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Trip Distribution Modelling Using Neural Network

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This thesis is presented for the Degree of
Doctor of Philosophy of
Curtin University

June 2014

Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made. This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

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Abstract

Trip distribution is the second step of the transport modelling process. Errors in trip distribution will propagate through the other stages of transport modelling and will leads to inaccurate projected traffic volumes. Finding a robust and efficient method of estimating trip distribution has therefore always been an objective for transport modellers. The problem of trip distribution is nonlinear and complex. Neural networks (NN) have been used effectively for solving nonlinear problems and have been used in different disciplines including traffic engineering. Accordingly, in this research a new NN model has been researched to estimate the distribution of journey to work trips. This research is unique in three aspects: (1) the training of the model was based on a generalized regression neural network (GRNN) algorithm while the majority of previous studies have used a back-propagation (BP) algorithm. The advantage of the GRNN model over other feed-forward or feedback neural network techniques is its simplicity and practicality. (2) The input data for the GRNN model was based on the land use data for each zone and the corresponding distance between a pair of zones, while previous NN models have used trip productions, trip attractions and the distance between a pair of zones as input. (3) The proposed GRNN model will establish a frame work for combined trip generation and distribution modelling. As a case study, the model was applied to the journey to work trips in the City of Mandurah in Western Australia. The results of the GRNN model were compared with the well-known doubly-constrained gravity model and the BP model. The modelling analysis indicated that a validated GRNN model could provide slightly better results than both the gravity and BP models, with a higher correlation coefficient and lower root mean square error (RMSE), and could be improved if the size of the training data set is increased.

Accordingly, the recommended GRNN model has been presented in the Australasian Transport Research Forum (ATRF) in 2013 and published in the *Road and Transport Research Journal in 2014*. Copies of the papers are also provided in **Appendix A** of this thesis.

Acknowledgements

I would like to express my gratitude to all those who gave me the possibility to complete this thesis. I owe my deepest gratitude to my research supervisor, Professor Hamid Nikraz for his supervision and support. His valuable guidance aided me to comprehend and to develop this research topic.

I appreciate the help of Transcore Pty Ltd, who gave me the opportunity of utilizing the strategic transport model developed for the Mandurah and Peel Region in Western Australia. I also would like to thank the City of Mandurah and the Department of Planning, Western Australia, who provided me with valuable information for my research.

To my parents, who have inspired my life in all manners,
for their continuous support which has helped me to pursue my higher education.

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Nomenclature

ANN	Artificial neural network
NN	Neural network
GRNN	Generalized regression neural network
BP	Back-propagation
LM	Levenberg-Marquardt
TAZ	Traffic analysis zone
PE	Processing element
RMSE	Root mean square error
MAE	Mean absolute error
R^2	Coefficient of determination
GM	Gravity model
MATLAB	Matrix Laboratory
VLR	Variable learning rate
NHB	None-home-based
DoP	Department of Planning
OD	Origin destination
SD	Standard deviation
ABS	Australian Bureau of Statistics
R	Pearson's correlation coefficient
P_j	Number of trips produced by zone j
A_i	Number of trips attracted to zone i
d_{ij}	Distance between zone i and zone j
T_{ij}	Number of trips produced in zone i and attracted to zone j
W_n	Weighting factor
δ	Spread factor

1 1. RESEARCH PROJECT INTRODUCTION

1.1 Introduction

This research has led to the development of a generalized regression neural network (GRNN) model as a recommended approach to estimating trip distribution, and the performance of this model has been compared with multi-layer feed forward back-propagation (hereafter referred to as BP model) and gravity models. The recommended approach is unique in three aspects:

- The input data for the GRNN model is based on land use data for each zone and the corresponding distance between the two zones, while previous neural models have used trip productions, trip attractions and the distance between a pair of zones as input to the neural models.
- The training of the neural model is based on a GRNN algorithm while previous studies have used a BP algorithm.
- The proposed model is providing a combined trip generation and distribution modelling frame work using neural networks.

As a case study, the new approach was applied to the journey to work (JTW) trips for the Mandurah area in Western Australia. Accordingly, three different models were developed: the GRNN, BP and gravity models. The recommended GRNN model was compared with gravity model method and previously established neural models based on a BP algorithm. The root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and the target data for training and the testing data set were used for the comparison of the models and advantage of the GRNN model over the previous models were demonstrated.

1.2 Research Background

1.2.1 Travel Demand Modelling

There are different approaches for travel demand modelling. The purpose of all of the travel demand approaches is to estimate the existing or future trips within the study area and project the traffic volumes on the existing or future road network. Four step modelling is the most common approach for travel demand modelling and is referred to as traditional or classical model. The traditional four step model consists of the following steps:

Trip Generation: The first model stage is to produce travel demand (trips) at zonal level, based on household characteristics or demographic data. The number of trips attracted to a zone is related to the number and size of the modeled activities available in each zone. These relationships apply to separate journey purposes.

Trip Distribution: The trip distribution stage distributes the trip productions amongst attraction zones according to the appropriate costs of travel and the model sensitivity parameters. Thus, the model creates trip matrices of travel by mode, demand and journey purpose for 24-hour trips.

Mode Choice: The mode choice model calculates the split between different modes of transport in the model (car, taxi and public transport).

Trip Assignment: The trip assignment will assign the trip matrices which are estimated during the trip distribution step for different modes of travel which was calculated at the mode choice step.

Traditional four step models have been used for many years since their development in 1950s in many countries, as they provide reliable and relatively simple method to estimate the future demand and traffic flows. However four step models have been criticised by many transport modellers. The major critics are related to its fixed sequential order and its aggregated level (Bates 2000). The traditional four step model, estimates each step independently and, therefore, some inconsistencies is likely to appear.

Relative simplicity of traditional models does not actually reflect the complexity of travel behaviour. Four step models are not “behavioural in nature” (Manoj et al., 2012).

They rely on statistical relationships between demographics and traffic flows. Usually those relationships are averaged over long time periods, or wide areas. Therefore four-step models would not be able to clearly model small scale changes, dynamic nature, and changes in travel behaviour that represent complex trade-offs of cost, convenience and time-savings under different constraints. The issues related to lack of flexibility and not being policy sensitive (McNally, 2000b, Ortuzar and Willumsen, 1994) resulted in emerging new generation of demand modelling called "Activity Demand Modelling" which was introduced in the late 1970s (McNally, 2000a, McNally, 2000b).

The activity-based model is a derivative of a traditional four stage transport model, which includes trip generation, destination choice, main mode choice (i.e. choice between car, taxi and public transport), allocation to time periods, and vehicles and public transport assignment models. The activity- based approach represents trip chains as travellers move from one activity to another, throughout a 24-hour period. For example, a member of a household may leave home and travel to work, then later pass from work to go shopping, finally returning from the shopping trip to home. These journeys represent three trips, with the destination of each leg or link in the activity chain being the origin of the next leg (Rasouli, 2013a).

1.2.2 Trip Distribution

Trip distribution is the second important stage in four-step travel demand forecasting. The purpose of trip distribution forecasting is to estimate trip linkages or interactions between traffic zones for trip-makers. The distribution of trips between traffic zones can be demonstrated by an OD matrix or Origin and Destination matrix (Taylor et al., 2000), as illustrated in **Figure 1-1**. The rows of the OD matrix represent the attraction zones (Destination, D) and the columns represent the zones of generation (origin, O). The number of trips indicated at the intersection of any zone of origin and attraction, e.g. T_{ij}, represents the number of trips originating in zone i and terminating in zone j. The total of any individual row, i, represents the total number of trips generated in a zone, i.e. P_j. Similarly the total of any individual column, i, represents the number of trips terminating in a zone i, i.e. A_i.

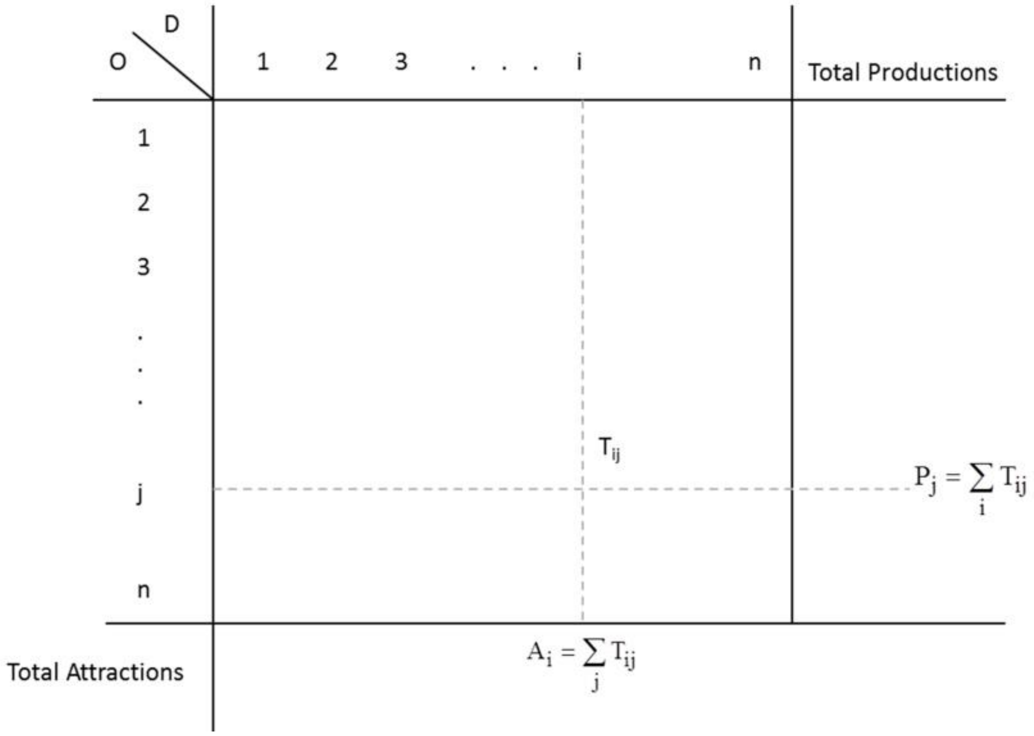

Figure 1-1: OD Matrix

There are two types of customary method for solving the problem of trip distribution: growth factor methods and synthetic methods. The Fratar method is the most well-known of the growth factor methods and the gravity model is a common synthetic approach.

The Fratar Model is reported as the first aggregate model, which was used about seven decades ago (Levinson and Kumar, 1994). Fratar model assumes that the future number of trips between a pair of zones can be estimated by proportioning the relative increases (growth) in trip ends in those zones. This proportioning process is iterative in nature. That means the first proportion is worked out based on initial conditions, then new trip end totals are computed and a new proportion established, and so on until stable numbers are obtained.

The Gravity Model is the most well-known synthetic model and is based on Newton's concept of gravity (Easa, 1993, Ortuzar and Willumsen, 1994). The gravity model assumes that the trip distribution between zones in an area is dependent upon the relative attraction between the zones and the spatial separation between them as measured by an appropriate function of distance. This function of spatial separation adjusts the relative attraction of each pair of zones and can be in the form of distance or time or a combination of different separation factors.

Neural network (NN) models were introduced as alternative methods for traditional modelling approaches, and have been increasing in use since the 1990s (Tillema et.al, 2006). Accordingly, the use of NN models for the prediction of trip distribution has been researched. Previous studies show that the NN approach is able to model commodity, migration and work trip flows. However, it does not perform as accurate as the well-known gravity model (Mozolin et al., 2000). According to a review of the literature, the majority of previous NN studies have utilized the back-propagation (BP) algorithm to solve the trip distribution problem. Most recent studies have tried to fix the performance of the BP neural network by training the models with different training algorithms such as the Levenberg-Marquardt (LM) or different activation functions (Yaldi et al., 2011).

1.3 Shortcomings of the Previous Models

The growth factor methods (Fratar model) are relatively simpler to use and understand. They are mostly used for small areas and for updating stable and uniform data. The following are some of the disadvantages of the growth factor methods:

1. Existing trip distribution matrix has to be prepared first, for large scale OD studies high sampling sizes are needed so as to estimate the smaller zone-to-zone movements accurately;
2. The error in original data collected on specific zone-to-zone movements gets magnified through the process;
3. It does provide a measure of the resistance to travel and will imply that resistance to travel will remain constant. It also ignores the effect of changes in travel pattern by the construction of new facilities or new network.

Traditional gravity models include constrained/unconstrained gravity models. The constrained models include production constrained; attraction constrained and fully constrained models. The most common model is the fully constrained gravity model because of its pattern recognition ability and accuracy. The rest of the gravity models have various problems, e.g. miss-specification, inconsistency, multi-co-linearity or data distortion due to the equations transforming into a linear and operational form. Other disadvantages of the gravity model are:

- it is very unlikely that the travel-time factors by trip purpose would remain constant throughout the urban area to the horizon period;
- the changing nature of travel times between zones with time of day makes the use of single values for the travel time factors questionable;
- it tends to overestimate short trips and underestimate long trips;
- it usually focuses on impedance (or zonal separation) which lacks a behavioural basis explaining the choices made by individuals among alternatives;
- it does not include variables that reflect the characteristics of the individuals or households who decide which destinations to choose in order to satisfy their activity needs (Tapkin, 2004).

1.4 Application of Neural Network in Trip Distribution

Neural Networks are another approach which is proposed for predicting travel demand modelling (Teodorovic and Vukadinovic, 1998). Neural Network models have been used in travel demand modelling since 1990. Dougherty (1995) provides extensive literature review for the application of NNs in different aspect of transportation modelling including trip distribution modelling. According to his research the NNs have been used for various steps of the tradition four-step modelling including trip generation, trip distribution and mode choice. Cantarella and de Luca (2005) used NNs for mode choice modelling. His study can be considered as a fundamental step in NN application for mode choice. Celikoglu (2007) also investigated the application of NNs in non-linear utility function specification for travel mode choice modelling.

The problem of trip distribution is of a nonlinear nature and neural networks are well suited for addressing nonlinear problems. This fact supports the use of artificial neural networks for trip distribution problems. Previous studies suggest that the neural network approach can be used to model the commodity, migration and work trip flows. However, its generalization performance is poor compared to the well-known doubly-constrained gravity model. The majority of previous studies have used a standard back-propagation algorithm. Recent studies have tried to improve the performance of the BP model by training the models with different training algorithms such as the Levenberg-Marquardt (LM) algorithm or different activation functions. According to the literature review, recent studies have improved the ability of the BP models to estimate the trip distribution and the modelling results indicated that the BP model can provide better estimations than the well-known gravity model.

In most of the relevant cited papers for trip distribution modelling using NNs, the feed-forward back propagation (FFBP) approach are investigated and proposed. The FFBP approach suffers from some disadvantages including their sensitivities to the selected initial weights and the local minima problem which will lead to in accurate outcomes.

However, the application of NNs for trip distribution modelling is limited in the literature; no work has been reported that investigate the application of generalized regression neural networks for modelling the trip distribution problem.

1.5 Advantage of GRNN Modelling

GRNNs are known for their ability to learn quickly (rapid training) with small number of data and their application have been investigate in various problems in different disciplines including Medical, hydrological and electrical science and in many studies GRNN provided better outcomes than FFBP. For example, the application of GRNN has been investigated by Celikoglu (2005) in travel mode choice modelling and its advantage over the FFBP model has been demonstrated. GRNN application is especially useful for function approximation with multi-dimensional inputs (Ariffin J. 2008). Other benefits of GRNN claimed by Specht (1991) include:

- The network is able to learn from the training data by ‘one-pass’ training in a fraction of the time it takes to train standard feed-forward networks.
- The spread, Sigma, is the only free parameter in the network, which often can be identified by the V-fold or split-sample cross validation.
- Unlike standard feed-forward networks, GRNN estimation is always able to converge to a global solution and won’t be trapped by a local minimum.

1.6 Purpose of this Research

The purpose of this research is to develop a generalised regression neural model to estimate the work trip distribution from the land use data of the origin and destination zones. The finding of this research provides an alternative simple and practical approach for trip distribution modelling. This research also establishes a frame work for a combined trip generation and distribution model using neural networks.

1.7 Research Approach

For the purpose of this research, three approaches/ models were developed for the estimation of trip distribution. GRNN modelling is the new approach and is the focus of this thesis, the BP and gravity models are the other approaches.

The BP model developed in this thesis is a two-layer feed-forward network, with a sigmoid transfer function in the hidden layer which is trained with the Levenberg-Marquardt algorithm (LM). The LM algorithm was used by previous researchers (Yaldi et al., 2011) in order to improve the performance of the BP models.

The gravity model used for the purpose of comparison has been developed for trip distribution of the internal zones in the strategic transport model established for the Mandurah Strategic Model (Rasouli, 2013b). The internal trips are distributed based on the gamma function in this model. The transport model is based on the traditional four-stage model process with five different categories for trip purpose: work, education, social, other and non-home-based (NHB) trips.

For the purposes of comparison, the results of the GRNN model were compared with those for the BP and gravity models. The root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and measures of the training and testing data set were used as indicators to provide a numerical description of the goodness of the model estimates. The model development and methodology is illustrated in **Figure 1-2** and includes the following steps:

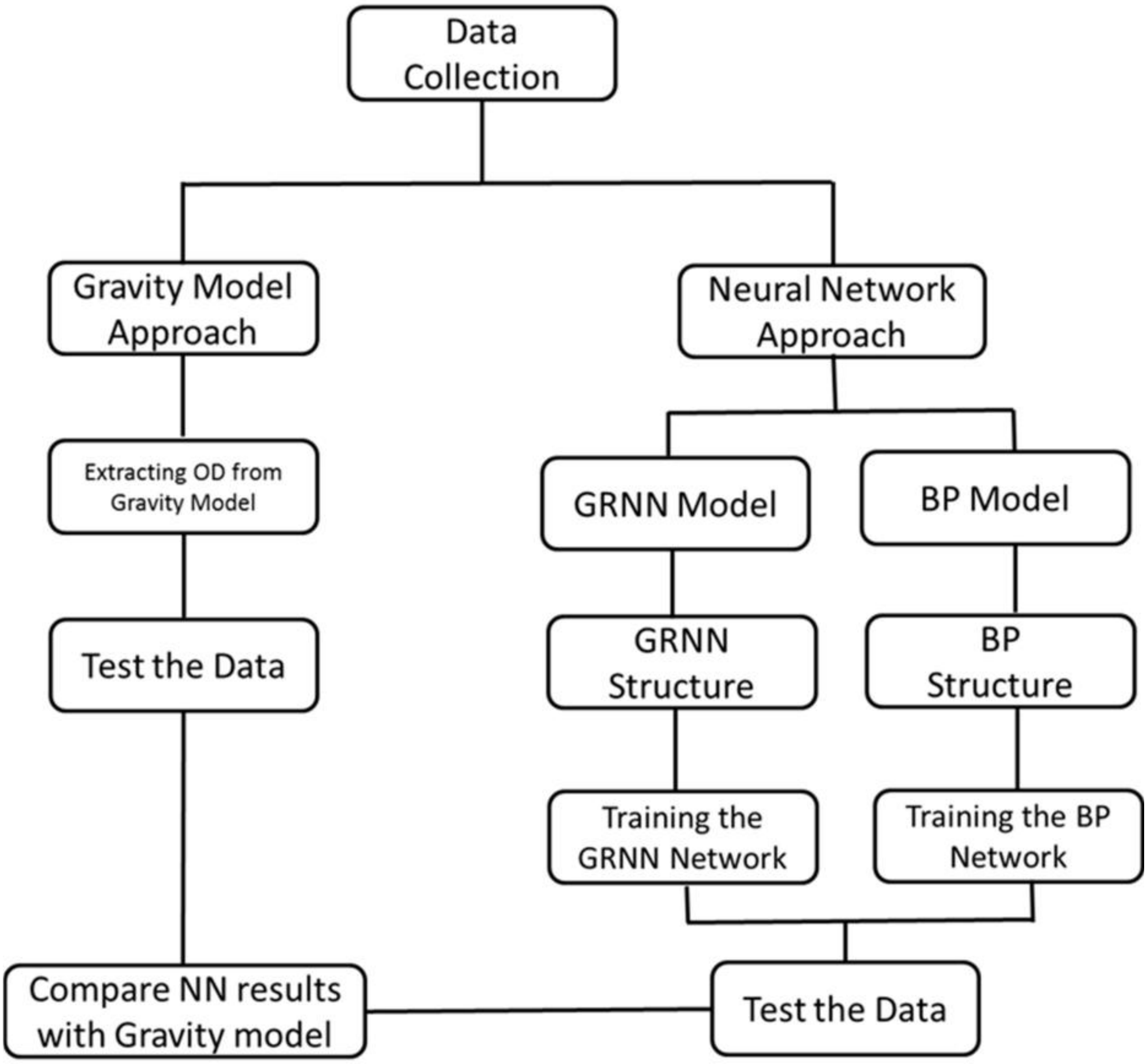

Figure 1-2: Model Development and Methodology

1. Undertaking a comprehensive literature review including library and internet search and direct contact with local and international people working in this field of research. The objective of this step is to discover whether similar studies had been undertaken using the neural network method for the prediction of trip distribution.
2. Collecting an appropriate original OD matrix for work trip distribution as a benchmark.
3. Estimating the original OD matrix based on a new NN approach known as the generalized regression neural network, and comparing the results with another common NN approach known as the back-propagation, and the customary gravity model method.

4. Developing a GRNN model to estimate the work trip distribution between a pair of zones using the land use data in each zone instead of trip production and attraction for that zone.
5. Investigating simple data normalization, linear transformation and statistical normalization methods for the NN input vectors to select the best format for the input data into the NN model.
6. Investigating the optimum spread (sigma) for the GRNN model by cross validation method.
7. Validating the proposed GRNN model and checking the satisfaction of gravity model constrains (total productions and attractions) by the validated GRNN model.
8. Developing a BP model using the Levenberg-Marquardt (LM) algorithm. Training the model with 10 different seeds and four different hidden layer neurons to select the best structure for the BP model.
9. Extracting the work trip matrix from the previously established strategic transport model for the Mandurah area.
10. Comparing the results with the commonly used doubly-constrained gravity model and BP model using the root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and measures of the training and testing data set as indicators to provide a numerical description of the goodness of the model estimates.

1.8 Research Objectives

The objective of this research is to investigate the application of the GRNN model for prediction of the work trip distribution using land use data between the OD zones and the distance between the zones. It is expected the outcome of this research, supported by comprehensive literature reviews, can establish guidelines for development of a combined trip generation and distribution model using neural networks. The proposed GRNN model can be used as an alternative tool for predicting trip distribution directly from land use data. The proposed model can be used by transport modellers, urban planners and software developers.

1.9 Research Significance

This research study is unique in its various aspects and a quantity of new knowledge was developed during the course of this work. The following are some of the major achievements of this study:

- Travel demand forecast is an essential element for transportation planning in order to evaluate the future needs of an urban area. A robust and efficient technique is required to predict the patterns of trips in the future, so that the desired outcomes and impacts can be achieved and anticipated.
- There is no technique in trip distribution that is universally applicable, so attempts to develop alternative methods are always needed. This includes the adoption of approaches from other disciplines. Neural networks are one possibility, and are proposed as an alternative method in this study.
- The problem of trip distribution is of a nonlinear nature and is complex. Neural networks have been used successfully for mapping nonlinear problems. This fact supports the use of artificial neural networks for trip distribution problem.
- According to the literature review, there have not been enough attempts to investigate the application of GRNN models for the trip distribution estimation. The advantage of the GRNN model over other feed-forward or feedback neural network techniques is its simplicity and practicality. This research aims to apply the GRNN model to improve the ability of neural networks to predict trip distribution problem.
- All the previous cited papers have used trip productions and attractions as input to the neural model. This research aims to predict trip distribution directly from the land use data instead of using trip production and attraction for traffic zones. This methodology will minimize the risk of error that normally happens during the trip production and attraction stage of the modelling and propagates through to other stages of the modelling.
- The proposed validated GRNN model is providing a combined trip generation and distribution modelling frame work which is another novelty and significance of this research.

1.10 Research limitations and Expected Outputs

This research aims to develop a new model for trip distribution using the GRNN method. The trip distribution modeling is undertaken for only work trip purposes. The data which have been used for this research is the 2006 Journey to Work (JTW) data set for the Mandurah area in Perth, Western Australia and sourced from the the Department of Planning (DoP).

The Neural Network modeling undertaken has been done by MATLAB software. The data preparation for the input to MATLAB software has been undertaken by Microsoft Excel. The Neural Network outputs are also transferred to Microsoft Excel for additional analysis and comparison.

The purpose of this research is to investigate the ability of the GRNN model to predict trip distribution. Therefore the focus of this research is utilization and improvement of GRNN model to be able to predict trip distribution of the work trips. The other purpose of this research is to estimate the trip distribution by the land use data. So the trip generation process is not required for the input to the neural model. In other words with the land use data for the origin and destination zones the GRNN model would be able to provide the trip distribution between a pair of zones in the model. This approach makes the proposed GRNN model very attractive in comparison with the other available NN approaches which have been undertaken so far, because

1. *Trip distribution will be predicted by land use data and there is no need to estimate the trip generation of the traffic zones. Therefore the errors associated with the trip generation step of the four step modelling will be minimised on this process.*
2. *The structure of the GRNN model is fixed and therefore does not have to be investigated by trial-and-error unlike the FFBP model, and then this will remove some of the uncertainty related to the NN model development process.*

The output of this research is a combined trip generation and distribution model which can be used by the transport modelers to estimate the second stage of the 4 step modeling with a simple and practical tool.

1.11 General Research Findings

The overall finding of this research is that neural network modeling is suitable for modeling very complicated functions. Trip distribution estimation is a complex problem. The traditional gravity model uses the simple assumptions through introducing the decay functions. The decay functions assume that the number of trips between a pair of zones depend on the generalized cost between the two zones. The generalized costs are often assumed to be the distance between the traffic zones or travel time or the combination of these two parameters. However this is not always the case in practice. The trip distribution depends on the behavior of the trip makers and their personal preferences as well as the road network facilities and the available mode of travel between pair of zones. Combining all these factors and parameters together and finding the relationship between these parameters are very complex. NNs are recommended for complex problems, when there is no simple mathematical approach to solve the problem. The use of NNs is not recommended if the conventional approach can provide satisfying result with an easily solvable and adequate mathematical model which already exists. Neural networks exhibit nonlinear behavior and learn through processing example data sets. The user must design training algorithms and provide proper input data sets for the automatic learning procedure to run the model successfully. Previous studies used the FFBP approach to estimate the trip distribution, the FFBP method includes some disadvantages such as their sensitivities to the selected initial weights and the local minima problem which will generate in accurate outcomes. GRNN modeling has already been investigated for the mode choice problem by Celikoglu (2007) and its superiority over the FFBB approach is reported but, there has not been any investigation for application of the GRNN approach for trip distribution problem.

The GRNN model is recommended for trip distribution estimation. The analysis undertaken indicates that a validated GRNN model can outperform the traditional gravity model and even the FFBP models in terms of RMSE, MAE and coefficient of determination (R^2) between the modelled output and measures of the training and testing data. . Simplicity and fixed structure of the GRNN model makes it a very powerful tool in practice and therefore it is recommended for modellers as an alternative tool to predict the trip distribution matrices.

The recommended GRNN model uses the land use data for origin and destination zones as the input to the model. GRNN will learn the relationship between the land use data of the origin and destination zones and the number of trips associated with them. Therefore the proposed GRNN model provides a combined trip generation and distribution technique.

1.12 Thesis Structure

The thesis comprises six chapters:

Chapter 1 is the introduction. This chapter describes the objectives, briefly reviews the methodology and also discusses the significance of the research. Background and limitations of the research and general findings of the research are also reviewed in this chapter.

Chapter 2 provided a brief review of the basic concepts in trip distribution. The concept of the gravity model, deterrence function and generalized cost are also reviewed in the trip chapter. Different methods of trip distribution will be reviewed and discussed. The advantages and disadvantages of the existing methods will be presented.

Chapter 3 reviews the concept of the neural network and its application in solving complex and nonlinear problems. The structure of an artificial neuron and the seven major components of an artificial neuron are also discussed in this chapter. A comprehensive literature review on the available research in the field of trip distribution estimation using neural networks are provided in this chapter and similar studies are reviewed and discussed. This chapter also reviews the back-propagation algorithm, since it is one of the most commonly used neural network models, and many others are based on it. The generalized regression neural network falls into the category of probabilistic neural networks which form the basis of analysis for this thesis and therefore will be discussed in this section as well.

Chapter 4 discuss the methodology adopted in the research, and consists of five categories for neural models: model specification/ structure, model training, model testing, model performance measurement, and the application of the proposed framework. The neural models are developed and trained with different sets of input data. The results are reported, analysed, and then discussed in detail in the next chapter. The Gravity model structure is also discussed in this chapter.

Chapter 5 includes the model development and analysis. This chapter discusses the methodology for the development of the GRNN, BP and gravity models. The process of data collection and the structure of the gravity model developed for the Mandurah area in WA will be provided in this chapter. The chapter also discusses model structures for the GRNN and BP models, normalization of the input data, training of the models and the modelling outcomes. Comparisons of the models will also be provided in this chapter.

Chapter 6 discuss the validation of the proposed GRNN model. Accordingly the GRNN model will be applied to ten different sample groups and the performance of the GRNN model will be investigated. The GRNN model outputs will be also investigated to evaluate the predictive ability of the GRNN model for satisfying the gravity model constraints.

Chapter 7 draws together the conclusions of this work, and also outlines proposals for further research in this subject.

Substantial references used in this study are given at the end of this thesis.

2 2. TRIP DISTRIBUTION MODELS

2.1 Introduction

In this chapter the literature review for trip distribution is reported. Different trip distribution techniques are reviewed and discussed. More discussions are provided for gravity model which is used in this thesis as a method which is compared with the neural models. The role of Origin Destination (OD) matrix for trip distribution is discussed and the concept of generalised costs and deterrence function is reviewed. The review from this chapter contributes to the development of the research and modelling methodology.

2.2 Trip Distribution

Trip distribution is the second step of the traditional modelling process, which has four steps (trip generation, trip distribution, mode split and assignment). The purpose of trip distribution is to estimate the number of trips distributed between traffic analysis zones (TAZ). Trip distribution depends on a general function of time and/or cost of travel between traffic zones and also the number of trips in both origin and destination zones. Outputs of trip generation, productions and attractions by trip purpose for each zone, and travel cost between each pair of zones are the inputs for the trip distribution model. Outputs of the trip distribution stage are the trips between each pair of zones for each trip purpose. Since different trip purposes correspond to different functions of time and cost of travel, trip distribution is applied for each trip purpose separately with a different cost function (National Cooperative Highway Research Program (NCHRP), 2012).

2.3 The Origin Destination (OD) Matrix

The new origin-destination (OD) trip matrix for the future is estimated using the trip distribution model, which specifies future trips resulting from demographic changes in the existing situation that reflect changes in people's choice of future destinations. The changes in demographic and land use data will be used in estimating the origin-destination pattern of future travel and generating the future OD matrix which can be assigned to the road network during the assignment step of the modelling process. The future OD matrix is expected to change due to changes in the land use data and the trip distribution model which models these changes and enhancements in the transport system (Davidson & Davidson, n.d.).

The distribution model requires data for the number of trips generated from and attracted to each traffic zone in order to indicate new levels of generation of future trips or any changes in land use data. These inputs are called 'trip ends'. Trip ends are determined from the OD matrix by summing up the row totals, which gives the total number of trips generated by each zone (named origin trip ends of that zone), and also by summing up the column totals for each zone, which gives the total number of trips attracted to that zone (named destination trip ends of that zone). Therefore, there is an origin and a destination trip end for each zone.

Base year trip ends should be adjusted for future year trip ends to reflect the future level of generation/attraction of trips. Therefore, future year trip ends will be used as inputs for the distribution model while estimating the OD matrix of the future year. Estimating the future year trip ends from the current year trip ends is the main goal of the trip end model (Davidson & Davidson, n.d.).

2.4 Methods to estimate the OD matrix

O-D matrices can be estimated by three different ways Taylor et al. (2000):

- Direct observation
- Synthesis
- Modelling procedures

Direct observation means that number of trips for each OD zone is obtained directly by traffic or questionnaire surveys. The most common method for direct survey includes the registration plate surveys, road side surveys or road side interview. The questionnaire surveys can be conducted by individual or home interview. Aerial photography can also be used as a direct technique in estimating the O-D matrices (Cremer and Keller, 1987). Regardless of its accuracy, the weaknesses of this method were reported in many studies, such as Nihan and Davis (1987), Cremer and Keller (1987), Sherali et al. (1994), Sherali et al. (2003), Nie et al. (2005) and Doblas and Benitez (2005). The drawbacks of the direct observation method include:

- Expensive and time-consuming;
- Not sensitive to the changes in trip patterns over time or the impact of land-use development; and
- Biased results

Synthesis approach is based on the traffic counts for the links in the transport road network, obtained from traffic counts survey for links. In this method the O-D matrices are established by using mathematical theory such as the work undertaken by VanZuilen and Willumsen (1980).

Modelling approach tries to estimate the OD matrices through modelling procedures. The most common modelling approaches for estimating the OD matrices are Furness and gravity models. Gravity model proposed by Wilson (1967) is widely used as a modelling procedure to establish the trip distribution matrices. The Furness and gravity models are discussed shortly in the next sections and will be reviewed in detail in section 2.5 (trip distribution techniques)

2.4.1 The Furness Distribution Model

The OD matrix for a future year can be extracted from the trip matrix of the base year in a way that the total values of the rows and columns match the future trip ends. The Furness distribution model is one of the simplest methods available for this practice. The procedure for the Furness model is explained in the following steps (Davidson & Davidson, n.d.):

1. Each row in the base year matrix is multiplied by the growth factor for its corresponding zone. The origin trip ends for the new matrix would match those of the future year; however the sum of the column values will not match the destination trip end for the future year, hence:
2. The cells of each column of the matrix are multiplied by the destination trip end ratio of the future year so that the total value of columns of the final matrix matches the column total of the destination trip end of the future year.
3. If the row total does not match the origin trip end for the future year, steps 1 and 2 must be repeated successively until both row and column totals become close to the future year origin and destination trip ends. This process should be repeated until the values are close enough.

The Furness model converges very quickly in most cases. Studies of this method have shown that if every matrix cell value is greater than zero, then the model converges to a unique answer. The Furness method is commonly used in transportation modelling and even in more complicated cases where advanced distribution models are employed, this method is usually used for modelling the external to external movements or estimation of goods vehicles and freight.

The Furness model has two major disadvantages, the first being that a cell in the matrix which is zero remains zero regardless of how many times it is factored. Assume a zone which is undeveloped in the existing year and therefore there is zero trip generation or attraction for that zone. As the zone becomes fully developed in the future with houses, shops, factories etc., trips will be generated and attracted to that zone. However in to the Furness model the trip distribution to/from the zone will remain at zero in the future if it is zero in the base year. One method for solving this problem is to 'seed' all the zero cells with a certain value (e.g. single-trip, or to consider a trip distribution from it to every other zone and from every other zone to it). Hence the resulting matrix of this zone would become dependent on input assumptions.

The second weakness of the Furness Model is that it is not sensitive to probable changes or enhancements that might occur in the transport system. Clearly if the transport system has been improved, people would change their routes and destinations due to these changes and would make the most of the additional options available to them. For example, if a new motorway is built which connects people to a big shopping centre; people are more likely to go to the new shopping centre. The situation is the same for job or education opportunities and people would even move to new places to gain better job opportunities available through the new transport infrastructure. This issue is more difficult to deal with, and a more complicated model is required, such as the gravity model, in order to solve it (Davidson & Davidson, n.d.).

2.4.2 The Gravity Distribution Model

This model takes its name from the theory of gravity; i.e. the 'pull' between two objects is proportional to their size and inversely proportional to (some function of) the distance between them. The concept is similar to the theory of travel between areas, where the frequency of travel between two areas can be proven to be proportional to their population and the number of jobs, schools, factories, offices etc., yet inversely proportional to the distance (or a function of separation or deterrence) between them. This relationship works quite well in general – the bigger the towns/zones, the more people travel there, and the greater the distance between towns/zones, the less people travel between them. The origin and destination trip ends of the origin and destination zones respectively are a measure of the amount of pull between them (Davidson & Davidson, n.d.).

As matrix cells of the trip ends that must be fitted, the deterrence function has also a coefficient which needs to be calibrated. The procedure is called 'calibration' and is performed during the process of building the transport model. The calibration process may quite change the matrix (the base year observed trip matrix) and proper calibration can guarantee the accuracy of the model.

The deterrence function can vary according to the different types of people, trips (e.g. journey to work, education, shopping, leisure, holiday etc.), times of day and modes of transport (e.g., travelling with/without a car). In order to embed these differences, various matrices are required which correspond to different types of trips and/or travellers. This can improve the calibration process. Additionally, different movements between areas can have diverse deterrence functions and sometimes different calibration coefficients. Because several variables are involved, calibration is sometimes a time-consuming procedure. There is always the possibility that no deterrence function can be determined, i.e. that calibration of the gravity model is not possible.

The gravity distribution model can be used only after it has been calibrated. In later stages, the same calibration coefficient can be employed in the deterrence function assuming it works for the future year. Generalized costs and trip ends for the future year are also required for forecasting the trip matrix for the future year.

The transport networks and future levels of travel between zones can affect the future year trip matrix. This can also solve the previously mentioned problem of the zero cells for the base year matrix in the Furness model, since the generalized cost matrix extracted from the transport network is used for the calculation of every cell.

The gravity model is more robust compared to the other trip distribution models since it considers the levels of travel between zones and the transport networks (Davidson & Davidson, n.d.).

2.5 The Concept of Generalized Cost

In order to express the separation between the origin and the destination, a measure is required than can reflect it properly. Common measures that can reflect separation include: distance, travel time, fare, waiting time, walking time, petrol cost, parking charge and toll.

Generalized cost combines all of these variables together. It is simply the weighted sum of the aforementioned factors; hence the generalized cost of travelling from origin to destination is defined as a weighted sum of those factors for the origin to destination zone in the model. These factors can be taken from the transport networks used in the assignment process. They can be ‘skimmed’ from the networks as a matrix, each cell of which represents the value of a variable. Therefore the in-vehicle time skim indicates the time needed to go from each zone to every other zone, the fare skim indicates the cost of travel from each zone to every other zone, and so on. These skim matrices are combined, by weighting each matrix and adding them all together, and form an overall measure of the ‘separation’ between every zone pair (Davidson & Davidson, n.d.).

2.6 The Concept of a Deterrence Function

The behaviour of travellers indicates that the further the destination, the less frequently they travel. Various research has been undertaken to determine a function that could best demonstrate the relationship between the distance of a trip and its frequency. A common practice is to consider a negative exponential (deterrence) function with generalized cost; hence in order to evaluate the measure of separation, one should calculate *e* to the power of generalized cost (the power is usually multiplied by a calibration constant). The constant has a negative value which indicates that the higher the generalized cost, the smaller the frequency of trips (Davidson & Davidson, n.d.).

2.7 Trip Distribution Techniques

Many mathematical models have been developed to describe and predict the distribution pattern of trips. They are generally divided into two groups:

- Growth factor methods, and
- Theoretically based methods.

In the first group, there are four basic types of model known as the Detroit, Fratar, uniform and average-factor methods. In the second group, the most well-known two models are the gravity model and the intervening opportunities model. These methods are explained briefly below.

2.7.1 Growth Factor Methods

Growth factor methods are claimed by Levinson and Kumar (1994) and Easa (1993) that are the first aggregated models introduced about seven decades ago. These methods assume that the future number of trips between a pair of zones can be estimated by proportioning the relative increases (growth) in trip ends in those zones. This proportioning process is iterative in nature, which means that the first proportion is worked out based on initial conditions, then the new trip end totals are computed, the new proportion established, and so on until stable numbers are obtained. Mathematically, this process is described below.

The initial growth factor for zone i, F_i^k, is computed by dividing the forecasted trips by actual trip ends:

$$F_i^k = \frac{T_i^*}{t_i^k} \qquad 2.1$$

For the whole study area, the trip ends over all zones are summed to calculate the corresponding area-wide growth factors, F^k.

$$F^k = \frac{\sum_i T_i^*}{\sum_i t_i^k} \qquad 2.2$$

Total trip ends in zone i are obtained using:

$$t_i^k = \sum_j t_{ij}^k \qquad i \neq j \qquad 2.3$$

By contrast, the Detroit model works as follows:

$$t'_{ij} = t_{ij} \frac{F_i F_j}{F} \qquad 2.4$$

$$t_{ij} = t_{ij}^{k-1} \frac{F_i^{k-1} F_j^{k-1}}{F^{k-1}} \qquad 2.5$$

where k denotes the k^{th} iteration, T_{ij} denotes the predicted trips, t_{ij}^k denotes the actual trip ends, F^k denotes corresponding area-wide growth factors. In these models, the number of trips between zones i and j increases in proportion to the growth of trip ends in the origin zone (i) and the growth of trip ends in the destination zone (j).

Another growth factor method is the Fratar model. This model is often used to estimate external trips, that is, trips that are either produced and/or attracted outside the boundaries of the region under study from outlying areas whose character is not explicitly analyzed (Tapkin, 2004). The Fratar model begins with the base year trip-interchange data. Usually this model does not distinguish between productions and attractions and considers the inter-zonal trips irrespective of their direction. Since no distinction is made between productions and attractions, the trip generation of each zone is denoted by T_i. The following trip balance equation provides the necessary relationship between the trip generation of a zone i and the trip interchanges that involve zone i:

$$T_i = \sum_j T_{ij} \qquad 2.6$$

The estimatation of the target-year trip generation T_i (t), which precedes the trip-distribution phase, is computed by multiplying the base year trip generation, T_i (b) by a simple growth factor, namely G_i. This growth factor is based on the anticipated land use changes that are expected to occur within the zone between the base year and the target year. Thus:

$$T_i(t) = G_i\left[T_i(b)\right] \qquad 2.7$$

Subsequently, the Fratar model estimates the target-year trip-distribution $T_{ij}(t)$ that satisfies the trip balance for that year. Mathematically, the model consists of successive approximations and a test of convergence in an iterative procedure. During each iteration, the target-year trip-interchange volumes are computed based on the anticipated growth of the two zones at either end of each interchange. The implied estimated target-year trip generation of each zone is then computed according to equation 2.6 and compared to the expected target-year trip generation from equation 2.7. A set of adjustment factors, R_i, is then computed by:

$$R_i = \frac{T_i(t)}{T_i(\text{current})} \qquad 2.8$$

If the adjustment factors are all sufficiently close to unity, the trip balance constraint is satisfied and the procedure is terminated. Otherwise the adjustment factors are used along with the current estimate of trip distribution Q_{ij} (current) to improve the approximation. A comparison of equations 2.7 and 2.8 shows that the adjustment factors used in all but the first iteration and the original growth factors applied during the first iteration play the same mathematical role. Their interpretation, however, is not the same: The growth factors constitute a prediction of the actual growth of each zone between the base year and the target year, but the subsequent adjustment factors are merely mathematical adjustments that facilitate the convergence of the solution to the predicted zonal trip generation.

The basic equation employed by the Fratar model to calculate the portion of the target-year generation of zone i that will interchange with zone j is:

$$T_{ij}(new) = \frac{\left(T_{ij}(current)\right)R_i}{\sum_j \left(T_{ij}(current)\right)R_j} T_i(t) \qquad 2.9$$

This equation is similar to that of the gravity model, which will be presented later in this chapter. The expected trip generation of zone i is distributed among all zones so that a specific zone j receives its share according to a zone-specific term divided by the sum of these terms for all 'competing' zones j. When equation 2.9 is applied to all zones, two estimated values result for each pair of zones: The first represents the portion of the generation of zone i chosen to the interchange due to the influence of zone j (or T_{ij}), and the second is the portion of the generation of zone j chosen to the interchange due to the influence of zone i (or T_{ij}).

An asymmetric form of the Fratar model begins with a base year trip table in the production-attraction format. In this case, the sum of each row represents the base year productions, whereas the sum of each column represents the base year attractions of the corresponding zone. Each zone is given two growth factors: one associated with the expected growth in residential activity (and therefore productions), whereas the second captures the zone's non-residential growth (i.e., attractions).

The uniform growth factor method can be summarized in a compact form as:

$$E = \frac{\sum T_i^G}{\sum\sum T_{ij}^T} \qquad 2.10$$

$$T_{ij}^F = T_{ij}^T \times E \qquad 2.11$$

where:

E = Uniform growth (adjustment) factor;

$T_i^{\,G}$ = Trip generation output for the future;

$T_{ij}^{\,T}$ = Total trips today; and

$T_{ij}^{\,F}$ = Flow from i to j in the future.

The steps that should be followed are straightforward. First the uniform growth factor will be calculated. Then this factor will be applied to all current flows. Also the average growth factor method can be presented mathematically as:

$$E_i^{k-1} = \frac{T_i^G}{T_i^{k-1}} = \frac{T_i^G}{\sum T_{ij}^{k-1}} \qquad 2.12$$

$$T_{ij}^k = T_{ij}^{k-1} \times \left[\frac{\left(E_i^{k-1} + E_j^{k-1}\right)}{2} \right] \qquad 2.13$$

where:

E_i^{k-1} = Average growth (adjustment) factor;

T_i^G= Total trip generation at i on future date;

T_i^{k-1} = Total trips for iteration k at I; and

T_{ij}^k = Flow from i to j for iteration k (represents the future).

The steps that should be followed to calibrate the model are:

1. Run a trip generation model;
2. Determine the first estimate of 'average growth factors';
3. Apply the first set of average growth factors to all current flows;
4. Check for closure.

The Fratar and Detroit models are considered to have better mathematical expressions and to be computationally more efficient than the uniform growth and average growth factor models. In any case, the growth factor models find most applications in estimating trips from external to internal or other external zones since there is no land use data available for the external areas outside the study area.

These models are advantageous (Tapkin, 2004) because they:

- Are simple, inexpensive and easy to apply;
- Are well-tested;
- Require no distance variables;
- Need no calibration;
- Can be applied to peak directional flows;
- Are useful in updating origin-destination surveys.

However, the disadvantages are:

- Only a single growth factor for each zone, and assumed stable to the horizon year;
- Inability to account adequately for major changes in land use or interzonal activity;
- No explicit term relating to any form of travel cost, time or other impedances;
- Zones having zero interchanges in the base will show zero interchanges in the horizon year;
- Errors in the original distribution due to sampling or other factors will be carried forward and magnified (Tapkin, 2004).

2.7.2 Theoretically Based Models

The gravity model gets its name from and is conceptually based on Newton's law of universal gravitation. This law concerns gravitation and states that the masses of two bodies divided by the square of the distance between them forms the gravitational force that exists between them:

$$F_{12} = G \times \frac{M_1 \times M_2}{d_{12}^2} \qquad 2.14$$

where:

F_{12} = the gravitational force between two bodies;

M_1 = mass of first body;

M_2 = mass of second body;

d_{12} = distance between two bodies; and

G = gravity constant.

Travel researchers found a notable analogy especially about shopping travel while analysing the gravity model: the available trips are represented by M_1 as the mass of trips in an inhabited area; the attractiveness of a shopping area is represented by M_2; the distance between these two points is represented by d_{12}; and the number of trips between these two areas is represented by F_{12}. Incorporating these interpretations through the gravity model implies that in order to increase the number of inter area trips, the attractiveness or size of the two areas should increase and the distance between them should decrease. So many situations in the real world resemble this equation. For instance, this equation can model the number of telephone calls that take place between cities that are far apart, while M_1 and M_2 represent the population sizes of the cities and d_{12} represents the distance or perhaps the cost of telephone calls between them. Applying this formula to trip distribution and replacing the items results in:

$$T_{ij} = k\frac{P_j A_i}{d_{ij}^c} \tag{2.15}$$

Where:

T_{ij} = trips produced from area i to j;

k = adjustment ratio between zones that inserts the impact on travel patterns of defined economic or social linkages;

P_j= trips that area j produces;

A_i = trips that area i attracts;

d_{ij} =distance between area i and area j; and

c = a tentative determined exponent which represents the average area-wide effect of spatial separation between areas on trip interchange.

Equation 2.15 indicates the interchange numbers between these two areas with area j producing and area i attracting trips. The trip distribution between a pair of zones increases when the attractiveness of area i or production of area j increases, and decreases if the distance between these areas increases.

There are dependent and independent variables in this model. The impedance, productions and attractions are independent and the inter-zonal trip is dependent. k and c are the model's constant parameters which are estimated in the calibration process using base year data. If the trip production balance constraint is considered then equation 2.15 can be rewritten without parameter k. This constraint states that the sum of the trips that area i attracts is equal to the trips that area j produces while considering specific interchange volumes:

$$P_j = \sum_i T_{ij} \tag{2.16}$$

$$A_i = \sum_j T_{ij} \tag{2.17}$$

After replacing these two formulae and carrying out the necessary mathematical operations, a new equation is achieved:

$$T_{ij} = P_j \left[\frac{A_j / d_{ij}^c}{\sum_j \left(A_j / d_{ij}^c \right)} \right] \qquad 2.18$$

The term inside the bracket is the trips that area i produces which are attracted by area j, proportional to all the trips that are attracted in the areas. It is notable that if we multiply a constant by all attraction terms, the numerical outcome value won't change. So it can be concluded that the relative attractiveness of areas can be measured by the attraction terms. For instance, one employment area can be said to be twice as attractive as another according to the available employment opportunities.

After applying these terms the following is obtained:

$$T_{ij} = P_j \left(\frac{A_j F_{ij}}{\sum_j A_j F_{ij}} \right) \qquad 2.19$$

where:

$$F_{ij} = \frac{1}{d_{ij}^c} \qquad 2.20$$

and F_{ij} is called the travel-time factor (or friction).

Hence the limited number of independent variables included in the model does not capture all the effects. In order to incorporate them, a set of Kij factors is introduced while calibrating which are inter-zonal socioeconomic adjustment factors.

Thus the formula can be expressed as:

$$T_{ij} = P_i \frac{A_j F_{ij} K_{ij}}{\sum_{j=1}^{n} A_j F_{ij} K_{ij}} \qquad 2.21$$

where:

K_{ij} = a specific adjustment factor between areas;

F_{ij} = travel-time factor;

A_j = number of trips that area j attracts;

P_i = number of trips that area i produces; and

T_{ij} = number of trips that area i produces and area j attracts.

Because the totals of the rows and columns are not constrained, this is called the unconstrained gravity model. Instead of unconstrained models we can use constrained gravity models.

In order to calibrate the gravity model, the numerical value of the parameter c must be determined. This parameter fixes the model to the one that replicates the observations of the base year. In order to fix the relationship between the inter-zonal impedance and the travel-time factor, the proper value of c must be known.

In order to calibrate the gravity formula, an iterative procedure is needed which is unlike a simple linear regression model calibration where a simple minimization of the sum squared deviations could solve the parameters. In order to compute T_{ij}, which are the inter-zonal volumes, the known base year productions, attractions and impedances are used in equation 2.21 while assuming an initial value of c. Then the observed results which are obtained for the base year are compared with these estimated results. The calibrated value will be the current value of c if the observed volumes and calculated volumes are sufficiently close to each other. Otherwise, the value of c is required to be adjusted and as long as the degree of convergence is not acceptable, the procedure needs to be continued. Most of the time, the friction-factor function F is used in the calibration procedure instead of parameter c.

Although the gravity model is the most common method for trip distribution, there are known advantages and disadvantages. Some of the advantages are:

- Its application in particular areas is easy because of being easy to understand;
- By emphasizing trip productions and attractions against each other, trips' competition between land uses is accounted for; and
- It is sensitive to the change in travel times between one zone and another (Tapkin, 2004).

The disadvantages of the gravity model are:

- The travel- time factors by trip purpose are unlikely to remain constant in future throughout the urban area;
- The reflection of the characteristics of the households or individuals who make the decision to choose destinations for satisfying their needs of activity is not included by variables.
- It usually focuses on impedance (or zonal separation) which lacks a behavioural basis explaining the choices made by individuals among the alternatives long trips are underestimated and short trips are overestimated;
- It is questionable that single values are used for the travel time factors because the nature of travel times changes between zones with time of day (Tapkin, 2004).

Another approach to this would be employing composite utility, computed with certain choice models. Recently a new practice has been proposed as an alternative for the gravity modelling approach, which abandons if there are destination choice models that are more behaviourally based.

Using K-factors in the adjustment of discrepancies observed between trip-length frequency distribution for the base year and the results of using only the final friction factors has become an interesting matter for two reasons. The first one relates to difficulties in the attempts made to interpret the effects captured via K-factors, and the second corresponds to these effects being true between the base year and the target year. Other results imply that K-factors are required in order to rectify possible mismatches between the engagement and job types of residents in the producing zones and the employment type that is available in the attracting zones; e.g. workers in the i-th zone could be employed in the jobs available in the j-th zone by the gravity model, because the j-th zone is the closest zone to the i-th zone. Some applications based on the gravity-model report on stratifying jobs based on their industry and employment type or income, which requires additional computational load. Research has revealed that this problem corresponds to the unique historical and cultural factors in each local area.

Invaluable insights can be gained from a good understanding of local conditions and the likelihood that they will persist over time. This insight can potentially be of help in interpreting and applying K-factors in the modeller.

Despite all of its shortcomings, gravity modelling has various applications in many urban transportation planning packages. There are two options for defining the distance variable: the first option is to consider the real physical distance, travel time, or cost of travel; and the second option is to employ a mathematical decay function of distance. The most common decay functions of distance are power functions in the form of:

$$f(d_{ij}) = d_{ij}^{-\beta} \tag{2.22}$$

and also exponential functions as in equations 2.23 and 2.24.

$$f(d_{ij}) = e^{-\beta d_{ij}} \tag{2.23}$$

$$f(d_{ij}) = d_{ij}\ e^{-\beta d_{ij}} \tag{2.24}$$

β is an empirical constant which expresses the severity of inhibiting distance effects (d_{ij}) on trip-makers. Increasing the value of beta, while other factors remain constant, means the number of trips decreases faster with distance. In the second form of negative exponential function, i.e. equation 2.24, distance decay due to distance increase is slower; the number of trips in this equation increases over short distances, but would decrease immediately afterwards. One problem with the power function is that it becomes zero when the distance is zero, while the gravity model predicts an infinite number of trips (Hanson, 1986). The intra-region, intra-city or intra-zonal trips are usually excluded from the spatial interaction modelling analysis when studying region-to-region, city-to-city or zone-to-zone trips, and their distance will be considered zero. Hence employing a power function (such as a distance decay function or any type of exponential functions) for such problems in the gravity model leads to an infinite number of trips in the intra-areas. A common solution to overcome this problem would be assigning large values to these intra-area distances, enough that the inhibiting effects become so powerful that trip distribution within these areas will become zero.

Traditional gravity models include the constrained/unconstrained gravity models; and the constrained type models include production constrained, attraction constrained and fully constrained models.

The traditional gravity models can be also categorized as follows:

- Unconstrained gravity model;
- Production constrained gravity model;
- Destination constrained gravity model;
- Doubly-constrained gravity model.

Among the models mentioned above, the most common model is the fully constrained gravity model because of its pattern recognition ability and accuracy. The rest of the gravity models have different problems, e.g. miss-specification, inconsistency, multi-co-linearity or data distortion due to the equations transforming into a linear and operational form (Tapkin, 2004).

The intervening opportunities model is an alternative for the gravity model. It is based on an interestingly simple assumption: there is always a probability that the traveller will be satisfied by the next opportunity that shows up. An even simpler hypothesis is that there is a proportional relationship between the number of trips that originate from an origin to a destination zone and the number of opportunities available at the destination zone; this relationship becomes inversely proportional to the number of intervening opportunities (Xie, 2000).

The order of destination zones that are far from the origin zone must be determined in order to compute the intervening opportunities. The following equation gives the number of trip origins inside the i-th zone (O_i) multiplied by the probability of the trip being terminated inside the j-th zone (Wilson, 1975):

$$T_{ij} = O_i \left(P(v_{j+1}) - P(v_j) \right) \qquad 2.25$$

$$T_{ij} = O_i \left(e^{-LV}{}_{j+1} - e^{-LV}{}_{j} \right) \qquad 2.26$$

where:

P(Vj) = total probability of the trip being terminated before the j-th possible destination;

Vj = included volume or already considered possible destinations that have been reached before the j-th zone; and

L = probability of acceptance of a considered possible destination.

Common statements of the intervening opportunities model are the above equations (equations 2.25 and 2.26). The major difference between the gravity model and the intervening opportunities model is that the latter is probability based while the former is a deterministic model.

2.8 Summary

This chapter provided a brief review of the basic concepts in trip distribution. The concept of the gravity model, deterrence function and generalized cost are reviewed in the trip distribution section of this chapter. This chapter also discussed the different techniques available for trip distribution problems. The advantages and disadvantages of the trip distribution techniques were reviewed. The gravity model was discussed as the most popular technique for trip distribution problems.

3

3. NEURAL NETWORKS

3.1 Introduction

This chapter starts with reviewing the concept of Neural Networks (NN) and their elements and learning algorithms. The Multilayer Feed Forward Neural Network (MLFFNN) which is the most common neural network architecture for forecasting purposes will be discussed and the Back Propagation (BP) algorithm for training the MLFFNN will be reviewed. The Probabilistic Neural Network (PNN) is another type of NNs which is discussed in this chapter. Generalised Regression Neural Networks (GRNN) fall into the category of PNNs and are used in this research as a recommended NN for forecasting the trip distribution. The MLFFNN and GRNN form the basis for the methodology used in this research and are explained in Chapter 4.

3.2 General Concept

As an artificial intelligence technique, the neural network (NN) simulates the functions of the human brain (including the nerves and neurons). It consists of parallel interconnected computer processor units that operate simultaneously. The concept of NN emerged with the discovery of Neuron in year 1836 (Skias, 2006). NN was first introduced by McCulloch et al. (1943) in the early 1940s (Haque & Sudhakar, 2002). They designed simple neural networks that could simulate basic logic functions.

Today neural networks are used for solving problems with complicated algorithmic solutions or even no algorithmic solutions; i.e. it is hard to develop a mathematical model for solving problems for which a relationship between its inputs and outputs is hard to distinguish. In order to solve this type of problem, NN employs sample sets and trains itself to learn the relationship between inputs and outputs. The ability to learn through samples makes NN a very flexible and powerful tool. Hence, it has been widely used in regression mapping and problem classification in various disciplines. Neural networks, in short, are nonlinear algorithms with the ability to learn and classify.

Artificial neural networks (ANNs) gather information about relationships between input data and learn (or get trained) through processing input data sets rather than programming. An ANN consists of a large number of smaller units, called artificial neurons or processing elements (PE) that are connected to each other with weighted connections. The neurons and the connections altogether form the neural network structure and are placed in different layers. Neural networks get their power from the interconnections between neurons of the network. Each PE is assigned some weighted inputs, a transfer function and one output. The transfer function, the learning rules, and the architecture of the neural network determine its performance. The weights of the connections are adjustable parameters, hence the neural network is considered to be a parameterized system. Activation of the neuron is based on the weighted sum of the inputs. When the activation signal is sent to the transfer function, the neural network generates an output. The non-linearity of the neural network is a result of the transfer function. The training procedure optimizes the inter-unit connections and must be repeated until the estimation errors are minimized and the required level of accuracy is achieved. With proper training, the neural network will be able to predict the output of new data sets. Various types of artificial neural networks have been introduced, and new ones are invented every week. All of these networks can be identified by three factors: the transfer functions, the learning rule, and the weights of the connections. ANNs are a capable means of modelling, especially for data sets with complex relationships. In the process of designing an artificial neural network model, no knowledge is needed about the data source; instead, many weights must be evaluated, therefore large training sets are required. ANNs can also combine and employ both literature-based and experimental data in problem solving. ANNs are employed in various applications, which can be divided into five major Categories (Anderson and McNeill, 1992):

- *Prediction,*
- *Classification,*
- *Data association,*
- *Data conceptualization, and*
- *Data filtering*

Jain et al. (1996) reported more categories to be solved by neural models. The advantages of application of NNs for solving complicated problems are:

Power: As a highly complex modelling technique, the ANN is suitable for modelling very complicated functions. Neural networks exhibit nonlinear behaviour. Since there are well-known optimization methods for linear models, they have been the most commonly used modelling technique; however they faced problems wherever linear approximation was not applicable.

Easy application: Neural networks learn through processing example data sets. The user must design training algorithms and provide proper input data sets for the automatic learning procedure to run successfully. In order to successfully employ an ANN, the user is required to have some level of knowledge about data selection and preparation, selection of an appropriate type of neural network, and also interpretation of the results. However, the required level of user knowledge is still very low compared with that required for using traditional nonlinear statistical models.

3.2.1 A Biological Neuron

A biological neuron is a large cell that receives input data from various sources. It combines the data and performs nonlinear operations on the results, and sends the final result to the output. **Figure 3-1** illustrates a biological neuron.

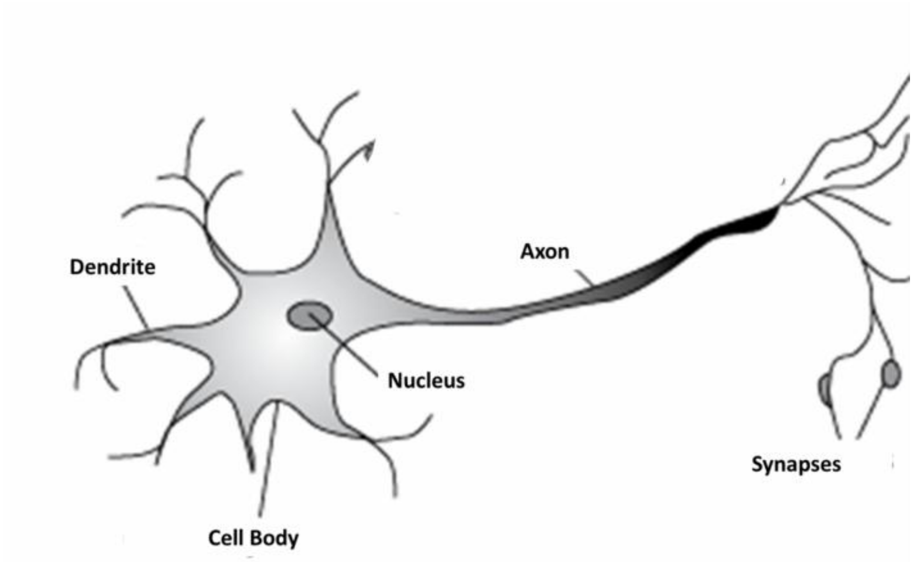

Figure 3-1: A Biological Neuron

There are numerous varieties of basic neuron types inside the human body, all of which share the same four basic components: the cell body (soma), dendrites, axon and synapses (Anderson & McNeill, 1992).

Cell body (soma): The soma contains the nucleus and performs vital biochemical reactions.

Dendrite: Dendrites are fine, hair-like tubular extensions connected to the Soma. They have several branches extending from them. Dendrites receive incoming signals.

Axon: The axon is a long, thin, tubular structure and works as a transmission line for sending data into other neurons.

Synapse: Synapses are complex spatial structures that connect neurons to each other.

Axons are divided into several branches at their extremities; this is called terminal arborisation. The highly complex and specialized arrangements at the end of the axons are the synapses which provide the connections between neurons. Dendrites accept incoming signals via the synapses of other neurons. The input data is then processed by the cell body over time and an output signal is generated, which is then sent to the axon to be delivered to other neurons through the synapses.

3.2.2 An Artificial Neuron

An artificial neuron is a PE that can perform the four basic functions of a biological neuron. **Figure 3-2** illustrates the basic structure of an artificial neuron.

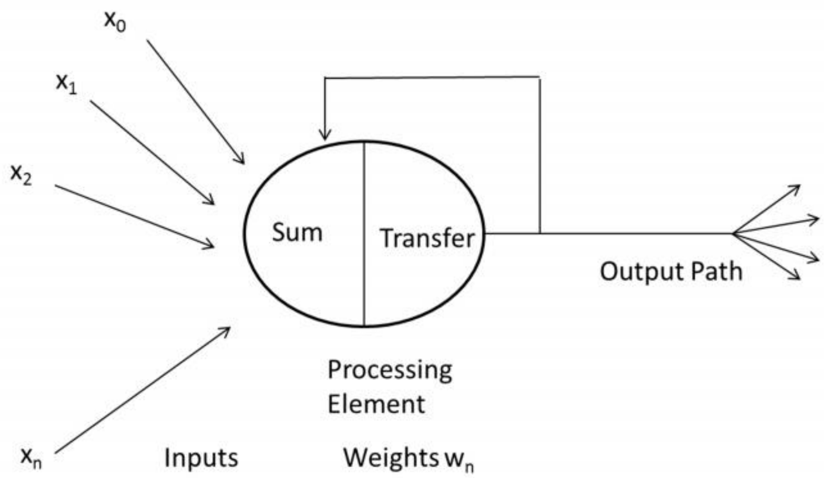

Figure 3-2: A Basic Artificial Neuron (Anderson & McNeill, 1992)

In **Figure 4**, input data from the network are represented by $x_{(n)}$. Each item of data has a corresponding weight, represented by $w_{(n)}$. In the simplest case, the sum of these products is sent to the transfer function which generates the final result. The result is then sent out. Realization of an artificial neural network is possible with various network structures, employing different summing functions and transfer functions.

In some applications like text recognition, speech identification and image processing, the neural network deals with binary data. In these applications, binary operations like ORing and ANDing may be performed in addition to summing operations. These functions can be implemented into the summation and transfer functions of the network. An artificial neuron is built up of seven major components, regardless of the neuron being used as input, output, or inside hidden layers (Anderson & McNeill, 1992).

Component 1, Weighting Factors: Most of the time, a neuron receives many simultaneous inputs, each having its own relative weight, giving them a corresponding impact on the summation result. The importance of each input is represented by the magnitude of its weight. The more important the input, the greater the weight, and the greater the effect on the output. These adaptive coefficients, i.e. the weights, determine the influence of the input and are a a measure showing the strength of the corresponding input connection. These strengths can be adjusted with respect to the training data sets and the network's topology and its learning rules.

Component 2, Summation Function: If the inputs and their corresponding weights are represented as (i_1, i_2 . . . i_n) and (w_1, w_2 . . . w_n) vectors, respectively, the total sum would be the dot product of the two vectors; i.e. $(i_1 * w_1) + (i_2 * w_2) + \ldots\ldots.. + (i_n * w_n)$.

The summation function, in many cases, is more complex than just a simple weighted sum of the inputs. Different combinations of the inputs and their corresponding weights can be defined and passed to the transfer function. Other functions like minimum, maximum, product, majority and other algorithms may also be implemented in the summation function. The selection of a proper algorithm for combining input data is based on the architecture and paradigm of the network. There is sometimes an additional 'activation function' implemented in the summation function, which allows the result to vary before it is sent to the transfer function.

Component 3, Transfer Function: The transfer function is an algorithm that processes the outcome of the summation function into a working output. In order to determine the neural output, the transfer function may compare the summation with a threshold. If the sum is out of range of the threshold, a signal would be generated but if it is within the range, the processing element would generate no signal or send an inhibitory one. The threshold function is usually a nonlinear function. Since the output of linear functions is proportional to the input, they are limited. If the transfer function is a step function, it would generate 0/1, 1/-1, or other combinations for the representation of the result. Another way is to mirror the input if it is within the given range and generate a step function if it is outside the range. In between the minimum and maximum values, the threshold function behaves linearly, but the clipping behaviour outside the threshold makes it a nonlinear function. An ‘S’ curve is another option with asymptotic minimum and maximum values, and is called a sigmoid within the range of 0 and 1, and a hyperbolic tangent between -1 and 1. The ‘S’ curve and all of its derivative functions are continuous functions.

Component 4, Scaling and Limiting: Being modified by the transfer function, the result passes through additional processes such as scaling and limiting. The scaling process multiplies a scale factor and adds an offset. In order to make sure that outcome of the scaling process does not exceed upper or lower limits, the limiting process is employed. This is in addition to the probable hard limits the main transfer function may have applied.

Component 5, Output Function (Competition): Each one of the processing elements can generate one single output signal and send it to many other neurons. The output is often equivalent to the result of the transfer function but sometimes the output of the transfer function is modified in order to incorporate competition between contiguous processing elements. In this case, the neurons can compete and even inhibit other processing elements, except the very strong ones. This type of competition may also occur at the input level. At the first level, the competition determines which neuron will provide the output. At the second level, competition is between inputs and determines which neuron participates in the learning or adaptation process.

Component 6, Back-Propagation and Error Function: The difference between the current and desired output is defined as error in most control schemes. The error is sent to the error function which must be designed according to the network architecture. Different architectures may use the error value directly or use its square, cube or other paradigms depending upon their specific purpose. The error or its modified paradigm is then propagated back to previous layers. This is called back-propagation. Before back-propagation, the error may be scaled or often fed to a derivative of the network transfer function depending on the network type. After being scaled by the learning function, this back-propagated value will usually be multiplied by the weights of each incoming connection and update their values before starting the next learning cycle.

Component 7, Learning Function: The learning function evaluates the weights of the inputs for each processing element based on a certain neural algorithm.

3.2.3 An Artificial Neural Network

An artificial neural network is illustrated in **Figure 3-3**. The incoming connection is on the upper left of the processing element. In the first step, each input is multiplied by its corresponding weighting factor ($w_{(n)}$). The modified values are then fed to the summing function, which may simply calculate their total sum or apply different operations, e.g. the largest input, the smallest input, the ORed and the ANDed values. Several other types of summing functions may also be implemented in addition to the activation function which is used to implement the time sensitivity option. In the next step, the output of the summing function is fed to the transfer function, and turned into an applicable output (e.g. 0/1, -1/+1) by certain algorithms. The output may also be scaled or threshold controls may be applied to it. The final output will be sent to other processing elements or probable outside connections, according to network structure (Anderson & McNeill, 1992).

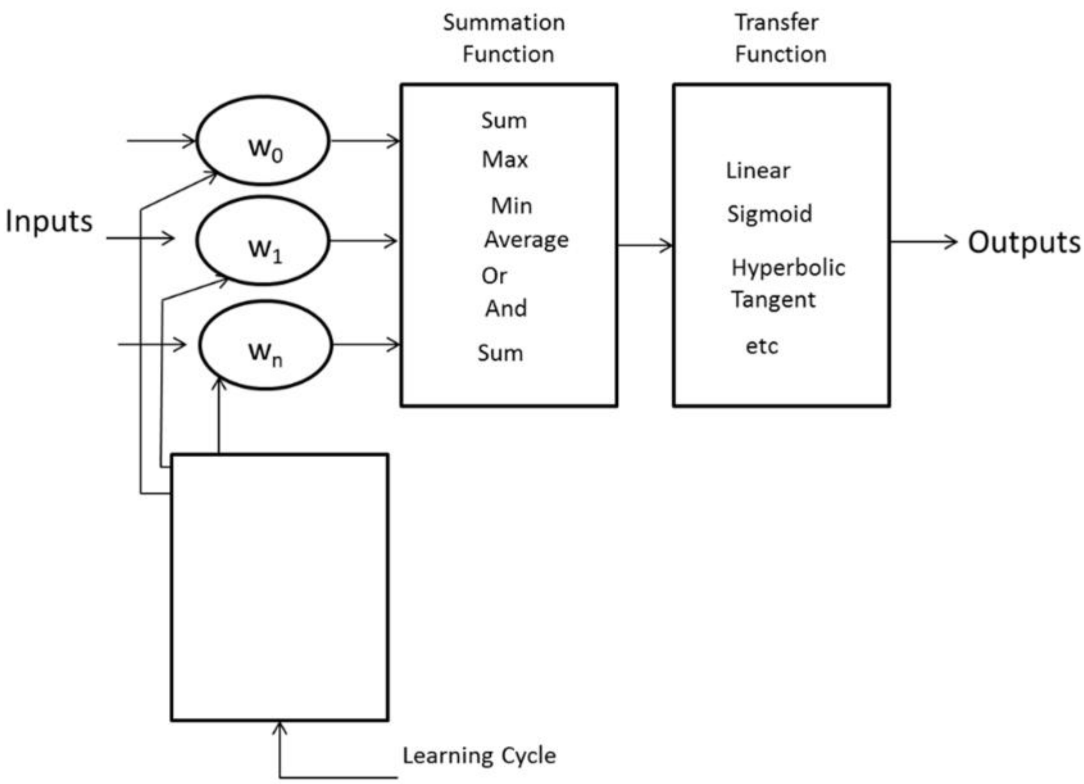

Figure 3-3: An Artificial Neural Network, (Anderson & McNeill, 1992)

3.2.4 Types of Artificial Neural Network

SINGLE LAYER FEED-FORWARD NETWORK

A neural network in which the input layer of source nodes projects into an output layer of neurons but not vice versa is known as a single feed-forward or cyclic network. In a single layer network, 'single layer' refers to the output layer of computation nodes as shown in **Figure 3-4**.

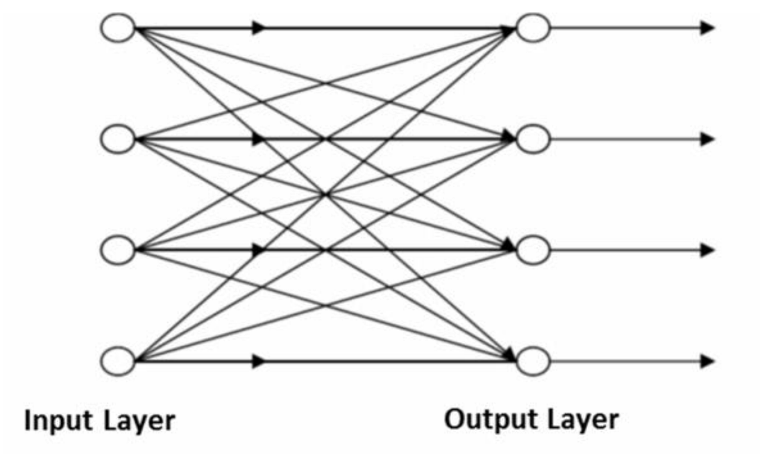

Figure 3-4: A Single Layer Feed-forward Network (Gershenson, 2003)

MULTILAYER FEED-FORWARD NETWORK

As **Figure 3-5** illustrates, there are one or several hidden layers in this type of network, and it has computation nodes which are called hidden units or hidden neurons. The function of hidden neurons is the interaction between the output and input of the network with some helpful methods, and the extraction of higher statistics orders. Due to being hidden from the first layer, the input signal to neurons for the second layer is supplied by the source nodes of the internal layer. The third layer receives the output signals of the second layer and so on. The overall response of the network to the activation pattern is constituted by the set of output signals from the external layer, which is supplied by the source nodes of the first layer input (Anderson & McNeill, 1992).

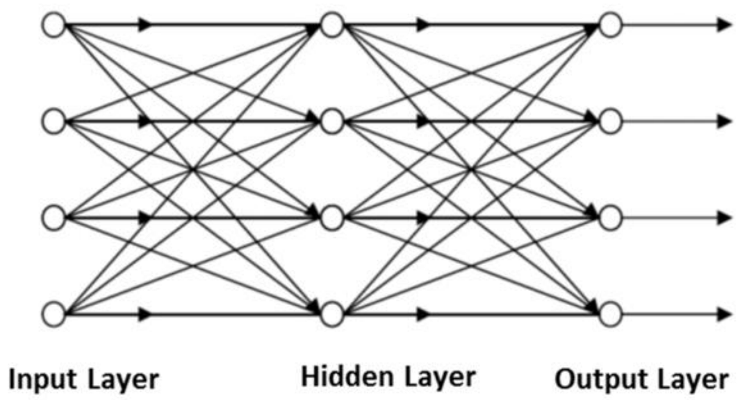

Figure 3-5: A Multilayer Feed-forward Network (Gershenson, 2003)

In brief, the features of feed-forward networks are as follows:

1. *Although there are many architectures, feeding of activation is typically done through ‘hidden layers’, from input to output.*
2. *Execution of static input-output mappings is mathematically performed by them.*
3. *Back-propagation algorithm is the most popular supervised training algorithm.*
4. *It has been use in many practical applications such as nonlinear function approximation, and also pattern classification.*

RECURRENT NETWORK

The feedback loop is known as a recurrent network, and there is at least one feedback loop as well as one or several hidden layers in a feed-forward neural network (**Figure 3-6**).When a neuron’s output is fed back to its own input, the feedback may be referred to as self-feedback. Unit delay elements are sometimes used in feedback loops, and assuming that the neural network has nonlinear units, this will result in nonlinear dynamic behaviour.

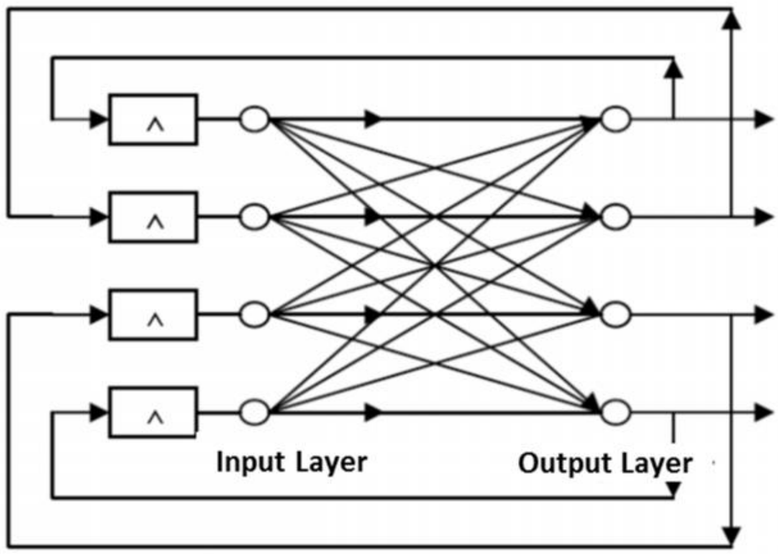

Figure 3-6: A Recurrent Network (Gershenson, 2003)

There are different types of network, such as Hamming, delta-bar-delta, Hopfield, probabilistic, counter propagation, adaptive resonance, vector quantization, bidirectional associative memory, Boltzman, recirculation, spacio-temporal pattern, self-organizing map etc. (Anderson & McNeill, 1992).

There is at least one cyclic path of synaptic connections in a recurrent neural network, and its basic characteristics are as follows:

1. All biological neural networks are recurrent;
2. Dynamic systems are mathematically implemented;
3. Without recognition of a clear winner, several types of training algorithms are known; and
4. So far practical applications have been prevented by theoretical and practical difficulties.

3.2.5 Training of Artificial Neural Networks

A network is ready for training when it has been constructed for a specific application. At first, the initial weights are randomly selected, and then the training starts. Two approaches are used for training: supervised and unsupervised.

SUPERVISED TRAINING

Both inputs and outputs are available in supervised training; the inputs are processed by the network and the results are compared with the desired outputs. Errors cause the system to adjust the weights by propagating back through the system, so the network can be controlled. This process of adjusting the weights occurs frequently. The ‘training set’ is the set of data used for the training process, The connections between the weights are continually refined, as the same set of data is processed several times during the training of a network. If specific information is lacking in the input data which leads to the desired output, the network may not learn. If the data is insufficient to enable complete learning, networks also will not converge. If there is sufficient data, a part of it can be taken to be tested (as a training data set). Many classified networks with multiple nodes can memorize data. It is essential to determine whether the system can simply memorize its own data in some unimportant way to monitor the network. This can be done by keeping back a set of data with which to test the system after it has undergone training.

If a network is simply unable to solve a problem, the designer will need to review the number of layers, the connections between the layers, the number of elements per layer, the input and outputs, transfer, training functions, the summation, and even the first weights. The training is governed by the designer's creativity. Adaptive feedback is required in order to adjust weights during training, and this can be achieved by using one of the laws (algorithms) which implement adaptive feedback. Back-propagation is known as the most common technique. The training acts as a kind of conscious analysis to ensure that the network is not over-trained. An artificial neural network can initially configure itself by using the general statistical trends of the data. The ANN continues to ‘learn’ from other aspects of the data; however this can be seen as spurious from a general viewpoint.

If desired, when no further learning is needed and the system is finally correctly trained, the weights can be ‘frozen’. In some systems, the finalized network can be converted into hardware to speed up the process; while other systems continue learning during the production phase because they are not locked in (Anderson & McNeill, 1992).

UNSUPERVISED OR ADAPTIVE TRAINING

The other type of training is unsupervised training. In this type of network there are only inputs available, and there are no desired outputs. To group the input data, the system itself decides what features will be used. This is often referred to as adaption or self-organization. These networks examine the performance of their weights internally as no external influence is involved. Adaptations are made according to the function of the network, and the networks also seek regularities in the input signals. The network should have some information about how to organize itself even if there is no awareness of the correct direction. This information determines the network's rules and its topology. Cooperation may be emphasized by an unsupervised learning algorithm among the clusters of processing elements, enabling the clusters to work together in such a scheme. If some external input motivates any node in a cluster, the entire cluster's activity could be increased. Moreover, external input could have an inhibitory effect on the entire cluster, if there is a decrease in the external input to the nodes.

Competition could also be a basis for learning between processing elements. The responses of specific groups to particular stimuli can be expanded by the training of competitive clusters. And also, those groups with a specific response will become associated with each other. When there is competition for learning, only the weights which belong to a prominent processing element will normally be updated. There is still a gap in the knowledge about unsupervised learning, so significant research is required (Anderson & McNeill, 1992).

LEARNING RATES

Several controllable factors affect the learning rate of ANNs. The slower the rate of learning, the more time is needed to produce an adequately trained system. Fine discriminations may not be made by a network undergoing a faster rate of learning, but should be possible by using a slow learning system. There is some provision for a learning rate (learning constant) in most learning functions. The rate of learning is usually positive and takes a number between 0 and 1. The learning algorithm easily overshoots to correct the weight if it is greater than 1, therefore the network will oscillate. Small values will not correct a current error quickly, but there will be a good chance for arriving at the best minimum convergence, if small steps are applied to correct errors.

LEARNING LAWS (ALGORITHMS)

There are many commonly used learning laws. Most of them are variations on a similar theme, and the oldest and most famous is ‘Hebb’s Rule’ (Anderson & McNeill, 1992).

Hebb’s Rule: If a neuron receives an input from another neuron while both of them are highly active with the same sign, the weight between them should be strengthened; this is defined as the basic rule. It was introduced by Donald Hebb in *Organization of Behaviour*.

Hopfield Law: There is an increment in the connection of weight by the learning rate if both desired output and input have the same state, otherwise it decreases the weight.

Delta Rule: According to this simple rule, in order to reduce the difference (the delta) between the actual output and the desired output value of a processing element, it continuously modifies the strengths of the input connections.

The Gradient Descent Rule: There are similarities between this and the Delta Rule. To modify the delta error before applying to the connection weights, there is still the derivative of the transfer function. However, an extra appropriate constant factor is appended to the final modifier factor which operates the weight.

Kohonen’s Law: Processing elements compete for the opportunity to update their weights. The element with the largest output is successful, and is capable of inhibiting its competitors and stimulating its neighbours. According to this rule, only the successful element is permitted to have an output, and only this element and its neighbours are allowed to adjust their connection weights.

Since the back-propagation algorithm is one of the most common methods used in ANNs, and many others are based on it, the back-propagation algorithm for learning the appropriate weights is discussed briefly here. Probabilistic neural networks are frequently used to classify patterns based on learning from the examples which will be reviewed briefly in this chapter. The generalized regression neural network (GRNN) falls into the category of probabilistic neural networks, which is the basis of the analysis in this thesis, and therefore will be discussed briefly in this section as well. More discussions on the GRNN model are provided in Chapter 4.

3.3 Back-Propagation Algorithm

The back-propagation algorithm is used to train a FFMLNN for a given set of input data with known classifications. When each data is presented to the neural network, the network compares its output response to the input data. The modelled output is then compared to the observed output and the error which difference between the modelled output and observed output is calculated. Based on the reported error, the connection weights will be adjusted. BP uses gradient descent similar to same algorithm which is used in solving mathematical optimization problems. In gradient descent algorithm a step size needs to be selected. This step size is called the learning rate in back-propagation algorithm. The learning rate indicates the adjustments to the weights at different iterations. Initial weights are normally selected randomly for the BP algorithm. The gradient descent based training algorithm is sensitive to the initial weights and often experiences local minima issue (Celikoglu, 2006), another disadvantage of BP algorithm is its convergence rate which is very slow (Rigler et al., 1991, Jacobs, 1988, Wilamowski et al., 2001) and therefore requires a number of iterations to converge (Vogl et al., 1988). Significant work has been undertaken to improve the convergence speed of BP through optimization techniques (Barnard, 1992, Hagan and Menhaj, 1994).

3.3.1 Pros and Cons of Back-Propagation Neural Networks

The flexibility of back-propagation neural networks is one of its attractive features. This feature is useful for decision-making or pattern recognition problems. Another advantage of BP is that the process is highly parallel, and the use of parallel processors could reduce the calculation time (Specht, 1991; Gupta & Rao, 1993; Cherkassky et al., 1993).

Back-propagation neural networks also have negative features as discussed in the above, such as the substantial amount of time required for training the BP network (Gupta & Rao, 1993). The network performs very fast as soon as the training is complete. The size of the training data for back-propagation neural networks should be very large, and in some respects this is a disqualifying aspect. Providing enough training samples is almost impossible (Zurada, 1962), for example, when the training samples are the result of very expensive experiments or when the data is from observations of nature which occur very rarely.

3.4 Probabilistic Neural Network

The classification of patterns is undertaken based on learning from examples using probabilistic neural networks. 'The Bayes Strategy for Pattern Classification' is the basis for probabilistic neural networks. The pattern of statistics is determined by the different rules from the training samples.

Back-propagation is not based upon statistical methods. Many feedback iterations and long time periods are required for the back-propagation to gradually approach the underlying function (Specht, 1991). It is desirable to approach the parameters by a one-step-only method. The Bayes Strategy for Pattern Classification is used to obtain characteristics from the training samples that reveal knowledge about the underlying function.

Figure 3-7 illustrates the general structure of a probabilistic neural network. There are two hidden layers and one input layer in the probabilistic neural network. The pattern units include the important functional form which is in the first hidden layer. Information on one training sample is represented by each pattern unit.

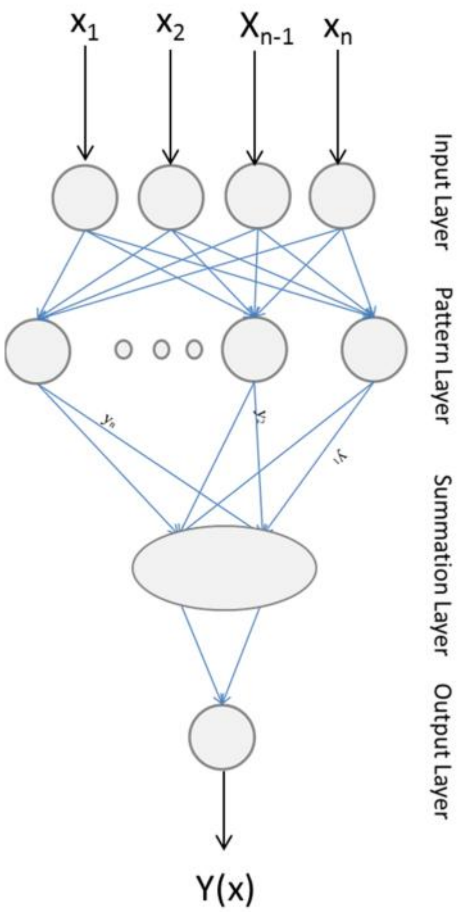

Figure 3-7: Block Diagram of a Probabilistic Neural Network

Figure 3-8 shows the calculations of the pattern unit. Each pattern unit performs an estimation of the probability on how well the input vector fits into the pattern unit. Deciding which pattern the input vector finally belongs to is done through the individual results for each pattern. Only one summation unit is in the second hidden layer. In order to give the output a physical meaning, a calculation is again performed by the output unit. Having multiple outputs is not always possible for a probabilistic neural network. There is a large difference between a probabilistic neural network and a back-propagation neural network which is defined as the process inside the neurons. Some functions are applied by the probabilistic neural network, based on knowledge from the Bayes Strategy for Pattern Classification. Therefore, fitting the data in the best way by the selection of weights is not a defined strength of the probabilistic neural network. This is used inside the neuron that lies in the function.

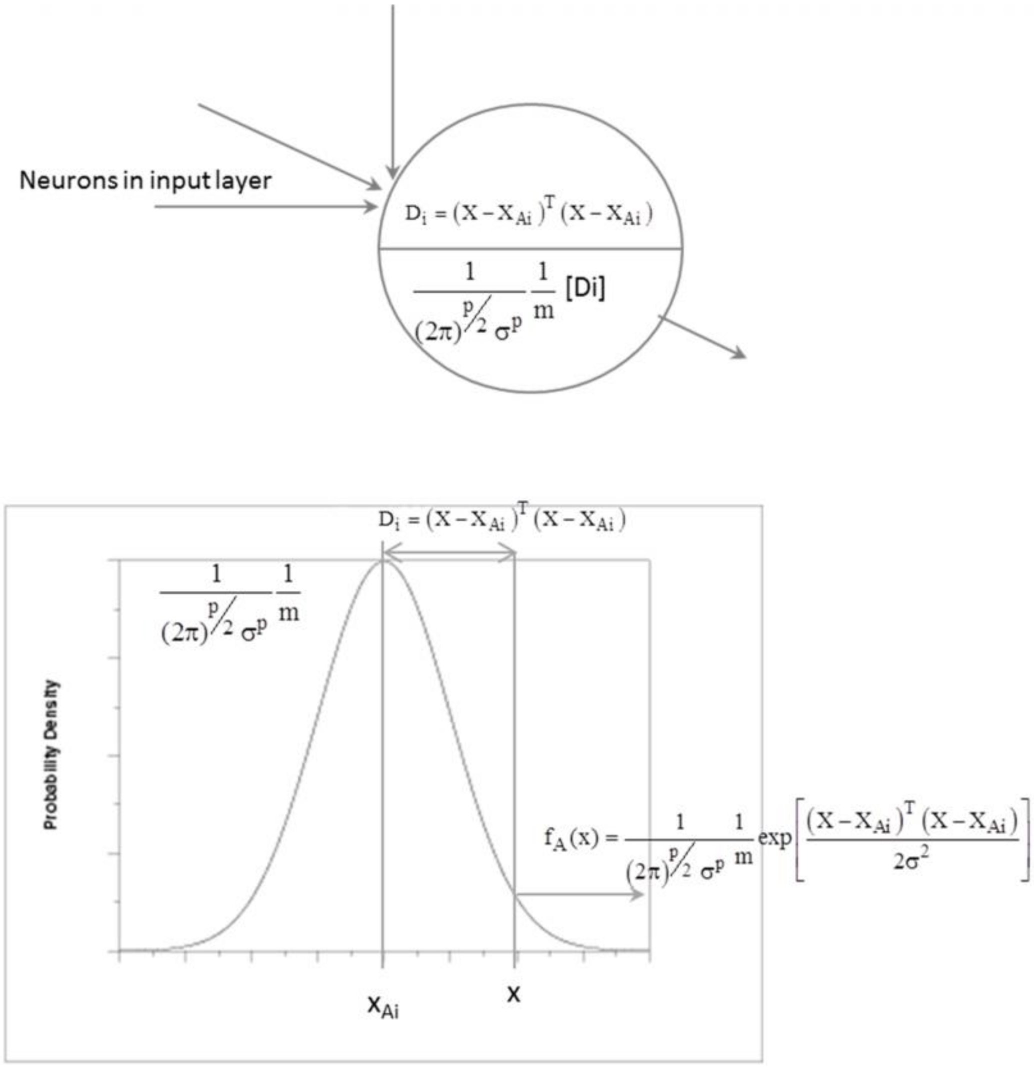

Figure 3-8: Process in a Pattern Unit (Anderson & McNeill, 1992)

A probability density function is used in the neuron of a pattern unit. As **Figure 3-9** illustrates, there should be a distance between the sample point and the position at which the prediction takes place that calculates the output. Actually, the probability density function needs this distance. In the summation unit, the output of each pattern unit is summed up and then transformed into a result with physical meaning.

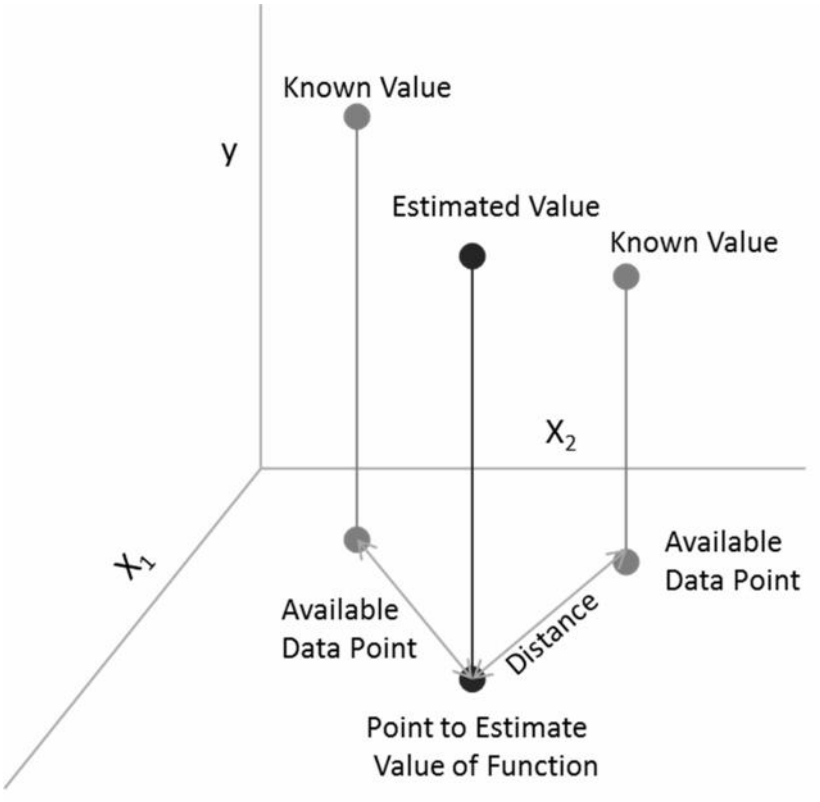

Figure 3-9: Distance between the Training Sample and the Point of Prediction (Anderson & McNeill, 1992)

The Bayes Strategy for Pattern Classification can also be used for prediction of continuous outputs because it is valid for continuous results as well (Parzen, 1962).

3.4.1 Generalized Regression Neural Network

Specht (1991) proposed that the generalized regression neural network (GRNN) falls into the category of probabilistic neural networks. Like other probabilistic neural networks, only a fraction of the training samples from a back-propagation neural network are required in this neural network (Specht, 1991). From the measurements of an operating system, the available and accessible data is generally never enough for a back-propagation neural network (Specht, 1990). Since the probabilistic neural network is capable of converging the underlying function of the data with only a few available training samples, it is considered as a powerful tool in practice.

3.5 Probabilistic Neural Networks vs Back-Propagation Neural Networks

According to Cherkassky et al. (Cherkassky, 1993) statisticians, researchers and neural network developers have different backgrounds and goals in analytical methods or designing algorithms, therefore a tension always exists between them. The structure of the data is the main focus in statistical methods, while it is secondary for neural network developers. Therefore, the neural network approach needs a greater quantity of data than the amount of data needed for statistical methods.

Most methods are asymptotically good (Cherkassky, 1993), while there are severe drawbacks to most of them. Back-propagation networks require a large number of training samples and a lot of time in order to gradually approach the good values of the weights. It is also computationally very expensive to add new information into BP models which require retraining, while this is not the case for probabilistic neural networks. The prediction algorithm in probabilistic neural networks works with only a few training samples, which is a great advantage, and the other main advantages are flexibility and the ability to add new information immediately without retraining. Therefore the advantage of PNNs over the BP can be summarised as below:

- *Fast training process;*
- *Always converge to an optimal;*
- *No local minima issues; and*
- *Training data can be added or removed without substantial retraining.*

3.6 Utilization of the neural networks to model trip distribution

The application of neural networks in the transport modelling area is growing fast. The literature indicates that NN have been used for driver behaviour simulation models, mode choice and trip distribution problems. **Table 1** summarizes the major studies undertaken so far to estimate trip distribution by applying the NN technique. **Table 3-1** indicates that all of the studies undertaken used trip production, trip attraction and distance between a pair of zones as the inputs to the neural network model. BP was the main training algorithm used for most of the studies and RMSE was the main performance measurement used in the majority of the research.

Table 3-1: Application of Neural Networks for Trip Distribution Estimation

		Network details			
Author	Date	Input Data	Network Structure	Training	Performance
Black	1995	P, A, D	MLF	BP	RMSE
Xie	2000	P, A, D	MLF	BP	RMSE, R
Mozolin et al.	2000	P, A, D	MLF	BP	RMSE, AE
Dantas et al	2000	GIS, REMOTE SENSING	MLF	BP	MSE
Tapkin	2004	P, A, D	Revised MLF	GD	RMSE
Celik	2004	P, A, D	MLF	BP, LM	RMSE
Tillema et al.	2006	P, A, D	NA	NA	RMSE
Yaldi et al.	2009	P, A, D	MLF	BP	RMSE, R
Yaldi et al.	2011	P, A, D	MLF	LM	R^2

Abbreviation definitions: P: production, A: attraction, D: distance, MLF: multi-layer feed-forward, BP: back-propagation, RMSE: root mean square error, AE: absolute error, NA: not available, R: correlation coefficient R^2: coefficient of determination, LM: Levenberg-Marquardt.GD: Gradient descent

A neural network is recognized by its key properties, including its learning algorithm, number of layers (input, hidden and output) and nodes inside each one, activation function, and learning rate (Teodorovic and Vukadinovic, 1998; Dougherty, 1995). The amount and split of the data used for training, validating and testing procedures are also important factors in the performance of the network (Carvalho et al., 1998). It was proposed by Zhang et al. (1998) that an NN model may be developed through trial and error methods if no appropriate guidelines are available. There is insufficient research on the behaviour of NN. Some researchers combine the application of NNs with other algorithms such as genetic algorithm to improve the performance of the modelling outcome. For example, Fischer and Leung (1998) developed different models of NN and combined them with the genetic algorithm (GA) in order to predict traffic flows in a region in Australia. Their results showed that combining GA and NN modelling leads to an improvement in results.

It should be noted that employing an NN must be the result of logical and theoretical considerations; otherwise it would be a naive tool. A neural network is an intelligent computer system that employs the processing capabilities of the human brain for its simulations (Black, 1995). It is a method able to forecast and solve problems through minimizing errors (the deviation between input and desired output) using complicated training processes (Black, 1995; Zhang et al, 1998).

Several studies have been undertaken in order to determine the advantages and disadvantages of using NN in transportation modelling. Studies have compared NN modelling results with the results of conventional methods; e.g. NN has been compared with the discrete choice model in research performed by Carvalho et al. (1998), Subba Rao et al. (1998) Hensher & Ton (2000) and Cantarella & de Luca (2005). According to the current literature, the application of NN in trip distribution is not as common as mode choice studies. Most of the cited papers in trip distribution modelling by NNs indicate the application of multi-layer feed forward NN trained by BP or LM. No researcher has investigated the application of the other NN structures such as Radial Basis Function (RBF) or Generalized Regression Neural Networks (GRNN) for trip distribution problem.

Celikoglu (2007) investigated the application of RBF and GRNN in non-linear utility function specification for travel mode choice modeling. The study undertaken by Celikoglu investigated the performance evaluation of three neural network methods, RBFNN, GRNN, and FFBPNN, and multivariate linear regression analysis during the calibration process of a binary logit model, in order to split daily home-based work trips into private car and public transport modes.

The neural network method established by Celikoglu was not used directly for model calibration. It was used as a sub-process for identifying an alternative to represent the non-linear utility function of the selected model. The neural mode choice model was developed for home-based work trips to split trips into private car and public transport modes. The calibrated outcomes were compared with a conventional statistical method, multivariate linear regression (MVLR), in terms of selected performance criteria. The results indicated that the all three NNs are able to predict utilities that provide reasonable estimates for mode choice calibration process. In particular, calibration involving NNs as a sub-process indicated slightly better performance.

Black (1995) has investigated the modelling of spatial interactions focusing on commodity flows with NN. The structure of the model was similar to that of the gravity model. The production and attraction of trips and distances between production zones and attraction zones were considered as the input for the NN model developed by Black. He designed an artificial neural network model that included a three-layer back-propagation network: The input, output and hidden layers. Three neurons were employed in each of the input and the hidden layers while the output layer only had one neuron; bias neurons were selected to be attached to neurons of the hidden and the output layers. The network structure contained 16 weighted connections, and every weight was evaluated during the training procedure.

Black analysed Commodity flow data between nine regions in order to make a comparison between the proposed model and a constrained/unconstrained version of the gravity model. Inputs for the gravity models included production/attraction of trips and distances between the regions, while input data for the proposed NN model included regional production and attraction of trips and interregional distance between the origins and destinations of the trips. All input data was normalized to between 0 and 1; the longest distance, the total production flows (row totals of the flow matrix) and the total attraction flows (column totals of the flow matrix) were also normalized. As the normalized data was fed into the proposed NN model, the model would generate normalized output flows though minimization of the errors via the back-propagation algorithm.

By comparing the root mean square values for errors (RMSE) of the aforementioned models, Black inferred that the proposed NN model could reduce error by between 30% and 50%. He concluded that the errors in the proposed NN model were 50% less than for gravity models. He also claimed that modelling accuracy increased from the unconstrained gravity model to the fully constrained gravity model and further to the proposed NN model.

Xie (2000) employed a neural network for the modelling and prediction of intercity passenger flows using the same architecture as the Black model, and this work can be considered as an extension of Black’s investigations which compared the predictive abilities of neural networks and conventional models. Xie (2000) also utilized the same normalization process that Black used for input data. Flow maps were generated after assigning the flows to a partial railroad network. Afterwards, the assigned flows and the flow maps were further statistically analysed.

Xie (2000) used actual passenger flow data from Amtrak for the prediction and analysis of regional passenger flow and its patterns for Amtrak. She stated that insufficient research had been undertaken on Amtrak passenger flows although there were several studies on region-to-region/city-to-city analysis of people and goods transportation by highway or air. She also noted that most of the studies excluded diagonal cells with zero values in the intra-city/intra-regional flows and also off-diagonal cells for these flows. She argued that the zero cells should also be predicted, which would help in comparing the prediction accuracy of different models. She therefore used the data set including all of its zero values.

In her study, Xie (2000) presented a neural network model with back-propagation and a descent gradient search algorithm. In order to assess the predictive ability of the model, it was employed to predict monthly inter-city Amtrak passenger flows between sample stations. Three gravity models were also simulated for comparison, including a regression model, a log-normal regression model, and a fully-constrained model. The predictions for passenger trips were designated to the railroad network in order to acquire the flow maps needed for further network flow pattern and link flow volume analyses. An additional study was performed to determine the relative order of importance of all of the variables that were defined in the neural network model. Xie addressed the temporal stability of the model by cross-validating the Amtrak passenger flow data for a 12-month period. The training data was a set of 97x97 cases tested with a sample size of 3104 cases. The root mean square errors were calculated for comparison with the gravity-based models.

Xie (2000) concluded that the neural network model performed satisfactorily when applied to large data sets and clearly outperformed the regression methods by minimizing errors and making more accurate predictions requiring no additional data. The interaction modelling by the neural network model showed the second best performance in minimizing the total root mean square error, compared with the fully-constrained gravity-based model. The neural network model also outperformed the fully-constrained gravity-based model in the minimization of root mean square error for certain volume groups.

Another study in this field was conducted by Mozolin et al. (2000), who researched the application of multilayer perceptron neural networks and doubly-constrained gravity models for the analysis of commuter trip distribution. They declared that different modelling approaches had been developed for modelling the distribution of trips/freight/information between origins and destinations, one successful example of which was the spatial interaction or the gravity-based model with interrelation between the matrix of flows and the matrix of inter-zonal impedances.

Several studies (Openshaw, 1993; Fischer & Gopal, 1994; Black, 1995) have encouraged the application of neural network architecture in modelling complex spatial interactions; therefore Mozolin et al. (2000) aimed to compare the performance of a perceptron neural network model with spatial interactions to the constrained gravity-based model. Journey-to-work patterns in metropolitan Atlanta were selected as an empirical case for this comparison.

In this study a detailed comparison was made between perceptron multi-layer neural network models and doubly-constrained models in predicting commuter trip distribution. Despite the results of the investigations done by Fischer and Gopal (1994) and Black (1995), which indicated that a neural network model using an iterative proportional fitting procedure might perform effectively in estimating spatial interaction flows, Mozolin et al. (2000) believe that it might better fit the data but accuracy of its predictions is not comparable to that of doubly-constrained models. They also noted that studies they have conducted show that neural network spatial interaction models display lower predictive accuracy than doubly-constrained models using an exponential function of distance decay. A number of probable reasons have been given for the under-performance of neural network models, including non-transferability of the model, its insufficient ability in generalization and dependency on sigmoid activation functions. Further investigations into the application of other perceptron formulations (i.e. spatial structure used as input for neural network) and other neural networks (e.g. radial basis functions) is recommended in order to perform highly accurate predictions of spatial interaction flows.

Dantas et al. (2000) used MLFF neural models to estimate travel demand where the data is mainly sourced from remote sensing (RS) images processed in geographical information system (GIS). Dantas et al. (2000) developed two different model structures for function approximation, and pattern classification. The developed model then applied to Boston Metropolitan Area (Massachusetts State – USA), the first model aimed to find the relations ship between the input data (which was sourced from the RS and GIS) and the output data (trip distributions). The second model structure was developed to forecast the levels of urban movements as main element for evaluation of strategic planning. The second model's output classifies the projected trips in different levels: high, medium-high, medium, medium-low and low.

Recent research by Tapkin (2004) recommended a neural trip distribution model (NETDIM) as a newly-developed approach, and a comparison was made with the predictive performance by three models: back-propagation neural, modular neural and unconstrained gravity models. The ultimate goal was to compare their levels of prediction rather than demonstrating how well they predicted a given set of data, in order to precisely investigate the models' predictive performances. The root mean square errors (RMSE) of predicted and observed zonal trips for different sizes of networks were used to compare the models' prediction levels.

In order to generate various sizes of networks, a network with a size of thirty nodes was chosen and data sets were taken from the Bursa Transportation Master Plan. The networks contained various nodes, each related to a network with a specific size, and the largest network with a size of thirty node zones was selected.

The test results gained for networks with different sizes from the trained neural models and calibrated gravity model yielded RMSE values for which the first, second and third lowest values respectively came from the NETDIM, modular model and gravity model. RMSE has the least predictive capability because of the significant fluctuations in values obtained from analysing the back-propagation model. The NETDIM therefore performed the best out of the models in the prediction of zonal trips, no matter what size the network.

Celik (2004b) used the US commodity flow by using the data from US 1993 Commodity Flow Survey (CFS) to develop and calibrate three different neural models. The neural models were constructed based on the condition of the input data and compared with the Box-Cox model. The Box-Cox model used in the comparison was an interregional commodity flow models based on the earlier study by Celik and Guldmann (2007). The study by Celik (2004b) reported that NN may improve the performance of the predictive models in freight distribution modeling, in the same way as they have for passenger flows. An NN with conventional flow distribution variables may provide moderate performance improvement in comparison with a regression based statistical model or a gravity model good performance of neural models. Then, the research was continued in the same year aiming at investigating the “predictive” capability of neural models (Celik, 2004a).

Tillema et al. (2006) have studied and compared the results of NN and the gravity model in order to predict trip distribution. This study revealed that neural networks in both synthetic and real situations transcend gravity models when data is scarce. These results show the future of trip distribution modelling and were obtained using both real-world and synthetic data sets, which provide the chance of controlling the test. There is a significant difference between this study and others such as that of Mozolin et al. (2000).

These studies clarify the performance of neural networks in various complex cases, and also show that neural networks perform better than other models in unusual cases. These results were achieved just using synthetic cases; when real-world cases are used, the results are stronger. In the study by Mozolin et al. (2000), both synthetic data and real-world data were not used for changing complexity. Moreover, the results of a statistical analysis in this research showed that more training samples are required for gravity models than for neural networks.

Finally, this study compared the performance of two models, doubly-constrained gravity and neural networks models, in the context of trip distribution. The results revealed that neural networks outperformed gravity models when data is scarce. According to the generation of the synthetic data and the research method, when there is a lot of data available, it is less certain that the gravity models outperform the neural networks.

Different studies have been implemented in order to improve the modelling ability of neural networks, also to satisfy the attraction and production constraints. Yaldi et al. (2009) have announced that NN modelling can satisfy production and attraction constraints, with average correlation coefficients (R) of 0.958 for production and 0.997 for attraction while using simple data normalization and a linear activation function (Purelin) in the output layer. Their research results also demonstrated that a reliable NN can generate a goodness of fit similar to that of a doubly-constrained gravity model. However, the average root mean square errors indicate that the NN error level is still greater than that for the gravity model, with the RMSE being 174 and 181 for the gravity model and NN respectively.

In another study, Yaldi et al. (2011) tried to improve the testing performance of the NN by training the models using the Levenberg-Marquardt (LM) algorithm, while standard back-propagation, Quickprop and variable learning rate (VLR) algorithms had been used in previous research. There is a significant difference between these algorithms and this is the method used to define the optimum connection weights.

The work trip data used in this study was based on the 2005 home interview survey conducted in Padang City, West Sumatra, Indonesia. The area of the study included 36 zones. In order to convert the input data to binary mode, a simple data normalization method was used. Matlab was the software used for developing the network, and the modelling tool set the initial values for the connection weights randomly.

As the authors claimed, the study was unique because the experiments were repeated 30 times (previous studies had repeated the experiments just five times, e.g. Mozolin et al. (2000)). Moreover, each experiment had a limit of 100 times for iteration number or epoch, while there had not been such a limit in previous studies. For example, Black (1995) iterated up to 150,000 epochs and Mozolin et al. (2000) up to 100,000 epochs. This high number in training leads to over-fitting of the models.

It was claimed that the error in NN models trained with the LM algorithm is much lower than in the doubly-constrained gravity model. It also had a higher goodness of fit (correlation coefficient/R). The production and attraction constraints are also only satisfied when the model is trained with the LM algorithm. As a result, none of the BP or VLR algorithms was suitable for training the problem of fully constrained spatial movement.

In this study RMSEs of 168, 152 and 125 were obtained for a model trained with BP, VLR and LM respectively, while the R^2 values were recorded as 0.194, 0.315 and 0.505 respectively. The forecasted total trip numbers estimated by the models which had been trained with BP and VLR were lower than the real ones, while the numbers for the LM algorithm were reported as being slightly higher. Yaldi et al. (2011) demonstrated that with the use of LM algorithm, the testing performance of the neural network model could be improved to the same level as the doubly-constrained gravity model.

3.7 Summary

This chapter provided basic information about neural networks and reviewed different types of neural networks. Back-propagation, probabilistic and generalized neural networks were discussed in this chapter. The advantages of the probabilistic neural networks over back-propagation neural networks were reviewed and a basic discussion on generalized neural networks was provided.

Reviewing the available research on trip distribution modelling using neural networks, indicates that neural networks are capable of predicting trip distribution and can be used as a method of trip distribution estimation. A number of studies claim that neural networks even outperform the gravity model in the prediction of trip distribution. NN is recognized by its important characteristics, such as the learning algorithm, activation function, number of layers (input, hidden and output), number of nodes inside each layer, and learning rate. The amount of data and the split of the data used for training, validating and testing purposes are also essential for NN performance. The literature review indicates that few studies have been undertaken that use land use data for a pair of zones as an input to the NN model instead of trip productions and attractions. There have also been no/few attempts to utilize a generalized regression neural network (GRNN) to estimate trip distribution. The advantage of the GRNN model over other feed-forward or feedback neural network techniques is its simplicity and practicality.

4 4. RESEARCH METHODOLOGY AND FRAMEWORK

4.1 Introduction

This chapter explains the methodology for developing three different models for estimation of the work trip distribution in Mandurah locality. The first two models are based on neural networks (GRNN and BP) and the third model is based on traditional gravity model. According to the literature a number of BP models have already been developed and tested for estimation of the trip distribution for commodity, migration and work trip flows. For the purpose of this research a BP model is developed with the proposed Levenberg-Marquardt (LM) algorithm which has been claimed by Yaldi (2011) that performs better than those neural models trained with other algorithms. Considering that GRNN is the focus of this research the structure and the theory of this model is discussed in this chapter and the role of spread factor (sigma) in GRNN models is also reviewed.

4.2 GRNN Model

The generalized regression neural network, as proposed by Specht (1991) falls into the category of probabilistic neural networks as discussed briefly in Chapter 3. The GRNN is a feed-forward network and is especially useful due to its ability to converge to the desired outcome with minimal available training data. Relatively little additional knowledge is required to train the network and develop the GRNN structure, and can be done without additional input by the user. This makes GRNN is a very powerful tool in practice. According to Specht (1991), other benefits of GRNN include:

- *The network is able to learn from the training data by 'one-pass' training in a fraction of the time it takes to train standard feed-forward networks.*
- *The spread, Sigma, is the only free parameter in the network, which often can be identified by split-sample cross validation.*

- *Unlike standard feed-forward networks, GRNN estimation is always able to converge to a global solution and won't be trapped by a local minimum.*

The fundamentals of the GRNN can be found in Specht (1991), Nadaraya (1964), Watson (1964), Tsoukalas and Uhrig (1997), and Schioler and Hartmann (1992). A schematic structure of the GRNN is illustrated in **Figure 4-1**. A GRNN does not require an iterative training procedure. It can estimate any nonlinear function between input and output vectors, learning the relationship between the input and output data directly from the training data. Furthermore, it has been found that with a larger training set size, the estimation error approaches zero, with minimum restrictions on the function. The GRNN is used to predict continuous variables as in standard regression methods.

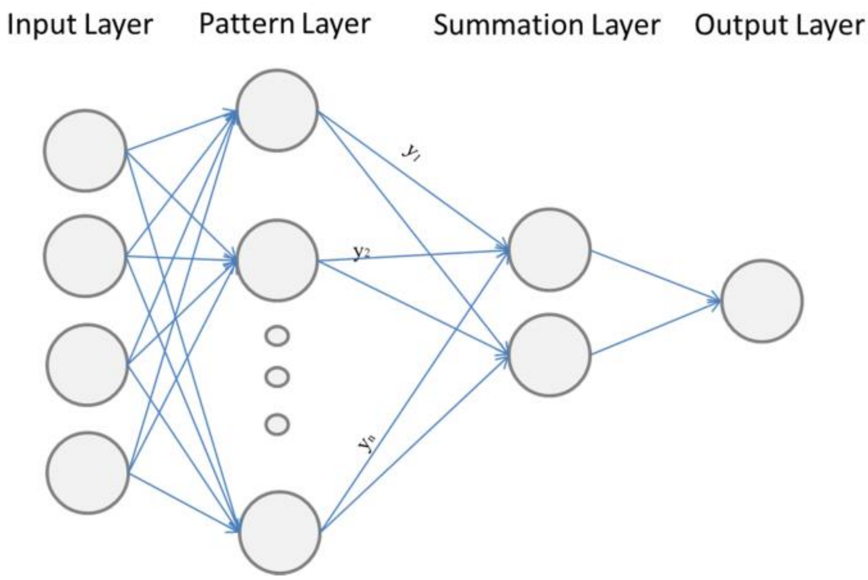

Figure 4-1: Schematic Structure of GRNN

The GRNN consists of four layers as shown in above figure: input layer, pattern layer, summation layer and output layer. The first layer which is the input layer is connected to the pattern layer. The total number of parameters in the input layer is identical to the number of input units. The second layer represents the training pattern and is called the pattern layer, and it calculates the distance between the input and the stored patterns. The third layer is the summation layer and entails two neurons: the S-summation neuron and the D-summation neuron. Each unit in the pattern layer connects to the summation layer. The S-summation layer calculates the sum of the weighted outputs of the pattern layer and the D-summation layer measures the unweighted output of the pattern neurons. The linkage weight y_i in above figure represents the calculated weight between the S-summation neuron and the i^th neuron in the pattern layer; the target output value links to the i^th input pattern. In the last layer or the output layer the output of each S-summation neuron will be split by the output of each D-summation neuron, which provides the predicted value to an unknown input vector x as:

$$yi(x) = \frac{\sum_{i=1}^{n} yi \; exp[-D(x, xi)]}{\sum_{i=1}^{n} exp[-D(x, xi)]} \qquad 4.1$$

in which n represents the number of training patterns. The Gaussian D function is calculated as follows:

$$D(x, xi) = \sum_{j=1}^{p} \left(\frac{xj - xij}{\delta}\right)^2 \qquad 4.2$$

p shows the number of elements of an input vector. The x_j and x_{ij} represent the j^{th} element of x and x_i respectively. The δ is generally known as the spread factor. The optimal value of δ is calculated experimentally (Specht, 1991). The larger the spread factor, the smoother is the function approximation. If the spread factor is too large, it means that many neurons are involved in function approximation. If the spread factor is too small then many neurons would be required to fit a smooth function, in which case the NN may not generalize well.

4.2.1 SIGMA Determination

The smoothness parameter or spread factor (σ) indicates the width and slope of the neurons functions. This factor is the only parameter in GRNN that needs to be adopted. The other parameters are provided by the training patterns.

According to figure **Figure 4-2** when σ is too high, the generalization ability is high as well and the MSE between the estimated training data and target data is significant. The higher values of σ are useful when the data is noisy or when it contains several significantly outstanding values because the spikes will be omitted successfully. However, the value of σ should not exceed 1 because the abilities of function approximation will be lost.

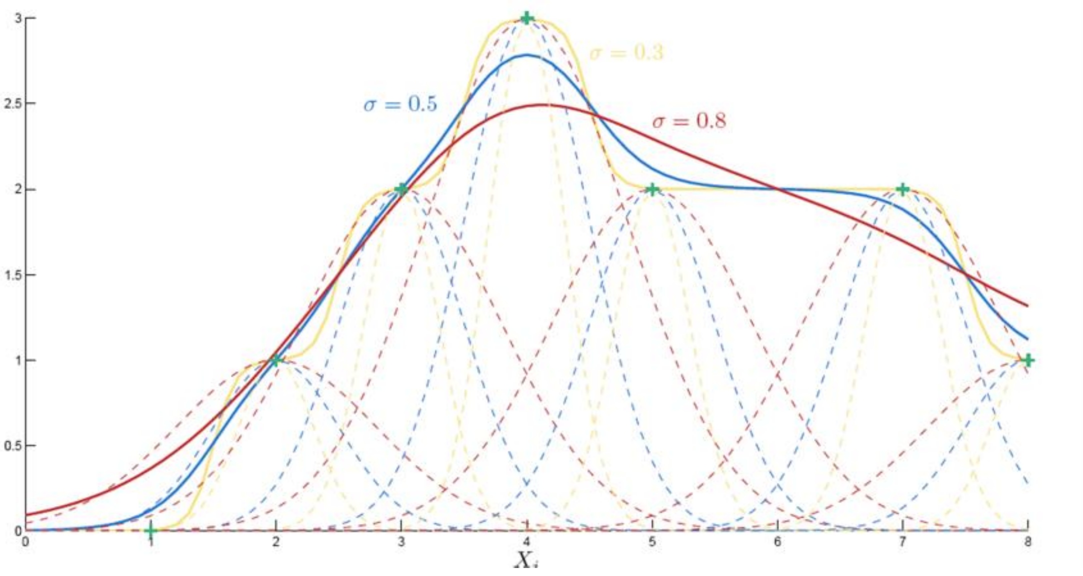

Figure 4-2: Dependence of Generalization Ability on the Spread Factor (Svobodova.J 2012)

The holdout method (Specht 1991) is more often used for selecting the σ value because of its simplicity. According to holdout method the training patterns will be divided into two groups, one third of the training dataset is used for testing while the rest are allocated to the training data set. After the network training, the MSE is calculated using the testing data set and will be saved. This process is repeated for a given number of passes with different division of the dataset (with less training data than in the previous run). Whole process is repeated for many different values of σ. The run with the smallest overall MSE value is picked and its σ is used for the whole network. More discussions on the hold out method and other cross validation techniques are provided in the GRNN model validation chapter of the thesis.

4.2.2 GRNN model variables

Input data into the GRNN model is in the form of a vector which the components of this vector reflects the land use data for the origin and destination zones. For the purpose of this research the following land use information were selected for the input to the GRNN model:

- *Residential dwellings: number of dwellings in each zone;*
- *Retail: Gross Floor Area (GFA) of retail in each zone;*
- *Commercial/ Office: Gross Floor Area of the office in each zone;*

- *Showroom: Gross Floor Area of showroom in each zone; and*
- *Schools: number of students in primary or high schools in each zone;*

The output of the GRNN model is the trip distribution between each pair of zones. For the purpose of this research only work trips are investigated. Therefore the output of the GRNN model is the number of work trips between each pair of zones.

It should be noted that, zones with the residential dwellings are generators of the traffic and zones including the non-residential land uses are considered to be attracting zones. Work trips generated from the residential zones will be attracted by the zones which entail retail/ shops, offices, showroom and schools. Therefore the work trip distribution between two purely residential zones is expected to be zero. There are a number of zones in practice that are purely residential (in particular if the zoning system is small and detailed for the modeling area) and therefore the work trip distribution between those zones is zero.

It is important that the neural models can predict the zero work trips within an OD matrix. Therefore for the purpose of this study the work trip distribution with zero values are not removed from the input vectors. The zero work trips sometimes would happen for diagonal cells of an OD matrix with zero values in the intra-zonal trips (Xie 2000) but it is recommended that intra-zonal trips are also included in the input data to neural models to be able to investigate the performance of the neural models for predicting the zero trips.

Accordingly the input layer of the GRNN model is represented by a vector including 11 components. The first 5 components reflect the land use data for the origin zone and the second 5 components (components 6 to 10) indicate the land use data for the destination zone. The Trips (T_{ij}) between a pair of zones are considered as the output layer of the neural network. On this basis vector (X) including the input data to the neural models is defined as:

X_{ij}=(RD_i, RE_i, CO_i, SH_i, SC_i,RD_j, RE_j, CO_j, SH_j, SC_j, D_{ij})

where i and j show the origin and destination zones, respectively.

D_{ij} is the last component of the Vector X_{ij} and it reflects the general cost between the origin and destination zone. The general cost indicates the separation between the origin and the destination zones and for private cars includes the following measures:

- *Operating costs (including fuel costs);*
- *in-vehicle time;*
- *parking costs;*
- *access time to and from the car;*
- *tolls or user charges;*

Generalized cost normally combines all of these variables together as a weighted sum of those factors for the origin to destination zone in the model.

For the purpose of this study distance between the origin and destination zones are used to reflect the generalised cost between the zones. Studies undertaken by Black (1995), Mozolin (2000), Tapkin (2006) and Yaldi (2011) are also used distance between the origin and destination zones as the generalised cost between the zones.

On this basis, total of 441 vectors was produced from 21 zones within the Mandurah and Murray study area. **Table 4-1** summarises the work trip distribution for the 21 destination zones in Mandurah. **Appendix B** of this thesis shows the destination zones and OD matrix for Mandurah and Murray. The work trips which are based on the 2006 ABS Census data are sourced from Department of Planning (DOP) in Western Australia. **Appendix C** of the thesis also shows the extracted 441 vectors that have been used for the development of the neural models (GRNN and BP).

Table 4-1: Work Trip Distributions for 21 Zones of Mandurah (2006 ABS Census Data)

O/D	zone 01	zone 02	zone 03	zone 04	zone 05	zone 06	zone 07	zone 08	zone 09	zone 10	zone 11	zone 12	zone 13	zone 14	zone 15	zone 16	zone 17	zone 18	zone 19	zone 20	zone 21
zone 01	352	3	235	48	37	318	115	43	81	138	82	169	187	18	68	30	8	37	0	0	20
zone 02	9	38	40	15	7	46	56	9	16	42	8	25	0	9	24	7	0	6	0	0	0
zone 03	19	0	143	12	19	65	51	32	26	39	29	52	37	13	21	15	4	22	0	3	8
zone 04	30	5	72	96	26	176	54	32	35	60	14	74	62	16	40	15	5	45	0	4	13
zone 05	64	6	176	46	247	327	140	51	66	140	47	177	100	23	64	45	7	49	0	12	11
zone 06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 07	74	19	148	0	0	0	273	19	51	65	46	37	19	0	65	14	0	56	0	0	0
zone 08	46	11	315	117	62	479	383	660	153	415	177	266	237	41	167	157	20	107	0	9	17
zone 09	19	0	57	18	14	29	37	0	70	27	16	31	6	6	33	19	0	19	0	6	8
zone 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 11	4	0	28	4	3	25	8	3	6	36	21	3	25	0	3	3	0	4	0	0	6
zone 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 13	24	0	82	28	6	47	9	39	34	52	30	71	120	15	45	13	0	28	0	6	0
zone 14	11	0	29	10	10	99	41	16	24	55	14	39	50	47	13	11	6	9	0	0	7
zone 15	30	8	109	46	23	213	77	43	58	161	29	120	144	13	176	38	11	45	4	6	14
zone 16	35	3	167	61	46	321	176	140	72	241	70	133	100	18	117	550	52	52	5	6	24
zone 17	13	0	89	33	8	74	87	56	37	106	45	38	37	15	31	91	220	22	0	6	0
zone 18	6	3	27	6	8	56	8	15	12	16	6	22	0	3	6	8	0	279	3	0	15
zone 19	0	0	52	20	9	17	12	20	0	0	9	40	0	0	9	0	0	164	69	17	17
zone 20	29	4	67	22	24	139	33	21	25	104	21	54	44	6	37	18	7	105	5	173	23
zone 21	15	3	67	19	20	136	48	17	19	36	21	58	81	6	28	9	0	76	3	12	193

4.2.3 GRNN Data Split Method

Data splitting is an important stage of the neural network development. The purpose of the data split is to produce separate, independent datasets for training, testing and validating NN models. There are different methods for splitting the data and generating the datasets for input to the NN models. The most common method is the random data splitting method. The random data splitting provides a data split using uniform random sampling to generate training, testing and validating datasets. There are other methods available for data splitting including systematic data splitting method (Baxter et al., 2000), SBSS-N data splitting method developed by Bowden (2002) and Kingston (2006) and duplex data splitting method developed by Snee (1977).

The study undertaken by Black (1995) did not report the data split method for each dataset. Mozolin et al. (2000) and Yaldi et al. (2009) reported that the random data split method was used for training, validation and testing.

For the purpose of this study random data split was used for the input dataset to NN model. The following steps were undertaken to prepare the training, testing and validation data sets:

- *All 441 vectors were stored in the Excel spread sheet;*
- *Through the random number generator function in excel software, random numbers were assigned in the first column for each vector;*
- *The vectors were sorted by the random numbers and the last 41 vectors were selected for the testing dataset.*
- *The 41 testing dataset were checked to insure that it includes all combination of land use data for the origin and destination zones (i.e. zones with purely residential land uses with zero work trip distribution and zones with high trip distributions are included in the testing data set);*
- *The process of random data selection for training and checking the testing data set was repeated a few times to insure that the testing data set represents a good sample of different trip conditions in Mandurah.*

4.2.4 GRNN Data Normalisation Method

Data normalisation means casting the data to a particular range, for example between 0 and 1 or between -1 and +1. The purpose of normalisation is to eliminate the influence of one model variable over the other variables and is used when the model variables are not is the same range. Theoretically, it is not necessary to normalize the x-data (independent data), however, practically it has been proven that when independent data are normalized, neural network training is more efficient and provides better estimations.

There are three different normalisation methods as follow:

- *Simple data normalization;*
- *Linear transformation; and,*
- *Statistical normalization.*

Simple normalization uses the following formula:

$$x_n = x_0 / x_{max} \qquad 4.3$$

Linear normalization will convert the input data to the range [0,1] with the following formula:

$$x_i^{scaled} = \frac{x_i^{actual} - x_{min}}{x_{max} - x_{min}} \qquad 4.4$$

Statistical normalization will convert the input data based on its mean and standard deviation using the following formula:

$$x_i = \left(x_0 - \overline{x}\right) / SD \qquad 4.5$$

Some researchers have used combination of the above normalisation methods for each independent data in the model. This is normally known as "mix sample" method. Black (1995) and Yaldi et al. (2011) have used mix sample method for normalisation of the x-data. Accordingly they have divided all the independent data, except the distance, by the summation of the number of trips for each x-data. The distance is normalized by its maximum value.

The study by Yaldi et al. (2009) showed that the neural model with simple normalization method performs better than the statistical and linear transformation methods for training or calibration.

For the purpose of this research all three methods of normalisation have been applied to the input data of the GRNN model and the performance of the model has been reported for each method. According to the Analysis undertaken the GRNN model provided very similar results for all three different normalization methods. The only difference was the value of the optimum spread factors. The optimum spread factor or σ value is reported to be different for each data normalisation method and needs to be adopted empirically or by cross validation techniques to get the best model performance.

The output layer of the GRNN model was not normalised in this study because generally, there is no need to normalize the output data, except in unusual situations. However the study undertaken by Black (1995) and Yaldi et al. (2011) have used simple normalisation for the neural model outputs (T_{ij}).

4.2.5 GRNN Model Testing

The model testing in this study applies to the 41 testing data set (about 10% of the total vectors). The testing data set are not used in the training process of the GRNN model and therefore are new to the GRNN model. **Table 4-2** summarises the 41 normalized vectors that are used in the testing data set.

When the GRNN model has been trained and the optimum spread factor has been adopted through the training process, then the model is applied to the testing dataset with the same spread factor calibrated during the training process and the modelled output are compared with the actual output to check the performance of the GRNN model. The performance measurement of the GRNN model is calculated and reported with three different methods explained in the next section.

Table 4-2: 41 Normalized Vectors in the Testing Data Set

Vectors	RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
1	0.0	0.3	0.1	1.0	0.1	0.1	0.0	0.0	0.0	0.0	0.3	0
2	0.1	0.2	0.0	0.1	0.2	0.3	0.0	0.0	0.0	0.0	0.0	0
3	0.0	1.0	0.2	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.5	0
4	0.0	0.9	0.1	0.2	0.0	0.8	0.6	0.0	0.0	0.2	0.2	0
5	0.2	0.3	0.8	0.4	0.4	0.4	0.0	0.4	0.0	0.6	0.1	21
6	0.1	0.2	0.0	0.1	0.2	0.1	0.0	0.0	0.0	0.2	0.0	3
7	0.2	0.0	0.0	0.0	0.0	0.2	0.3	0.4	0.4	0.0	0.2	0
8	1.0	0.4	0.1	0.1	0.8	0.4	0.0	0.0	0.0	0.3	0.2	20
9	0.0	0.3	1.0	0.4	0.0	0.2	0.3	0.8	0.4	0.4	0.0	0
10	0.4	0.0	0.0	0.0	0.7	0.0	0.3	0.1	1.0	0.1	0.1	47
11	0.1	0.0	0.0	0.0	0.0	0.2	0.3	0.8	0.4	0.4	0.0	57
12	0.2	0.0	0.0	0.0	0.0	0.2	0.3	0.4	0.4	0.0	0.6	81
13	0.4	0.0	0.4	0.0	0.6	0.8	0.6	0.0	0.0	0.2	0.2	38
14	0.1	0.2	0.0	0.1	0.2	0.0	0.3	1.0	0.4	0.0	0.0	56
15	0.4	0.0	0.0	0.0	0.7	0.1	0.0	0.0	0.0	0.0	0.5	0
16	0.2	0.3	0.4	0.4	0.0	0.0	0.3	1.0	0.4	0.0	0.1	47
17	0.0	0.9	0.1	0.2	0.0	0.1	0.0	0.0	0.0	0.0	0.5	0
18	0.0	1.0	0.2	0.1	0.0	1.0	0.4	0.1	0.1	0.8	0.1	0
19	0.2	0.3	0.8	0.4	0.4	0.1	0.0	0.0	0.0	0.0	0.0	26
20	0.1	0.2	0.0	0.1	0.2	0.0	0.9	0.1	0.2	0.0	0.0	16
21	0.2	0.3	0.8	0.4	0.4	0.2	0.0	0.0	0.0	0.0	0.3	8
22	0.0	0.3	1.0	0.4	0.0	0.4	0.0	0.4	0.0	0.6	0.1	0
23	0.0	1.0	0.2	0.1	0.0	0.4	0.0	0.0	0.0	0.3	0.4	0
24	0.2	0.0	0.4	0.0	0.2	0.8	0.6	0.0	0.0	0.2	0.3	15
25	0.1	0.0	0.0	0.0	0.0	0.0	1.0	0.2	0.1	0.0	0.5	40
26	0.1	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.4	0
27	0.2	0.0	0.0	0.0	0.0	0.0	0.3	0.1	1.0	0.1	0.3	21
28	0.2	0.0	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.4	0
29	0.4	0.0	0.0	0.0	0.7	0.0	0.3	1.0	0.4	0.0	0.1	327
30	0.4	0.0	0.0	0.0	0.3	0.2	0.2	0.0	0.4	0.0	0.4	87
31	0.2	0.0	0.4	0.0	0.2	0.2	0.0	0.0	0.0	0.0	0.2	5
32	0.1	0.0	0.0	0.0	0.2	0.0	0.3	0.1	1.0	0.1	0.0	14
33	1.0	0.4	0.1	0.1	0.8	0.8	0.6	0.0	0.0	0.2	0.1	157
34	0.0	0.9	0.1	0.2	0.0	0.4	0.0	0.0	0.0	0.3	0.3	0
35	0.0	0.9	0.1	0.2	0.0	0.4	0.0	0.4	0.0	0.6	0.1	0
36	0.2	0.0	0.4	0.0	0.2	0.1	0.0	0.0	0.0	0.2	0.2	16
37	0.1	0.0	0.0	0.0	0.0	0.1	0.2	0.0	0.1	0.2	0.0	19
38	0.2	0.0	0.0	0.0	0.0	0.0	0.3	0.1	1.0	0.1	0.1	8
39	0.0	1.0	0.2	0.1	0.0	0.4	0.0	0.4	0.0	0.6	0.1	0
40	0.6	0.3	0.0	0.0	1.0	0.4	0.0	0.4	0.0	0.6	0.2	68
41	0.4	0.0	0.4	0.0	0.6	1.0	0.4	0.1	0.1	0.8	0.1	43

4.2.6 GRNN Performance measurement method

The Root Mean Square Error (RMSE) (also called the root mean square deviation, RMSD) is a frequently used measure of the difference between values estimated by a model and the values actually observed. The RMSE has been used in the majority of the previous studies undertaken by Black (1995), Xie (2000), Mozolin et al. (2000) Tapkin (2004) Tillema et al. (2006) and Yaldi et al (2009). The individual differences between the modelled data and actual data are called residuals, and the RMSE aims to aggregate them into a single measure of predictive power. The RMSE formula is shown below:

$$\mathrm{RMSE} = \left(\frac{1}{N} \sum_{i=1}^{N} \left[A_i - T_i \right]^2 \right)^{1/2} \qquad 4.6$$

where:

N = number of observations;

Ti = observed value;

Ai = predicted value; and

The Mean Absolute Error (MAE) reflects the average magnitude of the errors, without considering their direction. The MAE is the average over the absolute values of the differences between modelled output and the actual output. The MAE is linear formula which means that all the individual differences are weighted equally in the average.

$$\mathrm{MAE} = \frac{1}{N} \sum_{i=1}^{N} \left| T_i - A_i \right| \qquad 4.7$$

The coefficient of determination indicates number of data points that falls within the results of the line formed by the regression equation. The higher the coefficient, the better is the fit. It means that when the data points and regression line are plotted, the regression line would pass through higher percentage of points. If the coefficient is 0.70, then 70% of the points would fall within the regression line. Values closer to 0 indicate that regression line represents none of the data. A higher coefficient is an indicator of a better goodness of fit for the observations. The formula for the coefficient of determination is as below:

$$R^2 = \frac{\sum_{i=1}^{N}\left[A_i - \bar{T}\right]^2}{\sum_{i=1}^{N}\left[T_i - \bar{T}\right]^2} \qquad 4.8$$

where:

N = number of observations;

T_i = observed value;

A_i = predicted value; and

T = average value of the explained variable on N observations.

For the purpose of this study the root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and measures of the training and testing data set have been used to provide a numerical description of the goodness of the model estimates.

4.2.7 Application of the proposed GRNN Model

The proposed GRNN model is applied to the work trip distribution in Mandurah area. The GRNN model is developed, trained, and tested according to the recommendations and specifications derived from the discussion in this chapter. For comparison purposes, the same testing dataset are estimated by using the BP model and doubly constrained gravity model.

4.3 BP Model

This section of the thesis discusses the model specification for the proposed Multilayered Feed Forward Neural Network MLFFNN which has been developed to predict the work trip distribution for Mandurah area and compared with the GRNN model. Because the BP algorithm is the most common algorithm for the training the MLFFNN, the proposed model is called BP model in this thesis. The model network architecture, training process and input to the BP model and testing the model is discussed in this section as well.

4.3.1 Training Algorithms

BP is the most famous training algorithm, widely used in previous studies. Black (1995), Mozolin et al. (2000) and Yaldi et al. (2009) have used BP for training the proposed neural networks. The Levenberg-Marquardt training algorithm is also used as an alternative for improving the performance of the BP algorithm by Yaldi et al (2011). The next sections of the thesis review briefly these two training algorithms.

4.3.2 BP Training Algorithm

The Multilayered Feed Forward Neural Network uses the back-propagation algorithm (Rumelhart and McClelland, 1986). This means that the artificial neurons are organized in layers; they send their signals 'forward', and then propagate errors backwards. The input and output of the network are received by the neurons in the input and output layers, respectively. One or more intermediate hidden layers are also provided. Supervised learning is utilized in the back-propagation algorithm, which means that the algorithm is provided with examples of the inputs and outputs; the network calculates the errors (the difference between the desired and actual results).The idea of the back-propagation algorithm is to reduce these errors until the network is trained. The training starts with random weights, and the model objective is to adjust the weights to minimize errors.

Implementing the i back-propagation algorithm which is a weighted sum (the sum of the inputs x multiplied by their respective weights w_{ij}) defined by the activation function of the artificial neurons in ANNs:

$$A_j\left(\overline{x},\overline{w}\right)=\sum_{i=0}^{n} x_i w_{ij} \qquad 4.9$$

It can be seen that only two factors, the inputs and the weights, control the activation. The neuron will be called linear if the output function is the identity (output = activation), the sigmoidal function is the most common output function:

$$O_j\left(\overline{x},\overline{w}\right)=\frac{1}{1+e^{A\left(\overline{x},\overline{w}\right)}} \qquad 4.10$$

For large positive numbers, the sigmoidal function would be very close to one, 0.5 at zero, and if the numbers are large negative, it would be very close to zero. Therefore, there will be a plain transition between the high and low output of the neuron (close to one or close to zero). Only the activation and subsequently the input values and their respective weights are factors which the output depends on them. To obtain a desired output when certain inputs are given is defined as the purpose of the training process. The error depends on the weights, because there is a difference between the desired and the actual output (error), so in order to minimize the errors, it is essential to adjust the weights. The following function defines the error of output for each neuron:

$$E_j(\bar{x}, \bar{w}, d) = \left(O_j(\bar{x}, \bar{w}) - d_j\right)^2 \qquad 4.11$$

The square of the difference between the desired target and the model output reflects the error. This value is always positive. If the difference is large, the error will be large, while a lower error value corresponds with smaller differences. The sum of the errors of all the neurons in the output layer will simply be the error of the network, which is defined as follows:

$$E\left(\bar{x}, \bar{w}, \bar{d}\right) = \sum_i \left(O_j(\bar{x}, \bar{w}) - d_j\right)^2 \qquad 4.12$$

The estimation of how the errors depend on input, weight and output is done by the back-propagation algorithm, and then the gradient descendent method is applied to adjust the weights:

$$\Delta w_{ji} = -\eta \frac{\partial E}{\partial w_{ji}} \qquad 4.13$$

The formula above is explained as follows. The multiplication of a constant eta (ŋ) by the dependence of the previous weight on the error of the network, which is the derivative of E in respect to w_{ji}, will make the adjustment of each weight (Δw_{ji}), so that Δw_{ji} will be a negative value. η and the contribution of the weight to the error of the function affect the size of the adjustment. This means that when there is a large error in the weight, the adjustment will be greater than if the weight contributes a smaller error. Until the appropriate weights are found (and the error is minimal), the function (4.13) is used.

Therefore, only finding the derivation of E in respect to w_{ji} is required. This is defined as the purpose of the back-propagation algorithm; to achieve this, we need to work backwards. First it should be defined as how much the error depends on the output, which d_j is the derivative of E in respect j to O_j.

$$\frac{\partial E}{\partial O_j} = 2\left(O_j - d_j\right) \qquad 4.14$$

The output depends on the activation, and subsequently depends on the weights. So for the next step, they need to be estimated by using (4.11) and (4.12):

$$\frac{\partial O_j}{\partial w_{ji}} = \frac{\partial O_j}{\partial A_j}\frac{\partial A_j}{\partial w_{ji}} = O_j\left(1-O_j\right)x_i \qquad 4.15$$

The following formula is derived from 4.14 and 4.15:

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial O_j}\frac{\partial O_j}{\partial w_{ij}} = 2\left(O_j - d_j\right)O_j(1-O_j)x_i \qquad 4.16$$

Therefore, the adjustment to each weight is calculated as:

$$\Delta w_{ji} = -2\eta\left(O_j - d_j\right)O_j\left(1-O_j\right)x_i \qquad 4.17$$

To train an ANN with two layers, the above formula (4.17) can be used. Some consideration needs to be given to training a network with more than one layer. To adjust the weights of a former layer v_{ik}, it is necessary to estimate how the error is influenced by the input of the earlier layer. To achieve this, it is necessary to transform the x_i to w_{ij} in (4.15), (4.16), and (4.17). Determining the effect of the network error on the adjustment of vik is achieved as follows:

$$\Delta v_{ik} = -\eta\frac{\partial}{\partial v_{ik}} - \eta\frac{\partial E}{\partial x_i}\frac{\partial x_i}{\partial v_{ik}} \qquad 4.18$$

where:

$$\frac{\partial E}{\partial w_{ji}} = 2\left(O_j - d_j\right)O_j\left(1-O_j\right)w_{ji} \qquad 4.19$$

It is assumed that inputs u are the neuron with v:

$$\frac{\partial x_i}{\partial v_{ik}} = x_i\left(1-x_i\right)v_{ik} \qquad 4.20$$

To add another layer, the influence of the weights and the inputs of the first layer upon the error need to be evaluated. It is essential to be careful with the indexes as there are different number of neurons in each layer, and they should not be confused. For practical reasons, ANNs implementing the back-propagation algorithm do not have too many layers, as the time for training the networks grows exponentially. According to Gershenson (2003), the speed of learning can be increased by making some refinements to the back-propagation algorithm.

4.3.3 Levenberg-Marquardt Training algorithm

The Levenberg-Marquardt algorithm is a simple and robust method for function approximation. The LM algorithm tries to solve the following equation:

$$(J^tJ + \lambda I)\delta = J^tE \quad 4.21$$

In this equation:

J is the Jacobian matrix for the system;

λ is the Levenberg's damping factor;

δ is the weight update vector which should be found; and

E is the error vector containing the output errors for each input vector used for training the network.

The δ is the parameter that indicates the changes to the network weights to achieve a better solution. The J^tJ matrix is known as the approximated Hessian.

The λ parameter is adjusted at each iteration. The adjustment of λ would guide the optimization process. The smaller value for λ leads to rapid reduction of E. changing the algorithm to the Gauss–Newton algorithm, larger value for λ, changes the algorithm to the gradient descent direction.

The Jacobian matrix is a N-by-W matrix, where N is the number of entries in the training set and W is the total number of parameters (weights + biases) of the neural network. The Jacobian matrix has the following form and can be created by taking the partial derivatives of each output in respect to each weight:

$$J = \begin{bmatrix} \frac{\partial F(x_1, w)}{\partial w_1} & \cdots & \frac{\partial F(x_1, w)}{\partial w_W} \\ \vdots & \ddots & \vdots \\ \frac{\partial F(x_N, w)}{\partial w_1} & \cdots & \frac{\partial F(x_N, w)}{\partial w_W} \end{bmatrix}. \quad 4.22$$

Where F (x_i, w) is the network function evaluated for the i^{th} input vector of the training set using the weight vector w and w_j is the j^{th} element of the weight vector w of the network.

4.3.4 BP Model Architecture

The standard network used for this study is a two-layer feed-forward network which is trained by LM training algorithm. The proposed BP model includes 11 input nodes reflecting the land use data for the origin and destination zones and the distance (d_{ij}) between the origin and destination zones. There is one node in the output layer which reflects the estimated trip numbers (T_{ij}). Each node is connected to hidden layer nodes by connection weights. Number of hidden layer nodes for the proposed BP model is set to 10. For the purpose of this study different BP models with 5, 10, 15 and 20 hidden layer nodes are investigated to assess the impact of different number of hidden layer nodes on the performance of the BP model. Higher numbers of nodes in hidden layer increases the computation time. Therefore, it is recommended to use a moderate number of nodes in hidden layer. The same consideration also is recommended for the number of hidden layers in the neural model (Yaldi 2012).

The activation function is used by the nodes in hidden layer and output layer to compute and transform the input information to the output. The activation function reflects relationship between the inputs and outputs of the neural model. **Figure 4-3** illustrates the most common activation functions.

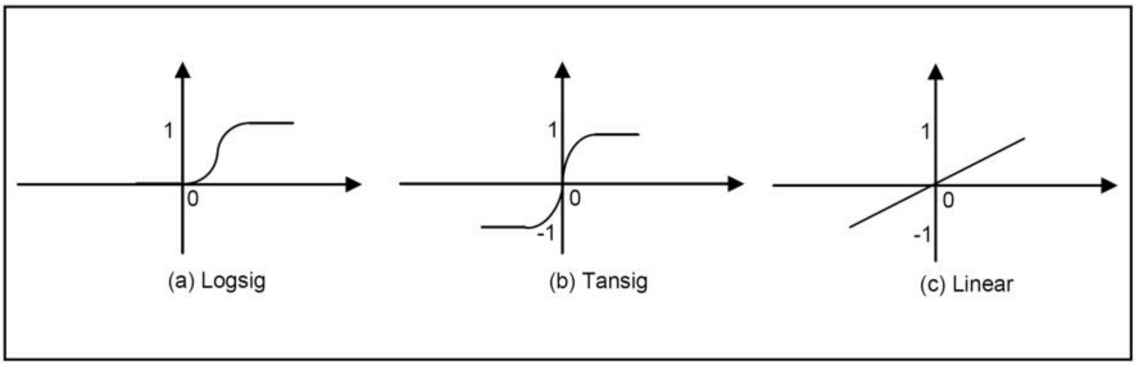

Figure 4-3: Common Activation Functions (Yaldi et al. 2012)

There is no standard rule for selection of the activation function. The sigmoid or logistic function is the most common activation function as it captures the nonlinear relationship among the model variables. The sigmoid function can be used for both hidden and output layer nodes.

In this study a sigmoid transfer function is used in the hidden layer and a linear transfer function is used in the output layer.

4.3.5 BP model variables

The model variables for the BP model are similar to the GRNN model. The inputs to the model are the land use data for the origin and destination zones and the distance between the two zones and the output is the number of trips between the origin and destination zones.

4.3.6 BP Data Split Method

Similar to the GRNN model random data split was used for the input dataset to NN model. Accordingly the testing dataset of 41 vectors which was prepared during the GRNN model development, were used as the testing data set for the BP model as well, so comparison between the two models be based on similar testing data set which are unseen by the neural models.

The 400 training vectors which were prepared during the GRNN model development were also used for the purpose of the BP model training and validating process. The validation process in BP model development is used to control the learning process. The learning process should be stopped when the error in validation data set is minimum. At this point the BP model generalizes best. If training continues, overtraining may occur and the performance of the BP model may decreases, while the error on the training data still reduces. When training process is finished, the BP model is ready and will be tested with the third data set, or the testing data set.

4.3.7 BP Data Normalisation Method

The normalisation of the BP model is only for the input data and is based on the simple normalisation. The output data are not normalised for the BP model similar to the GRNN model.

4.3.8 BP Model Testing

The model testing applies to the 41 testing data set (about 10% of the total vectors). Similar to the GRNN model the testing data set was hold-out and was not used in the training process.

4.3.9 BP Performance measurement method

The root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and measures of the training and testing data set have been used for the performance measurement of the BP model and comparison with the GRNN and the gravity models.

4.4 Gravity Model

This section briefly reviews the strategic transport model which is developed for Mandurah area. The trip distribution of the model is based on the doubly-constrained gravity model in the EMME software.

4.4.1 Mandurah Strategic Transport Model

The strategic transport model for the Mandurah area is based on the traditional four-stage model process developed for the City of Mandurah to assist the City in establishing future transport demand and testing the impact of land use growth, major developments and road network options (Rasouli, M & Claydon, A. 2012).

The modelled study area entails the Mandurah Local Government Area as described by the Australian Bureau of Statistics (ABS). **Figure 4-4** shows the boundary of the Mandurah Local Government Area and the corresponding modelling study area coded in the EMME software (Rasouli 2013). The EMME strategic model also includes the surrounding development of Mandurah locality including Pinjarra and all the other developments within the Shire of Murray and Peel region.

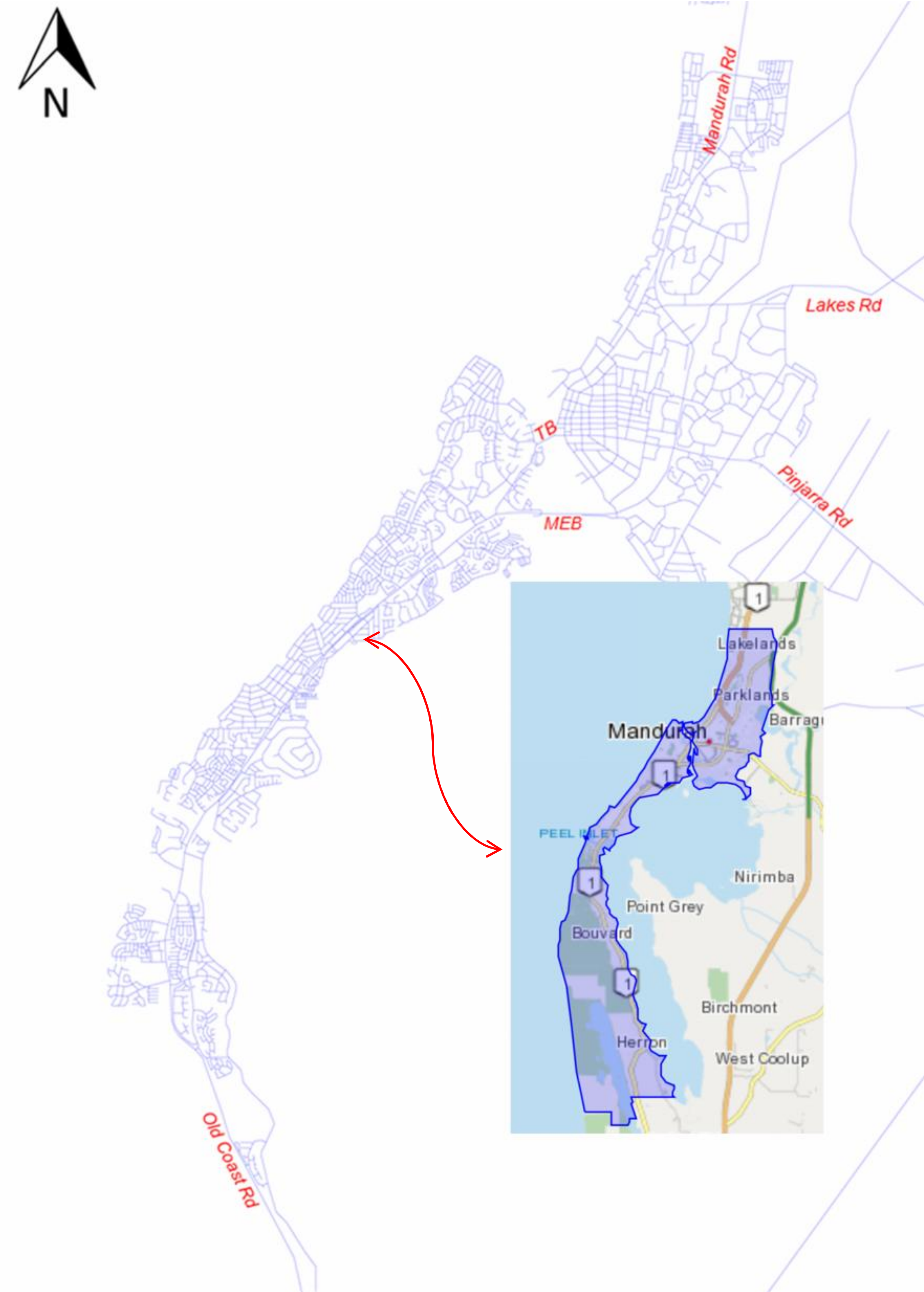

Figure 4-4: Mandurah Local Government Area and Modeling Study Area

Model Structure

The Mandurah strategic transport model is based on traditional four-stage model which includes the following stages:

- *Trip generation;*
- *Trip distribution;*
- *Mode Split and*
- *Trip assignment.*

Trip generation

The purpose of the trip generation step is to produce 24-hour trip productions and attractions from the existing land use data for input into the trip distribution step. The trips in the Mandurah strategic model are divided into 6 different trip purposes: work, education, shopping and personal business, social, other and non-home Based trips. **Table 4-3** indicates the percentage of car drivers for each trip purpose in City of Mandurah. The figures in Table 4 are derived from the Perth and Regions Travel Survey (Rasouli 2012).

Table 4-3: Percentage of Car Drivers for each Trip Purposes

Purpose of trips	**% of Total**
Home-Based Work	15.8%
Home-Based Education	12.2%
Home-Based Shopping and Personal Business	21.4%
Home-Based Social-Recreational	14.4%
Home-Based Other	6.5%
Non-Home-Based	29.7%
Total	**100.0%**

Trip Distribution

Trip distribution is the process that two-dimensional matrices of trips are produced from the one-dimensional production and attraction matrices. In this context, trip production generally refers to the number of trips starting or ending at residential land uses and trip attraction generally refers to trips starting or ending at other land uses (shops, offices, factories, schools, etc). Trips internal to the modelling area have been distributed based on the following gamma function:

$$Wij = a * dij^{b*\exp(-c*dij)} \qquad 5.9$$

where:

w_{ij} : weight between zone i and zone j; and

d_{ij} : distance between zone i and zone j.

Parameters a, b and c were calibrated for each trip purpose so that the model reflected the proportion of trips for each length as observed in the travel surveys. **Figure 4-5** illustrates the calibrated graphs for the gamma function. It is assumed that social and other trips would follow a similar graph.

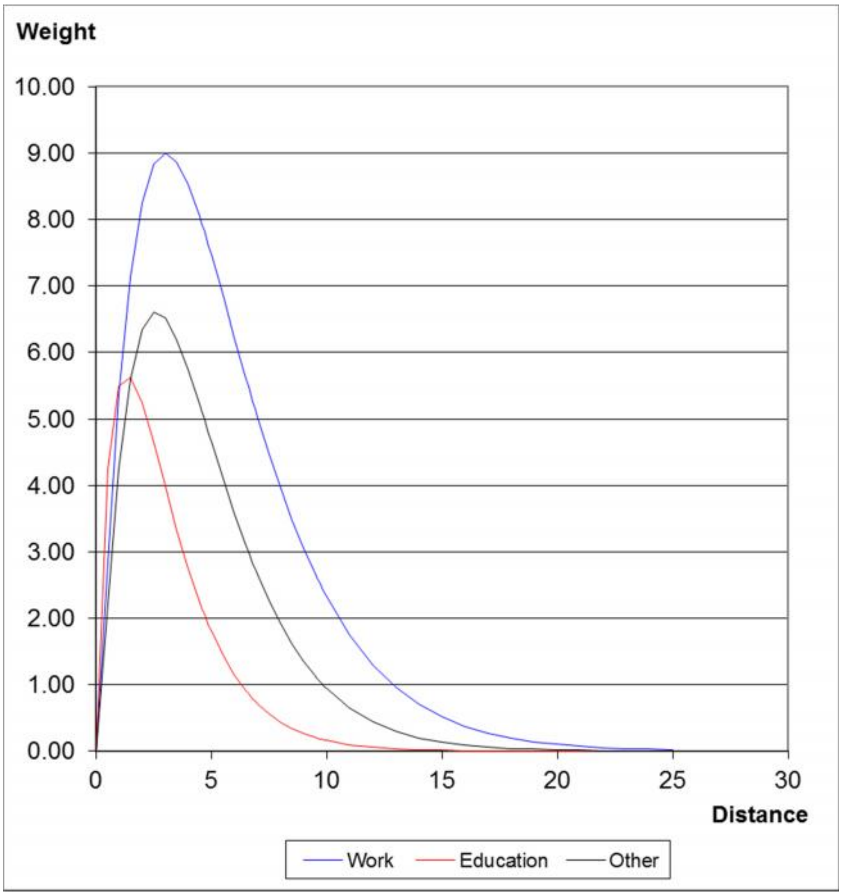

Figure 4-5: Friction Factors Calibrated for each Trip Purpose

The peak in each graph on the left hand side illustrates that private cars are not an attractive mode of transport for short distance trips, with other modes like walking or cycling being preferred. This figure also shows that long distance trips (more than 10km in the modelling area) are not attractive. The majority of car trips occur within a distance of three to five kilometres (Rasouli, 2012) and (Rasouli, 2013b).

Mode Split

The trip generation within the model is only based on private vehicle trips and therefore the mode split stage was not adopted. The mode split was taken into consideration when generating the trip production rates for the trip generation stage.

Trip Assignment

Assignment of the trips was based on the fixed demand traffic assignment module in EMME software. Accordingly, the trips are assigned to the modelled road network such that their total travel time is minimised. Travel time calculations for the road network take into consideration the road type, average speed and number of lanes along each route. Different road categories have been allocated different traffic capacities and speeds through the use of Volume Delay Functions (VDF). These functions vary the travel time based on the amount of traffic using each section of the road. Several iterations are undertaken to allow the effects of congestion to be included in travel time calculations.

Calibration

Calibration of the model was based on the existing traffic volumes on the road links. The actual traffic data was provided by the City of Mandurah. **Figure 4-6** shows the modelled traffic volumes against the actual traffic counts. The linear regression analysis for the 107 traffic count locations indicates that R^2 of the regression plot is 0.985 which shows how well the model is calibrated (Rasouli, 2012) and (Rasouli, 2013b).

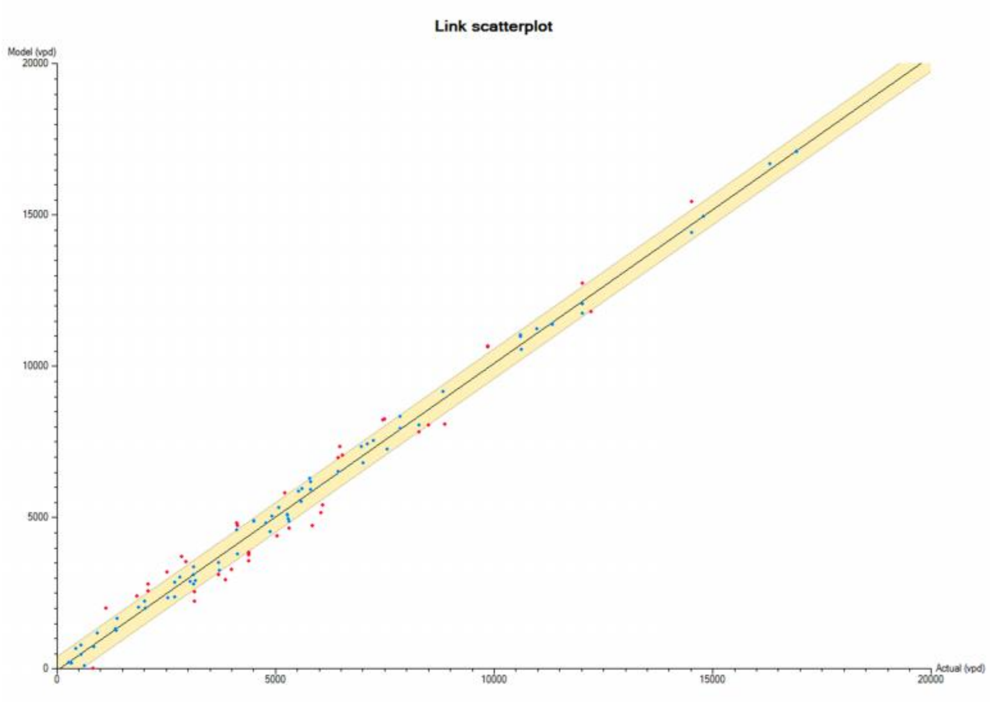

Figure 4-6: Regression Plot, Calibration

4.4.2 Land Use Data

The number of residential dwellings for the City of Mandurah was calculated for the 38 individual modelling zones as per **Figure 4-7**. The existing land use data for the attraction zones (retail, commercial, school, showroom, etc.) was sourced from City of Mandurah for the detailed zoning system illustrated in **Figure 4-7**. The zoning system for Mandurah modelled study area are much smaller than the Department of Planning zoning system (for the DoP zoning system refer to Appendix B), therefore the modelled smaller zones in EMME software needs to be aggregated to reflect the DoP zoning system to be able to compare the gravity modelling output by the previously established neural models. Aggregation of the land use data for the smaller zones and preparation of the land use data for the DoP zoning system was done by Microsoft Excel program. **Table 4-4** summarises the land use data for the 21 aggregated zones which correspond to the DoP zoning system. This land use data have also been used for the preparation of the input vectors to the neural models.

Figure 4-7: Mandurah Model Area and Zoning System

Table 4-4: Land use data for the aggregated zones (reflecting DoP Zones)

Zones	Residential Lots	Retail floor space (m2)	Office floor space (m2)	Show room (m2)	Primary+Secondary Students
Zone 1	4050	6964	106	192	2815
Zone 2	1007				
Zone 3	1560	6893	14705	5268	1027
Zone 4	1270		7500		683
Zone 5	2784				1868
Zone 6		7042	17365	5400	29
Zone 7	1634	4503	198	4407	
Zone 8	6653	10000	973	842	2147
Zone 9	689	1002	0	396	
Zone 10	248	23311	1900	2648	
Zone 11	223	7555	936	12485	201
Zone 12		27024	3354	930	
Zone 13	1187	7921	7614	4836	
Zone 14	900				435
Zone 15	2458		7500		1594
Zone 16	5227	15064	357	445	509
Zone 17	2660				823
Zone 18	612	4350		1300	600
Zone 19	916				
Zone 20	1907				
Zone 21	1231				

4.4.3 Extracting Work Trips from Gravity Model

Considering that the destination zones from the DoP are larger than the traffic zones coded in the EMME model for the gravity model, the following steps were undertaken to aggregate the data for the small zones in the gravity model to the same size for the DoP destination zones:

- *The relevant smaller traffic zones from the gravity model that are within each DoP destination zone are selected and allocated to a zone group;*
- *The 21 zone groups have been created in the gravity model using EMME platform; and*
- *A macro in EMME has been developed to extract the JTW OD matrix for the 21 zone groups.*

4.4.4 Gravity Model Testing

The model testing applies to the 41 origin destination zones which were prepared during the GRNN and BP model developments.

4.4.5 Performance measurement method

In order to compare the modelling outputs of the three models the same performance measurement method as per the GRNN and BP model was applied to the gravity model as well. The methods are root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2).

4.5 Summary

This chapter outlined the research frame work and model specifications. Three different models are developed, GRNN model, BP model and gravity model. The model specifications and key parameters of the neural models were discussed and explained how they have been utilised for the development of the neural models. The model variables, different methods of splitting and normalising data for input to the neural models were discussed in this chapter. The model testing and performance measured methods were also documented in detail in this chapter of thesis.

The Mandurah Strategic Transport model developed for Mandura area was also briefly reviewed. The model structure, trip distribution method and the land use data for the model development were also discussed. Aggregation of the Mandurah strategic model zoning system to reflect the DoP zoning system and extracting work trips from gravity model were also documented in this chapter.

5 5. MODEL DEVELOPMENT AND VALIDATION

5.1 Introduction

In the previous chapter the frame work of the neural model development has been established. In this chapter for each neural model the neural model properties and specifications will be presented. The neural models will be applied to the work trip distribution matrix for Mandurah area and the results will be compared with the gravity model.

5.2 Development of the GRNN Model

According to the literature review, the application of the GRNN model for work trip distribution is not reported yet. However the GRNN model has been applied for the mode choice step of the traditional four step model (Celikoglu, 2006) and its superiority over the BP model and gravity model is demonstrated. The proposed GRNN model in this research will use the land use data for the origin and destination zones and the distance between them as the input to the model and will estimate number of trips between the two zones. Since 1995 that Black developed a neural network for prediction of the commodity flow based on US Commodity Flow Survey (CFS), and used production, attraction and distance as the input to the model. All the other following studies also used the same proposed three inputs (production, attraction and distance) for the model development. The proposed GRNN model in this thesis aims to find the relationship between the land use data of origin and destination zones with respect to the distance between the two zones and estimate the number of trips between the two zones. The distance as the only factor for the separation between OD zones is not expected to be the best representation of the generalized cost between the two zones but as the existing gravity model developed for Mandurah strategic transport model used the distance for the purpose of the separation between the OD zones then neural models are also used the same parameter as input to the model to be able to compare the neural models with the gravity model.

5.2.1 Model Data

The 2006 Journey to Work data set for the Mandurah area was sourced from the Australian Bureau of Statistics (ABS). Journey to Work data is extracted from the five-yearly Census of Population and Housing conducted by the Australian Bureau of Statistics and includes data on employment by industry and occupation, and method of travel to work at a low geographical level known as the travel zone. The travel zones for the purpose of this study are 21 zones which will generate 441 (21 x 21 =441) input data for the purpose of model development. The 441 input vectors are provided in appendix C of this thesis.

5.2.2 GRNN Model Architecture

People's activities can be represented by land uses scattered over different zones that are separated by distance in an area. Therefore, trip distribution relates to the land use patterns in different zones inside that area. For instance, one zone which is typically occupied by residential land use patterns generates trips that are attracted to another zone which is formed by retail, industrial, commercial, etc.

On this basis the input layer of the neural network is represented by land use data in each zone, which is assigned to RD (residential dwellings), RE (retail), CO (commercial land use), SH (showroom) and SC (schools). In order to represent the spatial distribution of a pair of zones, the distance D_{ij} (meters) between zones i and j is defined. Accordingly the input vector (X) is defined as:

$X_{ij}=(RD_i, RE_i, CO_i, SH_i, SC_i,,RD_j, RE_j, CO_j, SH_j, SC_j, D_{ij})$

where i and j show the origin and destination, respectively.

Trips (T_{ij}) between a pair of zones are considered as the output layer of the neural network. The GRNN has to be able to model the relation between trips T_{ij} and input vector X_{ij}. The model is developed to forecast the work trips. MATLAB software is used to develop the network where the optimum spread factor is selected by cross validation technique.

The model structure used in the MATLAB software is illustrated in **Figure 5-1**. The model has 11 input nodes (P) representing the land uses for zone i and zone j, and the distance between zone i and j (as defined in the above X_{ij} input vector). There is one node in the output layer (T) which represents the estimated trip number (T_{ij}). The preferred spread was chosen through a trial and error process. Different spread factors were tested through a macro program in MATLAB and for each spread the relevant RMSE was recorded. The spread that provided the minimum RMSE was used as the preferred spread factor.

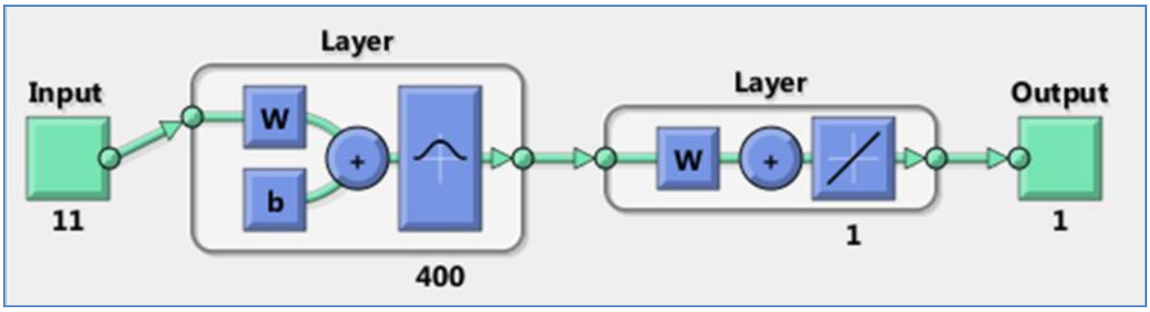

Figure 5-1: GRNN Model Structure Used in MATLAB Software

Simple data normalization, linear transformation and statistical normalization methods were used in this study for the input vectors. Simple normalization uses the following formula:

$$x_n = x_0 / x_{max} \tag{5.3}$$

Linear normalization will convert the input data to the range [0,1] with the following formula:

$$x_i^{scaled} = \frac{x_i^{actual} - x_{min}}{x_{max} - x_{min}} \tag{5.4}$$

Statistical normalization will convert the input data based on its mean and standard deviation using the following formula:

$$x_i = \left(x_0 - \bar{x}\right) / SD \tag{5.5}$$

There are usually two kinds of input data sets in neural networks, namely training and testing data sets. The training data set is used in estimating the model parameters/variables while the testing data set is for evaluating the forecasting ability of the model. For the purpose of this study, about 90% of the data (400 input vectors) was used for training and abou10% was used for testing (41 vectors). **Table 5-1** summarises the GRNN model properties.

Table 5-1: GRNN Model Property

Model architecture	Generalise Regression Neural Network
Number of layers	3 layers (Input, hidden and output layers)
Number of input nodes	11 nodes (land use data for OD zones and distance between the zones)
Number of output nodes	1 (Trip distribution)
Optimum spread factor	Cross validation technique
Data split	Random zone based
Data normalisation	Simple data normalisation
Performance measurement	RMSE, MAE, R^2

5.2.3 GRNN Modelling Results

The performance of the GRNN model is investigated in both calibration level and testing level and are presented in the next sections.

5.2.4 GRNN Calibration Performance

The root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and measures of the training and testing data set are the most common indicators to provide a numerical description of the goodness of the model estimates. They are calculated and defined according to equations 5.6, 5.7, and 5.8, respectively (Sousa et al., 2007):

$$\mathrm{RMSE} = \left(\frac{1}{N} \sum_{i=1}^{N} \left[A_i - T_i \right]^2 \right)^{1/2} \qquad 5.6$$

$$\mathrm{MAE} = \frac{1}{N} \sum_{i=1}^{N} \left| T_i - A_i \right| \qquad 5.7$$

$$R^2 = \frac{\sum_{i=1}^{N}\left[A_i - \bar{T}\right]^2}{\sum_{i=1}^{N}\left[T_i - \bar{T}\right]^2} \qquad 5.8$$

where:

N = number of observations;

T_i = observed value;

A_i = predicted value; and

T = average value of the explained variable on N observations.

RMSE and MAE indicate the residual errors, which give a global idea of the difference between the observed and predicted values. R^2 is the proportion of variability (sum of squares) in a data set that is accounted for by a model. When the RMSE and MAE are at a minimum and R^2 is high ($R^2 > 0.80$), a model can be judged as very good (Kasabov, 1998).

The GRNN model was trained using a data set with 400 randomly selected vectors and with different spread factors. **Table 5-2** summarizes the modelling results for the training data set. Analysis undertaken indicates that the GRNN model can produce the same results for different normalization methods with different optimum spread factors as indicated in **Table 5-2**. Therefore for the sake of simplicity, simple normalization has been used for the testing data set.

Table 5-2: GRNN Modelling Results for the Training Data Set

Indicators	RMSE	MAE	R^2	Optimum Spread
Simple Normalization	10	4	0.984	0.1
Linear Transformation	10	4	0.984	1.0
Statistical Normalization	10	4	0.983	0.7

Figure 5-2 illustrates the goodness of fit for the trained GRNN model based on simple normalization; an R^2 of 0.984 was obtained from the training process which shows how well the network is trained.

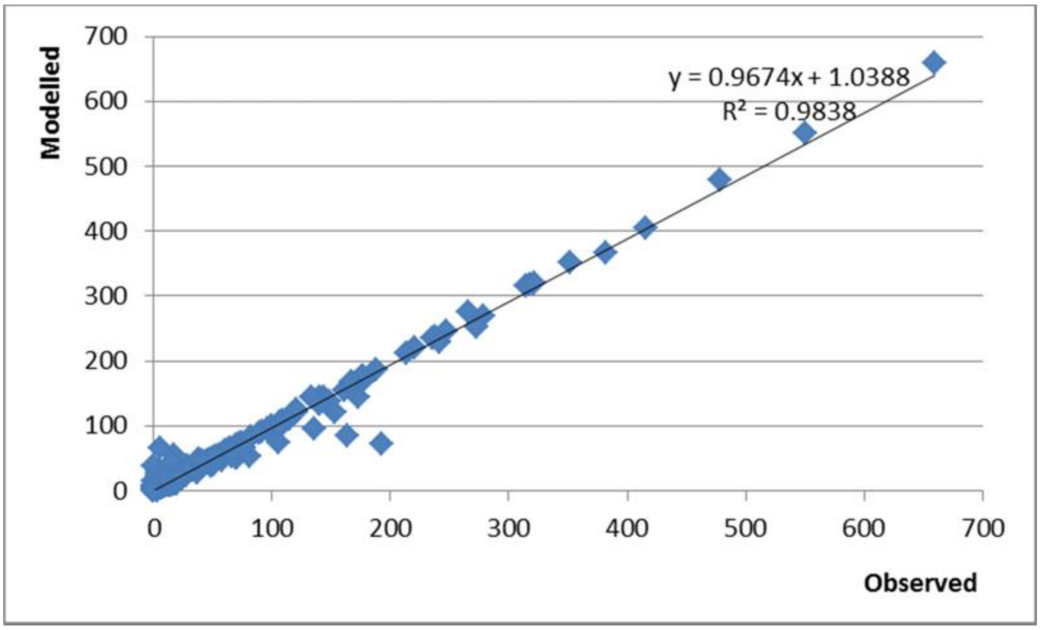

Figure 5-2: Modelled T_{ij} Through the Training Process Against the Observed T_{ij}

Review of the training dataset (400 vectors) indicates that out of the 400 training vectors 98 vectors (24.5%) include zero trip distributions, which mean there is no interaction between the OD zones. This could be the case when both zones are purely residential and there are no work trips between the two zones. In order to investigate how the GRNN model predicts the zero trips between residential zones, the modelled trip distribution (T_{ij}) by GRNN model and the actual trip distributions (in this case all the actual trip distributions are equal to zero) were compared. **Figure 5-3** illustrates the predicted zero trip distribution between the residential OD zones. Analysis undertaken indicates that the GRNN model could predict 76 vectors correctly and only 26 vectors are predicted incorrect. The highest difference between the predicted trip distribution and the actual zero trip distribution is 38 and it happened only in one occasion. The rest of the non-zero estimated vectors are below 15 trips.

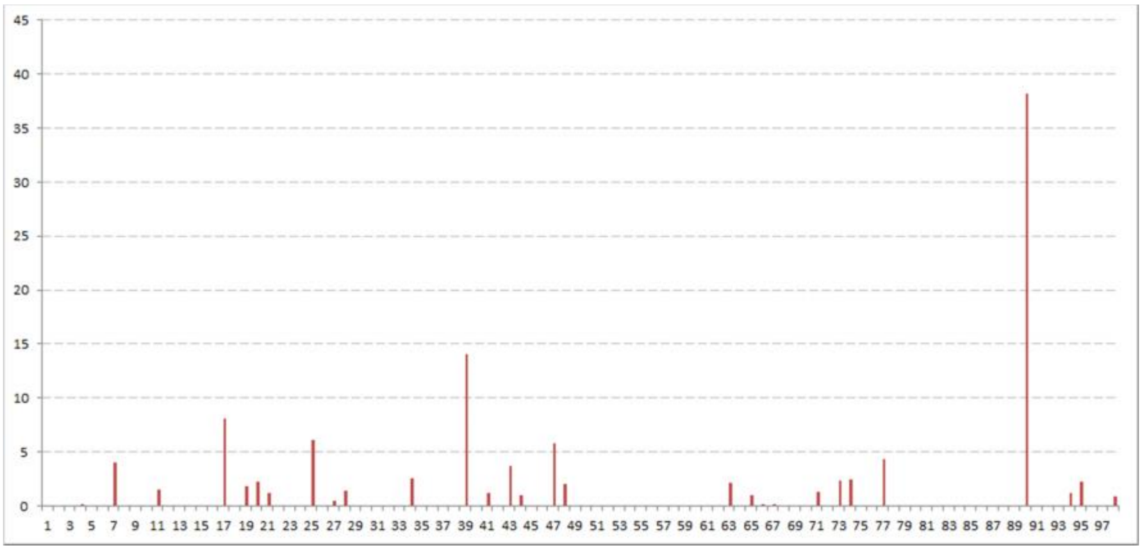

Figure 5-3: Zero Trip Distribution Estimation by GRNN Model, Training Data Set

5.2.5 GRNN Testing Performance

The trained GRNN model was then used to test the 41 unseen vectors. **Table 5-3** summarizes the modelling results for the testing data set.

Table 5-3: GRNN Modelling Results for the Testing Data Set

Indicators	RMSE	MAE	R^2
Simple Normalization	38	22	0.575

Figure 5-4 illustrates the modelled trip distribution against the observed data. The average RMSE for the tested data was recorded as 38.

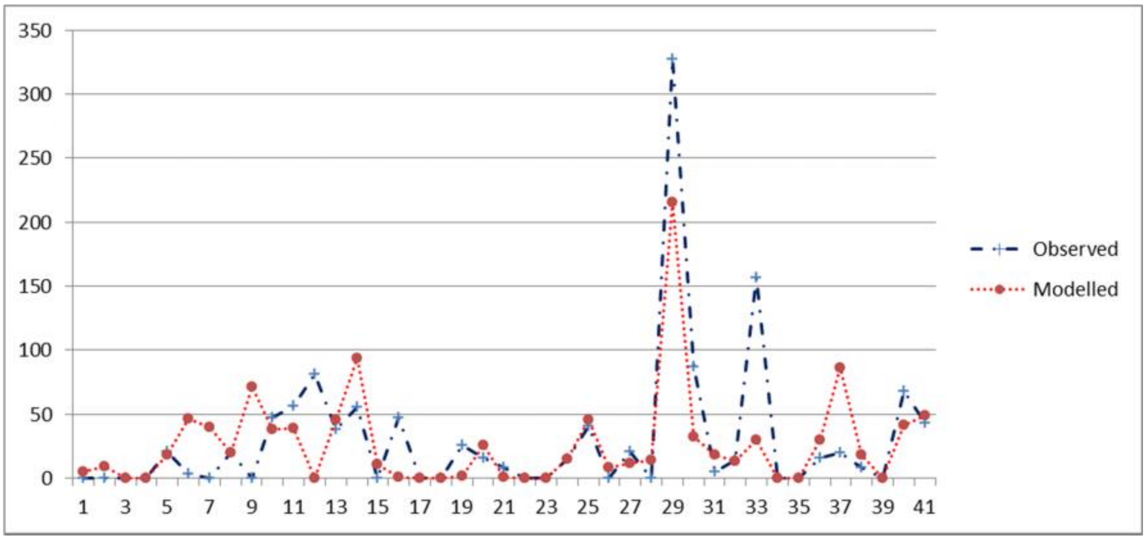

Figure 5-4: Error Estimation between the GRNN Modelled and Observed Data

The R^2 of the tested model is reported as 0.575 as shown in **Figure 5-5**.

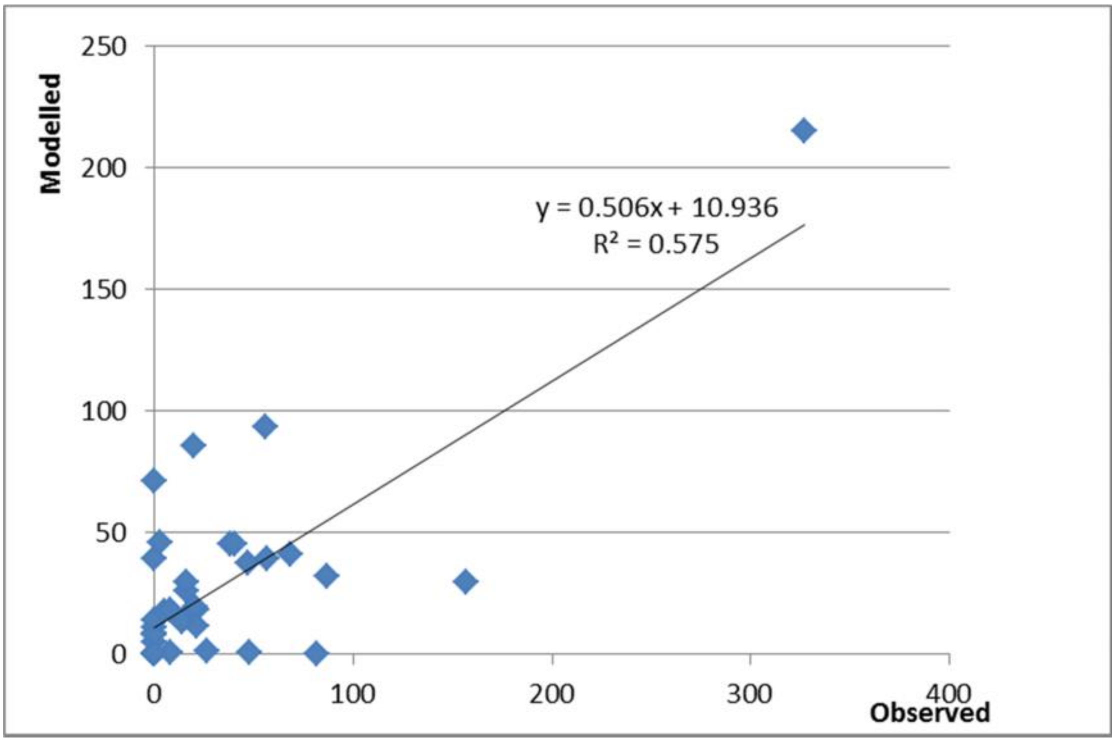

Figure 5-5: Modelled and Observed T_{ij} for the Testing Data, GRNN Model

The analysis undertaken for the testing dataset indicates that out of 41 testing vectors 16 vectors (39%) have zero trip distribution. The modelled GRNN could predict 9 vectors out of 16 vectors correctly and 9 vectors are not predicted correctly by trained GRNN model. **Figure 5-6** illustrates the predictions for the zero trip distribution vectors in the testing data set. The highest error is reported as about 70 trips which are about twice the highest error of the training process of the GRNN model. The second highest is about 40 and the rest are below 15.

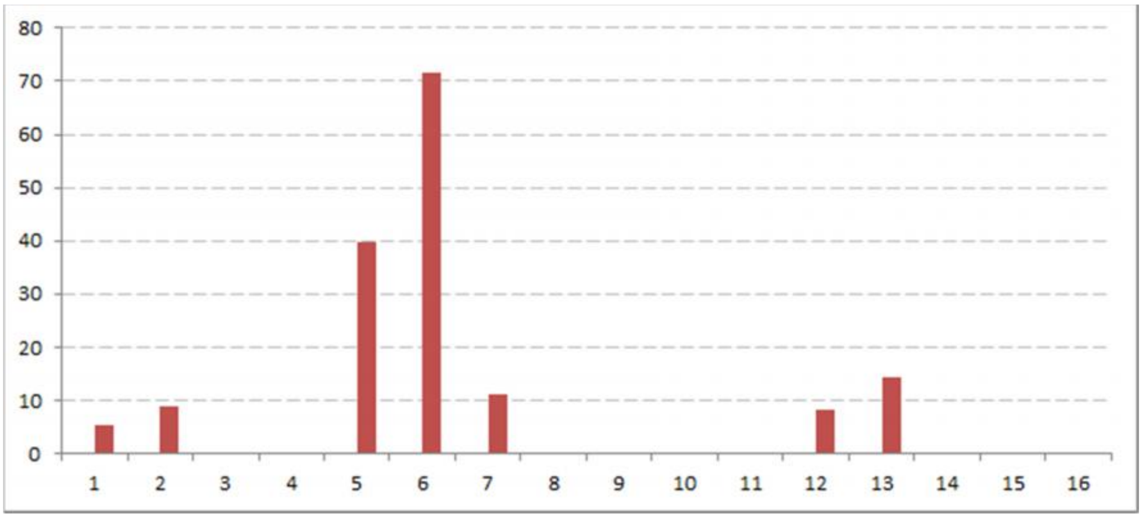

Figure 5-6: Zero Trip Distribution Estimation by GRNN Model, Testing Data Set

Review of the GRNN model output also indicates that GRNN generates a very small number close to zero for the zero value trip vectors and this small number is always positive.

In order to investigate the ability of the GRNN model to predict the non-zero trip distribution vectors, the difference between the modeled trips and the actual trips was calculated and illustrated in **Figure 5-7**. This figure indicates that GRNN model overestimates the trips for 10 non-zero trip vectors (about 38%), it correctly predict 1 non-zero vector and underestimate the 15 non-zero trip vectors (about 58%).

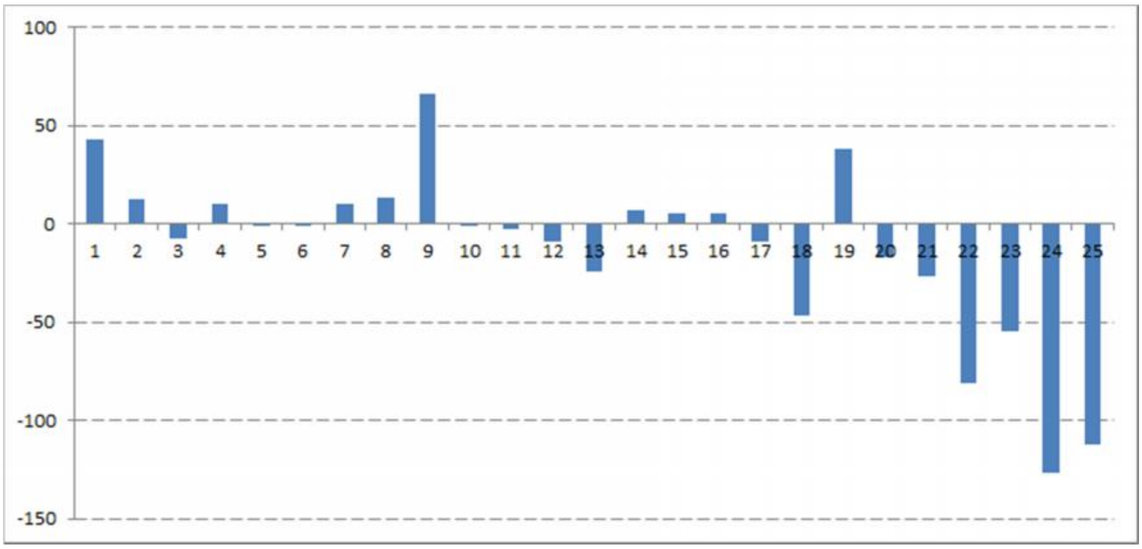

Figure 5-7: Difference Between the GRNN Model Output and Actual Trip Distributions for Non-Zero Trip Vectors

5.2.6 Observation and Discussions

One of the issues with the neural models discussed in the previous studies is the zero trip vectors. Zero trips can happen for the internal zones (diagonal cells in the OD matrix) if the internal zones are small and the model do not estimate any trips for the internal zones.

This issue has been raised in a research by Xie (2000), and she noted that most of the studies exclude diagonal cells with zero values in the intra-city/intra-regional flows. She argued that the zero cells should also be predicted, which would help in comparing the prediction accuracy of different models. She therefore used the data set including all of the zero values for the internal zones.

The OD matrix used in this study aggregates the small zones within the Mandurah EMME model to reflect the DoP zoning system and therefore because of this aggregation the internal zones in the aggregates matrix would provide some trips for the internal zones and these trips are even larger than some of the non-diagonal trips in the OD matrix. The reason is that more work trips are expected within the shorter distances.

Yaldi et al (2011) claimed that neural models are unable to predict zero value trips perfectly. The NN models estimate the zero trips as numbers very close to zero. Same observation also is expected from the GRNN model as discussed in the above section. However the study undertaken by Yaldi et al. (2011) indicates that the zero value observation is estimated as either positive or negative with very small number close to zero while the GRNN model output always predict positive small values for the zero value trips.

The distribution of points in the regression plot (**Figure 5-5**) indicates that the majority of the points are clustered at low values, with one or two at much higher levels (which represent the variety of the work trip conditions in Mandurah for the testing dataset). Therefore, the regression parameters are dependent on these points.

The x parameter is reported 0.506 in the regression plot, which means that the GRNN model is underestimating the observed values. This fact is also shown in **Figure 5-7** which indicates that for non-zero trips the majority of the estimated trips are lower than the actual trips.

5.3 Development of the BP Model

5.3.1 Introduction

Previous studies suggest that the neural network approach is able to model commodity, migration and work trip flows. However, its generalization performance is poor compared to the well-known doubly-constrained gravity model. Various studies have subsequently been undertaken to improve the performance of the NN models. Most of the previous analyses are based on back-propagation algorithm for training the NN. The latest studies undertaken in this regard aimed to fix the testing performance of NN by training the models with the Levenberg-Marquardt (LM) algorithm, and compare the results with standard back-propagation, Quickprop and variable learning rate (VLR) algorithms (Yaldi et al., 2011). The literature review indicates that NN models trained with the LM algorithm perform better than those trained with other algorithms. Therefore, for the purpose of this thesis the BP model has been trained with the Levenberg-Marquardt algorithm and the performance of the BP model has been compared with the GRNN and the gravity model.

5.3.2 BP Model Architecture

The input and output to the BP model were kept the same as for the GRNN model. The BP network specifications are as below:

- One hidden layer;
- The hidden units have a sigmoidal activation function (tansig or logsig) while the output units have a linear activation function; and
- The training algorithm is back-propagation based on a Levenberg-Marquardt minimization method.

The learning process is controlled by a cross-validation technique based on a random division of the initial set of data in three subsets: for training (weights adjustment), for learning process control (validation) and for evaluation of the quality of approximation (testing). The quality of the approximation can be evaluated by:

- Mean squared error (MSE) which expresses the difference between the correct outputs and those provided by the network; the approximation is better if MSE is lower (closer to 0);
- Pearson's correlation coefficient (R) which measures the correlation between the correct outputs and those provided by the network; the closer R is to 1, the better the approximation.

The model structure used in the MATLAB software is illustrated in **Figure 5-8**.

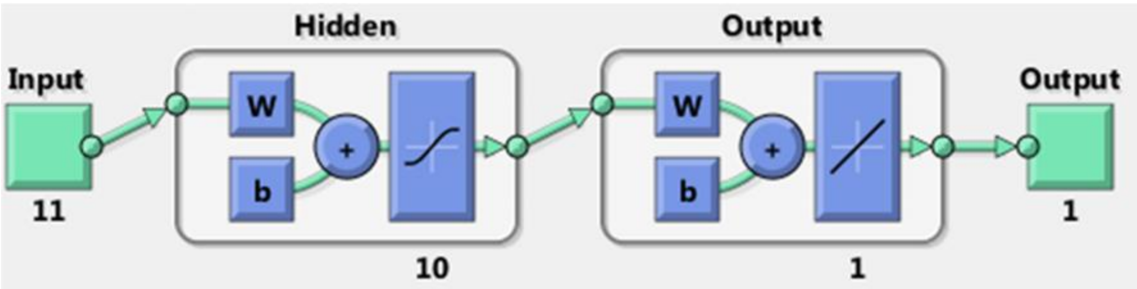

Figure 5-8: BP Model Structure Used in MATLAB Software

The standard network used for this study is a two-layer feed-forward network, with a sigmoid transfer function in the hidden layer and a linear transfer function in the output layer. The number of hidden neurons was set to 10. Simple data normalization was used for the input vectors. In order to be consistent with the GRNN modelling, 90% of the data (400 input vectors) was used for training and validating and about 10% was used for testing. The testing vectors were not used in the training or validation process. **Table 5-5** summarises the BP model properties.

Table 5-4: BP Model Property

Model architecture	Multi-layer feed forward neural network/MLFFNN
Number of layers	3 layers (Input, hidden and output layers)
Number of input nodes	11 nodes (land use data for OD zones and distance between the zones)
Number of output nodes	1 (Trip distribution)
Training Algorithm	Levenberg-Marquardt (LM)
Activation Function in Hidden Layer	Sigmoid (Logsig)
Activation Function in Output Layer	Linear
Data split	Random zone based
Data normalisation	Simple data normalisation
Performance measurement	RMSE, MAE, R^2

5.3.3 BP Modelling Results

The performance of the BP model is investigated in both calibration level and testing level and is presented in the next sections.

5.3.4 BP Calibration Performance

The 400 training vectors which have been selected randomly for the purpose of the GRNN model development were also used for the purpose of training the BP model. The BP model needs three sets of data for training:

- *Training data set: the training data set will be presented to the network during the training process;*
- *Validation data set: the validation data set are used to stop training when the generalisation of the NN network stops improving for the testing data set; and,*

- *Testing data set: the testing data set is unseen data set which does not affect the training or validation performance. They just provide independent measure of network performance.*

For the purpose of this study The 400 vectors are divided into the above datasets using the following data split:

- *70% for training or 280 vectors;*
- *15% for validation or 60 vectors; and,*
- *15% for testing or 60 vectors.*

The testing data set which was used out of the 400 vectors and explained above are different than the 41 testing data set which was selected and was hold out during the development of the GRNN model. In order to compare the results of the GRNN model and BP model the same 41 testing data set will be used to assess the performance of the neural networks (BP and GRNN) at the testing level.

There is no standard rule for the data split for training, validation and testing and therefore the testing data set could be assumed to be zero for the training purpose of the BP model, because the 41 testing data set has already been hold out and not included in the 400 vectors which was used to develop the BP model. However, in order to investigate the performance of the BP model for different random set of testing vectors 15% of the total 400 vectors were used for the testing.

The selection of the data sets (training, validation and testing) are random based and is controlled by seed numbers. Different seed numbers will generate different data sets. In order to investigate the performance of the BP model with different sets of data, the BP network was trained with 10 different seeds (10 different data stets) and the performance of the training, validation and testing data set is reported in **Table 5-5.** According to the analysis undertaken for the different data sets, the reported R^2 for the training data set was between 0.17 and 0.77. The highest R^2 recorded was 0.77 for seed number 9. The corresponding R^2 for the validation and testing data was reported as 0.42 and 0.48. Table 4 indicates that the expected R^2 for the testing data was between 0.3 and 0.5 and only in one case (seed number 2) the BP model not well trained (i.e. very poor correlation for training), and subsequently produced poor validation and testing results. The unsuccessful train could be due to the data split for seed number 2 and the initial selected weights. The range of RMSE for the testing data set was expected to be between 40 and 80. The corresponding RMSE to seed number 9 (best training data set) is reported as 64.

Table 5-5: Performance of the BP Model for Different Seeds

Seeds	Training Data		Validation Data		Testing Data	
	RMSE	R^2	RMSE	R^2	RMSE	R^2
1	47	0.74	54	0.50	52	0.46
2	73	0.17	84	0.20	72	0.02
3	49	0.62	60	0.62	53	0.45
4	50	0.67	65	0.46	46	0.47
5	51	0.62	46	0.45	39	0.46
6	52	0.59	60	0.55	56	0.35
7	43	0.76	47	0.59	64	0.38
8	60	0.46	61	0.34	49	0.32
9	**45**	**0.77**	**57**	**0.42**	**64**	**0.48**
10	45	0.74	46	0.48	72	0.37

In order to investigate the impact of the different number of nodes in hidden layer upon the performance of the BP model, different umber of nodes were tested and the performance of the model was reported in **Table 5-6** for each scenario. The analysis is undertaken for same seed number (seed number 9) for all scenarios.

Table 5-6: Performance of the BP Model for Different Nodes in Hidden Layer

Number of nodes	Training Data		Validation Data		Testing Data	
	RMSE	R^2	RMSE	R^2	RMSE	R^2
5	50	0.62	39	0.40	49	0.47
10	**45**	**0.77**	**57**	**0.42**	**64**	**0.48**
15	65	0.41	73	0.22	45	0.40
20	59	0.56	62	0.29	94	0.25

Table 5 indicates that the best performance was demonstrated by the BP network with 10 nodes in the hidden layer. Increasing the number of nodes to 15 or 20 nodes did not improve the performance of the BP model.

Figure 5-9 illustrates the BP model outputs against the actual trip distributions for training and validation data sets for the prefered BP model structure with 10 nodes in hidden layer.

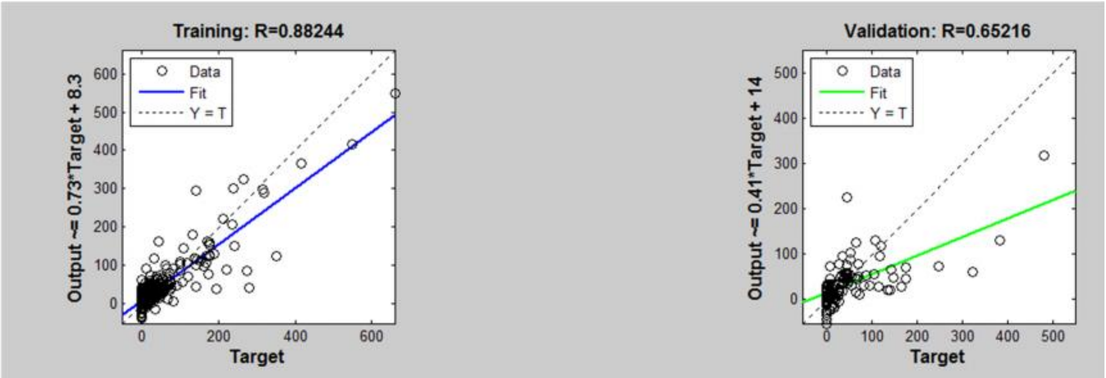

Figure 5-9: Performance of the BP model for Training and Validation Data Sets for Seed Number 9

In the above graphs, Pearson's correlation coefficient (R) is illustrated whereas in Tables 5-5 and 5.6 the R^2 was reported to be consistent with that for the GRNN and gravity model outputs.

5.3.5 BP Testing Performance

The trained BP model was then used to test the 41 unused vectors. Table 5-7 summarizes the BP modelling results for the testing data set.

Table 5-7: BP Modelling Results for the Testing Data Set

Indicators	RMSE	MAE	R^2
BP Model	64	31	0.485

Figure 5-10 illustrates the modelled and observed trip distributions of the testing data set for the preferred BP model structure.

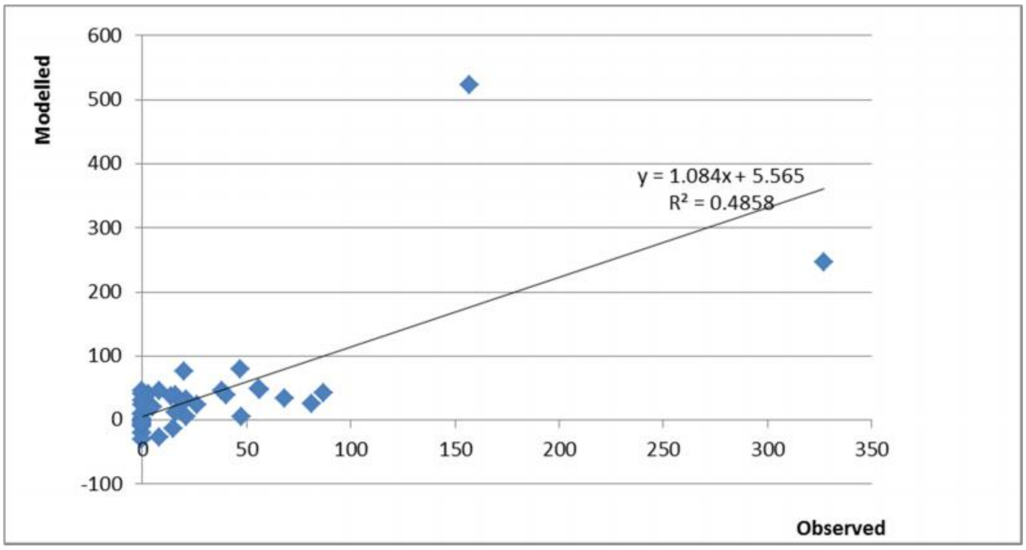

Figure 5-10: Modelled and Observed T_{ij} for the Testing Data, BP Model

The regression plot in Figure 5-10 provides lower R^2 than the similar graph for the GRNN model (refer **Figure 5-5**), however the x parameter of the BP model (1.084) better estimates the observed data than the GRNN model.

In order to investigate the ability of the BP model for estimation of the observed data more detailed analysis are undertaken. Similar to the GRNN model the analysis is undertaken separately for the zero trips and non-zero trips in the testing data set. Analysis undertaken for the 16 zero trip vectors indicate that BP model ability to estimate the zero trips is significantly lower than the GRNN model. The GRNN model could predict 9 vectors out of 16 vectors correctly while the BP model did not estimate even one zero trip vector correctly. **Figure 5-11** illustrates the predictions for the zero trip vectors in the testing data set. This figure also indicates that BP model will predict negative trips for some of the zero trip vectors. This is due to the linear activation function used in the output layer.

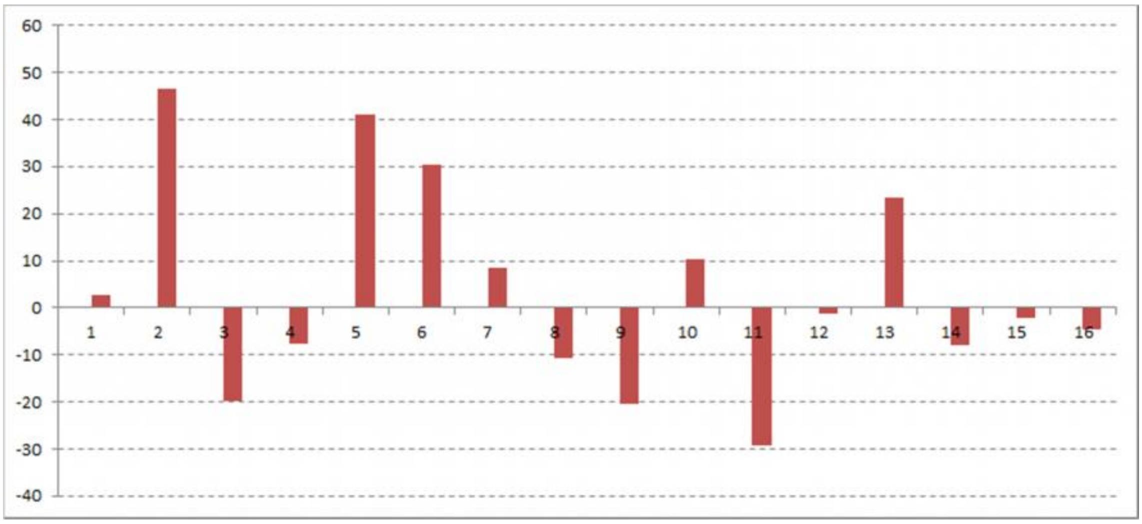

Figure 5-11: Zero trip distribution estimation by BP model, testing data set

Comparing the non-zero trip estimations with the observed trips (refer **Figure 5-12**) indicates that BP model slightly overestimate the observed non-zero trips. Only for two cases the BP model predicted negative trips.

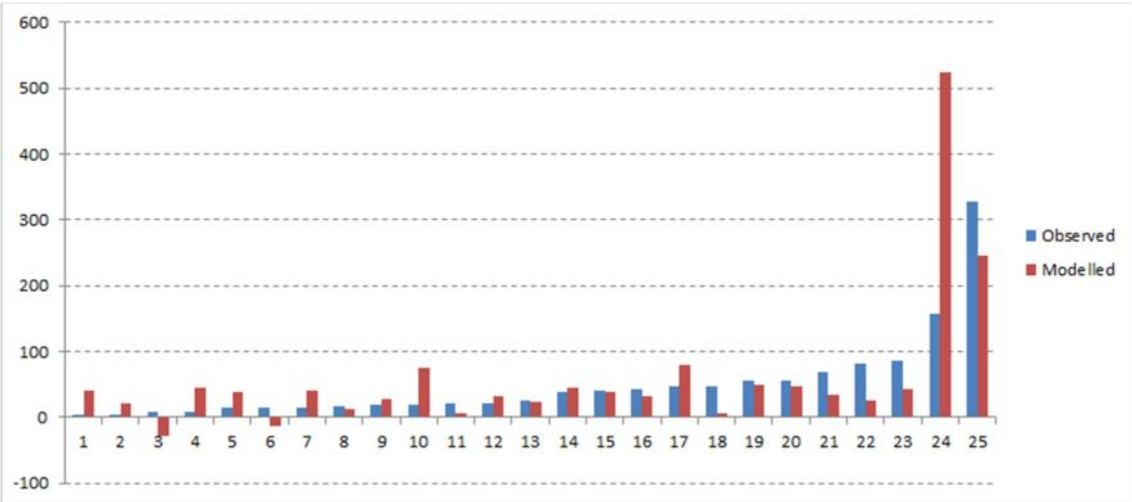

Figure 5-12: Comparing Modeled Non-Zero Trips with the Observed Non-Zero Trips for Testing Data Set.

5.3.6 Observation and Discussions

Reviewing the analysis undertaken for the training data sets indicates that increasing the number of nodes in the hidden layer would not necessarily improve the performance of the BP model. Analysis undertaken for 4 different sets of number of nodes (5, 10, 15 and 20) indicated that the BP model performed better with 10 nodes in the hidden layer and increasing the number of nodes in the hidden layer to 15 or 20 nodes did not improve the performance of the BP model.

The BP model provides negative predictions for some of the observed trips in the testing data set (about 27%). The negative predictions are mostly related to the zero trip zones (about 56% of zero trips are predicted with negative values) which mean that BP model ability to predict zero trips is poor. Using the linear activation function in the output layer is the reason for producing negative values for the trip estimations. The linear transfer function do not change the summation results and transfers them after the summation process, therefore the outputs (predictions) have no limits and can also be negative. This issue was also raised in the study undertaken by Yaldi et al (2009). Yaldi et al. suggested to change the negative values to zero as the majority of the negative predictions were related to the zero trips. He also tested the other most common activation functions such as “Transig” and “Logsig” in the output layer, however he concluded that the linear function in output layer in combination with “Logsig” activation function in the hidden layer (Logsig-Purelin) is more suitable for forecasting the work trips.

In order to investigate the impact of replacing the negative trips with zero trips in the testing data set, the below regression plot (**Figure 5-13**) is prepared. According to this plot R^2 of the BP model has been improved slightly from 0.48 to 0.50.

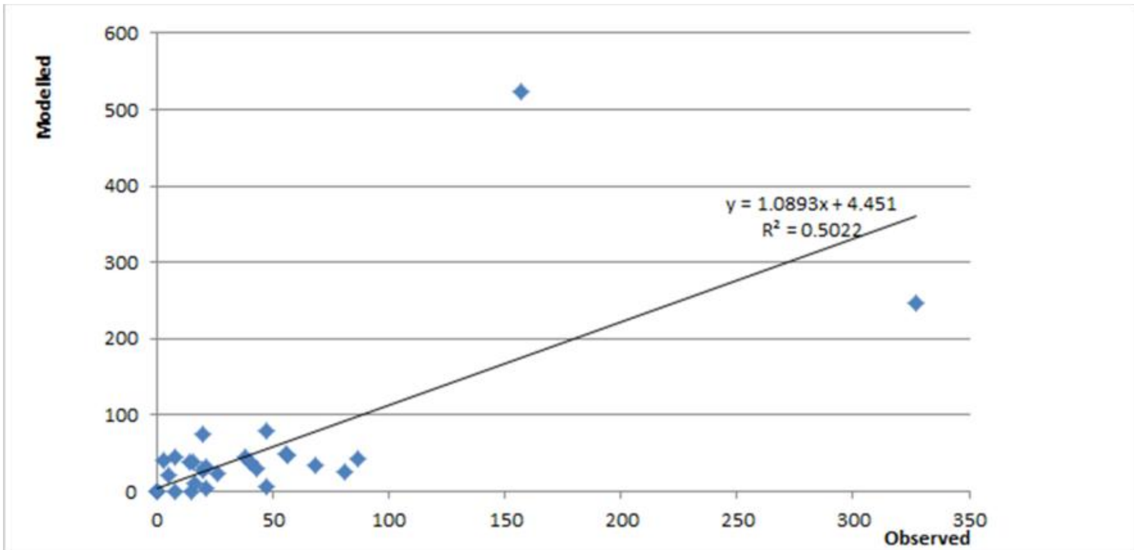

Figure 5-13: Updated Regression Plot for the Testing Data Set with Replacing the Negative Trips by Zero Trips in BP Model Output.

The distribution of points in the regression plot prepared for the BP model (testing data set) indicates that, similar to the GRNN model, the majority of the points are clustered at low values, with one or two at much higher levels (which represent the variety of the work trip conditions in Mandurah for the testing dataset). Therefore, the regression parameters are dependent on these points.

The x parameter is reported 1.084 in the regression plot, which means that the modelled values match the observed values over the range of data and therefore it is expected that BP model provides better match than the GRNN model. The GRNN model x parameter for the testing data set was reported 0.506 which resulted in underestimation of the observed data, however, the GRNN model provided better R^2 and RMSE for the testing data set.

5.4 Development of the Gravity Model

5.4.1 Introduction

In this section of the thesis, the estimated trip distribution for the 400 training and 41 testing OD zones that were used for the training and testing of the neural models will be extracted from the gravity model and will be compared with the actual trip distribution figures for the training and testing OD zones. The results of the analysis then will be compared with the GRNN and BP modelling results.

The strategic transport model for the Mandurah area is based on the traditional four-stage model process. The trips on this model are divided into five different categories based on trip purpose: work, education, social, other and non-home based (NHB) trips. Trip distribution of the model is based on the doubly-constrained gravity model with following gamma function (Rasouli 2012):

$$Wij = a * dij^{b*\exp(-c*dij)} \qquad 5.9$$

where:

w_{ij} : weight between zone i and zone j; and

d_{ij} : distance between zone i and zone j.

5.4.2 Gravity Modelling Results

The journey to work OD matrix was extracted from the Mandurah strategic transport model and has been aggregated to reflect the same zoning system that has been used for the DoP JTW matrix. **Table 5-8** summarises the extracted work trip OD matrix from the gravity model.

Table 5-8: Modeled work trip distribution (Gravity Model)

O/D	zone 01	zone 02	zone 03	zone 04	zone 05	zone 06	zone 07	zone 08	zone 09	zone 10	zone 11	zone 12	zone 13	zone 14	zone 15	zone 16	zone 17	zone 18	zone 19	zone 20	zone 21
zone 01	150	0	376	79	84	484	185	0	3	179	93	231	148	10	91	13	1	37	0	0	0
zone 02	14	10	67	13	13	81	46	0	1	44	23	48	37	3	15	4	0	6	0	0	0
zone 03	21	0	99	40	28	139	82	0	1	84	43	84	73	6	48	13	1	22	0	0	0
zone 04	16	0	105	40	33	143	51	0	1	55	30	101	57	5	58	9	0	45	0	0	0
zone 05	38	0	230	86	100	306	125	0	2	136	73	223	136	11	103	21	1	49	0	0	0
zone 06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 07	20	0	121	30	24	142	125	0	1	94	47	116	88	6	40	17	1	12	0	0	0
zone 08	28	0	197	56	36	204	212	100	3	249	126	251	209	15	91	82	6	107	0	0	0
zone 09	10	0	58	19	13	74	44	0	10	38	17	52	32	3	24	10	1	10	0	0	0
zone 10	3	0	19	5	4	22	13	0	0	5	3	19	12	1	7	4	0	0	0	0	0
zone 11	3	0	20	6	4	25	15	0	0	10	3	19	12	1	8	4	0	4	0	0	0
zone 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 13	12	0	83	30	17	100	74	0	1	69	36	93	56	4	42	18	1	13	0	0	0
zone 14	9	0	59	21	13	76	46	0	1	50	25	53	35	2	28	12	1	9	0	0	0
zone 15	22	0	154	76	38	208	99	0	2	138	69	195	132	11	91	31	2	45	0	0	0
zone 16	21	0	165	68	32	203	203	0	3	316	150	248	231	18	119	509	90	52	0	0	0
zone 17	6	0	51	21	9	61	68	0	1	120	55	83	82	6	40	409	148	22	0	0	0
zone 18	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1	0	0	200	0	0	0
zone 19	4	0	33	40	13	48	14	0	0	19	9	45	22	2	45	3	0	57	100	0	0
zone 20	16	0	130	138	46	183	58	0	1	74	37	170	86	8	156	11	1	105	0	100	0
zone 21	10	0	77	80	28	109	35	0	1	44	22	101	51	5	91	7	0	76	0	0	100

The OD matrix for the JTW extracted from the gravity model was compared with the OD matrix from ABS data. **Table 5-9** summarizes the modelling results for the testing data set (400 vectors).

Table 5-9: Gravity Modelling Results for Training Dataset

Indicators	RMSE	MAE	R^2
Gravity Model	50	23	0.59

Figure 5-14 illustrates the comparison between the trip distribution (T_{ij}) extracted from the gravity model and the ABS data. The R^2 for the trend line in is 0.59.

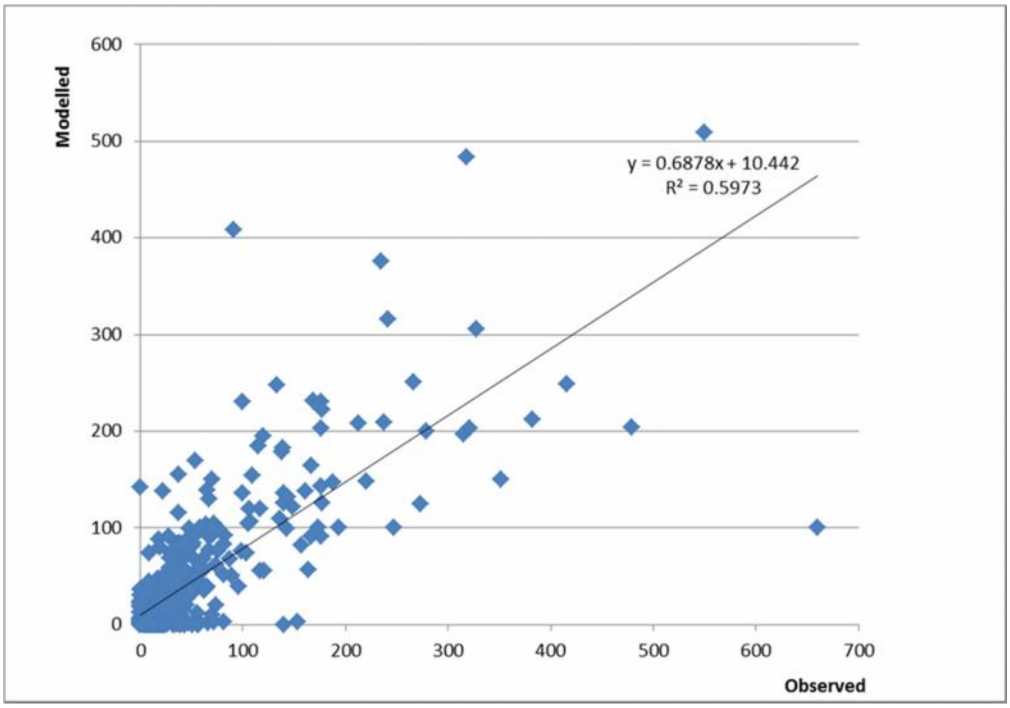

Figure 5-14: Observed and Modelled Work Trips Based on the Gravity Model (Training Dataset)

The gravity model developed for the Mandurah area was then used to estimate the trip distribution for the testing data set used in the GRNN and BP models. **Figure 5-15** illustrates the modelled and observed trip distributions for the testing data set. The R^2 from the gravity model to predict the trip distribution of the testing data set was reported as 0.446.

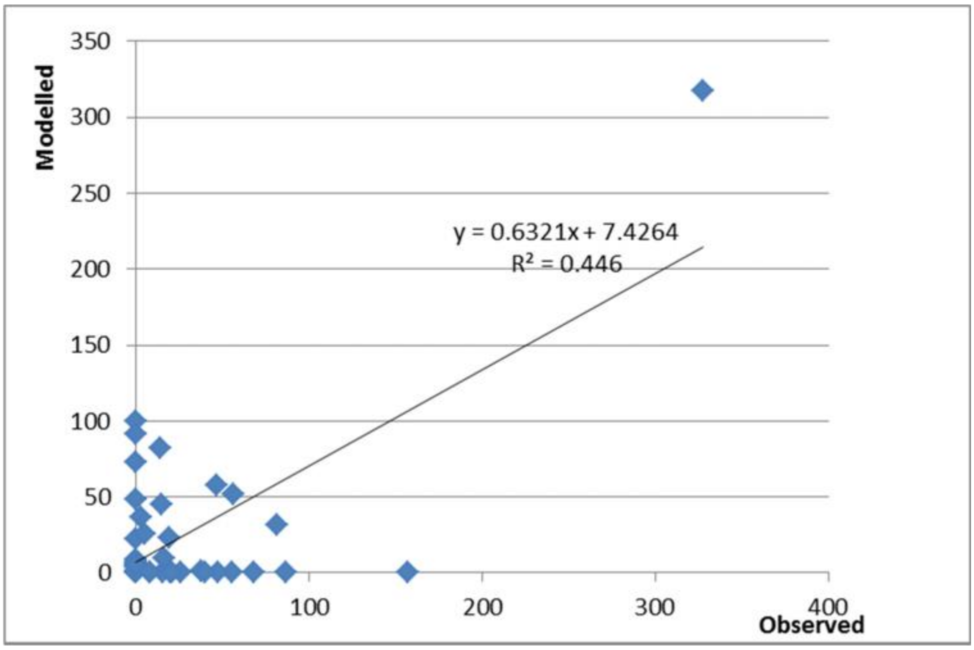

Figure 5-15: **Modelled and Observed T_{ij} for the Testing Data, Gravity Model**

5.5 Comparing the GRNN, BP and Gravity Models

In order to compare the performance of the GRNN, BP and gravity models, the tested data set was used to estimate the trip distribution based on the various models. The RMSE, MAE and R^2 indicators were calculated for each model and are compared in **Table 5-10.**

Table 5-10: NN and Gravity Modelling Results for the Testing Data Set

Models	RMSE	MAE	R^2	Regression Parameter
GRNN Model	38	22	0.575	0.51
BP Model	64	31	0.485	1.08
Gravity Model	46	31	0.446	0.63

Table above indicates that the GRNN model provides slightly better results than the BP and gravity models in term of RMSE, MAE and R^2. However the x parameter in the regression plot for BP model is closest to 1 which means that BP model provides better match for the observed data. The R^2 for the BP model is slightly higher than for the gravity model, while the reported RMSE is higher for the gravity model. The mean average error for both the BP and gravity model is reported as being 31.

Figure 5-16, Figure 5-17 and **Figure 5-18** illustrate the goodness of fit for the GRNN, BP and gravity models respectively.

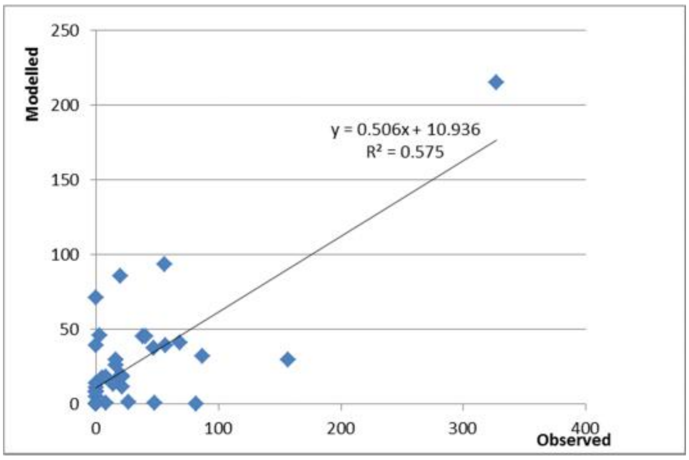

Figure 5-16: Modelled and Observed T_{ij} for the Testing Data, GRNN Model

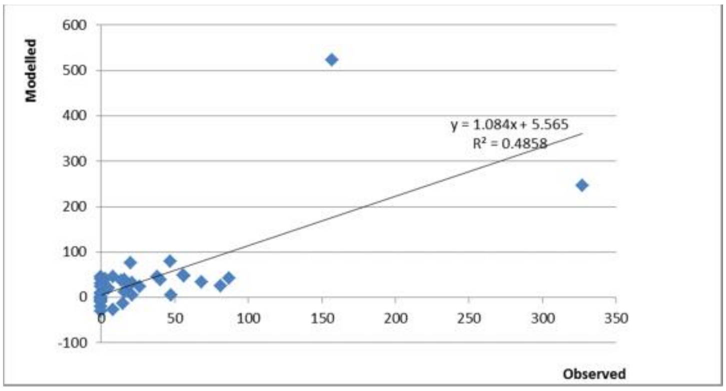

Figure 5-17: Modelled and Observed T_{ij} for the Testing Data, BP Model

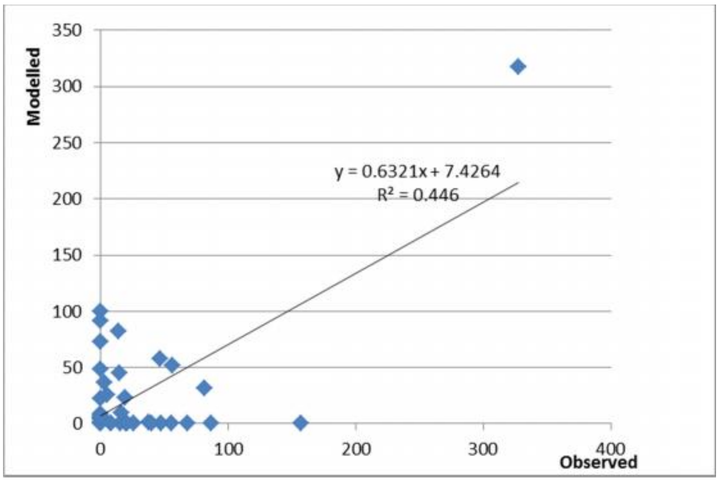

Figure 5-18: Modelled and Observed T_{ij} for the Testing Data, Gravity Model

5.6 Conclusions and Discussions

Comparing the performance of the neural models (GRNN and BP) and gravity model at training and testing level indicates that:

Neural network models can be used to forecast trip distribution, especially for work trips. The neural models are able to forecast work trip distribution based on the land use data for each pair of traffic zones and the corresponding distance between the two zones.

GRNN model could provide a slightly better goodness of fit than the BP and gravity models with a lower error level than BP and gravity models, as indicated by the average root mean square error (RMSE), where the RMSE for the GRNN, BP and gravity models are 38, 64 and 46 respectively. The estimated R^2 for the GRNN, BP and gravity models is reported as being 0.557, 0.485 and 0.446 respectively.

The distribution of points in the regression plot for all models indicates that the majority of the points are clustered at low values, with one or two at much higher levels therefore, the regression parameters are dependent on these points. As discussed before the testing data set was selected through the random split method and checked to insure that testing data represent the variety of the work trip conditions in Mandurah. Therefore the testing data set includes range of different work trips including zero trips and higher work trip generators.

The x parameter in the regression plots indicates the slope of the regression line. Reviewing the regression plots for the 3 models indicates that BP model provides closer x parameter to 1 and therefore can provide better match for the observed work trips. However BP model performance measures are lower than the GRNN model.

Considering that x parameter for both GRNN and gravity models are lower than 1, then it is expected that these models underestimate the observed data.

Analysis of the zero work trip vectors indicates that neural models (both GRNN and BP) are unable to predict zero value trips perfectly. The NN models estimate the zero trips as numbers very close to zero.

The BP model provides negative predictions for some of the observed trips in the testing data set (about 27%). The negative predictions are mostly related to the zero trip zones. The negative value predicted in BP model for the testing data set is due to the selection of the linear transfer function in the output layer. Linear transfer function do not change the summation results and transfers them after the summation process, therefore the predictions have no limits and can also be negative. This issue was also raised in the study undertaken by Yaldi et al (2009). He also tested the other most common activation functions such as "Transig" and "Logsig" in the output layer however he concluded that the linear function in output layer in combination with "Logsig" activation function in the hidden layer (Logsig-Purelin) is more suitable for forecasting the work trips.

GRNN model performance in predicting zero trips is better than the BP model. BP model mostly generated a negative value for the zero work trips because of the linear activation function in its output layer. Replacing the negative value predictions for the BP model with zero trips will not improve the performance of the BP model significantly.

5.7 Summary

In this chapter, model development for the neural models and gravity model were discussed. For each neural model, the neural model properties and specifications were presented. The neural models then were applied to the work trip distribution matrix for Mandurah area and the results were compared with the gravity model. Analysis undertaken indicated that neural models can estimate the work trips between OD zones based on land use data for the OD zones and the separation distance between the two zones. The performance of the neural models was investigated at both training and testing levels and the results were compared with the gravity model. Analysis undertaken also indicated that BP model provides closer x parameter to 1 and therefore could provide better match for the observed work trips; however GRNN model could provide a slightly better goodness of fit than the BP and gravity models with a lower error level than BP and gravity models, as indicated by the average root mean square error (RMSE), where the RMSE for the GRNN, BP and gravity models are 38, 64 and 46 respectively. The estimated R^2 for the GRNN, BP and gravity models is reported as being 0.557, 0.485 and 0.446 respectively.

6 6. GRNN MODEL VALIDATION

6.1 Introduction

In the previous chapter the development of the proposed GRNN model has been discussed and the performance of the proposed GRNN was compared with the BP and Gravity models in calibration and testing levels. The analysis undertaken indicated that the performance of the GRNN model in calibration level is very good (with R^2 of 0.984) However the performance of the proposed GRNN model in testing level was slightly better than the BP and gravity models (with R^2 of about 0.575).

In this chapter more detailed analysis will be undertaken for the proposed GRNN model to investigate the validation of the GRNN model by different datasets. Also the performance of the proposed GRNN model will also be investigated to see if the proposed GRNN model would be able to satisfy the gravity model constraints for total productions and attractions. The performance of the GRNN model for satisfying the gravity model constraints will be investigated by number of different data sets.

Previous studies by Mozolin et al. (2000) and Yaldi et al. (2009b) indicated that that the neural model is unable to satisfy the production and attraction constraints of the gravity model. However, in a different study by Yaldi et al. (2010), he claimed that training the neural models with the Levenberg-Marquardt algorithm can satisfy the trip production and trip attraction constraints. Therefore the performance of the GRNN model for satisfying the above constrains needs to be investigated.

6.2 Cross Validation

The purpose of NN training is to find a set of NN weights so that the input data estimates output data with best match for the target data. A simplistic approach is using all the available data to train the neural network. However, this approach would likely lead to over fitting issue which means that the data match would be extremely well but when tested with unseen set of input data (or testing data), the neural network would perform poor. In order to avoid over fitting, the idea is to separate the available data into a training data set and a test set. Training data set will be used for finding NN weights and the test set is used to evaluate the performance of neural network.

Since no independent dataset (demand matrix) is available, it is not possible to provide the external validation and validate the proposed GRNN model by external independent set of data. Therefore the validation of the GRNN model is assessed using the cross validation technique.

6.2.1 Cross Validation Techniques

Cross validation techniques (Refaeilzadeh et al., 2009) and (Picard and Cook, 1984) is used to insure good generalisation of a model and avoid over fitting. There are different methods available for cross validation. The two most common methods are explained here.

Hold out cross validation is a widely used technique due to its simplicity and efficiency. This method randomly divides the available data into a training data set and a test data set. An advantage of this method is that the proportion of these two data subsets is not restricted. The disadvantage of this approach is that the data split significantly affects the performance of the model. Therefore an unlucky split of the data could result in poor neural network performance. One possibility is to repeat the hold out validation several times. This method is called repeated sub-sampling validation.

K-fold cross-validation uses the combination of more tests for cross validation (Mitchell, 1997). The idea of k-fold cross-validation is to divide all the available data into roughly same size data sets. Each data set is used once as the test set and the remaining data is used as the training set. Unlike the hold out method, there is not a separate testing set in this method and proportion of the training and validation subsets is dictated by the number of folds k. In most applications k = 10 is selected. The important parameter of K-fold cross validation is the data split method.

In this research, a variation of hold out method is used to validate the GRNN model and also check the constrain satisfaction of the GRNN model. The data split and selection of the sample groups for cross validation is explained in next section.

6.3 Sample Groups

Validation of the GRNN model should be based on a set of vectors which are not used for the training of the GRNN model. In order to investigate the constraint satisfaction (productions and attractions), the sample data should include all the trips generated from or attracted to a set of zones, so the total trip generation or trip attraction for that zones could be calculated. On this basis and considering the limitation of the available vectors (total of 441 vectors which is extracted from 21 zones) the validation was investigated separately for total productions and attractions.

Accordingly 10 different data sets from 5 sample groups were identified for validating the GRNN model. 5 data sets were used to check the model ability to satisfy the total productions and other 5 data sets were used to check the model performance for estimation of the total attractions. For checking the total productions the rows of the OD matrix were used for training and validation but for the total attractions the columns of the OD matrix were used for training and validation purposes. The rows of the OD matrix include all the trip generations and the columns of the OD matrix include all the trip attractions. **Figure 6-1** illustrates the data split (zone split) for checking the total productions of sample group 1 and **Figure 6-2** shows the same figure for sample group 1 which has been used for checking the total attractions. Therefore each sample group provides two different data sets, one will be used to check the row totals and the other one will be used to check the column totals.

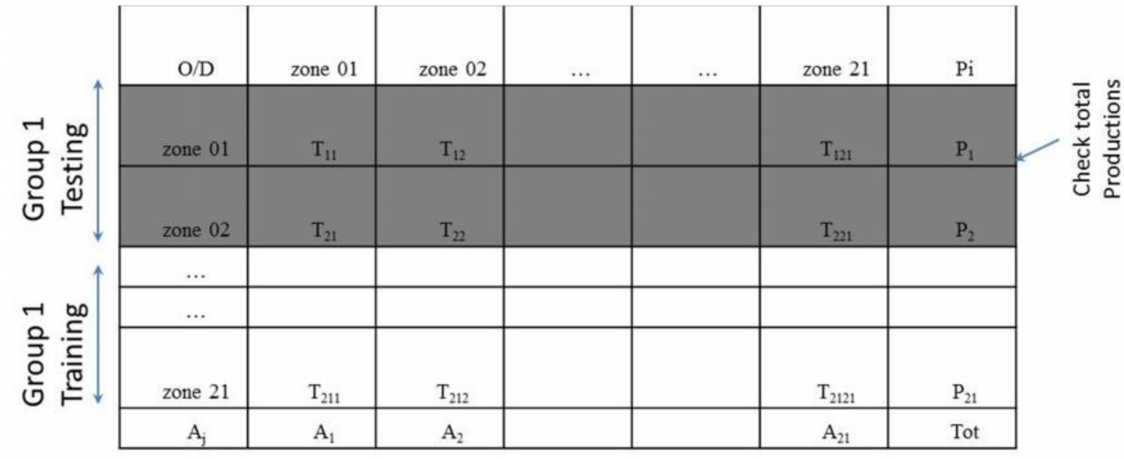

Figure 6-1: Sample Group 1 for Checking Total Productions, Training and Testing Data Sets

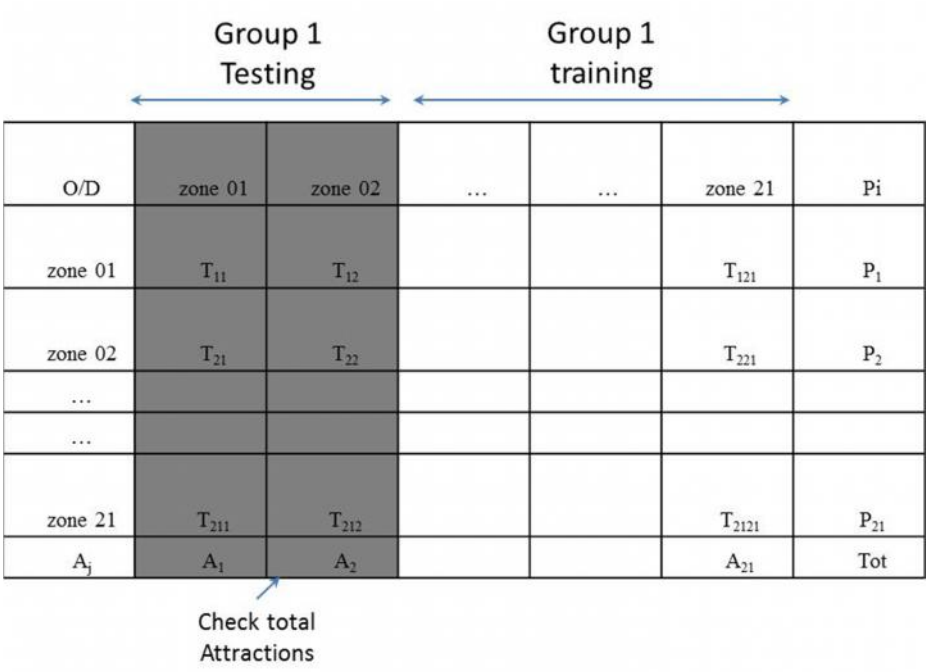

Figure 6-2: Sample Group 1 for Checking the Total Attractions, Training and Testing Data Sets

On this basis sample group 2 assumed to use the first 4 zones for testing and the rest of the zones (17 zones) for training. **Table 6-1** shows the sample group zones which are used for validation of the GRNN model and checking the production and attraction constraints.

Table 6-1: Various Groups for GRNN Validation

Groups	Sample Zones (validation)	Validation Zones (Percentage)	Training Zones	Training Zones (Percentage)
G1	1,2	About 10%	3 to 21	About 90%
G2	1,2,3,4	About 20%	5 to 21	About 80%
G3	1,2,3,4,5,6	About 30%	7 to 21	About 70%
G4	1,2,3,4,5,6,7,8	About 40%	9 to 21	About 60%
G5	1,2,3,4,5,6,7,8,9,10	About 50%	11 to 21	About 50%

The zones in each sample group were selected to represent various land uses within the Mandurah area. The training zones (the reminder of the zones which will be used for model training) also include various land use data so the model can find the relationship between the land use data and the trip distribution between the OD zones. **Table 6-2**shows the distribution of the land use data for the 21 zones within the study area.

Table 6-2: Land use Distribution for the 21 Zones in Mandurah

Zones	Residential	Retail	Office	Showroom	School	Comments
Zone 1	Y	Y	Y	Y	Y	Mixed Use
Zone 2	Y					Residential
Zone 3	Y	Y	Y	Y	Y	Mixed Use
Zone 4	Y		Y		Y	Mixed Use
Zone 5	Y				Y	Residetial + School
Zone 6		Y	Y	Y	Y	Mixed Use
Zone 7	Y	Y	Y	Y		Mixed Use
Zone 8	Y	Y	Y	Y	Y	Mixed Use
Zone 9	Y	Y		Y		Mixed Use
Zone 10	Y	Y	Y	Y		Mixed Use
Zone 11	Y	Y	Y	Y	Y	Mixed Use
Zone 12		Y	Y	Y		Mixed Use
Zone 13	Y	Y	Y	Y		Mixed Use
Zone 14	Y				Y	Residetial + School
Zone 15	Y		Y		Y	Mixed Use
Zone 16	Y	Y	Y	Y	Y	Mixed Use
Zone 17	Y				Y	Residetial + School
Zone 18	Y	Y		Y	Y	Mixed Use
Zone 19	Y					Residential
Zone 20	Y					Residential
Zone 21	Y					Residential

6.4 Validation Results

The analyses were undertaken separately for each data set. The optimum spread factor (σ) for each data set was searched through a macro developed in MATLAB software. Accordingly different spread factors (σ) between (0-1) were tested with 0.02 step increase for the σ value. Therefore the NN is trained 50 times by 50 different spread factors and the trained network is simulated for the testing data set for each spread factor. The Sum Square Error (SSE) of the testing data (sample zones) is recorded for each σ and the plot showing the SSE against σ is prepared to easily select the optimum σ.

The sample groups then are simulated again with the optimum σ and the model outputs were compared with the target data and the regression plot is prepared for each sample group to investigate the goodness of fit for the trip distribution. The same regression plot is also prepared for the total productions and attractions for each sample group to investigate the GRNN performance for estimating total productions and attractions for each sample zone. It should be noted that the production plots are prepared for the sample groups that are trained by rows of the OD matrices and attraction plots are prepared for the sample zones that are trained with the columns of the OD matrix.

Appendix D provides the σ determination plots, trip distribution regression plots and total production and attraction regression plots for each dataset. According to the analysis undertaken the validated GRNN provided better results for sample group 2. The optimum spread factor is reported as 0.42 for both datasets trained by rows and columns of the OD matrix for sample group 2. **Figure 6-3** illustrates the reported SSE for different spread factors used to train separately the rows and columns of the OD matrix for sample group 2.

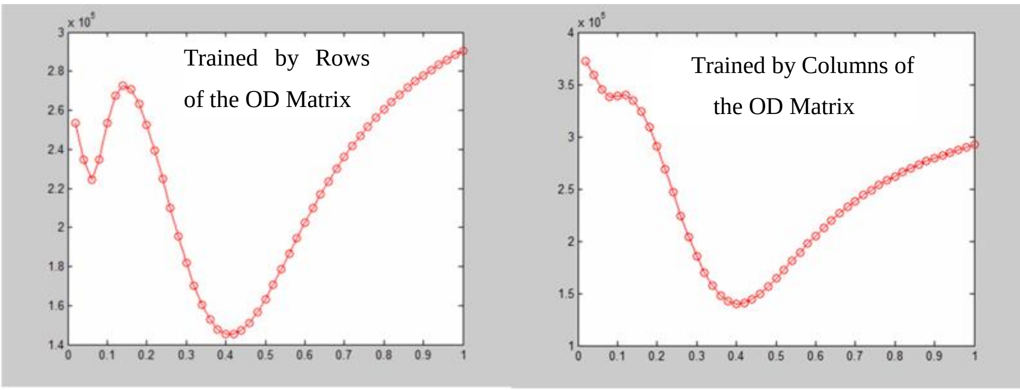

Figure 6-3: Optimum Spread Factor for Sample Group 2 Trained by Rows and Columns of the OD Matrix

Table 6-3 and **Table 6-4** summarise the results of the analysis for the 5 sample groups trained separately by the rows and columns of the OD matrix.

Table 6-3: Validation Results for Production Zones (trained by rows of the OD matrix)

Groups	Optimum Sigma (σ) trained by OD Rows	RMSE for Testing Data	R^2 for T_{ij} trained by OD Rows	R^2 for total Productions
G1	0.42	53.1	0.62	1
G2	0.42	29.4	0.59	0.92
G3	0.45	49.8	0.48	0.67
G4	0.18	78	0.39	0.94
G5	0.24	70.8	0.41	0.90

Table 6-4: Validation Results for attraction Zones (trained by columns of the OD matrix)

Groups	Optimum Sigma (σ) trained by OD Columns	RMSE for Testing Data	R^2 for T_{ij} trained by OD Columns	R^2 for total Attractions
G1	0.46	48.5	0.26	1
G2	0.42	40.1	0.61	0.95
G3	0.44	66.1	0.06	0.02
G4	0.36	76.4	0.44	0.79
G5	0.36	71.2	0.48	0.66

6.5 Observations and Discussions

Analysis undertaken indicates that:

- *GRNN predictive ability for total productions is better than total attractions. The reported R^2 for total productions are more than 0.9 for all the sample groups except sample group 3 which is 0.67.However for the attraction zones lower R^2 than production zones is reported for the majority of the sample groups.*
- *There is a poor R^2 reported for sample group 3 in attraction zones (0.02). This could be due to an unsuccessful training for the GRNN model which would be associated with data split pattern for training and validation vectors.*
- *The optimum σ is greater than 0.4 for the first three groups in both production and attraction zones. Then it drops to less than 0.4 for the samples in group 4 and 5. One possible reason for adopting a lower spread factor for the last two sample groups could be the smaller size of the training data set in comparison with the first three sample groups. With lower number of training vectors the NN may not generalize well. This is more obvious for production zones than attraction zones.*
- *The best results for both production and attraction zones are related to Group 2 with two sample zones and similar optimum σ of 0.42.*
- *The reported RMSE for both total productions and attractions is lower for sample group 2 (refer* ***Figure 6-4***)
- *The data split for Sample group 2 included about 80% data for training and about 20% for validation. There is no standard rule for the data split but the 80% / 20% training and testing (validation) split seems to be more efficient.*
    - *Sample group 2 includes 4 zones for validation which reflects different land use data available in the study area, which is combination of purely residential (zone 2) and Mixed Use(Zones 1,3and 4) and therefore provides a good data split for training and testing.*
- *The predictive ability of the GRNN model for satisfying the production and attraction constraints for sample group 2 is very well with R^2more than 0.9 and RMSE Less than 41.*

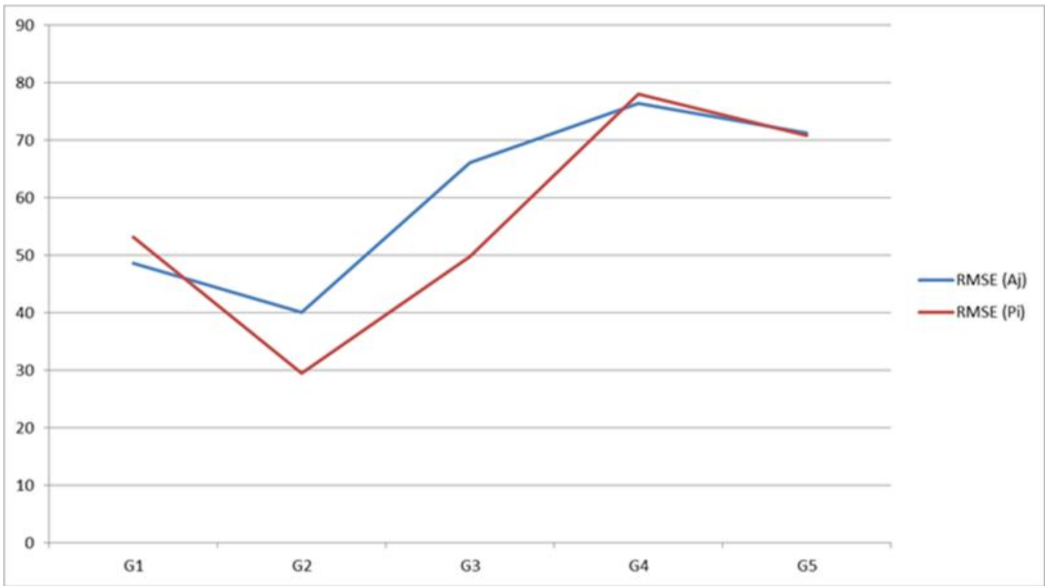

Figure 6-4: Reported RMSE for Total Productions (P_i) and Attractions (A_j) for Each Sample Group

6.6 Satisfying the Gravity Model Constrains

In order to investigate the predictive ability of the validated GRNN model for estimating the entire OD matrix and evaluating GRNN model ability for satisfying the gravity model constraints , the whole 441 vectors were used by the validated GRNN model and the modeled output were simulated by the optimum spread factor of 0.42. **Table 6-3** summarizes the GRNN model outputs for the entire OD matrix. **Figure 6-5** illustrates the regression plot for all trips (T_{ij}) within the 21 x 21 Mandurah OD matrix. The reported R^2 in this plot is 0.686.

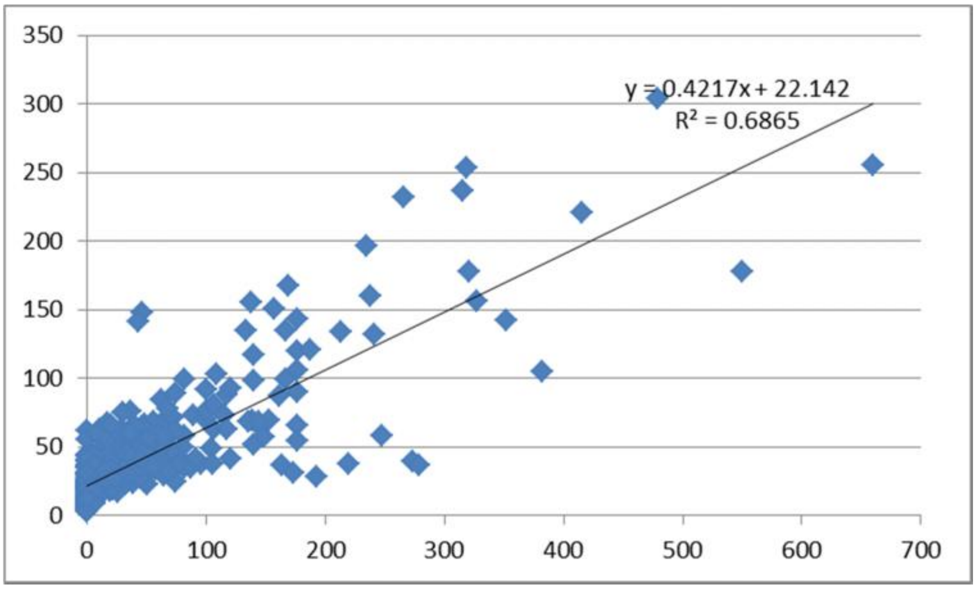

Figure 6-5: Trip Distribution Estimation for Mandurah OD Matrix by Validated GRNN Model (Regression Plot).

The x parameter in the above regression plot is 0.422 which indicates that the validated GRNN model would underestimate the observed data. Analysis undertaken indicates that the total trips in the observed OD matrix is 19,636 vehicles and the validated GRNN model projects about 18,046 vehicles with calculated RMSE of 50.34.

Table 6-5: Estimating the OD matrix for Mandurah Area by the Validated GRNN Model

O/D	zone 01	zone 02	zone 03	zone 04	zone 05	zone 06	zone 07	zone 08	zone 09	zone 10	zone 11	zone 12	zone 13	zone 14	zone 15	zone 16	zone 17	zone 18	zone 19	zone 20	zone 21
zone 01	142	47	197	67	76	254	73	141	49	156	99	168	121	46	78	75	42	60	36	40	34
zone 02	21	33	55	30	25	65	36	21	32	47	27	47	40	30	28	25	21	36	19	23	17
zone 03	20	16	67	23	18	69	23	23	17	42	27	46	38	16	25	19	14	19	13	14	13
zone 04	27	27	72	38	31	90	35	28	30	59	31	62	49	29	38	29	22	35	22	26	22
zone 05	66	36	120	51	58	157	51	62	39	98	54	106	76	39	58	51	32	47	28	33	27
zone 06	11	8	36	12	9	35	12	12	9	22	15	24	20	8	12	9	6	10	6	7	5
zone 07	24	29	58	30	25	62	40	24	32	48	32	48	43	30	30	29	22	35	19	24	18
zone 08	148	63	237	89	85	304	105	255	69	221	143	232	160	65	100	150	66	81	56	60	54
zone 09	21	31	54	31	26	63	37	21	33	46	27	46	40	32	29	27	25	36	24	27	23
zone 10	8	8	18	9	7	20	11	9	9	15	10	16	13	9	9	13	7	10	6	7	6
zone 11	11	10	33	12	9	32	16	10	11	26	25	26	23	10	12	10	9	12	9	9	9
zone 12	4	4	9	5	4	10	5	5	4	8	5	8	7	4	5	7	3	5	3	3	3
zone 13	22	22	58	27	21	61	31	24	25	46	30	47	42	24	27	24	18	27	15	18	14
zone 14	23	29	58	32	28	70	37	23	33	50	28	50	43	33	31	29	24	36	21	25	20
zone 15	45	30	103	47	44	133	44	45	34	87	43	93	69	34	55	42	29	41	25	29	24
zone 16	61	40	135	55	51	177	65	117	46	132	73	135	91	44	63	177	50	57	30	37	27
zone 17	29	24	73	31	28	89	35	34	30	63	36	64	48	28	34	41	37	40	16	20	12
zone 18	24	31	58	33	29	68	38	24	33	49	29	50	43	32	32	32	31	37	32	31	32
zone 19	19	19	54	25	20	67	26	21	25	44	27	46	35	21	25	20	14	36	32	24	25
zone 20	22	24	58	30	25	70	31	23	29	48	29	50	39	26	29	26	18	37	25	31	26
zone 21	19	17	55	26	20	69	25	21	24	44	27	47	35	20	25	19	11	37	25	25	28

Figure 6-6 and **Figure 6-7** also illustrate the performance of the GRNN model for estimating the total trip productions and attractions for each traffic zone in the OD matrix. Analysis undertaken indicates that the estimated R^2 for both productions and attractions are more than 0.8 which indicates a good fit.

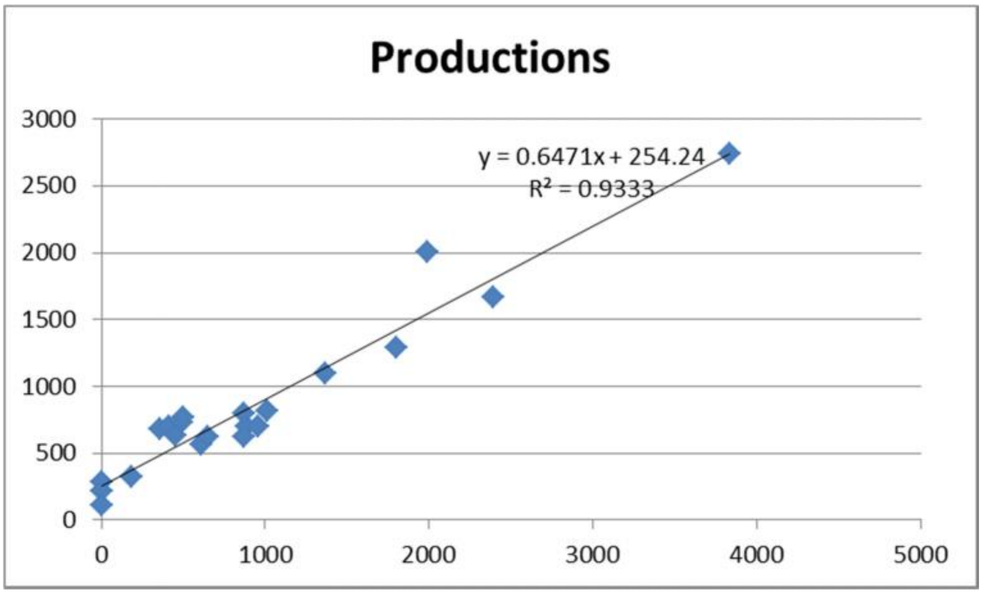

Figure 6-6: Validated GRNN Model Performance for Total Productions of Each Traffic Zone

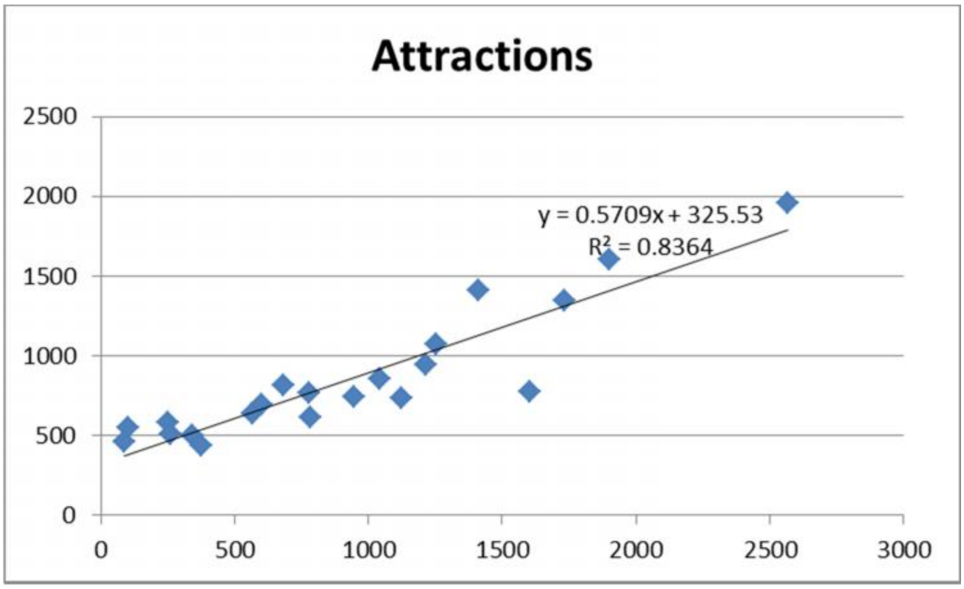

Figure 6-7: Validated GRNN Model Performance for Total Attractions of Each Traffic Zone

In order to undertake more detailed analysis and investigate the errors for total productions and attractions for each zone **Table 6-6** summarises the percentage error for both productions and attractions for each individual zone and also for the total OD matrix. Figure 6-8 also shows the **percentage error** for each zone in a plot format.

Table 6-6 and **Figure 6-8** are prepared. **Table 6-6** summarises the percentage error for both productions and attractions for each individual zone and also for the total OD matrix. **Figure 6-8** also shows the percentage error for each zone in a plot format.

Table 6-6: Error (gap) Calculation for Total Productions and Attractions

Zones	Total Productions Observed	Total Productions Modeled	Productions Error (%)	Total Attractions Observed	Total Attractions Modeled	Attractions Error (%)
Zone 1	1989	2001	-1	780	767	2
Zone 2	357	678	-90	103	547	-434
Zone 3	610	562	8	1902	1607	16
Zone 4	874	800	8	601	700	-16
Zone 5	1799	1289	28	569	641	-13
Zone 6	0	286	NA	2568	1963	24
Zone 7	885	703	20	1606	776	52
Zone 8	3839	2742	29	1215	945	22
Zone 9	415	699	-68	786	613	22
Zone 10	0	220	NA	1732	1351	22
Zone 11	182	323	-78	685	818	-19
Zone 12	0	110	NA	1409	1410	0
Zone 13	650	621	4	1251	1074	14
Zone 14	490	726	-48	249	581	-133
Zone 15	1367	1097	20	947	743	22
Zone 16	2390	1665	30	1043	856	18
Zone 17	1009	812	20	340	502	-48
Zone 18	498	769	-54	1125	735	35
Zone 19	454	627	-38	89	463	-420
Zone 20	961	697	27	261	514	-97
Zone 21	868	620	29	376	439	-17
Total	**19636**	**18046**	**8**	**19636**	**18046**	**8**

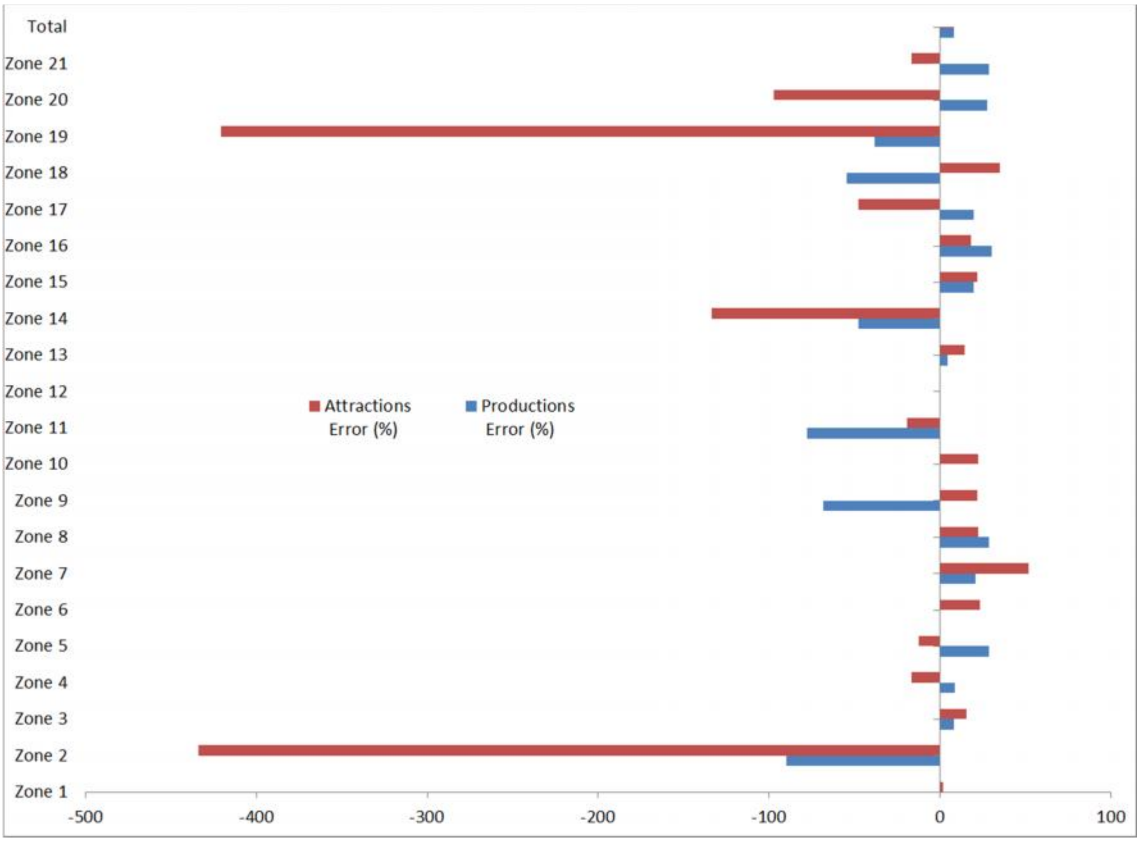

Figure 6-8: Percentage Error for Total Productions and Attractions

Reviewing the gap (error) between the modelled and observed total productions and attractions for each zone indicates that performance of the validated GRNN model for predicting the total productions is better that predicting the total attractions. The reported error for total attractions of zone 2 and zone 19 are more than 100%. Reviewing the total productions also indicates that predictive ability of the GRNN model is poor for the zones with zero work trip productions (Zones 6, 10 and 12). The calculated RMSE for the total productions (total row) and total attractions (total column) are 364 and 326 respectively.

According to the analysis undertaken the validated GRNN model could not perfectly satisfy the total production and attraction constraints for each individual zone. In order to address this issue, similar to the gravity model, it is proposed to balance the modelled OD matrix by the total productions and attractions of the original observed OD matrix (2006 ABS work trip matrix). **Table 6-7** summarises the balanced OD matrix for the validated GRNN model.

Table 6-7: Balanced OD Matrix for Mandurah Area estimated by the Validated GRNN Model

O/D	zone 01	zone 02	zone 03	zone 04	zone 05	zone 06	zone 07	zone 08	zone 09	zone 10	zone 11	zone 12	zone 13	zone 14	zone 15	zone 16	zone 17	zone 18	zone 19	zone 20	zone 21
zone 01	141	47	195	67	75	252	72	140	49	155	99	167	120	46	77	75	42	60	36	40	34
zone 02	11	18	29	16	13	34	19	11	17	25	14	25	21	16	15	13	11	19	10	12	9
zone 03	22	17	73	25	20	75	25	25	18	46	30	49	41	18	27	21	15	21	14	15	14
zone 04	30	29	78	41	34	98	38	30	33	64	34	68	53	32	41	32	24	38	24	28	24
zone 05	92	50	167	71	81	219	72	87	55	137	76	148	106	54	80	71	45	66	39	46	38
zone 06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 07	30	37	73	37	32	77	50	30	41	61	41	61	54	38	37	37	28	44	25	30	23
zone 08	207	88	332	124	119	425	147	357	97	310	200	325	225	91	140	211	92	113	78	83	76
zone 09	13	18	32	18	16	37	22	12	20	27	16	27	24	19	17	16	15	21	14	16	14
zone 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 11	6	6	19	7	5	18	9	5	6	15	14	14	13	6	7	6	5	7	5	5	5
zone 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zone 13	23	23	61	28	22	64	32	25	26	48	31	49	44	25	29	25	18	29	16	19	15
zone 14	16	20	39	21	19	47	25	16	22	33	19	34	29	23	21	20	17	25	14	17	14
zone 15	56	38	129	58	55	166	54	57	43	108	54	116	85	43	68	53	36	51	31	36	30
zone 16	88	57	194	79	74	255	94	168	66	189	105	193	131	63	91	255	72	81	43	53	39
zone 17	36	30	91	38	35	111	44	43	38	78	44	80	59	34	42	52	47	50	20	25	14
zone 18	16	20	38	21	19	44	25	16	21	32	19	32	28	21	21	20	20	24	21	20	21
zone 19	14	14	39	18	15	49	19	15	18	32	19	34	26	15	18	15	10	26	23	18	18
zone 20	30	33	79	41	34	96	43	32	40	67	39	69	54	36	40	36	25	51	35	43	36
zone 21	27	24	77	36	29	96	35	30	34	62	38	66	49	28	35	26	15	51	36	35	40

Analysis undertaken for the outcome of the matrix balancing indicates that the R^2 of the trip distribution plot for balance matrix will improve from 0.686 to 0.705 (refer **Figure 6-9**).

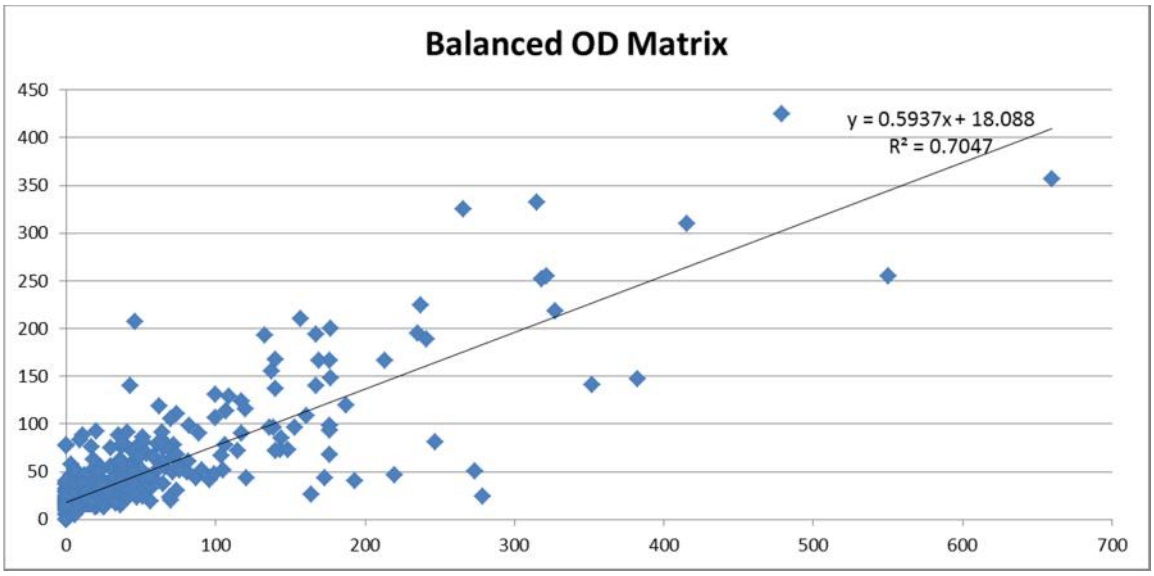

Figure 6-9: Regression Plot for the Balanced OD Matrix

The balance matrix will not only satisfy the model constrains for the total rows and columns but also will improve the RMSE of the total OD matrix from 50.34 to 43.55.

6.7 Summary

Since no independent dataset (demand matrix) was available, it was not possible to provide the external validation. Therefore the validation of the GRNN model was performed using the cross validation technique.

In this research, a variation of hold-out method was used to validate the GRNN model and also check the production and attraction constrain satisfaction of the GRNN model. The validation process was applied to 10 different sample groups.

Analysis undertaken indicated that GRNN predictive ability for total productions was better than total attractions for the majority of the sample data sets. The predictive ability of the GRNN model for production and attraction constraints for a sample group which included about 20% testing data set and about 80% training data set was very well with R^2more than 0.9 for both productions and attractions.

The validated GRNN model was applied to the entire OD matrix and the modelling output indicated that the validated GRNN model could estimate the trip distributions of the OD matrix with R^2 of 0.686 and RMSE of 50.34.

The validate GRNN model was able to satisfy both total productions and attractions for each zone, although not perfectly. In order to improve the performance of the GRNN modelling results (similar to the gravity model) the projected OD matrix by validated GRNN model was balanced with the total production and attractions of the original 2006 ABS OD matrix.

The analysis undertaken indicated that the balance matrix will satisfy the model constrains for the total rows and columns and also will improve the RMSE of the total OD matrix from 50.34 to 43.55.

The validated GRNN model with balanced OD matrix would outperform the gravity model in terms of R^2 and RMSE.

Since the trip production and attraction is not a direct input into the validated GRNN model, it seems that the GRNN model would not be able to perfectly satisfy the total production and attraction constraints; hence matrix balancing is required to satisfy the constraints.

7 7. CONCLUSIONS AND RECOMMENDATIONS

Trip distribution is behavioural in nature and is more complicated to be estimated by statistical relationships between the socio economic and household demographic data. The traditional gravity models have been used widely in different countries for many years since they provide reliable and relatively simple method to estimate trip distribution, however their simplicity does not reflect complexity of the travel behaviour in trip distribution. The proposed GRNN model tries to capture the behavioural nature of the trip distribution and keep the simplicity and practicality of the gravity model.

Since 1995 that Black developed a neural network for prediction of the commodity flow based on US Commodity Flow Survey (CFS), and used production, attraction and distance as the input to the model, most of the other following studies also used the proposed three inputs (production, attraction and distance) to develop a neural model for predicting the trip distribution. Neural networks are known as powerful tools for their ability for solving problems with complicated algorithmic solutions or even no algorithmic solutions through establishing the relationship between number of different inputs and outputs for a system. Therefore in this study instead of using the three common inputs (production, attraction and distance) for estimation of number of trips between origin and destination zones, 10 inputs in the form of a vector which its components include the land use data for the origin and destination zones were used. The separation distance between the OD zones are also added to the input vector to reflect the generalised cost between the two zones. The other novelty of this study is the application of generalised regression neural networks for predicting the trip distribution between the OD zones. The application of NN to trip distribution modelling is limited in the literature and according to the knowledge of the author; no study has been reported to investigate the potential of the GRNN model for better estimation of the trip distribution.

The application of GRNN model has been investigated for travel mode choice modelling (Celikoglu, 2007); however this type of neural network has not been used for trip distribution modelling. the common feed-forward back propagation (FFBP) neural network approach which has been used in most of the relevant cited papers for trip distribution modelling inherit some disadvantages including their sensitivities to the selected initial weights and the local minima problem which will generate inaccurate outcomes.

GRNNs are known for their ability to learn quickly with small number of data and their application has been investigated in different studies including Medical, hydrological and electrical and many more applications. GRNN application is especially useful for function approximation with multi-dimensional inputs and therefore in this thesis the ability of the GRNN model for trip distribution modelling has been investigated with multiple inputs in the form of land use data into the GRNN model. The performance of the GRNN model has been compared with the traditional gravity model and the FFBP model. In order to validate the GRNN model the performance of the GRNN model has been investigated by 10 different data sets and the performance of the GRNN model to satisfy total production and attraction constraints were assessed.

In summary the modelling and analysis undertaken indicated that the accuracy of the three developed models (GRNN, BP an Gravity models) for estimation of the work trips for a small area such as Mandurah is almost similar. The reported R^2 for all three models is in the range of 0.4 to 0.7; however the GRNN model proposed in this research provides a simple and practical methodology which can be used by the traffic and transport modellers or software developers to estimate the trip distribution matrices for the strategic transport models. The proposed GRNN model has been applied to the work trips in this thesis and provided promising results for work trip estimations. Analysis undertaken also indicated that the validated GRNN model could outperform gravity model.

The proposed GRNN model is expected to predict other trip purposes such as education trips, shopping trips and other trips and therefore the master matrix which would be the combination of these matrices can be assigned to the road network to estimate the projected traffic volumes on road network within the modelling study area.

NN is still in its infancy in the field of transportation and more guidelines and research work are required to improve the performance of the NN models to be able to effectively utilise their ability in practice. The proposed GRNN model proposes a simple and practical approach for estimation of trip distribution by land use data. The proposed approach needs to be investigated further with larger dataset if available and could be a recommendation for future research work in this field.

7.1 Combined Trip Generation and Distribution Modelling

The proposed validated GRNN model is providing a combined trip generation and distribution modelling frame work which is another novelty of this research. The transportation planning process has traditionally suggested a four step models for trip generation, distribution, modal split, and assignment. The traditional four step model, estimates each step independently and, therefore, some inconsistencies would appear.

The decision to take a trip by an individual trip maker, involves travel to a particular destination, using a particular mode, and traverses a particular route. This process is made simultaneously rather than sequentially. In the last two decades, this issue has been studied by many researchers. In order to reflect the joint nature of these decisions and improve the behavioural nature of the trip makers different studies has been undertaken to simulate these steps simultaneously and provide consistent outcomes. Nabil et al. (1988) developed a transportation equilibrium model and an algorithm for the simultaneous prediction of trip generation, trip distribution, modal split, and trip assignment on large-scale networks. Using the random utility theory framework, Zhong et al. (2009) recommended an alternative formulations, including mathematical programming and variational inequality formulations for a combined travel demand model that integrates trip generation, trip distribution, modal split, and traffic assignment.

Accordingly, similar studies have been undertaken for combining two or more of the above components of the four step modelling. For example, Tomlin (1971) developed a model that simultaneously estimated trip distribution and assignment; Wilson (1969) has researched a combined trip distribution and modal choice model; and the presented model of Quandt et al. (1966) simultaneously estimated trip generation and modal split. Frank (1975) also recommended a combined trip generation and distribution model at the aggregate level.

According to the literature no work has been reported that employ neural networks to investigate trip generation and distribution simultaneously. Therefore the potential of the GRNN for better addressing this issue is recommended and investigated in this research.

7.2 Summary of the Neural Network Modelling

Neural network (NN) models were introduced as alternative methods for traditional modelling approaches, and have been increasing in use since the 1990s (Tillema et.al, 2006). A neural model is able to learn the relationship between input and output data for a system. According to Shmueli (1998), NNs can overcome the problems faced by the behavioural or disaggregate models because the neural model learns the relationship between variables of a model automatically and discovers the best fit which is a complicated task for a disaggregate model; and also the neural model directly works on the data without the aid of additional models.

The main limitation of NN models is related to its ‘‘black-box’’ nature of NNs (Dougherty, 1995, Fu and Rilett, 1995, Cantarella and de Luca, 2005). NNs cannot establish a causal relationship between the model parameters. Therefore it is impossible to measure the elasticity of the parameters unlike regression models. Also, the outputs of the NNs are connection weights for model variable. These weights do not provide a clear elasticity measure unlike regression models and it is difficult to interpret the meaning of the final connection weights. However it is always possible to explore the behaviour of neural models towards different properties of the NNs. In this thesis the behaviour of the NNs have been investigated against the format of the input data into the model, selection of the best NNs for solving the trip distribution problem, and specifically for the BP model the number of nodes in hidden layers were investigated.

7.3 Summary of the GRNN Modelling

GRNN is a feed-forward neural network and is known as a powerful tool in practice because of the following reasons:

- *Its ability to converge to the desired outcome with minimal available training data;*
- *Its flexibility to train the network and develop the NN structure with relatively little additional knowledge by the user.*

The input layer of the GRNN was represented by land use data in each zone, which was assigned to RD (residential dwellings), RE (retail), CO (commercial land use), SH (showroom) and SC (schools). In order to represent the spatial distribution of a pair of zones, the distance D_{ij} (metres) between zones i and j is defined. Accordingly the input vector (X) is defined as:

X_{ij}=(RD_i, RE_i, CO_i, SH_i, SC_i,,RD_j, RE_j, CO_j, SH_j, SC_j, D_{ij})

where i and j show the origin and destination, respectively.

Trips (T_{ij}) between a pair of zones were considered to be the output layer of the neural network. Simple data normalization, linear transformation and statistical normalization methods were used in this study for the input vectors to normalize the input data into the NN model. Analysis undertaken indicated that GRNN performance in training would be similar with all three different normalisation methods. For simplicity the simple data normalisation was used for the modelling the testing data set.

There are usually two kinds of input data sets in neural networks, namely training and testing data sets. The training data set is used in estimating the model parameters/variables while the testing data set is for evaluating the forecasting ability of the model. For the purpose of this study, 90% of the data (400 input vectors) was used for training and about 10% (41 vectors) was used for testing.

The smoothness parameter or spread factor indicates the width and slope of the neurons functions. This factor is the only parameter in GRNN that needs to be adopted. For developing the preferred GRNN model structure, the optimum spread factor was obtained through cross validation technique.

As a case study the proposed GRNN model was applied to the 2006 Journey to Work data set for the Mandurah Area. Data was sourced from the Department of planning (DoP). The travel zones for the purpose of this study were 21 zones which generated 441 (21 x 21 =441) input data for the purpose of the model development.

The modelling and analysis undertaken indicated that:

- *The GRNN model could estimate the work trip distribution by land use data and distance between the OD zones. The model performance indicators at calibration level were reported as 10 for RMSE, 10 for MAE and 0.984 for R^2, these figures for testing level were reported as 38 for RMSE, 22 for MAE and 0.575 for R^2.*
- *Analysis of the zero work trip vectors indicated that GRNN model would be able to estimate zero trips. The calibrated GRNN could predict 9 zero trip vectors out of 16 zero trip vectors correctly.*

7.4 Summary of the BP Modelling

The standard BP network used for this study was a two-layer feed-forward network with a sigmoid transfer function in the hidden layer and a linear transfer function in the output layer. The number of hidden neurons was set to 10. The training algorithm was based on a Levenberg-Marquardt (LM) minimization method. Simple data normalization was used for the input vectors. In order to be consistent with the GRNN modelling, 90% of the data (400 input vectors) was used for training and validation and 10% was used for testing. The testing vectors were not used in the training or validation process. The BP network was trained with 10 different seeds and five various hidden neurons. The BP model was applied to the same training and testing data set as GRNN model and the following observation were reported:

- *The BP model could estimate the work trip distribution by land use data and distance between the OD zones. The model performance indicators at calibration level were reported as 45 for RMSE and 0.77 for R2, these figures for testing level were reported as 64 for RMSE, and 0.485 for R^2.*
- *Reviewing the analysis undertaken for the training data sets indicated that increasing the number of nodes in the hidden layer would not necessarily improve the performance of the BP model. Analysis undertaken for 4 different sets of number of nodes (5, 10, 15 and 20) indicated that the BP model performed better with 10 nodes in the hidden layer and increasing the number of nodes in the hidden layer to 15 or 20 nodes did not improve the performance of the BP model.*
- *The BP model provided negative predictions for some of the observed trips in the testing data set (about 27%). The negative predictions are mostly related to the zero trip zones (about 56% of zero trips are predicted with negative values) which mean that BP model ability to predict zero trips is poor.*
- *Using the linear activation function in the output layer is the reason for producing negative values for the trip estimations. The linear transfer function do not change the summation results and transfers them after the summation process, therefore the outputs (predictions) have no limits and can also be negative.*
- *The x parameter in BP model for testing data set is reported 1.084 in the regression plot, which means that the modelled values match the observed values over the range of data and therefore it is expected that BP model provides better match than the GRNN model.*

7.5 Summary of the Gravity Modelling

The gravity model used in this desertion was based on the strategic transport model developed for the Mandurah and Peel Region. The transport model was based on the traditional four-stage model process (trip generation, trip distribution, mode split and traffic assignment). The trips were divided into five different categories based on trip purposes: work, education, social, other and non-home-based (NHB) trips. Trips internal to the modelling area were distributed based on the gamma function.

The number of residential dwellings for the City of Mandurah was calculated for the 38 individual modelling zones. The existing land use data for the attraction zones (retail, commercial, school, showroom, etc.) was sourced from City of Mandurah. The zoning system for Mandurah modelled study area were much smaller than the Department of Planning zoning system which was the base of the observed data, therefore the modelled smaller zones were aggregated to reflect the DoP zoning system to be able to compare the gravity modelling output with the previously established neural models.

The gravity model was applied to the same training and testing data set as neural models and the model performance indicators at calibration level were reported as 50 for RMSE, 23 for MAE and 0.59 for R^2, these figures for testing level were reported as 46 for RMSE, 31 for MAE and 0.446 for R^2.

7.6 Models Performance Comparison

The root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and measures of the training and testing data set are the most common indicators used to provide a numerical description of the goodness of the model estimates. Accordingly these indicators were calculated and reported for the three models at the calibration and testing levels. In order to compare the models' performance, these indicators at the testing level were summarised in **Table 7-1.**

Table 7-1: GRNN, BP and Gravity Modelling Results for the Testing Data Set

Models	RMSE	MAE	R^2	Regression x-Parameter	Comments
GRNN Model	38	22	0.575	0.51	Better results in terms of performance indicators but low regression x-parameter
BP Model	64	31	0.485	1.08	Similar results in terms of performance indicators as gravity model but better x-parameter than both models
Gravity Model	46	31	0.446	0.63	Low performance indicators and low regression x-parameter

This table indicates that the GRNN model provided slightly better results than the BP and gravity models in terms of the performance indicators however its x parameter is reported lower than BP model. The R^2 of the BP model was slightly higher than for the gravity model, while the reported RMSE was higher than for the gravity model. The mean average error of 31 is reported for both BP and gravity models.

Comparing the three developed models indicated that:

- *GRNN model could provide a slightly better goodness of fit than the BP and gravity models with a lower error level than BP and gravity models, as indicated by the average root mean square error (RMSE), where the RMSE for the GRNN, BP and gravity models was 38, 64 and 46 respectively. The estimated R^2 for the GRNN, BP and gravity models was reported as being 0.557, 0.48 and 0.446 respectively.*
- *The distribution of points in the regression plot for all models indicated that the majority of the points are clustered at low values, with one or two at much higher levels therefore, the regression parameters are dependent on these points. The testing data set was selected through the random split method and checked to insure that testing data represent the variety of the work trip conditions in Mandurah. Therefore the testing data set included*

range of different work trips including zero trips and higher work trip generators.

- *Reviewing the regression plots for the 3 models indicated that BP model provides closer x parameter to 1 and therefore can provide better match for the observed work trips. However BP model performance measures are lower than the GRNN model.*
- *Considering that x parameter for both GRNN and gravity models are lower than 1, then it is expected that these models underestimate the observed data.*

7.7 Summary of the GRNN Model Validation

The purpose of the GRNN model validation was to validate the performance of the GRNN model with different number of sample groups and check the predictive ability of the GRNN model for satisfying the gravity model constraints (total productions and attractions). Since no independent dataset (demand matrix) was available, it was not possible to provide the external validation. Therefore the validation of the GRNN model was performed using the cross validation technique. Cross validation techniques are normally used to insure good generalisation of a neural model and avoid over fitting. There are different methods available for cross validation. In this research a variation of hold-out method was used to validate the GRNN model. Accordingly the cross validation technique was applied to the following 5 sample groups:

- G1 :About 10% of the data (zones 1 and 2) used for testing and about 90% was used for training ;
- G2 : About 20% of the data (zones 1 to 4) used for testing and about 80% was used for training ;
- G3: About 30% of the data (zones 1 to 6) used for testing and about 70% was used for training ;
- G4: About 40% of the data (zones 1 to 8) used for testing and about 60% was used for training ; and,
- G5: About 50% of the data (zones 1 to 10) used for testing and about 50% was used for training ;

The above sample groups were trained separately by the rows and columns of the OD matrix. Analysis undertaken for the above sample groups indicated that:

- *The best results obtained were related to Group 4 with two sample zones for validation. The reported RMSE for both total productions and attractions was lower for sample group 2;*
- *The data split for Sample group 2 included about 80% data for training and about 20% for validation. There is no standard rule for the data split but the 80% / 20% training and testing (validation) split seems to be more efficient.*
- *The validated GRNN model could not perfectly satisfy the total production and attraction constraints for each individual zone in the OD matrix. In order to address this issue, similar to the gravity model, it was proposed to balance the modelled OD matrix by the total productions and attractions with the original observed OD matrix (2006 ABS work trip matrix);*
- *The analysis undertaken indicated that the balanced matrix could satisfy the model constrains for the total rows and columns and also could improve the RMSE of the total OD matrix from 50.34 to 43.55.and,*
- *The validated GRNN model with balanced OD matrix would outperform the gravity model in terms of R^2 and RMSE.*

7.8 Future Research

This research aimed to estimate the work trip distribution by generalised regression neural networks. The trip distribution was estimated by the land use data for the OD zones and the distance between the zones. The results of the analysis were also compared with the BP model and traditional gravity model.

Despite the efforts devoted to the analysis of all of the approaches discussed in this dissertation, there are major areas that still need to be researched. The following sections provide recommendations for the future research in this area.

7.8.1 Using a larger dataset

The GRNN model outputs rely greatly on the amount of data available and the variety of the training data set vectors. The greater the number of input vectors in the training data set, the more accurate the results in the output vector. Therefore it is recommended that the efficiency of the GRNN model be tested and improved with a larger data set if available. This research used 441 input vectors which were derived from a 21x21 OD matrix for the Mandurah and Peel Region. It is recommended that work trip distribution be estimated for a wider area and with larger dataset which would provide variety of land use data for the OD zones. The GRNN model performance is expected to improve with the larger input vectors.

7.8.2 Using Different Generalised Costs

The proposed GRNN model in this thesis aimed to find the relationship between the land use data of origin and destination zones with respect to the distance between the two zones and estimate the number of trips between the two zones. The distance as the only factor for the separation between OD zones is not expected to be the best representation of the generalized cost between the two zones but as the existing gravity model used for Mandurah strategic transport model utilised the distance for the purpose of the separation between OD zones then neural models were also used the same parameter as input to the model to be able to compare the neural models with the gravity model.

The general cost indicates the separation between the origin and the destination zones and for private cars includes operating costs, in-vehicle time, parking costs, access time to and from the car and tolls or user charges. Generalized cost normally combines all of these variables together as a weighted sum of those factors for the origin to destination zone in the model. Therefore it is recommended that different generalized costs be tested as an input to the model rather than the distance between a pair of zones, in order to investigate the sensitivity of the model to different generalized costs.

7.8.3 Estimating trip Purposes other than Work Trips

This research concentrated on work trip purposes only, however the other trip purposes such as education trips, shopping trips, non-home bases trips and other trips also need to be investigated. According to the Perth and Regions Travel Survey (PARTS) data the percentage of car drivers for work trip purpose in City of Mandurah is about 15.8% (refer **Figure 7-1**).

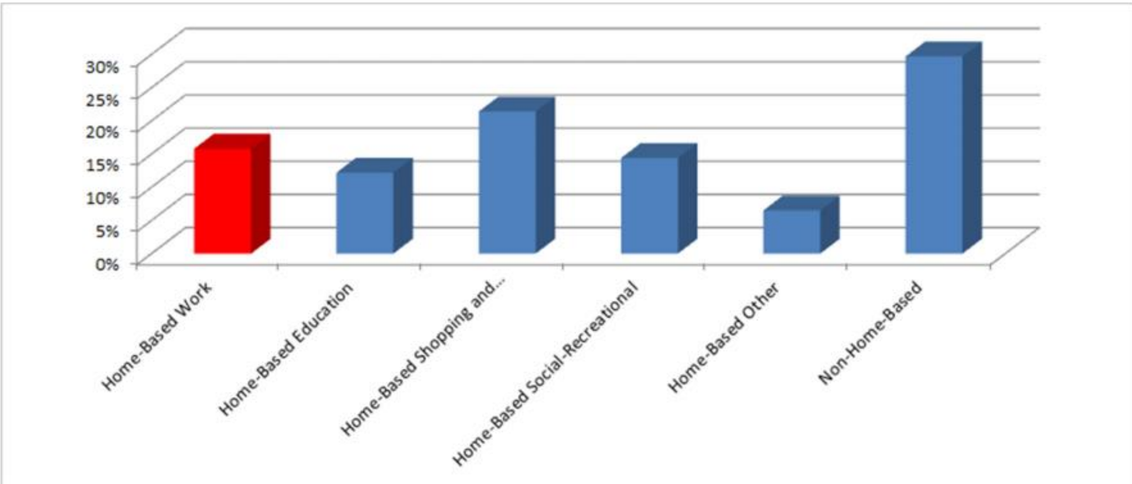

Figure 7-1: Percentage of Car Drivers for each Trip Purposes in Mandurah

In order to estimate the total traffic in a study area the master matrix which combines all the trip matrices should be assigned to the road network. It is therefore recommended that different trip purposes be estimated by the GRNN model and the total trip distribution matrix which is the combination of all the trip matrices be assigned to the model road network and the projected traffic volumes at links be compared with the projected traffic volumes by the gravity model distribution.

7.8.4 Trip Production, Attraction and Distance

All the previous cited papers used trip production, trip attraction and distance as the input to the neural models. In this study it was suggested to use land use data as input to the model. In order to investigate the performance of the GRNN model with BP model and gravity model and be consistence with previous studies, it is recommended that the GRNN performance be investigated by the common inputs to the model as trip production, trip attraction and distance between the OD zones.

7.8.5 Combining Land Use Data with Trip Production, Attraction and Distance

According to the analysis undertaken for the neural models in this thesis, the predictive ability of the neural models for total productions and attractions or satisfying the gravity model constrains is not very good and the projected OD matrix needs to be balanced to be able to satisfy the constraints. One possible reason for this issue could be related to the input data which has been used for training the neural models. The input data includes the land uses activities within the study area which reflects the distributions of the activities, however the size of the activities which is identified by trip generation and attraction of the zones are not direct input to the neural models and therefore the neural model would not be able to predict perfectly the total production and attractions for each zone. Therefore it is recommended that trip production and attractions are also included into the input data in combination with the land use data to improve the predictive ability of the neural models for total production and attractions.

7.8.6 Household Demographic Data

The proposed GRNN model in this thesis tries to take into account the behavioural nature of the trip distribution (by land use data instead of trip production and attractions and also the generalised cost between the origin and destination zones) and keep the simplicity and practicality of the gravity model. The established strategic model for Mandurah reflected private cars only and did not include the mode choice step of the traditional four step model. The trip generation step of the model also estimated from number of residential dwellings per each traffic zone and therefore the input to neural models were also kept the same as inputs to the gravity model to be able to compare the models with similar inputs. However, the trip generation and mode choice of the traditional 4-step model uses additional information related to the socio-economic data and household demographic data at the zonal level. Therefore it is recommended that the proposed GRNN model be utilised and compared with a more sophisticated strategic transport models (4-step model) and inputs to the GRNN model be increased to include additional household demographic data including household car owner ship and house hold income.

7.8.7 Software Development

In this study all the input vectors to the GRNN model was prepared by Microsoft Excel program and the developed GRNN model trained by MATLAB software. It is therefore recommended that all this process be automatic through development of new software or linking the Excel and MATLAB software packages. Extracting the land use data information and recording them into excel file, data normalisation and preparation of the input vectors can be done easily through Excel program. Some modellers also prefer to use Excel program to prepare the trip generation and attraction files for each trip purposes. Then the prepared files will be used by strategic transport software packages such as EMME for trip distribution process including balancing the matrices and adding the matrices together and preparing them for assignment. This process can be automatically undertaken without suing those software packages through developing simple software or program for training the GRNN model. Considering that structure of the GRNN model is fixed, therefore does not have to be investigated by trial-and-error unlike the BP model then this will remove some of the uncertainty related to the NN model development process.

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Appendix A:

Papers

Application of generalised regression neural networks in trip distribution modelling

Mohammad Rasouli and Hamid Nikraz

Peer reviewed paper

This paper has been critically reviewed by at least two recognised experts in the relevant field.

Originally submitted: February 2014.

Abstract

Trip distribution is the second step of the transport modelling process. Errors in this trip distribution step will propagate through the other stages of the transport modelling process and will affect the reliability of the model outputs. Therefore, finding a robust and efficient method for trip distribution has always been an objective of transport modellers. The problem of trip distribution is non-linear and complex. Neural networks (NNs) have been used effectively in different disciplines for solving non-linear problems. Accordingly, in this paper, a new NN model has been researched to estimate the distribution of the journey to work trips. This research is unique in two aspects: firstly, the training of the model was based on a generalised regression neural network (GRNN) algorithm, while the majority of previous studies have used a back-propagation (BP) algorithm. The advantage of the GRNN model over other feed-forward or feed-back neural network techniques is the simplicity and practicality of the model. The second unique aspect is that the input data for the GRNN model was based on land use data for each pair of zones and the corresponding distance between them, while the previous NN models used trip productions, trip attractions and the distance between a pair of zones as inputs. As a case study, the model was applied to the journey to work trips in the City of Mandurah in Western Australia. The results of the GRNN model were compared with the well-known doubly-constrained gravity model and the BP model.

INTRODUCTION

Neural network (NN) models were introduced as alternative methods to traditional modelling approaches, and have been increasingly used since the 1990s (Tillema, van Zuilekom & van Maarseveen 2006). The use of NN models has been researched for the prediction of trip distribution.

Previous studies show that the NN method has been used successfully to model commodity flows (Black 1995), inter-city passenger flows (Xie 2000) and work trip flows. Other researches indicated that the NN performance is not as good as the well-known gravity model (Mozolin, Thill & Lynn 2000). According to our review of the literature, the majority of previous studies utilised a back-propagation (BP) algorithm to solve the trip distribution problem. Most recent studies tried to improve the performance of neural networks by training the models with different training algorithms, such as the Levenberg-Marquardt (LM) algorithm or different activation functions (Yaldi, Taylor & Yue 2011).

Although the recent studies were able to improve the performance of the NN models, there have not been enough attempts to utilise other NN models such as the generalised regression neural network (GRNN). The advantage of GRNN models over other NN models is their ability to converge to the target data with only limited training data available. Also, the additional knowledge needed to develop and train the GRNN is relatively small and can be done without additional input by the user (Specht 1991). This makes the GRNN a very useful tool in practice. In this research, a GRNN model has been developed as a new approach and the performance of this model has been compared with back-propagation and gravity models. This study is unique in two aspects:

- The input data for the GRNN model was based on the land use data for each zone and the corresponding distance between a pair of zones, while the previous NN models used trip productions, trip attractions and the distance between a pair of zones as input into the model.
- The training of the model was based on a GRNN algorithm, while the previous studies used a BP algorithm.

As a case study, the new approach was applied to journey to work (JTW) trips for the Mandurah area in Western Australia. The 2006 JTW data set for the Mandurah area was sourced from the Australian Bureau of Statistics (ABS). Accordingly, three different models were developed: the GRNN, BP and gravity models. MATLAB[1] was used to train and develop the GRNN and BP models. The gravity model used in this research was based on the strategic transport model developed for the Mandurah and Peel Region in Western Australia with EMME software (Rasouli 2012).

Simple data normalisation, linear transformation, and statistical normalisation methods were used in this study for the input vectors. The root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) between the modelled output and target data for training and testing data sets were used as indicators of goodness-of-fit of the model estimates.

BACKGROUND

The application of neural networks in the transport modelling area is growing fast. The literature indicates that NNs have been used for driver behaviour simulation models, mode choice and trip distribution problems. *Table 1* summarises the major studies undertaken so far to estimate trip distribution by applying the NN technique. This table indicates that all the studies undertaken used trip production, trip attraction and distance between a pair of zones as the inputs to the NN model. BP was the main training algorithm used for the majority of studies undertaken, and RMSE was the main performance measurement used in the majority of research.

Black (1995) investigated the application of NNs for commodity flows. Black's model was developed the same as the gravity model, with trip production, trip attraction and distances between each pair of zones as inputs to the NN model. The model developed by Black was a back-propagation model with three layers (input, output and hidden layers). He compared the RMSE of the NN model with the gravity model for the data of commodity flows between nine regions. Based on this comparison, he demonstrated that the errors from the proposed NN model were as much as 50% lower than those from the gravity model.

Xie (2000) undertook an NN approach to model inter-city passenger flows. Xie extended the work undertaken by Black by using the same NN architecture. In this study, a back-propagation neural network model with a gradient descent search algorithm was used to predict monthly intercity Amtrak passenger flows between various stations in order to evaluate the model's predictive ability. According to the analysis, the application of neural networks to large data sets produced satisfactory performance results and the neural network model outperformed the fully-constrained gravity model in terms of RMSE for some volume groups.

Mozolin et al. (2000) researched the performance of NNs and doubly-constrained gravity models for the distribution of commuter trips. Their research indicated that the NN models performed better to fit the data, but their accuracy in predicting

1 http://www.mathworks.com.au/

Table 1
Application of neural networks for trip distribution estimation

		Network detail			
Author	Date	Input Data	Network Structure	Training	Performance
Black	1995	P, A, D	MLF	BP	RMSE
Xie	2000	P, A, D	MLF	BP	RMSE, R
Mozolin et al.	2000	P, A, D	MLF	BP	RMSE, AE
Tapkin	2004	P, A, D	Revised MLF	GD	RMSE
Tillema et al.	2006	P, A, D	NA	NA	RMSE
Yaldi et al.	2009	P, A, D	MLF	BP	RMSE, R
Yaldi et al.	2011	P, A, D	MLF	LM	R^2

Abbreviation definitions: P: Production, A: Attraction, D: Distance, MLF: Multi-Layer Feed-forward, BP: Back-propagation, RMSE: Root Mean Square Error, AE: Absolute Error, NA: Not Available, R: Correlation Coefficient R^2: Coefficient of Determination, LM: Levenberg-Marquardt, GD: Gradient Descent.

the target data was not as good as the doubly-constrained models. They further claimed that the analysis undertaken proves that the accuracy of the NN models was poorer in comparison with that of doubly-constrained gravity models with the distance decay of exponential function format. They referred to different reasons for NN under-performance, including 'model non-transferability, insufficient ability to generalise, and reliance on sigmoid activation functions'.

In a study by Tapkin (2004), a recommended neural trip distribution model (NETDIM) was developed and its performance was compared with three different models, including the back-propagation network, modular neural network and unconstrained gravity model. The objective of this research was to demonstrate the performance of the three models by comparing their levels of prediction, rather than by comparing outputs of the models for a specific data set. RMSE has been used as an indicator for comparison of the levels of prediction of the models. The analysis undertaken indicated that NETDIM provided more accurate predictions than the modular approach, unconstrained gravity model and the back-propagation neural network.

Tillema et al. (2006) undertook a study to compare the results of the NN and the gravity model in predicting trip distribution. This study researched both synthetic data and real-world data. Calibration of the neural network and gravity models was based on different percentages of hold-out data. This research demonstrated that neural networks outperformed gravity models in both synthetic and real situations. The modelling results indicated that the gravity model only gives better results when the model is very well calibrated. But in reality, with scarce data, neural networks showed their capabilities and outperformed the gravity model.

Yaldi et al. (2009) reported that in order to satisfy the production and attraction constraints in NN modelling, a linear activation function can be used in the output layer of the model. Their recommended model used simple data normalisation for the inputs of the NN. Their analysis proved that a validated NN model could perform the same as a doubly-constrained gravity model with a similar R^2. However, the error level of an NN model is still more than the gravity model in terms of the average RMSE.

In another study, Yaldi, Taylor and Yue (2011) used the Levenberg-Marquardt (LM) algorithm to improve the performance of NN models. They compared the results of the new model with standard back-propagation, Quickprop and variable learning rate (VLR) algorithms. Their research demonstrated that with the use of the LM algorithm, the testing performance of the NN model can be improved to the same level as the doubly-constrained gravity model.

A brief description of the neural network

The neural network is an artificial intelligence method that simulates the operation of the human brain (nerves and neurons). The NN approach was developed by Warren S. McCulloch and co-workers in the early 1940s (Haque & Sudhakar 2002). They developed simple neural networks to model simple logic functions.

Nowadays, neural networks are used for complex problems that do not have algorithmic solutions. In

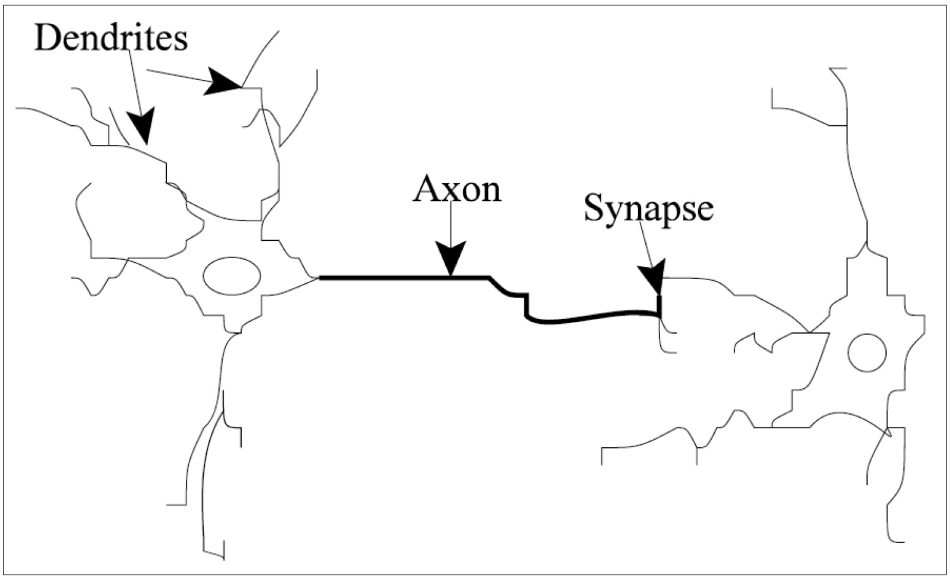

Figure 1
Natural neuron

other words, it is not easy to establish a mathematical model for problems with no clear relationship between the inputs and outputs of a system. To solve this sort of problem, the NN uses input samples and is trained to learn the relationship between the input and output data.

The ability of an NN to learn by samples makes this a very flexible and powerful tool. Accordingly, neural networks have been largely used for mapping regression and classification problems in many disciplines, and their usage is growing fast.

There have been a number of different NN models developed since McCulloch's first NN model. The differences in the NN models are related to the activation functions, the topology, the learning algorithms, etc. The back-propagation algorithm is one of the most common methods used in NN modelling, and many others are based on it. The GRNN is a feed-forward network. The advantage of a GRNN over the other NN models is simplicity and practicality of the GRNN. The required knowledge for a user to develop a GRNN model is relatively small. Another advantage of the GRNN is its ability to converge to the desired outcome with only limited training data.

Basic concept of neural networks

The artificial neural network (ANN) is a computational approach inspired by real neurons. Real neurons have synapses located on their dendrites or membrane to receive input signals (*Figure 1*). Once the received signal becomes strong enough (exceeds a certain threshold), it can activate the neuron, which then generates an output signal and transfers it through the axon of the neuron. The output signal can be received by other synapses, which might activate other neurons successively (Gershenson 2003).Artificial neurons are highly abstracted models of complex real neurons. These neurons consist of three basic parts: inputs (as synapses), which are multiplied by weights corresponding to the strength of the signals; a mathematical function, determining the activation of the neuron; and an output layer (*Figure 2*).

The higher the weight of an artificial neuron, the stronger the input multiplication result will be. There are negative weights, so signal inhibition becomes possible. The computation inside each neuron is different, depending on its weight. Through adjustment of the weight of an artificial neuron, any desired output can be obtained for specific inputs. However, it would be quite difficult to manually determine all of the necessary weights in an ANN with hundreds or even thousands of artificial neurons. There are algorithms which can calculate the weights for an ANN in order to generate the desired output. This weight adjustment process is known as the learning or training procedure (Gershenson 2003).

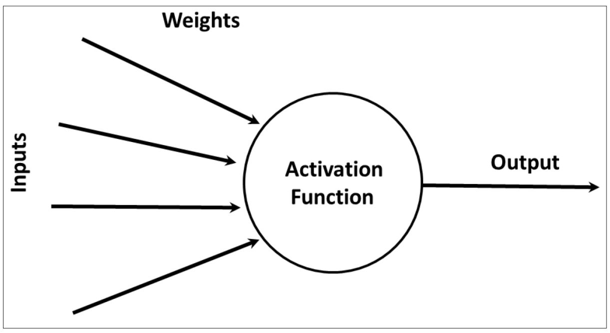

Figure 2
Artificial neuron (Gershenson 2003)

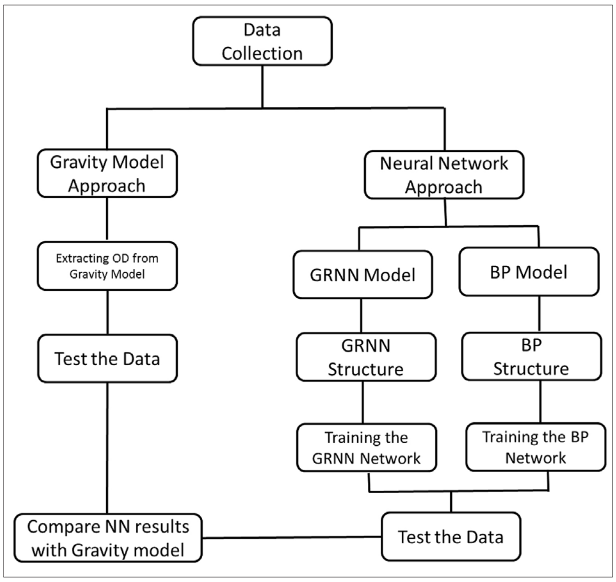

Figure 3
Model development and methodology

MODEL DEVELOPMENT AND METHODOLOGY

For the purposes of this research, three models were developed for estimation of the trip distribution. GRNN modelling is the new model, which is the focus of this research. The BP and gravity models are the other approaches. The results of the GRNN model have been compared with the BP and gravity model. The root mean square error (RMSE), mean absolute error (MAE) and coefficient of determination (R^2) between the modelled output and measures of the training and testing data set have been used to compare the modelling results.

At the time of preparation of this paper, the 2011 JTW data was not available; hence, the 2006 JTW data was used. Taking into consideration that the strategic transport model for the Mandurah area was developed and calibrated for the year 2011, the 2011 JTW data was estimated from the 2006 data assuming the same travel pattern for the JTW in 2006. The model development and methodology is illustrated in *Figure 3* and is discussed in the following sections.

GRNN model architecture

The input layer of the GRNN model is represented by land use data in each zone (Rasouli & Nikraz 2013), which is assigned to RD (residential dwellings), RE (retail), CO (commercial land use), SH (showroom) and SC (schools). In order to represent the spatial distribution of a pair of zones, the distance D_{ij} (metres) between zones i and j is also defined. Accordingly, the input vector (X_{ij}) is defined as:

$$X_{ij} = (RD_i, RE_i, CO_i, SH_i, SC_i, RD_j, RE_j, CO_j, SH_j, SC_j, D_{ij})$$

where i and j show the origin and destination, respectively.

Trips (T_{ij}) between a pair of zones are considered to be the output layer of the neural network. The GRNN has to be able to model the relationship between trips T_{ij} and the input vector X_{ij}. The model was developed to forecast the work trip. MATLAB R2011a was used to develop the network, where the optimum spread factor was selected through a trial and error process. Simple data normalisation, linear transformation and statistical normalisation methods were used in this study for the input vectors. Simple normalisation uses the following formula:

$$x_{\mathrm{n}} = \frac{x_0}{x_{\max}}$$

where:

x_{n} = normalised input

x_0 = each data input

$x_{\max}$ = the maximum among all the data.

Linear transformation will convert the input data to the range [0, 1] with the following formula:

$$x_{(\mathrm{scaled})} = \frac{x_{(\mathrm{actual})} - x_{(\mathrm{min})}}{x_{\max} - x_{\min}}$$

where:

$x_{(\mathrm{scaled})}$ = normalised input

$x_{(\mathrm{actual})}$ = each data input

$x_{\max}$ = the maximum among all the data

$x_{\min}$ = the minimum among all data.

Statistical normalisation will convert the input data based on its mean and standard deviation using the following formula:

$$x_{\mathrm{i}} = \frac{x_0 - x_{(\mathrm{mean})}}{\mathrm{SD}}$$

where:

x_{i} = normalised input

x_0 = each data input

$x_{(\mathrm{mean})}$ = the mean value of all data

SD = standard deviation of all data.

Table 2
GRNN modelling results for the training data set

Indicators	RMSE	MAE	R^2	Optimum Spread
Simple Normalisation	10	4	0.984	0.1
Linear Transformation	10	4	0.984	1
Statistical Normalisation	10	4	0.984	0.7

There are two kinds of input data sets in neural networks: the training data set and the testing data set. The training data set is used to calibrate the model parameters, while the testing data set is used to evaluate the forecasting ability of the model. For the purpose of this study, out of a total 440 vectors, which cover all the origins and destinations in the City of Mandurah, 90% (400 input vectors) were used for training and 10% were used for testing. The training data set was selected randomly, and because it contained 90% of the data, it would cover a wide range of work trip conditions in Mandurah. The remaining 41 vectors were checked to ensure that they also covered a different range of work trips (a few to a large number of trips between different pairs of zones). The process of random data selection for training and checking the testing data set was repeated a few times to insure that the testing data set represents a good sample of different trip conditions.

The testing data set was hold-out and was not used in the training process. This set of training data was used for BP and gravity modelling as well, to compare the results for one set of testing data. The RMSE, MAE and R^2 between the modelled output and measures of the training and testing data set were used to demonstrate the performance of the model according to the following equations:

$$RMSE = \sqrt[2]{\frac{1}{N}\sum_{i=1}^{n}(A_i - T_i)^2}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{n}|A_i - T_i|$$

$$R^2 = \frac{\frac{1}{N}\sum_{i=1}^{n}(A_i - T)^2}{\frac{1}{N}\sum_{i=1}^{n}(T_i - T)^2}$$

where:

N = number of observations

T_i = observed value

A_i = predicted value

T = average value of the explained variable on N observations.

RMSE and MAE provide a general idea of the difference between the observed and predicted values and, therefore, are used as an indication of the residual errors. R^2 is the proportion of variability or sum of squares. When the RMSE and MAE are at a minimum, and R^2 is high ($R^2 > 0.80$), a model can be judged as very good (Kasabov 1998).

The training data set (400 vectors selected randomly) was used for training by the GRNN model and with different spread factors. *Table 2* summarises the modelling results for the training data set.

Analysis indicates that the GRNN model can produce the same results for different normalisation methods with different optimum spread factors as indicated in *Table 2*. Therefore, for simplicity, the simple normalisation method has been used for the testing data set. *Figure 4* illustrates the modelled T_{ij} through the training process against the observed data. The R^2 of 0.984 was obtained from the training process, which shows how well the network is trained.

The trained GRNN model was then used to test the 41 unused vectors. *Table 3* summarises the modelling results for the testing data set. *Table 3* indicates that the average RMSE for the tested data was 38.

BP model architecture

The input and output vectors to the BP model were kept the same as for the GRNN model. The standard network used for this study was a two-layer feed-forward network, with a sigmoid transfer function in the hidden layer and a linear transfer function in the output layer. The number of hidden neurons was set to 10. The training algorithm was back-propagation based on a Levenberg-Marquardt minimisation method.

The initial set of data was divided into three subsets: training, validation and testing. For the purpose of

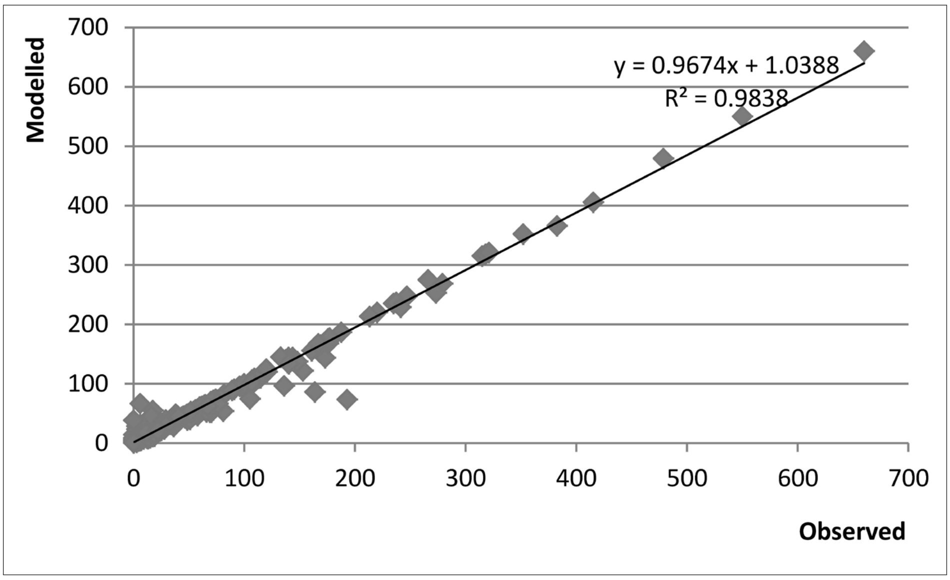

Figure 4
Modelled T_{ij} through the training process against the observed data

Table 3
GRNN modelling results for the testing data set

Indicators	RMSE	MAE	R^2
Simple Normalisation	38	22	0.575

Table 4
Performance of the BP model for different seeds

	Training data		Validation data		Testing data	
Seeds	RMSE	R^2	RMSE	R^2	RMSE	R^2
1	47	0.74	54	0.50	52	0.46
2	73	0.17	84	0.20	72	0.02
3	49	0.62	60	0.62	53	0.45
4	50	0.67	65	0.46	46	0.47
5	51	0.62	46	0.45	39	0.46
6	52	0.59	60	0.55	56	0.35
7	43	0.76	47	0.59	64	0.38
8	60	0.46	61	0.34	49	0.32
9	45	0.77	57	0.42	64	0.48
10	45	0.74	46	0.48	72	0.37

validation, 15% of the total 400 training data set was selected randomly. The testing data set was similar to the data set used in GRNN modelling. The testing vectors were not used in the training or validation process. Simple data normalisation was used for the input vectors. The BP network was trained with 10 different seeds and the performance of the training, validation and testing data sets is reported in *Table 4*.

According to the analysis of the different seeds, the reported R^2 for the training data set was between 0.17 and 0.77. The highest R^2 recorded was 0.77 for seed number 9. The corresponding R^2 for the validation and testing data was reported as 0.42 and 0.48. *Table 4* indicates that only in one case (seed number 2) was the BP model not well trained (i.e. very poor correlation for training), and subsequently produced poor validation and testing results.

Table 5
Performance of the BP model for different hidden layers

Hidden layers	Training data		Validation data		Testing data	
	RMSE	R^2	RMSE	R^2	RMSE	R^2
5	50	0.62	39	0.40	49	0.47
10	45	0.77	57	0.42	64	0.48
15	65	0.41	73	0.22	45	0.40
20	59	0.56	62	0.29	94	0.25

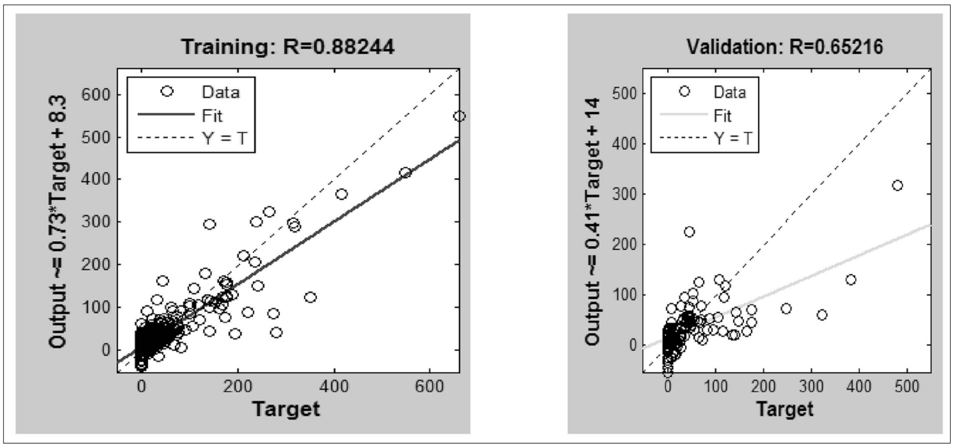

Figure 5
Reported R^2 for training and validation data sets for Seed Number 9 and 10 layers

The best training results are related to seed number 9 with an R^2 of 0.77 and RMSE of 45. Accordingly, better validation and testing results are also produced by seed number 9. The reported R^2 and RMSE for seed 9 are 0.48 and 64, respectively.

In order to investigate the impact of the different number of hidden layers on the performance of the BP model, different hidden layers were tested, with the performance of the model being reported in *Table 5* for the various hidden layers.

Table 5 indicates that the best performance is related to the BP network with 10 hidden layers. Increasing the number of hidden layers to 15 or 20 did not improve the performance of the BP model. *Figure 5* illustrates the BP model outputs against the actual trip distributions for the training and validation data sets for the preferred BP model structure with 10 hidden layers.

Gravity model structure

The strategic transport model for the Mandurah area is based on the traditional four-stage model process (trip generation, trip distribution, mode split and traffic assignment); however, the trip generation within this model considered only private vehicle trips and, therefore, the mode split stage was not adopted. The mode split was taken into consideration when generating the trip production rates for the trip generation stage (Rasouli 2012). For the purpose of this study, the trips were divided into five different categories based on trip purpose: work, education, social, other and non-home based (NHB) trips. Trip distribution of the model was based on the doubly-constrained gravity model in the EMME software. The following gamma function was used to reflect deterrence in the gravity model:

$$W_{ij} = a * d_{ij}{}^{b*\exp(-c*d_{ij})}$$

where:

W_{ij} = weight between zone i and zone j

d_{ij} = distance between zone i and zone j.

Parameters a, b and c were calibrated for each trip purpose so that the model reflected the proportion of trips for each length, as observed in the travel surveys. Assignment of the trips was based on the fixed demand traffic assignment module in the EMME software. Calibration of the model was based on the existing traffic volumes on the road links. The actual traffic data was provided by the City of Mandurah. *Figure 6* shows the modelled traffic volumes against the actual traffic counts. The R^2 for

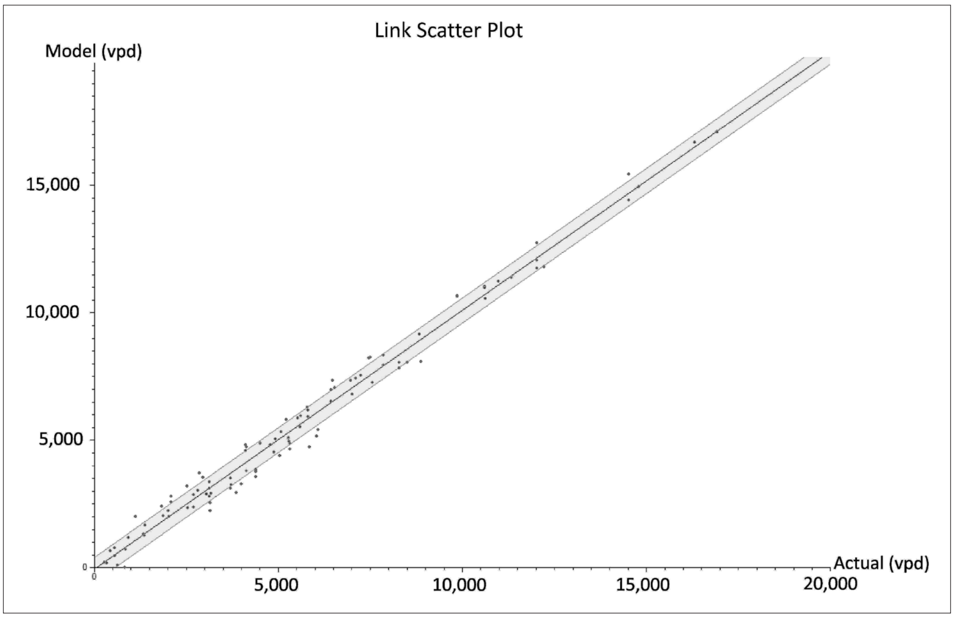

Figure 6
Regression plot, calibration of the base case (2011)

Table 6
Gravity modelling results

Indicators	RMSE	MAE	R^2
Gravity Model	50	23	0.59

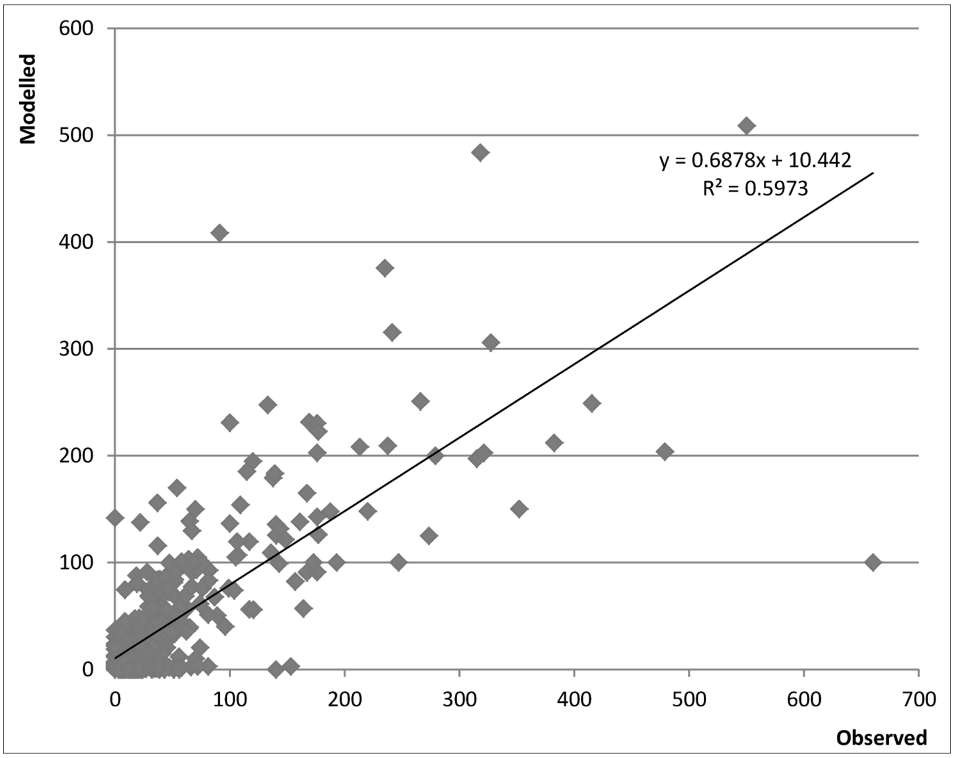

Figure 7
Observed and modelled work trips based on the gravity model

the 107 traffic count locations is 0.985, which shows how well the model is calibrated (Rasouli 2012).

The JTW origin-destination (OD) matrix was extracted from the Mandurah strategic transport model and compared with the 2011 JTW OD matrix obtained from the ABS data. The extracted OD matrix for JTW from the gravity model was compared with the OD matrix from the ABS data. *Table 6* summarises the modelling results for the gravity model.

Figure 7 illustrates the comparison between the trip distribution (T_{ij}) extracted from the gravity

Table 7
Gravity modelling results for the testing data set

Indicators	RMSE	MAE	R^2
Gravity Model	46	31	0.446

Table 8
GRNN, BP and Gravity modelling results for the testing data set

Indicators	RMSE	MAE	R^2	Regression parameter
GRNN Model	38	22	0.575	0.51
Gravity Model	46	31	0.446	0.63
BP Model	64	31	0.48	1.08

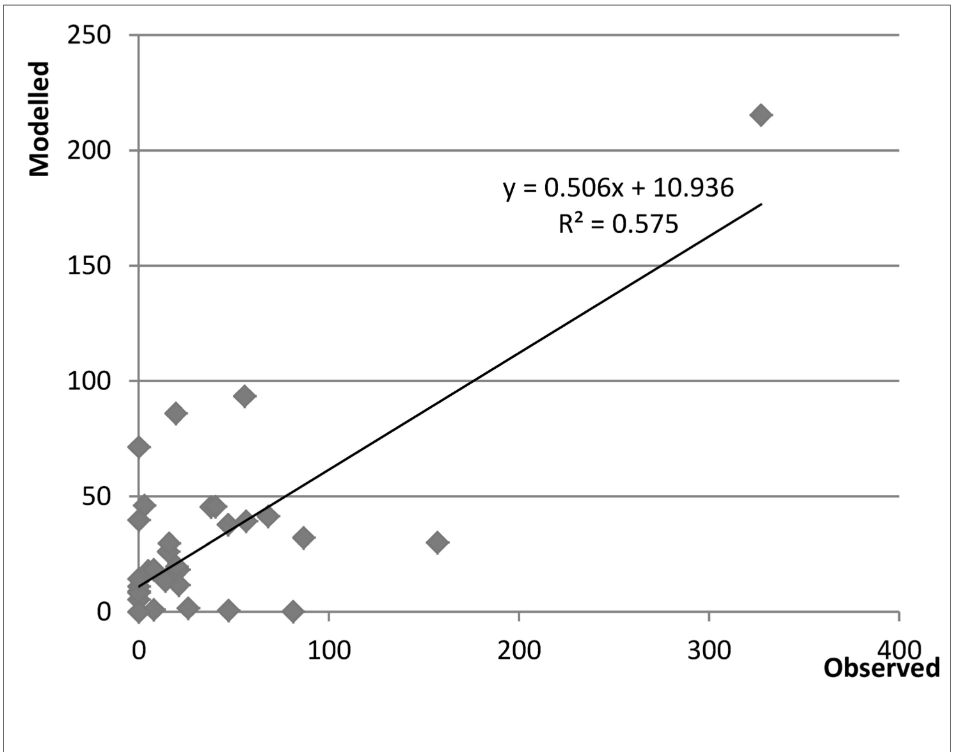

Figure 8
Modelled and observed T_{ij} for the testing data, GRNN model

model and the ABS data. The R^2 is reported as 0.59. According to the analysis undertaken, the average RMSE of the modelled trips is estimated to be 51.

The gravity model developed for the Mandurah area was then used to estimate the trip distribution of the testing data set used in the GRNN and BP models. *Table 7* summarises the modelling results for the testing data set.

COMPARISON OF MODELS

In order to compare the performance of the GRNN, BP and gravity models, the tested data set was used to estimate the trip distribution based on the various models. The RMSE, MAE and R^2 indicators were calculated for each model and are compared in *Table 8*.

Table 8 indicates that the GRNN model provides slightly better results than the BP and gravity models for all the performance indicators. However, the regression parameter value for the GRNN model is lower than that for the BP and gravity models, which means that the GRNN model would underestimate the observed value.

The R^2 of the BP model is slightly higher than that of the gravity model, while the reported RMSE for the BP model is higher than for the gravity model. The MAE for both the BP and gravity model is reported as 31. Therefore, it is expected that the BP

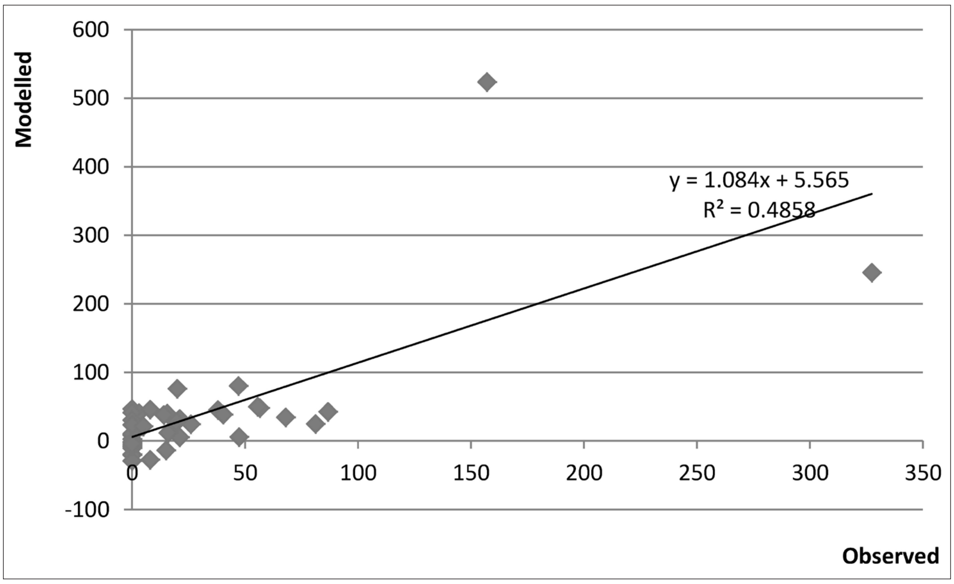

Figure 9
Modelled and observed T_{ij} for the testing data, BP model

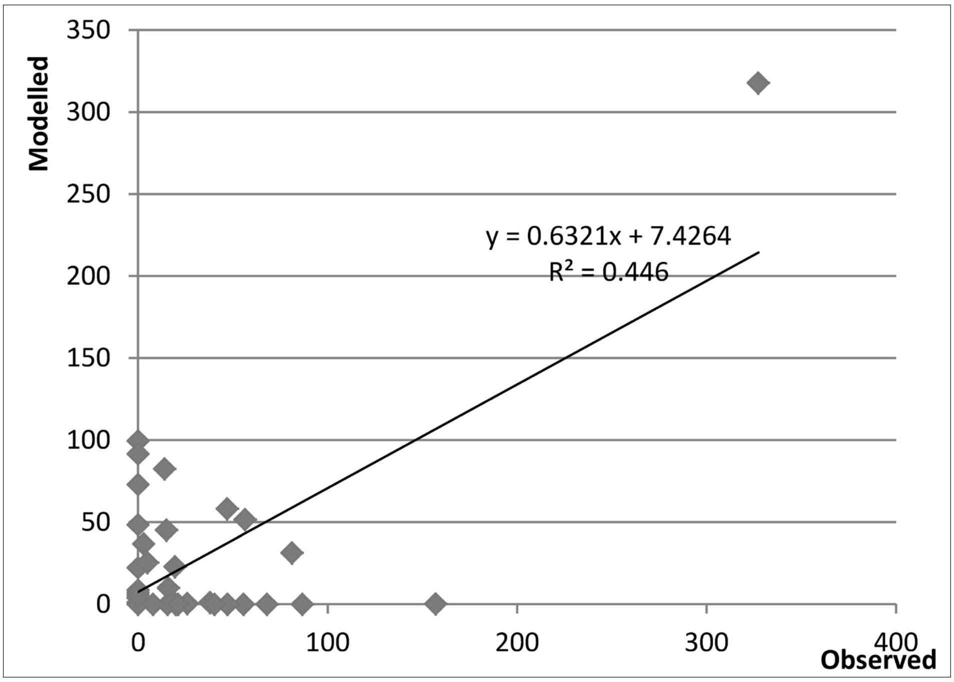

Figure 10
Modelled and observed T_{ij} for the testing data, Gravity model

and gravity models will perform the same. The BP model provides the closest regression parameter (x parameter) to 1, indicating that the modelled values match the observed values over the range of data.

Figures 8, *9* and *10* illustrate the modelled and observed T_{ij} for the testing data set for the GRNN, BP and gravity models, respectively. The distribution of points in these figures indicates that the majority of the points are clustered at low values, with one or two at much higher levels, which represent the variety of the work trip conditions in Mandurah. Therefore, the regression parameters (and thus the level of bias) are strongly dependent on these points.

CONCLUSION AND RECOMMENDATIONS

In this paper, a generalised regression neural network (GRNN) model was developed as a new approach, and the performance of this model was compared with the back-propagation and gravity models. The modelling and analysis undertaken indicate that:

- The neural network (NN) models can be used to forecast trip distribution directly from the land use data for each pair of traffic zones, instead of production and attraction for each pair of zones.
- The modelling results indicated that a validated GRNN model could provide a slightly lower

error level than the BP and gravity models, as indicated by the average root mean square error (RMSE); however, it might underestimate the observed values compared with the BP and gravity models.

Despite the efforts devoted to analysing all of the approaches discussed in this paper, there are major areas that still need to be researched. The following recommendations are put forward for future studies:

- The GRNN outputs rely heavily on the amount of data available and the variety of the training data set vectors. The greater the number of input vectors in the training data set, the more accurate the results in the output vector. Therefore, it is recommended that the efficiency of the GRNN model be tested and improved with a larger data set if available.
- The GRNN model needs to be tested with trip generation, trip attraction, and the distance between pairs of zones as inputs to the model, instead of the land use data, and be compared with the gravity and BP models.

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Australasian Transport Research Forum 2013 Proceedings
2 - 4 October 2013, Brisbane, Australia

Trip Distribution Modelling Using Neural Network

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ABSTRACT

Trip distribution is the second important stage in the 4-step travel demand forecasting. The purpose of the trip distribution forecasting is to estimates the trip linkages or interactions between traffic zones for trip makers. The problem of trip distribution is of non-linear nature and Neural Networks (NN) are well suited for addressing the non-linear problems. This fact supports the use of artificial neural networks for trip distribution problem. In this study a new approach based on the Generalised Regression Neural Network (GRNN) has been researched to estimate the distribution of the journey to work trips. The advantage of GRNN models among other feed-forward or feedback neural network techniques is the simplicity and practicality of these models. As a case study the model was applied to the journey to work trips in City of Mandurah in WA. Keeping in view the gravity model, the GRNN model structure has been developed. The inputs for the GRNN model are kept same as that of the gravity model. Accordingly the inputs to the GRNN model is in the form of a vector consist of land use data for the origin and destination zones and the corresponding distance between the zones. The previous studies generally used trip generations and attractions as the inputs to the NN model while this study tried to estimate the trip distribution based on the land uses. For the purpose of comparison, gravity model was used as the traditional method of trip distribution. The modelling analysis indicated that the GRNN modelling could provide slightly better results than the Gravity model with higher correlation coefficient and less root mean square error and could be improved if the size of the training data set is increased.

Keywords:
Trip Distribution, Neural Network, Generalised Regression Neural Network, Gravity Model.

1 INTRODUCTION

Conventional transport modelling, known as 4-step modelling is highly depending on the input data used in different modelling steps. The trip distribution process is relatively complex in nature and difficult to model without adequate amounts of data. Errors that are generated during the trip distribution stage, distribute through the other stages of modelling which in turn affects the reliability of the modelling results. Therefore it is important to ensure that the trip distribution techniques are able to estimate accurate results.

A robust and efficient technique to estimate the trip distribution is always an essential part of the modelling process. There is no technique in trip distribution that is universally applicable, so attempts to develop alternative techniques are always needed. This includes the utilisation of approaches from other disciplines. Neural Networks are one of them and are proposed as an alternative method in this study. The problem of trip distribution is of non-linear nature and complex. Neural networks have been used successfully for solving the non-linear problems. This fact supports the use of artificial neural networks for trip distribution problem.

Since the beginning of nineties, neural network models were introduced as alternatives for traditional modelling approaches. The previous studies suggest that the NN approach is able to model the commodity, migration and work trip flows. However, its performance is not as good as the well-known gravity model. According to the literature review, the majority of the previous studies utilised the standard Back Propagation (BP) algorithm and there have not been enough attempts to utilise the GRNN approach. The knowledge required to develop the GRNN structure is relatively small and can be done without additional input by the user. This makes GRNN a very powerful tool in practice. This research aims to apply the GRNN model to test the ability of the neural network in prediction of the trip distribution problem. One of the differences in this approach with the previous studies is the use of land use data as an input to the NN model instead of using the trip generation and attraction. There is direct relation between the land use data and trip distribution between different land uses in a modeled area. Sometimes estimation of trip productions and attractions from the land use data involves simplistic assumptions that generate errors in the trip production and attraction stage. This error would distribute to the other stages of the modeling process including trip distribution stage which in turn affects the reliability of the modeling results. Therefore estimation of the trip distribution directly from the land use data would remove the errors related to the trip production and attraction stage. This study also compares the GRNN approach with the gravity model and documents the outcomes of this comparison.

2 BACKGROUND

The use of NN is growing fast and covers many disciplines, including transport modelling. The literature indicates that NN were used in some 13 areas of transport modelling studies up to year 1990 where driver behaviour simulation models had the highest usage of NN applications (Dougherty, 1995). However, more recent research indicates a growing application of NN in travel demand modelling, mostly by Mode Choice and Trip Distribution problems.

It must be noted that the NN approach must be followed by logic and sensible theory, otherwise NN is just a naive tool. According to Black (1995), NN is an intelligent computer system that simulates the processing capabilities of the human brain. It is a forecasting method that generates output by minimizing an error calculated by the deviation between input and output through the use of a complex training process (Black, 1995; Zhang et al, 1998).

Various studies in transportation modelling prove the advantages and disadvantages of using NN. It is usually compared with the existing methods in relevant studies. For example, the neural network has been compared with the Discrete Choice Model as reported by Cantarella & de Luca (2005), Hensher & Ton (2000), Carvalho et al. (1998), and Subba Rao et al. (1998). Reviewing the literature indicates that there is less application of NN in trip distribution problem compared to mode choice studies. Black (1995) investigated the spatial interaction modelling using NN focusing on commodity flows. This model was structured similarly to the gravity model. Mozolin et al. (2000) utilised NN to model trip distribution for passenger flow modelling. The studies by Black and Mozolin et al. were based on multilayer perceptron neural networks.

NN is recognised by its important characters, such as learning algorithm, activation function, number of layers (input, hidden and output), number of nodes inside each layer, and learning rate (Teodorovic and Vukadinovic, 1998, Dougherty, 1995). The amount of data and the split of the data which is used for training, validating and testing purpose are also essential for NN performance (Carvalho et al., 1998). Zhang et al. (1998) suggested that if there is not any appropriate guideline then NN model can only be developed through trial and error procedures. There is also a lack of reported researches on the behaviour of NN with respect to these properties. Lack of knowledge in structuring the main properties of NN could lead to disadvantages in using NN models, for example if the modeller is not able to enforce the network to simulate according to the existing constraints. This problem has happened in the study by Mozolin et al (2000). They reported that NN was not able to meet the double constraints and they provided adjustment factors for the output of the NN model so that the model satisfied the Production and Attraction constraints. They also reported that NN had slightly poor generalization capability. Although this was not comprehensively reported, Black (1995) provided a small report about this issue in commodity flow estimation using NN. It was not clearly reported if the model can properly satisfy the constraints.

Accordingly a number of different studies were undertaken to improve the ability of the NN to satisfy the production and attraction constrains. Gusri Yaldi, M A P Taylor and Wen Long Yue (2009) reported that a NN with simple data normalization and a linear activation function (Purelin) in the output layer could satisfy the two constraints, with average correlation coefficients (r) of 0.958 and 0.997 for Production and Attraction respectively. The test results of their research also proved that a validated NN could generate a similar goodness of fit as a doubly-constrained gravity model. However, the error level is still more than the gravity model as indicated by the average Root Mean Square Error (RMSE), where the RMSE for the NN and gravity model are reported 181 and 174 respectively.

In another research they tried to fix the testing performance of NN by training the models with the Levenberg-Marquardt (LM) algorithm, while the previous studies used standard Back propagation (BP), Quickprop and Variable Learning Rate (VLR) algorithms. The main difference between those algorithms is the method used in defining the optimum connection weights. The research results suggest that the RMSE are 168, 152 and 125 for model trained with BP, VLR and LM respectively, while the R^2 values are 0.194 0.315, 0.505. The models trained by BP and VLR have underestimated the forecasted total trip numbers, while the LM algorithm has slightly higher numbers. The research concluded that the testing performance of NN approach can be improved to the same level as doubly constrained gravity model when the model is trained by LM algorithm.

Fischer and Leung (1998) developed different models of NN by the use of different learning algorithms, and in conjunction with Genetic Algorithm (GA), to forecast traffic flows in a region in Australia. They found that GA can improve the NN modelling results.

3 A BRIEF DESCRIBTION OF NEURAL NETWORK

Neural Network is an artificial intelligence method that simulate the operation of the human brain (nerves and neurons), and consist of number of interconnected computer processors that perform simultaneously in parallel. NN was founded by McCulloch and co-workers in the early 1940s (Haque ME, Sudhakar KV, 2002). They developed simple neural networks to model simple logic functions.

Nowadays, neural networks are used for problems that do not have algorithmic solutions or problems that algorithmic solutions are too complex to be developed. In other words, it is not easy to establish a mathematical model for problems that with no clear relationship between inputs and outputs. To solve this sort of problems, NN uses the samples and will be trained to learn the relationship of such systems. The ability of NN to learn by samples makes them very flexible and powerful. Therefore, neural networks have been largely used for mapping regression and classification problems in many disciplines. In short, neural networks are nonlinear algorithms that perform learning and classification.

In general, neural networks are adjusted/ trained to reach from a particular input to a desired output. Therefore the neural network can learn the system. This type of learning is called supervised learning. The learning ability of a neural network depends on its structure and the training algorithm. Training algorithm can be stopped if the difference between the network output and actual output is less than a certain tolerance value. When the NN was learned, the network is then ready to estimate outputs based on the new inputs that are not used in the training data set. A neural network is usually consisting of three parts: the input layer, the hidden layer and the output layer. The information saved in the input layer is transferred to the output layers through the hidden layers. Each unit can transfer its output to the units on the higher layer only and receive its input from the lower layer.

3. 1 Generalised Regression Neural network

The Generalised Regression Neural Network (GRNN) is a feed-forward network. The use of a GRNN is especially helpful because it has the ability to converge to the desired outcome with only few training data available. The additional knowledge required to train the network and develop the NN structure is relatively small and can be done without additional input by the user. This makes GRNN a very powerful tool in practice.

The fundamentals of the GRNN can be found from Specht, (1991); Nadaraya–Watson kernel regression (1964), Tsoukalas and Uhrig (1997), also Schioler and Hartmann (1999). A schematic structure of the GRNN is illustrated in **figure 1**. A GRNN does not require an iterative training procedure. It can estimate any non-linear function between input and output vectors, learning the relationship between the input and output data directly from the training data. Furthermore, it is found that if the training set size becomes large, the estimation error approaches zero, with minimum restrictions on the function. The GRNN is used to predict the continuous variables as in standard regression methods.

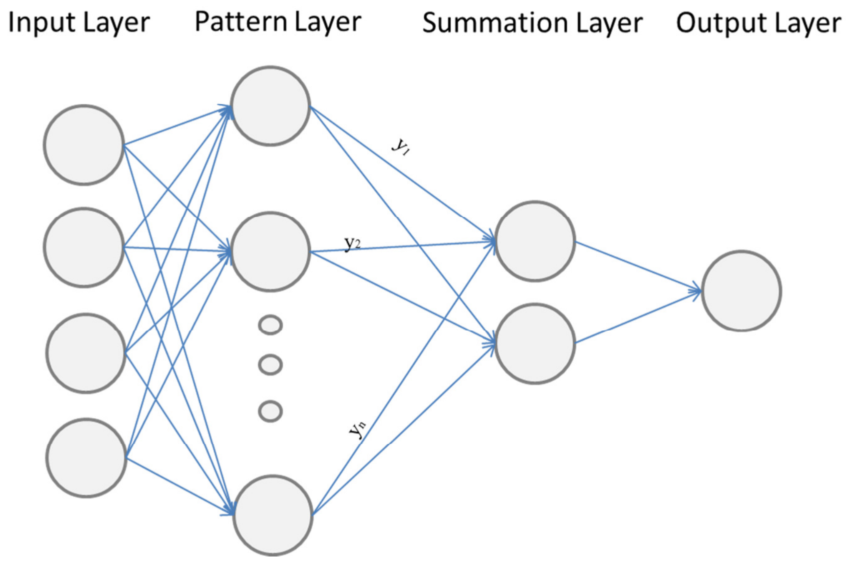

Figure 1: Schematic structure of GRNN

The GRNN consist of four layers: Input layer, pattern layer, summation layer, and output layer. The total number of parameters is identical to the number of input units in the input layer. The first layer is connected to the second, pattern layer. In pattern layer, each unit represents a training pattern, and its output calculates the distance between the input and the stored patterns. Each pattern layer unit is joined to the two neurons in the summation layer: S- summation neuron and D- summation neuron. Here, the sum of the weighted outputs of the pattern layer is measured by the summation and the un-weighted output of the pattern neurons is calculated by the D-summation. The linkage weight between the S-summation neuron and the *i*th neuron in the pattern layer is called y_i ; the target output value joint to the *i*th input pattern. The output layer just splits the output of each S-summation neuron by the output of each D-summation neuron, providing the predicted value to an unknown input vector x as:

$$yi(x) = \frac{\sum_{i=1}^{n} yi\ exp[-D(x, xi)]}{\sum_{i=1}^{n} exp[-D(x, xi)]}$$

In which the number of training patterns is specified by n and the Gaussian D function is calculated as:

$$D(x, xi) = \sum_{j=1}^{p} (\frac{xj - xij}{\delta})^2$$

In which p represents the number of element of an input vector. The x_j and x_{ij} show the *j*th element of x and x_i, respectively. The δ is generally known as the spread factor, whose optimal value is often calculated experimentally for the problems. If the spread factor becomes larger, the function approximation will be smoother. If spread factor is too large, then a lot of neurons will involve fitting a fast changing function. If the spread factor is small then many neurons will be required to fit a smooth function, and the network may not generalize well.

4 MODEL DEVELOPMENT AND METHODOLOGY

The model development and methodology is illustrated in **Figure 2** and is described in the following sections.

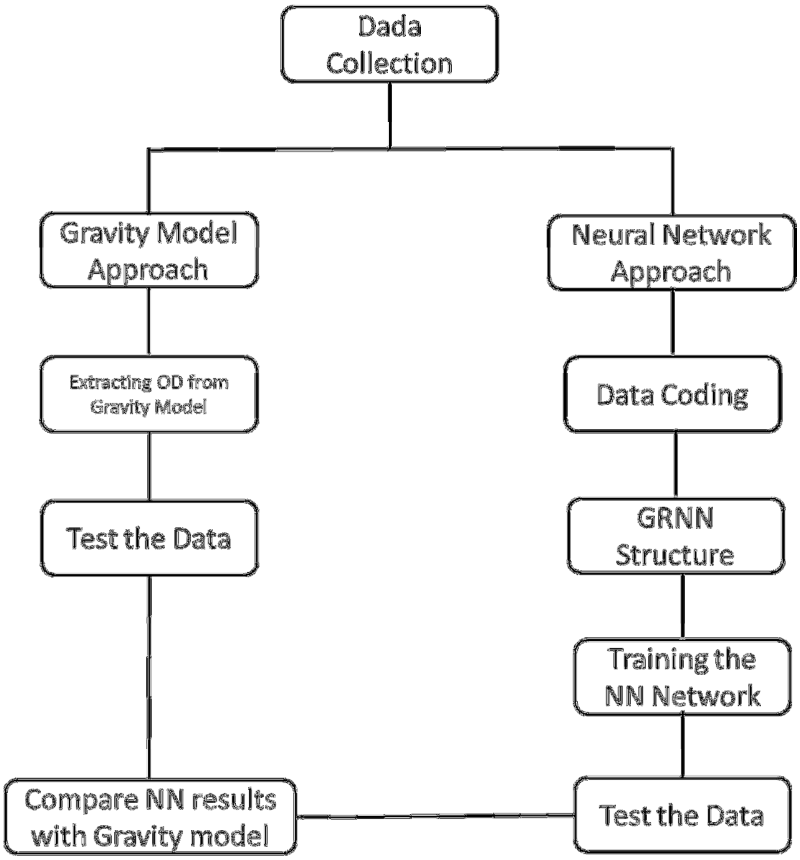

Figure 2: Model Development and Methodology

5 DATA COLLECTION

The 2006 Journey to Work dataset for the Mandurah Area in Perth WA was sourced from Australian Bureau of Statistics (ABS). Journey to Work (JTW) data are extracted from the five-yearly Census of Population and Housing conducted by the Australian Bureau of Statistics. It includes data on employment by industry and occupation, and method of travel to work at a small geographical level known as the travel zone.

At the time of preparation of this paper the 2011 JTW data was not available and therefore the 2006 JTW data was used. Considering that the strategic transport model for Mandurah area was developed and calibrated for year 2011, then the 2011 JTW data was estimated from the 2006 data assuming the same travel pattern for the JTW in 2006.

6 O-D MATRIX ESTIMATION USING GRAVITY MODEL

6. 1 Mandurah strategic transport model

Due to significant growth in recent years and anticipated future growth the City of Mandurah is faced with a number of challenges with planning and managing its movement network and transport system particularly within the City Centre. The City has ambitious plans for the future to deliver an attractive, dynamic and vibrant City. These plans will generate significant transport demand which will put pressure on the existing transport infrastructure and systems, particularly the road network within the City Centre.

In order to assist with its decision-making process, the City has engaged Transcore Pty Ltd to develop a strategic transport model for the greater Mandurah area. The strategic transport model will assist the City in establishing the future transport demand and test the impact of land use growth, major developments and road network options.

The modelled study area entails the Inner Peel Region including Mandurah, Pinjarra and Yunderup. The number of residential dwellings for the City of Mandurah was calculated for the 38 individual modelling zones as per **Figure 3**. According to the Australian Bureau of Statistics census results for 2011 the total number of dwellings in Mandurah is estimated to be about 35,372 with about 69,903 people residing in the municipality.

Zones	Dwellings
1	294
2	411
3	331
4	597
5	665
6	777
7	612
8	1341
9	1044
10	1339
11	1165
12	2323
13	1035
14	985
15	806
16	1592
17	495
18	622
19	669
20	522
21	457
22	270
23	675
24	1534
25	1021
26	1212
27	1136
28	785
29	751
30	1321
31	1381
32	1706
33	1095
34	1079
35	1341
36	851
37	200
38	933
Total	35372

Figure 3: Mandurah Model Area and Zoning System

6. 2 Model Structure

The traffic model is based on the traditional four-stage model process (trip generation, trip distribution, mode split and traffic assignment) however, the trip generation within this model considered only private vehicle trips and therefore the mode split stage was not adopted. The mode split was taken into consideration when generating the trip production rates for the trip generation stage. For the purpose of this study the trips were divided into 5 different categories based on the trip purposes: Work, Education, Social, Other and Non Home Based (NHB) trips.

Trips internal to the modelling area have been distributed based on the following gamma function:

$$Wij = a * dij^{b*\exp(-c*dij)}$$

where:

w_{ij} : weight between zone i and zone j

d_{ij} : distance between zone i and zone j

Parameters a, b and c were calibrated for each trip purpose so that the model reflects the proportion of trips for each length as observed in the travel surveys. Assignment of the trips was based on the fixed demand traffic assignment module in EMME software.

Calibration of the model was based on the existing traffic volumes on the road links. The actual traffic data was provided by City of Mandurah. **Figure 4** shows the modelled traffic volumes against the actual traffic counts. The linear regression analysis for the 107 traffic count locations indicates that R^2 of the regression plot is 0.985 which shows how well the model is calibrated.

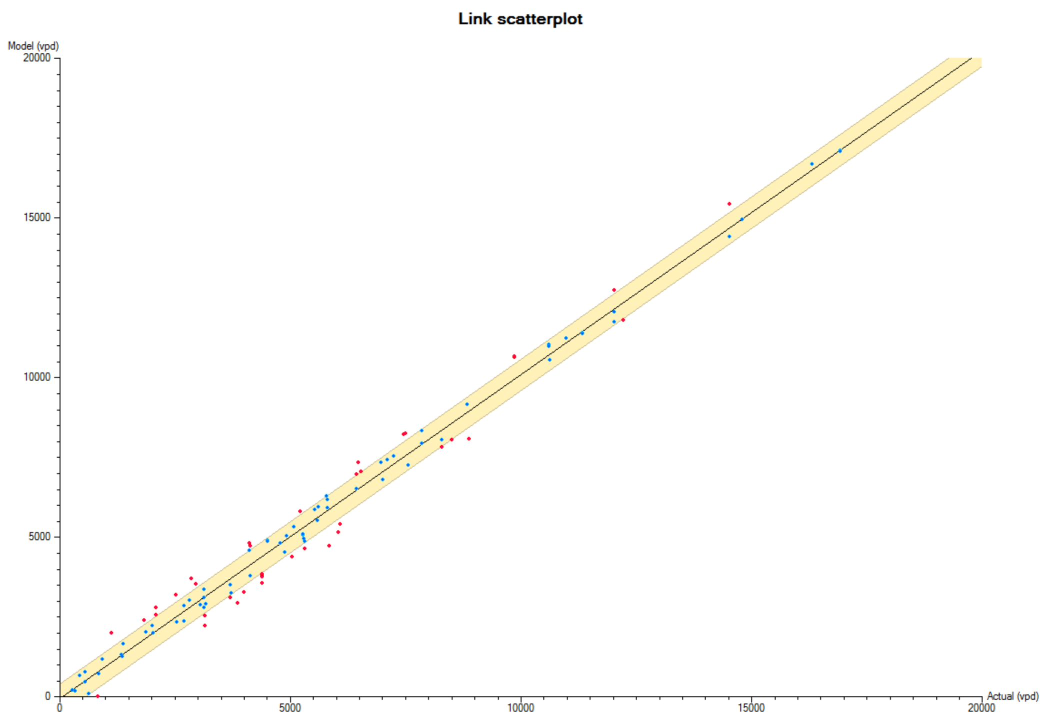

Figure 4: Regression Plot, Calibration of the Base Case (2011)

6. 3 Extracting and comparing the journey to work OD matrix from Gravity Model

The journey to work OD matrix was extracted from the Mandurah strategic transport model and compared with the 2011 JTW OD matrix obtained from the ABS data. The R^2 for the trend line in Figure 5 is 0.59. According to the analysis undertaken the average Root Mean Square Error (RMSE) of the modelled trips were estimated to be 51.

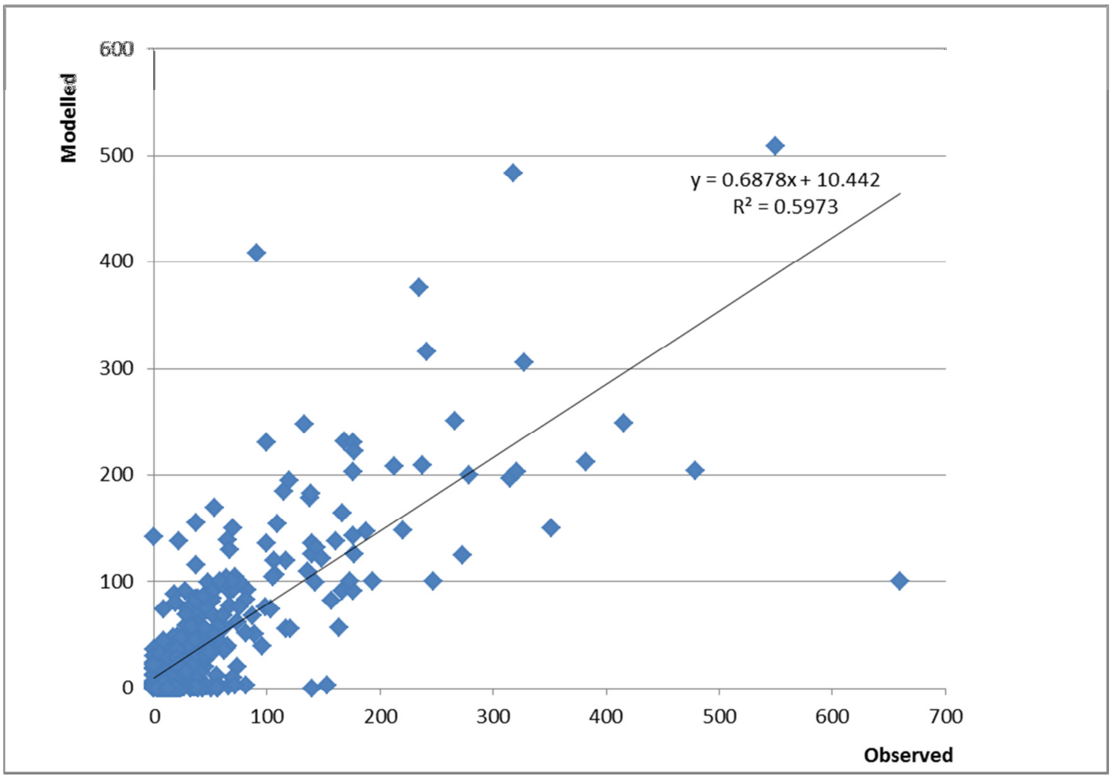

Figure 5: Observed and Modelled work Trips Base on Gravity Model

7 O-D MATRIX ESTIMATION USING NEURAL NETWORK

7. 1 Neural Network Model Architecture

People's activities can be represented by land uses scattered on different zones that are separated by distance in an area. Therefore, trip distribution relates to the land use patterns in different zones inside that area. For instance, one zone which is typically occupied by residential land use patterns generates trips that are attracted to another zone which is formed by retail, industrial, commercial, etc.

On this basis the input layer of the neural network is represented by land use data in each zone, which is assigned to RD (Residential Dwellings), RE (Retails), CO (Commercial Land use), SH (showroom) and SC (Schools). In order to represent the spatial distribution of a pair of zones, the distance Dij (meters) between zones i and j is defined. Accordingly the input vector (X) is defined as:

Xij=(RDi, REi, COi, SHi, SCi,,RDj, REj, COj, SHj, SCj, Dij)

Where i and j shows the origin and destination, respectively.

Trips (Tij) between a pair of zones are considered as the output layer of the neural network. The GRNN has to be able to model the relation between trips *Tij* and input vector *X* ij. The model is developed to forecast the work trip. MATLAB R2011a is used to develop the network where the optimum spread factor was selected through try and error process. The model structure used in MATLAB software is illustrated by **Figure 6**. It has 11 input nodes representing the land uses for zone i and zone j, and distance between zone i and j (as defined in the above Xij input vector). There is one node in the output layer which represents the estimated trip number (Tij).

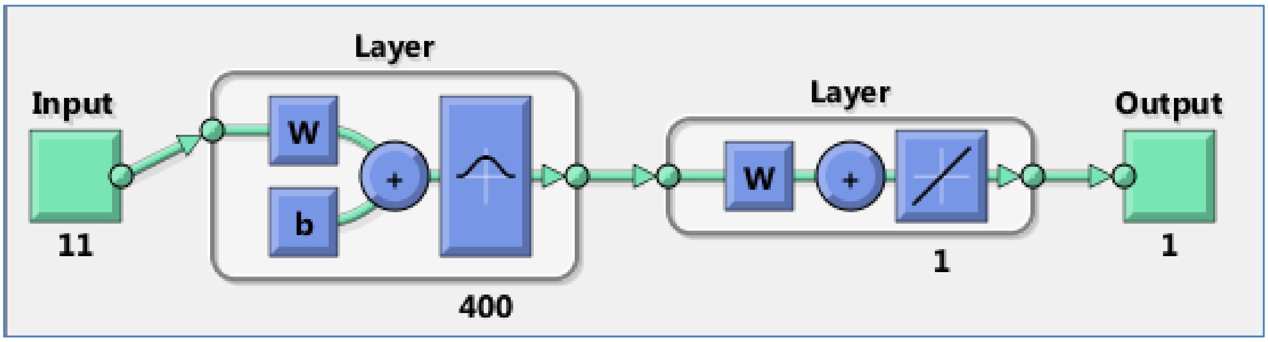

Figure 6: GRNN Model Structure Used in MATLAB Software

Simple data normalization method is used in this study for the input vectors. Simple normalization will convert the input data to the range [0,1].

There are usually two kinds of input data sets in neural networks, namely training and testing data sets. The training data set is used in estimating the model parameters/variables while the testing data set is for evaluating the forecasting ability of the model. For the purpose of this study 90% of the data (400 input vectors) were used for training and 10% were used for testing.

7. 2 GRNN modelling results

The training data set (400 vectors selected randomly) were trained using the GRNN model and with different spread factors. The optimum spread factor of 1 was selected through try and error process. **Figure 7** illustrates the goodness of fit for the trained GRNN model; R^2 of 0.984 was obtained from the training process which shows how well the network is trained.

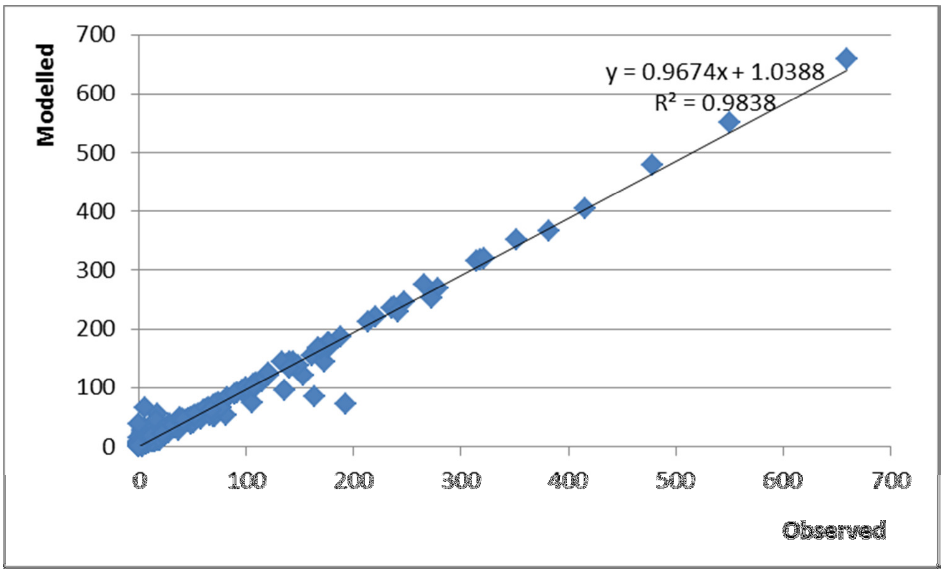

Figure 7, Modeled Tij through the Training Process against the Observed Ones

The trained GRNN model was then used to test the 41 unused vectors. **Figure 8** illustrates the modeled trip distribution against the observed data. The absolute difference (error) is also shown in this figure. The average RMSE for the tested data recorded as 38.

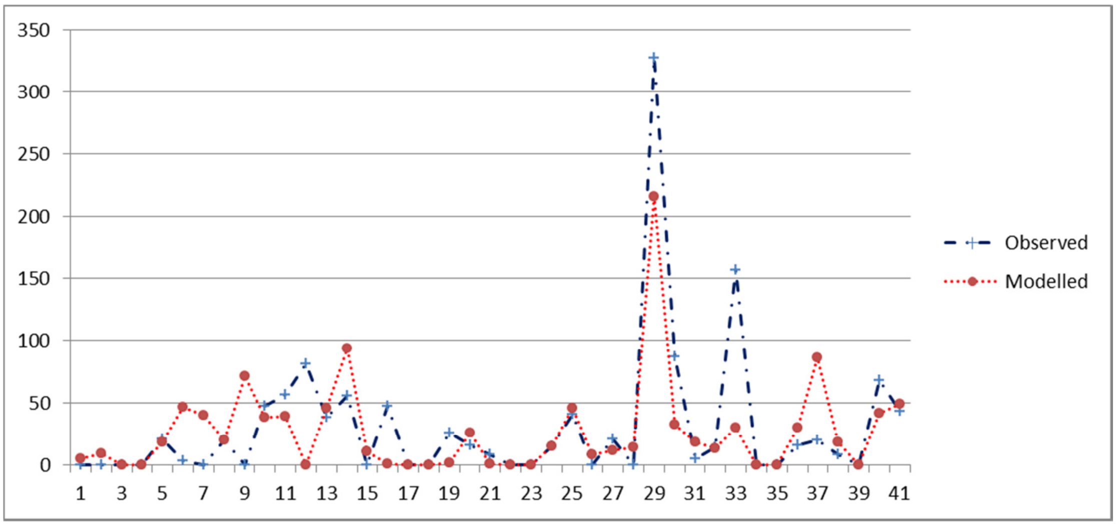

Figure 8, Error Estimation between the GRNN Modeled and Observed data

The R^2 of the tested model is reported as 0.575 as shown in **Figure 8.**

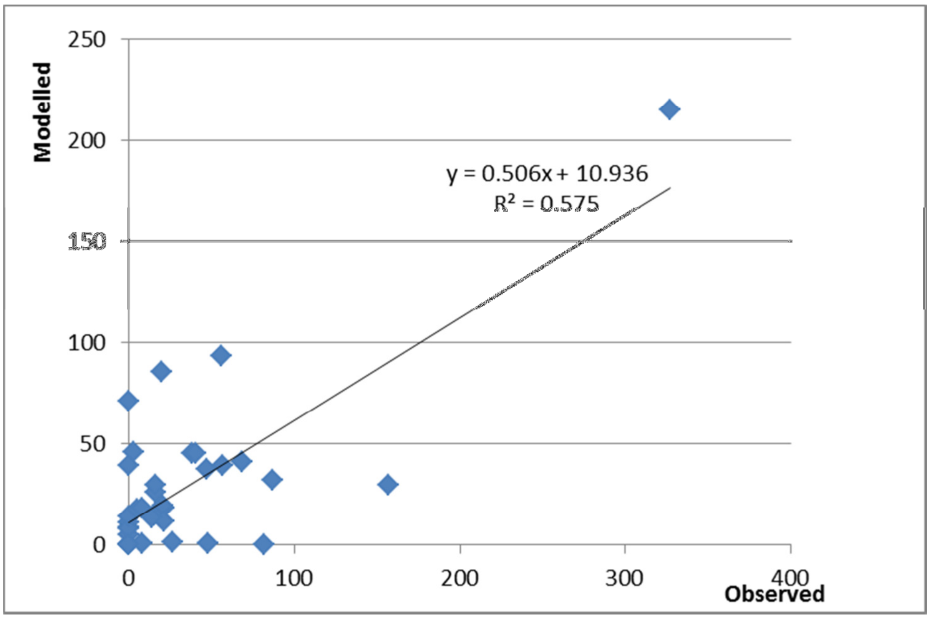

Figure 8, Modeled and Observed Tij for the Testing Data, GRNN Model

The R^2 of the tested data based on the Gravity model is estimated to be 0.446 (refer **Figure 9**) with the corresponding average RMSE of 46.

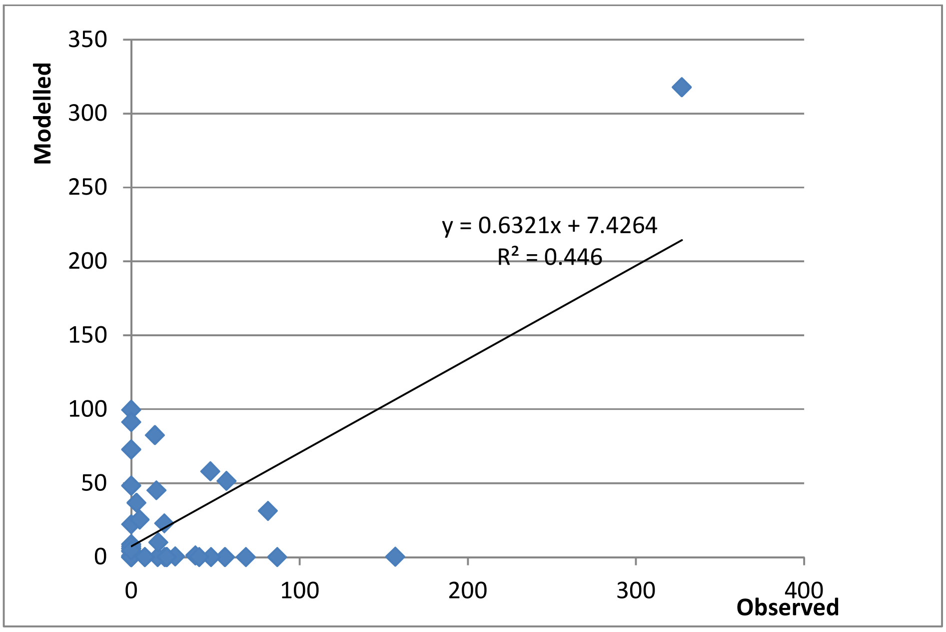

Figure 9, Modeled and Observed Tij for the Testing Data, Gravity Model

8 CONCLUSION AND RECOMMENDATION

Based on the results of the analysis undertaken, it can be concluded that the Neural Network model can be used to forecast trip distribution, especially for work trips. GRNN model could forecast the work trip distribution based on the land use data for each pair of traffic zones and the corresponding distance between the two zones.

The modeling results have also provided evidence that a validated GRNN could provide slightly better goodness of fit than a gravity model with the error level less than the gravity model as indicated by the average Root Mean Square Error (RMSE), where the RMSE for the NN and Gravity Model are 38 and 45 respectively. The estimated R^2 for the GRNN model and gravity model is reported 0.557 and 0.446 respectively.

The GRNN outputs highly rely on the amount of data available and the variety of the training data set vectors. The more the number of input vectors in the training data set the more accurate results in the output vector. Therefore it is recommended that the efficiency of the GRNN model be tested and improved with a bigger data set if available.

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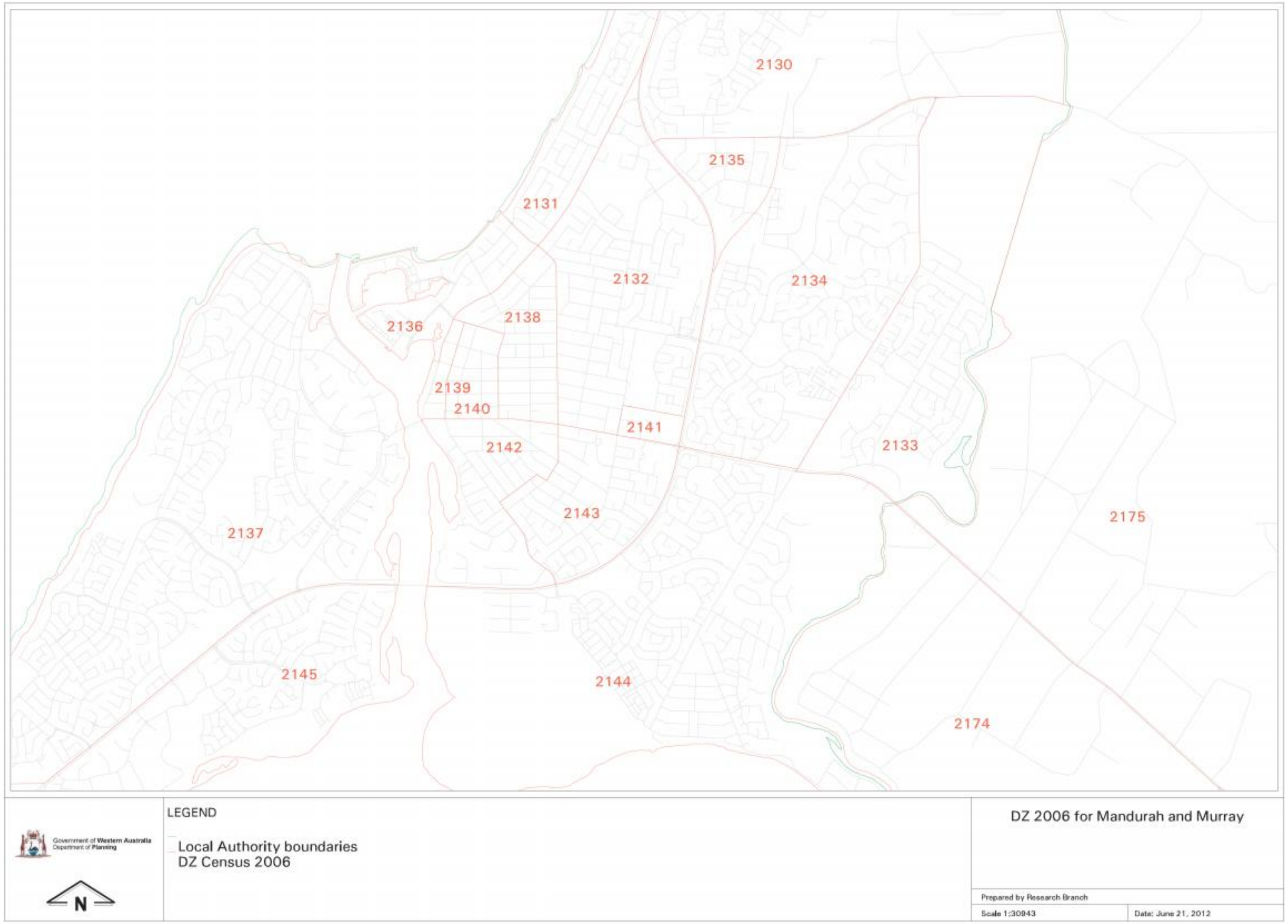
Appendix B:

Destination Zones and OD Matrix for Mandurah and Murray (2006 Census Data)


2130
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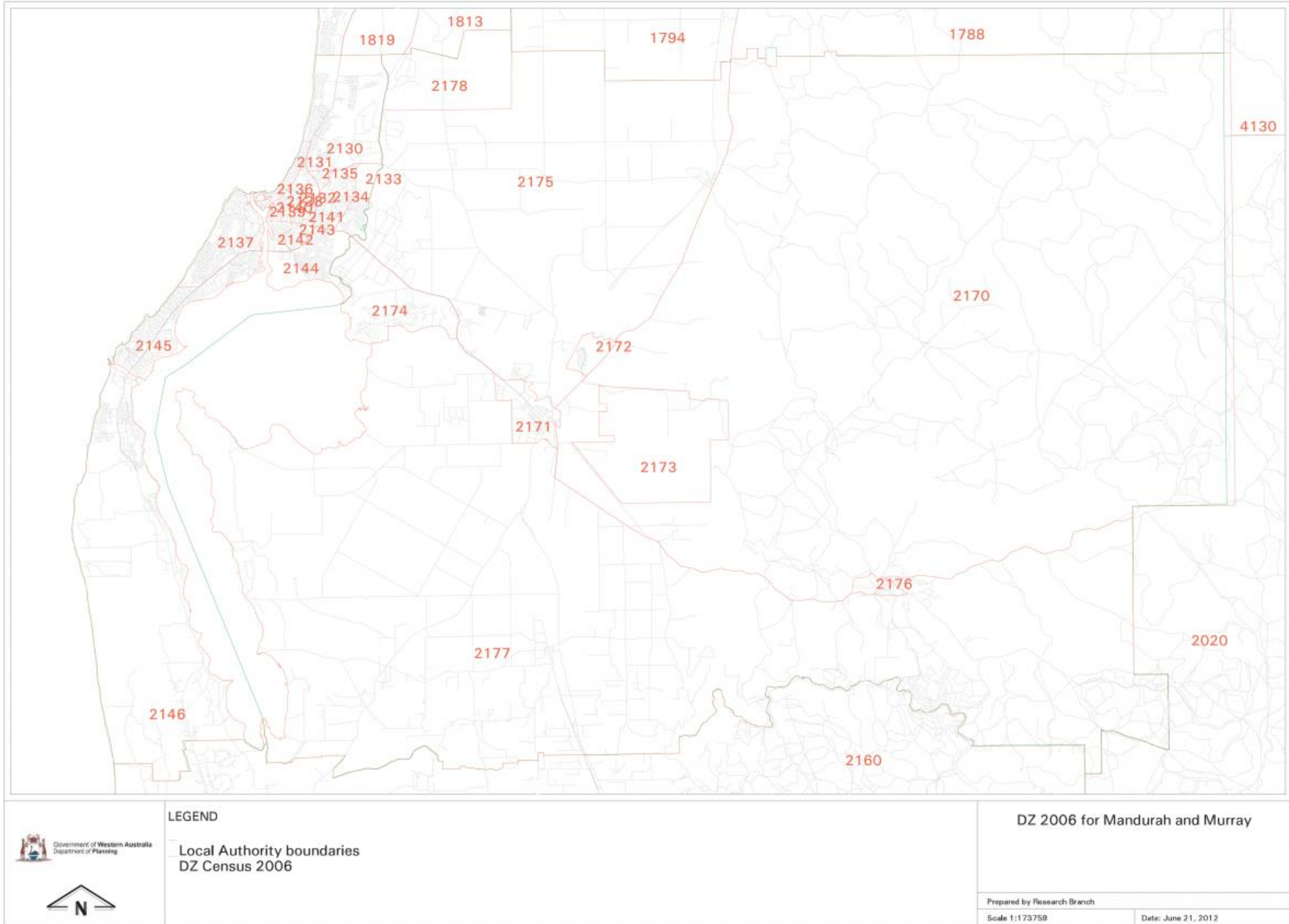
2175
2137
2145
2144
2174
LEGEND
Local Authority boundaries
DZ Census 2006
Government of Western Australia
Department of Planning
N
DZ 2006 for Mandurah and Murray
Prepared by Research Branch
Scale 1:30943
Date: June 21, 2012


1813
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LEGEND
Local Authority boundaries
DZ Census 2006
DZ 2006 for Mandurah and Murray
Government of Western Australia
Department of Planning
N
Prepared by Research Branch
Scale 1:173759
Date: June 21, 2012

ABS Census 2006 JtW

Peel North and Rockingham

DZ_06	SLA	2130	2131	2132	2133	2134	2135	2136	2137	2138	2139	2140	2141	2142	2143	2144	2145	2146	2170	2171	2172	2173	2174	2175	2176	2177	2178
		Mandurah																	Murray								
2130	**5110**	352	3	235	48	37	103	45	43	81	53	82	169	30	18	68	30	8	12	37	0	84	0	20	3	3	0
2131	**5110**	9	38	40	15	7	15	22	9	16	16	8	25	0	9	24	7	0	0	6	0	15	0	0	0	0	0
2132	**5110**	19	0	143	12	19	21	20	32	26	15	29	52	6	13	21	15	4	6	22	0	31	3	8	0	0	0
2133	**5110**	30	5	72	96	26	57	21	32	35	23	14	74	10	16	40	15	5	0	45	0	45	4	13	3	5	0
2134	**5110**	64	6	176	46	247	106	55	51	66	54	47	177	16	23	64	45	7	19	49	0	78	12	11	9	6	0
2135	**5110**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2136	**5110**	16	4	32	0	0	0	59	4	11	14	10	8	4	0	14	3	0	3	12	0	8	0	0	0	0	0
2137	**5110**	46	11	315	117	62	155	150	660	153	160	177	266	38	41	167	157	20	15	107	0	229	9	17	9	0	0
2138	**5110**	10	0	29	9	7	15	19	0	36	14	8	16	3	3	17	10	0	6	10	0	9	3	4	3	0	0
2139	**5110**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2140	**5110**	4	0	28	4	3	8	3	3	6	14	21	3	4	0	3	3	0	3	4	0	8	0	6	0	0	0
2141	**5110**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2142	**5110**	11	0	38	13	3	22	4	18	16	24	14	33	56	7	21	6	0	3	13	0	3	3	0	0	0	0
2143	**5110**	11	0	29	10	10	32	16	16	24	21	14	39	8	47	13	11	6	0	9	0	43	0	7	0	0	0
2144	**5110**	30	8	109	46	23	69	30	43	58	62	29	120	23	13	176	38	11	11	45	4	66	6	14	0	0	6
2145	**5110**	35	3	167	61	46	104	69	140	72	93	70	133	16	18	117	550	52	4	52	5	103	6	24	3	0	0
2146	**5110**	13	0	89	33	8	24	34	56	37	41	45	38	6	15	31	91	220	4	22	0	46	6	0	0	3	0
2170	**6230**	0	0	10	3	0	0	0	0	3	0	0	0	0	0	0	3	0	31	19	0	7	0	0	10	3	0
2171	**6230**	6	3	27	6	8	18	3	15	12	6	6	22	0	3	6	8	0	21	279	3	95	0	15	3	10	0
2172	**6230**	0	0	18	7	3	6	4	7	0	0	3	14	0	0	3	0	0	7	57	24	28	6	6	6	3	0
2173	**6230**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2174	**6230**	29	4	67	22	24	45	13	21	25	40	21	54	7	6	37	18	7	19	105	5	78	173	23	0	8	0
2175	**6230**	15	3	67	19	20	44	19	17	19	14	21	58	13	6	28	9	0	24	76	3	64	12	193	3	8	10
2176	**6230**	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	10	11	0	5	0	0	63	5	0
2177	**6230**	0	0	7	3	4	3	4	3	3	3	7	3	0	6	6	0	0	11	77	0	42	0	12	37	121	0
2178	**6230**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2180	**6300**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2160	**8820**	0	0	0	3	3	3	0	10	4	6	5	7	0	3	7	4	8	3	21	0	37	0	4	6	10	0
2161	**8820**	0	0	7	0	0	6	3	0	4	3	3	3	0	0	3	0	0	0	30	0	18	0	0	0	3	0
Total		**700**	**88**	**1705**	**573**	**560**	**856**	**593**	**1180**	**707**	**676**	**634**	**1319**	**240**	**247**	**866**	**1023**	**348**	**212**	**1108**	**44**	**1142**	**243**	**377**	**158**	**188**	**16**

Source: DoP Research June 2012;

ABS Census 2006 JtW

DZ_06	SLA	2160 Waroona	2161	1797 Rockingham	1798	1799	1800	1801	1802	1803	1804	1806	1807	1808	1809	1810	1811	1812	1813	1814	1815	1816	1817	1818	1819	Total North Peel-Rock	Reminder WA	WA Total Employment
2130	**5110**	50	3	37	0	16	7	17	7	16	13	3	9	16	0	3	7	6	0	4	7	10	17	18	0	**1830**	**1053**	**2883**
2131	**5110**	3	3	0	0	6	0	3	0	0	3	3	0	3	8	0	5	0	0	0	3	0	0	3	0	**324**	**214**	**538**
2132	**5110**	7	3	13	0	3	0	7	4	0	0	0	4	0	0	0	0	0	0	3	3	0	0	0	0	**564**	**300**	**864**
2133	**5110**	19	0	6	0	6	0	4	0	4	4	0	0	0	0	0	0	0	0	4	0	0	0	5	3	**741**	**326**	**1067**
2134	**5110**	31	0	18	4	3	0	3	3	0	3	4	5	12	0	7	0	3	0	7	3	3	7	9	0	**1559**	**760**	**2319**
2135	**5110**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**
2136	**5110**	7	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	**213**	**116**	**329**
2137	**5110**	131	9	33	9	6	6	12	12	13	11	0	18	3	0	7	4	7	0	11	17	8	18	31	3	**3450**	**1441**	**4891**
2138	**5110**	4	0	0	0	4	0	0	0	0	0	0	0	3	0	0	0	0	0	3	0	0	3	4	0	**252**	**109**	**361**
2139	**5110**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**
2140	**5110**	4	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**136**	**68**	**204**
2141	**5110**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**
2142	**5110**	7	0	3	0	3	0	0	0	0	0	0	0	7	0	0	4	0	0	0	0	0	6	4	0	**342**	**192**	**534**
2143	**5110**	7	0	0	0	0	4	0	0	0	0	0	0	4	0	0	3	0	0	0	0	0	0	4	0	**388**	**204**	**592**
2144	**5110**	27	0	17	0	3	0	7	0	0	0	4	0	3	0	6	0	6	0	0	4	3	3	3	4	**1130**	**520**	**1650**
2145	**5110**	54	11	24	0	10	0	3	6	0	14	0	3	6	0	6	10	4	0	3	0	6	0	12	0	**2115**	**1197**	**3312**
2146	**5110**	35	4	7	4	3	0	0	0	0	11	4	7	4	0	3	0	0	0	4	0	0	0	6	0	**954**	**581**	**1535**
2170	**6230**	3	3	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**99**	**70**	**169**
2171	**6230**	29	10	10	0	0	0	0	0	3	0	0	0	0	0	3	0	3	0	0	3	0	0	0	0	**636**	**230**	**866**
2172	**6230**	12	0	0	0	0	0	3	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**220**	**68**	**288**
2173	**6230**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**
2174	**6230**	32	5	16	0	0	0	0	0	0	0	0	0	3	0	7	0	3	0	0	0	3	3	3	0	**926**	**536**	**1462**
2175	**6230**	25	0	12	5	0	0	5	0	4	4	0	4	3	3	11	0	0	0	0	0	0	7	4	0	**852**	**377**	**1229**
2176	**6230**	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**103**	**47**	**150**
2177	**6230**	47	9	0	0	0	0	6	0	4	0	0	0	0	0	3	0	0	0	0	0	0	3	5	0	**429**	**136**	**565**
2178	**6230**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**
2180	**6300**	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**4**	**537**	**541**
2160	**8820**	304	81	0	0	3	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	**535**	**179**	**714**
2161	**8820**	271	208	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**562**	**200**	**762**
Total		**1117**	**349**	**200**	**22**	**66**	**17**	**70**	**32**	**44**	**70**	**18**	**50**	**71**	**11**	**56**	**33**	**35**	**0**	**39**	**40**	**33**	**67**	**111**	**10**	**18364**	**9461**	**27825**

Source: DoP Research June 2012;

Appendix C:
Input Vectors Extracted from the Mandurah and Murray Study Area

RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
4050	6964	106	192	2815	4050	6964	106	192	2815	2	352
4050	6964	106	192	2815	1007	0	0	0	0	4	3
4050	6964	106	192	2815	1560	6893	14705	5268	1027	2	235
4050	6964	106	192	2815	1270	0	7500	0	683	6	48
4050	6964	106	192	2815	2784	0	0	0	1868	5	37
4050	6964	106	192	2815	0	7042	17365	5400	29	3	318
4050	6964	106	192	2815	1634	4503	198	4407	0	5	115
4050	6964	106	192	2815	6653	10000	973	842	2147	4	43
4050	6964	106	192	2815	689	1002	0	396	0	4	81
4050	6964	106	192	2815	248	23311	1900	2648	0	5	138
4050	6964	106	192	2815	223	7555	936	12485	201	3	82
4050	6964	106	192	2815	0	27024	3354	930	0	5	169
4050	6964	106	192	2815	1187	7921	7614	4836	0	6	187
4050	6964	106	192	2815	900	0	0	0	435	6	18
4050	6964	106	192	2815	2458	0	7500	0	1594	6	68
4050	6964	106	192	2815	5227	15064	357	445	509	10	30
4050	6964	106	192	2815	2660	0	0	0	823	15	8
4050	6964	106	192	2815	612	4350	0	1300	600	1	37
4050	6964	106	192	2815	916	0	0	0	0	16	0
4050	6964	106	192	2815	1907	0	0	0	0	11	0
4050	6964	106	192	2815	1231	0	0	0	0	18	20
1007	0	0	0	0	4050	6964	106	192	2815	4	9
1007	0	0	0	0	1007	0	0	0	0	0	38
1007	0	0	0	0	1560	6893	14705	5268	1027	3	40
1007	0	0	0	0	1270	0	7500	0	683	8	15
1007	0	0	0	0	2784	0	0	0	1868	7	7
1007	0	0	0	0	0	7042	17365	5400	29	4	46
1007	0	0	0	0	1634	4503	198	4407	0	4	56
1007	0	0	0	0	6653	10000	973	842	2147	4	9
1007	0	0	0	0	689	1002	0	396	0	3	16
1007	0	0	0	0	248	23311	1900	2648	0	4	42
1007	0	0	0	0	223	7555	936	12485	201	3	8
1007	0	0	0	0	0	27024	3354	930	0	6	25
1007	0	0	0	0	1187	7921	7614	4836	0	6	0
1007	0	0	0	0	900	0	0	0	435	6	9
1007	0	0	0	0	2458	0	7500	0	1594	8	24
1007	0	0	0	0	5227	15064	357	445	509	11	7
1007	0	0	0	0	2660	0	0	0	823	18	0
1007	0	0	0	0	612	4350	0	1300	600	1	6
1007	0	0	0	0	916	0	0	0	0	21	0
1007	0	0	0	0	1907	0	0	0	0	15	0
1007	0	0	0	0	1231	0	0	0	0	24	0
1560	6893	14705	5268	1027	4050	6964	106	192	2815	2	19
1560	6893	14705	5268	1027	1007	0	0	0	0	3	0
1560	6893	14705	5268	1027	1560	6893	14705	5268	1027	1	143
1560	6893	14705	5268	1027	1270	0	7500	0	683	3	12
1560	6893	14705	5268	1027	2784	0	0	0	1868	2	19
1560	6893	14705	5268	1027	0	7042	17365	5400	29	1	65
1560	6893	14705	5268	1027	1634	4503	198	4407	0	2	51
1560	6893	14705	5268	1027	6653	10000	973	842	2147	2	32
1560	6893	14705	5268	1027	689	1002	0	396	0	1	26
1560	6893	14705	5268	1027	248	23311	1900	2648	0	2	39
1560	6893	14705	5268	1027	223	7555	936	12485	201	1	29
1560	6893	14705	5268	1027	0	27024	3354	930	0	2	52
1560	6893	14705	5268	1027	1187	7921	7614	4836	0	2	37
1560	6893	14705	5268	1027	900	0	0	0	435	2	13
1560	6893	14705	5268	1027	2458	0	7500	0	1594	3	21
1560	6893	14705	5268	1027	5227	15064	357	445	509	6	15
1560	6893	14705	5268	1027	2660	0	0	0	823	10	4
1560	6893	14705	5268	1027	612	4350	0	1300	600	1	22
1560	6893	14705	5268	1027	916	0	0	0	0	11	0
1560	6893	14705	5268	1027	1907	0	0	0	0	7	3
1560	6893	14705	5268	1027	1231	0	0	0	0	12	8
1270	0	7500	0	683	4050	6964	106	192	2815	6	30

RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
1270	0	7500	0	683	1007	0	0	0	0	8	5
1270	0	7500	0	683	1560	6893	14705	5268	1027	3	72
1270	0	7500	0	683	1270	0	7500	0	683	1	96
1270	0	7500	0	683	2784	0	0	0	1868	3	26
1270	0	7500	0	683	0	7042	17365	5400	29	5	176
1270	0	7500	0	683	1634	4503	198	4407	0	6	54
1270	0	7500	0	683	6653	10000	973	842	2147	4	32
1270	0	7500	0	683	689	1002	0	396	0	4	35
1270	0	7500	0	683	248	23311	1900	2648	0	6	60
1270	0	7500	0	683	223	7555	936	12485	201	3	14
1270	0	7500	0	683	0	27024	3354	930	0	5	74
1270	0	7500	0	683	1187	7921	7614	4836	0	6	62
1270	0	7500	0	683	900	0	0	0	435	6	16
1270	0	7500	0	683	2458	0	7500	0	1594	4	40
1270	0	7500	0	683	5227	15064	357	445	509	10	15
1270	0	7500	0	683	2660	0	0	0	823	17	5
1270	0	7500	0	683	612	4350	0	1300	600	1	45
1270	0	7500	0	683	916	0	0	0	0	16	0
1270	0	7500	0	683	1907	0	0	0	0	10	4
1270	0	7500	0	683	1231	0	0	0	0	16	13
2784	0	0	0	1868	4050	6964	106	192	2815	5	64
2784	0	0	0	1868	1007	0	0	0	0	7	6
2784	0	0	0	1868	1560	6893	14705	5268	1027	2	176
2784	0	0	0	1868	1270	0	7500	0	683	3	46
2784	0	0	0	1868	2784	0	0	0	1868	3	247
2784	0	0	0	1868	0	7042	17365	5400	29	3	327
2784	0	0	0	1868	1634	4503	198	4407	0	5	140
2784	0	0	0	1868	6653	10000	973	842	2147	4	51
2784	0	0	0	1868	689	1002	0	396	0	3	66
2784	0	0	0	1868	248	23311	1900	2648	0	5	140
2784	0	0	0	1868	223	7555	936	12485	201	3	47
2784	0	0	0	1868	0	27024	3354	930	0	4	177
2784	0	0	0	1868	1187	7921	7614	4836	0	6	100
2784	0	0	0	1868	900	0	0	0	435	5	23
2784	0	0	0	1868	2458	0	7500	0	1594	4	64
2784	0	0	0	1868	5227	15064	357	445	509	10	45
2784	0	0	0	1868	2660	0	0	0	823	17	7
2784	0	0	0	1868	612	4350	0	1300	600	1	49
2784	0	0	0	1868	916	0	0	0	0	17	0
2784	0	0	0	1868	1907	0	0	0	0	11	12
2784	0	0	0	1868	1231	0	0	0	0	18	11
0	7042	17365	5400	29	4050	6964	106	192	2815	3	0
0	7042	17365	5400	29	1007	0	0	0	0	4	0
0	7042	17365	5400	29	1560	6893	14705	5268	1027	1	0
0	7042	17365	5400	29	1270	0	7500	0	683	4	0
0	7042	17365	5400	29	2784	0	0	0	1868	3	0
0	7042	17365	5400	29	0	7042	17365	5400	29	0	0
0	7042	17365	5400	29	1634	4503	198	4407	0	4	0
0	7042	17365	5400	29	6653	10000	973	842	2147	4	0
0	7042	17365	5400	29	689	1002	0	396	0	3	0
0	7042	17365	5400	29	248	23311	1900	2648	0	4	0
0	7042	17365	5400	29	223	7555	936	12485	201	2	0
0	7042	17365	5400	29	0	27024	3354	930	0	3	0
0	7042	17365	5400	29	1187	7921	7614	4836	0	5	0
0	7042	17365	5400	29	900	0	0	0	435	5	0
0	7042	17365	5400	29	2458	0	7500	0	1594	5	0
0	7042	17365	5400	29	5227	15064	357	445	509	10	0
0	7042	17365	5400	29	2660	0	0	0	823	17	0
0	7042	17365	5400	29	612	4350	0	1300	600	1	0
0	7042	17365	5400	29	916	0	0	0	0	18	0
0	7042	17365	5400	29	1907	0	0	0	0	12	0
0	7042	17365	5400	29	1231	0	0	0	0	19	0
1634	4503	198	4407	0	4050	6964	106	192	2815	5	74
1634	4503	198	4407	0	1007	0	0	0	0	4	19

RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
1634	4503	198	4407	0	1560	6893	14705	5268	1027	2	148
1634	4503	198	4407	0	1270	0	7500	0	683	7	0
1634	4503	198	4407	0	2784	0	0	0	1868	6	0
1634	4503	198	4407	0	0	7042	17365	5400	29	4	0
1634	4503	198	4407	0	1634	4503	198	4407	0	1	273
1634	4503	198	4407	0	6653	10000	973	842	2147	2	19
1634	4503	198	4407	0	689	1002	0	396	0	1	51
1634	4503	198	4407	0	248	23311	1900	2648	0	1	65
1634	4503	198	4407	0	223	7555	936	12485	201	1	46
1634	4503	198	4407	0	0	27024	3354	930	0	3	37
1634	4503	198	4407	0	1187	7921	7614	4836	0	3	19
1634	4503	198	4407	0	900	0	0	0	435	4	0
1634	4503	198	4407	0	2458	0	7500	0	1594	6	65
1634	4503	198	4407	0	5227	15064	357	445	509	8	14
1634	4503	198	4407	0	2660	0	0	0	823	15	0
1634	4503	198	4407	0	612	4350	0	1300	600	1	56
1634	4503	198	4407	0	916	0	0	0	0	19	0
1634	4503	198	4407	0	1907	0	0	0	0	13	0
1634	4503	198	4407	0	1231	0	0	0	0	22	0
6653	10000	973	842	2147	4050	6964	106	192	2815	3	46
6653	10000	973	842	2147	1007	0	0	0	0	4	11
6653	10000	973	842	2147	1560	6893	14705	5268	1027	2	315
6653	10000	973	842	2147	1270	0	7500	0	683	4	117
6653	10000	973	842	2147	2784	0	0	0	1868	4	62
6653	10000	973	842	2147	0	7042	17365	5400	29	3	479
6653	10000	973	842	2147	1634	4503	198	4407	0	2	383
6653	10000	973	842	2147	6653	10000	973	842	2147	1	660
6653	10000	973	842	2147	689	1002	0	396	0	1	153
6653	10000	973	842	2147	248	23311	1900	2648	0	1	415
6653	10000	973	842	2147	223	7555	936	12485	201	1	177
6653	10000	973	842	2147	0	27024	3354	930	0	2	266
6653	10000	973	842	2147	1187	7921	7614	4836	0	2	237
6653	10000	973	842	2147	900	0	0	0	435	2	41
6653	10000	973	842	2147	2458	0	7500	0	1594	3	167
6653	10000	973	842	2147	5227	15064	357	445	509	3	157
6653	10000	973	842	2147	2660	0	0	0	823	6	20
6653	10000	973	842	2147	612	4350	0	1300	600	1	107
6653	10000	973	842	2147	916	0	0	0	0	10	0
6653	10000	973	842	2147	1907	0	0	0	0	7	9
6653	10000	973	842	2147	1231	0	0	0	0	11	17
689	1002	0	396	0	4050	6964	106	192	2815	4	19
689	1002	0	396	0	1007	0	0	0	0	3	0
689	1002	0	396	0	1560	6893	14705	5268	1027	1	57
689	1002	0	396	0	1270	0	7500	0	683	4	18
689	1002	0	396	0	2784	0	0	0	1868	4	14
689	1002	0	396	0	0	7042	17365	5400	29	3	29
689	1002	0	396	0	1634	4503	198	4407	0	1	37
689	1002	0	396	0	6653	10000	973	842	2147	1	0
689	1002	0	396	0	689	1002	0	396	0	0	70
689	1002	0	396	0	248	23311	1900	2648	0	1	27
689	1002	0	396	0	223	7555	936	12485	201	0	16
689	1002	0	396	0	0	27024	3354	930	0	1	31
689	1002	0	396	0	1187	7921	7614	4836	0	1	6
689	1002	0	396	0	900	0	0	0	435	2	6
689	1002	0	396	0	2458	0	7500	0	1594	3	33
689	1002	0	396	0	5227	15064	357	445	509	6	19
689	1002	0	396	0	2660	0	0	0	823	10	0
689	1002	0	396	0	612	4350	0	1300	600	1	19
689	1002	0	396	0	916	0	0	0	0	14	0
689	1002	0	396	0	1907	0	0	0	0	9	6
689	1002	0	396	0	1231	0	0	0	0	15	8
248	23311	1900	2648	0	4050	6964	106	192	2815	5	0
248	23311	1900	2648	0	1007	0	0	0	0	4	0
248	23311	1900	2648	0	1560	6893	14705	5268	1027	2	0

RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
248	23311	1900	2648	0	1270	0	7500	0	683	6	0
248	23311	1900	2648	0	2784	0	0	0	1868	5	0
248	23311	1900	2648	0	0	7042	17365	5400	29	4	0
248	23311	1900	2648	0	1634	4503	198	4407	0	1	0
248	23311	1900	2648	0	6653	10000	973	842	2147	1	0
248	23311	1900	2648	0	689	1002	0	396	0	1	0
248	23311	1900	2648	0	248	23311	1900	2648	0	0	0
248	23311	1900	2648	0	223	7555	936	12485	201	0	0
248	23311	1900	2648	0	0	27024	3354	930	0	3	0
248	23311	1900	2648	0	1187	7921	7614	4836	0	2	0
248	23311	1900	2648	0	900	0	0	0	435	2	0
248	23311	1900	2648	0	2458	0	7500	0	1594	4	0
248	23311	1900	2648	0	5227	15064	357	445	509	6	0
248	23311	1900	2648	0	2660	0	0	0	823	12	0
248	23311	1900	2648	0	612	4350	0	1300	600	1	0
248	23311	1900	2648	0	916	0	0	0	0	17	0
248	23311	1900	2648	0	1907	0	0	0	0	12	0
248	23311	1900	2648	0	1231	0	0	0	0	19	0
223	7555	936	12485	201	4050	6964	106	192	2815	3	4
223	7555	936	12485	201	1007	0	0	0	0	3	0
223	7555	936	12485	201	1560	6893	14705	5268	1027	1	28
223	7555	936	12485	201	1270	0	7500	0	683	3	4
223	7555	936	12485	201	2784	0	0	0	1868	3	3
223	7555	936	12485	201	0	7042	17365	5400	29	2	25
223	7555	936	12485	201	1634	4503	198	4407	0	1	8
223	7555	936	12485	201	6653	10000	973	842	2147	1	3
223	7555	936	12485	201	689	1002	0	396	0	0	6
223	7555	936	12485	201	248	23311	1900	2648	0	0	36
223	7555	936	12485	201	223	7555	936	12485	201	0	21
223	7555	936	12485	201	0	27024	3354	930	0	1	3
223	7555	936	12485	201	1187	7921	7614	4836	0	1	25
223	7555	936	12485	201	900	0	0	0	435	1	0
223	7555	936	12485	201	2458	0	7500	0	1594	2	3
223	7555	936	12485	201	5227	15064	357	445	509	4	3
223	7555	936	12485	201	2660	0	0	0	823	7	0
223	7555	936	12485	201	612	4350	0	1300	600	1	4
223	7555	936	12485	201	916	0	0	0	0	10	0
223	7555	936	12485	201	1907	0	0	0	0	7	0
223	7555	936	12485	201	1231	0	0	0	0	11	6
0	27024	3354	930	0	4050	6964	106	192	2815	5	0
0	27024	3354	930	0	1007	0	0	0	0	6	0
0	27024	3354	930	0	1560	6893	14705	5268	1027	2	0
0	27024	3354	930	0	1270	0	7500	0	683	4	0
0	27024	3354	930	0	2784	0	0	0	1868	4	0
0	27024	3354	930	0	0	7042	17365	5400	29	3	0
0	27024	3354	930	0	1634	4503	198	4407	0	3	0
0	27024	3354	930	0	6653	10000	973	842	2147	2	0
0	27024	3354	930	0	689	1002	0	396	0	1	0
0	27024	3354	930	0	248	23311	1900	2648	0	3	0
0	27024	3354	930	0	223	7555	936	12485	201	1	0
0	27024	3354	930	0	0	27024	3354	930	0	0	0
0	27024	3354	930	0	1187	7921	7614	4836	0	2	0
0	27024	3354	930	0	900	0	0	0	435	2	0
0	27024	3354	930	0	2458	0	7500	0	1594	3	0
0	27024	3354	930	0	5227	15064	357	445	509	8	0
0	27024	3354	930	0	2660	0	0	0	823	15	0
0	27024	3354	930	0	612	4350	0	1300	600	1	0
0	27024	3354	930	0	916	0	0	0	0	18	0
0	27024	3354	930	0	1907	0	0	0	0	12	0
0	27024	3354	930	0	1231	0	0	0	0	19	0
1187	7921	7614	4836	0	4050	6964	106	192	2815	6	24
1187	7921	7614	4836	0	1007	0	0	0	0	6	0
1187	7921	7614	4836	0	1560	6893	14705	5268	1027	2	82
1187	7921	7614	4836	0	1270	0	7500	0	683	6	28

RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
1187	7921	7614	4836	0	2784	0	0	0	1868	5	6
1187	7921	7614	4836	0	0	7042	17365	5400	29	5	47
1187	7921	7614	4836	0	1634	4503	198	4407	0	3	9
1187	7921	7614	4836	0	6653	10000	973	842	2147	2	39
1187	7921	7614	4836	0	689	1002	0	396	0	1	34
1187	7921	7614	4836	0	248	23311	1900	2648	0	2	52
1187	7921	7614	4836	0	223	7555	936	12485	201	1	30
1187	7921	7614	4836	0	0	27024	3354	930	0	2	71
1187	7921	7614	4836	0	1187	7921	7614	4836	0	1	120
1187	7921	7614	4836	0	900	0	0	0	435	2	15
1187	7921	7614	4836	0	2458	0	7500	0	1594	4	45
1187	7921	7614	4836	0	5227	15064	357	445	509	7	13
1187	7921	7614	4836	0	2660	0	0	0	823	14	0
1187	7921	7614	4836	0	612	4350	0	1300	600	1	28
1187	7921	7614	4836	0	916	0	0	0	0	19	0
1187	7921	7614	4836	0	1907	0	0	0	0	13	6
1187	7921	7614	4836	0	1231	0	0	0	0	21	0
900	0	0	0	435	4050	6964	106	192	2815	6	11
900	0	0	0	435	1007	0	0	0	0	6	0
900	0	0	0	435	1560	6893	14705	5268	1027	2	29
900	0	0	0	435	1270	0	7500	0	683	5	10
900	0	0	0	435	2784	0	0	0	1868	5	10
900	0	0	0	435	0	7042	17365	5400	29	4	99
900	0	0	0	435	1634	4503	198	4407	0	4	41
900	0	0	0	435	6653	10000	973	842	2147	2	16
900	0	0	0	435	689	1002	0	396	0	2	24
900	0	0	0	435	248	23311	1900	2648	0	3	55
900	0	0	0	435	223	7555	936	12485	201	1	14
900	0	0	0	435	0	27024	3354	930	0	2	39
900	0	0	0	435	1187	7921	7614	4836	0	2	50
900	0	0	0	435	900	0	0	0	435	1	47
900	0	0	0	435	2458	0	7500	0	1594	4	13
900	0	0	0	435	5227	15064	357	445	509	7	11
900	0	0	0	435	2660	0	0	0	823	14	6
900	0	0	0	435	612	4350	0	1300	600	1	9
900	0	0	0	435	916	0	0	0	0	18	0
900	0	0	0	435	1907	0	0	0	0	12	0
900	0	0	0	435	1231	0	0	0	0	20	7
2458	0	7500	0	1594	4050	6964	106	192	2815	6	30
2458	0	7500	0	1594	1007	0	0	0	0	8	8
2458	0	7500	0	1594	1560	6893	14705	5268	1027	3	109
2458	0	7500	0	1594	1270	0	7500	0	683	4	46
2458	0	7500	0	1594	2784	0	0	0	1868	4	23
2458	0	7500	0	1594	0	7042	17365	5400	29	5	213
2458	0	7500	0	1594	1634	4503	198	4407	0	6	77
2458	0	7500	0	1594	6653	10000	973	842	2147	3	43
2458	0	7500	0	1594	689	1002	0	396	0	4	58
2458	0	7500	0	1594	248	23311	1900	2648	0	5	161
2458	0	7500	0	1594	223	7555	936	12485	201	3	29
2458	0	7500	0	1594	0	27024	3354	930	0	3	120
2458	0	7500	0	1594	1187	7921	7614	4836	0	4	144
2458	0	7500	0	1594	900	0	0	0	435	4	13
2458	0	7500	0	1594	2458	0	7500	0	1594	2	176
2458	0	7500	0	1594	5227	15064	357	445	509	8	38
2458	0	7500	0	1594	2660	0	0	0	823	15	11
2458	0	7500	0	1594	612	4350	0	1300	600	1	45
2458	0	7500	0	1594	916	0	0	0	0	17	4
2458	0	7500	0	1594	1907	0	0	0	0	10	6
2458	0	7500	0	1594	1231	0	0	0	0	17	14
5227	15064	357	445	509	4050	6964	106	192	2815	9	35
5227	15064	357	445	509	1007	0	0	0	0	11	3
5227	15064	357	445	509	1560	6893	14705	5268	1027	6	167
5227	15064	357	445	509	1270	0	7500	0	683	10	61
5227	15064	357	445	509	2784	0	0	0	1868	10	46

RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
5227	15064	357	445	509	0	7042	17365	5400	29	10	321
5227	15064	357	445	509	1634	4503	198	4407	0	8	176
5227	15064	357	445	509	6653	10000	973	842	2147	3	140
5227	15064	357	445	509	689	1002	0	396	0	6	72
5227	15064	357	445	509	248	23311	1900	2648	0	6	241
5227	15064	357	445	509	223	7555	936	12485	201	4	70
5227	15064	357	445	509	0	27024	3354	930	0	8	133
5227	15064	357	445	509	1187	7921	7614	4836	0	7	100
5227	15064	357	445	509	900	0	0	0	435	7	18
5227	15064	357	445	509	2458	0	7500	0	1594	8	117
5227	15064	357	445	509	5227	15064	357	445	509	4	550
5227	15064	357	445	509	2660	0	0	0	823	7	52
5227	15064	357	445	509	612	4350	0	1300	600	1	52
5227	15064	357	445	509	916	0	0	0	0	22	5
5227	15064	357	445	509	1907	0	0	0	0	16	6
5227	15064	357	445	509	1231	0	0	0	0	26	24
2660	0	0	0	823	4050	6964	106	192	2815	15	13
2660	0	0	0	823	1007	0	0	0	0	18	0
2660	0	0	0	823	1560	6893	14705	5268	1027	10	89
2660	0	0	0	823	1270	0	7500	0	683	17	33
2660	0	0	0	823	2784	0	0	0	1868	17	8
2660	0	0	0	823	0	7042	17365	5400	29	16	74
2660	0	0	0	823	1634	4503	198	4407	0	14	87
2660	0	0	0	823	6653	10000	973	842	2147	6	56
2660	0	0	0	823	689	1002	0	396	0	10	37
2660	0	0	0	823	248	23311	1900	2648	0	12	106
2660	0	0	0	823	223	7555	936	12485	201	7	45
2660	0	0	0	823	0	27024	3354	930	0	15	38
2660	0	0	0	823	1187	7921	7614	4836	0	14	37
2660	0	0	0	823	900	0	0	0	435	14	15
2660	0	0	0	823	2458	0	7500	0	1594	15	31
2660	0	0	0	823	5227	15064	357	445	509	7	91
2660	0	0	0	823	2660	0	0	0	823	3	220
2660	0	0	0	823	612	4350	0	1300	600	2	22
2660	0	0	0	823	916	0	0	0	0	30	0
2660	0	0	0	823	1907	0	0	0	0	23	6
2660	0	0	0	823	1231	0	0	0	0	37	0
612	4350	0	1300	600	4050	6964	106	192	2815	1	6
612	4350	0	1300	600	1007	0	0	0	0	1	3
612	4350	0	1300	600	1560	6893	14705	5268	1027	1	27
612	4350	0	1300	600	1270	0	7500	0	683	1	6
612	4350	0	1300	600	2784	0	0	0	1868	1	8
612	4350	0	1300	600	0	7042	17365	5400	29	1	56
612	4350	0	1300	600	1634	4503	198	4407	0	1	8
612	4350	0	1300	600	6653	10000	973	842	2147	1	15
612	4350	0	1300	600	689	1002	0	396	0	1	12
612	4350	0	1300	600	248	23311	1900	2648	0	1	16
612	4350	0	1300	600	223	7555	936	12485	201	1	6
612	4350	0	1300	600	0	27024	3354	930	0	1	22
612	4350	0	1300	600	1187	7921	7614	4836	0	1	0
612	4350	0	1300	600	900	0	0	0	435	1	3
612	4350	0	1300	600	2458	0	7500	0	1594	1	6
612	4350	0	1300	600	5227	15064	357	445	509	1	8
612	4350	0	1300	600	2660	0	0	0	823	2	0
612	4350	0	1300	600	612	4350	0	1300	600	0	279
612	4350	0	1300	600	916	0	0	0	0	0	3
612	4350	0	1300	600	1907	0	0	0	0	1	0
612	4350	0	1300	600	1231	0	0	0	0	1	15
916	0	0	0	0	4050	6964	106	192	2815	16	0
916	0	0	0	0	1007	0	0	0	0	22	0
916	0	0	0	0	1560	6893	14705	5268	1027	11	52
916	0	0	0	0	1270	0	7500	0	683	16	20
916	0	0	0	0	2784	0	0	0	1868	17	9
916	0	0	0	0	0	7042	17365	5400	29	18	17

RD i	RE i	CO i	SH i	SC i	RD j	Rej	CO j	SH j	SC j	Dij	Tij
916	0	0	0	0	1634	4503	198	4407	0	20	12
916	0	0	0	0	6653	10000	973	842	2147	10	20
916	0	0	0	0	689	1002	0	396	0	14	0
916	0	0	0	0	248	23311	1900	2648	0	18	0
916	0	0	0	0	223	7555	936	12485	201	10	9
916	0	0	0	0	0	27024	3354	930	0	18	40
916	0	0	0	0	1187	7921	7614	4836	0	19	0
916	0	0	0	0	900	0	0	0	435	18	0
916	0	0	0	0	2458	0	7500	0	1594	17	9
916	0	0	0	0	5227	15064	357	445	509	22	0
916	0	0	0	0	2660	0	0	0	823	30	0
916	0	0	0	0	612	4350	0	1300	600	0	164
916	0	0	0	0	916	0	0	0	0	1	69
916	0	0	0	0	1907	0	0	0	0	13	17
916	0	0	0	0	1231	0	0	0	0	12	17
1907	0	0	0	0	4050	6964	106	192	2815	11	29
1907	0	0	0	0	1007	0	0	0	0	16	4
1907	0	0	0	0	1560	6893	14705	5268	1027	8	67
1907	0	0	0	0	1270	0	7500	0	683	10	22
1907	0	0	0	0	2784	0	0	0	1868	11	24
1907	0	0	0	0	0	7042	17365	5400	29	12	139
1907	0	0	0	0	1634	4503	198	4407	0	14	33
1907	0	0	0	0	6653	10000	973	842	2147	7	21
1907	0	0	0	0	689	1002	0	396	0	9	25
1907	0	0	0	0	248	23311	1900	2648	0	12	104
1907	0	0	0	0	223	7555	936	12485	201	7	21
1907	0	0	0	0	0	27024	3354	930	0	12	54
1907	0	0	0	0	1187	7921	7614	4836	0	13	44
1907	0	0	0	0	900	0	0	0	435	12	6
1907	0	0	0	0	2458	0	7500	0	1594	10	37
1907	0	0	0	0	5227	15064	357	445	509	16	18
1907	0	0	0	0	2660	0	0	0	823	23	7
1907	0	0	0	0	612	4350	0	1300	600	1	105
1907	0	0	0	0	916	0	0	0	0	13	5
1907	0	0	0	0	1907	0	0	0	0	5	173
1907	0	0	0	0	1231	0	0	0	0	12	23
1231	0	0	0	0	4050	6964	106	192	2815	18	15
1231	0	0	0	0	1007	0	0	0	0	25	3
1231	0	0	0	0	1560	6893	14705	5268	1027	12	67
1231	0	0	0	0	1270	0	7500	0	683	16	19
1231	0	0	0	0	2784	0	0	0	1868	18	20
1231	0	0	0	0	0	7042	17365	5400	29	20	136
1231	0	0	0	0	1634	4503	198	4407	0	22	48
1231	0	0	0	0	6653	10000	973	842	2147	12	17
1231	0	0	0	0	689	1002	0	396	0	15	19
1231	0	0	0	0	248	23311	1900	2648	0	20	36
1231	0	0	0	0	223	7555	936	12485	201	11	21
1231	0	0	0	0	0	27024	3354	930	0	19	58
1231	0	0	0	0	1187	7921	7614	4836	0	21	81
1231	0	0	0	0	900	0	0	0	435	20	6
1231	0	0	0	0	2458	0	7500	0	1594	17	28
1231	0	0	0	0	5227	15064	357	445	509	26	9
1231	0	0	0	0	2660	0	0	0	823	37	0
1231	0	0	0	0	612	4350	0	1300	600	1	76
1231	0	0	0	0	916	0	0	0	0	12	3
1231	0	0	0	0	1907	0	0	0	0	12	12
1231	0	0	0	0	1231	0	0	0	0	8	193

Appendix D:

Validation Analysis for Different Sample Groups

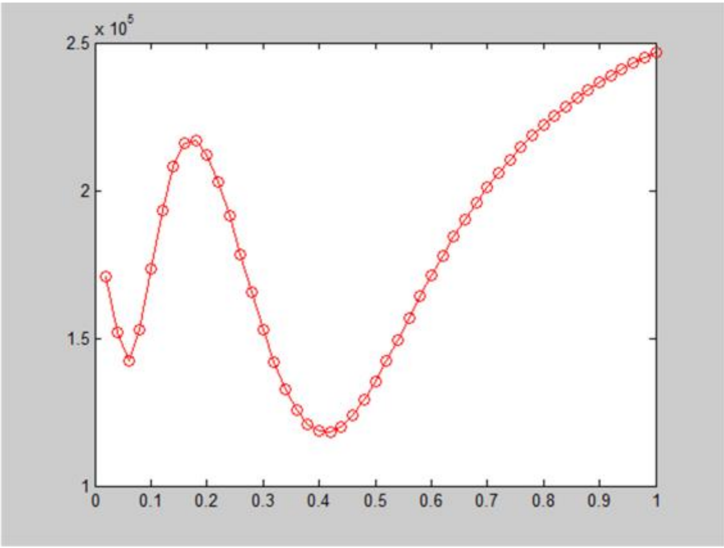

Figure D-1: Sigma determination for sample group 1 (trained by rows of the OD matrix)

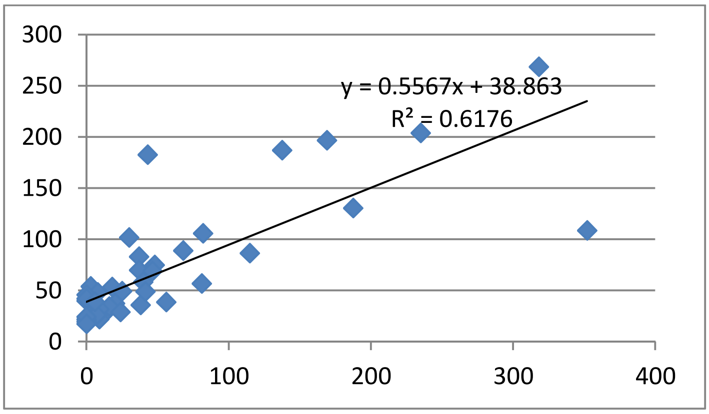

Figure D-2: Trip distribution regression plot for sample group 1 (trained by rows of the OD matrix)

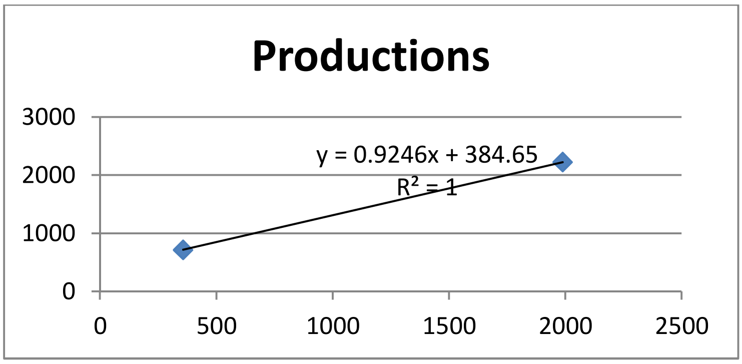

Figure D-3: Production plot for sample group 1 (trained by rows of the OD matrix)

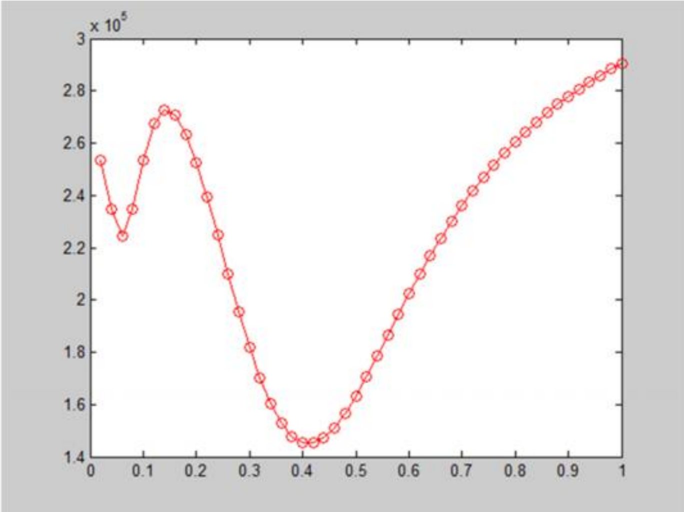

Figure D-4: Sigma determination for sample group 2 (trained by rows of the OD matrix)

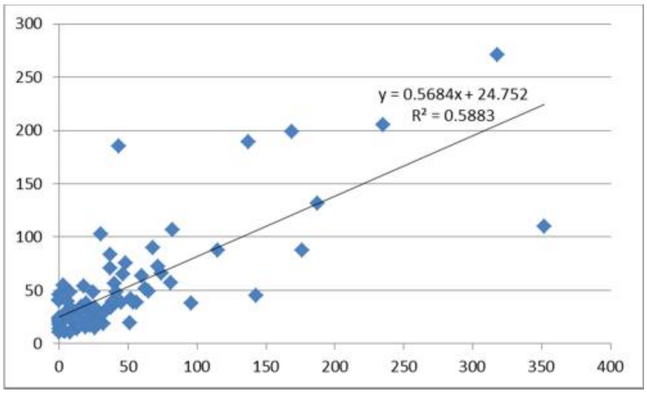

Figure D-5: Trip distribution regression plot for sample group 2 (trained by rows of the OD matrix)

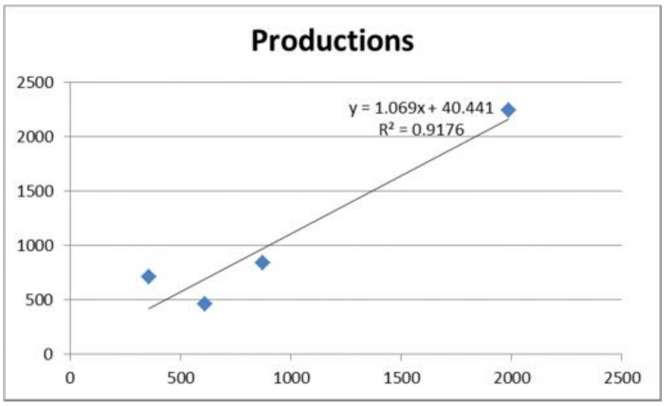

Figure D-6: Production plot for sample group 2 (trained by rows of the OD matrix)

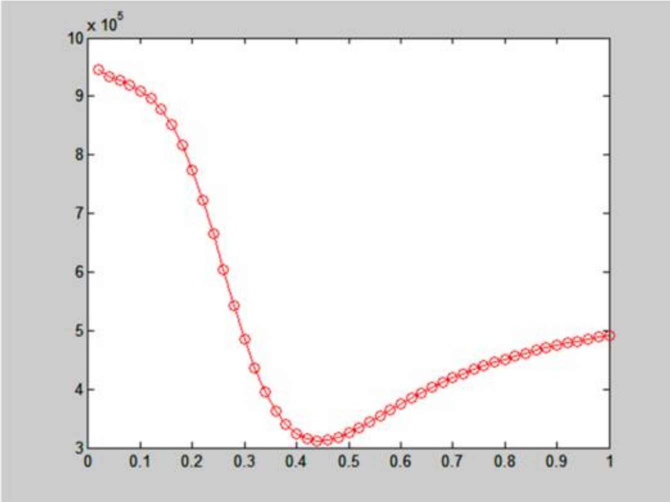

Figure D-7: Sigma determination for sample group 3 (trained by rows of the OD matrix)

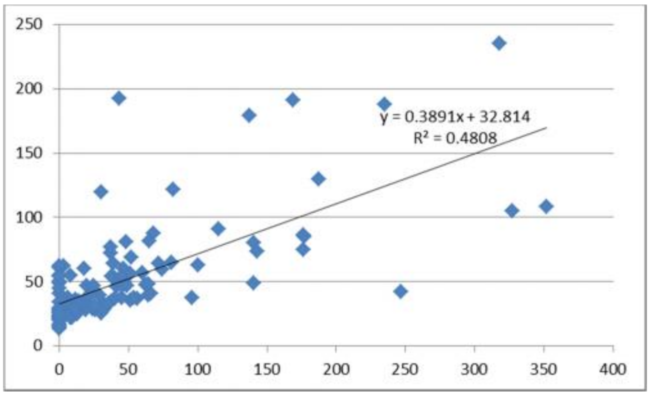

Figure D-8: Trip distribution regression plot for sample group 3 (trained by rows of the OD matrix)

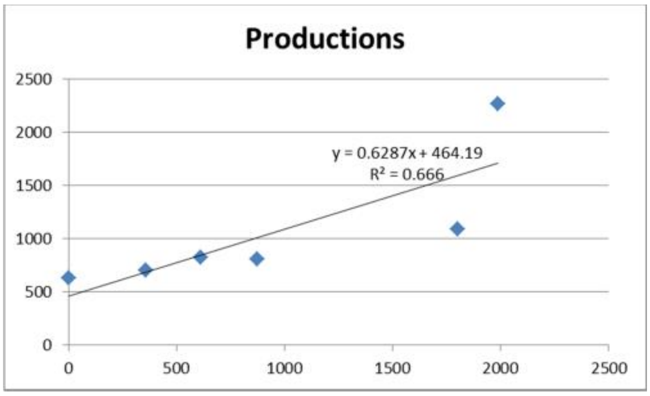

Figure D-9: Production plot for sample group 3 (trained by rows of the OD matrix)

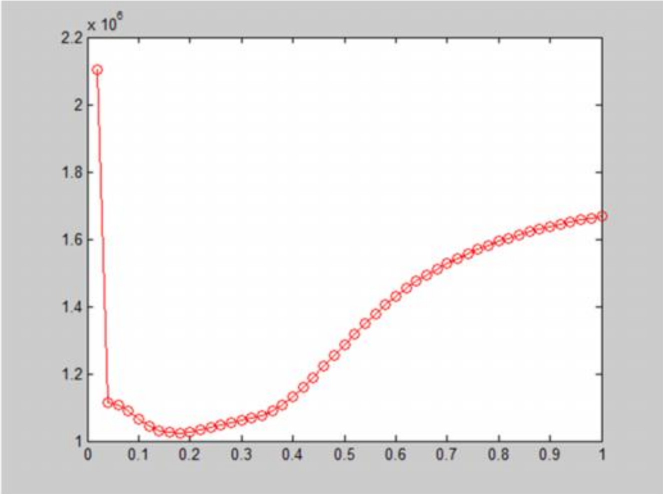

Figure D-10: Sigma determination for sample group 4 (trained by rows of the OD matrix)

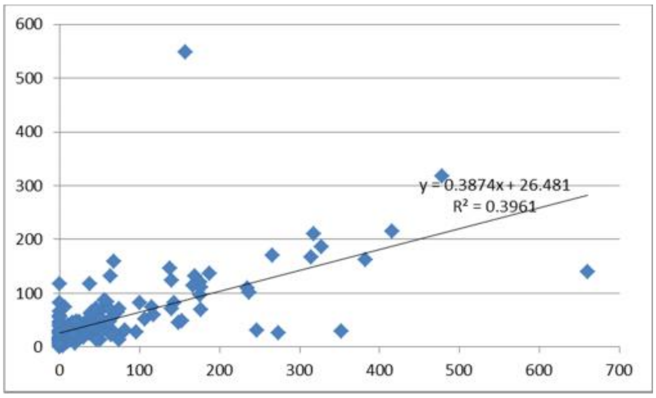

Figure D-11: Trip distribution regression plot for sample group 4 (trained by rows of the OD matrix)

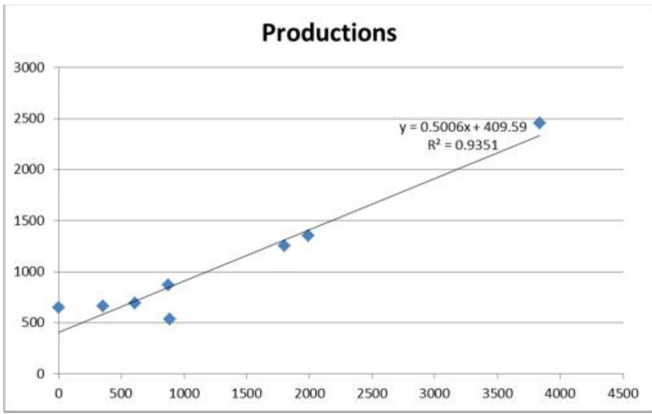

Figure D-12: Production plot for sample group 4 (trained by rows of the OD matrix)

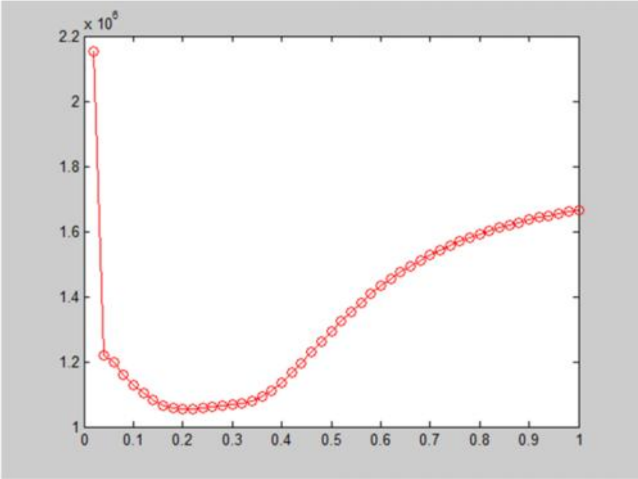

Figure D-13: Sigma determination for sample group 5 (trained by rows of the OD matrix)

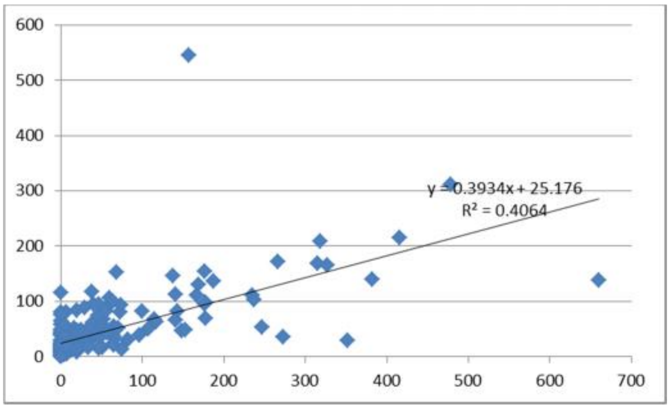

Figure D-14: Trip distribution regression plot for sample group 5 (trained by rows of the OD matrix)

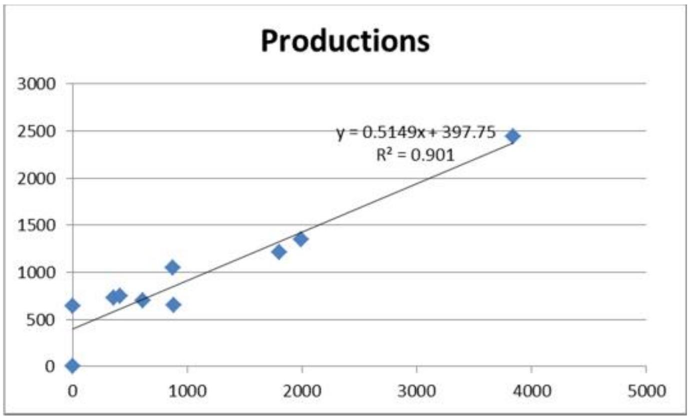

Figure D-15: Production plot for sample group 5 (trained by rows of the OD matrix)

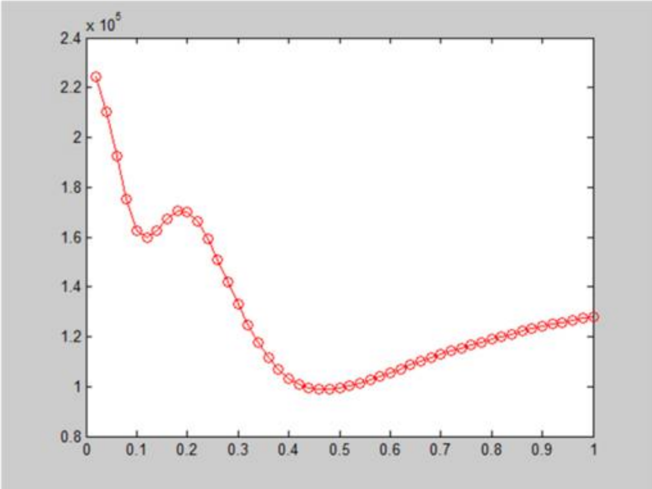

Figure D-16: Sigma determination for sample group 1 (trained by columns of the OD matrix)

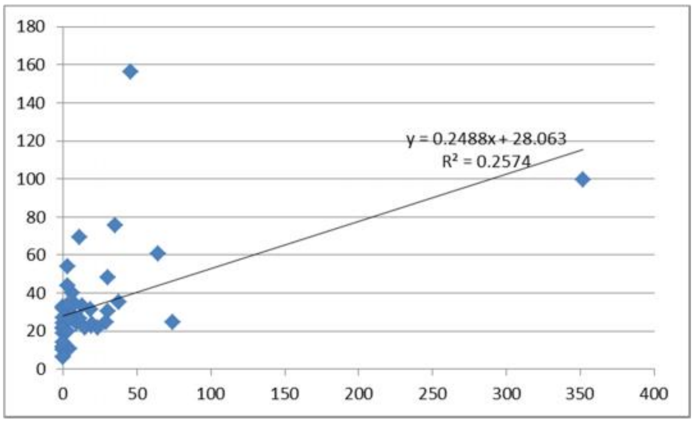

Figure D-17: Trip distribution regression plot for sample group 1 (trained by columns of the OD matrix)

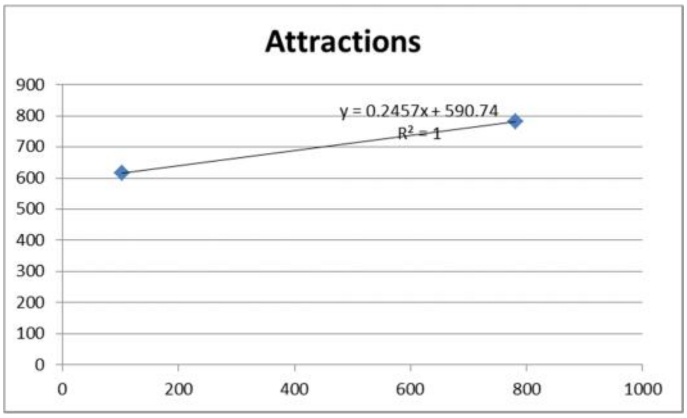

Figure D-18: Attraction plot for sample group 1 (trained by columns of the OD matrix)

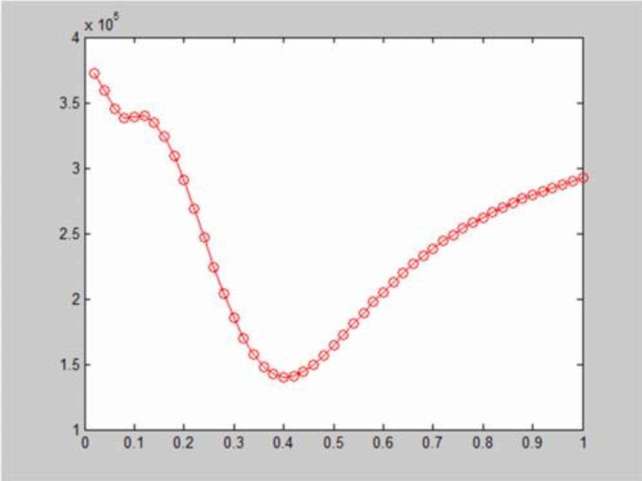

Figure D-19: Sigma determination for sample group 2 (trained by columns of the OD matrix)

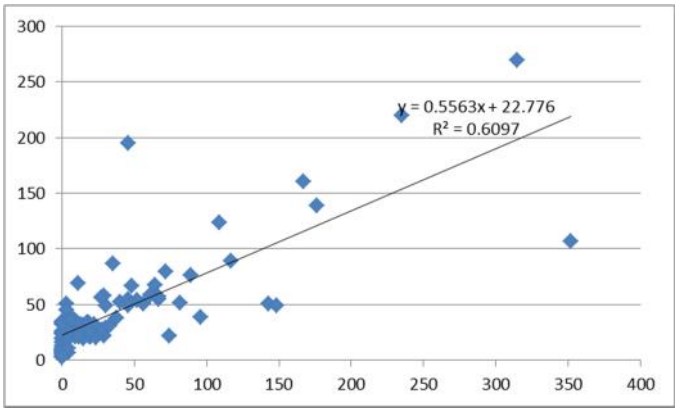

Figure D-20: Trip distribution regression plot for sample group 2 (trained by columns of the OD matrix)

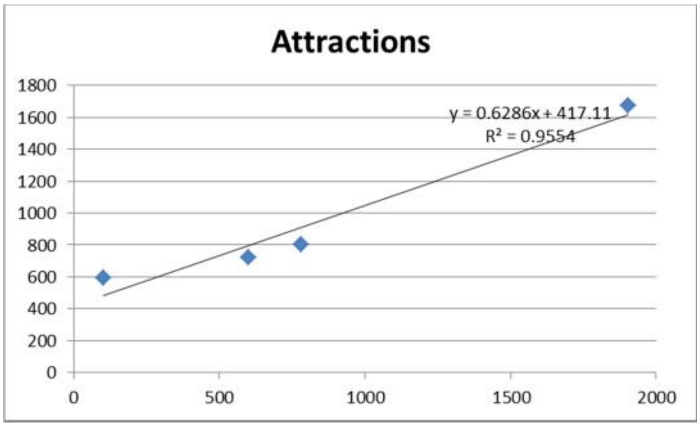

Figure D-21: Attraction plot for sample group 2 (trained by columns of the OD matrix)

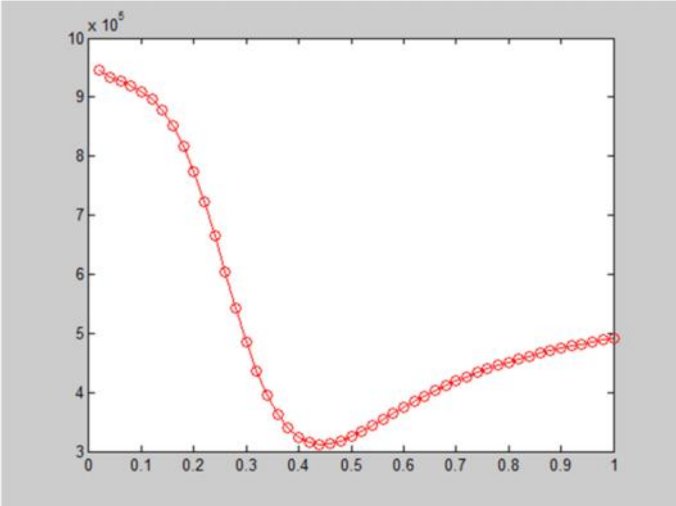

Figure D-22: Sigma determination for sample group 3 (trained by columns of the OD matrix)

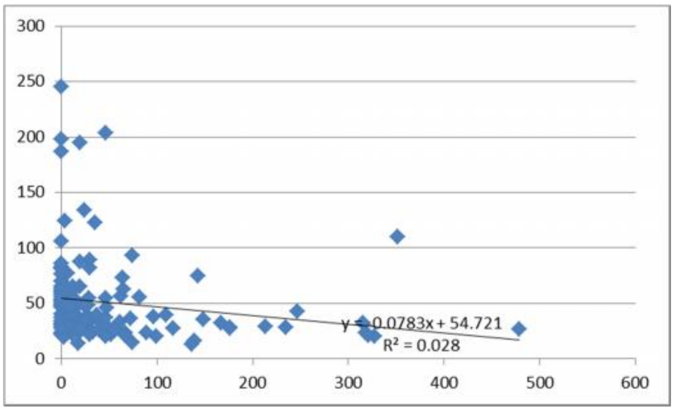

Figure D-23: Trip distribution regression plot for sample group 3 (trained by columns of the OD matrix)

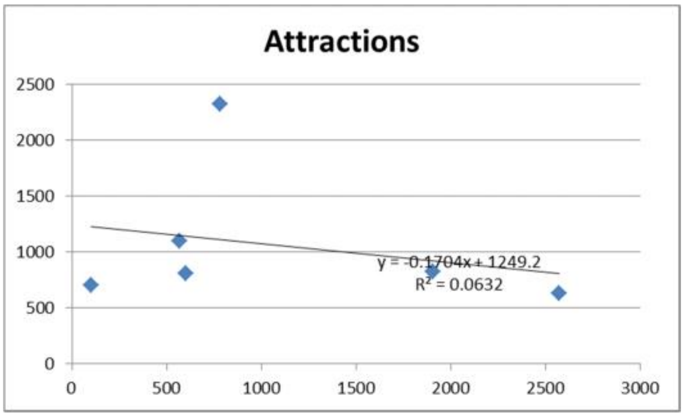

Figure D-24: Attraction plot for sample group 3 (trained by columns of the OD matrix)

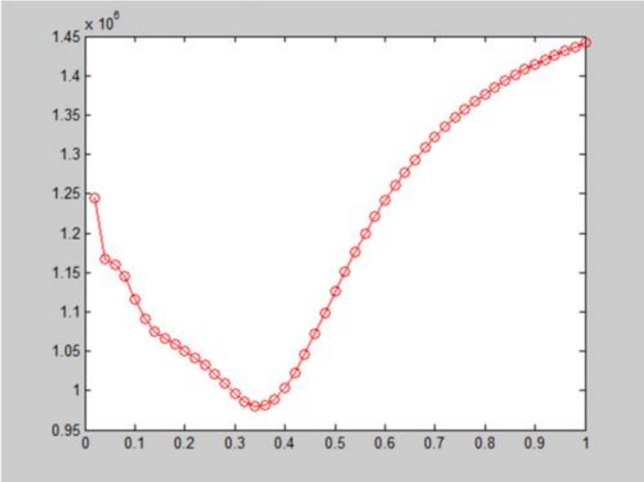

Figure D-25: Sigma determination for sample group 4 (trained by columns of the OD matrix)

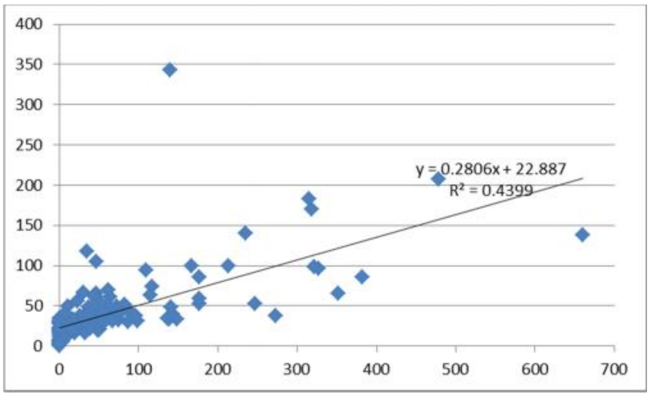

Figure D-26: Trip distribution regression plot for sample group 4 (trained by columns of the OD matrix)

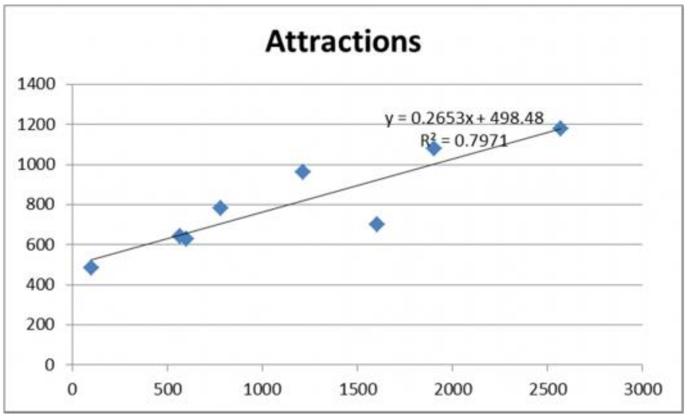

Figure D-27: Attraction plot for sample group 4 (trained by columns of the OD matrix)

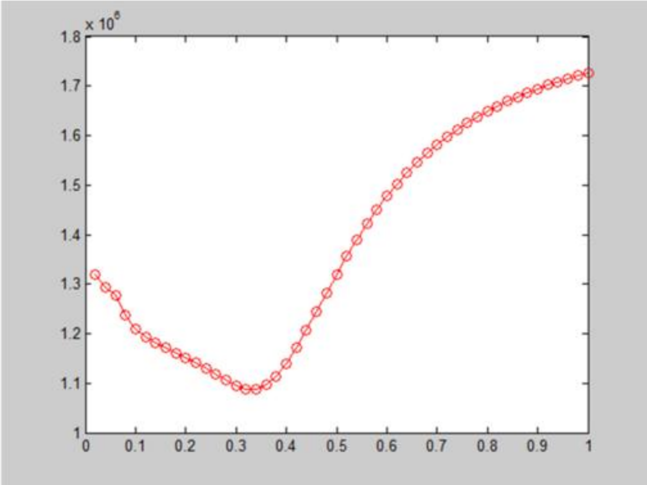

Figure D-28: Sigma determination for sample group 5 (trained by columns of the OD matrix)

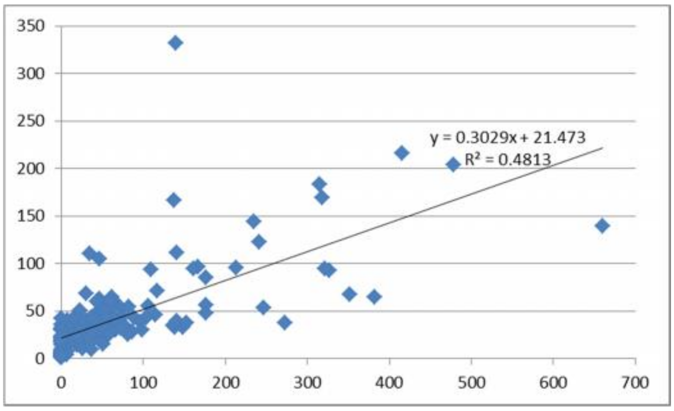

Figure D-29: Trip distribution regression plot for sample group 5 (trained by columns of the OD matrix)

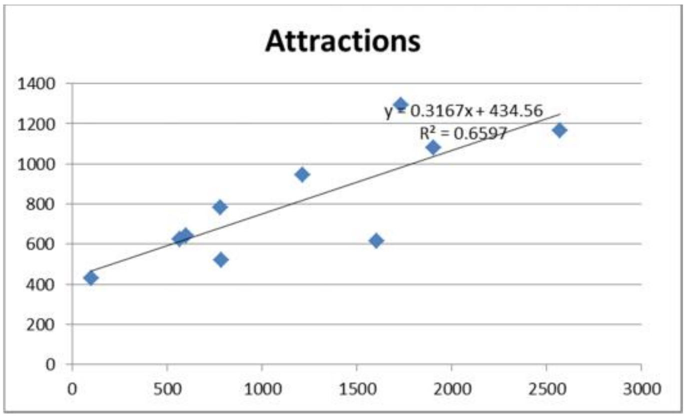

Figure D-30: Attraction plot for sample group 5 (trained by columns of the OD matrix)