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Abstract

A general solving method for the plate-type model called M4-5n, relying on mixed finite elements is presented hereafter. In particular, we explain how embedded discontinuities in a multilayer pavement structure can be easily taken into account using the numerical approach developed.
M4-5n stands for Multi-Particle Model for Multilayer Materials (M4) considering 5 equilibrium equations per layer (n being the total number of layers) [1]. This model can be viewed as a piling of linear elastic plates whose equilibrium equations per layer bear on the bending moments, the torques and the shear resultants of the tridimensional stresses. The load transfer between two layers is ensured by interface normal and shear stresses. This model is derived from the Hellinger-Reissner variational principle considering a polynomial representation of the stress field (3D) in the direction orthogonal to the layers [2]. This model allows us to diminish by one dimension the modeling of the actual problem, which in particular facilitates the geometrical description of embedded discontinuities (cracks or debonding). Another advantage of M4-5n is that it leads to finite values of the generalized stress fields at the location of discontinuities. In the case of cracks, propagation criteria can thus rely straightly on the values of the generalized interface stress fields or on other quantities such as the individual strain energy release rates which can be computed from these values [1][3-4]. M4-5n is quite well adapted to the study of the bending of elastic pavement structures incorporating cracks or interlayer debonding.

Recently, we extended previous research related to M4-5n to the systematic solving of the model equations by means of the mixed finite element method. The approach developed relies on the complementary energy theorem applied to M4-5n, the condition of statically admissible stress being taken into account using Lagrange multipliers which can be identified to components of the generalized displacement fields. The resulting functional, L, is proved to be equivalent to that of Hellinger-Reissner expressed in terms of the generalized fields [5]. The solution of the M4-5n problem is a saddle point for L with regards to the generalized stress and displacement fields. The solution of the mixed formulation ($dL=0$) is approximated using FEM paying a specific attention to the functional spaces of interpolation to avoid ill-conditioned systems of algebraic equations after discretization. The numerical procedure was implemented using the FreeFem++ environment [6] and then validated.

This approach makes it possible to incorporate in a simple way different types of discontinuity in a structure. These are vertical cracks (or joints) and debonding at an interface

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between layers. Incorporating the first type of discontinuity for instance consists in splitting a cracked layer into two sub-layers of similar mechanical properties one of which is used to fit the crack shape and the other to represent the healthy part of the layer. In terms of FE modeling, the vertical plane the crack belongs to is represented by its contour projected on the 2D-mesh along which nodes are doubled. Then, penalty terms are added to L to ensure the traction free boundary condition applying for the cracked sub-layer and continuity of the other layers on this doubled-node (curved line) segment. A crack growing vertically can thus be handled without remeshing just by making the variable thickness of the cracked sub-layer evolve according to some fracture mechanics criteria. This type of treatment with some adaptations can be considered also to introduce debonding between layers of a structure or be generalized to take into account cracks lying in non-vertical planes still taking advantage of the 2D discretization.

As an illustration, the approach developed is used to simulate the mechanical response of a pavement structure with cracks on which a full-scale accelerated fatigue test was performed.

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Keywords: mixed FEM, layerwise model, crack, debonding, pavement