and generally her performance in the HAT and the AMST seemed to be consistently weak.

Regarding correlations between positions on the meta-criteria and mathematics performance, there appear to be strong links in some cases such as P4 and P5, but moderately weak links for others such as P7.

The same can be said for the correlation between the HAT and school performance, and the AMST and school performance. The study would need to be expanded to a much larger sample to formulate any general inferences.

Analysis of S2

Table 6.7 displays a global overview of the performance of S2.

Table 6.7 Meta-profile, HAT, AMST and school performance for S2.

NAME	WORLD-VIEW INTERVIEWS (META-LEVEL)															HAT		AMST				SCHOOL PERFORMANCE			
	Abstraction			Imagination			Critical			Insightful			Complexity			Spatial orientation	Spatial visualisation	Spatial orientation		Spatial visualisation		Algebra		Geometry	
	l	m	h	l	m	h	l	m	h	l	m	h	l	m	h	Rank	Rank	R	S 54,7	R	S 46,2	year	exam	year	exam
P8		*		*			*			*			*			12	6	8	70	9	58,3	72,4	45	54	41
P9			*			*		*				*			*	20	16	17	50	22	33,3	68	56	76,6	71
P10		*				*		*				*			*	12	1	1	100	5	66,7	56	34	42,5	29
P11		*				*		*				*		*		3	15	6	80	11	54,2	43,5	44	38,3	abs
P12			*		*		*					*		*		7	7	13	60	26	20,8	73	68	68,6	50
P13		*				*		*				*			*	7	2	4	90	2	75	79,4	66	85,3	64

Both P9 and P13 were singled out in section 6.6 on page 234 for their high levels of thinking skills in terms of the meta-criteria. It is interesting to note that their corresponding performance in the mathematics examinations can also be regarded as high. Their teacher commented as follows about P9:

> *Good pupil*
> *Confident and determined*
>
> *Enjoys geometry*
> *Participates freely*
> *Enjoys a challenge*

… and about P13:

> *Generally performs very well.*
> *Although capable of excellent results in geometry, lacks confidence and needs continuous guidance.*
>
> *Needs constant reassurance that she is on the right track.*
>
> *Does not participate in class willingly.*

The above consistency is, however, not apparent if we compare their meta-levels with their relative performance in the HAT and the AMST. P9, for example, ranked in the bottom third for both the HAT and the AMST, whereas P13 was ranked very highly for those tests.

P10 is also an interesting case in that her performance in mathematics also does not appear to correlate very well with her performance in the HAT and the AMST or her position with regards to the meta levels. Her teacher had this to say about her mathematics:

> *Lacks insight and would probably do really well on the Standard Grade*
>
> *Gives up before she even begins.*
> *No participation*
> *Daydreamer.*

P8's profile perhaps displayed the most inconsistencies. In terms of her meta-skills she was positioned low for four of the criteria, yet her performance in the HAT and the AMST was in the top third and her mathematics performance suggests a fair mark. Her teacher confirms:

> *Could do better, but lacks confidence*
>
> *Average performance*
>
> *Lacks confidence*
> *Enjoys geometry, but struggles to express herself*

Analysis of S3

Table 6.8 displays a global overview of the performance of S3.

Table 6.8 Meta-profile, HAT, AMST and school performance for S3

NAME	WORLD-VIEW INTERVIEWS (META-LEVEL)															HAT		AMST				SCHOOL PERFORMANCE			
	Abstraction			Imagination			Critical			Insightful			Complexity			Spatial orientation	Spatial visualisation	Spatial orientation		Spatial visualisation		Algebra		Geometry	
	l	m	h	l	m	h	l	m	h	l	m	h	l	m	h	Rank	Rank	R	S 54,7	R	S 46,2	year	exam	year	exam
P14		*			*		*				*		*			10	31	17	50	20	41,7	86	82	79	61
P15	*			*			*			*			*			29	29	17	50	22	33,3	71	92	58	61
P16		*			*			*				*		*		24	28	23	40	15	50	86	98	100	86
P17	*			*			*			*			*			24	19	28	20	18	45,8	66	91	54	66
P18	*			*			*			*			*			27	11	25	30	20	41,7	64	72	55	49
P19	*			*			*			*			*			24	25	17	50	18	45,8	75	86	56	74

P16 was often singled out in section 6.6 on page 234 as having the most insight, and being able to think abstractly and critically and with certain imaginative skills compared with his classmates. This is illustrated clearly in Table 6.8. This reasonably high level of skill is consistent with his mathematics performance which by all accounts is excellent. His teacher's experience in the mathematics classroom was articulated as follows:

> *His maths performance is always above average.*
> *His understanding and application of geometry theorems and analytical thinking skills are far above average. He is generally very good in geometry.*

The same apparent link is, however, not evident in P16's performance in the HAT and the AMST where he was placed in the bottom third of the sample.

P14 fell into a similar category as P16 in being placed higher than the rest of the class in his meta-levels. His mathematics performance was above average, as his teacher confirmed:

> *His performance in mathematics is generally just above average. He is good in geometry. He understands theorems and can apply them in solving riders.*

His performance in the HAT and AMST relative to the sample was not strong, however, as Table 10.3 illustrates.

P18's weak performance in the mathematics classroom appeared to correlate well with her low rankings in the HAT and AMST. Her teacher suggested:

> *In general she is just average in her mathematics performance. She takes much longer to assimilate concepts and apply them.*
>
> *The application of geometry theorems is not very good. She struggles in her geometry especially when it comes to applying a theorem she has just learnt.*

She also scored consistently low in her meta-levels.

Analysis of S4

Refer to Table 6.9 for a global overview.

Table 6.9 Meta-profile, HAT, AMST and school performance for S4

NAME	WORLD VIEW-INTERVIEWS (META -LEVEL)															HAT		AMST				SCHOOL PERFORMANCE			
	Abstraction			Imagination			Critical			Insightful			Complexity			Spatial orientation	Spatial visualisation	Spatial orientation		Spatial visualisation		Algebra		Geometry	
	l	m	h	l	m	h	l	m	h	l	m	h	l	m	h	Rank	Rank	R	S 54,7	R	S 46,2	year	exam	year	exam
P20		*			*			*			*			*		20	22	17	50	9	58,3	53,7	42	57,5	25
P21		*			*		*			*				*				8	70	4	70,8	59,7	37	64	38
P22		*			*				*			*		*		3	3	8	70	8	62,5	33,3	53		65
P23			*			*		*				*		*		1	21	8	70	5	66,7	48	40	35	14
P24			*			*			*			*			*	3	11	1	100	11	54,2	86,7	67	79	69
P25		*			*			*			*			*		12	14	1	100	2	75	37,5	54	34,5	28
P26		*		*			*			*			*			4	23	4	90	15	50	83,7	80	77,5	66

P24 featured prominently in what he said and how he said it when talking about space. He was very eloquent and was positioned highly in all five of the meta-levels (the only participant in the sample who achieved that). In the HAT and the AMST he also features prominently and was placed in the top third for all the tests. His mathematics performance seemed to correlate well with the above and according to his teacher he was very competent although she pointed out that geometry was not his strong point:

> *His general performance is very good. He is methodical, neat and determined.*
>
> *Geometry is not his strong point, but he does try hard – takes advice on how to learn it and is determined to succeed.*

P23 was a very interesting case in that he featured highly in the world-view profiles. He was very imaginative in his deliberations, theorising with insight and often using abstract imagery and concepts to illustrate his perceptions. He was placed in the top third for the HAT and AMST test yet his performance in the mathematics classroom was very weak. His teacher had these perceptions:

> [P23] *always tries his best and really believes that he is able to do tasks assigned to him. Whenever offered help, he always declines, saying that he is on top of it, which he is obviously not. His general maths performance is weak.*

The general profile of P22 showed reasonable consistency across all the tasks. He was positioned relatively highly on the meta-levels and his performance in the HAT and AMST was consistently in the top third. Compared to his peers his performance in mathematics was solid. This is confirmed by his teacher:

> *P22 leads a full life. Time allocation seems a problem at times. He needs time to wrestle with concepts and make them his own. If this is done successfully, things are rosy. If he has not had the opportunity to get to grips with a topic thoroughly, he tends to get frustrated with the subject. Luckily so far, there does not appear to be too much accumulated negativity as he is a fairly buoyant character and has achieved well in many of his interests.*
>
> *Geometry does not appear too problematic. Has developed spatial awareness.*

The above consistency cannot be found in the case of P25, however, who, despite a very high ranking in the AMST, was only placed in the middle of the sample for the HAT. His mediocre position in the meta-levels correlates with the ranking in the HAT, but his mathematics performance, particularly in geometry, appeared to be very weak. His teacher concurred:

Not up to standard as he has not really focused on the task at hand. [P25] *does not contribute at all during problem solving and prefers to try and figure out on his own, which is not generally possible for him.*
He battles with geometry and finds it difficult to solve problems.

Analysis of S5

Table 6.10 displays a global overview of the performance of S5.

Table 6.10 Meta-profile, HAT, AMST and school performance for S5

NAME	WORLD-VIEW INTERVIEWS (META -LEVEL)															HAT		AMST				SCHOOL PERFORMANCE			
	Abstraction			Imagination			Critical			Insightful			Complexity			Spatial orientation	Spatial visualisation	Spatial orientation		Spatial visualisation		Algebra		Geometry	
	l	m	h	l	m	h	l	m	h	l	m	h	l	m	h	Rank	Rank	R	S 54,7	R	S 46,2	year	exam	year	exam
P27	*			*			*			*			*			20	26	32	0	26	20,8		43		38
P28	*			*			*			*			*			18	11	29	10	31	12,5		41		39
P29	*				*			*			*		*			29	30	29	10	26	20,8		49		48
P30	*				*		*				*		*			10	18	25	30	5	66,7		50		43
P31	*			*			*			*			*			31	20	29	10	31	12,5		47		46
P32	*			*			*			*			*			20	8	23	40	25	25		47		45

The average year marks have been omitted for S5 due to the fact that, for all their tests throughout the year, algebra and geometry items were integrated.

P30 stood out on numerous occasions in the word-view analyses as someone who articulated his ideas eloquently as compared to his classmates. His position on the meta-levels is reflected in Table 6.10. He clearly ranked higher in the HAT and the AMST than his peers, yet his performance in the mathematics classroom appeared to be similar to those of his peers. His teacher reflected:

He has an outstanding performance, he loves his mathematics. He always challenges and also tries to win the challenge. He likes homework and sometimes even asks me to do the same task for him comparing his computative skills to mine. He seldom discusses with other learners not unless they ask him.

Geometry and Trig are not the sections which boost his interests although he likes them. He enjoys Algebra more than other sections. He tackles the simplifying part of geometry with more confidence than the theorems.

In terms of her world-view profile and the way she responded during the interviews, P29 is similar to P30. Her rankings in the HAT and AMST were very low, however. Her marks for her mathematics examination were relatively high and her teacher spoke positively of her performance:

> *She has an interest in mathematics. She can calculate but joking at times. She likes competition, she early finishing group; top achievers group. She is an encouraging member in her group. Very rare to find her not doing her homework.*
>
> *She is not a shy learner, she is curious, wants to know everything. She always to be given a time to try her best.*

6.11.2 Spatial capacity and mathematics performance

Relationships between spatial visualisation and orientation factors, and mathematical performance

Referring back to Fennema and Tartre's (1985) study cited in 2.2.1 where it was suggested that a correlation between 0,3 and 0,6 exists between spatial visualisation skill factors and mathematical tasks that contained overtly spatial tasks, this case study obtained similar results for two of its schools, S1 and S3. A correlation between AMST visualisation tasks and overtly spatial school tasks (geometry examination) of 0,45 and 0,66 respectively was recorded for those schools. For details see Table 6.11. In correlations of a similar dimension between the AMST spatial orientation factors and overtly spatial school tasks (geometry examination), S4 and S5 obtained a correlation coefficient of 0,44 and 0,34 respectively.

Table 6.11 Correlation coefficients between AMST and school performance.

SCHOOL	AMST **sv** and **algebra**	AMST **sv** and **geometry**	AMST **so** and **algebra**	AMST **so** and **geometry**
S1	0,3	0,45	0,25	0,14
S2	-0,42	-0,27	-0,46	-0,56
S3	0,24	0,66	0,1	0,18
S4	-0,69	-0,63	0,68	0,44
S5	0,61	0,02	0,51	0,34

Interestingly, the correlations (of similar dimension to above ie. 0,3-0,6) between AMST spatial visualisation factors and school tasks that are devoid of overtly spatial ideas (performance in algebra examinations) showed similar trends. For example, the correlation coefficient of the AMST visualisation factors and performance for algebra for S1 and S5 were 0,3 and 0,61 respectively. In terms of the correlation between the AMST orientation factors and performance in algebra the results for S4 and S5 were 0,68 and 0,51 respectively.

In the context of this case study the correlations above do not allow for widespread generalisations that suggest that students with high spatial visualisation and orientation factors (as reflected in the AMST) necessarily perform well in school mathematics. I would thus concur with Fennema and Tartre's conclusion (1985) that students who had a high spatial visualisation skill factor solved no more mathematical problems than students with a low spatial skill factor. See 2.2.1 on page 11. A similar conclusion can be drawn in the case of the spatial orientation factor.

In the case for the correlations between the HAT and spatial capacity, see Table 6.12.

Table 6.12 Correlation coefficients between HAT and school performance.

SCHOOL	HAT **so** and **algebra**	HAT **so** and **geometry**
S1	0,39	0,64
S2	0,22	-0,14
S3	-0,22	0,11
S4	0,34	0,41
S5	-0,21	-0,65

The correlation of the HAT orientation factors and performance for non-geometry (algebra) tasks for S1 and S4 were 0,39 and 0,34 respectively. Interestingly the correlation for the same schools between HAT orientation factors and geometry tasks were 0,64 and 0,41 respectively. It was not possible to do a correlational analysis between HAT visualisation factors and school performance because achievement in the

HAT visualisation tasks was measured in terms of time taken and not as marks achieved (see 3.5.3 on page 107). As was the case with the AMST, the evidence above is not conclusive enough to suggest that high spatial orientation factors in general, as reflected in the HAT, necessarily implies that students will perform well in school mathematical problems.

Gender differences and equity in spatial capacity, and mathematical performance

Focusing specifically on gender differences, it was interesting to note that the positive correlation coefficients for the HAT and school performance of the single-sex schools were noticeably higher than for the co-ed schools. For S1 (the all-girls school) the correlation coefficient between the HAT orientation factor and non-geometric school tasks (algebra) was 0,39. In contrast the correlation coefficient between the HAT orientation factors and the school tasks containing obvious spatial elements (geometry) was 0,64. A similar trend was evident for the S4, the all-boys school. Consistent with S1, the correlation between the orientation factors and the school task containing geometrical problems was higher (0,41) than the correlation between the orientation factors and the school tasks containing no obvious spatial dimension (0,34). Overall, the HAT orientation tasks were a moderately good indicator of school performance for S1 and S4.

Comparing the correlations of the results on the AMST orientation factors and school performance revealed a different pattern. S3 (the all-boys school) recorded the highest correlation coefficient between the AMST orientation factors and school performance for algebra and geometry (0,68 and 0,44 respectively). S5, the rural co-ed school, recorded correlation coefficients of 0,51 and 0,34 respectively for the same comparison. The all-girls school S1 also recorded positive correlation coefficients, but they were noticeably lower than those of S3 and S5.

Comparing the visualisation factors of the AMST and school performance, the all-girls school showed positive correlation coefficients (albeit relatively low) whereas the all-boys school displayed relatively high negative correlation coefficients. This suggests that for the boys' school, the AMST visualisation test tasks were not good indicators in terms of school performance.

Relationship between spatial ability and mathematical ability

This refers to Battista *et al.*'s (1982) note that "the role that spatial thinking plays in mathematical performance has not been adequately described. It is still not known how important spatial ability is for learning various topics in mathematics, nor are the spatial components of mathematical thinking understood" (see section 2.2.1 on page 11). In the context of this study, spatial ability is defined in terms of spatial visualisation and orientation (referred to as spatial capacity) and, as noted above, a case is made that although some pupils displayed a high correlation between spatial capacity and performance (in both geometric and algebraic tasks), it was not necessarily true for all pupils in this sample. While this study identifies with Battista *et al.*'s (1982) sentiments, it has illustrated that the relationship between spatial capacity and mathematical performance is not a straightforward linear one – at least for the participants in this study.

6.11.3 Summary of findings

Although on an intuitive level there appear to be high correlations between the various tests, meta-levels and school performance for some of the participants, the same cannot be said for the sample as a whole. Tentatively, the following trends can be isolated:

- Some of the participants who were highly rated in the meta-levels were also ranked relatively highly in the HAT and the AMST tests. They were in turn usually relatively good performers in the mathematics classroom;
- Most of the participants who rated low in the meta-levels invariably were also ranked in the bottom third of the HAT and AMST and in turn performed poorly in their mathematics;

- Both the township school S3, and the rural school S5, recorded low rankings and positions in the spatial tests and the meta levels respectively. Those participants who recorded relatively high positions compared to their peers were, however, also the relatively high achievers in their respective mathematics classrooms;
- Generally, students who had a high spatial visualization skill factor in the AMST solved no more mathematical problems than students who had a low spatial skill factor. In particular, only the all-girls and the all-boys schools recorded correlations between 0,3 and 0,7 between spatial visualisation factors and mathematical tasks that contained overtly spatial problems (geometry);
- A similar trend was observed for students who had a high spatial orientation factor in the AMST. In particular, only S4 and S5 recorded correlations between 0,3 and 0,7 between spatial orientation factors and mathematical tasks that contained overtly spatial problems (geometry);
- In terms of correlating spatial visualisation factors in the AMST and mathematical tasks that contained no spatial problems (algebra), only S1 and S5 recorded correlation coefficients between 0,3 and 0,7;
- In terms of correlating spatial orientation factors in the AMST and mathematical tasks that contained no spatial problems (algebra), only S4 and S5 recorded correlations coefficients between 0,3 and 0,7;
- Only S1 recorded correlation coefficients higher than 0,3 between spatial visualisation factors in the AMST and performance in geometry and algebra;
- Only S4 and S5 recorded correlation coefficients higher than 0,3 between spatial orientation factors in the AMST and performance in geometry and algebra;
- In the HAT, once again both S1 and S4 recorded a correlation coefficient between spatial visualisation and orientation factors, and performance in algebra and geometry, greater than 0,3;
- For this sample, therefore, the scores on the spatial capacity tests (AMST and HAT) were a relatively good indicator for mathematics performance only for participants in S1;
- For S2 the correlation coefficients between the scores on the AMST and mathematics performance in general were consistently negative, suggesting that a high score on the

spatial capacity test most certainly did not imply high performance in mathematical tasks;

- The same can be said for S4 as far as the AMST visualisation factor is concerned;
- A similar deduction can be made in the case of S5 in the HAT;
- In terms of the spatial orientation construct there is no conclusive evidence to suggest that either the AMST or the HAT is a better indicator for mathematics performance.

As suggested earlier, these findings are tentative and should be considered in the context of small samples.

6.12 CHAPTER CONCLUSION

The central assertion of this project, that spatial conceptualization is a complex and multifaceted blend of competencies ranging from spatial visualisation and orientation skills that manifest themselves not only in pen-and-paper tests but also in hands-on activities, to world-view perceptions, is illustrated in the above analysis. It would be incorrect to suggest for this sample that there are high correlations between the various spatial dimensions of pen-and-paper performance, hands-on performance and world-view profiles. Firstly, the relatively small sample (n=32) precludes making broad generalizations. Secondly, most of the analyses was done *within* the participating schools, which in effect meant that the sample consisted of at most six participants at any one time. Thirdly, the paradigm within which this research was situated would suggest that a broad generalization is inappropriate. This case study has, however, provided a glimpse into the complexity of spatial conceptualization of learners in five schools representing the diverse social background of South Africa. It has further highlighted the need for further research using larger samples over longer periods of time.

CHAPTER SEVEN

CONCLUSION

7.1 INTRODUCTION

This study aimed to explore spatial capacity through pen-and-paper and hands-on activity tests, and explore world-view perceptions of space in an attempt to show that spatial conceptualisation is a rich and complex blend of spatial capacity and world view. The research presented in this study is described as a case study, oriented in an interpretive-naturalistic paradigm and characterised by multidimensional quantitative and qualitative methods. The research, set in five secondary schools in the Eastern Cape, was carried out with 32 Grade 11 learners. The study was structured around seven stages and focused primarily on exploring spatial capacity and investigating world views of space. Many of the research findings have been discussed and related to the literature review of chapter two, in chapters four and five. This chapter offers a summary of those findings and specifically addresses the following:

- review of the study objectives;
- a summary of the findings that have the greatest significance from this study;
- significance of the study;
- a description and discussion of the limitations of the study;
- areas for future research;
- implications and recommendations for teaching and learning;
- a personal reflection.

7.2 REVIEW OF THE STUDY OBJECTIVES

In chapter one the four main objectives of this study were articulated as follows:

1. to explore spatial capacity (spatial visualisation and orientation skills) using both pen-and-paper test items and hands-on activities;
2. to explore spatial conceptualisation by engaging in world-view profiles of space using a logico-structural approach;
3. to investigate consistencies and relationships between spatial capacity, world views and aspects of mathematics performance of Grade 11 learners;
4. to show that spatial conceptualisation is a rich and complex blend of spatial capacity and world view.

First objective

This objective was achieved by employing an adapted pen-and-paper test (AMST – see section 3.5.2 on page 85) and a hands-on activities test (HAT – see section 3.5.3 on page 107). The findings of this study pertaining to gender differences were consistent with those of Wattanahawa (1977) upon whose test the AMST was modelled. On the whole, the AMST and the HAT were effective tests to explore spatial capacity in varying contexts. The former relied on pen-and-paper strategies and skills, whereas the latter demanded hands-on techniques.

Second objective

This objective was achieved by engaging in a logico-structuralist approach which relied on conversations that were structured around nine bipolar themes (refer to chapter five on page 181). The resultant world views of space that emerged were mostly rich, intricate and complex perceptions that facilitated a meta-analysis (see section 5.4 on page 228) in order to establish profiles of thinking skills such as capacity to abstract, to be insightful, to deal with complex ideas, to critically engage, and to be imaginative.

Third objective

This objective was achieved by superimposing the world view meta-levels on the participants' performances in the AMST, the HAT and school mathematics (refer to section 5.5 on page 274).

Fourth objective

On reflection this objective should have been incorporated into the overall research question as it encompasses the central theme of the project. This objective was achieved by working through the previous three objectives – that is, by describing and engaging with the samples' spatial capacity in terms of spatial visualisation and orientation in the context of the AMST and the HAT, and by engaging in individual world views of space which extended into profiles of thinking skills.

The objectives of the study were framed by one central research question:

> Can understanding a world-view-theory approach contribute towards the exploration of an individual's spatial conceptualisation?

As discussed in chapter two, most investigations into spatial capacity and spatial conceptualisation were conducted within a framework of tests, mostly of the pen-and-paper type. Although the notion of prior knowledge and presuppositions forms a cornerstone of a constructivist paradigm there is very little evidence of documentation which relates the broad presupposition of space (world view of space) to spatial conceptualisation as defined in this study.

Spatial conceptualisation was seen in this study as a union of spatial capacity and *Weltanschauung.* This study, through a rigorous logico-structuralist approach advocated by world-view theorists such as Kearney (1984) and Cobern (1991), has shown that world views form an integral part of an individual's epistemological make-up. The world-view profiles established through this approach were rich reflections of the participants' epistemological macrostructures. This approach proved to be an effective technique in understanding the world-view aspect of an individual's spatial conceptualisation as defined in this study.

7.3 SUMMARY OF DEFINITIONS

As was alluded to in chapter two on page 12, there exists little consensus in the terminology and definitions used in much of the documented research on spatial conceptualisation. It was therefore deemed important that the concepts used in this study were appropriately defined from the outset and used consistently throughout. The concepts which framed this research (in terms of spatial conceptualisation and world view) are summarized for the reader's convenience:

a) **spatial visualisation -** the ability to mentally manipulate, rotate, twist, or invert a pictorially or physically presented stimulus object. The underlying ability in *spatial visualisation* appears to be connected to movement, transformation and manipulation. It is dynamic and involves motion (see page 18);

b) **spatial orientation** - the ability to recognise the identity of an object when it is seen from different angles. It is the ability to make sense of spatial orientations of objects relative to different positions of itself or other objects (see page 19);

c) **spatial capacity -** the all-encompassing concept which embraces spatial vizualization and orientation (see page 18);

d) **spatial conceptualisation -** the fundamental concept that ultimately incorporates spatial capacity and world view of space (see page 19);

e) **world view -** presupposition about the world; the epistemological macrostructure (see section 2.3 on page 30);

f) **capacity to abstract -** the capacity to explain in theory rather than practice, in intangible rather than concrete terms; the capacity to denote a quality or condition or intangible thing rather than a concrete object (see page 230);

g) **capacity of insight -** the capacity of understanding hidden truths; a view into anything, enlightenment (see page 230);

h) **capacity to deal with complex issues -** the capacity to identify related parts, to deal with composites; the ability to see more than one, or many parts: not simple but intricate (see page 230);

i) **capacity to engage critically -** the capacity to finding fault, being able to censure; able to express criticism; discerning and reflective (see page 231);

j) **being imaginative -** having or showing in a high degree the faculty of imagination; being able to form a mental image or concept; being able to conjecture; to think vainly or falsely (see page 231).

7.4 SUMMARY OF FINDINGS

Since the findings were viewed as integral to the data narrative, they were, in conjunction with the literature reviewed in chapter two, explicitly referred to in chapters four and five. A summary of the findings of greatest significance to this study are presented under the following headings:

- spatial capacity and the AMST;
- spatial capacity and the HAT;
- relationship between the AMST and the HAT;
- word view profiles of space;
- spatial conceptualisation and mathematics performance.

7.4.1 Spatial capacity and the AMST

Gender differences

- For each item of the AMST the males performed better than the females (see page 129).
- The extent of this performance varied, however. In two-dimensional problems, the females were not as outperformed by the males as in three-dimensional problems (refer to page 130);

- In terms of the spatial orientation and visualisation constructs, there was a higher mean between the difference of performance of the males and females in the spatial orientation problems than in the spatial visualisation problems (refer to page 130).

School differences

- The rural school learners found it difficult to cope with three-dimensional problems and those characterised by the spatial orientation construct (see page 131).
- The township school learners found it difficult to cope with three-dimensional problems and those characterised by the spatial visualisation construct (refer to page 132).

Individual differences

- There was a high correlation between poor performance and poor socio-economic background (refer to page 133).

7.4.2 Spatial capacity and the HAT

Gender differences

- In terms of the spatial visualisation construct, there were no observable differences (refer to page 153);
- In terms of the spatial orientation construct, the males performed better than the females (refer to page 153).

School differences

- In both the spatial visualisation and the orientation items, the township and the rural school performed poorly compared to the other schools of the sample (refer to page 154).

Individual differences

- Despite the strong performance of P14 and P30 from S3 and S5 respectively, the participants from a poor socio-economic background did not perform as well as their counterparts who came from more advantaged homes (refer to page 154).

7.4.3 Relationship between the AMST and the HAT

For this sample, performance in the AMST was not necessarily a good predictor for performance in the HAT. In particular, the correlation between the spatial visualisation construct items of the two tests was weak to moderate in the case of the males and moderate in the case of the females. A similar scenario was experienced for the orientation construct items (see section 4.4 on page 173).

7.4.4 World view profiles of space

The general world view of space of this sample was characterised by rich, complex and multifaceted ideas (see section 5.3.11 on page 202). Space was seen as:

- having a strong religious element;
- relatively orderly, peaceful and underpinned by design;
- somewhat obscure and mysterious;
- special and beautiful;
- something which has direction;
- making us insignificant and small;
- having no beginning and no end;
- something which we cannot touch, yet feel;
- something that is visible in the sense that we can see that there is nothing there.

The overall perception of space of the sample leant towards a Newtonian division of absolute and relative space. In terms of magnitude it regarded space as infinite thereby refuting the Platonic position which asserts that space had length, depth and breadth. In terms of the Cartesian reductionist and grid-like outlook on space, this sample preferred to view space as mysterious, infinite and somewhat obscure. It often referred to space in Kantian terms and related to space in terms of subjective feelings (refer to page 203).

Gender differences

The theme of personal space, the personal 'space bubble', was emphasized by numerous female members of the sample (refer to page 204).

School differences

There was no one school with a particular strong or unique world view in response to the various vector pair themes. The world views of space were similar across all the schools, yet the extent and depth of articulation and discussion varied greatly. Participants from the township and the rural schools generally found it difficult to articulate, describe and explain their world views (refer to page 206).

On a meta-level it was found that generally the sample engaged at a low level of critical engagement. The females were rated at a higher level than their male counterparts for their capacity to abstract and be complex, whereas the males rated higher for their capacity to show insight, be imaginative and be critical (refer to section 5.4.6 on page 267).

7.4.5 Spatial conceptualisation and mathematics performance

Although at first glance there appeared to be high correlations between the various tests, meta-levels and school performance for some of the participants, the same cannot be said for the sample as a whole. Tentatively, the following trends were identified (refer to section 5.5.1 on page 276):

- A number of the participants who were highly rated in the meta-levels were also ranked relatively highly in the HAT and the AMST tests. They were in turn usually relatively good performers in the mathematics classroom;
- Most of the participants who rated low in the meta-levels invariably were also ranked in the bottom third of the HAT and AMST and in turn performed poorly in their mathematics;
- Both the township school S3, and the rural school S5, recorded low rankings and positions in the spatial tests and the meta-levels respectively. Those participants who recorded relatively high positions compared to their peers were, however, also the relatively high achievers in their respective mathematics classrooms;
- Mostly, students who had a high spatial visualisation skill factor in the AMST solved no more mathematical problems than students who had a low spatial skill factor;
- A similar trend was observed for students who had a high spatial orientation factor in the AMST;
- In terms of correlating spatial visualisation factors in the AMST and mathematical tasks that contained no spatial problems (algebra), only S1 and S5 recorded correlation coefficients between 0,3 and 0,7 (when correlations are around 0,4 only crude predictions may be possible, Cohen *et al.*, 2000);
- In terms of correlating spatial orientation factors in the AMST and mathematical tasks that contained no spatial problems (algebra), only S4 and S5 recorded correlations coefficients between 0,3 and 0,7;
- Only S1 recorded correlation coefficients higher than 0,3 between spatial visualisation factors in the AMST and performance in geometry and algebra;
- Only S4 and S5 recorded correlation coefficients higher than 0,3 between spatial orientation factors in the AMST and performance in geometry and algebra;
- In terms of the HAT, once again both S1 and S4 recorded a correlation coefficient between spatial visualisation and orientation factors, and performance in algebra and geometry, greater than 0,3;
- For this sample, therefore, the scores on the spatial capacity tests (AMST and HAT) were a relatively good indicator for mathematics performance only for participants in S1;

- For S2 the correlation coefficients between the scores on the AMST and mathematics performance in general was consistently negative, suggesting an inverse relationship between the variables;
- The same can be said for S4 as far as the AMST visualisation factor is concerned;
- In terms of the spatial orientation factor, there was a negative correlation coefficient between the scores on the HAT and mathematics performance for S5 in general, suggesting an inverse relationship between the variables;
- In terms of the spatial orientation construct there is no conclusive evidence to suggest that either the AMST or the HAT is a better indicator for mathematics performance.

7.5 SIGNIFICANCE OF THE STUDY

The significance of the study is two-fold:

- in incorporating world-view in its understanding of spatial conceptualization;
- in its research approach and design.

Taking cognizance of Duit's (1995) reference to the scant attention to meta-knowledge (views about the nature of the world) in educational research in general and in mathematics education in particular (see chapter two, page 41), and his claim alluding to the importance of meta-knowledge, this study embraced the notion that world view of space is a necessary ingredient in the understanding of an individual's global conceptualisation of space. This study therefore incorporated Kearney's (1984) theoretical notions of world view (see section 2.3 on page 30) in its investigation of students' conceptualisation of space. This study has contributed to the world-view literature by attempting to apply a rigorous methodology to explore aspects of world view, namely those of space. This study has broadened the outlook on and understanding of spatial conceptualisation beyond the traditional pen-and-paper approach to include world-view perspectives.

The methodology in the exploration of world views was based on and facilitated by a logico-structuralist approach (refer to page 32). In educational research, this approach, modelled after Kearney (1984), was pioneered from a science education perspective by Cobern (1991, 1993) in the United States of America, and Slay (2000) in Australia. As far as I could ascertain, this methodology has not yet been applied in a mathematics education context. This study is a first of its kind in South Africa and has opened up a number of issues which could be researched further. One of the strengths of this study lies in its innovative approach in attempting to incorporate Kearney's world-view model in understanding spatial conceptualisation. The methodology of the study provides an effective and rigorous enabling framework around which to explore the *Weltanschauung* of learners, an integral component of our epistemological macrostructure. It has hopefully shown that this kind of research is useful in the area of exploring preconceived perceptions and suppositions. In its attempt to break new ground, this study has opened up possibilities for further research which are discussed in section 7.7 on page 305. Further methodological strengths which are of significance pertained to the research design:

- The involvement of a consensual validation team (refer to page 67 and section 5.4.1 on page 228) paid dividends in terms of ensuring quality and credibility.
- The inclusion of the hands-on activities test as described in section 3.5.3 on page 107 added a practical and hands-on dimension to the tests;
- The use of a portfolio of artworks as discussed in section 5.3.14 on page 213 enabled the participants to illustrate, support and complement their previous comments in articulating their world view of space.

7.6 LIMITATIONS

The limitations of the study can be divided into the following categories:

- contextual matters;
- research implementation;
- research methodology.

Contextual matters

It is acknowledged that the findings of this study are limited and tentative in their generalisability and wider application, given the relatively small sample that was used. This study constituted a case study which, although providing insight into a specific situation, at best can only provide a frame of reference for further research.

In its attempt to capture and incorporate the diversity of South African culture, school culture and socio-economic backgrounds (for the objectives of this research project), this multi-sited case study involved five 'varying' types of schools. Five different ethnographic type studies would have perhaps been more appropriate to appreciate the subtleties and complexities of the various school cultures. The strength of the multi-sited approach, however, was that it provided a 'broad' picture from which further, more specific, research could emanate.

As identified in earlier chapters, language is a problematic issue that cannot be ignored in a study of this nature, which particularly relied on interviews and conversations as data-collecting techniques. Language limitations that related to this study were two-fold. Firstly, for many of the participants in this sample, English was their second language. Although an interpreter was present for the rural school, it cannot be assumed that the language problem necessarily diminished. Issues of power relations between the interpreter, the researcher and the participants could, for example, have affected the outcome of the interviews (see page 212). Secondly, many of the participants (also the English first-language speakers) found it difficult to articulate their ideas and perceptions.

The notion that what was said was a true reflection of what was thought and perceived is not necessarily true. Many may well have given the impression of weak and limited perceptions due mainly to their weak and limited ability to articulate their ideas. Their words may well not have reflected their thoughts entirely.

Research implementation

Despite the rigorous planning and careful implementation of the research process, unforeseen circumstances such as unrest in the township school hampered the flow of the data-gathering process. The effect was that I had to rearrange meetings and wait for long periods of time for data to come in. Although I was conscious not to let this affect my mindset, this caused frustration and anxiety on my part.

Research methodology

The following points relating to research methodology could be perceived as limitations:

- This case study incorporated five sites and it could be argued its scope therefore did not allow for a sufficiently deep analysis. As the study unfolded I became aware that each school could easily constitute a unique case to be investigated. So, although a case study is, by definition, an in-depth study of a particular case, it could be argued that this study evolved into five case studies;
- Inherently problematic in a case study is the issue of generalisability and globality of its findings. The scope of a case study is relative to a particular case and can therefore only be considered as microscopic (Hamel, Dufour and Fortin, 1993). This may be seen as a limitation of this study, although it was not the objective of the study to generalize;
- Hamel *et al.* (1993) acknowledge that the strength of a case study lies in its wealth of empirical material. Case-studies are based on a great wealth of empirical materials, notably because of their variety. This is particularly characteristic of this study. Hamel *et al.* (1993) warn that this can also present problems, however, particularly in the analysis of the diverse data. I have attempted to overcome potential difficulties

with handling the materials of different origins by incorporating the diverse findings into a single narrative and in this way create a sense of cohesion;

- On validity, Cohen *et al.* (2000) argue that the researcher's discussion of validity needs to be located within the research paradigm that is being used (see page 70). Although the dominant paradigm of this study was an interpretive one, the validity of one of its tests (the AMST) was based on a positivist position. This could be seen as a contradiction. I did argue for a multi-paradigm approach, however, as advocated by Hammersley (1992) on page 70;
- Two items in Guba and Lincoln's (1989) framework for ensuring quality and credibility as outlined on page 71, which in my view were not fully exploited were *peer debriefing* and *member checks;*
- On peer debriefing, the consensual validation team that was established (see page 67) particularly to assist in and validate the meta-analysis process (refer to section 5.4 on page 226) could, in hindsight, have been engaged in the initial content analysis (see section 5.3 on page 182) of the interviews as well. As far as member checks are concerned, more use could have been made of the participants for reviewing and 'checking' my analyses;
- Although the involvement of the consensual validation team and the participants strengthened the objectivity of this study, it could still be argued that there was excessive subjective intervention on my part. This dilemma is characteristic of qualitative research. I took encouragement from Zonabend (1992:53), who asserted that "we must be aware that the most rigorous objectivity is only possible through the most intrepid subjectivity";
- The extent of utilising data-reduction processes of the interview transcripts posed a constant dilemma to me. I did not want to 'over reduce' the data as that would diminish the rich realism of the raw data (hence the extensive use of the raw 'voices'), and at the same time I did not want to compromise on the rigour and the credibility of the data;
- In terms of a more global and comprehensive world view, it could be argued that this study focused too narrowly on the space universal at the expense of the other six universals given on page 32. This study was particularly concerned with the pre-

suppositions of space in conjunction with spatial capacity, and hence focused only on the space universal. It is thus acknowledged that the world-view claims made in this study pertain only to the understanding of space and consequently reflects only a small aspect of a more global world view;

- Kearney's (1984) logico-structuralist model is not without its problems. Although the framework of bipolar codes (refer to section 5.2 on page 182) was meant to enable the establishment of world-view profiles, it can be argued that the framework was too rigid, prescriptive and linear in its approach and did not allow for deviation and flexibility. It can also be argued that the bipolar codes in themselves were too limiting and restrictive. For example, in the first bipolar code (refer to page 182), the assumption was made that the antitheses of naturalism is religion. Although the notion of a bipolar continuum (see page 183) provided for compromise and a softening of mutually exclusivity, it did not challenge the fundamental assumption of bipolarity. The same can be said for the other codes. By the same token it can be reasoned that the strength of the logico-structuralist approach lies in its structure and rigour. World view is a complex concept to describe and explore, and Kearney's model provided a workable framework and point of reference for researching world views;
- This study focused on only one of Kearney's (1984) seven world-view 'universals' (refer to section 2.3 on page 32), namely, that of space. In terms of a global world-view analysis, this can be viewed as a limitation.
- In terms of gender differences, it could be argued that child-rearing practices could have an effect on performance. This study did not include an exploration into child-rearing experiences of the participating pupils, and this can be viewed as a limitation.

7.7 AREAS FOR FUTURE RESEARCH

As Le Compte and Preissle-Goetz (1982:35) suggest, "because human behaviour is never static, no study can be replicated exactly, regardless of the methods and designs employed". This applies to certain aspects to this work such as the study of world views.

A duplication of this study in another context might, however, add credibility to the results of this work. As suggested earlier, a more ethnographic approach focusing on only one school could be adopted to produce a more in-depth and comprehensive world view study, for example.

Investigating prior knowledge and presuppositions using a logico-structuralist methodology has considerable research potential in a mathematics-education context, particularly when investigating the constructivist tenet of epistemological macrostructures. In a broader educational context this methodology could be effectively utilised for the exploration of other world-view universals.

7.8 IMPLICATIONS AND TENTATIVE RECOMMENDATIONS

Implications and tentative recommendations emerging out of this study for teaching and learning are described in the following points:

- The notion of prior knowledge and world views (epistemological macro-structures) is central to the constructivist epistemology (see section 2.5 on page 39), and the logico-structuralist approach used in chapter five on page 181 could provide the teacher and researcher with an enabling methodological framework to investigate learners' prior knowledge and world views (refer to section 2.5 on page 39). For example, the bi-polar framework used in the logico-structuralist approach could be as the guiding framework for classroom discussion and debate. Other universals in Kearny's model could be used to establish more comprehensive world views on topics such as nature in the science classroom (Slay, 2000);
- Grimison and Pegg (1995:199) remark that in terms of gender equity, "for committed teachers it is [often] difficult to face the possibility that the students in their classes are not treated equally". They suggest that research has revealed that unequal treatment does occur "even in the classrooms of teachers who are aware and concerned about gender issues in mathematics". The gender differences in

performance of spatial tasks as confirmed by this study reinforce and emphasise the importance of equity in the mathematics classroom. As suggested by Grimison and Pegg (1995) the following steps can be taken by teachers to contribute to creating a gender-equitable classroom situation:

- critically examine and evaluate classroom displays, resources and texts for gender imbalances, stereotyping and sexist language;
- examine the language of the students and your own for gender biases and stereotyping;
- facilitate appropriate cross-gender interactions;
- ensure equitable gender contexts when dealing with occupation-related discussions and materials;
- incorporate materials and resources which extend and reinforce spatial skills.

- This study makes the point that scores on a pen-and-paper test do not always tell the whole story and should therefore be viewed with circumspection. This could have implications for the teacher in numerous contexts, such as summative assessment where pen-and-paper scores have traditionally been regarded as important indicators of achievement and performance. Assessment strategies should therefore be extended beyond the traditional pen-and-paper tests and include hands-on activity tests and assessment strategies in order to evaluate student's mathematical development and progress. The use of journals and portfolios could be used to get insight into different world views of learners;
- Activities and programmes that develop spatial capacity should be extended beyond pen-and-paper items and include hands-on activities appropriately contextualised in the world of the learners. The teaching and learning of geometry, for example, should include activities that require hands-on skills and make use of physical apparatus and equipment. Learners need to be encouraged to operate in their physical world and solve problems that are appropriately contextualised;
- Learners from deprived socio-economic backgrounds would benefit from supplementary support to facilitate:
  - the development of more sophisticated and informed world views;
  - the development of spatial capacity.

Schools need to be appropriately equipped with materials and resources, and teachers need to ensure access to that support;

- This study reinforces the need for educators to recognise and acknowledge that cognition in general is not merely reflected in a mark on a pen-and-paper test but also involves a complex process of incorporating presuppositions and world views.

7.9 PERSONAL REFLECTIONS AND FINAL WORD

For me the end of this thesis represents an end of one of the many cycles that have determined the course of my professional life. Unlike some of the other cycles, this one started with a definite and clear objective – to conduct worthwhile research that would culminate in a PhD thesis. Little did I know that this cycle would grow into a task that would challenge my own world view and my comfort zones to the core. After many years of classroom practice in many corners of the world, my academic life started relatively late. I was still basking in the afterglow of my Master's research when I embarked on this project, conceiving of a research project which I naively believed was neatly packaged with a clear overview and a precisely defined strategy. Little did I realize how "recursive and reflective" (Ely *et al.*, 1991) the entire process would become, and how it would dominate the next three years of my life. It has truly been a remarkable experience. The many troughs of self-doubt, loss of confidence, exhaustion, anxiety and desperation have been outweighed by the incredible highs, with feelings of enlightenment, growth, satisfaction and achievement. It has truly been a 'roller-coaster ride'. The realisation that research is not neat but messy and often very obscure has been a big learning experience for me. Ely *et al.* (1991:198) believe that "the processes of qualitative research also become processes of professional growth". This resonates particularly strongly with my experience. My relationship with the participants made me review and rethink my own world view many times over. My limited insight into their minds has made me reflect on my own practice – it has made me realise that understanding cognition is very complex and cannot be rationalised on the basis of test results only.

The generosity with which the participants shared their perceptions and the trust they afforded me has renewed my respect for the learners we seek to teach. Often entangled in the routine of teaching, the requirements of the school bureaucracy, the demands of the parents and the expectations of the profession, we as teachers lose sight of the needs of our learners. We thus often reduce them to mere receptacles of arbitrary content, ignoring the notion that they come to the classroom with a wealth of preconceptions, with wonderful world views and a well-developed epistemological macrostructure.

Although this dissertation represents the end of one of my life cycles, I would not like it become an end in itself. As much as I hope that this study will contribute to the knowledge base of my field, I also hope that it will remind me to "keep on working, keep on speaking, keep on writing, and, in the process, keep on refining and refining the nature of the information that one is contributing" (Ely *et al.*, 1991:228).

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APPENDIX 1

QUESTIONNAIRE 1

---

1. Name: ____

2. a) Gender: Male Female b) Date of birth: _______________

3. Home language: Xhosa English Afrikaans Other

---

4. How often do you speak English at home?

Circle either A, B or C.

Always or almost always ………………………….. A

Sometimes …………………………………………. B

Never ………………………………………………. C

---

5. Altogether, including yourself, how many people live in your home?

……………………….

---

6. Altogether, how many brothers and sisters do you have?

Number of brothers: …………………….. Number of sisters: …………………..

7. Which of these people live at home with you most or all of the time?

Circle either A or B on each line

		Yes	No
a)	mother ………………………	A	B
b)	father ……………………….	A	B
c)	one or more brothers ……….	A	B
d)	one or more sisters ……….	A	B
e)	one or more grandparents .	A	B
f)	another relative or relatives .	A	B
h)	another person or persons .	A	B

---

8. How far did your mother and father study at school?

Circle one letter in each column

		Mother	Father
a)	completed junior primary (Grade 4) ……..	A	A
b)	completed senior primary (Grade 7) ……..	B	B
c)	completed junior secondary (Grade 9) ……	C	C
d)	completed senior secondary (Grade 12) ….	D	D
e)	completed college/technikon …………….	E	E
f)	completed university study ……………..	F	F
g)	I don’t know ……………………………..	G	G

9. Which of these items do you have at your home?

Circle either A or B on each line

			Yes	No
a)	calculator	…………………………..	A	B
b)	computer	…………………………...	A	B
c)	television	……………………………	A	B
d)	study desk/table for your use	……………	A	B
e)	dictionary	……………………………	A	B
f)	encyclopedia	……………………………	A	B
g)	electricity	……………………………	A	B
h)	running tap water in your house	……	A	B

---

10. About how many books are there in your house?
(Do not count magazines, newspapers, or your school text books)

Circle A, B, C, D or E

None or very few (0 – 10 books) ………………………	A
Enough to fill one shelf (11 – 25 books) ………………	B
Enough to fill one bookcase (26 – 100 books) ………	C
Enough to fill two bookcases (101 – 200 books) ………	D
Enough to fill three or more bookcases (more than 200) …..	E

11. Did you attend pre-school/kindergarten/nursery school when you were small?

Yes **No**

---

12. When you were a small child did you play with the following?

Circle either A or B on each line

			Yes	**No**
a)	Jigsaw puzzles	……..…………………..	A	B
b)	Building blocks	………………………….	A	B
c)	Lego	………………………….	A	B
d)	Peg boards	………………………….	A	B
e)	Threading beads	………………………….	A	B
f)	Clay modeling	………………………….	A	B
g)	Climbing jungle gyms and ropes	………….	A	B
h)	Sandpit	………………………….	A	B
i)	Shape-sorter	………………………….	A	B

13. Do you remember your parents or brothers and sisters playing games like those listed in 12 with you when you were a small child?

Provide details.

---

14. What is your favourite shape?

Describe it and say why it is your favourite shape.

15. **What do you feel when you think about GEOMETRY?**

Complete the sentence:

When I think about Geometry I ____

____

____

____

____

____

____

16. **Why do you think we study Geometry at school?**

17. Do you like doing Geometry at school?

---

18. What do you feel like when you think about SPACE?

Complete the sentence:

When I think of space I ____

____

____

____

____

____

____

____

APPENDIX 2

QUESTIONNAIRE 2

SCHOOL PROFILE 2001

Name:	School:	Grade
	Number of Maths classes in this Grade:	Class size
Subjects: 1 2 3 4 5 6 7 Other:	Mathematics: HG SG Extra mural activities:	
TERM 1 Tests		

Topic	Mark (%)	
		Term mark:
		Class position:
Examination I		Class position:
Examination II		Class position:

TERM 2 Tests		
Topic	Mark (%)	
		Term mark:
		Class position:
Examination I		Class position:
Examination II		Class position:

TERM 3 Tests		
Topic	Mark (%)	
		Term mark:
		Class position:
Examination I		Class position:
Examination II		Class position:

TERM 4 Tests		
Topic	Mark (%)	
		Term mark:
		Class position:
Examination I		Class position:
Examination II		Class position:

Comments on this learner's general Mathematics performance

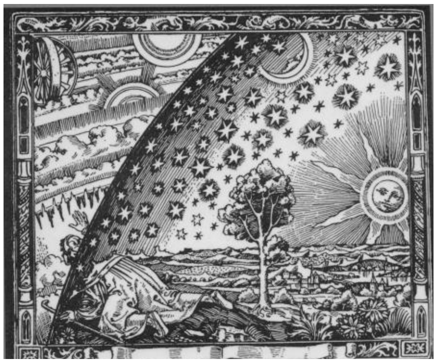
Comments on this learner's general performance in the Geometry class

Comments on the learner's social skills and interaction in the class

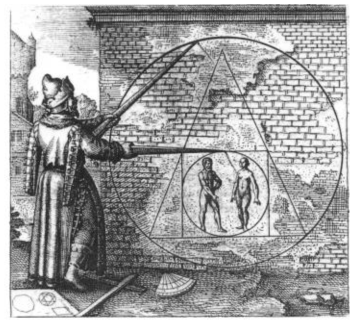
General comments on the learner's outlook on life (world view)

APPENDIX 3

PORTFOLIO OF ILLUSTRATIONS



PICTURE 1



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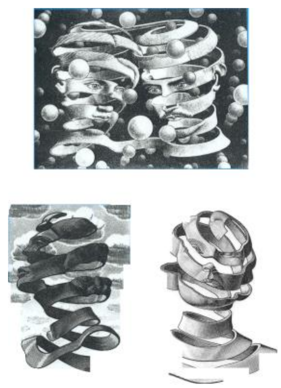

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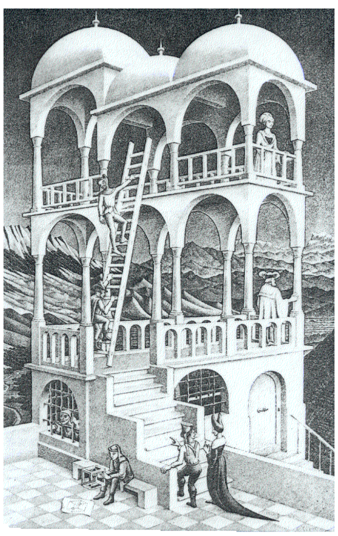
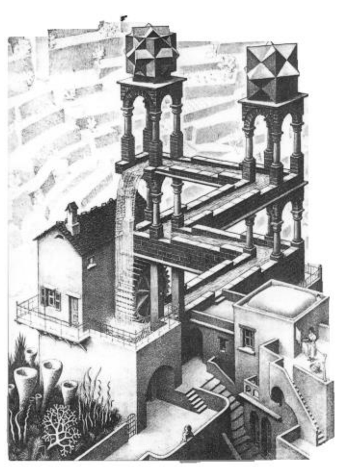
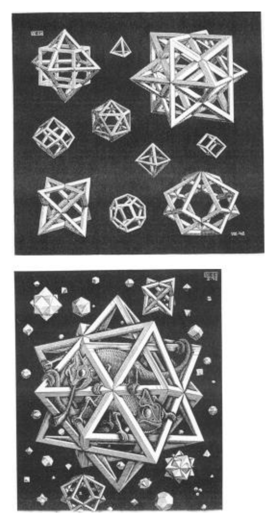


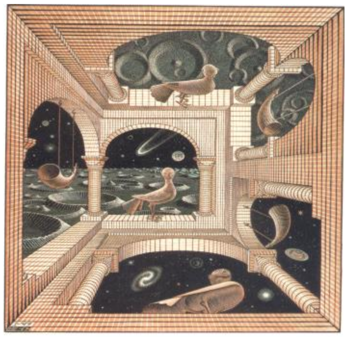
PICTURE 2



PICTURE 3

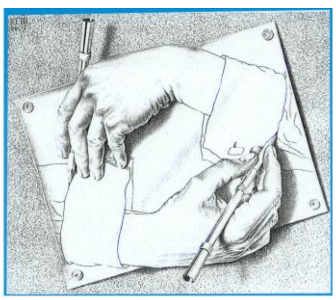


PICTURE 6



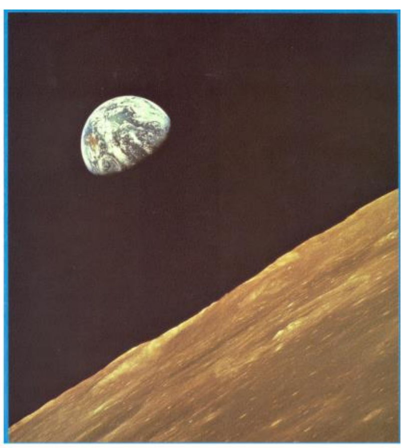
PICTURE 7

PICTURE 10



PICTURE 8

PICTURE 11



PICTURE 9

APPENDIX 4

ADAPTED MONASH SPACE TEST

Note: For copyright reasons Appendix 4, pp. 340-365, has not been reproduced

(Co-ordinator, ADT Project, Curtin University of Technology, 01.12.03)

APPENDIX 5

ADAPTED MATHEMATICAL PROCESSING QUESTIONNAIRE

On this questionnaire you are asked to consider how you solved the tasks in the test that you have just written. Every task is accompanied by at least two possible solution strategies.

For every task you are asked to indicate which solution strategy is the one you applied when you solved the tasks (place a ✓ in the appropriate box).

If, for any task you think that none of the suggested solution strategies is the one that you used, or is very similar to the one you used, you are asked to explain, in the space provided, the method you used when you first solved the task.

Even if you did not get the ‘correct’ answer to the task you are still asked to articulate your solution strategy.

Name:

TASK 04

☐ SOLUTION 1

To answer this question I imagined what the ‘F’ would look like once I stamped the stamp.

☐ SOLUTION 2

I drew a mental diagram and worked out the reflection of the ‘F’ in that way.

☐ SOLUTION 3

I looked at the five possible solutions and matched them to the original stamp.

☐ SOLUTION 4

I looked at the five possible solutions and matched them to the original stamp by reflecting each one.

☐ SOLUTION 5

I took a guess.

SOLUTION 6

I did not use any of the above methods.

I attempted the problem in this way:

TASK 05

☐ SOLUTION 1

I took each cube of the stack and unfolded them.

☐ SOLUTION 2

I looked at the unfolded cube at the bottom of the page and worked out the letters of each face by matching them to the stack.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 06

☐ SOLUTION 1

I stretched out each path and visualised which one was the longest.

☐ SOLUTION 2

I measured each path to see which one was the longest.

☐ SOLUTION 3

I could immediately see which one was the longest.

☐ SOLUTION 4

I took a guess.

SOLUTION 5

I did not use any of the above methods.

I attempted the problem in this way:

TASK 07

☐ SOLUTION 1

I went through the entire process in my head and imagined the solution.

☐ SOLUTION 2

I took each of the five solutions and folded them.

☐ SOLUTION 3

I folded a piece of paper and followed the instructions.

☐ SOLUTION 4

I took a guess.

SOLUTION 5

I did not use any of the above methods.

I attempted the problem in this way:

TASK 08

☐ SOLUTION 1

I went through the whole process in my head and imagined the solution.

☐ SOLUTION 2

I looked at the sequence of diagrams and worked backwards to work out the final solution.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 09

☐ SOLUTION 1

I did it by trial and error

☐ SOLUTION 2

I started by shading one square and then built up my pattern from there.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 10

☐ SOLUTION 1

I folded up the net and visualised what the house looked like in reality. I then matched that image to the correct answer.

☐ SOLUTION 2

I looked at the five solutions and eliminated the impossible options.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 11

☐ SOLUTION 1

I drew in all the main axes and then counted them.

☐ SOLUTION 2

I counted all the corners and divided by two.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 12

☐ SOLUTION 1

I imagined myself on the other side of the window and visualised the picture.

☐ SOLUTION 2

I just knew what it was going to look like.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 13

☐ SOLUTION 1

I looked at Figure A and Figure B and worked out that Figure B is obtained by moving the shapes in Figure A down by one block.

I then applied that rule to Figure C and Figure D.

☐ SOLUTION 2

I took a guess.

SOLUTION 3
I did not use any of the above methods.

I attempted the problem in this way:

TASK 14

☐ SOLUTION 1

I looked at Figure A and Figure B and worked out that Figure B is obtained by reflecting and rotating Figure A.

I then applied that rule to Figure C and D.

☐ SOLUTION 2

I took a guess.

SOLUTION 3

I did not use any of the above methods.

I attempted the problem in this way:

TASK 15

☐ SOLUTION 1

I took A and rotated it to obtain the other figures. I then eliminated the odd one out.

☐ SOLUTION 2

I imagined them all on top of each other and eliminated the odd one out.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 16

☐ SOLUTION 1

I looked at the first two columns and saw that the second column is obtained by halving the shapes in the first column.
I then looked at the last two columns and saw that the third column is obtained by again halving the shapes in the second column.
The shapes in the first column are shaded vertically. The shapes in the second column are shaded horizontally and the shapes in the third column are shaded grid-like.

☐ SOLUTION2

I looked at the third column and saw that each shape consists of the bottom right-hand quarter of the original shape.
The shading of the shapes in the third column is grid-like.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 17

☐ SOLUTION 1

I worked out the pattern on the left and chose the correct answer from the right.

☐ SOLUTION 2

I took each of the options on the right and tested them against the statements on the left. I eliminated the incorrect ones.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 18

☐ SOLUTION 1

I imagined the whole stack and counted each block.

☐ SOLUTION 2

I imagined each horizontal level of the stack and counted the blocks in each level.
I then added up the number of blocks for each level.

☐ SOLUTION 3

I imagined each vertical level of the stack and counted the blocks in each level.
I then added up the number of blocks for each level.

☐ SOLUTION 4

I took a guess.

SOLUTION 5

I did not use any of the above methods.

I attempted the problem in this way:

TASK 19

☐ SOLUTION 1

I took each of the four shapes below and divided them up into the two objects above.

☐ SOLUTION 2

I took the two objects above and combined them in many different ways to obtain the four shapes below.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 20

☐ SOLUTION 1

I imagined what the shape would look like from behind.

☐ SOLUTION 2

The shape looks the same from behind as it does from the front.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 21

☐ SOLUTION 1

I looked at each shape, covered up the one half and imagined the resultant mirror image.

☐ SOLUTION 2

I physically folded each shape along the dotted line.

☐ SOLUTION 3

I covered up the one half of the shape and drew the mirror image.

☐ SOLUTION 4

I took a guess.

SOLUTION 5

I did not use any of the above methods.

I attempted the problem in this way:

TASK 22

☐ SOLUTION 1

I took each of the five solutions and pushed the two pieces together to see whether they would form a cube as shown above.

☐ SOLUTION 2

I took the cube and imagined what the two pieces would look like once I pushed them apart. I then selected the correct answer.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

TASK 23

Please explain how you went about drawing your path.

TASK 24

☐ SOLUTION 1

I turned the figure ninety degrees to the right and then inserted the letters as instructed.

☐ SOLUTION 2

I turned the figure ninety degrees to the left and then inserted the letters as instructed.

☐ SOLUTION 3

I took a guess.

☐ SOLUTION 4

I turned the figure ninety degrees to the right and then reflected it around a central axis.

SOLUTION 5

I did not use any of the above methods.

I attempted the problem in this way:

TASK 25

☐ SOLUTION 1

I searched for the five pieces in the big square and eliminated the ones that did not fit any of the five pieces.

☐ SOLUTION 2

I took each of the five pieces and searched for the matching piece in the square.

☐ SOLUTION 3

I took a guess.

SOLUTION 4

I did not use any of the above methods.

I attempted the problem in this way:

APPENDIX 6

HANDS-ON ACTIVITY TEST

Each of the activities below were printed out on separate cards.

Activity 1

SHAPE SORTER

- Each yellow shape fits through ONE of the openings in the ball
- Your task is to place the shapes through the openings into the ball AS QUICKLY AS POSSIBLE
- You can have two attempts
- Both attempts will be timed
- You will be asked to reflect on your strategy in solving this problem.

Activity 2

PUZZLE

- Your task is to complete this puzzle AS SOON AS POSSIBLE
- You will be asked to reflect on your strategy in solving this problem
- You will be timed.

Activity 3

TANGRAM

- Assemble the <u>seven pieces</u> into a perfect square
- None of the pieces may overlap and there may be no gaps in between the pieces
- You will be timed
- You will be asked to reflect on your strategy in solving this problem.

Activity 4

SOMA CUBE

- Assemble the <u>seven pieces</u> into a cube
- You will be timed
- You will be asked to reflect on your strategy in solving this problem.

Activity 5a

FRONT VIEW

- The two objects in front of you must not be moved
- Draw the objects on the table as you see them in front of you as accurately as possible.

Activity 5b

OBSCURED REVERSE VIEW

- Without moving from your present position, draw the same two objects as they would appear to you if you were sitting on the opposite side of the table.

Activity 6

AERIAL VIEW

- The picture shows a number of objects which were photographed from the side
- Draw an aerial view of the same objects; ie draw them as they would appear from above.

Activity 7

ROUTE MAP

- Draw an accurate map of the route that you would take from your school to the Standard Bank building in the centre of town
- Provide as mush detail as you think is necessary for a stranger to follow your map

Activity 7 (Alternative for the rural school)

ROUTE MAP

- Draw an accurate map of the route that you would take from your school to Scott's Petrol Station in the centre of Tarkastad
- Provide as much detail as you think is necessary for a stranger to follow your map.

Activity 8

SCHOOL HALL

- Imagine that you are standing in the middle of your empty school hall facing the stage
- There are stairs leading up to the stage on either side
- Just in front of the stage are two long rectangular tables and just to the right of you is a round table
- Just behind you is a trapezoidal table (in the shape of a trapezium)
- Draw the above scene.

Activity 8 (Alternative for the rural school)

SCHOOL HALL

- Imagine that you are standing in the middle of an empty classroom facing the chalkboard
- There is a window on either side of the chalkboard
- Just in front of the chalkboard are two long rectangular tables and just to the right of you is a round table
- Just behind you is a trapezoidal table (in the shape of a trapezium)
- Draw the above scene.

APPENDIX 7

WORLD VIEW OF SPACE: A FRAMEWORK FOR INTERVIEWS USING BIPOLAR CODES

Naturalism – Religion

1. Do you think that space was created?
    a. If so, by whom/what?
    b. Why do you say that?

2. Do you think that space evolved?
    a. If so, from where/to where?
    b. Why do you say that?

3. Do you think space was always there – a big unchanging ‘playground’?

Chaos – Order

1. Do you think that space is the same everywhere?
    a. If so explain your answer.

2. Do you think that space works according to the same order principles (rules) no matter where you are?
    a. If so, try and explain what order you observe?
    c. Why do you say that?

3. Do you think that space is haphazard and unorderly?
   Do you think space changes its rules?

    a. If so, explain what you mean?

Mystery - Knowledge

1. Do you think that we can ever understand what space is or is it something mysterious?
    a. If you think the latter, try and explain what about space do you find mysterious?
    b. Why do you say that?

2. Do you think that space is something certain, something that can be defined or explained?
    a. If so, can you articulate this explanation?

Function - Purpose

Function – what it **does** … utility
Purpose – what it **is** … why it is … transcendental

1. What do you think is the function of space? What does it do? Does space serve any function?
2. What is space? Why is there space?

Mundane - Special

1. Do you see space as something beautiful or not really?
   a. If so, try and describe this beauty.

2. Do you think that space is something special?
   b. If so, what is special about space?

3. Do you think that space is just something ordinary that we walk around in?

Mathematical – Non-Mathematical (Cartesian – non-cartesian)

1. Where in space do you locate earth?
2. Where in space do you locate yourself?
3. What do you understand by 2-dimensional space?
4. What do you understand by 3-dimensional space?
5. Do you think space has other dimensions?

Finite - Infinite

1. Where do you think space starts?
2. Where do you think space ends?
3. How big do you think space is?
4. How small do you think space is?
5. What does space rest on?

Tangible – Non-tangible

1. Can you see space? Explain.
2. Can you touch space? Explain.

Internal - External

1. Do you have space inside of you? Explain.
2. Does your mind have space? Explain.

APPENDIX 8

THE DIPT CLASSIFICATION MODEL

Note: For copyright reasons Appendix 8, p. 394, has not been reproduced

APPENDIX 9
ANALYSIS OF THE AMST

INTRODUCTION

This appendix provides graphic details of the analysis of each test item (from task 4 onwards) of the AMST. For each item the following descriptive information is provided:

- The acceptable response
- A diagram of each item to remind the reader what each item was about
- Where appropriate, a list of all the responses given by the participants
- The maximum mark obtainable
- The internal difficulty factor (see explanation below)
- Classification in terms of spatial visualisation and orientation (shaded accordingly)
- Classification in terms of two-dimension and three-dimension perspective (shaded accordingly)
- A table indicating the performance of the participating schools and gender performance differences.

This is followed by a bar graph which illustrates the performance of each participant.

Internal difficulty factor

The internal difficulty factor is a useful indicator as to how difficult each item was perceived by the participants. It is simply a rank based on the performance of the entire sample. For example, the internal difficulty factor of 1 for task 5 suggests that this was the most difficult item because only 22% of the sample managed the correct answer.

ITEM-BY-ITEM ANALYSIS

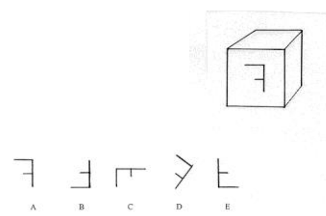
TASK 4

Acceptable response: B

The response to this question is either correct or incorrect.

Other responses given:
(with frequency)

A	11
C	3
D	3
E	3



Reminder

Maximum mark: 1

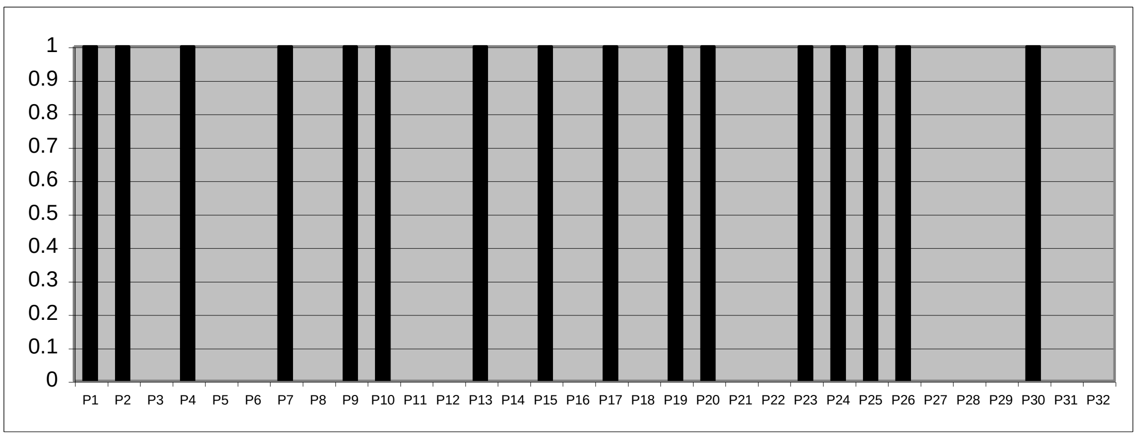
Internal difficulty factor: 10

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

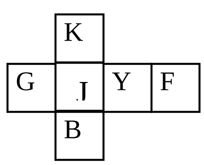
SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	4/7	4/7
S2	0/1	3/5	3/6
S3	0/2	3/4	3/6
S4	5/7	NA	5/7
S5	1/1	0/5	1/6
	6/11 55%	10/21 48%	16 50%



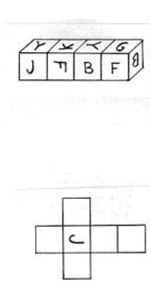
Overall performance in task 4 by the entire sample

TASK 5

Acceptable response:



The response to this question entails providing the correct five letters of the alphabet in the 5 empty squares



Reminder

5

Maximum mark:

Internal difficulty factor:

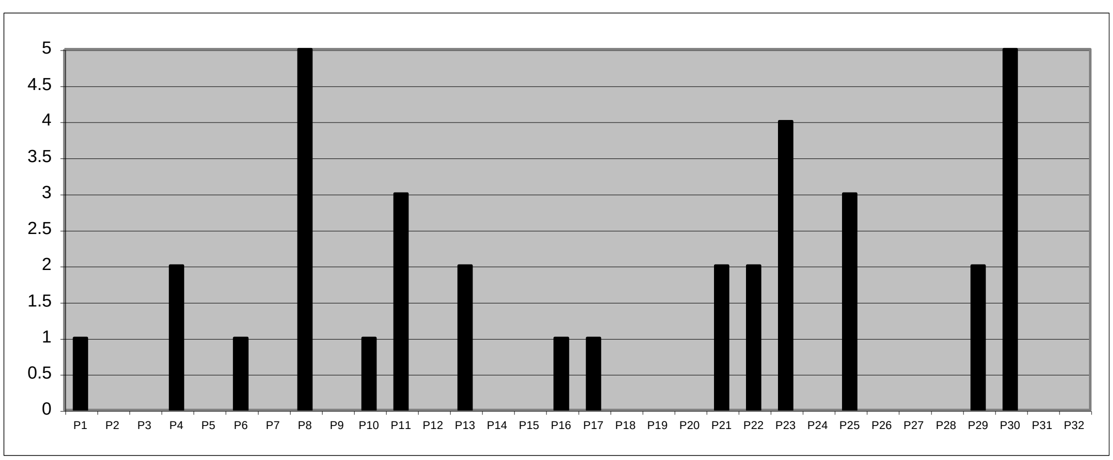
1

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=160
S1	NA	4/35	4/35
S2	3/5	8/25	11/30
S3	1/10	1/20	2/30
S4	11/35	NA	11/35
S5	5/5	2/25	7/30
	2/55 36%	15/105 14%	35 22%



Overall performance in task 5 by the entire sample

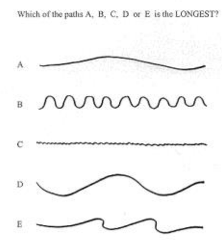
TASK 6

Acceptable response: B

The response to this question is either correct or incorrect.

Other response given:
(with frequency)

C	2
D	1



Reminder

Maximum mark: 1

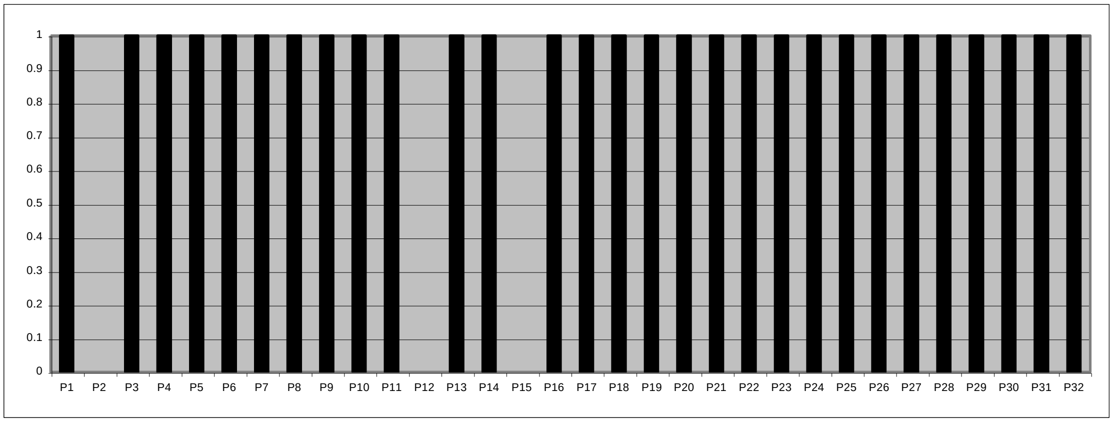
Internal difficulty factor: 22

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	6/7	6/7
S2	1/1	4/5	5/6
S3	2/2	3/4	5/6
S4	7/7	NA	7/7
S5	1/1	5/5	6/6
	11/11 100%	18/21 86%	29 91%

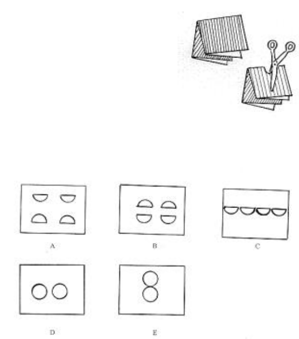


Overall performance in task 6 by the entire sample

TASK 7

Acceptable response: **D**

The response to this question is either correct or incorrect.



Reminder

Other responses given:
(with frequency)

A	4
B	7
C	5
E	5

Maximum mark: **1**

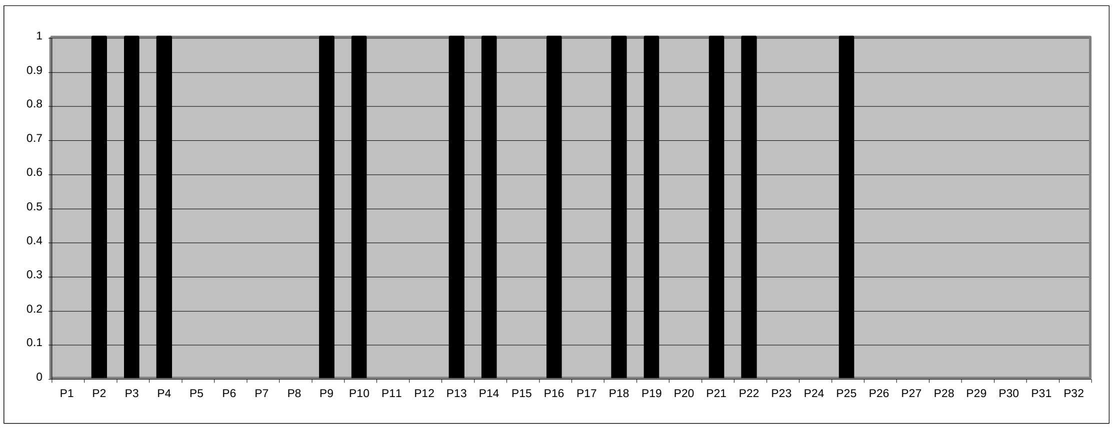
Internal difficulty factor: **6**

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

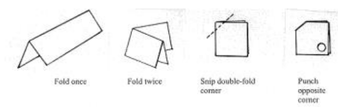
SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	3/7	3/7
S2	0/1	3/5	3/6
S3	2/2	2/4	4/6
S4	3/7	NA	3/7
S5	0/1	0/5	0/6
	5/11 45%	8/21 38%	13 41%



Overall performance in task 7 by the entire sample

TASK 8

Acceptable response: Their own drawing



Reminder

The response to this question entails drawing a diamond shape in the centre of square with a circle in each corner.

Maximum mark: 2

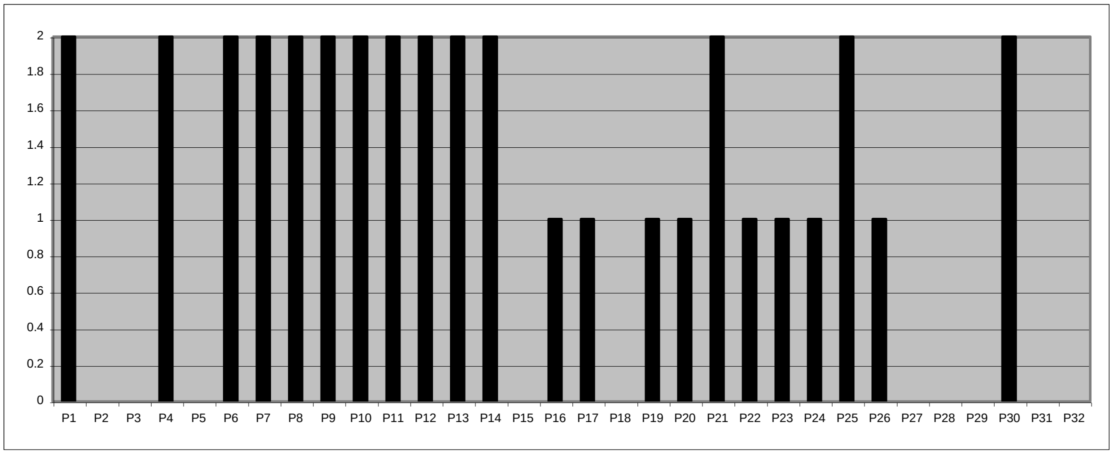
Internal difficulty factor: 15

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=64
S1	NA	8/14	8/14
S2	2/2	10/10	12/12
S3	3/4	2/8	5/12
S4	9/14	NA	9/14
S5	2/2	0/10	2/12
	16/22 73%	20/42 48%	36 56%

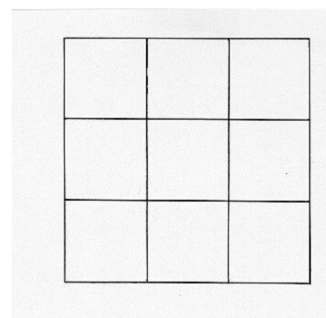


Overall performance in task 8 by the entire sample

TASK 9

Acceptable response: Their own drawing.

The response to this question entails shading in six squares so that no three shaded squares are in a straight line.



Reminder

Maximum mark: 1

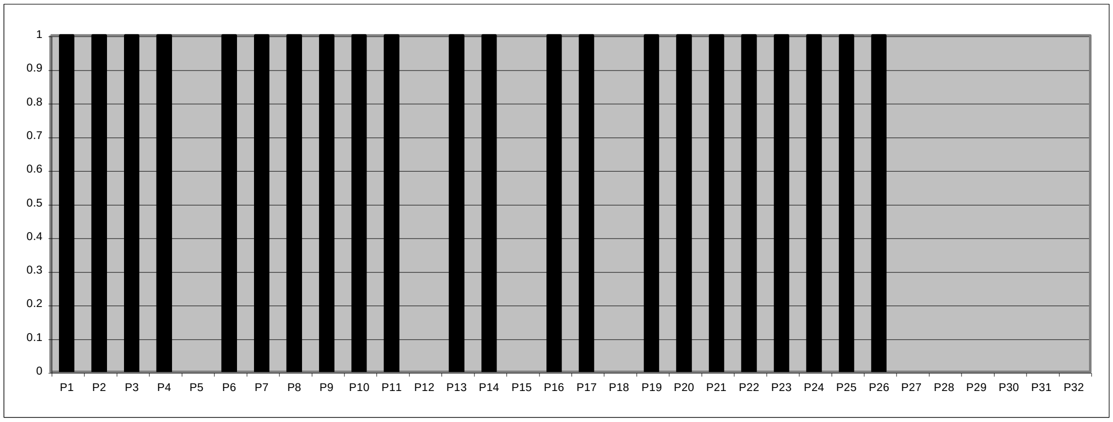
Internal difficulty factor: 19

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	6/7	6/7
S2	1/1	4/5	5/6
S3	2/2	2/4	4/6
S4	7/7	NA	7/7
S5	0/1	0/5	0/6
	10/11 91%	12/21 57%	22 69%



Overall performance in task 9 by the entire sample

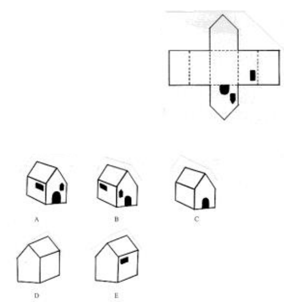
TASK 10

Acceptable response: D

The response to this question is either correct or incorrect.

Other responses given:
(with frequency)

A	6
B	14
E	3



Reminder

Maximum mark: 1

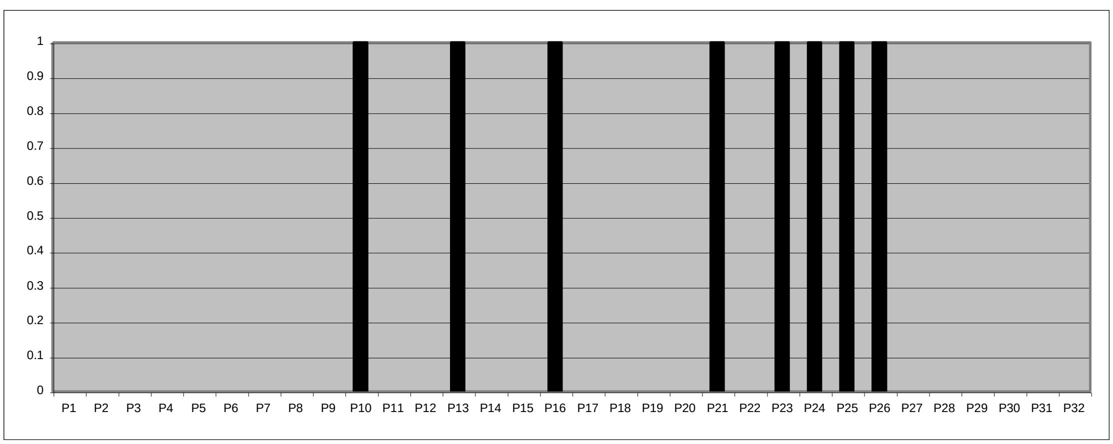
Internal difficulty factor: 2

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	0/7	0/7
S2	0/1	2/5	2/6
S3	1/2	0/4	1/6
S4	5/7	NA	5/7
S5	0/1	0/5	0/6
	6/11 55%	10/21 9%	8 25%

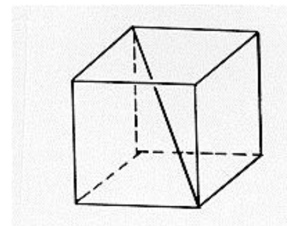


Overall performance in task 10 by the entire sample

TASK 11

Acceptable response: 4

The response to this question is either correct or incorrect.



Reminder

Other responses given:
(with frequency)

2	2
3	1
5	1
6	2
8	2
10	2
12	1

Maximum mark: 1

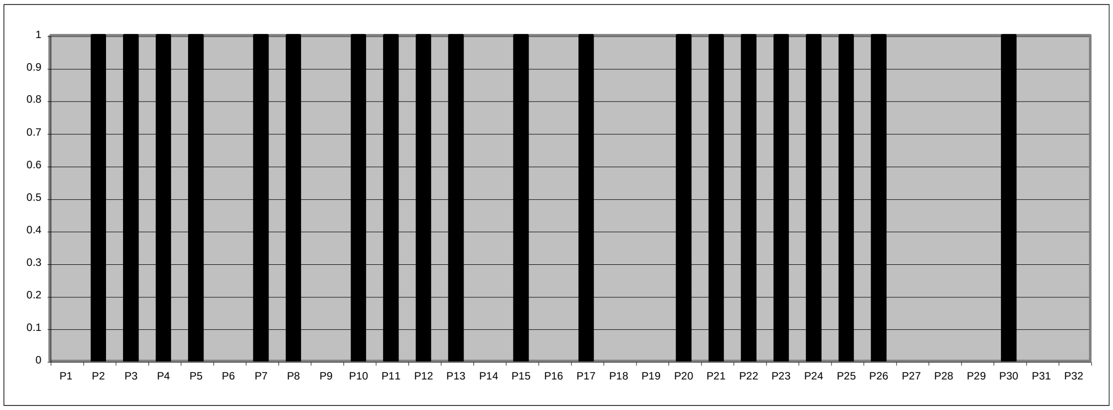
Internal difficulty factor: 16

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	5/7	5/7
S2	1/1	4/5	5/6
S3	0/2	2/4	2/6
S4	7/7	NA	7/7
S5	1/1	0/5	1/6
	9/11 82%	11/21 52%	20 62%

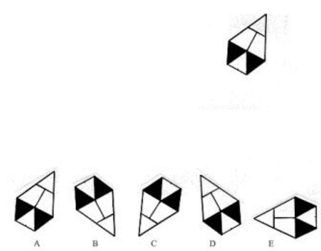


Overall performance in task 11 by the entire sample

TASK 12

Acceptable response: **D**

The response to this question is either correct or incorrect.



Reminder

Other responses given:
(with frequency)

A	13
B	2
C	1
E	1

Maximum mark: **1**

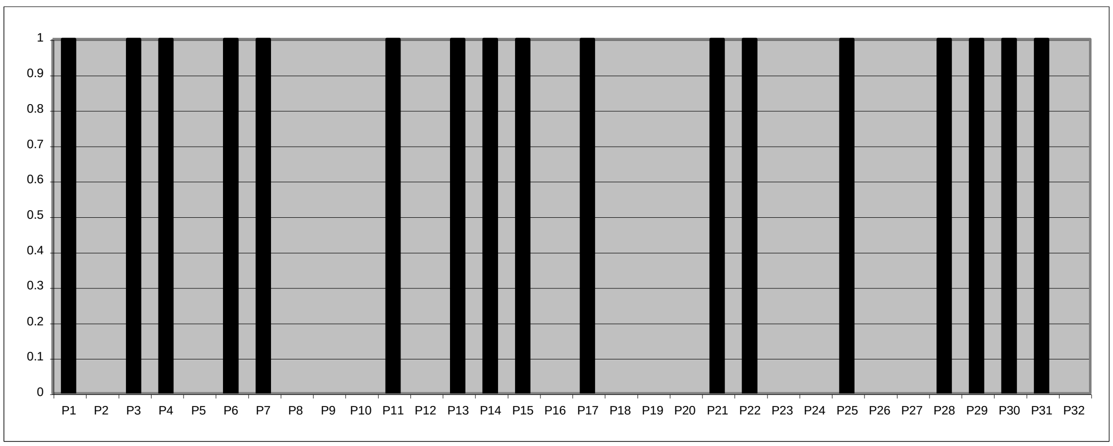
Internal difficulty factor: **13**

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	5/7	5/7
S2	1/1	1/5	2/6
S3	1/2	2/4	3/6
S4	3/7	NA	3/7
S5	1/1	3/5	4/6
	6/11 54%	11/21 52%	17 53%

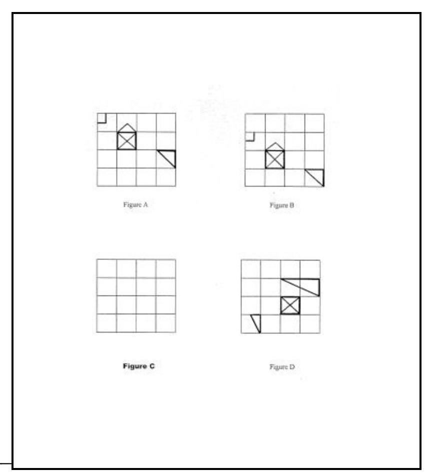


Overall performance in task 12 by the entire sample

TASK 13

Acceptable response: Their own drawing.

The response to this question entails drawing the three given shapes in the correct position (translated 1 unit up).



Reminder

Maximum mark: 3

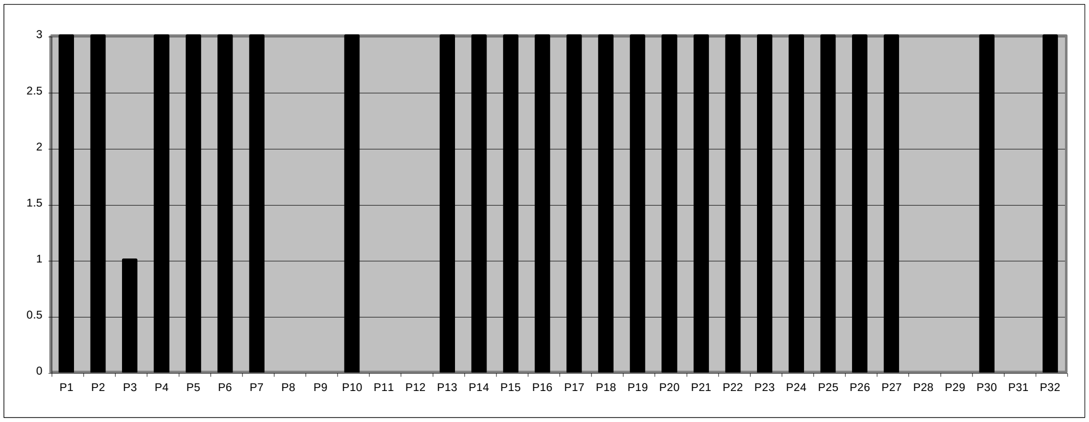
Internal difficulty factor: 21

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=96
S1	NA	19/21	19/21
S2	0/3	6/15	6/18
S3	6/6	12/12	18/18
S4	21/21	NA	21/21
S5	3/3	6/15	9/18
	30/33 91%	43/63 68%	73 76%

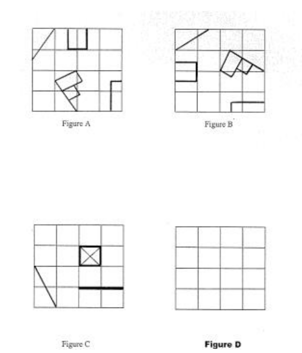


Overall performance in task 13 by the entire sample

TASK 14

Acceptable response: Their own drawing.

The response to this question entails drawing the three given shapes in the correct position (reflection and rotation).



Reminder

Maximum mark: 3

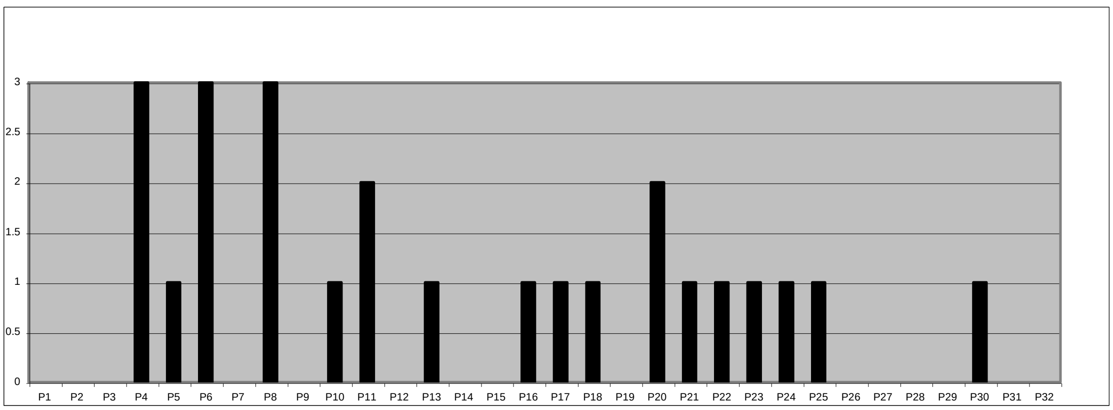
Internal difficulty factor: 3

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=96
S1	NA	7/21	7/21
S2	2/3	5/15	7/18
S3	1/6	2/12	3/18
S4	7/21	NA	7/21
S5	1/3	0/15	1/18
	11/33 33%	14/63 22%	25 26%



Overall performance in task 14 by the entire sample

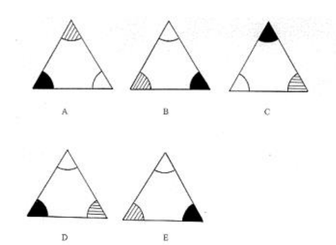
TASK 15

Acceptable response: D

The response to this question is either correct or incorrect.

Other responses given:
(with frequency)

A	11
B	5
C	9



Reminder

Maximum mark: 1

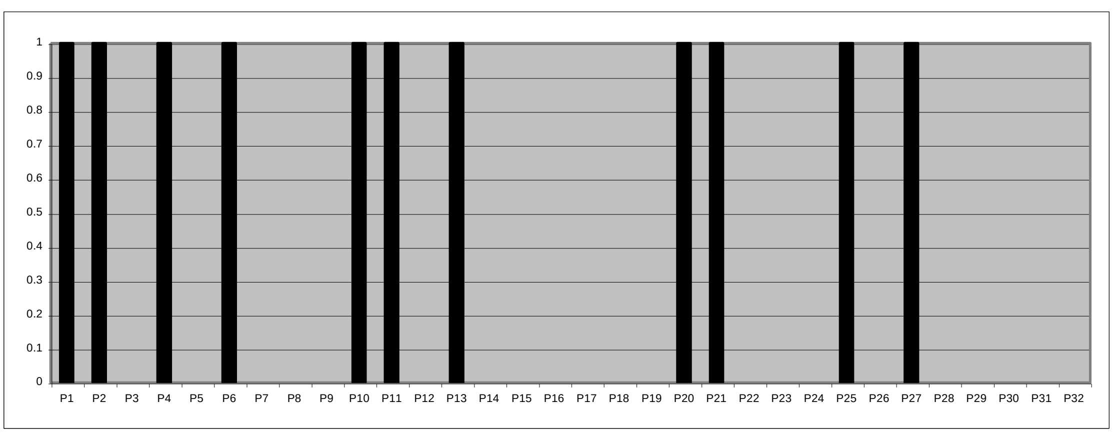
Internal difficulty factor: 4

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	4/7	4/7
S2	1/1	2/5	3/6
S3	0/2	0/4	0/6
S4	3/7	NA	3/7
S5	0/1	1/5	1/6
	4/11 36%	7/21 33%	11 34%

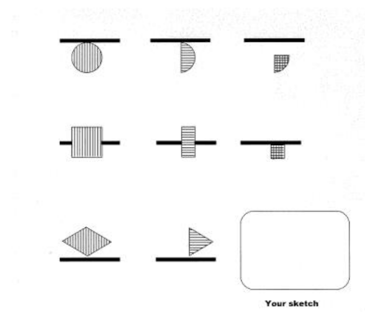


Overall performance in task 15 by the entire sample

TASK 16

Acceptable response: Their own drawing.

The response to this question entails drawing a new figure once the pattern has been established.



Reminder

Maximum mark: 1

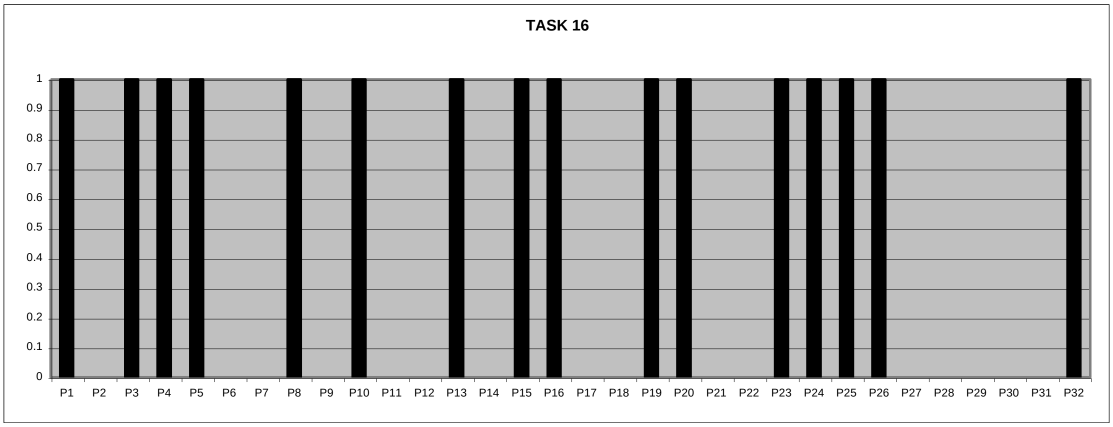
Internal difficulty factor: 10

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	4/7	4/7
S2	0/1	3/5	3/6
S3	1/2	2/4	3/6
S4	5/7	NA	5/7
S5	0/1	1/5	1/6
	6/11 54%	10/21 48%	16 50%



Overall performance in task 16 by the entire sample

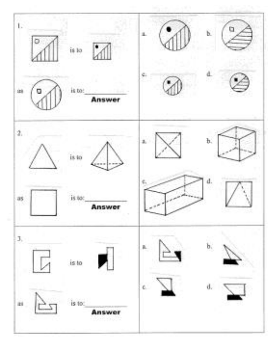
TASK 17

Acceptable response: c,b,d

The response to this question is either correct or incorrect.

Other responses given:
(with frequency)

a 2, a 9, a 5
b 4, c 1, b 6
d 0, d 1, c 1



Reminder

Maximum mark: 3

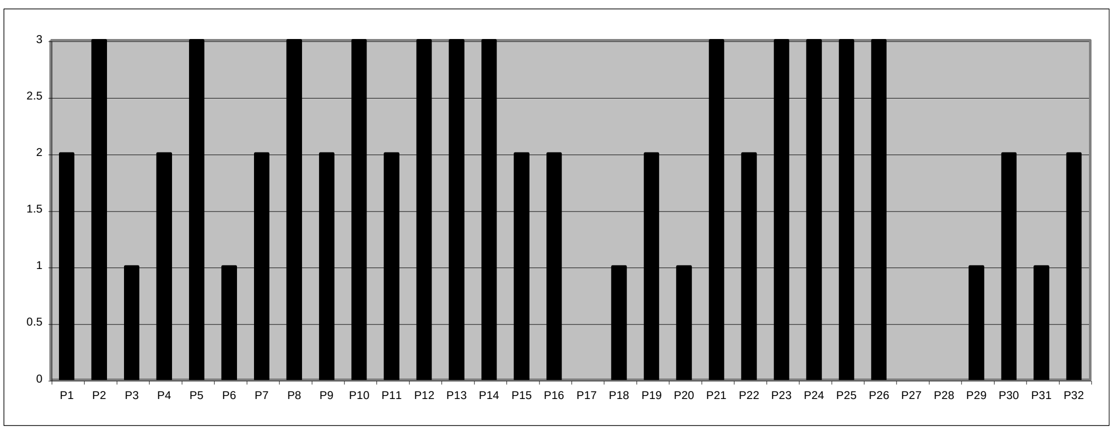
Internal difficulty factor: 18

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=96
S1	NA	14/21	14/21
S2	2/3	14/15	16/18
S3	5/6	5/12	10/18
S4	18/21	NA	18/21
S5	2/3	4/15	6/18
	27/33 82%	37/63 59%	64 67%



Overall performance in task 17 by the entire sample

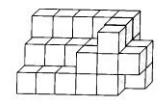
TASK 18

Acceptable response: 37

The response to this question is either correct or incorrect.

Other responses given:
(with frequency)

36,78,21,44,45,
41,33,35,20,
42,43,40,116,
70,24,40,22



Reminder

Maximum mark: 1

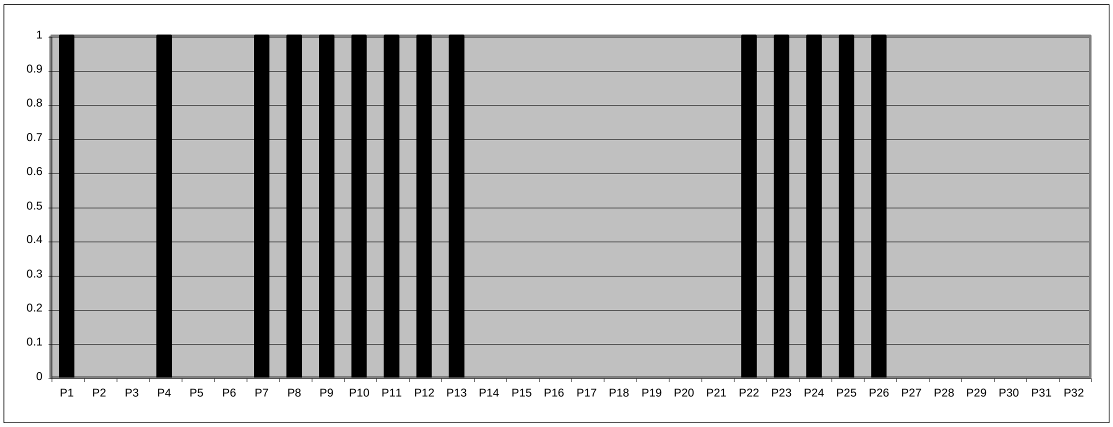
Internal difficulty factor: 8

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	3/7	3/7
S2	1/1	5/5	6/6
S3	0/2	0/4	0/6
S4	5/7	NA	5/7
S5	0/1	0/5	0/6
	6/11 54%	8/21 38%	14 44%

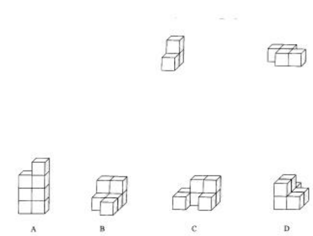


Overall performance in task 18 by the entire sample

TASK 19

Acceptable response: C

The response to this question is either correct or incorrect.



Reminder

Other responses given:
(with frequency)

A	3
B	3
C	2

Maximum mark: 1

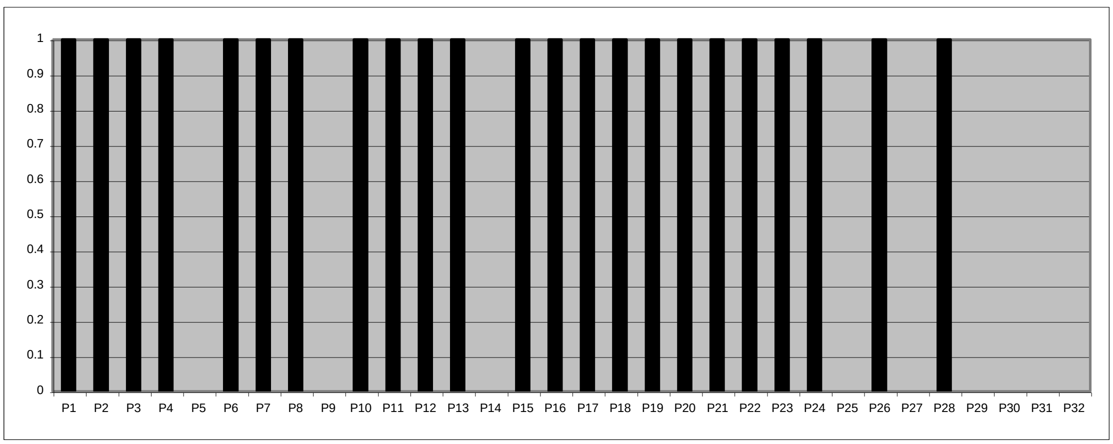
Internal difficulty factor: 20

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	6/7	6/7
S2	1/1	4/5	5/6
S3	1/2	4/4	5/6
S4	6/7	NA	6/7
S5	0/1	1/5	1/6
	8/11 73%	15/21 71%	23 72%

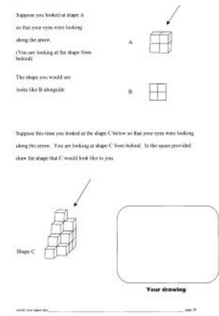


Overall performance in task 19 by the entire sample

TASK 20

Acceptable response: Their own drawing.

The response to this question entails drawing a new shape of the figure as seen from the back.



Reminder

Maximum mark: 1

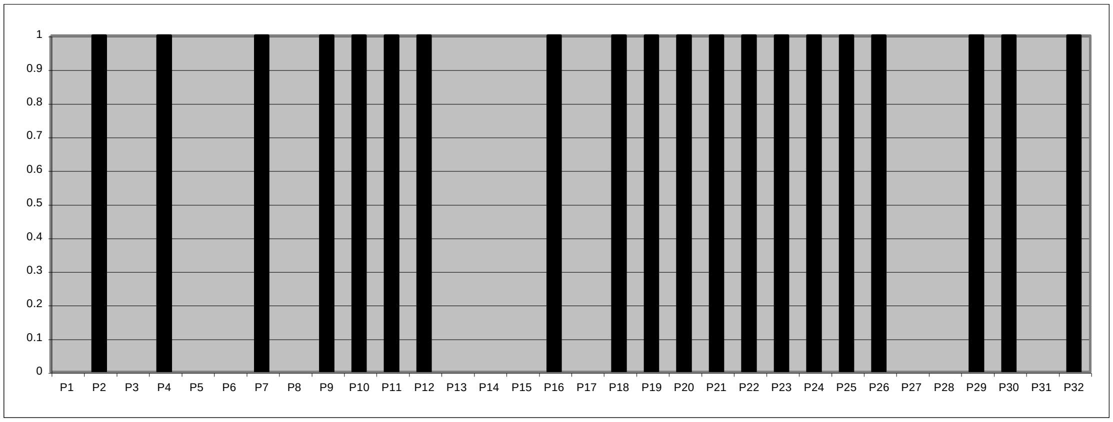
Internal difficulty factor: 5

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	3/7	3/7
S2	1/1	2/5	3/6
S3	0/2	0/4	0/6
S4	4/7	NA	4/7
S5	0/1	0/5	0/6
	5/11 45%	5/21 24%	10 31%



Overall performance in task 20 by the entire sample

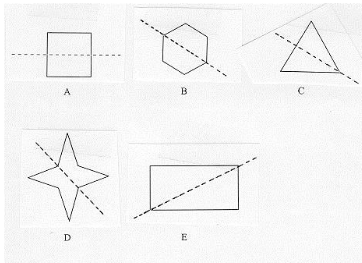
TASK 21

Acceptable response: E

The response to this question is either correct or incorrect.

Other responses given:
(with frequency)

B	3
C	4
D	2



Reminder

Maximum mark: 1

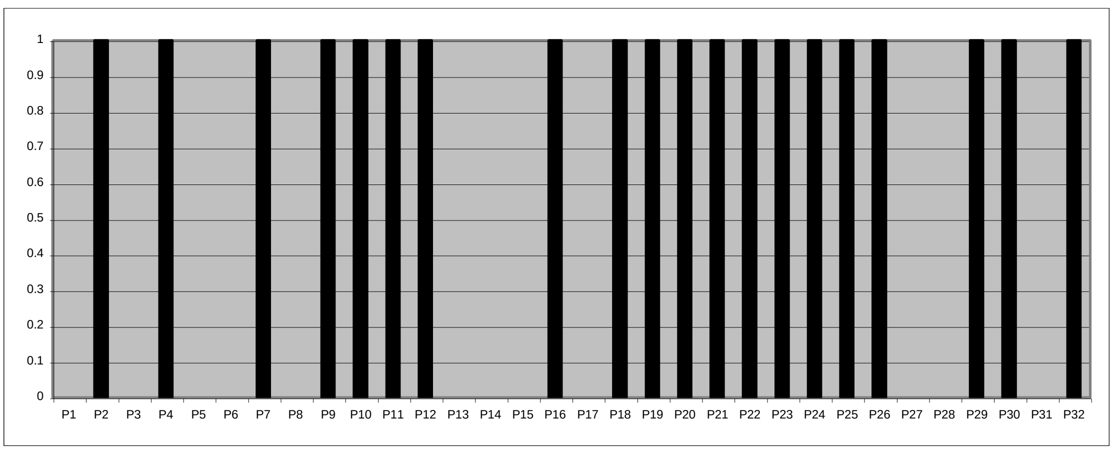
Internal difficulty factor: 16

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	3/7	3/7
S2	1/1	3/5	4/6
S3	1/2	2/4	3/6
S4	7/7	NA	7/7
S5	1/1	2/5	3/6
	10/11 91%	11/21 52%	20 62%



Overall performance in task 21 by the entire sample

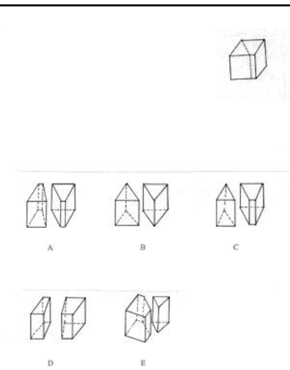
TASK 22

Acceptable response: C

The response to this question is either correct or incorrect.

Other responses given:
(with frequency)

A	2
B	5
D	2
E	5



Reminder

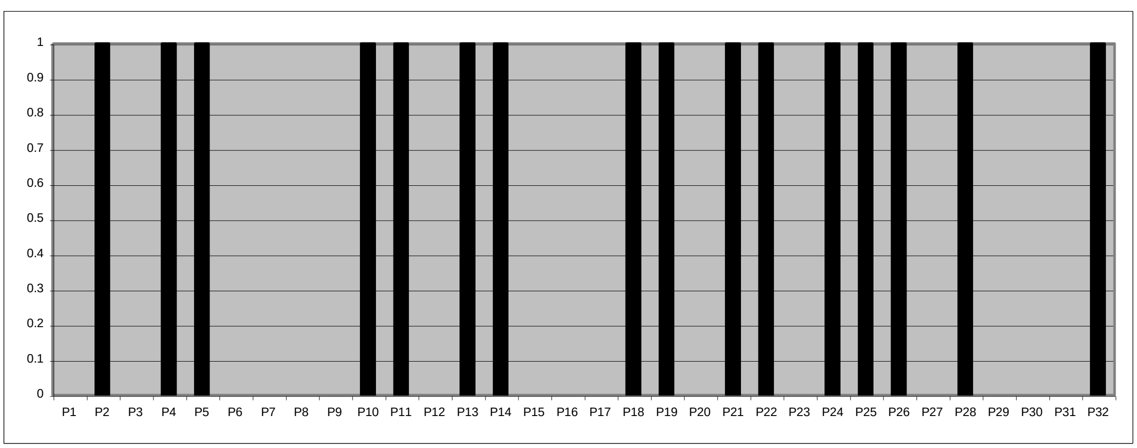
Maximum mark: 1

Internal difficulty factor: 10

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

SCHOOL	CORRECT		
	Males	Females	Total N=32
S1	NA	3/7	3/7
S2	1/1	2/5	3/6
S3	1/2	2/4	3/6
S4	5/7	NA	5/7
S5	0/1	2/5	2/6
	7/11 64%	9/21 43%	16 50%

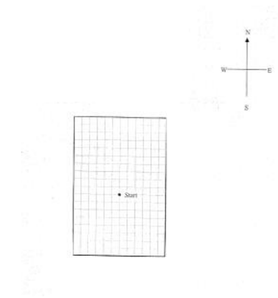


Overall performance in task 22 by the entire sample

TASK 23

Acceptable response: Their own drawing.

The response to this question entails drawing a path according to given directions



Reminder

Maximum mark: 1

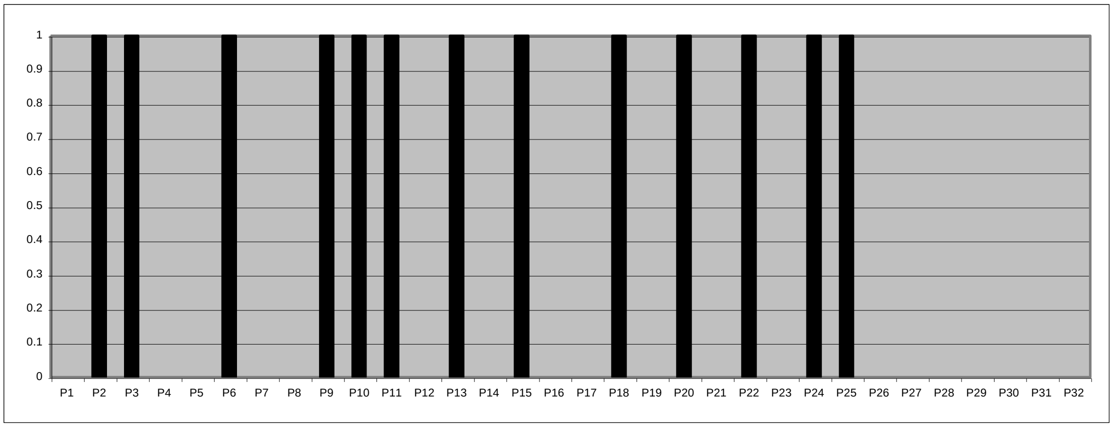
Internal difficulty factor: 6

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

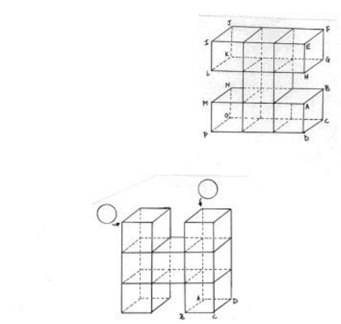
SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	3/7	3/7
S2	1/1	3/5	4/6
S3	0/2	2/4	2/6
S4	4/7	NA	4/7
S5	0/1	0/5	0/6
	5/11 45%	8/21 38%	13 41%



Overall performance in task 23 by the entire sample

TASK 24

Acceptable response: J,M



Reminder

The response to this question is either correct or incorrect.

Other responses given: (with frequency)

LM, IP, FJ,
JO, EB,LP,KM,
IM,FH, LD,
FM, IC,
KO - 3

Maximum mark: 2

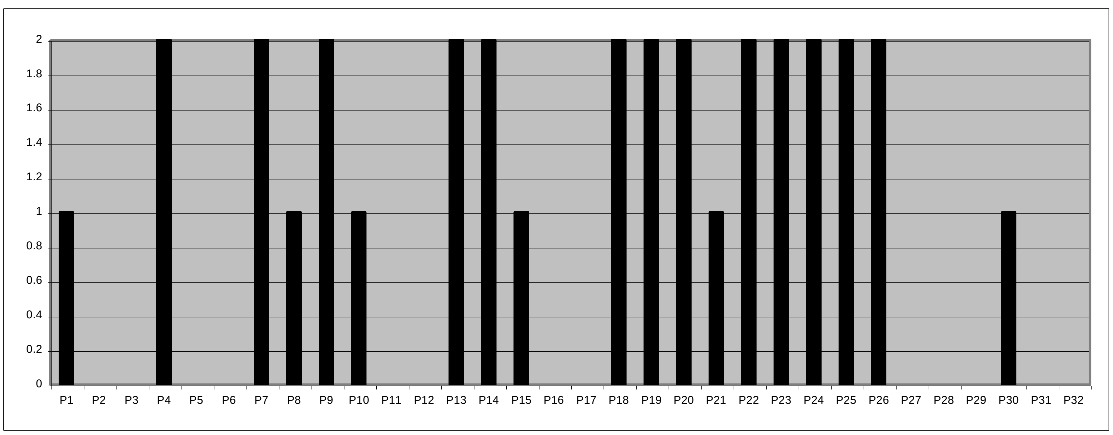
Internal difficulty factor: 9

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=64
S1	NA	5/14	5/14
S2	0/2	6/10	6/12
S3	2/4	5/8	7/12
S4	13/14	NA	13/14
S5	1/2	0/10	1/12
	16/22 73%	16/42 38%	32 50%

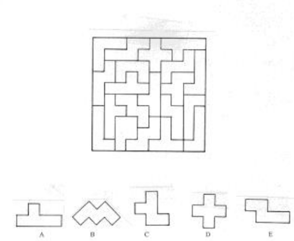


Overall performance in task 24 by the entire sample

TASK 25

Acceptable response: C

The response to this question is either correct or incorrect.



Reminder

Other responses given:
(with frequency)

A	3
B	9
D	2
E	6

Maximum mark: 1

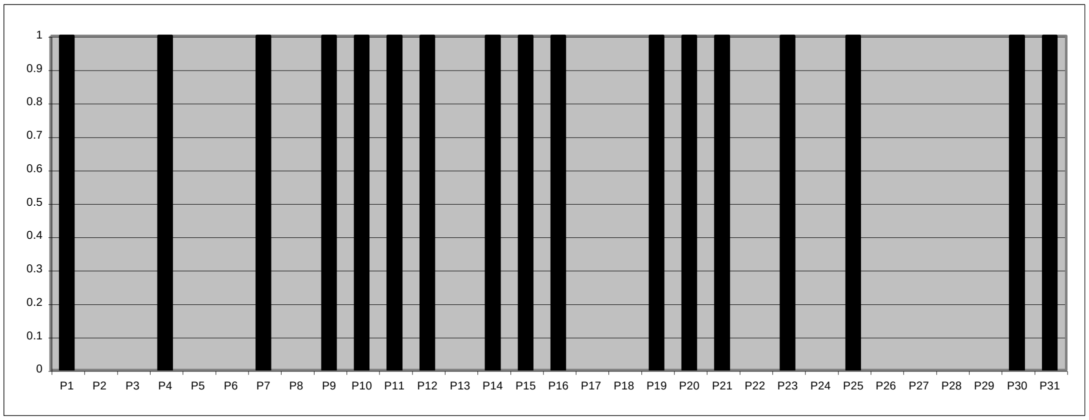
Internal difficulty factor: 13

Classification:

Spatial visualisation	Spatial orientation	Two dimensions	Three dimensions

Performance:

SCHOOL	CORRECTLY ANSWERED		
	Males	Females	Total N=32
S1	NA	2/7	2/7
S2	1/1	4/5	5/6
S3	1/2	2/4	3/6
S4	5/7	NA	5/7
S5	0/1	2/5	2/6
	7/11 64%	10/21 48%	17 53%



Overall performance in task 25 by the entire sample