ECONOMETRIC MODELLING OF THE RELATIONSHIP BETWEEN MONEY, INCOME AND INTEREST RATES IN THE U.K. : 1963 TO 1978

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SUMMARY

This thesis investigates empirically the relationship between money, income and interest rates in the U.K. over the period 1963 to 1978. After developing univariate models of the time series' proxying these theoretical variables, the paradox existing between the conventional theoretical model, the IS/LM framework, and the usual empirical practice of directly estimating the demand for money function is investigated. It is shown that the crucial issues are the exogeneity assumptions placed on the IS/LM framework. As such assumptions cannot be tested in a static framework, a dynamic analogue of the IS/LM model is developed, along with appropriate methods for testing exogeneity in dynamic multivariate systems. Empirical tests show that the assumptions of the exogeneity of money and government expenditure are invalid, but that the direct estimation of demand for money functions is appropriate. This leads to an investigation of the dynamic structure and functional form of this function using recently developed techniques based on specification search procedures.

A major conclusion of this study is that the IS/LM model is an invalid framework for empirical research, and in particular money cannot be regarded as being exogenously determined. Indeed, there is no evidence of feedback from money to either real income or prices, although both statistical and economic reasons are advanced for the possibility that such feedback cannot be detected by the techniques employed. Important short run dynamic effects are found on the demand for money with respect to real income, prices and interest rates. Furthermore, both the wage rate and

an own rate of interest variable are also important determinants of money demand. The demand for narrow money function also exhibits sensible long run behaviour and has an adequate predictive performance but, unfortunately, the broad money function has no long run properties and predicts unsatisfactorily.

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INTRODUCTION AND OVERVIEW

In this thesis an attempt is made to empirically model the relationship between money, income and interest rates in the United Kingdom over the period 1963 to 1978. As these variables are among the most important appearing in aggregate macroeconomic models of the economy, it was felt that a detailed investigation of their interaction over a period of great upheaval in the U.K. economy using recently developed quantitative techniques would be of interest and use to researchers in the field of macroeconomic modelling.

Thus chapter 1 considers the decision as to which of the numerous published series should best represent these theoretical variables and embarks on an initial statistical investigation of the time series so chosen. This culminates in the development of univariate models describing the behaviour of the individual series over the data period. Two money and interest rate series' were actually investigated and (nominal) income was split into its real and price components, thus allowing six series to be analysed.

Such univariate models can only be regarded as a first step to analysing what is essentially a multivariate problem, and to model such interactions between variables a theoretical framework is essential. An available framework is the conventional text book IS/LM model (see, e.g. Chick (1973)), but the great majority of empirical research in this area appears to have concentrated on estimating one of the structural equations of this model in isolation, namely the demand for money function. In view of the numerous published studies of this function, a review of this literature is undertaken in section 2 of chapter 2, from which it is clear that unresolved problems of instability do still exist. Furthermore, it is a curious paradox that direct estimation of the demand for money function is, in fact, inconsistent with one of the assumptions of the IS/LM framework; the exogeneity of money. To show this and to develop its consequences further, the standard IS/LM framework is set out in section 3 of the chapter, with the familiar policy conclusions being drawn in terms of the partial derivatives of the model. Having set out this framework, the empirical consequences of the underlying exogeneity assumptions are developed and compared with the conventional approach of estimating the demand for money function directly. It is shown that under the assumptions of the IS/LM framework, inconsistent estimators of the income and interest elasticities of the demand for money result from direct estimation of the function. Such a finding either throws into doubt the theoretical underpinnings of the IS/LM model or seriously questions the validity of the empirical research on the demand for money. The crucial issue is the exogeneity assumptions of the IS/LM paradigm, the validity of which cannot be empirically tested within the static framework of this chapter. However, a dynamic extension of the framework, which should also be welcomed on theoretical grounds, is capable of allowing these exogeneity assumptions to be tested.

The theoretical construction of techniques designed to test exogeneity in multivariate models using the framework supplied by Zellner and Palm (1974) and Wallis (1977) is undertaken in the first section of chapter 3. The close connection between the definition of exogeneity employed here and the concept of causality advanced by Granger (1969) is drawn in the second section, while the recent criticism of this concept and the tests based upon it (see Zellner (1978)) is analysed in section 3. A dynamic analogue of the static IS/LM framework based on Laidler (1973) is then developed to enable these techniques to test the crucial assumptions of the exogeneity of money and government expenditure. Such tests are performed in section 5, where it is shown that both definitions of money and government expenditure are rejected as being exogenously determined with respect to the endogenous variables of the model, i.e. real income, prices and the interest rate. This, therefore, invalidates the dynamic IS/LM model as a suitable framework for empirical research. Direct estimation of demand for money functions are shown to be appropriate, however, and the implications of these results and some reinterpretation of previous research on the causal patterns existing between money and income in the U.K. is discussed in the final section of this chapter.

As we have obtained empirical verification of the conventional approach of directly estimating the demand for money function, a detailed empirical study of this function is undertaken in the remaining chapters of this thesis.

Having emphasised the important dynamic nature of the demand for money function as a consequence of the tests of exogeneity, a methodology proposed by Hendry and Mizon (1978, 1979) for empirically determining the appropriate dynamic structure of a regression model is developed in chapter 4 in the context of the above function. This methodology is then employed on the four alternative combinations of variables available. An important feature of this work, apart from yielding an in depth application of this recently developed methodology, is to determine the empirical validity of a number of additional variables that have been theoretically proposed as determinants of the demand for money but have not, until now, been included empirically in the function.

Further modelling of the demand for money function is undertaken in chapter 5, where the assumption of a logarithmic functional form employed in previous chapters is relaxed by introducing the Box and Cox (1964) family of power transformations in the manner of Zarembka (1968) and Mills (1978). Particular attention is also paid to the joint consideration of the problems of functional form and autocorrelation, as proposed by Savin and White (1978).

Based on the models developed in these two chapters, chapter 6 selects the most appropriate models for both the demand for broad and narrow money functions and subjects these to post sample predictive tests. The final section of this chapter draws together the results of the modelling procedure and presents overall conclusions and suggestions for further research.

GLOSSARY

In subsequent chapters the following notation and test statistics are employed when analysing either the regression

$$y_t = \underset{\sim}{X}_t \underset{\sim}{\beta} + \varepsilon_t \quad , \qquad t = 1, 2, \ldots, T$$

where $\underset{\sim}{X}_t$ will typically contain a constant and lagged values of y, or the time series model

$$\phi(L)y_t = \theta(L)\varepsilon_t \; , \qquad t = 1, 2, \ldots, T$$

where $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator L.

T is the number of observations used for estimation.

k is the length of the coefficient vector $\underset{\sim}{\beta}$ or the number of estimated coefficients in $\phi(L)$ and $\theta(L)$. Estimated standard errors of the parameters are given in parantheses ( ).

t ratios are given in brackets [ ].

$\hat{\varepsilon}_t$ is the estimated residual in time period t.

$S = \sum_{t=1}^{T} \hat{\varepsilon}_t^2$ is the sum of squared residuals.

$\hat{\sigma} = (S/(T - k))^{\frac{1}{2}}$ is the error standard deviation.

$\hat{r}_s = \Sigma\hat{\varepsilon}_t\hat{\varepsilon}_{t-s}/\Sigma\hat{\varepsilon}_t^2$ is the sth order residual autocorrelation. It is also used in chapter 1 to denote the sth order sample autocorrelation of a time series.

$\hat{\phi}_{ss}$ is the sth order partial autocorrelation.

d_1 is the Durbin and Watson (1950, 1951) statistic for testing for first order serial correlation in the residuals from a static regression.

$h_1 \sim N(0, 1)$ is Durbin's (1970) generalisation of d_1 to dynamic regressions.

d_4 is Wallis' (1972) statistic for testing for simple fourth order serial correlation in the residuals from a static regression.

$h_4 \sim N(0, 1)$ is the generalisation of d_4 to dynamic regressions (see Breusch (1978)).

$Q(V) = T \sum_{s=1}^{V} \hat{r}_s^2 \sim \chi^2(V)$ is the Pierce (1971) portmanteau statistic for testing departures of the residuals from white noise. (See also Box and Pierce (1970)).

$\tilde{Q}(V) = T(T + 2) \sum_{s=1}^{V} (T - s)^{-1}\hat{r}_s^2 \sim \chi^2(V)$ is the modified portmanteau statistic proposed by Ljung and Box (1978).

$\phi(n) = T R_n^2 \sim \chi^2(n)$, where R_n^2 is the multiple correlation coefficient from the regression of $\hat{\varepsilon}_t$ on $[\hat{\varepsilon}_{t-1}, \ldots \hat{\varepsilon}_{t-n}, \underset{\sim}{X}_t]$ is the Lagrange multiplier test statistic proposed by Godfrey (1978 b, c) for testing general nth order serial correlation in the regression residuals.

$F(r, T - k) = \left(\frac{S_R}{S} - 1\right)\left(\frac{T - k}{r}\right) \sim F(r, T - k)$ is the test of the imposition of r linear restrictions on $\underset{\sim}{\beta}$, where S_R is the sum of squared residuals from the restricted regression.

$F_c(k, T - 2k) = \left(\frac{S}{S_1 + S_2} - 1\right)\left(\frac{T - 2k}{k}\right) \sim F(k, T - 2k)$ is the Chow (1960) test for the stability of the $\underset{\sim}{\beta}$ vector over two sub-periods of the sample period, where S_1 and S_2 are the sum of squared residuals from the two subperiod regressions.

$\chi^2(r) = T \ln(S_R/S) \sim \chi^2(r)$ is the likelihood ratio test of the imposition of r nonlinear restrictions on $\underset{\sim}{\beta}$.

μ_j are the roots of either the $\phi(L)$ polynomial or the lag polynomial attached to y in the regression model.

ABBREVIATIONS

AD	Autoregressive dynamic
ARMA	Autoregressive - moving average
ARIMA	Autoregressive - integrated - moving average
CCC	Competition and credit control
IV	Instrumental variable
LR	Likelihood ratio
OLS	Ordinary least squares
SURE	Seemingly unrelated regression equations
2SLS	Two stage least squares
3SLS	Three stage least squares.

CHAPTER ONE: INITIAL DATA ANALYSIS

1.1 THE CHOICE OF DATA

In investigating the relationship between money, income and interest rates in the U.K., a researcher is immediately confronted with the decision as to which of the available published series are the most appropriate empirical proxies for these theoretical variables.

Two money series have been published consistently since the early 1960's; the M1 series, defined as notes and coin in cirulation with the public plus private sector sterling sight deposits, and the M3 series, defined as M1 plus private sector sterling time deposits, public sector sterling deposits and U.K. residents deposits in other currencies. The M1 series has typically been regarded as the most suitable proxy for the theoretical construct of "transactions money balances" while empirical testing of theories based on the precautionary and speculative motives for holding money have usually employed the more widely defined M3 series. Both series are used in this study, with the former series being termed narrow money and the latter series broad money, thus reflecting their relative definitions.

Various measures of national income are available and the GDP at factor cost series was actually chosen, this being probably the most popular proxy for income used in the U.K. However, rather than simply using this nominal measure of national income, the series was split into its real and price components, thus empirically following the conventional theoretical distinction between real income (or output) and the price level. In effect, then, we are actually modelling the relationship between money, interest rates, real income and prices and accordingly the published GDP at constant price series proxies for real income with the price series being the implicit deflator of nominal income, defined as the ratio of GDP to GDP at constant prices.

A number of interest rates are published, of which two were chosen. The theoretical literature often makes the somewhat arbitrary distinction between short and long term interest rates and accordingly rates were chosen to reflect this simplification of the term structure. The rate on three month Local Authority loans was used as a proxy for the short interest rate in preference to the historically conventional Treasury Bill rate, which has been found to be a poor reflection of the opportunity cost of holding money over the period to be analysed. However, following convention the yield on $2\frac{1}{2}$ per cent Consolidated Stock was employed as the representative long interest rate.

The real income series (and hence the derived price series) is only available on a quarterly basis, with the two money series only available monthly since 1970. Hence quarterly data was used with the data period chosen to be 1963 I to 1977 IV, a total of 60 observations, this being the longest period for which consistent money series were available. As the interest rate series are available on a more frequent basis, quarterly series' were derived as three month averages of the monthly observations.

These six data series, accompanied by exact definitions and sources, are given in the Appendix to this chapter, where all additional series used in this study may also be found.

1.2 MAIN FEATURES OF THE DATA

The data series are plotted in Figures 1.1, 1.2 and 1.3, where in the latter RC denotes the Consol Yield and RL the Local Authority rate. Summary statistics of the series and the correlation matrix are presented as Table 1.1, while the correlograms for each of the series are shown in Table 1.2

TABLE 1.1

Variable	$\bar{X}$	S	V	%Δ
M1	11226	4423	.39	5.9
M3	23218	12124	.52	7.9
Y	10795	1116	.10	2.6
P	1.255	0.561	.45	5.8
RC	0.095	0.031	.33	3.1
RL	0.082	0.029	.35	2.4

$\bar{X}$: mean   S : standard deviation   V : coefficient of variation

%Δ : average percentage change per quarter.

CORRELATION MATRIX

Variable	M1	M3	Y	P	RC	RL
M1	1.00	.98	.82	.99	.83	.54
M3		1.00	.85	.97	.90	.67
Y			1.00	.77	.87	.69
P				1.00	.82	.54
RC					1.00	.81
RL						1.00

M1, M3, REAL INCOME

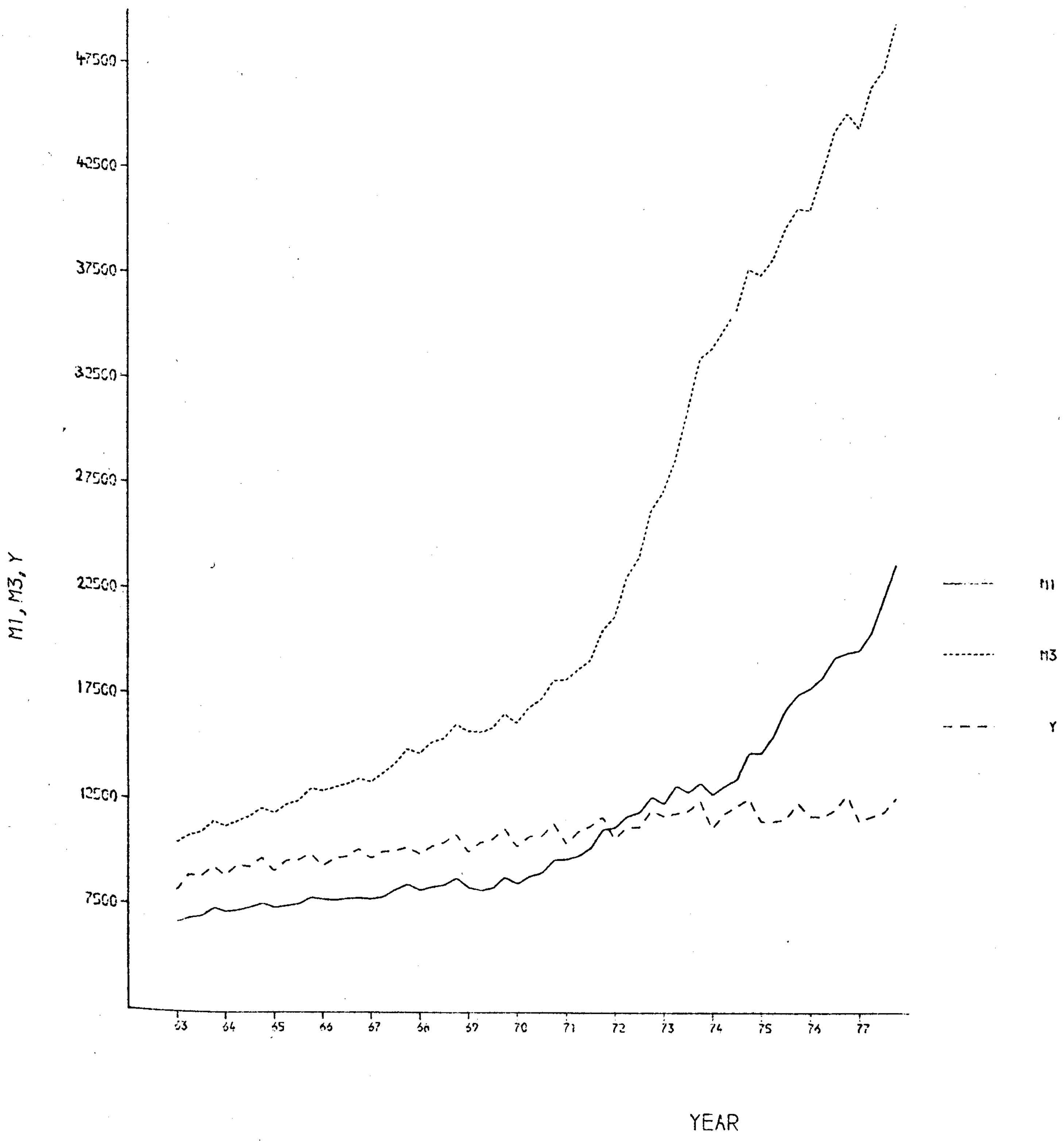


FIGURE 1.1

PRICE

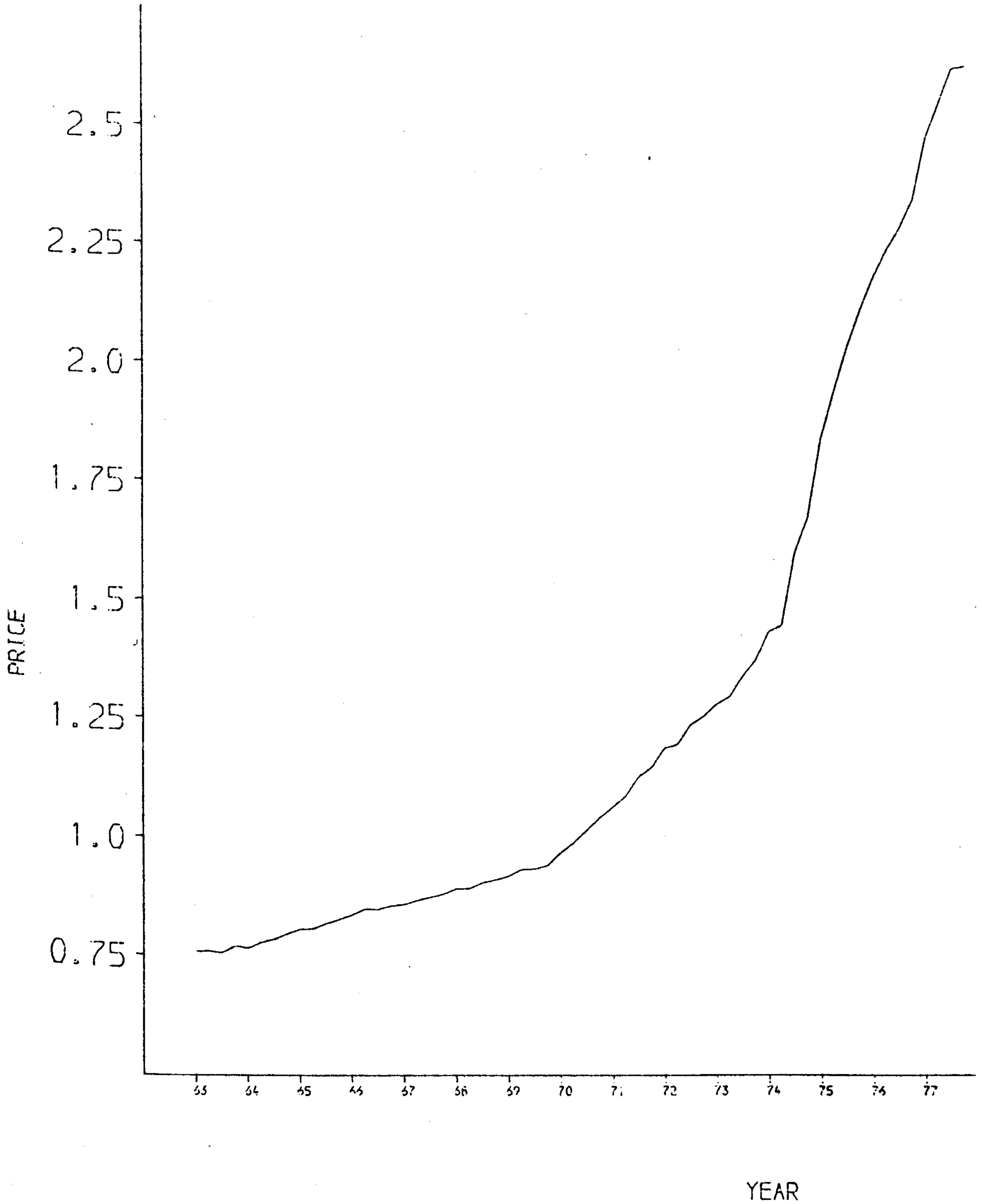


FIGURE 1.2

CONSOL YIELD/LOCAL AUTHORITY RATE

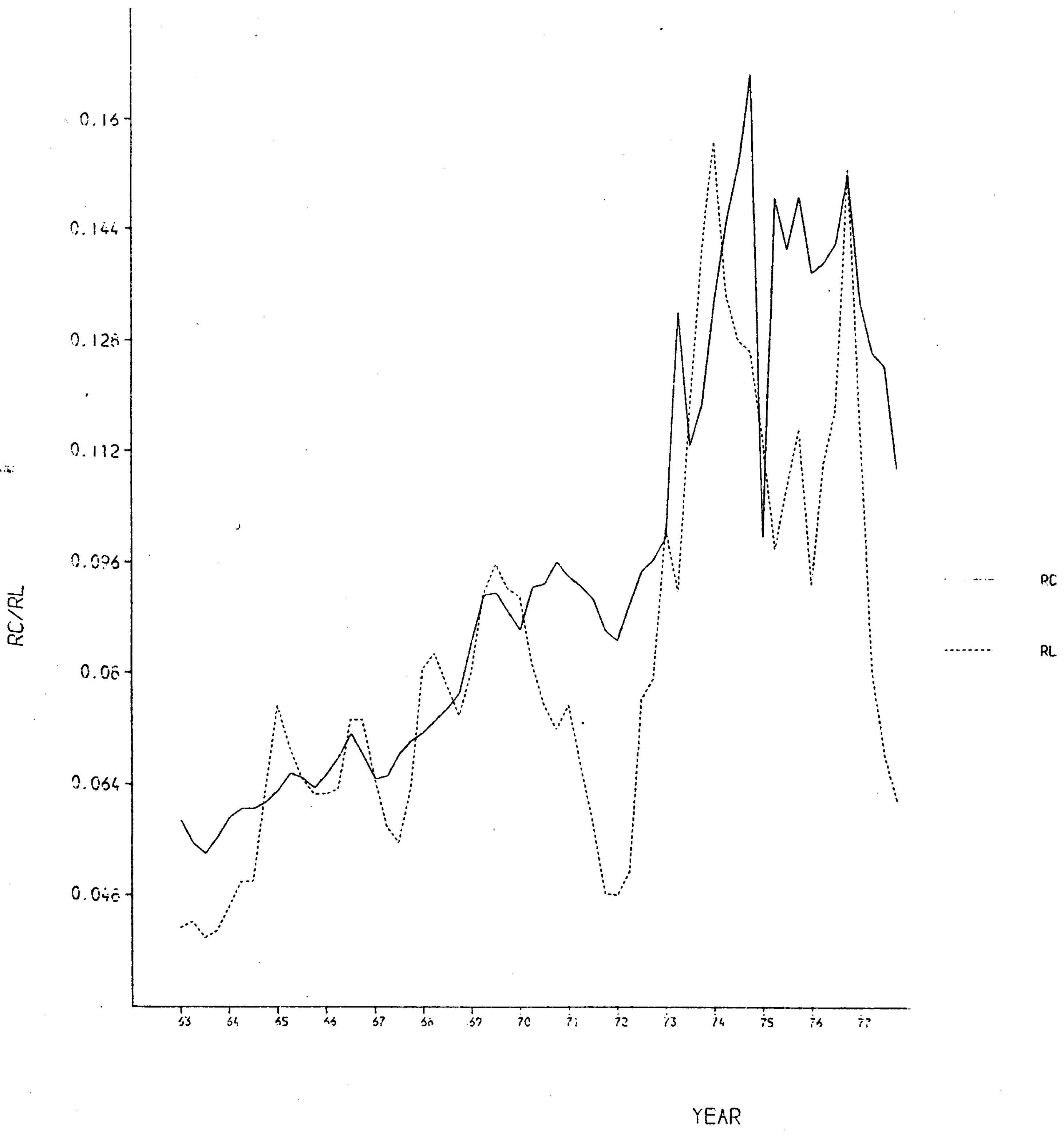


FIGURE 1.3

TABLE 1.2

CORRELOGRAMS FOR UNTRANSFORMED VARIABLES

Autocorrelations / Variables	$\hat{r}_1$	$\hat{r}_2$	$\hat{r}_3$	$\hat{r}_4$	$\hat{r}_5$	$\hat{r}_6$	$\hat{r}_7$	$\hat{r}_8$
M1	.92	.86	.81	.77	.72	.67	.62	.57
M3	.95	.91	.88	.85	.80	.76	.71	.67
Y	.82	.79	.75	.84	.70	.69	.63	.72
P	.94	.89	.84	.78	.73	.67	.62	.56
RC	.90	.89	.85	.83	.79	.74	.65	.64
RL	.87	.73	.58	.50	.41	.31	.23	.16

From this information the following features emerge. The macro-aggregates, M1, M3, real income and prices, are highly correlated with almost perfect correlation between the two money series and prices. This is obviously strongly influenced by the pronounced upward trends of the three series with M3 having the largest average rate of increase and M1 and prices having almost identical average rates of increase. The rates of increase in all three series have accelerated in the second half of the data period, this acceleration appearing almost contemporaneously around 1970.

Real income has increased at a much slower rate than the other macro-aggregates with the rate of increase being fairly constant throughout the entire period.

The long interest rate, RC, is more strongly related to the macro-aggregates than the short rate, RL, but the behaviour of the two interest rates are reasonably similar. Although RL exhibits greater fluctuations, both series have a general upward trend, with the dip in the early 1970's being followed by a sharp acceleration. There is evidence in the last year of the data period of another downward movement in the two rates.

As might be expected, there is evidence of seasonal fluctuations in the macro-aggregates, particularly real income, but no such evidence in either of the interest rate series.

As a consequence of these features the series were transformed to make them more amenable to further statistical analysis. All series were transformed logarithmically, the reasons for such a transformation being threefold. The transformation scales the series more appropriately by making them all the same order of magnitude. As well as allowing easier comparison of the series, this has the additional benefit of increasing computational efficiency. Transforming logarithmically is a well known method of helping to stabilise the variance and induce stationarity of a time series and is particularly useful in the present case where all the series display marked upward trends and, particularly the interest rates, greater variation in the second half of the data period. Finally, in subsequent econometric modelling of the interactions between the series, the logarithmic transformation is theoretically desirable and allows parameter inferences to be drawn more easily. It may also be noted that after the transformation real income and prices are additive components of nominal income, and such an aggregation may be particularly useful.

Since the macro-aggregates display four period seasonality, the logarithmically transformed money, real income and price series were seasonally adjusted by the method of the ratio of the series to a moving average. The interest rate series' were not adjusted.

Plots of the transformed time series are shown in Figures 1.4, 1.5 and 1.6, with the corresponding summary statistics and correlation matrix displayed as Table 1.3 and the correlograms as Table 1.4, noting that the transformed M1 and M3 series are now denoted as narrow (MN) and broad (MB) money respectively. As expected, the basic features of the data remain with the exception that seasonality appears to have been effectively removed from the macro-aggregates.

The time series are now in the form on which the main body of statistical analysis is performed and it is to such initial analysis that we now turn to.

1.3 UNIVARIATE MODELLING OF THE TIME SERIES

It is now well recognised that univariate modelling of individual time series is a useful first stage in the building of a multivariate (i.e. econometric) model describing the interactions between a set of economic variables, although it should not be regarded as a substitute (see, e.g. Zellner and Palm (1974), Prothero and Wallis (1976), Wallis (1977) and Granger and Newbold (1977)).

The conventional approach to univariate time series modelling has become the Box-Jenkins (1970) cycle of identification, estimation and diagnostic checking using the class of autoregressive - integrated - moving average models customarily denoted as ARIMA (p, d, q) and defined as

$$\phi(L)\Delta^d x_t = \theta_0 + \theta(L)\varepsilon_t \quad , \quad t = 1, \ldots T \tag{1.1}$$

where x_t is the time series under consideration,

TABLE 1.3

Variable	$\bar{X}$	S
MN	9.260	.354
MB	9.928	.494
Y	9.281	.100
P	0.145	.391
RC	-2.411	.323
RL	-2.563	.340

CORRELATION MATRIX

Variable	MN	MB	Y	P	RC	RL
MN	1.00	.98	.89	.99	.89	.60
MB		1.00	.93	.98	.94	.70
Y			1.00	.86	.93	.74
P				1.00	.89	.61
RC					1.00	.81
RL						1.00

TABLE 1.4

CORRELOGRAMS FOR TRANSFORMED VARIABLES

Autocorrelations / Variables	$\hat{r}_1$	$\hat{r}_2$	$\hat{r}_3$	$\hat{r}_4$	$\hat{r}_5$	$\hat{r}_6$	$\hat{r}_7$	$\hat{r}_8$
MN	.94	.90	.86	.82	.78	.74	.69	.64
MB	.96	.93	.90	.87	.83	.79	.75	.71
Y	.92	.88	.85	.82	.78	.76	.72	.68
P	.95	.91	.87	.83	.78	.74	.69	.63
RC	.93	.91	.87	.84	.80	.76	.69	.66
RL	.89	.74	.59	.49	.40	.30	.20	.11

BROAD MONEY/NARROW MONEY/REAL INCOME

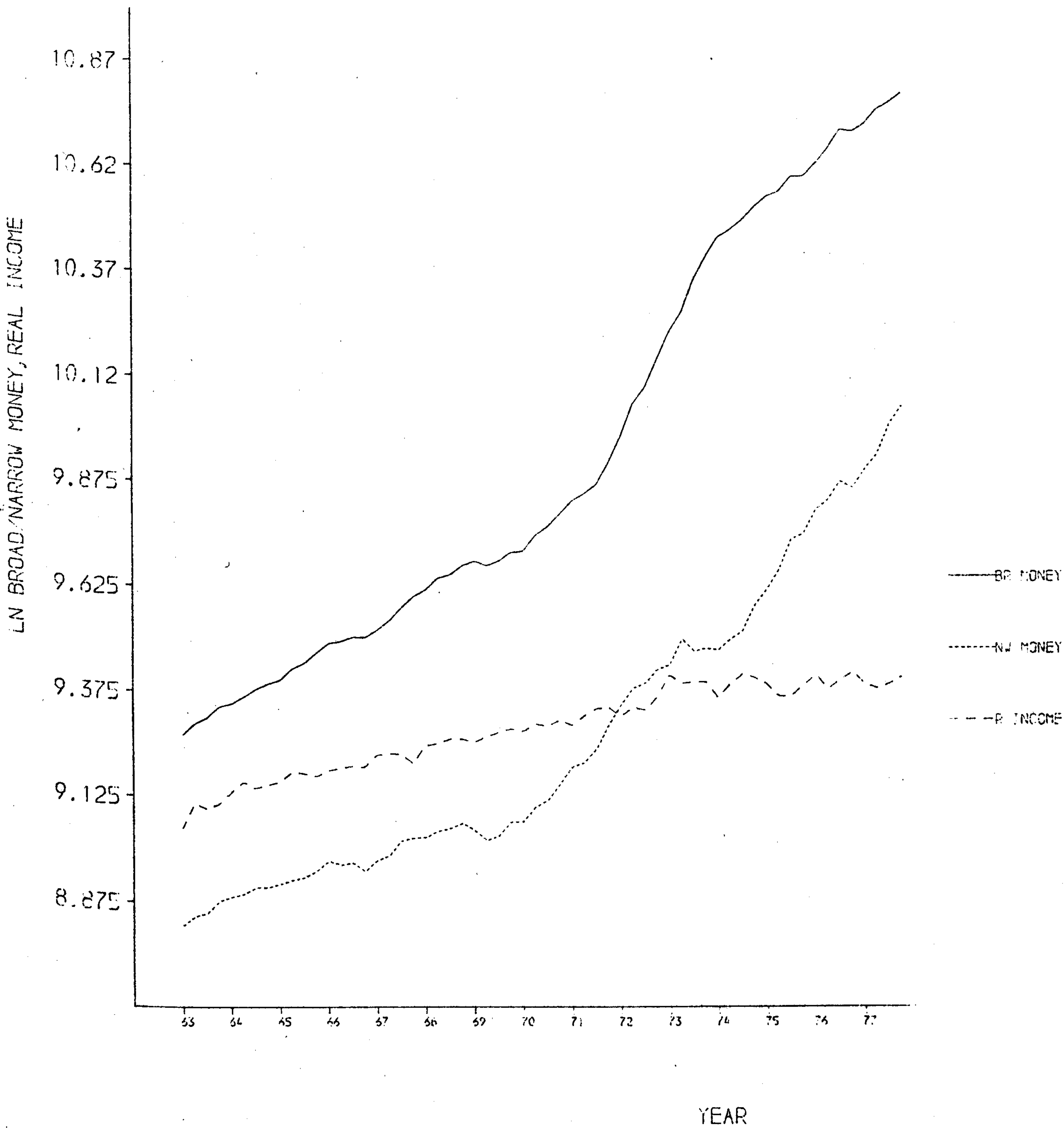


FIGURE 1.4

PRICE LEVEL

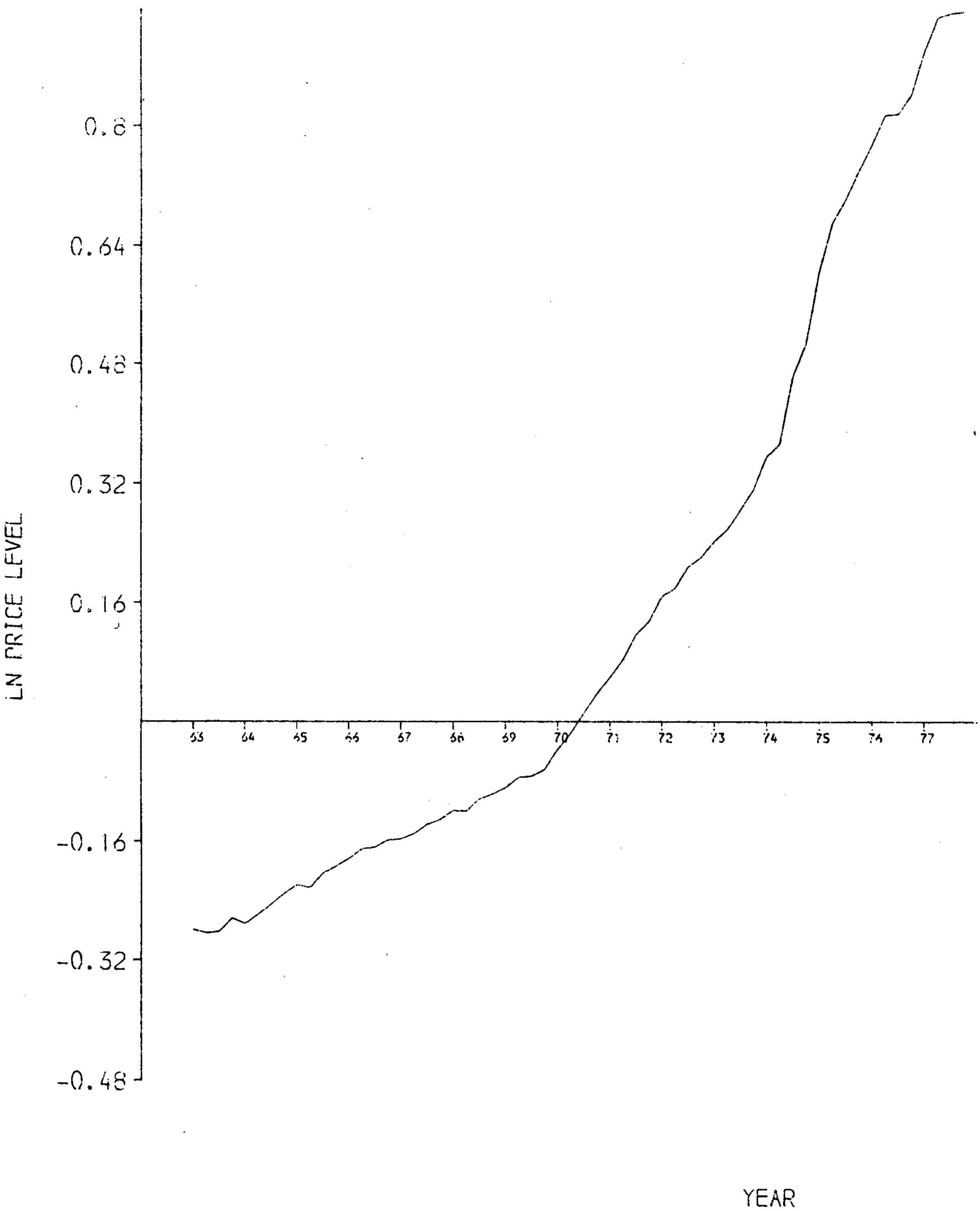


FIGURE 1.5

LOCAL AUTHORITY RATE/CONSOL YIELD

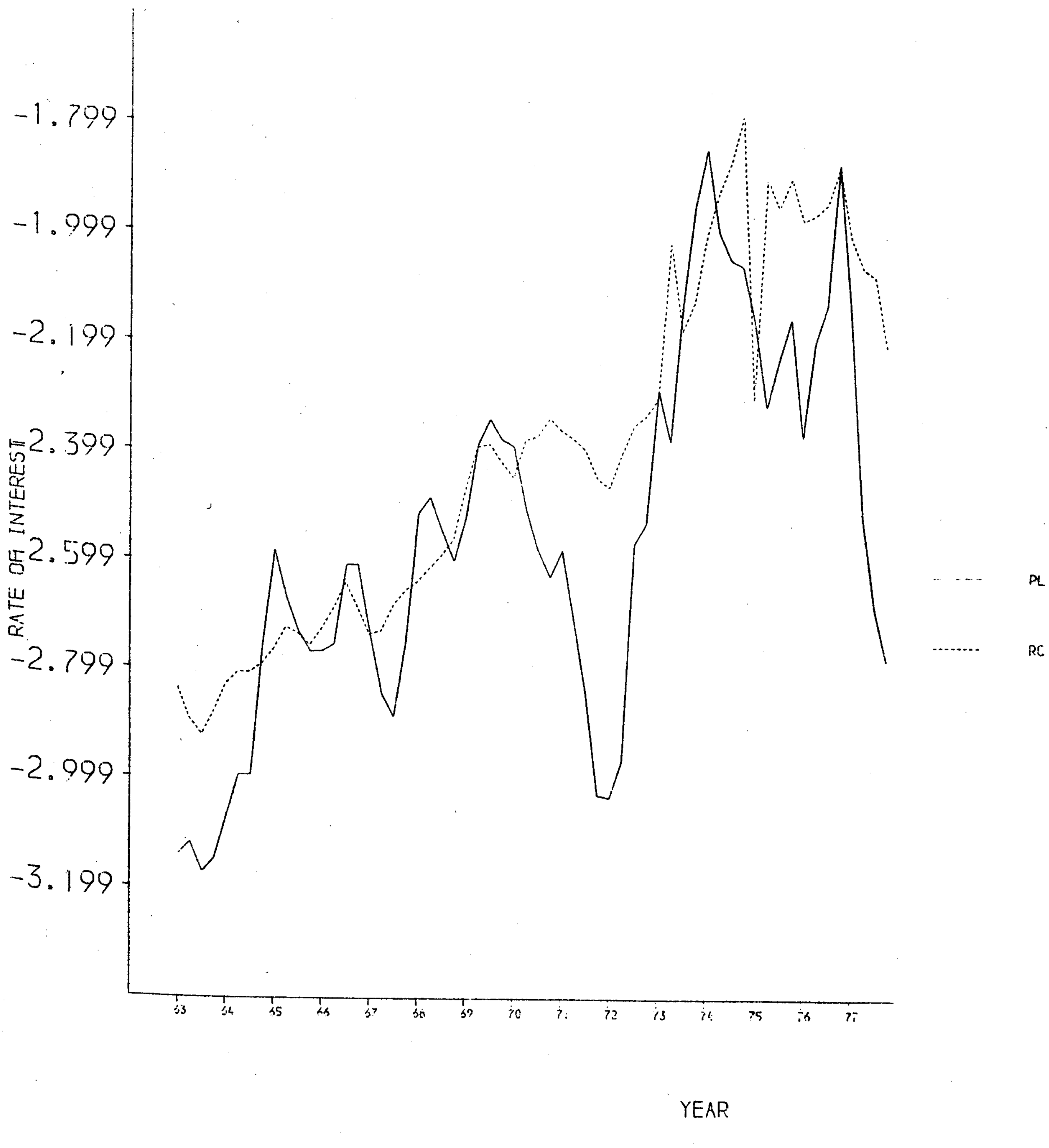


FIGURE 1.6

$\phi(L) = (1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p)$ is a pth order polynomial in the lag operator L (where $L^r x_t = x_{t-r}$), θ_0 is a constant, $\theta(L) = (1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_q L^q)$ is a qth order polynomial in L, $\Delta^d = (1 - L)^d$ is a dth order differencing operator in L and ε_t is a white noise error series. Accordingly, this methodology was employed on each of the transformed series discussed in Section 1.2.

Typically the process (1.1) is assumed to be stationary and invertible, necessary and sufficient conditions for which are that the roots of $\phi(z)$ and $\theta(z)$ must all lie outside the unit circle. To insure stationarity, a property not commonly found in economic time series, the differencing operator Δ^d is applied to the original series. This, in effect, assumes that the original series is homogeneous nonstationary with d roots lying on the unit circle. Extracting these roots therefore allows the differenced series to satisfy the stationarity conditions.

Following Box and Jenkins (1970), the degree of differencing, d, required to render a series stationary is selected by considering whether the sample autocorrelation function (i.e. the correlogram) dies out quickly for a particular value of d. A useful ancillary method for determining d is to consider the sample variances of the series obtained for alternative values of d. Anderson (1976) suggests that the minimum sample variance will occur at the value of d that induces stationarity. Inspection of the correlograms presented as Table 1.4 reveals that none of the six sample autocorrelation functions die out rapidly, the gentle linear decline in the autocorrelationsbeing typical of nonstationary time series. Thus differencing is required for each series and the sample autocorrelation function and the partial autocorrelation function (denoted by $\hat{\phi}_{ii}$) of each of the series for d = 1 and 2 are shown in Tables 1.5 to 1.10, with plots of the first and second differences of each of the series being shown as Figures 1.7 to 1.12.

On the basis of this information the Box-Jenkins methodology was employed on each of the series and the results of such modelling are discussed below.

(a) <u>Series MN</u>

Inspection of Table 1.5 and Figure 1.7 indicates that the appropriate degree of differencing is d = 1. Both the sample and partial autocorrelation functions appear to cut off after two lags although $\hat{r}_4$ is also just significant, possibly due to some inadequacy of the seasonal adjustment procedure. With this rather limited information both the ARIMA (2, 1, 0) and ARIMA (0, 1, 2) models were tentatively entertained, yielding on estimation

$$(1 - \underset{(.123)}{.131}L - \underset{(.130)}{.439}L^2)\Delta MN_t = \underset{(.004)}{.010} + \hat{\varepsilon}_t, \quad \hat{\sigma} = .0211, \quad Q(15) = 19.2 \tag{1.2}$$

$$\Delta MN_t = \underset{(.005)}{.021} + (1 + \underset{(.111)}{.286}L + \underset{(.112)}{.582}L^2)\hat{\varepsilon}_t, \quad \hat{\sigma} = .0207, \quad Q(18) = 17.0 \tag{1.3}$$

On the basis of the Box-Pierce (1970) portmanteau Q statistics, both models pass the diagnostic check. However, this statistic is somewhat unsatisfactory in view of the finding of Davies, Triggs and Newbold (1977) that it has an actual size in finite samples much smaller than the nominal size expected from asymptotic theory. As an additional diagnostic check, visual inspection of the individual residual correlations may be recommended as this should reveal any serious model deficiencies. No low order residual correlations

TABLE 1.5

SERIES ΔMN

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	.20	.42	.02	.26	.10	.13	-.04	.00	.28	.03	.20	.01
$\hat{\phi}_{ii}$	.20	.40	-.14	.14	.11	-.06	-.12	-.02	.43	-.17	-.03	.18

$S.E(\hat{r}_i) = .13$ for $i > 1$ $S.E(\hat{\phi}_{ii}) = .13$ $Var(\Delta MN) = .0005$

SERIES $\Delta^2 MN$

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.65	-.41	-.41	.28	-.15	.14	-.16	-.12	.30	-.26	.24	-.27
$\hat{\phi}_{ii}$	-.65	-.04	-.27	-.17	-.01	.02	-.09	-.49	.07	-.07	-.19	-.10

$S.E(\hat{r}_i) = .18$ for $i > 1$ $S.E(\hat{\phi}_{ii}) = .13$ $Var\ (\Delta^2 MN) = .0008$

TABLE 1.6

SERIES ΔMB

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	.55	.53	.48	.38	.35	.29	.19	.12	-.06	.05	-.04	-.17
$\hat{\phi}_{ii}$	.55	.33	.17	-.00	.01	-.00	-.09	-.10	0.27	.17	-.00	-.19

$S.E(\hat{r}_i) = .16$ for $i > 1$ $S.E.(\hat{\phi}_{ii}) = .13$ $Var(\Delta MB) = .0004$

SERIES $\Delta^2 MB$

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.47	.04	.05	-.08	.03	.04	-.03	.11	-.32	.23	.03	-.19
$\hat{\phi}_{ii}$	-.47	-.24	-.06	-.09	-.05	.03	.02	.14	-.28	-.08	.10	-.13

$S.E.(\hat{r}_i) = .15$ for $i > 1$ $S.E.(\hat{\phi}_{ii}) = .13$ $Var(\Delta^2 MB) = .0003$

TABLE 1.7

SERIES ΔY

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.16	-.24	-.02	.05	-.01	.18	-.06	.07	.08	-.39	.04	.20
$\hat{\phi}_{ii}$	-.16	-.28	-.13	-.06	-.06	.18	.02	.19	.18	-.34	-.07	-.06

$S.E.(\hat{r}_i) = .13$ for $i > 1$ $\quad S.E.(\hat{\phi}_{ii}) = .13 \quad Var(\Delta Y) = .0004$

SERIES $\Delta^2 Y$

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.42	-.16	.06	.05	-.07	.18	-.18	.04	.25	-.39	.09	.11
$\hat{\phi}_{ii}$	-.42	-.40	-.29	-.19	-.23	.08	-.07	.01	.38	-.10	-.06	-.13

$S.E.(\hat{r}_i) = .15$ for $i > 1$ $\quad S.E.(\hat{\phi}_{ii}) = .13 \quad Var(\Delta^2 Y) = .0009$

TABLE 1.8

SERIES ΔP

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	.48	.51	.38	.41	.35	.19	.23	.29	.18	.23	.07	.15
$\hat{\phi}_{ii}$	.48	.37	.07	.14	.06	-.18	.05	.21	-.10	.08	-.13	-.05

$S.E.(\hat{r}_i) = .16$ for $i > 1$, $\quad S.E.(\hat{\phi}_{ii}) = .13 \quad Var(\Delta P) = .0004$

SERIES $\Delta^2 P$

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.54	.22	-.19	.09	.07	-.17	-.02	.15	-.14	.23	-.21	.06
$\hat{\phi}_{ii}$	-.54	-.10	-.16	-.09	.12	-.12	-.25	.09	-.07	.14	.09	-.13

$S.E.(\hat{r}_i) = .16$ for $i > 1$, $\quad S.E.(\hat{\phi}_{ii}) = .13 \quad Var(\Delta^2 P) = .0004$

TABLE 1.9

SERIES ΔRC

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.41	.09	-.07	.07	.01	.23	-.46	.23	-.02	-.07	.02	.01
$\hat{\phi}_{ii}$	-.41	-.10	-.09	.01	.06	.32	-.31	-.09	.06	-.16	-.06	.06

$S.E.(\hat{r}_i) = .15$ for $i > 1$, $S.E.(\hat{\phi}_{ii}) = .13$ $Var(\Delta RC) = .0115$

SERIES $\Delta^2 RC$

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.68	.24	-.11	.09	-.10	.32	-.50	.35	-.09	-.01	-.04	.07
$\hat{\phi}_{ii}$	-.68	-.40	-.35	-.25	-.33	.33	.01	-.12	.08	.03	-.16	-.19

$S.E.(\hat{r}_i) = .18$ for $i > 1$, $S.E.(\hat{\phi}_{ii}) = .13$ $Var(\Delta^2 RC) = .0327$

TABLE 1.10

SERIES ΔRL

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	.26	.04	-.25	.02	-.02	.02	-.07	-.15	-.31	-.17	.11	.20
$\hat{\phi}_{ii}$	.26	-.03	-.28	.18	-.06	-.05	-.00	-.17	-.27	-.03	.16	.00

$S.E.(\hat{r}_i) = .14$ for $i > 1$ $S.E.(\hat{\phi}_{ii}) = .13$ $Var(\Delta RL) = .0201$

SERIES $\Delta^2 RL$

i	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{r}_i$	-.36	.03	-.38	.24	-.06	.09	-.04	.08	-.21	-.11	.13	.10
$\hat{\phi}_{ii}$	-.36	-.11	-.47	-.13	-.15	-.16	.00	.06	-.19	-.36	-.17	-.25

$S.E.(\hat{r}_i) = .14$ for $i > 1$ $S.E.(\hat{\phi}_{ii}) = .13$ $Var(\Delta^2 RL) = .0300$

NARROW MONEY

FIRST, SECOND DIFFFRENCES

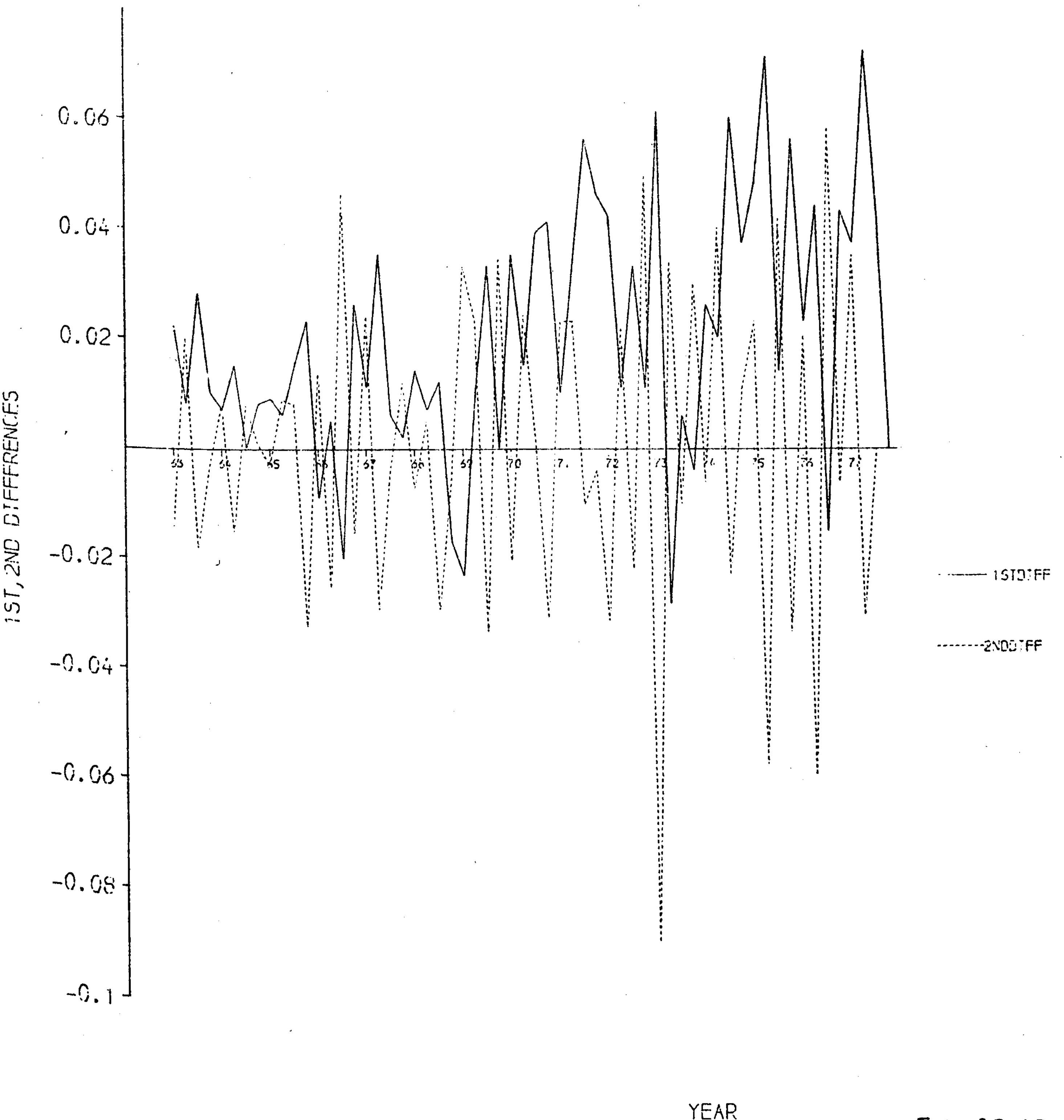


FIGURE 1.7

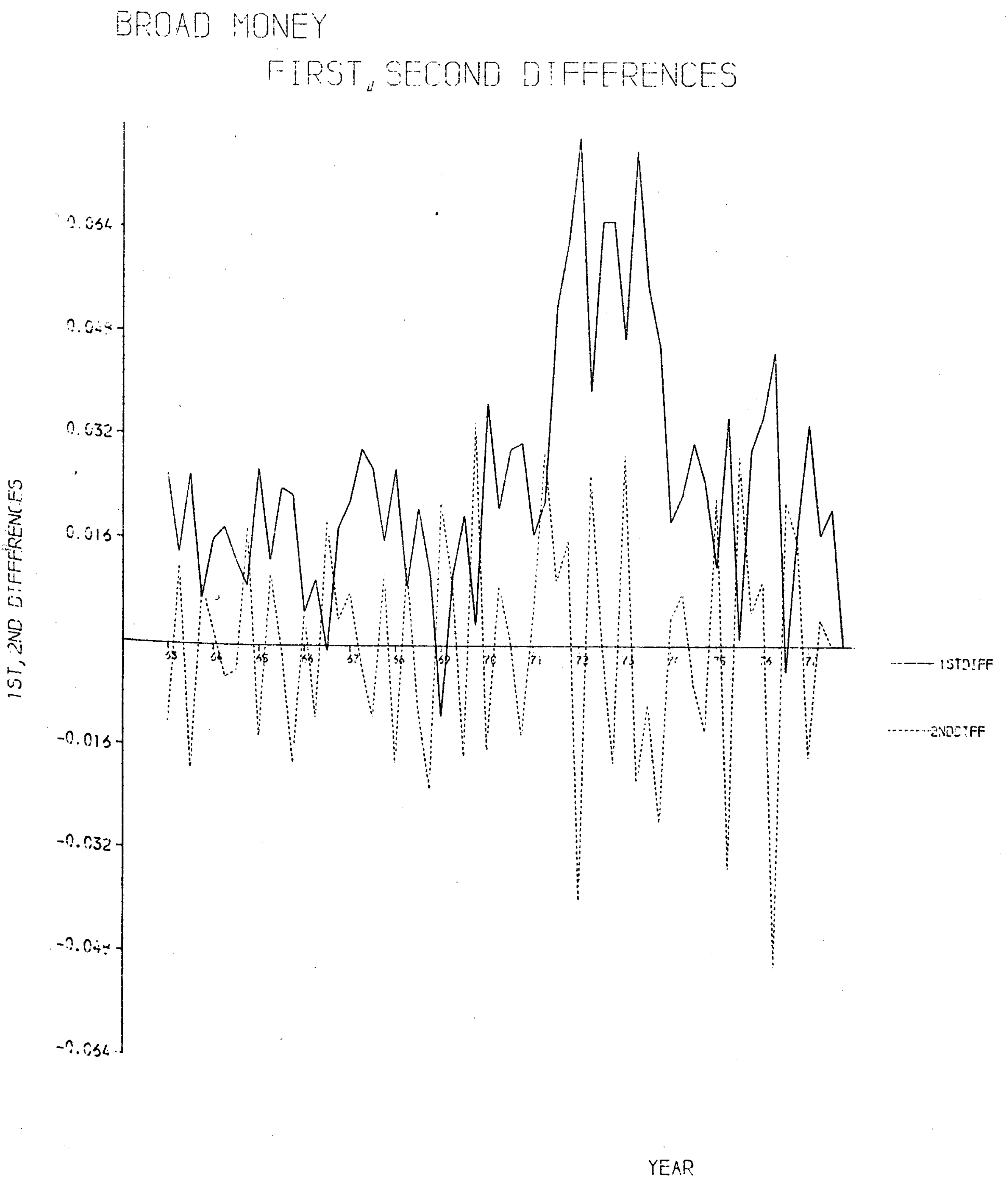

BROAD MONEY
FIRST, SECOND DIFFERENCES
0.064
0.048
0.032
0.016
-0.016
-0.032
-0.048
-0.064
1ST, 2ND DIFFERENCES
1STDIFF
2NDDIFF
70
71
72
73
75
YEAR

FIGURE 1.8

REAL INCOME

FIRST, SECOND DIFFERENCES

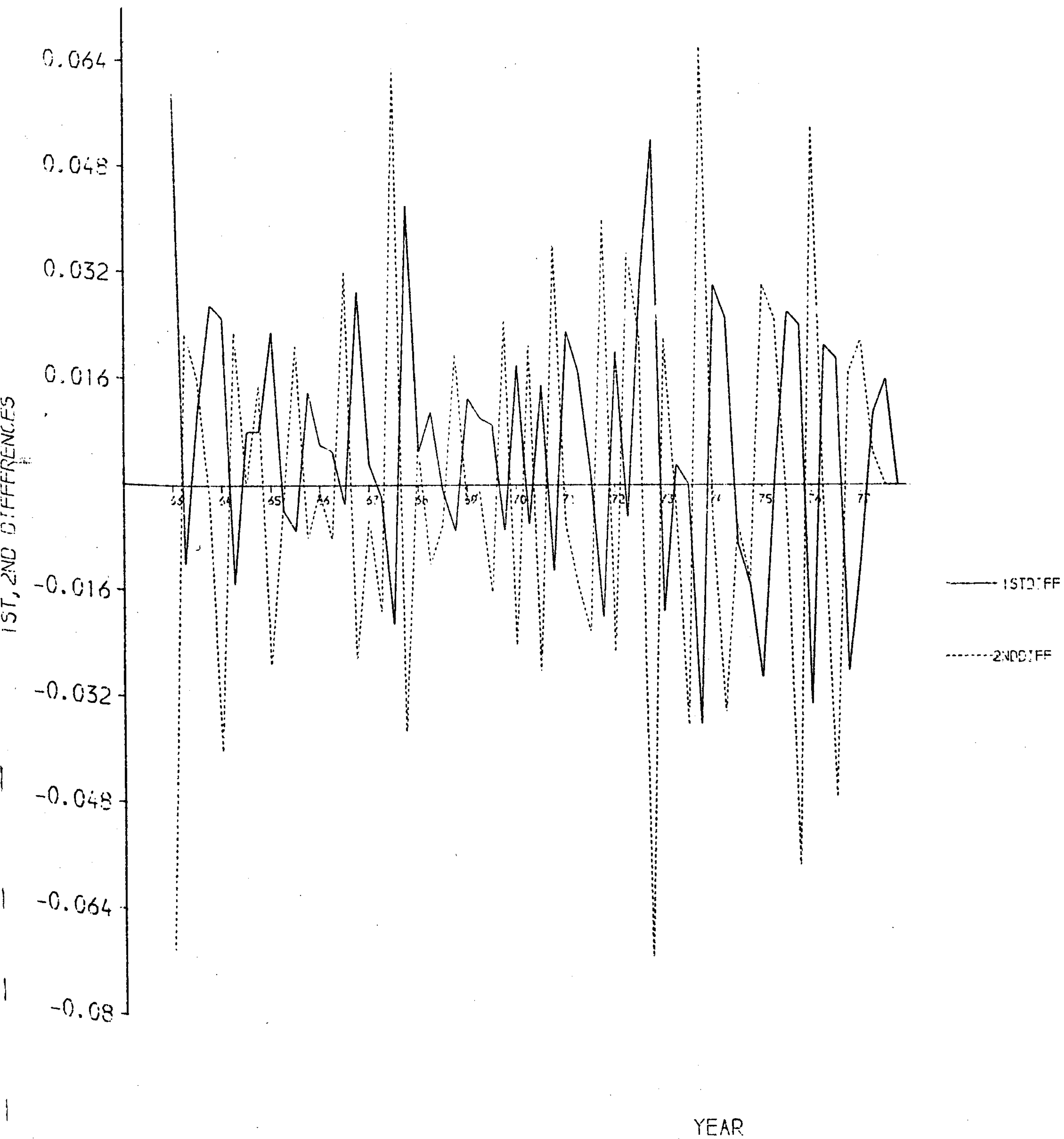


FIGURE 1.9

PRICE-FIRST, SECOND DIFFERENCES

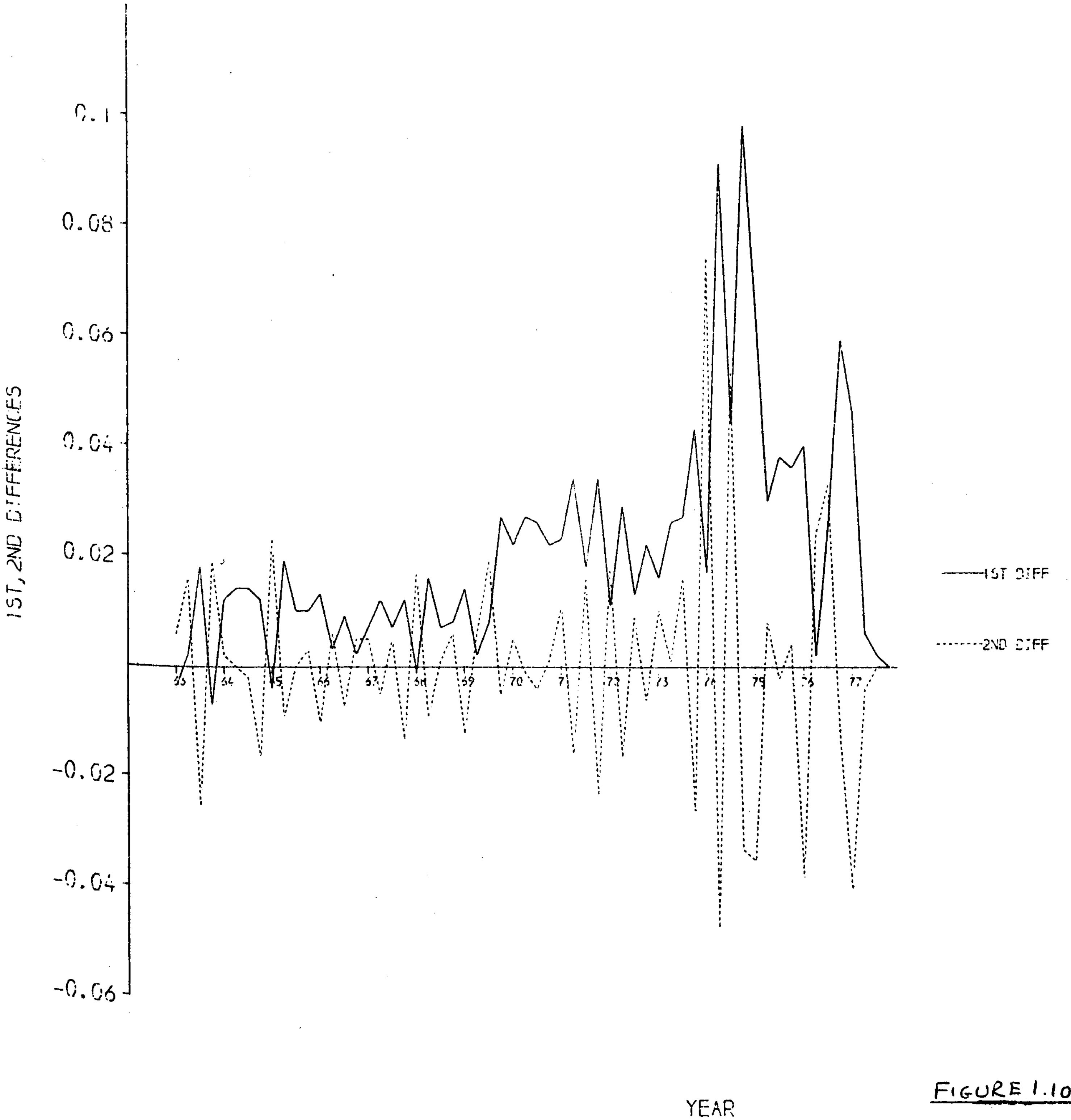


FIGURE 1.10

CONSOL YIELD -
FIRST, SECOND DIFFERENCES

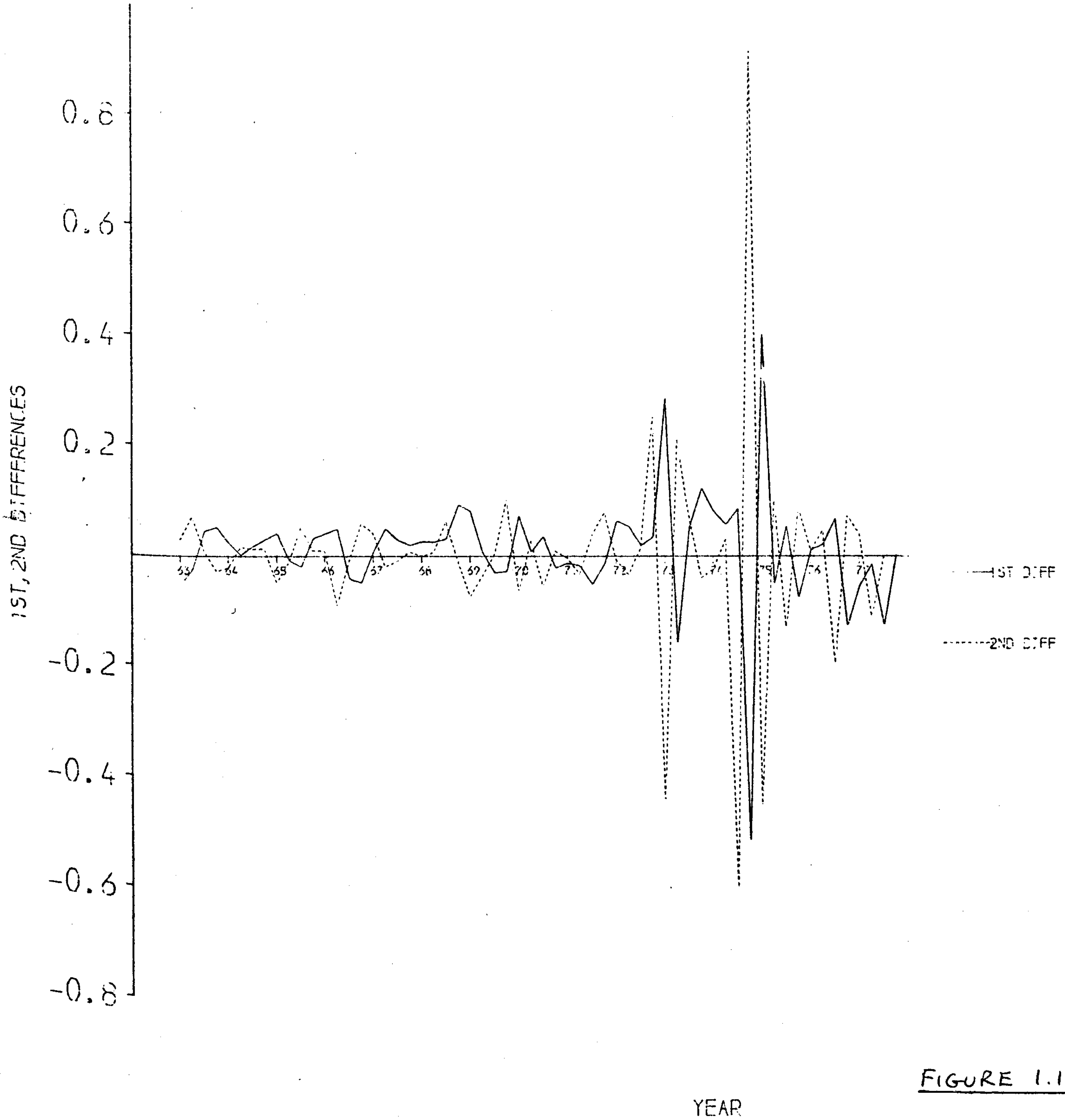


FIGURE 1.11

LOCAL AUTH. RATE-
FIRST, SECOND DIFFERENCES

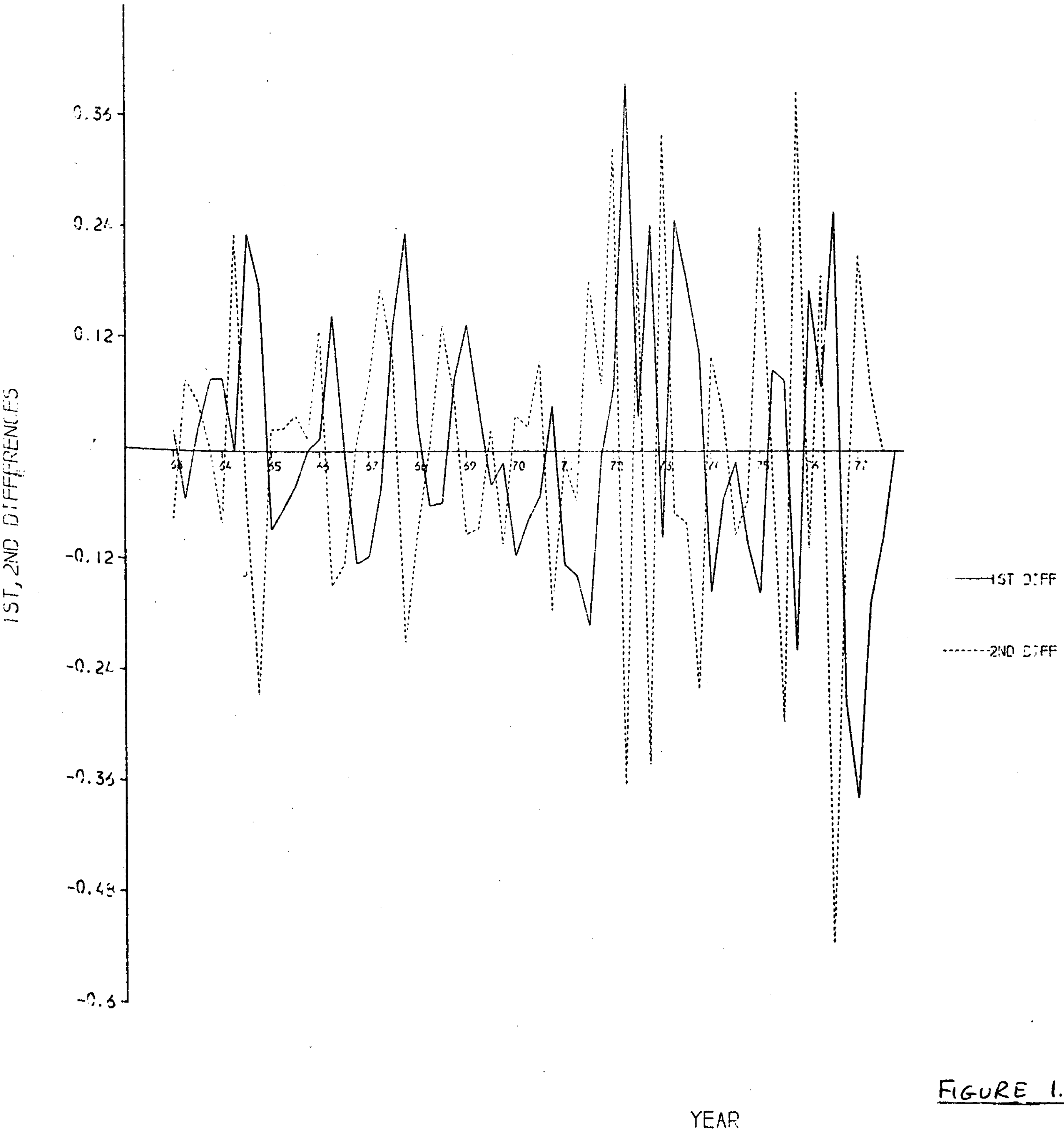


FIGURE 1.12

were found to be significant although for both models $\hat{r}_7$ and $\hat{r}_9$ were reasonably large, perhaps again as a consequence of the prior seasonal adjustment. Nevertheless, no serious inadequacies are revealed in either model, with the moving average model (1.3) being preferred as it has more precisely determined coefficients and a smaller error standard deviation.

(b) Series MB

Inspection of Table 1.6 and Figure 1.8 indicates that, on both the criteria of rapidly declining autocorrelation function and minimum sample variance, the appropriate degree of differencing is $d = 2$. Although only the first lags of the sample and partial autocorrelation functions are significant (apart from those at lag nine), $\hat{\phi}_{22}$ is quite large, thus lending more of a "tailing off" appearance to the partial autocorrelation function. Consequently an ARIMA (0, 2, 1) model was tentatively entertained, yielding on estimation

$$\Delta^2 MB_t = \underset{(.108)}{(1 - .580L)}\varepsilon_t \qquad \hat{\sigma} = .0154 \quad Q(19) = 11.0 \tag{1.4}$$

No serious inadequacies appear to be revealed by the Q statistic, which is well inside the .05 critical value. However, if the correlation plots for the ΔMB series are considered, an ARIMA (2, 1, 0) model is identified. This yields on estimation

$$(1 - \underset{(.129)}{.382}L - \underset{(.129)}{.312}L^2)\Delta MB_t = \underset{(.004)}{.008} + \hat{\varepsilon}_t\ , \ \hat{\sigma} = .0153 \quad Q(15) = 15.3 \quad \mu_1 = .78,\ \mu_2 = -.40 \tag{1.5}$$

This model also passes the diagnostic checks, with Q insignificant and only $\hat{r}_9$ significant. Furthermore, it has a slightly smaller error standard deviation than (1.4) and suggests that the appropriate degree of differencing may indeed be one. Corroborating evidence of this is supplied by the fact that neither of the roots of the $\phi(L)$ polynomial are close to unity, and indeed imposing a unit root, i.e. estimating the ARIMA(1, 2, 0) model, gives

$$\underset{(.118)}{(1 + .466L)}\Delta^2 MB_t = \hat{\varepsilon}_t \,, \qquad \hat{\sigma} = .0159 \tag{1.6}$$

which is clearly inferior to (1.5), as it is to (1.4) as well. These models highlight a potential difficulty in determining the most appropriate degree of differencing, and leave us with two models, an ARIMA (2, 1, 0) and an ARIMA (0, 2, 1), between which it is very difficult to discriminate.

(c) <u>Series Y</u>

From Table 1.7 and Figure 1.9 d = 1 is shown to be the appropriate degree of differencing. Although there are no significant low order sample correlations, perhaps suggesting that ΔY is white noise, $\hat{\phi}_{22}$ is significant thus identifying an ARIMA (2, 1, 0) model. Estimation obtained

$$(1 + \underset{(.130)}{.190L} + \underset{(.122)}{.272L^2})\Delta Y_t = \underset{(.003)}{.008} + \varepsilon_t \qquad \hat{\sigma} = .0179 \quad Q(15) = 23.7 \tag{1.7}$$

Although the Q statistic is only just inside the .05 critical value, there are only two significant residual correlations, $\hat{r}_{10}$ and $\hat{r}_{13}$, and hence the model appears acceptable. The coefficient ϕ_1 is rather imprecisely determined but its deletion worsens the fit of the model and reduces the significance of ϕ_2. Thus (1.7) is, somewhat hesitatingly, accepted as the appropriate model for the Y series.

(d) Series P

It is seen from Table 1.8 and Figure 1.10 that although both ΔP and $\Delta^2 P$ have the same sample variances, the sample autocorrelation function of the latter series dies out much more rapidly. Thus d is initially taken to be 2. The only significant correlations are at lag one and with such a paucity of information an ARIMA (1, 2, 1) model was tentatively entertained. By obtaining initial estimates of the coefficients (see, e.g. Anderson (1976, ch. 7)) the following approximate model results

$$(1 + .4L)\Delta^2 P_t = (1 - .2L)\varepsilon_t$$

This can be rewritten as either

$$(1 + .2L - .04L^2 + .008L^3 - \ldots)(1 + .4L)\Delta^2 P_t = \varepsilon_t$$

or

$$\Delta^2 P_t = (1 - .4L - .16L^2 - .064L^3 - \ldots)(1 - .2L)\varepsilon_t.$$

By dropping terms in L^2 or higher, these two models can be well approximated by either

$$(1 + .6L)\Delta^2 P_t = \varepsilon_t$$

or

$$\Delta^2 P_t = (1 - .6L)\varepsilon_t$$

i.e. either an ARIMA (1, 2, 0) or an ARIMA (0, 2, 1) model would appear to fit the data equally well. This is indeed found to be the case, for on estimation we obtain

$$(1 + .540L)\Delta^2 P_t = \hat{\varepsilon}_t \qquad \hat{\sigma} = .0177 \tag{1.8}$$
$$\quad (.113) \qquad\qquad Q(9) = 8.4$$

$$\Delta^2 P_t = (1 - .631L)\hat{\varepsilon}_t \qquad \hat{\sigma} = .0172 \tag{1.9}$$
$$\quad (.100) \qquad\qquad Q(19) = 10.3$$

Both models pass the diagnostic checks, with the moving average model (1.9) perhaps being preferred as it has a slightly smaller error standard deviation.

However, in view of the modelling of the MB series, consideration of the ΔP series would allow identification of an ARIMA (2, 1, 0) model, which on estimation yields

$$(1 - .291L - .377L^2)\Delta P_t = .007 + \hat{\varepsilon}_t, \qquad \hat{\sigma} = .0170 \tag{1.10}$$
$$(.126) \quad (.125) \qquad\qquad (.004) \qquad Q(15) = 15.5$$
$$\mu_1 = .78, \ \mu_2 = -.49$$

This model passes the diagnostic checks, with Q insignificant and only $\hat{r}_6$ significant. Neither of the roots of $\phi(L)$ are close to unity and the error standard deviation is smaller than for either (1.8) or (1.9). Again, it would appear that the possibility of overdifferencing may again be a problem, for (1.10) is certainly superior to either of the models identified for the $\Delta^2 P$ series.

(e) <u>Series RC</u>

Inspection of Table 1.9 and Figure 1.11 shows that d should be set at one. Again only the lag one correlations (of the early lags) are significant and following the analysis of the price series above, the ARIMA (1, 1, 0) and ARIMA (0, 1, 1) models were both considered. Estimation of the two models resulted in

$$(1 + .404L)\Delta RC_t = \hat{\varepsilon}_t , \qquad \hat{\sigma} = .0994 \qquad (1.11)$$
$$\quad (.123) \qquad\qquad Q(15) = 19.8$$

$$\Delta RC_t = (1 - .396L)\hat{\varepsilon}_t , \qquad \hat{\sigma} = .0990 \qquad (1.12)$$
$$\quad (.120) \qquad\qquad Q(19) = 11.9$$

Both Q statistics are insignificant and for both models only $\hat{r}_7$ is insignificant, as might be expected from the sample autocorrelation function. Thus both models pass the diagnostic checks, with some preference being shown for the moving average model (1.12).

(f) Series RL

From Table 1.10 and Figure 1.12 the appropriate degree of differencing is shown to be d = 1. Both $\hat{\phi}_{11}$ and $\hat{\phi}_{33}$ are significant and as none of the early sample autocorrelations are significant this suggests that an ARIMA (3, 1, 0) model should be considered. Estimation yields

$$(1 - .260L - .034L^2 + .304L^3)\Delta RL_t = .009 + \varepsilon_t , \quad \hat{\sigma} = .1383 \qquad (1.13)$$
$$(.133) \quad (.138) \quad (.138) \qquad\qquad (.019) \quad Q(15) = 22.3$$

The Q statistic is only just inside the .05 critical value but as only $\hat{r}_9$ and $\hat{r}_{14}$ are significant (1.13) is deemed to be acceptable. However, the coefficient ϕ_2 is insignificant, as might be expected from the partial autocorrelation function, and deletion of this coefficient, along with the insignificant constant, yields on re-estimation,

$$(1 - .271L + .288L^3)\Delta RL_t = \hat{\varepsilon}_t , \quad \hat{\sigma} = .1370 \qquad (1.14)$$
$$(.127) \quad (.137)$$

This more parsimonious model is obviously accepted as the appropriate explanation of the RL series.

From these descriptions it appears that the Box-Jenkins methodology has obtained adequate fits to each of the six time series considered. The finally chosen models are shown as Table 1.11. Nevertheless, for a number of the series the identification of an overall superior model is quite difficult as the information, in terms of interpretable patterns in the correlation functions, contained in the short series of sixty observations is somewhat limited. In particular, the modelling of the broad money and price series has highlighted potential difficulties in determining the appropriate degree of differencing required to render a series stationary. Even so, reasonably parsimonious models have been obtained notwithstanding these difficulties.

However, a number of cavaets must be considered with respect to the above analysis. The logarithmic transformation applied to each of the series may not be appropriate, although with regard to subsequent analysis it is certainly the most convenient. In any case, the role of transformations are explicity considered within a multivariate framework in chapter 5.

The prior seasonal adjustment of the series may be criticised as such considerations can be easily incorporated within the modelling framework anyway. Furthermore, lag structures may be distorted by prior adjustment and, indeed, the repeated findings of significant residual correlations around the seasonal lags may be regarded as evidence of such distortions. However, in mitigation, subsequent chapters develop and estimate complicated multivariate models and it was felt that prior adjustment, in the overall context of the study,

TABLE 1.11

ARIMA MODELS FOR THE INDIVIDUAL TIME SERIES

Series	Model	Estimate
MN	ARIMA (0, 2, 1)	$\Delta MN_t = .02 + (1 + .29L + .58L^2)\varepsilon_t$
MB	ARIMA (2, 1, 0)	$(1 - .38L - .31L^2)\Delta MB_t = .01 + \varepsilon_t$
	or	or
	ARIMA (0, 2, 1)	$\Delta^2 MB_t = (1 - .58)\varepsilon_t$
Y	ARIMA (2, 1, 0)	$(1 + .19L + .27L^2)\Delta Y_t = .01 + \varepsilon_t$
RC	ARIMA (0, 1, 1)	$\Delta RC_t = (1 - .40L)\varepsilon_t$
RL	ARIMA (3, 1, 0)	$(1 - .27L + .29L^3)\Delta RL_t = \varepsilon_t$

was the most convenient method of modelling seasonality.

Finally, the aim of this chapter has been to provide an initial, exploratory modelling of the major variables under consideration and, in particular, the univariate models thus derived have not been used for forecasting purposes. Indeed, forecasting is not the intention of this study, the prime aim being the development of empirical models successfully explaining the interaction between these important macroeconomic variables within the chosen sample period. Obviously, one would expect a multivariate analysis of these variables to be superior to the univariate analysis developed here, which should be regarded as a useful reference base to which more complex models can be compared. For a multivariate analysis to be performed an essential prerequisite is a theoretical framework and it is to the development of such that attention is now turned.

APPENDIX TO CHAPTER ONE

DATA DEFINITIONS AND SOURCES

M1 : Money Stock M1, FS(7.1, column 6).

M3 : Money Stock M3, FS(7.1, column 12).

Y : Real Gross Domestic Product at factor cost (expenditure based), revalued at 1970 prices, MDS (1.1).

P : Implicit deflator of Y.

RC : British Government securities $2\frac{1}{2}\%$ Consols yield. Quarterly average of monthly series, ET(66).

RL : Deposits with local authorities (3 months). Quarterly average of monthly series, ET(66).

W : Basic weekly wage rate (manual workers), all industries and services, July 1972 = 100, ET(40).

F = L - B, where L = Minimum Lending Rate FS(13.11) and B = London Clearing Banks Deposit Account Rate (7 days notice), FS(13.10).

G = Total general government expenditure, ET(56).

$V = \ln\left(\frac{IM + EX}{P.Y}\right)$, where IM = Imports (f.o.b.) and EX = Exports (f.o.b.).

ET : Economic Trends.

FS : Financial Statistics

MDS: Monthly Digest of Statistics.

All table numbers refer to May 1979 issues.

M1	M3	Y	P	RC	RL
6672.00000	10476.00000	8215.00000	0.75557	0.05860	0.04330
6855.00000	10835.00000	8964.00000	0.75602	0.05540	0.04420
6947.00000	11002.00000	8914.00000	0.75275	0.05390	0.04190
7322.00000	11516.00000	9348.00000	0.76690	0.05620	0.04290
7146.00000	11271.00000	8940.00000	0.76253	0.05910	0.04630
7223.00000	11536.00000	9431.00000	0.77439	0.06040	0.05000
7376.00000	11765.00000	9353.00000	0.78146	0.06040	0.05000
7557.00000	12155.00000	9778.00000	0.79239	0.06130	0.06310
7358.00000	11914.00000	9170.00000	0.80273	0.06300	0.07530
7461.00000	12337.00000	9662.00000	0.80335	0.06550	0.06920
7548.00000	12514.00000	9682.00000	0.81429	0.06480	0.06500
7848.00000	13083.00000	9977.00000	0.82239	0.06340	0.06260
7764.00000	12992.00000	9417.00000	0.83190	0.06530	0.06260
7728.00000	13164.00000	9748.00000	0.84499	0.06780	0.06340
7806.00000	13312.00000	9854.00000	0.84392	0.07110	0.07330
7844.00000	13555.00000	10205.00000	0.85213	0.06810	0.07330
7773.00000	13414.00000	9763.00000	0.85476	0.06470	0.06490
7899.00000	13825.00000	10082.00000	0.86312	0.06510	0.05790
8225.00000	14270.00000	10126.00000	0.87014	0.06820	0.05560
8478.00000	14949.00000	10291.00000	0.87640	0.07010	0.06370
8210.00000	14748.00000	9984.00000	0.88792	0.07130	0.08050
8356.00000	15270.00000	10333.00000	0.88851	0.07300	0.08280
8461.00000	15444.00000	10507.00000	0.90054	0.07470	0.07810
8784.00000	16092.00000	10897.00000	0.90686	0.07690	0.07380
8339.00000	15790.00000	10063.00000	0.91474	0.08420	0.07980
8188.00000	15737.00000	10496.00000	0.92845	0.09110	0.09140
8312.00000	15934.00000	10670.00000	0.92943	0.09150	0.09570
8812.00000	16596.00000	11179.00000	0.93667	0.08870	0.09220
8507.00000	16161.00000	10319.00000	0.96298	0.08600	0.09100
8852.00000	16893.00000	10816.00000	0.98465	0.09230	0.08130
9032.00000	17281.00000	10824.00000	1.01072	0.09280	0.07540
9635.00000	18175.00000	11409.00000	1.03786	0.09590	0.07180
9691.00000	18192.00000	10458.00000	1.06081	0.09380	0.07530
9831.00000	18662.00000	11025.00000	1.08354	0.09240	0.06660

M1	M3	Y	P	RC	RL
10210.00000	19112.00000	11284.00000	1.12363	0.09050	0.05820
11088.00000	20541.00000	11742.00000	1.14393	0.08590	0.04820
11197.00000	21206.00000	10691.00000	1.18314	0.08450	0.04800
11729.00000	23105.00000	11234.00000	1.19245	0.08980	0.05130
11930.00000	24060.00000	11250.00000	1.23262	0.09460	0.07620
12657.00000	26245.00000	12039.00000	1.24977	0.09630	0.07910
12333.00000	27142.00000	11778.00000	1.27517	0.09950	0.10090
13175.00000	28674.00000	11903.00000	1.29068	0.13200	0.09190
12882.00000	31008.00000	12023.00000	1.33303	0.11270	0.11780
13303.00000	33478.00000	12478.00000	1.36905	0.11860	0.14110
12772.00000	33938.00000	11187.00000	1.42764	0.13370	0.15660
13175.00000	34880.00000	11876.00000	1.44182	0.14480	0.13450
13513.00000	35758.00000	12253.00000	1.59504	0.15310	0.12790
14739.00000	37698.00000	12608.00000	1.66672	0.16640	0.12630
14735.00000	37422.00000	11534.00000	1.83258	0.09950	0.11420
15538.00000	38190.00000	11548.00000	1.93324	0.14830	0.09790
16773.00000	39628.00000	11831.00000	2.02502	0.14090	0.10670
17481.00000	40573.00000	12393.00000	2.10304	0.14850	0.11500
17801.00000	40471.00000	11788.00000	2.17153	0.13760	0.09270
18290.00000	42280.00000	11748.00000	2.22829	0.13900	0.11010
19227.00000	44272.00000	12073.00000	2.27425	0.14180	0.11790
19467.00000	45124.00000	12765.00000	2.33453	0.15160	0.15250
19566.00000	44421.00000	11580.00000	2.46649	0.13330	0.11630
20410.00000	46367.00000	11770.00000	2.53798	0.12600	0.08000
11048.00000	47239.00000	11969.00000	2.61125	0.12400	0.06790
23660.00000	49362.00000	12637.00000	2.61700	0.10940	0.06170

W	F	G	V
53.10000	0.02000	1644.00000	-1.08517
53.70000	0.02000	2557.00000	-1.14574
53.90000	0.02000	2877.00000	-1.16452
54.50000	0.02000	1875.00000	-1.14779
55.50000	0.01660	3005.00000	-1.03483
56.10000	0.02000	2781.00000	-1.09861
56.70000	0.02000	3096.00000	-1.16313
57.10000	0.02000	3111.00000	-1.13355
57.80000	0.02000	3253.00000	-1.11201
58.30000	0.02000	3124.00000	-1.12063
59.20000	0.02000	3496.00000	-1.16712
59.80000	0.02000	3436.00000	-1.13283
60.80000	0.02000	3530.00000	-1.06005
64.40000	0.01000	3383.00000	-1.15698
62.00000	0.02000	3732.00000	-1.16459
62.00000	0.02000	3803.00000	-1.16260
62.50000	0.02000	4109.00000	-1.07329
63.00000	0.02000	3838.00000	-1.11098
64.50000	0.02000	4416.00000	-1.22087
65.40000	0.01750	4309.00000	-1.19870
67.20000	0.02000	4865.00000	-0.96750
67.60000	0.02000	4212.00000	-1.01808
68.20000	0.02000	4627.00000	-1.04521
69.40000	0.02000	4586.00000	-1.03225
70.70000	0.02000	4792.00000	-0.96299
71.00000	0.02000	4238.00000	-0.96061
71.80000	0.02000	4967.00000	-1.01554
73.10000	0.02000	4957.00000	-0.99820
75.70000	0.02000	5101.00000	-0.93503
77.50000	0.02000	4747.00000	-0.93191
78.30000	0.02000	5420.00000	-1.05874
82.60000	0.02000	5438.00000	-0.99520
85.80000	0.02000	5749.00000	-0.96965

W	F	G	V
87.60000	0.02000	5371.00000	-0.95043
89.80000	0.02000	5956.00000	-1.06702
92.80000	0.02420	6123.00000	-1.05970
96.00000	0.02500	6571.00000	-0.99129
98.10000	0.02670	6082.00000	-0.99901
103.40000	0.00540	6644.00000	-1.17206
107.90000	0.02090	6957.00000	-0.97451
108.70000	0.01250	7882.00000	-0.91008
113.40000	0.01080	7182.00000	-0.88130
118.10000	0.02660	7387.00000	-0.89603
120.60000	0.02920	8049.00000	-0.80573
124.30000	0.03080	9505.00000	-0.61773
131.70000	0.02580	8402.00000	-0.55784
143.20000	0.02170	10513.00000	-0.70779
152.70000	0.02000	10768.00000	-0.72016
162.70000	0.01960	12897.00000	-0.74325
175.30000	0.03670	11862.00000	-0.76660
184.30000	0.04460	13049.00000	-0.82730
192.60000	0.04630	13602.00000	-0.82664
204.20000	0.03820	15149.00000	-0.75586
211.50000	0.05000	13745.00000	-0.65794
217.80000	0.04730	14968.00000	-0.71067
219.30000	0.03710	14533.00000	-0.66601
223.30000	0.03830	16186.00000	-0.57209
225.90000	0.04100	14650.00000	-0.56913
228.70000	0.03500	15565.00000	-0.60418
231.10000	0.02920	15561.00000	-0.70266

CHAPTER TWO :

THE EMPIRICAL CONSEQUENCES OF THE IS/LM PARADIGM AND THE DEMAND FOR MONEY

2.1 INTRODUCTION

The standard theoretical framework employed in investigating the interactions between money, income and interest rates has been the conventional IS/LM model popularised in many macroeconomic textbooks. However, for the period under consideration the most popular method of empirical analysis has been to investigate simply one of the structural relationships in isolation, namely the demand for money function, rather than analyse the whole model. Indeed, few attempts have been made to investigate other possible single equation relationships between the variables; exceptions being Artis and Nobay (1969) and Matthews and Ormerod (1978), who have estimated reduced form relationships relating income to money and a measure of fiscal policy à la St. Louis, and Demery and Duck (1978), who have developed a model in which interest rates are determined by, amongst other variables, money and prices.

In view of the overwhelming popularity of demand for money estimation, a brief review of the relevant literature is given in section 2.2. However, it is a curious paradox that direct estimation of the demand for money function is inconsistent with the assumptions underlying the IS/LM framework and as a consequence the empirical properties of this framework require close investigation. This necessitates the setting out of the standard static IS/LM model in section 2.3, in which the conventional policy conclusions are obtained in terms of partial derivatives. Section 2.4 considers a stochastic extension of the framework and obtains consistent estimators of the important monetary parameters of the model. These are contrasted with the estimators of the same parameters obtained directly from estimation of the demand for money function, these latter estimators being shown to be inconsistent under the assumptions of the model.

We are therefore faced with the problem of reconciling the assumptions of the IS/LM framework with a large body of empirical work on the demand for money function. The problem is, in fact, that of the imposition of a priori exogeneity assumptions, which are impossible to test in a static model. Section 2.5 considers both the problems of testing exogeneity assumptions and the inappropriateness of the static IS/LM framework and suggests that the generalisation to a dynamic framework is essential on both counts, thus supplying the necessary groundwork for the development of exogeneity tests in dynamic models that is the subject of chapter 3.

2.2 REVIEW OF THE LITERATURE ON THE DEMAND FOR MONEY FUNCTION

The last decade has witnessed a proliferation of empirical research on the U.K. demand for money function utlising post-war quarterly data.[1] The early studies, those of Fisher (1968), Laidler and Parkin (1970), Goodhart and Crockett (1970) and Price (1972) have been well surveyed by Goodhart and Crockett (1970), Laidler (1971), and more recently Goodhart (1975) and Fisher (1978). By the early 1970's the common consensus seemed to be that the accumulated evidence did not contradict the existence of a stable demand for money function comprising a small

1 This has not been the case for the longer run of annual data; the most notable studies being Kavanagh and Walters (1966), Laidler (1971) and Mills and Wood (1977).

number of explanatory variables; the most noticeable features of the research being the importance of time lags in the function and the apparent inability to discover an appropriate short interest rate to include as an explanatory variable measuring the opportunity cost of holding money. Nevertheless, estimated elasticities tended to vary quite considerably; for example, income (real G.D.P.) elasticities ranged from .42 found by Price (1972) using nominal narrow money to 2.64 found by Goodhart and Crockett (1970) using real broad money. Interest elasticities were generally found to lie between zero and minus unity although, as stated above, they were rather small for short interest rates. Although usually constrained to unity, when estimated freely price elasticities tended to be greater than this value.[2]

However, later studies undertaken by Haache (1974) and Artis and Lewis (1974) found that the inclusion of observations from the 1970's resulted in the apparent breakdown of the stability for broad money functions in which a partial adjustment lag mechanism (the standard specification) was employed. This instability manifested itself in the form of dynamically unstable adjustment processes resulting in economically meaningless parameter estimates and a systematic underprediction of M3 in the post 1971 period. While their empirical

2 Most studies have used the narrow (M1) and/or the broad (M3) definitions of money, although Haache (1974) in a later study disaggregated M3 into personal sector and commercial and industrial sector holdings. The conventional income variable has been G.D.P., although both total final expenditure and personal disposable income have occasionally been used. Often money and income have been deflated by a price level (usually the deflator of the income series being employed) and sometimes by population as well. If not used as a deflator the price level has been included as a separate explanatory variable. A range of interest rates have been employed, the most popular being the treasury bill yield and the local authority rate as representative short rates while the consol yield has normally proxied for the long term rate. Very few, if any, additional variables have been introduced into the function.

findings were similar, the explanations of this instability were rather different. Haache's view was that the observed instability was due to a shift in the demand function caused by a change in the Bank of England's method of financial operations in 1971 - the introduction of Competition and Credit Control (CCC), an institutional reform aimed at the removal of restraints on competition and innovation in the banking sector (as outlined in Bank of England (1971) and discussed at greater length in chapter 4.) Artis and Lewis, on the other hand, suggested that an excess supply of money (possibly caused by CCC) created a disequilibrium in which recent observations had been "off the demand curve", presumably lying above it.[3]

While the lack of sufficient data after 1971 prevented adequate testing of the competing hypotheses at the time, Artis and Lewis (1976) subsequently found the same form of instability in the broad money function even after the inclusion of additional variables modelling the institutional developments. Furthermore, they claimed that instability was also present in the narrow money function, thus prompting them to make a radical departure in methodology by reversing the implicit assumption that the money stock is demand determined - an assumption that is required for ordinary least squares to provide consistent estimates of the parameters of the demand function. By considering the money stock to be exogenously determined by the authorities, Artis and Lewis developed single equation models in which either the money income ratio or the interest rate responded to changes in money, thus reversing the causal relationship implied by conventional demand for money functions.

3 Mills (1975) provides additional evidence for the structural change hypothesis.

Notwithstanding the success claimed by Artis and Lewis for these alternative models, recent studies have continued to employ the conventional approach. Mills (1978) generalised the functional form by using the Box and Cox (1964) family of power transformations but restricting the specification to a partial adjustment lag mechanism. The narrow money function was found to be stable and relatively insensitive to functional form, while the broad money function displayed either instability or an unacceptably slow lag adjustment for all functional forms. Coghlan (1978), although employing the conventional logarithmic functional form, generalised the lag structures when analysing just the narrow money function. Using tests based upon the cumulative sums of squares of recursive residuals it was claimed that this generalised dynamic function also exhibited stability when including data from the 1970's. Further evidence of the stability of the narrow money function is supplied by Laumas (1978), although here the data period was split into two at 1971. Using the varying parameter technique proposed by Cooley and Prestcott (e.g. 1973) Laumas also presented evidence of a stable broad money function specified with a partial adjustment lag mechanism when the data was split into two periods.

The exchange of views in the September 1978 issue of the <u>Economic Journal</u> was also concerned in part with dynamic specification, particularly the discussions by Hendry and Mizon (1978) and Williams (1978). Courakis (1978), however, was primarily interested in pointing out the implications, both statistical and theoretical, of the imposition of untested a priori assumptions, e.g. the restriction to unitary price elasticities, partial adjustment processes and first order serial correlation. As these studies were primarily of a theoretical and expositional nature only limited supporting empirical evidence was provided, but it is clear from these numerous recent papers that there is still a great deal to be resolved before it can be concluded with any confidence whether a stable demand for money function exists.

2.3 MACROECONOMIC THEORY AND THE DEMAND FOR MONEY

While the wealth of empirical evidence has obviously been of great importance in settling some issues and clarifying others it is curious to note that, apart from the alternative models developed by Artis and Lewis (1976), the conventional approach to estimating the demand for money function is at odds with the assumptions of the customary theoretical framework, the IS/LM paradigm, within which the function plays the fundamental role of the transmission mechanism. Although this framework has lost its dominant position in macroeconomic theory in recent years as a result of both its inability to account for interactions of price and output fluctuations (the stagflation phenomenon so apparent in the last decade) and its avoidance of the dynamic nature of the economy, nevertheless it has been accepted by Keynesians and monetarists alike as providing a common framework of analysis. (This is apparent from the "Symposium on Friedman's Theoretical Framework" in the September/October 1972 issue of the Journal of Political Economy.) It is therefore of interest to set out this framework here to show that even such an admittedly simple theoretical structure is capable of producing a number of results that are important in assessing both the statistical and policy implications of empirical studies of the demand for money function.[4]

To develop such a framework let us introduce the following notation. Let Y be real national income, E real private expenditure, G real government expenditure, P the price level, R the interest rate, M actual nominal money balances, M^d nominal money balances demanded and M^s nominal money balances supplied. A linear static IS/LM model may then be constructed in the following manner:

Expenditure Function

$$E = aY - bR \tag{2.1}$$

Government expenditure

$$G = \bar{G} \tag{2.2}$$

Real (goods) sector equilibrium

$$Y = E + G \tag{2.3}$$

Demand for money function

$$M^d = P(cY - dR) \tag{2.4}$$

Supply of money function

$$M^s = \bar{M} \tag{2.5}$$

Monetary sector equilibrium

$$M^d = M^s \tag{2.6}$$

Bars above variables denote that they are exogenously determined and all parameters are positive. In the interests of simplicity we have ignored any autonomous investment and consumption (which would introduce a constant into the expenditure function) and we have abstracted from taxation. Note that the aggregation of investment

4 For detailed treatment of the IS/LM framework see e.g. Chick (1973), Goodhart (1975) and Laidler (1977).

and consumption in the expenditure function allows the possibilities that interest rates affect consumption and income affects investment.

Equations (2.1), (2.2) and (2.3) may be solved to yield the familiar IS curve

$$R = \frac{-(1 - a)}{b} Y + \frac{\overline{G}}{b} \tag{2.7}$$

while solving equations (2.4), (2.5) and (2.6) yields the LM curve

$$R = \frac{c}{d} Y - \frac{1}{d} \frac{\overline{M}}{P} \tag{2.8}$$

These two equations are underdetermined as there are three endogenous variables Y, P and R contained within them. A common solution to this "missing equation" problem is to assume that in conditions of unemployment the price level may be considered fixed at P^*, in which case equations (2.7) and (2.8) determine Y and R, while if full employment attains, there is an associated full employment level of income Y^* allowing equations (2.7) and (2.8) to determine P and R. Although such a solution is difficult to justify in the light of recent experiences of coexisting inflation and unemployment, it does capture the essential elements of both the classical and Keynesian doctrines and will be retained here on the grounds of convenience.[5]

If R is eliminated from these two equations the following solutions are obtained in conditions of unemployment and full employment respectively,

$$Y = \frac{d}{bc + d(1 - a)} \overline{G} + \frac{b}{bc + d(1 - a)} \cdot \frac{\overline{M}}{P*} \tag{2.9}$$

5 A similar model to this is developed in Laidler (1977, ch.2.)

$$P = \frac{b\overline{M}}{(bc + d(1 - a))Y^* - d\overline{G}} \tag{2.10}$$

The comparative static properties of the model are then obtained by taking partial derivatives of (2.9) and (2.10) with respect to the exogenous variables $\overline{M}$ and $\overline{G}$. In unemployment conditions we obtain

$$\frac{\partial Y}{\partial \overline{M}} = \frac{b}{bc + d(1 - a)} > 0 \tag{2.11}$$

$$\frac{\partial Y}{\partial \overline{G}} = \frac{d}{bc + d(1 - a)} > 0 \tag{2.12}$$

while in conditions of full employment

$$\frac{\partial P}{\partial \overline{M}} = \frac{b}{(bc + d(1 - a))Y^* - d\overline{G}} = \frac{P}{\overline{M}} > 0 \tag{2.13}$$

$$\frac{\partial P}{\partial \overline{G}} = \frac{dP^2}{b\overline{M}} > 0 \tag{2.14}$$

Thus we obtain the familiar results that, at less than full employment, increasing the money supply and government expenditure both lead to a higher equilibrium level of income while at full employment similar policies will lead to increases in the equilibrium price level. Furthermore changes in the price level are proportional to changes in the quantity of money, the strict Quantity Theory result.

Also of importance are the following results concerning the behaviour of the parameter d, which measures the sensitivity of the demand for money to interest rate changes and is of course proportional to the interest elasticity of the demand for money. If d equals zero then

$$\frac{\partial Y}{\partial \bar{G}} = \frac{\partial P}{\partial \bar{G}} = 0 \;,\; \frac{\partial Y}{\partial \bar{M}} = \frac{1}{c} \;,\; \frac{\partial P}{\partial \bar{M}} = \frac{1}{cY*}$$

and as d tends to infinity [6]

$$\frac{\partial Y}{\partial \bar{G}} \rightarrow \frac{1}{1 - a} \;,\; \frac{\partial P}{\partial \bar{G}} \rightarrow \infty \;,\; \frac{\partial Y}{\partial \bar{M}} \rightarrow 0 \;,\; \frac{\partial P}{\partial \bar{M}} \rightarrow 0$$

Furthermore,

$$\frac{\partial Y}{\partial \bar{M}} \Big/ \frac{\partial Y}{\partial \bar{G}} = \frac{b}{d} \quad \text{and} \quad \frac{\partial P}{\partial \bar{M}} \Big/ \frac{\partial P}{\partial \bar{G}} = \frac{b}{dP}$$

From these results we see that the absolute effectiveness of monetary and fiscal policies depend on the absolute size of d, while the relative effectiveness of monetary policy to fiscal policy depends on the relative magnitudes of d and b, the parameter measuring the sensitivity of private expenditure to interest rate changes. It is also apparent that the predictability of monetary and fiscal policies requires the stability of all the parameters in the model.

These results offer an explanation as to why both Keynesians and monetarists have come to accept the IS/LM paradigm - debate between the groups centres on the values taken by the parameters of the model. For example, the extreme monetarist contention that income can be affected directly and only by changes in the money supply requires

6 One should be careful in interpreting the approach of $\partial P/\partial \bar{G}$ to infinity as d approaches infinity. It means that, so long as the demand for money remains perfectly elastic with respect to the interest rate, the price level will rise without limit after an expansionary fiscal policy undertaken at full employment. However the price rise is indefinite rather than infinite since this "liquidity trap" region of the demand for money function becomes irrelevant as the price level rises.

setting d to zero. The converse extreme Keynesian contention that monetary policy is ineffective (the liquidity trap argument) implies that d tends to infinity, or at least that d is very much greater than zero. The influential Radcliffe report (Radcliffe 1959) may be crudely summarised in this framework as stating that c and d are inherently unstable.[7]

Even allowing for the extreme simplicity of this framework and its obvious drawbacks, it does yield a number of interesting results that are capable of empirical resolution. A considerable amount of research has thus been conducted in an attempt to estimate the parameters of the model, particularly the parameters of the demand for money function and it is to the model's implications for estimation techniques that we now turn.

2.4 ESTIMATION WITHIN THE STATIC IS/LM FRAMEWORK

As we have shown in the preceding section, both the absolute and relative magnitudes of the parameters of the IS/LM model are required in evaluating the effectiveness of monetary and fiscal policy. It is also important that the parameters are stable if policy effects are to be predictable. While it is obvious that the parameters of both the real and the monetary sectors are required to analyse these matters, the majority of empirical research has been directed at obtaining estimates of c and d, the parameters of the demand for money function. As our interest also lies primarily with estimating this demand function, we will consider the implications of the IS/LM model for obtaining consistent and

7 See Chick (1972, ch. 3 and 4) for a very extensive treatment of these matters.

efficient estimators of c and d.[8]

A prerequisite of estimation is to render the deterministic model presented in the previous section stochastic. We achieve this by introducing random errors into the expenditure and demand for money functions,

$$E = aY - bR + \eta_1 \tag{2.15}$$

$$\frac{M^d}{P*} = cY - dR + \eta_2 \tag{2.16}$$

where η_1 and η_2 are random errors and for convenience the price level is assumed constant (i.e. the unemployment solution to the missing equation problem). The stochastic IS and LM equations can now be written as

$$R = \frac{-(1 - a)}{b}Y + \frac{\overline{G}}{b} + \frac{\eta_1}{b} \tag{2.17}$$

$$Y = \frac{d}{c} R + \frac{1}{c}\frac{\overline{M}}{P*} + \frac{\eta_2}{c} \tag{2.18}$$

A more convenient notation is the following

$$r_t = \beta_1 y_t + \gamma_1 g_t + u_{1t} \tag{2.19}$$

$$y_t = \beta_2 r_t + \gamma_2 m_t + u_{2t} \tag{2.20}$$

in which time subscripts have been added, lower case letters denote variables expressed as deviations from their respective means (thus

8 Consistency and efficiency are desirable properties of estimators in their own right, of course, and a considerable amount of research in econometric theory has been undertaken in developing estimators with such properties. However, we can see here that these properties are also essential for drawing correct inferences regarding policy efficiency: consistency being required as both absolute and relative magnitudes of estimates must be compared while efficient estimators are needed to determine policy stability.

alleviating the need for the inclusion of constants) and where we make the conventional assumptions,

$$Eu_{it} = Eu_{it}u_{j,t-s} = Eu_{it}g_t = Eu_{it}m_t = 0$$

$$Eu^2_{it} = \sigma^2_i \qquad ; \quad i,j = 1, 2.$$

We see that r and y may be regarded as endogenous variables determined within a two equation simultaneous model with g and m exogenously determined.[9] The relationships between the parameters of the original model and this redefined model are easily found to be

$$a = (\beta_1 + \gamma_1)/\gamma_1 \quad ; \quad b = 1/\gamma_1 \quad ; \quad c = 1/\gamma_2 \quad ; \quad d = \beta_2/\gamma_2$$

Both (2.19) and (2.20) are therefore just identified and consistent and efficient estimators of the parameters may be obtained by the use of indirect least squares. Rewriting (2.19) and (2.20) as

$$\begin{pmatrix} 1 & -\beta_1 \\ -\beta_2 & 1 \end{pmatrix}\begin{pmatrix} r_t \\ y_t \end{pmatrix} = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}\begin{pmatrix} g_t \\ m_t \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

leads to the reduced form

$$\begin{pmatrix} r_t \\ y_t \end{pmatrix} = \frac{1}{1 - \beta_1\beta_2}\begin{pmatrix} 1 & \beta_1 \\ \beta_2 & 1 \end{pmatrix}\begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}\begin{pmatrix} g_t \\ m_t \end{pmatrix} + \frac{1}{1 - \beta_1\beta_2}\begin{pmatrix} 1 & \beta_1 \\ \beta_2 & 1 \end{pmatrix}\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

i.e.

$$\begin{pmatrix} r_t \\ y_t \end{pmatrix} = \frac{1}{1 - \beta_1\beta_2}\begin{pmatrix} \gamma_1 & \beta_1\gamma_2 \\ \beta_2\gamma_1 & \gamma_2 \end{pmatrix}\begin{pmatrix} g_t \\ m_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

9 g is exogenous by assumption while m, being the ratio of the assumed exogenous $\overline{M}$ and the constant P* must also be exogenous.

or

$$r_t = \pi_{11} g_t + \pi_{12} m_t + \varepsilon_{1t} \tag{2.21}$$

$$y_t = \pi_{21} g_t + \pi_{22} m_t + \varepsilon_{2t} \tag{2.22}$$

where

$$\pi_{11} = \frac{\gamma_1}{1 - \beta_1\beta_2}, \quad \pi_{12} = \frac{\beta_1\gamma_2}{1 - \beta_1\beta_2}, \quad \pi_{21} = \frac{\beta_2\gamma_1}{1 - \beta_1\beta_2}, \quad \pi_{22} = \frac{\gamma_2}{1 - \beta_1\beta_2}$$

Since $\beta_2 = \dfrac{\pi_{21}}{\pi_{11}}$ and $\gamma_2 = \dfrac{\pi_{11}\pi_{22} - \pi_{12}\pi_{21}}{\pi_{11}}$

the expressions for the parameters of interest are given by

$$c = \frac{\pi_{11}}{\pi_{11}\pi_{22} - \pi_{12}\pi_{21}} \quad \text{and} \quad d = \frac{\pi_{21}}{\pi_{11}\pi_{22} - \pi_{12}\pi_{21}}$$

Ordinary least squares applied to the reduced form yields

$$\hat{\pi}_{11} = \frac{\Sigma rg\Sigma m^2 - \Sigma rm\Sigma gm}{\Sigma g^2\Sigma m^2 - (\Sigma gm)^2}, \quad \hat{\pi}_{12} = \frac{\Sigma rm\Sigma g^2 - \Sigma rg\Sigma gm}{\Sigma g^2\Sigma m^2 - (\Sigma gm)^2}$$

$$\hat{\pi}_{21} = \frac{\Sigma yg\Sigma m^2 - \Sigma ym\Sigma gm}{\Sigma g^2\Sigma m^2 - (\Sigma gm)^2}, \quad \hat{\pi}_{22} = \frac{\Sigma ym\Sigma g^2 - \Sigma yg\Sigma gm}{\Sigma g^2\Sigma m^2 - (\Sigma gm)^2}$$

and therefore the indirect least squares estimator of c is given by

$$\hat{c} = \frac{(\Sigma rg\Sigma m^2 - \Sigma rm\Sigma gm)(\Sigma g^2\Sigma m^2 - (\Sigma gm)^2)}{(\Sigma rg\Sigma m^2 - \Sigma rm\Sigma gm)(\Sigma ym\Sigma g^2 - \Sigma yg\Sigma gm) - (\Sigma rm\Sigma g^2 - \Sigma rg\Sigma gm)(\Sigma yg\Sigma m^2 - \Sigma ym\Sigma gm)}$$

$$= \frac{\Sigma rg\Sigma m^2 - \Sigma rm\Sigma gm}{\Sigma rg\Sigma ym - \Sigma rm\Sigma yg}$$

Similarly we obtain

$$\hat{d} = \frac{\Sigma yg\Sigma m^2 - \Sigma ym\Sigma gm}{\Sigma rg\Sigma ym - \Sigma rm\Sigma yg}$$

The $\hat{\pi}_{ij}$'s are, of course, consistent and efficient estimators of the π_{ij}'s. Although c and d are non linear functions of these parameters the asymptotic properties of these estimators carry over to the estimators $\hat{c}$ and $\hat{d}$, which are therefore consistent and efficient. (The small sample distribution of $\hat{c}$ and $\hat{d}$ are very difficult to obtain algebraically, but an approximate method is given in Kmenta (1971 p.442-8)).

This structural approach may be contrasted with the conventional method of obtaining estimators of c and d directly from the demand for money function

$$m_t = cy_t - dr_t + \eta_{2t} \tag{2.23}$$

The ordinary least squares estimator of c derived from (2.23) is

$$\tilde{c} = \frac{\Sigma ym\Sigma r^2 - \Sigma mr\Sigma yr}{\Sigma y^2\Sigma r^2 - (\Sigma yr)^2} \tag{2.24}$$

which is seen to be numerically different from the indirect least squares estimator $\hat{c}$. Substituting (2.23) into (2.24) yields

$$\tilde{c} = c + \frac{\Sigma y\eta_2\Sigma r^2 - \Sigma r\eta_2\Sigma yr}{\Sigma y^2\Sigma r^2 - (\Sigma yr)^2} \tag{2.25}$$

It is clear that $\tilde{c}$ will, in general, only be unbiased if $Ey\eta_2 = Er\eta_2 = 0$. Now since $\eta_2 = cu_2$ and Eyu_2 and Eru_2 are non zero from the simultaneous nature of equations (2.19) and (2.20), $\tilde{c}$ will be a biased (and inconsistent) estimator of c. An analogous argument yields

$$\tilde{d} = \frac{\Sigma my\Sigma yr - \Sigma rm\Sigma y^2}{\Sigma y^2\Sigma r^2 - (\Sigma yr)^2}$$

$$= d + \frac{\Sigma y\eta_2\Sigma yr - \Sigma r\eta_2\Sigma y^2}{\Sigma y^2\Sigma r^2 - (\Sigma yr)^2} \tag{2.26}$$

It is not possible to determine a priori the directions of the biases inherent in the estimators (2.25) and (2.26).

The problem of inconsistency cannot be solved by employing a simultaneous equation technique such as two stage least squares to (2.23), for this would require that m be an endogenous variable, which is in conflict with the assumptions of the IS/LM model. If m was to be endogenised the present model would be incomplete and would have to be modified by incorporating a money supply function relating m to, say, r and at least one further exogenous variable, perhaps the monetary base. For ordinary least squares to produce consistent estimates of (2.23) we require not only that m be an endogenous variable but also that y and r be themselves exogenously determined with respect to m, thus requiring a theoretical model radically different from the present IS/LM framework.[10]

As a corollary to the structural approach, consider the stochastic counterpart of equation (2.9), obtained by eliminating the interest rate from the system comprising equations (2.19) and (2.20)

$$y_t = \theta_1 g_t + \theta_2 m_t + v_t$$

where $\theta_1 = \beta_2\gamma_1/(1 - \beta_1\beta_2)$, $\theta_2 = \gamma_2/(1 - \beta_1\beta_2)$ and $v_t = \beta_2 u_{1t} + u_{2t}$.

10 An analogous argument holds if we consider the monetarist solution to the "missing equation" problem, in which Y is held fixed at its full employment level Y* and P is allowed to vary. The Appendix to this chapter sets out the algebra.

Although unique estimates of the structural parameters cannot be obtained we observe that

$$\frac{\theta_1}{\theta_2} = \frac{\beta_2\gamma_1}{\gamma_2} = \frac{d}{b}$$

i.e. the ratio of the estimated coefficients, $\hat{\theta}_1/\hat{\theta}_2$, is a measure of the relative efficiency of monetary to fiscal policy. This result is thus a rationale for the test proposed by Friedman and Meiselman (1964), although we must emphasise that we have presented here a purely theoretical exposition which ignores the practical and empirical considerations around which this famous debate has revolved.[11] Again such an approach rests on the assumptions that g and m are exogenous.

This analysis of the IS/LM framework has emphasised the fact that if one wishes to work within the confines of this framework, consistent estimates of the parameters of the demand for money function cannot be obtained by direct estimation of the function in the conventional manner. For direct estimation of the demand function to produce consistent estimates a fundamentally different theoretical framework is required, in particular an alternative set of exogeneity assumptions is required.

11 For a recent contribution to the debate containing references of previous work see Poole and Kornblith (1973).

2.5 EXOGENEITY AND STATIC MODELS

The above analysis has highlighted the crucial nature of assumptions of exogeneity for both theoretical and empirical research in macroeconomics in general and the demand for money in particular. These assumptions may be regarded as the foundation stones of model building and as such determine the choice of estimation technique employed. It is therefore essential that such assumptions be capable of empirical verification.

The models developed in the previous section may be considered as cases of the general static simultaneous model comprising g equations,

$$BY_t + \Gamma X_t = U_t \tag{2.27}$$

where Y_t is a vector of g endogenous variables, X_t is a vector of k exogenous variables, B and Γ are coefficient matrices of dimensions (g x g) and (g x k) respectively and U_t is a vector of errors of length g. This model represents an a priori classification of the variables (Y_t X_t) into endogenous and exogenous groups and as such will impose restrictions upon the matrices B and Γ, namely normalizing unity restrictions on the diagonal elements of B and identifying restrictions on both matrices.

It has been shown by Engle, Hendry and Richard (1979) that exogeneity can be tested in static simultaneous equation models, but as the static nature of the IS/LM model developed above has been one of its major criticisms, an extension of the model to incorporate dynamic behaviour would appear to be a justifiable modification. Thus, following Geweke (1978), the next chapter develops a methodology for testing the assumptions of exogeneity within a dynamic simultaneous equation model and uses this methodology for testing the assumptions of exogeneity in dynamic IS/LM models. However, it should be emphasised that it has only been chosen to test for strong, rather than weak, exogeneity. (For definitions and discussion of these twin concepts of exogeneity, see Engle, Hendry and Richard (1979)).

APPENDIX TO CHAPTER TWO

Consider an IS/LM model in which real income is determined exogenously and the price level is now determined within the system. To retain linearity the expenditure and demand for money functions will be written in the following form

$$E = a\bar{Y} - bR + \eta_1$$

$$\bar{M} = c\bar{Y} - dR + eP + \eta_2$$

where $\bar{Y}$ denotes exogenous real income. Analogous to equations (2.19) and (2.20) we may obtain

$$r_t = \delta_{11}y_t + \delta_{12}g_t + v_{1t} \tag{2.28}$$

$$p_t = \delta_{21}m_t + \delta_{22}y_t + \delta_{23}r_t + v_{2t} \tag{2.29}$$

with the assumptions

$$Ev_{it} = Ev_{it}g_t = Ev_{it}m_t = Ev_{it}v_{jt-s} = 0$$

$$Ev_{it}^2 = \sigma_{iv}^2 \qquad i, j = 1, 2.$$

and where

$$a = \frac{\delta_{11} + \delta_{12}}{\delta_{12}}, \quad b = \frac{1}{\delta_{12}}, \quad c = \frac{-\delta_{22}}{\delta_{21}}, \quad d = \frac{\delta_{23}}{\delta_{22}}.$$

The equations (2.28) and (2.29) are seen to form a recursive system in which consistent and efficient estimates of the δ_{ij}'s may be obtained by applying ordinary least squares to each equation in turn. This is a consequence of the classical dichotomy in which the interest rate is determined within the real sector and the price level in the monetary sector.

Ordinary least squares applied to the demand for money function directly yields

$$E\begin{pmatrix} \tilde{c} \\ -\tilde{d} \\ \tilde{e} \end{pmatrix} = \begin{pmatrix} c \\ -d \\ e \end{pmatrix} + E\begin{pmatrix} \Sigma y^2 & \Sigma yr & \Sigma yp \\ \Sigma ry & \Sigma r^2 & \Sigma rp \\ \Sigma py & \Sigma pr & \Sigma p^2 \end{pmatrix}^{-1} \begin{pmatrix} \Sigma y\eta_2 \\ \Sigma r\eta_2 \\ \Sigma p\eta_2 \end{pmatrix}$$

Now, since $\eta_2 = ev_2$ and Σpv_2 is non zero, $E\Sigma p\eta_2$ is non zero, and hence these estimators are biased (and inconsistent).

CHAPTER THREE :

DYNAMIC MACROECONOMIC MODELS AND TESTING FOR EXOGENEITY

3.1 TESTING EXOGENEITY IN DYNAMIC ECONOMETRIC MODELS

An appropriate framework within which tests of exogeneity may be constructed is supplied by the linear multivariate mixed autoregressive moving average (ARMA) process discussed by Zellner and Palm (1974), Wallis (1977) and Granger and Newbold (1977),

$$H(L)z_t = F(L)\varepsilon_t \quad , \quad t = 1, 2, \ldots, T. \tag{3.1}$$

where $z_t' = (z_{1t}, \ldots, z_{pt})$ is a vector of random variables of length p, $\varepsilon_t' = (\varepsilon_{1t}, \ldots, \varepsilon_{pt})$ is a vector of random errors of length p, and H(L) and F(L) are both (p x p) matrices, assumed of full rank, whose elements are finite polynomials in the lag operator L. Typical elements of H(L) and F(L) are given by

$$h_{ij} = \sum_{l=0}^{r_{ij}} h_{ijl}L^l \qquad f_{ij} = \sum_{l=0}^{q_{ij}} f_{ijl}L^l.$$

and the assumptions placed upon the error process are

$$\begin{aligned} E\varepsilon_t &= 0 \qquad & E\varepsilon_t\varepsilon_s &= \Sigma \text{ (positive definite)} \qquad & t &= s \\ & & &= 0 & t &\neq s \end{aligned}$$

for all t and s with the normalization $F_0 = I$. It is further assumed that the roots of the determimental equations $|H(L)| = 0$ and $|F(L)| = 0$ all lie outside the unit circle, so that the process is both stationary and invertible.

For our present purposes it is convenient to consider the autoregressive restriction of (3.1) obtained by setting $F(L)$ to a polynomial of degree zero in L,[1]

$$H(L)z_t = \varepsilon_t \tag{3.2}$$

Consider an arbitrary partitioning of z_t such that $z_t' = (y_t' \; x_t')$ where y_t and x_t are vectors of lengths g and k respectively, $g + k = p$. Partitioning ε_t and $H(L)$ conformably, we may then write

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tag{3.3}$$

where H_{11} and H_{22} are square matrices of dimensions g and k, and H_{12} and H_{21} are matrices of dimensions $(g \times k)$ and $(k \times g)$. Using the partitioned inverse rule we obtain

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} J & -JH_{12}H_{22}^{-1} \\ -H_{22}^{-1} H_{21}J & G \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tag{3.4}$$

where $J = (H_{11} - H_{12}H_{22}^{-1}H_{21})^{-1}$ and $G = H_{22}^{-1} + H_{22}^{-1}H_{21}JH_{12}H_{22}^{-1}$. From the system (3.4) we may solve for y_t in terms of x_t and ε_{1t}, yielding

$$y_t = -JH_{12}H_{22}^{-1}G^{-1}x_t + J(I - H_{12}H_{22}^{-1}G^{-1}H_{22}^{-1}H_{21}J)\varepsilon_{1t} \tag{3.5}$$

or solve for x_t in terms of y_t and ε_{2t}

1 Alternatively, equation (3.2) may be considered as a finite approximation to the infinite AR process obtained by the transformation,

$$F^{-1}(L)H(L)z_t = \varepsilon_t.$$

$$x_t = -H_{22}^{-1}H_{21}y_t + H_{22}^{-1}\varepsilon_{2t} \tag{3.6}$$

Now, if it is assumed that x_t is determined exogenously to the system, then $H_{21} = 0$ and since this implies that $J = H_{11}^{-1}$ and $G = H_{22}^{-1}$, we have

$$y_t = -H_{11}^{-1}H_{12}x_t + H_{11}^{-1}\varepsilon_{1t} \tag{3.7}$$

$$x_t = H_{22}^{-1}\varepsilon_{2t} \tag{3.8}$$

Equations (3.3) to (3.8) form the basis for testing the assumption that x_t is exogenous, i.e. that H_{21} is identically zero. We see from (3.6) and (3.8) that if x_t is exogenous, then in the dynamic multivariate regression,

$$\theta_1(L)x_t = \theta_2(L)y_t + u_t \tag{3.9}$$

$\theta_2(L)$ should be identically zero, i.e. that no present or past values of y should enter the regression.

From equation (3.4) we also see that in general x_t is correlated with past values of ε_{1t} whereas if x_t is exogenous then it is correlated with ε_{2t} alone. This has the implication that in the dynamic multivariate regression,

$$y_t = \phi_1(L)x_t + \phi_2(L)v_t \tag{3.10}$$

the error term will be uncorrelated with all values of x_t only if x_t is exogenously determined. Hence if $\phi_1(L)$ is allowed to be two sided, the assumption of exogeneity requires that all coefficients on future values of x_t must be zero.

These implications of exogeneity in multivariate dynamic models have also been derived in slightly different fashion by Geweke (1978), who also shows that they hold even if x_t is nonstationary. Geweke's approach can be developed using the present framework and yields some interesting insights on the implications of exogeneity. Under the assumption of exogeneity, the process (3.3) can be written as

$$\begin{pmatrix} H_{11} & H_{12} \\ 0 & H_{21} \end{pmatrix} \begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tag{3.11}$$

or

$$H_{11}y_t + H_{12}x_t = \varepsilon_{1t} \tag{3.12}$$

$$H_{21}x_t = \varepsilon_{2t} \tag{3.13}$$

Recalling that the H_{ij} are polynomials in the lag operator L, we see that equation (3.12) is the dynamic simultaneous equation model considered by Geweke (1978), to which has been appended the process (3.13) generating the exogenous variables. This process is usually ignored in econometric applications, in which case Granger and Newbold (1977) point out that (3.12) can only yield, for example, conditional forecasts for the endogenous variables. From this system we see that equation (3.7), viz

$$y_t = -H_{11}^{-1}H_{12}x_t + H_{11}^{-1}\varepsilon_{1t}$$

is the final form of the system (3.12) expressing the current endogenous variables as functions of only present and past values of the exogenous variables. Thus one implication of exogeneity is that no future values of the exogenous variables can appear in the final form equations. The second implication of exogeneity is that no endogenous variables can appear in the process generating the exogenous variables, as is apparent from the equivalent equations (3.8) and (3.13).

While the two implications of exogeneity are equivalent, (as shown by Geweke (1978)), the practical problems of testing suggest that the multivariate regression of (3.9) will probably be the most convenient procedure. Estimation of (3.10) is a multivariate regression containing g equations. The serial correlation in the error vector will, however, present difficulties and although Geweke (1978) provides an asymptotically efficient method of estimation, it does require Fourier techniques that are not always readily available. The direct regression of (3.9) does not suffer from serial correlation problems as long as the order of $\theta_1(L)$ is kept generous.

Thus a test of exogeneity may be constructed in the following manner. In the regression model (3.9) we wish to test the null hypothesis $H_0 : \theta_2(L) \equiv 0$. This model can be written more fully as

$$\begin{pmatrix} \theta_{111} & \cdots & \theta_{11k} \\ \vdots & & \vdots \\ \theta_{1k1} & \cdots & \theta_{1kk} \end{pmatrix} \begin{pmatrix} x_{1t} \\ \vdots \\ x_{kt} \end{pmatrix} = \begin{pmatrix} \theta_{211} & \cdots & \theta_{21g} \\ & & \\ \theta_{2k1} & \cdots & \theta_{2kg} \end{pmatrix} \begin{pmatrix} y_{1t} \\ \\ y_{gt} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ \\ u_{kt} \end{pmatrix} \qquad (3.14)$$

where $\theta_{1ij} = \sum_{l=0}^{m_{ij}} \theta_{1ijl} L^l$ and $\theta_{2ie} = \sum_{l=0}^{n_{ie}} \theta_{2iel} L^l$

$i, j = 1 \ldots k \quad , \qquad e = 1 \ldots g.$

A more convenient representation is

$$\begin{pmatrix} 1 & \theta_{1120} & \cdots & \theta_{11k0} \\ \theta_{1210} & 1 & \cdots & \theta_{12k0} \\ \vdots & & \ddots & \vdots \\ \theta_{1k10} & \cdots & & 1 \end{pmatrix} \begin{pmatrix} x_{1t} \\ \vdots \\ x_{kt} \end{pmatrix} = \begin{pmatrix} \theta^*_{111} & \cdots & \theta^*_{11k} \\ \vdots & & \vdots \\ \theta^*_{1k1} & & \theta^*_{1kk} \end{pmatrix} \begin{pmatrix} x_{1t} \\ \vdots \\ x_{kt} \end{pmatrix}$$

$$+ \begin{pmatrix} \theta_{211} & \cdots & \theta_{21g} \\ \vdots & & \vdots \\ \theta_{2k1} & \cdots & \theta_{2kg} \end{pmatrix} \begin{pmatrix} y_{1t} \\ \vdots \\ y_{gt} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ \vdots \\ u_{kt} \end{pmatrix} \qquad (3.15)$$

where $\theta^*_{1ij} = \sum_{l=1}^{m_{ij}} \theta_{1ijl} L^l = \theta_{1ij} - \theta_{1ij0}$ and the normalisation $\theta_{1ii0} = 1$ has been made. The ith equation in the system (3.15) can be written as

$$x_{it} = (-\theta_{1i10}, \ldots, -\theta_{1i,\, i-1,\, 0}, -\theta_{1i,i+1,0}, \ldots, -\theta_{1ik0}) \begin{pmatrix} x_{1t} \\ \vdots \\ x_{i-1,\, t} \\ x_{i+1,\, t} \\ \vdots \\ x_{kt} \end{pmatrix}$$

$$+ (\theta^*_{1i1} \; .. \; \theta^*_{1ik}) x_t + (\theta_{2i1} \; .. \; \theta_{2ig}) y_t + u_{it}$$

or $$x_{it} = (\beta_{i0} : \beta_{i1} : \beta_{i2}) \begin{pmatrix} x_{i0t} \\ x_t \\ y_t \end{pmatrix} + u_{it}$$

where $\beta_{i0} = (-\theta_{1i10}, \ldots, -\theta_{1i,i-1,0}, -\theta_{1i,i+1,0}, \ldots, -\theta_{1ik0})$

$\beta_{i1} = (\theta^*_{1i1} \ldots \theta^*_{1ik})$

$\beta_{i2} = (\theta_{2i1} \ldots \theta_{2ig})$

and $x'_{i0t} = (x_{1t}, \ldots, x_{i-1,t}, x_{i+1,t}, \ldots x_{kt})$

or $x_{it} = \beta_i Y_{it} + u_{it}$

where $\beta_i = (\beta_{i0}\beta_{i1}\beta_{i2})$ and $Y'_{it} = (x'_{i0t}x'_t y'_t)$.

If we define $x'_i = (x_{i1} \ldots x_{iT})$

$Y'_i = (Y'_{i1} \ldots Y'_{iT})$

$u'_i = (u_{i1} \ldots u_{iT})$

then we have

$$x_i = \beta_i Y_i + u_i \qquad i = 1, \ldots, k$$

Combining all k equations using the definitions

$$x' = (x'_1 \ldots x'_k) \; ; \; Y = \text{diag}(Y_1, \ldots Y_k)$$

$$\beta' = (\beta'_1 \ldots \beta'_k); \; U' = (u'_1 \ldots u'_k)$$

yields
$$x = Y\beta + U \tag{3.16}$$

Since in general $EU'U = \Omega \otimes I_k$, where Ω is a positive definite matrix, the transformed system (3.16) can be consistently and efficiently estimated by three stage least squares (3SLS) assuming the appropriate identification conditions are met. If we do in fact assume that these identification conditions are met and therefore that (3.16) is estimable, then since

$$\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} = \begin{pmatrix} \beta_{10} & \beta_{11} & \beta_{12} \\ \vdots & \vdots & \vdots \\ \beta_{k0} & \beta_{k1} & \beta_{k2} \end{pmatrix} = (\beta_1 : \beta_2)$$

where $\beta_2' = (\beta_{12} \ldots \beta_{k2})$, partitioning Y conformably allows us to write

$$x = Y_1\beta_1 + Y_2\beta_2 + U \tag{3.17}$$

The hypothesis $\theta_2 = 0$ is now seen to correspond to the null hypothesis H_0: $\beta_2 = 0$, asymptotically equivalent tests of which may be constructed using either the Wald, Lagrange Multiplier or Likelihood Ratio criteria, e.g. the Likelihood Ratio statistic would be

$$\lambda = \frac{|\hat{\Omega}|^{T/2}}{|\hat{\Omega}_{\beta_2=0}|^{T/2}}$$

which is distributed as χ^2 with $Q^* = \sum_i \sum_j n_{ij} + kg$ degreés of freedom under the null hypothesis and where $\hat{\Omega}$ is the unrestricted estimate of the variance-covariance matrix and $\hat{\Omega}_{\beta_2=0}$ is the restricted estimate under this null hypothesis.

For the identification of equation (3.16) it is important to note that since this system is not required to be a parsimonious representation of any particular underlying process, the orders of the lag polynomials making up the regressors are at our disposal. Hence, for instance, sufficient zero restrictions may be placed upon any individual equation in the system by appropriate a priori determination of the values taken by m_{ij} and n_{ie}, the orders of the lag polynomials θ_{1ij} and θ_{2ie} respectively (see 3.14). Recalling the necessary order condition for identification of the ith equation under only zero restrictions - i.e. that the number of excluded variables should be at least (k - 1) (there being k equations in the system), this suggests a particularly useful a priori setting of these orders. If m_{ii} is set equal to $\bar{m}$, the remaining m_{ij} set equal to $\bar{m} - 1$ and the n_{ie} all set to $\bar{n}$ then there will be k - 1 restrictions placed upon the ith equation if this same procedure is used on all k equations. Not only do we now have to determine just the two orders $\bar{m}$ and $\bar{n}$ but as each equation is just identified consistent and efficient estimates of the parameter vector can be obtained by using two stage least squares (2SLS) rather than 3SLS. (See, e.g. Schmidt (1976, p.212)). Thus both an easier parameterisation and a computational saving is achieved by the strategy.

In determining $\bar{m}$ and $\bar{n}$ two factors suggest themselves. The orders should be set high enough to allow for all reasonably strong lag effects and to eliminate serial correlation, the presence of which would make the F tests invalid (see Granger and Newbold (1974)). On the other hand the orders should not be set too high so that degrees of freedom and multicollinearity problems are avoided. These factors are seen to be the criteria of unbiasedness, which demands a generous parameterisation, and power, which necessarily diminishes as the parameter space is expanded. Geweke's (1978) favoured reconciliation of these criteria is to set $\bar{m}$, the order of the lagged x_t, generously while restricting the size of $\bar{n}$. However we feel here that subject matter considerations must play an important role and hence the chosen orders must reflect and be based upon the nature of the data available.

In any case, since typically the data series available for the majority of macroeconomic variables are quite short, even a modest setting of the orders of the lag polynomials will leave precious few degrees of freedom available for estimation in models containing more than two or three variables. An alternative approach to the testing for exogeneity that leads to an increase in power, albeit at some cost to unbiasedness, coupled with a further computational saving is to set the θ_{1ij0}'s to zero in (3.15), thus assuming that there is no simultaneous coupling of the exogenous variables. Under this assumption we have

$$\begin{pmatrix} x_{1t} \\ \vdots \\ x_{kt} \end{pmatrix} = \begin{pmatrix} \theta^*_{111} & \cdots & \theta^*_{11k} \\ \vdots & & \vdots \\ \theta^*_{1k1} & \cdots & \theta^*_{1kk} \end{pmatrix} \begin{pmatrix} x_{1t} \\ \vdots \\ x_{kt} \end{pmatrix} + \begin{pmatrix} \theta_{211} & \cdots & \theta_{21g} \\ \vdots & & \vdots \\ \theta_{2k1} & \cdots & \theta_{2kg} \end{pmatrix} \begin{pmatrix} y_{1t} \\ \vdots \\ y_{gt} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ \vdots \\ u_{kt} \end{pmatrix}$$

or

$$x = Y^*\beta^* + U \tag{3.18}$$

where $Y^{*}_{,} = \text{diag}\ (Y^{*}_{1}, \ldots Y^{*}_{k})$; $Y^{'*}_{i} = (Y^{'*}_{i1}, \ldots Y^{'*}_{iT})$; $Y^{*'}_{iT} = (x'_t y'_t)$

$\beta^{*} = (\beta^{'*}_{1} \ldots \beta^{'*}_{,})$; $\beta^{*}_{i} = (\beta_{i1}\beta_{i2})$

Now (3.18) is seen to be a system of seemingly unrelated regression equations (SURE - see Zellner (1962)) and furthermore, as no identification problem exists, a more convenient simplification may be obtained by setting the lag polynomials of each variable to the same order in each equation, i.e. setting $m_{ij} = \bar{m}_j$ and $n_{ie} = \bar{n}_e$. In this case estimation of (3.18) by SURE is equivalent to estimating each of the k equations individually by ordinary least squares (OLS) as each equation now contains the same set of explanatory variables.

The appropriate test statistic for testing the null hypothesis H_0: $\beta_2 = 0$ is now given by

$$F = \left(\frac{S_R}{S_u} - 1 \right) \cdot \frac{kT - S}{Q}$$

where $S_u = \sum_{i=1}^{k} S_{ui}$, where S_{ui} is the sum of squared residuals obtained by estimating the ith equation of (3.18) by OLS and $S_R = \sum_{i=1}^{k} S_{Ri}$, where S_{Ri} is the sum of squared residuals obtained by estimating the ith equation of (3.18) by OLS under the restriction $\beta_{i2} = 0$. In this statistic $Q = \sum_j \bar{n}_j + kg$ and $S = \sum_j \bar{m}_j + Q$ and F is distributed as $F(Q, kT - S)$ with calculated values larger than the appropriate critical α value leading to the rejection of exogeneity at the α level of significance.

An individual member of the x vector, say the ith, may be tested for exogeneity with respect to the endogenous vector y_t by the statistic

$$F_i = \left(\frac{S_{Ri}}{S_{ui}} - 1 \right) \cdot \frac{T - S_i}{Q_i}$$

where $Q_i = \sum_j \bar{n}_j + g$ and $S_i = \sum_j \bar{m}_j + Q_i$, and where F_i is distributed as $F(Q_i, T - S_i)$.

As is apparent from the papers by Zellner and Palm (1974) and Wallis (1977) the partitioning of the z_t vector into endogenous and exogenous subvectors is normally decided by utilising prior information, usually in the form of received economic theory. It is our contention here that, like Geweke (1978), such a priori classifications should be tested before constructing a dynamic economic model based upon them. By employing the tests proposed here whole classes of possible models may be rejected by the data before the costs of specification and estimation are incurred.

> It makes no sense to discuss and test the detailed dynamics of an econometric model within a framework which can be rejected in its entirety by the data.
>
> Geweke (1978, p.182).

3.2 EXOGENEITY AND THE CONCEPT OF CAUSALITY

The definition of exogeneity on which the development of the above tests were based is intimately linked with the concept of causality proposed by Granger (1969) and empirically implemented by Sims (1972).[2]

Granger's definition of causality is usually stated for the

2 For a survey of both the theoretical and empirical literature see Pierce and Haugh (1977).

bivariate case. Consider a stationary stochastic process A_t. Let $\overline{A}_t$ represent the set of past values of $A_t\{A_{t-j}, j = 1, 2, .., \infty\}$ and let $\overline{\overline{A}}_t$ represent the set of past and present values $\{A_{t-j}, j = 0, 1, 2, ..., \infty\}$. Denote the optimum, unbiased, linear least squares predictor of A_t using the set of values B_t by $P_t(A/B)$, the predictor error series by $\varepsilon_t(A/B) = A_t - P_t(A/B)$ and the variance of this series by $\sigma^2(A/B)$.

Suppose that the relevant information set comprises of just the two series X_t and Y_t. Then we have the following results

$$P_t(X/\overline{X}, \overline{Y}) = \sum_{i=1}^{\infty} h_{1i}X_{t-i} + \sum_{i=1}^{\infty} h_{2i}Y_{t-i} = h_1^*(L)X_t + h_2^*(L)Y_t$$

using the notation introduced in section 3.1.

$$P_t(X/\overline{X}, \overline{\overline{Y}}) = \sum_{i=1}^{\infty} h_{1i}X_{t-1} + \sum_{i=0}^{\infty} h_{2i}Y_{t-i} = h_1^*(L)X_t + h_2(L)Y_t$$

$$P_t(X/\overline{X}) = \sum_{i=1}^{\infty} h_{1i}X_{t-i} = h_1^*(L)X_t$$

$$\varepsilon_t(X/\overline{X}, \overline{Y}) = X_t - h_1^*(L)X_t - h_2^*(L)Y_t$$

$$\varepsilon_t(X/\overline{X}, \overline{\overline{Y}}) = X_t - h_1^*(L)X_t - h_2(L)Y_t$$

$$\varepsilon_t(X/\overline{X}) = X_t - h_1^*(L)X_t$$

with $\sigma^2(X/\overline{X}, \overline{Y})$, $\sigma^2(X/\overline{X}, \overline{\overline{Y}})$ and $\sigma^2(X/\overline{X})$ being the variances of the respective predictor error series. We can now state the following definitions

(1) <u>Causality</u>: If $\sigma^2(X/\bar{X}, \bar{\bar{Y}}) < \sigma^2(X/\bar{X})$ then Y is said to cause X i.e. we are better able to predict X_t by using the information contained in the series Y than by just using the series X.

<u>Corrollary</u>: If $\sigma^2(X/\bar{X}, \bar{Y}) < \sigma^2(X/\bar{X}, \bar{\bar{Y}}) < \sigma^2(X/\bar{X})$ then Y is said to cause X but not instantaneously. If $\sigma^2(X/\bar{X}, \bar{\bar{Y}}) < \sigma^2(X/\bar{X}, \bar{Y}) < \sigma^2(X/\bar{X})$ then there is said to be instantaneous causality.

(2) <u>Feedback</u>: If $\sigma^2(X/\bar{X}, \bar{\bar{Y}}) < \sigma^2(X/\bar{X})$ and $\sigma^2(X/\bar{Y}, \bar{\bar{X}}) < \sigma^2(Y/\bar{Y})$ then there is said to be feedback between X and Y.

From these definitions we see that Y will cause X if $h_2(L)$ is non zero and this supplies the relationship between causality and exogeneity. To make this apparent rewrite the model (3.2) in a form comparable to (3.15),

$$\begin{pmatrix} 1 & h_{120} & \cdots & h_{1p0} \\ \\ h_{p10} & \cdots\cdots & & 1 \end{pmatrix} \begin{pmatrix} z_{1t} \\ \\ z_{pt} \end{pmatrix} = \begin{pmatrix} h^*_{11} & \cdots & h^*_{1p} \\ \\ h^*_{p1} & \cdots & h^*_{pp} \end{pmatrix} \begin{pmatrix} z_{1t} \\ \\ z_{pt} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \\ \varepsilon_{pt} \end{pmatrix} \tag{3.19}$$

The system (3.19) can be seen to be the multivariate generalisation of the bivariate instantaneous causality model given by Granger (1969, p.431). The above definitions of causality may be easily extended to this system. Thus let $\sigma_i^2(z_i/z)$ be the variance of ε_{it} and let $\sigma_i^2(z_i/z - z_j)$ be the variance of ε_{it} after the deletion of the jth variable from the ith equation. Then if $\sigma_i^2(z_i/z) < \sigma_i^2(z_i/z - z_j)$, z_j causes z_i. This can be seen to imply that $h_{ij} = (h_{ij0}, h_{ij}^*)$ must be non zero, i.e. z_j causes z_i if z_i is better predicted by including the variable z_j in the predicting equation than by omitting it. It follows then that instantaneous causality exists if h_{ij0} is non zero, and feedback exists if both h_{ij} and h_{ji} are non zero.

In this framework any single variable, z_j say, will be exogenous to the rest of the z vector if $h_{ij} \equiv 0$ for all $i \neq j$, i.e. that no values of any other variables apart from lagged values of z itself appear in the predicting equation. We see then that a test for exogeneity of a single variable may be regarded as a joint test that the other (p - 1) variables do not cause that particular variable. The test of the exogeneity of a subvector of z can be seen as a joint test of there being no causal relationships running from the "endogenous" subvector to the "exogenous" subvector.

3.3 ZELLNER'S CRITIQUE OF GRANGER'S DEFINITION OF CAUSALITY

The tests of causality developed in 3.1 have been shown to be multivariate extensions of the bivariate causality tests developed by Granger (1969), although they are somewhat more specialised in that they test for only one direction of causality - that of feedback from the endogenous to the exogenous variables, causality existing from the exogenous to the endogenous variables being implicitly assumed in the model building procedure.

As Zellner (1978) has recently criticised Granger's definition of causality and the empirical tests based upon it, e.g. those associated with Sims (1972), it is obviously important to discuss Zellner's criticisms in the context of exogeneity tests in dynamic simultaneous models. Zellner criticises this definition of causality on three grounds. The use of the criterion of the forecast error of a linear, unbiased, least squares predictor is criticised on the grounds that it may not always be available and, on occasions, may even be inadmissable. The definition of causality is particularly criticised for being devoid of "subject matter considerations", for example, no mention of relevant economic laws is made. Finally, bivariate causality tests, the type that have been usually investigated in the literature, are inadequate as they restrict analysis to the investigation of causal patterns existing between just two variables, thus leaving the results open to spurious causality. This problem, also discussed by Granger and Newbold (1977, p.225), is analogous to the omitted variable problem in regression analysis. A further criticism of the early empirical investigations based on this criteria is the mechanical, a priori, prefiltering of the actual data series which has been found to inadequately render series white noise (see Pierce and Haugh (1977)).

In the context of the exogeneity tests proposed for dynamic simultaneous equation models these criticisms do not seem particularly applicable. The prediction error variance criterion, which is equivalent to testing whether the coefficients of the appropriate lag polynomials are zero, is seen to be a natural criterion to use. More importantly, the criticism that economic theory plays little part in the testing procedure is clearly invalid as it is the use of such theory that determines the testable classification of endogenous and exogenous variables. The bivariate framework criticism is obviously invalidated by the multivariate framework, an essential feature of the whole analysis. With regard to the empirical testing procedures, Geweke (1978) has shown that causality tests based on Granger's definition do not require stationarity and furthermore, prefiltering techniques that are open to criticism are avoided in the tests developed in the previous section.

We therefore feel that although Zellner's criticisms may well be valid for tests of causality applied to two series in isolation, they are not appropriate in the multivariate framework of dynamic simultaneous models that are being explicitly considered here. In particular economic theory is used to suggest both the form of the relationships under consideration and the endogenous - exogenous variable classifications, which, of course, is subjected to empirical verification.

3.4 A DYNAMIC IS/LM MODEL

As we have argued in chapter two, the IS/LM model outlined there, apart from not enabling exogeneity assumptions to be tested, is inadequate in (at least) two theoretical respects.

The static framework of the model is unduly restrictive in that the possible importance of dynamic behaviour cannot be ascertained; in particular such matters as the importance of time lags in the structural relationships, the possibility of cyclical disequilibrium behaviour and the consequences of stabilisation policies must all be automatically excluded from analysis. Secondly, the treatment of the price level - real income interaction as obeying one of two extreme assumptions, either the Keynesian assumptions that the price level is fixed exogenously and real income is determined endogenously, or the monetarist contention which reverses these assumptions, is obviously inadequate.

The modification of the IS/LM model to incorporate dynamic behaviour has been considered by Tucker (1966), Tanner (1969) and further advanced by Laidler (1971, 1973 and 1977). Even so, the most sophisticated of the models developed only incorporate partial adjustment and adaptive expectations lag mechanisms with the price level assumed fixed. An intermediate position in which both the price level and real income are endogenised has been proposed by McCallum (1975, 1976) and Turnovsky (1977) whereby a price adjustment equation is posited allowing price to respond to current disequilibria in the product market. The derivation of such an equation may be sketched in the following way for a static model. Let W denote the percentage change in money wages and U the unemployment rate and consider the Phillip's curve

$$W = \alpha_0 + \alpha_1 U$$

Since we are dealing with a short run model in which the capital stock is given, we may assume that real income and employment (N) are related by

$$Y = F(N).$$

Assuming that the unemployment rate is sufficiently small so that F(N) can be adequately represented by a linear approximation about the full employment level $\overline{N}$ we obtain

$$U = \frac{\overline{N} - N}{N} = h(Y - Y^*)$$

where Y^* is the corresponding full employment output level. Labour productivity will remain constant if zero technical change is assumed so that P = W and therefore that

$$P = \alpha_0 + \alpha_1 h(Y - Y^*). \qquad (3.20)$$

A dynamic IS/LM model can now be constructed in which general lag mechanisms are incorporated and where both the price level and real income are treated endogenously. Thus analogous to the static model incorporating equations (2.1) to (2.6), we have

$$e(L)E = a(L)Y - b(L)R + f(L)P + \varepsilon_1 \tag{3.21}$$

$$G = \overline{G}$$

$$Y = E + G$$

$$g(L)M^d = c(L)Y - d(L)R + h(L)P + \varepsilon_2 \tag{3.22}$$

$$M^s = \overline{M}$$

$$M^s = M^d$$

$$i(L)P = j(L)Y + \varepsilon_3 \tag{3.23}$$

Equations (3.21) and (3.22) are stochastic expenditure and demand for money functions in which the endogenous price level appears as an explanatory variable, all coefficients being scalar polynomials in the lag operator L. Equation (3.23) is a linearised and dynamic version of (3.20) in which the exogenous full employment level of income, being a theoretical concept without an empirical counterpart, is conveniently omitted. These three structural equations can be combined with the equilibrium conditions and exogeneity assumptions to yield the following system

$$(e(L) - a(L))Y = f(L)P - b(L)R + e(L)\overline{G} + \varepsilon_1$$

$$d(L)R = c(L)Y + h(L)P - g(L)\overline{M} + \varepsilon_2$$

$$i(L)P = j(L)Y + \varepsilon_3$$

or in more conventional form

$$\beta_{11}(L)Y_t + \beta_{12}(L)P_t + \beta_{13}(L)R_t + \gamma_1(L)\overline{G}_t = \varepsilon_{1t} \tag{3.24}$$

$$\beta_{21}(L)Y_t + \beta_{22}(L)P_t + \beta_{23}(L)R_t + \gamma_2(L)\overline{M}_t = \varepsilon_{2t} \tag{3.25}$$

$$\beta_{31}(L)Y_t + \beta_{32}(L)P_t = \varepsilon_{3t} \tag{3.26}$$

and by making the definitions

$$y_t' = (Y_t P_t R_t) \qquad x_t' = (G_t M_t) \qquad \varepsilon_t' = (\varepsilon_{1t}\varepsilon_{2t}\varepsilon_{3t})$$

$$B(L) = \begin{pmatrix} \beta_{11}(L) & \beta_{12}(L) & \beta_{13}(L) \\ \beta_{21}(L) & \beta_{22}(L) & \beta_{23}(L) \\ \beta_{31}(L) & \beta_{32}(L) & 0 \end{pmatrix} \qquad C(L) = \begin{pmatrix} \gamma_1(L) & 0 \\ 0 & \gamma_2(L) \\ 0 & 0 \end{pmatrix}$$

the system (3.24) to (3.26) can be expressed as

$$B(L)y_t + C(L)x_t = \varepsilon_t \tag{3.27}$$

which is a dynamic simultaneous equation model as considered by Wallis (1977) and Geweke (1978). The model will be stable if the roots of $|B(L)| = 0$ all lie outside the unit circle. In general $B(L)$ and $C(L)$ can be expressed as

$$B(L) = B_0 + B_1L + \ldots + B_rL^r \qquad C(L) = C_0 + C_1L + \ldots + C_sL^s$$

and hence the reduced form of the system, expressing each endogenous variable as a function of predetermined variables, is

$$y_t = -B_0^{-1}(B_1y_{t-1} + \ldots + B_ry_{t-r} + C(L)x_t) + B_0^{-1}u_t. \tag{3.28}$$

The final form expresses each endogenous variable as an infinite distributed lag function of the exogenous variables, together with an error term comprising moving averages of the original disturbances,

$$y_t = -B(L)^{-1}C(L)x_t + B(L)^{-1}u_t \tag{3.29}$$

The coefficients in the expansion of $B(L)^{-1}C(L)$ provide dynamic multipliers, describing the response of y_{it} to a unit change in $x_{j,\,t-1}$.

The final equations may be obtained by first writing $B(L)^{-1} = b^*(L)/|B(L)|$, where $b^*(L)$ is the adjoint matrix of $B(L)$. Equation (3.29) therefore becomes

$$y_t = -\frac{b^*(L)C(L)}{|B(L)|}x_t + \frac{b^*(L)}{|B(L)|}u_t \tag{3.30}$$

Multiplying through by $|B(L)|$ yields

$$|B(L)|y_t = -b^*(L)C(L)x_t + b^*(L)u_t \tag{3.31}$$

in which each equation relates a given endogenous variable to its own past values and to the exogenous variables, but to no other endogenous variable. While, in general, each of the endogenous variables will have a common autoregressive operator, $|B(L)|$, this will not be the case in this particular system under consideration. From its definition we see that $B(L)$ is block diagonal (there being no feedback from R to P) and as Wallis (1977 p.1482-3) points out, this will result in cancellation of factors across equations, in fact resulting in the equations for Y and P having a simpler autoregressive operator than that of the R equation.

If the exogenous variables $x_t = (G_tM_t)$ have a vector autoregressive representation

$$\Phi(L)x_t = \eta_t \tag{3.32}$$

then (3.32) can be combined with (3.27) to yield

$$\begin{pmatrix} B(L) & C(L) \\ 0 & \Phi(L) \end{pmatrix} \begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \tag{3.33}$$

which is, of course, in the form of the multivariate autoregressive process discussed in section 3.1 with the exogeneity assumption of x_t made explicit. Using the procedures just developed the assumption of the exogeneity of the x_t vector may be subjected to empirical verification, and it is to such tests that we now turn.

3.5 EXOGENEITY TESTS FOR THE DYNAMIC IS/LM MODEL

The dynamic IS/LM model denoted by equation (3.33) is a formal framework depicting the assumed interactions existing between money, real income, prices and the interest rate and, therefore, the data series utilised in chapter one are available for testing the exogeneity assumptions of the model.

However, an empirical counterpart of the government expenditure variable G is required and consequently the nominal general government expenditure series was employed, suitably logarithmically transformed and seasonally adjusted.

As two money series and two interest rate series are available, four alternative vectors of variables may be examined, viz.

$$z_1 = (Y, RC, P, M1, G)$$
$$z_2 = (Y, RL, P, M1, G)$$
$$z_3 = (Y, RC, P, M3, G)$$
$$z_4 = (Y, RL, P, M3, G)$$

For each vector we wish to test the appropriateness of the partitioning $z = (y, x)$ where $x = (M, G)$ is the exogenous variable vector, i.e. in the framework of equation (3.3) we wish to test the null hypothesis that the (2 x 3) matrix H_{21} is identically zero. If we make the assumption that there is no simultaneity existing between G and M then the model (3.18) is the appropriate one to use for testing exogeneity and can be written here as

$$\begin{pmatrix} M_t \\ G_t \end{pmatrix} = \begin{pmatrix} \theta^*_{111} & \theta^*_{112} \\ \theta^*_{121} & \theta^*_{122} \end{pmatrix} \begin{pmatrix} M_t \\ G_t \end{pmatrix} + \begin{pmatrix} \theta_{211} & \theta_{212} & \theta_{213} \\ \theta_{221} & \theta_{222} & \theta_{223} \end{pmatrix} \begin{pmatrix} Y_t \\ R_t \\ P_t \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \qquad (3.34)$$

in which we require to test the null nypothesis

$$H_0 : \theta_2 = \begin{pmatrix} \theta_{211} & \theta_{212} & \theta_{213} \\ \theta_{221} & \theta_{222} & \theta_{223} \end{pmatrix} \equiv 0$$

The recommendations for enabling this bivariate regression system to be consistently and efficiently estimated by the application of OLS to each equation individually were that the orders m_{ij} and n_{ie} of the lag polynomials θ^*_{1ij} and θ_{2ie} should be set equal to $\bar{m}_j$ and $\bar{n}_e$ respectively. Taking into account the number of observations available, the number of variables in the system and the nature of the data, i.e. quarterly, this led to the decision to set the $\bar{m}_j$ equal to four and the $\bar{n}_e$ equal to two, thus leading to the specification of the following regression equations, constants also being included,

$$M_t = \sum_{j=1}^{4} \theta_{111j} M_{t-j} + \sum_{j=1}^{4} \theta_{112j} G_{t-j} + \sum_{e=0}^{2} \theta_{211e} Y_{t-e}$$

$$+ \sum_{e=0}^{2} \theta_{212e} R_{t-e} + \sum_{e=0}^{2} \theta_{213e} P_{t-e} + \phi_1 + u_{1t} \qquad (3.35)$$

$$G_t = \sum_{j=1}^{4} \theta_{121j} M_{t-j} + \sum_{j=1}^{4} \theta_{122j} G_{t-j} + \sum_{e=0}^{2} \theta_{221e} Y_{t-e}$$

$$+ \sum_{e=0}^{2} \theta_{222e} R_{t-e} + \sum_{e=0}^{2} \theta_{223e} P_{t-e} + \phi_2 + u_{2t} \qquad (3.36)$$

Denoting the sums of squared residuals obtained by estimating (3.35) and (3.36) by OLS and S_{u1} and S_{u2} respectively as the sums of squared residuals obtained by estimating these equations under the restriction to the null hypothesis as S_{R1} and S_{R2}, i.e. from the regressions

$$M_t = \sum_{j=1}^{4} \theta_{111j} M_{t-j} + \sum_{j=1}^{4} \theta_{112j} G_{t-j} + \phi_1 + u_{1t} \qquad (3.37)$$

$$G_t = \sum_{j=1}^{4} \theta_{121j} M_{t-j} + \sum_{j=1}^{4} \theta_{122j} G_{t-j} + \phi_2 + u_{2t} \qquad (3.38)$$

the appropriate test statistics can then be constructed. Referring to the development of such test statistics in Section 3.1 we have

$$S_u^* = S_{u1} + S_{u2} \text{ , } S_R^* = S_{R1} + S_{R2}$$

and furthermore $g = 3$, $k = 2$, $Q^* = 18$ and $S^* = 36$ (including the two constant terms)[4] Thus, to test $H_0 : \theta_2 = 0$ the statistic

$$F = \left(\frac{S_R^*}{S_u^*} - 1 \right) \cdot \left(\frac{76}{18} \right)$$

will be distributed as $F(18, 76)$, high values of F leading to rejection

4 The number of observations, T, is equal to 56 as the maximum lag length has been set to 4.

of the null hypothesis. The exogeneity of M or G may be tested individually by constructing

$$F_i = \left(\frac{S_{Ri}}{S_{ui}} - 1\right) \cdot \frac{38}{9}$$

which will be distributed as F (9, 38), also following the development of section 3.1.

The calculated F statistics and the accompanying sums of squared residuals for each of the four z vectors are shown in Table 3.1. We see that in all four variable specifications the hypothesis of exogeneity of the x vector is rejected and indeed in every case the exogeneity of both money and government expenditure individually must be rejected. We must therefore conclude that this general dynamic IS/LM model is misspecified in that its exogeneity assumptions are found to be rejected by the data - money, however defined, and government expenditure cannot be considered as being exogenously determined with respect to the other variables in the model. Therefore, the model may be regarded as being unsuitable for further empirical research.

However, as a major concern of this thesis is the empirical investigation of the demand for money function we must consider the implications of these results for the estimation of this function. Clearly, from the results presented in chapter 2, a finding that money was actually exogenously determined would prohibit the conventional estimation of the demand for money function. While the finding that money is in fact endogenously determined does indeed allow the function to be estimated conventionally, it does not imply that OLS is the appropriate estimation technique. Consider therefore a dynamic demand for money function,

TABLE 3.1

EXOGENEITY TEST STATISTICS FOR DYNAMIC IS/LM MODEL

$z_1 = (Y, RC, P, M1, G)$

$S_{u1} = .00566031$	$S_{R1} = .0173297$	$F_1 = 8.70^{**}$
$S_{u2} = .0389612$	$S_{R2} = .0724108$	$F_2 = 3.62^{**}$
$S_u = .0446215$	$S_R = .0897405$	$F = 4.27^{**}$

$z_2 = (Y, RL, P, M1, G)$

$S_{u1} = .00450173$	$S_{R1} = .0173297$	$F_1 = 12.03^{**}$
$S_{u2} = .0376449$	$S_{R2} = .0724108$	$F_2 = 3.90^{**}$
$S_u = .04214663$	$S_R = .0897405$	$F = 4.77^{**}$

$z_3 = (Y, RC, P, M3, G)$

$S_{u1} = .00452968$	$S_{R1} = .0115987$	$F_1 = 6.59^{**}$
$S_{u2} = .0363049$	$S_{R2} = .0604761$	$F_2 = 2.81^{*}$
$S_u = .0408346$	$S_R = .0720748$	$F = 3.23^{**}$

$z_4 = (Y, RL, P, M3, G)$

$S_{u1} = .00523536$	$S_{R1} = .0115987$	$F_1 = 5.13^{**}$
$S_{u2} = .0336189$	$S_{R2} = .0604761$	$F_2 = 3.37^{**}$
$S_u = .0388543$	$S_R = .0720748$	$F = 3.61^{**}$

$F_{.95}(9, 38) \simeq 2.13$ $\quad$ $F_{.99}(9, 38) \simeq 2.93$

$F_{.95}(18, 76) \simeq 1.78$ $\quad$ $F_{.99}(18, 76) \simeq 2.22$

* denotes significance at .05 level

** denotes significance at .01 level

$$\beta_1(L)M_t = \beta_2(L)Y_t + \beta_3(L)R_t + \beta_4(L)P_t + u_t \tag{3.39}$$

where the $\beta_i(L)$ are lag polynomials in L with $\beta_{10} = 1$. For OLS to produce consistent and efficient estimates of (3.39) we require that Y, R and P must be exogenous with respect to M (and, of course, that M be endogenously determined with respect to Y, R and P). Thus (3.39) can be considered part of a multivariate system in which the variable vector, z say, is partitioned in usual notation as $z = (yx)$ with $y = M$ and $x = (Y, R, P)$. The assumption of exogeneity of x can again be tested by using the techniques just developed. If we again assume that no simultaneity exists between exogenous variables then in the multivariate regression

$$\begin{pmatrix} Y_t \\ R_t \\ P_t \end{pmatrix} = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{pmatrix} \begin{pmatrix} Y_t \\ R_t \\ P_t \end{pmatrix} + \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} M_t + u_t \tag{3.40}$$

where $\delta_{ij} = \sum_{l=1}^{d_{ij}} \delta_{ijl} L^l$ and $\gamma_i = \sum_{l=0}^{c_i} \gamma_{il} L^l$

testing the assumption that $x = (Y, R, P)$ is exogenous is equivalent to testing the null hypothesis $H_0 : \gamma' \equiv (\gamma_1 \gamma_2 \gamma_3) = 0$.

In this framework we also note that the test of the endogeneity of M simply requires the estimation of (3.39) and testing whether the vector $\beta = (\beta_2(L), \beta_3(L), \beta_4(L))$ is zero. If β is non zero then M is endogenously determined, i.e. that if a dynamic regression model relating M to any or all of Y, R, and P exists then M must be endogenous. Hence the existence of a dynamic demand for money function implies that money is endogenously determined.

However, exogeneity of the regressors in equation (3.39) is a sufficient condition for OLS to be used appropriately. A necessary condition is that no simultaneous feedback should exist, i.e. in equation (3.40) a necessary condition is that $\gamma_{10} = \gamma_{20} = \gamma_{30} = 0$. Thus while there may be feedback from M to the regressors (in which case the sufficient exogeneity restrictions are violated), the absence of simultaneity will produce a purely recursive model in which OLS will still produce consistent and efficient estimates of the parameters of equation (3.39).

To perform these exogeneity tests the orders of the lag polynomials in (3.40) were again set so as to allow the individual equations to be estimated by OLS. In this case the d_{ij}'s were set to four and the c_i's set to three, with constants again being included in the regressions. Rather than jointly test the exogeneity of the complete x vector, as we were primarily interested in which of the regressors, if any, violated the exogeneity assumptions, tests were performed on the regressors individually. Analogous to the construction of the tests for equation (3.34), the exogeneity of the ith member of x was tested by calculating

$$F_i = \left(\frac{S_{Ri}}{S_{ui}} - 1\right) \cdot \frac{39}{4}$$

which is distributed as $F(4, 39)$ and where, as usual, S_{ui} is the unrestricted sum of squared residuals from the ith equation in (3.40) and S_{Ri} is the sum of squared residuals from the ith equation obtained under the restriction $\gamma_i = 0$. For these variables whose exogeneity was rejected, simultaneity was tested by computing the conventional t statistic associated with the parameter γ_{i0}. For completeness the assumption of the exogeneity of money was also tested by estimating (3.39) with the order of β_1 set to four and the orders of the regressor polynomials set to three. In this case the test statistic is

$$F_M = \left(\frac{S_{RM}}{S_{uM}} - 1\right) \cdot \frac{39}{12}$$

which is distributed as $F(12, 39)$ and where S_{uM} is the sum of squared residuals from estimation of (3.39) and S_{RM} is the sum of squared residuals from estimation under the restriction $\beta = 0$.

Once again there are four alternative vectors of variables, denoted

$$z_1 = (M1, Y, RC, P)$$
$$z_2 = (M1, Y, RL, P)$$
$$z_3 = (M3, Y, RC, P)$$
$$z_4 = (M3, Y, RL, P)$$

The F statistics, and where appropriate t statistics, obtained from these four alternatives are shown in Table 3.2. As would be expected from previous results, both definitions of money are found to be endogenously determined, thus confirming our earlier conclusion that the demand for money can be estimated in conventional fashion. The assumption of exogeneity is rejected for two regressors, the local authority yield when M1 is the endogenous variable and the consol yield when M3 is the endogenous variable. However, only in the former case is there evidence, from the t statistics, of simultaneity. Thus we may conclude that all four specifications of the demand for money function can be conventionally estimated, with OLS able to produce consistent and efficient estimates for three of the specifications but because of simultaneity between M1 and the local authority yield the specification containing

TABLE 3.2

DEMAND FOR MONEY EXOGENEITY TESTS

$z_1 = (M1, Y, RC, P)$

$F_1 = 2.27$

$F_2 = 1.83$

$F_3 = 0.95$

$F_M = 6.20^*$

$z_2 = (M1, Y, RL, P)$

$F_1 = 1.61$

$F_2 = 5.47^*$ $\quad t_2 = 4.02^*$

$F_3 = 0.67$

$F_M = 9.26^*$

$z_3 = (M3, Y, RC, P)$

$F_1 = 1.30$

$F_2 = 3.69^*$ $\quad t_2 = 1.49$

$F_3 = 2.32$

$F_M = 3.76^*$

$z_4 = (M3, Y, RL, P)$

$F_1 = 1.15$

$F_2 = 2.38$

$F_3 = 1.37$

$F_M = 3.21^*$

$F_{.95}(4, 39) = 2.62$ $\quad F_{.95}(12, 39) = 2.00$ $\quad t_{.975}(39) = 2.02$

(*) denotes significance at .05 level.

these two alternatives requires estimation by some form of instrumental variables (IV) technique to produce consistent estimates.

3.6 THE IMPLICATIONS OF THE RESULTS AND SOME REINTERPRETATION OF PREVIOUS WORK

As we have stated above, the results of these exogeneity tests do not seem to conflict with the conventional practice of estimating demand for money functions similar to equation (3.39). However, we have highlighted the possibility of simultaneity in the relationship, which, although it has been mentioned before (see e.g. Hendry and Mizon (1978)), has not been explicitly modelled. Nevertheless, there seems strong evidence for continuing with the usual approach of estimating the function by OLS.

Also interesting is the comparison of these results with the findings of two recent papers by Williams, Goodhart and Gowland (1976) and Mills (1979). Williams et al employed Sims' (1972) test to investigate the relationship between money and income over the period 1958 to 1971, finding no clear evidence of causality in either direction. Mills, using data almost identical to that used here, found causality running from real GDP to both M1 and M3 when using the Sims test. As serial correlation was present in the regressions used for this test, thus invalidating the test statistics (a common occurrence in such tests, see Pierce and Haugh (1977)), Mills re-examined the relationship using a direct causality test proposed by Sargent (1976). In this case serial correlation problems were avoided but the direction of causality was reversed, causality now running from money to real income. Mills suggested that the inability of Williams et al to discover any causal relationship between money and income may, to some extent, have been due to serial correlation problems but may have also been caused by using data from a period in which the exchange rate was pegged. This argument has been developed more fully in Mills and Wood (1978), where it is argued that as the monetary theory of the balance of payments predicts that the monetary authorities in a non reserve centre can only fully control domestic monetary conditions under a freely floating exchange rate regime, under pegged exchange rates the authorities control is limited by the extent to which they are willing to allow their exchange rate to change or their willingness to change their stock of international reserves. This implies that the causal relationship between money and income, rather than not existing, as suggested by Williams et al, may have varied within their period of analysis.

> On some occasions, when the monetary fluctuation either originated from the reserve center, was in line with a monetary fluctuation in the reserve center, or was accommodated by an exchange rate change, money influenced income in the U.K. At other times, the monetary stimulus, of domestic origin, led to balance of payments pressure which induced the monetary authorities to reverse their previous monetary policy with sufficient rapidity that the initial monetary stimulus did not persist long enough to have a discernible effect on income. If this occurred, no causality from money to income would be observed. Further, when the U.K. monetary authorities were pegging the exchange rate and resisting interest rate movements - as they were for a substantial part of the data period used by Williams et al - an exogenous income fluctuation would induce an accommodating monetary response.
>
> (Mills and Wood 1978, p.23).

The data used here and by Mills (1979) is less susceptible to such problems as a substantial part of the data is post float (June 1972) although this floating rate has not been free of official intervention. The results of these papers strongly suggest that both serial correlation and "subject matter considerations" play an important role in detecting causal relationships, as advanced by Zellner (1978).

However, the results presented in this chapter suggest a third reason for the apparent divergencies in the observed causal relationship between money and income. As we have shown in section 3.2 there is an intimate connection between the definition of exogeneity used here and Granger's (1969) definition of causality on which Sims' type tests are based. The results presented here may be interpreted as suggesting the presence of unidirectional causality from real income and prices to money with no evidence of feedback. More importantly these "causality patterns" have been obtained by explicitly using a multivariate framework, unlike the bivariate frameworks employed by all previous studies. As Granger and Newbold (1977, p.225) note, it will always be possible to obtain spurious causality between two variables because a third variable, causal to both, has been omitted from the analysis. This is directly analogous to the omitted variable problem in classical regression analysis and a contender is immediately apparent - the interest rate. Thus is seems very likely, then, that previously observed causal relationships may have been spuriously achieved by the omission of the interest rate from the analysis.

The other consequence of the results presented here is the seeming demise of the IS/LM model, at least as a framework for empirical research. This cannot really be said to be a surprising conclusion, for support for the proposition that the money supply is endogenously determined has been widespread. The usual justification for the endogeneity of money has been the behaviour of the monetary authorities in pursuing the objective of pegging, or supporting, interest rates. If this policy is pursued (and it is widely accepted that, at least up until about five years ago, the targets of the authorities were set in interest rate terms, see Goodhart (1975)) then it will lead the authorities to allow the money stock to vary in line with the level of income.

A further justification for the endogeneity of money may be found from the "new view" of money supply determination (see Chick (1973 ch.5)). In the "new view" banks are seen to be responsive to private sector portfolio choice; hence the money supply is dictated by the wants of the private sector. Since it is true that the Bank of England supplies whatever currency the general public wants and that current account holdings can be rearranged quickly with little cost, this suggests that a priori M1 should be seen as demand determined. From our results it seems as though M3 may also be regarded as demand determined as well.

The feature of the exogeneity tests on the regressors entering the demand for money function is the absence of any feedback from money to either real income or prices, thus rejecting any direct transmission mechanism of monetary policy. (It is possible that since there is some evidence of feedback from money to interest rates, an indirect "Keynesian" mechanism may operate).[5] Two objections to such a conclusion may be raised. The first is that it is often advanced that the lag effect on prices of changes in the money supply is notoriously long and variable. If this is the case then the lag lengths used in this analysis may be too short to pick up significant feedback effects. Unfortunately this is the price that one must pay when conducting a multivariate analysis on data series of limited length.[6]

5 See Goodhart and Crockett (1970) for extended discussion of the respective transmission mechanisms.

6 Note also that apart from test's power problems, increasing the number of lagged regressors may even prevent the regression from being computable, a situation quite easily achieved on the TSP package.

Secondly, while it may be accepted that no feedback from money to income existed during the early part of the data period, i.e. when both interest rates and the exchange rate were pegged, neither situation has been in operation since about 1972. With both the publication of money supply targets and a floating exchange rate, implying greater control of monetary conditions, it may be argued that a more conducive environment for feedback from money now exists but has not been in operation long enough to be able to be detected when analysing the full data period. Parkin (1978) has recently argued, though, that the monetary control technique usually practised is to specify a growth range for the money supply and try to achieve this by using the best available estimated demand for money function and then manipulating the interest rate to achieve the desired money supply. Again, the money supply will be endogenously determined in this case with the interest rate exogenous.

Nevertheless, the results of this chapter confirm that the demand for money function may be estimated in its conventional form, but consideration must be paid to the possibility of simultaneity, the presence of which must necessitate using an IV estimation technique rather than the commonly used OLS.

CHAPTER FOUR :

DYNAMIC SPECIFICATION OF THE DEMAND FOR MONEY FUNCTION

4.1 INTRODUCTORY REMARKS

The results of the previous chapter confirm the appropriateness of estimating conventional demand for money functions but have highlighted two important features inadequately covered by previous research, namely the importance, albeit shown indirectly, of dynamic specification and the possibility of feedback from money to the regressors (i.e. simultaneity).

The purpose of this chapter is to investigate in detail dynamic specification within the demand for money function using appropriate estimation techniques that take into account any previously encountered feedback. The methodology employed is that proposed by David Hendry and Grayham Mizon for determining empirically the appropriate specification of a dynamic autoregression model. (See Hendry (1974, 1977, 1978), Mizon (1977a) and Hendry and Mizon (1978, 1979)). The plan of the chapter is as follows. The methodology is first outlined within the framework of a dynamic demand for money function containing conventional regressors, with the function being subsequently extended to incorporate additional explanatory variables proposed by various authors as potentially important determinants of the demand for money. The methodology is then employed to empirically determine the appropriate dynamic specification of the demand function for each of the four combinations of variables analysed in the previous chapter. Emphasis is placed on the detailed description of the specification searches undertaken to show how an acceptable model may be obtained by appropriate respecification based on empirical performance. The theoretical implications of the finally chosen specifications are analysed, with particular attention being focused on the long run (steady state) solutions of the models.

4.2 DYNAMIC SPECIFICATION METHODOLOGY

Davidson et al (1978) have suggested three principles on which a constructive research strategy for applied econometric modelling might profitably be based. Firstly, any new model should be related to existing models, with these previous explanations only being supplanted if new proposals account for previously understood results and also explain some new phenomena. Secondly, to be empirically acceptable a model must account for the properties of the data and finally, to avoid directionless research and uninterpretable measurements, a theoretical framework is essential. Unfortunately, economic theory is notorious for the shortage of detailed information it yields on the dynamic, short run, structure of economic relationships, concentrating as it does on long run, steady state conditions. This is particularly so in monetary theory, where disequilibrium behaviour in the demand for money is typically modelled by assuming ad hoc adjustment mechanisms, usually of a partial adjustment or adaptive expectations form.

In view of these principles, a model of the demand for money is required embodying the following characteristics. It should both model short run disequilibrium dynamics and yield an acceptable steady state solution, it should account for the autocorrelation function of the money series and enable, as far as possible, previous models to be derived as special cases of it.

Following the notation of the previous chapters a model embodying these characteristics is the autoregressive distributed lag demand for money function

$$\theta_0(L)M_t = \theta_1(L)Y_t + \theta_2(L)R_t + \theta_3(L)P_t + w_t \tag{4.1}$$

where the $\theta_i(L) = \theta_{i0} + \theta_{i1}L + \ldots + \theta_{im_i}L^{m_i}$ are polynomials in the lag operator L of orders m_i respectively with θ_{00} normalised to unity. The orders m_i are taken to be sufficiently large so that the error w_t may be treated as serially independent with zero mean and constant variance. Thus by reducing the error process to white noise the model (4.1) accounts for the autocorrelation function of M and the unrestricted lag structure is capable of modelling many forms of short run disequilibrium behaviour.

A steady state solution is obtained by noting that in the steady state $x_t = x_{t-s}$ for all s, and thus (4.1) can be rewritten, with $w_t = 0$, as

$$\left(\sum_{j=0}^{m_0} \theta_{0j}\right)M_t = \left(\sum_{j=0}^{m_1} \theta_{1j}\right)Y_t + \left(\sum_{j=0}^{m_2} \theta_{2j}\right)R_t + \left(\sum_{j=0}^{m_3} \theta_{3j}\right)P_t$$

or

$$M_t = \lambda_1 Y_t + \lambda_2 R_t + \lambda_3 P_t \tag{4.2}$$

where the $\lambda_i = \dfrac{\sum_{j=0}^{m_i} \theta_{ij}}{\sum_{j=0}^{m_0} \theta_{0j}}$ are the long run elasticities of the demand for money with respect to real income, the interest rate and the price level respectively and a priori have the signs $\lambda_1 \geq 0$, $\lambda_2 \leq 0$, $\lambda_3 \geq 0$.

The model (4.1) will be denoted $AD(m_0, m_1, m_2, m_3)$ and it can be seen that the conventional partial adjustment of money and adaptive expectation of income formulations of the demand function, which yield the reduced forms

$$M_t = \theta_{01}M_{t-1} + \theta_{10}Y_t + \theta_{20}R_t + \theta_{30}P_t + w_{1t} \tag{4.3}$$

$$M_t = \theta_{01}M_{t-1} + \theta_{10}Y_t + \theta_{20}R_t + \theta_{21}R_{t-1} + \theta_{30}P_t + \theta_{31}P_{t-1} + w_{2t} \tag{4.4}$$

(where strictly w_{2t} is a moving average process of order one) are, in fact, special cases of (4.1), being AD(1, 0, 0, 0) and AD(1, 0, 1, 1) specifications respectively.

The model (4.1) can be written more compactly as

$$\underset{\sim}{\theta}(L)\underset{\sim}{X}_t = w_t \tag{4.5}$$

where $\underset{\sim}{\theta}(L) = (\theta_0(L), -\theta_1(L), -\theta_2(L), -\theta_3(L))$ is a vector polynomial in L and $\underset{\sim}{X}_t' = (M_tY_tR_tP_t)$. The dynamic specification problem is therefore that of determining the simplest model contained within (4.5) that is consistent with the sample data. However, an important simplification of (4.5) is the "factored" model

$$\rho(L)\underset{\sim}{\alpha}(L)\underset{\sim}{X}_t = \varepsilon_t \tag{4.6}$$

where $\rho(L)$ is a scalar polynomial in L of order r, $\underset{\sim}{\alpha}(L)$ is a vector polynomial in L of orders $l_0, \ldots, l_3$ such that $m_i = r + l_i$, and ε_t is white noise. The factorisation in (4.6) will be valid if

$$\rho(L)\underset{\sim}{\alpha}(L) = \underset{\sim}{\theta}(L)$$

in which case $\underset{\sim}{\theta}(L)$ can be said to have a common factor $\rho(L)$. An alternative and equivalent way of looking at the two models is to rewrite (4.6) as

$$\underset{\sim}{\alpha}(L)\underset{\sim}{X}_t = u_t \tag{4.7a}$$

$$\rho(L)u_t = \varepsilon_t \tag{4.7b}$$

which is seen to be a dynamic linear model with errors generated by an autoregression of order r so that $\underset{\sim}{\alpha}(L)$ represents sytematic dynamics and $\rho(L)$ error dynamics. Since (4.6) typically requires the estimation of 3r fewer parameters than (4.5)(generally if there are k variables in the unrestricted model then there are (k - 1)r fewer parameters in the factored model) the specialisation to (4.6), if true, represents a convenient simplication of the model (see Hendry and Mizon (1978)).

It is possible, of course, that the serial correlation in (4.7) may arise through an underlying moving average or mixed error process rather than the pure autoregression assumed here. However, since moving average processes pose difficult identification, estimation and testing problems and as Hendry (1977) has demonstrated that the correlogram of a moving average process can be adequately approximated by an autoregressive process, the error specification of (4.7) appears to be a reasonable assumption.

The problem of determining dynamic specification is, therefore, that of not only determining the order of $\underset{\sim}{\theta}(L)$, the overall order of dynamics, but also of testing whether a factorisation of $\rho(L)\underset{\sim}{\alpha}(L) = \underset{\sim}{\theta}(L)$ is also appropriate. However, as there is no unique ordering of the alternative hypotheses contained within the model (4.5) various specification testing procedures may be employed. The approach adopted here is a two stage procedure in which

(a) the overall order of dynamics $\hat{m} = \min(\hat{m}_i)$ is initially determined, and

(b) conditional on $\hat{m}$ the factorisation $\rho(L)\underset{\sim}{\alpha}(L) = \underset{\sim}{\theta}(L)$ is tested to determine the order of error dynamics $\hat{r}$ and the order of systematic dynamics $\hat{l}$ such that $\hat{r} + \hat{l} = \hat{m}$.

Stage (a) of the procedure is implemented by setting the m_i at some pre chosen maximum value, $\bar{m}$ say, and estimating this maintained hypothesis by an appropriate technique (e.g. OLS, IV). The orders $\hat{m}_i$ of the component polynomials of $\underset{\sim}{\theta}(L)$ can then be determined by separate t tests on the sequence of hypotheses

$$H_1 : \theta_{i\bar{m}} = 0$$

$$H_2 : \theta_{i\bar{m}} = \theta_{i\bar{m}-1} = 0$$

$$\vdots$$

$$H_{\bar{m}-m_i} : \theta_{i\bar{m}} = \theta_{i\bar{m}-1} = \dots = \theta_{im_i+1} = 0$$

Having determined $\hat{m}$, stage (b) tests the autoregressive error factorisation using the sequence of ordered and nested hypotheses

$$\rho_r(L)\underset{\sim}{\alpha}_{\hat{m}-r}(L) = \underset{\sim}{\theta}_{\hat{m}}(L) \ , \ r = 0, 1, \dots, \hat{m}$$

where $\rho_r(L)$ is a scaler polynomial in L of order r, $\underset{\sim}{\alpha}_{\hat{m}-r}(L)$ is a vector polynomial in L having constituent polynomials of orders between $(\hat{m} - r)$ and $(\bar{m} - r)$ and $\underset{\sim}{\theta}_{\hat{m}}(L)$ is a vector polynomial in L having constituent polynomials of orders between $\hat{m}$ and $\bar{m}$, the testing procedure moving from the least restricted hypothesis in the nest $(r = 0)$ to successively more restricted ones until a significant test is encountered. The tests in stage (b) are performed by estimating the models

$$\rho_r(L)\underset{\sim}{\alpha}_{\hat{m}-r}(L)\underset{\sim}{X}_t = \varepsilon_t \qquad (4.8)$$

for $r = 0, 1, .., \hat{m}$ by autoregressive least squares (see Hendry (1976, section 7) for alternative methods) and constructing the set of likelihood ratio statistics

$$\chi_j^2 = T\ln\left(\frac{S_{j+1}}{S_j}\right) \qquad j = 0, 1, \ldots \hat{m} - 1$$

where S_j is the residual sum of squares obtained from estimating (4.8) with $r = j$ and T is the number of observations. On $H_j : r = j + 1$, χ_j^2 is asymptotically distributed as chi square with three degrees of freedom. (An alternative test would be to perform a Wald test using the COMFAC algorithm (see, e.g. Hendry and Mizon (1978, 1979)). The non availabilty of this algorithm led to the use of the asymptotically equivalent likelihood ratio approach).

As stated above, the non unique ordering of the hypotheses contained within (4.5) allows the possibility of alternative testing procedures to be used, with the subsequent need for their relative evaluation. The two stage procedure does have the advantage that for each of the stages the hypotheses to be tested are uniquely ordered sequences and as the testing proceeds from the most general model and sequentially tests the need for more restricted models, the tests induced at each stage will have high power asymptotically. (An alternative approach proposed by Hendry and Mizon (1978, 1979) is to use the COMFAC algorithm directly on the maintained hypothesis and then test for zero roots among the $\hat{r}$ common roots extracted. This approach is analogous to performing stage (b) of our procedure first and then testing for zero roots in the $\hat{r}$th order polynomial $\rho_{\hat{r}}(L)$ thus chosen).

Since the two stage procedure is only designed to determine the appropriate orders of systematic and error dynamics, it is likely that certain of the individual coefficients within the $\rho_{\hat{r}}(L)$ and $\alpha_{\hat{m}-\hat{r}}(L)$ polynomials chosen at the end of the second stage will be insignificantly different from zero. The procedure is also likely to have low power against alternative specifications with larger values of m arising from higher order error dynamics with perhaps lower order systematic dynamics. This lack of power may be overcome to some extent by employing residual diagnostic tests on the chosen specification and respecifying the model as appropriate in the event of significant test statistics. Hence reestimation of the specification so chosen at the end of stage (b) of the procedure will often be required. (However one should note that if stage (a) selects $\hat{m} = 0$ then stage (b) is necessarily redundant.)

An important consideration in the application of sequential testing procedures such as the one considered here is the appropriate choice of significance levels for each particular test. To control the probability of a Type I error for the procedure as a whole it is necessary to consider the significance levels for each individual test in the two sequences carefully. If ε_i is the significance level of the ith test in a sequence, then the significance level of the jth test against the maintained hypothesis is

$$1 - \prod_{i=1}^{j} (1 - \varepsilon_i).$$

Hence if a constant significance level ε is set for each of a sequence of n tests, the overall significance level of the sequence will be $1 - (1 - \varepsilon)^n$. In the present circumstances, the specification of a fairly general maintained hypothesis, accomplished by setting $\bar{m}$ quite large may not necessitate each hypothesis in the stage (a) sequence being treated symmetrically. In this case the appropriate setting may be ε_1 small with subsequent ε_i increasing with i. Similar considerations will apply to stage (b) of the procedure but because the two stages are not statistically independent the overall significance level of the procedure is difficult to derive. The implication of these considerations is that the use of conventional significance levels for individual tests will imply very large overall significance levels for each stage. For example, if there are four tests in each of the sequences, conducting each test at the .05 significance level would imply an overall significance level for each sequence of almost 19%. However, it may be argued that the choice of (implicitly) large significance levels may well be reasonable since this will improve the power of the procedure against unconsidered alternatives involving higher order error dynamics, thus helping to alleviate the problem discussed earlier. In any event, such choices can only be made by fully taking into account specific subject matter considerations.

The testing procedure outlined above has been termed here a specification testing procedure, as it proceeds from a general maintained hypothesis and sequentially simplifies the model in the light of sample evidence. Much of applied econometric modelling is concerned, however, with tests of mis-specification, i.e. in estimating a very restricted maintained hypothesis and considering the need to modify this hypothesis in the light of sample evidence. The distinction between these two approaches to econometric modelling has been drawn by Mizon (1977b). The latter approach has been the one typically followed in demand for money studies, with the conventional maintained hypothesis being the partial adjustment or adaptive expectation reduced forms (4.3) and (4.4) which have been shown to be particular cases of the general maintained hypothesis (4.5). Both Courakis (1978) and Hendry and Mizon (1978) have criticised such an approach for the uncritical acceptance of imposed a priori parameter restrictions and the inadequate use of residual diagnostic tests. However, the recent development of residual diagnostic tests other than the conventional d and h statistics, making it now possible to test for higher order error processes (see Pierce (1971), Wallis (1972), Ljung and Box (1978) and Godfrey (1978, b, c)), suggests that the estimation of restricted models plus the full use of such diagnostic tests should yield more useful evidence on the direction in which the model might be respecified. Hence prior to the implementation of the specification testing procedure the AD(1, 0, 0, 0) model was estimated for each combination of variables and a battery of residual diagnostic tests performed to suggest possible directions in which this model could be profitably respecified. The AD(1, 0, 0, 0) model may also be regarded as a baseline to which the models obtained by the two stage procedure may be compared and indeed the explanation of the deficiencies of this restricted model is one of the principles stated by Davidson et al (1978).

While the specification searches undertaken in the application of the two stage procedure to each combination of variables are discussed in detail in subsequent sections, two overall features emerge. The narrow money specifications seem capable of adequate modelling and yield sensible long run elasticities but the broad money specifications suffer from a combination of instability and unacceptable long run properties. This general inadequacy of the demand for broad money function has also been observed by Haache (1974) and Artis and Lewis (1974, 1976) and has been attributed by these authors, at least in part, to the introduction of Competition and Credit Control, a monetary policy which operated between May 1971 and December 1973. (See Bank of England (1971) for the official view of the policy at its introduction and Gowland (1978) for a detailed analysis of its failure and subsequent replacement).

Competition and Credit Control (henceforth known as CCC) was designed to redress the loss of competitiveness that had been incurred by the banking system vis-a-vis other channels of financial intermediation in the late 1960's. It operated by altering reserve requirements, removing ceilings on bank lending and, in particular, freeing the clearing banks borrowing and lending rates from their rigid link with Bank Rate. As a consequence the clearing banks were able to adopt more flexible policies in bidding for funds, the increased competitiveness of deposit bank liabilities thereby increasing the attractiveness of money, as measured by its own interest rate. Thus asset holders were attracted into holding interest bearing money balances as the interest paid on these balances rose. Such considerations prompted the above authors to introduce an own rate of money variable, variously defined, into the demand for broad money function in an attempt to explain the acceleration of the broad money series during the period of CCC.

However, this emphasis on the own rate of money does not reflect the full intention of the policy. As noted above, the banks were also able to be more flexible with their lending rates and such "price changes" were envisaged as the weapon by which credit (and presumably the money supply) could be controlled. To fully model the implications of CCC, then, variables measuring both the clearing banks borrowing and lending rates should be included in the demand function. The introduction of two additional interest rate variables, bearing in mind that they would appear as lag polynomials, was felt to be undesirable on both multicollinearity and degrees of freedom considerations. It was, therefore, decided to employ the differential between the two rates as the appropriate variable, defining this to be

$$F_t = \ln(L_t - B_t)$$

where L_t is the London Clearing Banks base lending rate (Bank Rate until its abolishment in October 1972) and B_t is their deposit account (7 days notice) rate. As well as its empirical justification, this variable is able to model the rigid behaviour between lending and borrowing rates before the introduction of CCC, when there was an almost constant two per cent differential, the more flexible behaviour of the rates during CCC and their subsequent behaviour after the abandonment of the policy in 1973, when it was replaced by a ceiling on bank liabilities (as opposed to the ceiling on bank assets that operated before 1971). Moreover, it is also capable of capturing one of the major problems of CCC, the relative inflexibility of the clearing banks lending rates when compared with their borrowing rates, a consequence of the unwillingness of the monetary authorities to allow nominal rates to increase sufficiently to counteract rising inflationary expectations. At times the differential between the rates was narrowed to such an extent that some bank customers were provided with a significant incentive to borrow in order to build up their balances of interest bearing deposits. Indeed, possibilities of pure arbitrage arose on brief occasions when the differential even became negative.

The above analysis suggests that interest bearing money balances are positively related to borrowing rates and negatively related to lending rates. It therefore follows that the long run elasticity of these money balances with respect to F_t should be negative.

Two further variables have recently been proposed as additional arguments in the demand for money function. Tsiang (1977) has argued that the appropriate constraint variable in the transactions demand for money function is planned national expenditure rather than total income received or output produced, and that in a short run dynamic framework aggregate planned expenditure need not be identical to aggregate income. Unfortunately, an ex ante magnitude such as aggregate planned expenditure is difficult to measure statistically but this problem may be circumvented in a closed economy by using ex post magnitudes, since ex post aggregate expenditure and income are necessarily identical. In an open economy such as the U.K., however, aggregate expenditure can be different from aggregate income in an ex post sense, the difference between the two measures being the balance of trade.

Moreover, Tsiang (1977) also argues that the volume of trade itself has an independent influence on the transactions demand for money. Exports, even when financed by foreign credit, create a demand for transaction cash balances since domestic means of payment are required throughout the domestic production process. Similarly, imports must be purchased in the home market by the domestic means of payment and they also create a series of intermediate transactions from the time of unloading to actually reaching the consumer which also require transactions cash balances.

Such considerations lead Tsiang to suggest that if a measure of national income such as GDP is used as the constraint variable in the demand for money function then the volume of trade relative to national income should also be included as an argument in the function. In view of this the volume of trade variable V_t was constructed as

$$V_t = \ln\left(\frac{I_t + EX_t}{Y_t}\right)$$

where I_t is the volume of imports, EX_t is the volume of exports and Y_t is nominal GDP, and included as an additional regressor in the demand for money function with an a priori expectation that the long run elasticity of the demand for money with respect to V_t should be positive.

Secondly, two papers by Dutton and Gramm (1973) and Karni (1974) have considered the inclusion of the wage rate as an argument in the demand function, finding that it appears significantly as a regressor in the function when using annual U.S. data. Dutton and Gramm interpret the wage rate as a proxy variable for the brokerage fee of the transactions and precautionary theory of the demand for money, in particular regarding the wage rate as measuring the consumer's valuation of time, i.e. the use of money is assumed to save transactions time and hence increases the amount of leisure time available. In equilibrium the marginal valuation of an hour of leisure time must, therefore, be equal to the wage rate.

Karni places particular emphasis on this relationship between the value of time and the demand for money and, using an inventory-theoretic approach, develops a model in which the demand for money is positively related to the value of time as measured by the wage rate, reflecting the attempts on the part of households and businesses to save time in conducting their exchange activities.

Both these papers develop long run equilibrium models based on explicit theories of the demand for money and hence are not designed to explain short run dynamic fluctuations between the money supply and the wage rate. In accordance with the present framework a wage rate variable W_t, defined as the log of the basic weekly wage rate of manual workers in all industries and services, was included in the demand function in the form of a lag polynomial. A priori, one should expect the long run elasticity of the demand for money with respect to the wage rate to be positive.

Introducing these three additional variables into the autoregressive distributed lag demand for money framework developed above leads to the extended model

$$\underset{\sim}{\theta}^*(L)\underset{\sim}{X}_t^* = w_t^* \tag{4.9}$$

where $\underset{\sim}{\theta}^*(L) = (\underset{\sim}{\theta}(L) : -\theta_4(L),\ \theta_5(L),\ \theta_6(L))$ and $\underset{\sim}{X}_t^{*'} = (\underset{\sim}{X}_t' : F_t,\ V_t,\ W_t)$, which may be denoted as $AD(m_0,\ m_1,\ \ldots,\ m_6)$. The dynamic specification of (4.9) was investigated analogously to that of (4.5) using the two stage testing procedure.

The additional regressors were included in all four combinations of variables, even though there may seem theoretically a case for including F_t only in the broad money specifications, as this money aggregate corresponds most closely to "interest bearing money balances", and including V_t and W_t only in the narrow money specifications, the money aggregate corresponding most closely to transactions balances. Such restrictions were ignored for the following reasons. As Gowland (1978) points out, the narrow money aggregate (M1) does in fact include a substantial amount of interest bearing deposits and also contains current account deposits which earn implicit (tax-free) interest in the form of remitted bank charges. Furthermore, it may also be argued that the increased bank borrowing rates attracted funds from current accounts to deposit accounts, in which case there should be a positive relationship between narrow money balances and F_t. Although the narrow money aggregate may well correspond most closely to transaction balances, it does oomprise a substantial part of the broad money series and in any case cheques can be traditionally written against deposit accounts. Thus, one may well expect broad money to be related to both V_t and W_t. Indeed, in view of the explicit targets adopted for the growth of M3 in 1976, it seems essential to identify possible important determinants of the broad money series.

Having determined the most appropriate dynamic specifications of the extended demand function, these were then subjected to further empirical tests. In response to the widespread view that CCC caused a structural shift in the demand function, the data period was split in half (which corresponds very closely with the introduction of CCC) and Chow (1960) tests for structural stability performed on each specification. If this test rejected stability for any particular specification then this specification was re-estimated to take account of the structural shift by employing the dummy variable approach suggested by Gujarati (1970 a, b). Finally, noting that an individual long run elasticity is defined as

$$\lambda_i = \frac{\sum_{j=0}^{m_i} \theta_{ij}}{\sum_{j=0}^{m_0} \theta_{0j}} \qquad i = 1, .., 6$$

tests of unitary ($\Sigma\theta_{ij} = \Sigma\theta_{0j}$) or zero ($\Sigma\theta_{ij} = 0$) elasticity restrictions were performed where appropriate. It may also be noted that in addition to the a priori expectations regarding λ_1, λ_2 and λ_3, the preceding analysis suggests that $\lambda_4 \leq 0$, $\lambda_5 \geq 0$, $\lambda_6 \geq 0$.

Thus, the most appropriate, parsimonious specifications consistent with the sample evidence may be identified and estimated through the use of these empirical procedures.

4.3 SPECIFICATION SEARCHES: GENERAL CONSIDERATIONS

For the searches undertaken on the conventional model (4.5) the maximum lag $\bar{m}$ was set at four, this choice being determined by the need to allow any remaining seasonality to be picked up, thus ensuring that the error is indeed white noise, to incorporate the most important lag effects and to allow a suitable number of degrees of freedom to be available for estimation. The maintained hypothesis thus defined requires the estimation of $k = 20$ coefficients, noting the normalisation of θ_{00} and the inclusion of a constant as an additional regressor. This choice for $\bar{m}$ loses four observations from the data series and therefore sets the estimation period as 1964 I to 1977 IV, a total of $T = 56$ observations. This became the standard estimation period over which all regressions in the searches were performed.

The extended model (4.9) has seven constituent lag polynomials and initial analysis determined that choosing $\bar{m}$ equal to two was the maximum possible setting that would allow estimation to be performed, the maintained hypothesis thus defined requiring $k = 21$ coefficients to be estimated. In view of this relatively restricted maintained hypothesis, the qualifications placed on the two stage testing procedure regarding unconsidered alternative specifications are particularly appropriate and careful analysis of the regression parameters and residuals is essential to indicate directions of mis-specification.

For the conventional model (4.5) four combinations of variables are available and are denoted as

$$\underset{\sim}{X}_1' = (MN, Y, RC, P)$$
$$\underset{\sim}{X}_2' = (MN, Y, RL, P)$$
$$\underset{\sim}{X}_3' = (MB, Y, RC, P)$$
$$\underset{\sim}{X}_4' = (MB, Y, RL, P)$$

Analogously the combinations available for the extended model (4.9) are denoted as

$$\underset{\sim}{X}_i^{*'} = (\underset{\sim}{X}_i' : F, V, W) \qquad i = 1, 2, 3, 4$$

Using the notation of the previous section, the maintained hypothesis corresponding to the models contained within (4.5) are denoted as AD(4, 4, 4, 4) and those contained within (4.9) are denoted as AD(2, 2, 2, 2, 2, 2, 2). The specificafication obtained after the factorisation of a common polynomial of order r from an AD(m_0, ..., m_6) specification, say, is denoted as AD(r)(m_0 - r, ..., m_6 - r).

All "Chow tests", the F_c statistics, were performed by splitting the data period into two equal sub-periods with $T_1 = T_2 = 28$ observations in each; the first period therefore being defined as 1964 I to 1970 IV, and the second period as 1971 I to 1977 IV. Note that the break corresponds very closely to the introduction of CCC.

As T = 56 all residual correlations have a standard error of .13 attached to them and as almost every regression reported in the following sections had an R^2 in excess of .99 this summary statistic is not shown, "goodness of fit" being summarised by the error standard deviation $\hat{\sigma}$.

4.4 SPECIFICATION SEARCH 1

In this section specification searches of the models (4.5) and (4.9) for the vectors

$$\underset{\sim}{X}_1' = (MN,\ Y,\ RC,\ P) \text{ and } \underset{\sim}{X}_1^{*'} = (\underset{\sim}{X}_1' : F,\ V,\ W)$$

are investigated and discussed.

The AD(1, 0, 0, 0) specification was first estimated by OLS to provide both a baseline for subsequent comparison and an example of the "conventional" approach to modelling the demand for money.

Estimation yields

$$\widehat{MN}_t = -.051 + .770MN_{t-1} + .221Y_t - .045RC_t + .216P_t \qquad (4.10)$$
$$\quad (.809) \quad (.073) \qquad (.098) \quad (.026) \quad (.060)$$

$T = 56 \quad S = .0189742 \quad \hat{\sigma} = .0193 \quad \hat{\lambda}_1 = .96,\ \hat{\lambda}_2 = -.20,\ \hat{\lambda}_3 = .94,\ \mu_1 = .77$

$d_1 = 2.32 \quad h_1 = -1.42 \quad d_4 = 1.59 \quad h_4 = 1.29 \quad Q(15) = 23.7 \quad \tilde{Q}(15) = 28.5$

$\phi(1) = 1.92 \quad \phi(2) = 4.68 \quad \phi(4) = 5.42 \quad F_c(5, 46) = 1.88.$

There is no evidence of either first or fourth order serial correlation, the only significant diagnostic statistic being $\tilde{Q}$. Inspection of the residual correlogram revealed that only $\hat{r}_2 = .26$ was (just) significant, thus suggesting the possibility of mis-specified second order error dynamics. The F_c and μ_1 values suggest neither structural instability nor nonstationarity and the coefficient estimates are correctly signed and reasonably precisely determined, implying approximately unitary income and price elasticities but a rather small interest elasticity. Thus, apart from some tenuous evidence of dynamic misspecification, (4.10) appears to be an adequate description of the data and may well satisfy the requirements of many researchers.

On testing dynamic specification more generally, estimation of the maintained hypothesis AD(4, 4, 4, 4) obtained the estimates shown in Table 4.1. Performing the sequence of t tests comprising stage (a) of the specification testing procedure with individual significance levels set at $\varepsilon_1 = .01$, $\varepsilon_2 = .02$, $\varepsilon_3 = \varepsilon_4 = .05$ (thus implying a significance level for the fourth test ($m_i = 0$) against the maintained hypothesis ($m_i = 4$) of .125) led to the acceptance of an AD(4, 1, 2, 1) specification. Stage (b) of the procedure is therefore that of testing whether the factorisation of a common first order lag polynomial, representing error dynamics, from the above specification is consistent with the sample evidence. This test yielded the statistic $\chi^2(3) = 4.98$, in the light of which we accept the factorisation to the specification AD(1)(3, 0, 1, 0). Estimates of this model were

$$
\begin{aligned}
\hat{MN}_t = &\underset{(.472)}{-.817} + \underset{(.130)}{1.428MN_{t-1}} - \underset{(.203)}{.920MN_{t-2}} + \underset{(.115)}{.257MN_{t-3}} + \underset{(.066)}{.297Y_t} \\
&\underset{(.023)}{-.005RC_t} - \underset{(.028)}{.079RC_{t-1}} + \underset{(.040)}{.222P_t} - \underset{(.085)}{.772\hat{\varepsilon}_{t-1}} \qquad (4.11)
\end{aligned}
$$

$$S = .0123941 \qquad \hat{\sigma} = .0161$$

and deletion of the insignificant RC_t regressor leads to the respecification

TABLE 4.1

j	0	1	2	3	4	$\Sigma\theta_j$
MN_{t-j}	1	.745[4.76]	.236[1.39]	-.479[2.86]	.279[2.51]	.22
Y_{t-j}	.246[1.70]	.492[2.93]	-.244[1.39]	-.233[1.44]	.127[.91]	.39
RC_{t-j}	-.055[2.01]	-.030[1.02]	-.075[2.08]	.018[.52]	.009[.28]	-.13
P_{t-j}	-.022[.13]	.945[3.99]	-.484[1.77]	-.305[1.01]	.088[.42]	.22

constant = -1.925[1.95]   S = .00694294 $\hat{\sigma}$ = .0139

TABLE 4.2

j	0	1	2	$\Sigma\theta_j$
MN_{t-j}	1	.575[3.87]	.110[.84]	.32
Y_{t-j}	.331[2.43]	.482[3.07]	-.227[1.64]	.59
RC_{t-j}	-.106[3.58]	-.034[1.02]	-.093[2.99]	-.23
P_{t-j}	-.229[1.08]	1.120[4.77]	-.629[2.72]	.26
F_{t-j}	.005[.52]	.005[.56]	.004[.41]	.01
V_{t-j}	.004[.07]	.004[.07]	.003[.53]	.01
W_{t-j}	.682[3.19]	-.770[2.45]	.133[.52]	.05

constant = -3.243[2.98]   S = .00637693 $\hat{\sigma}$ = .0135

$$\hat{MN}_t = -.784 + 1.435MN_{t-1} - .931MN_{t-2} + 262MN_{t-3} + .294Y_t$$
$$(.445) \quad (.127) \quad (.198) \quad (.113) \quad (.064)$$

$$-.083RC_{t-1} + .223P_t - .774\hat{\varepsilon}_{t-1} \tag{4.12}$$
$$(.020) \quad (.039) \quad (.085)$$

$\hat{\lambda}_1 = 1.26 \quad \hat{\lambda}_2 = -.35 \quad \hat{\lambda}_3 = .95 \quad \hat{\mu}_1 = .61, \quad \mu_2, \mu_3 = 4.1 \pm .51i$

$S = .0124080 \quad \hat{\sigma} = .0159 \quad Q(15) = 21.7 \quad F_c(11, 34) = 3.20 \quad \chi^2(12) = 32.5$

The Q statistic, although not significant, is no doubt inflated by $\hat{r}_{14} = .38$, the only significant residual correlation, thus affording little evidence of dynamic mis-specification. Interestingly, the evidence of mis-specified second order dynamics in the AD(1, 0, 0, 0) specification has been modelled by a third order lag polynomial on MN, a lagged rather than a contemporaneous interest rate and first order error dynamics. There would seem to have been little hope of achieving such a respecification on the information yielded by the diagnostic test statistics from (4.10) alone, even assuming that any respecification would have been deemed necessary anyway. Thus the importance of tests of specification is readily apparent.

The function is now income elastic while remaining interest inelastic, with the approximately unitary price elasticity being consistent with a real money formulation of the function. Unfortunately we cannot stop here, for the Chow test on the unrestricted version of (4.12) yields a significant F_c statistic, thus indicating structural instability which was not found in (4.10). Moreover, the test of (4.12) against the maintained hypothesis yields a significant χ^2 statistic, suggesting further difficulties.

The extension to the model (4.9) is therefore required and estimates of the extended maintained hypothesis AD(2, 2, 2, 2, 2, 2, 2) are given in Table 4.2. Peforming stage (a) of the procedure at significance levels $\varepsilon_1 = \varepsilon_2 = .02$ leads to the acceptance of an AD(1, 1, 2, 2, 0, 0, 1) specification, rendering stage (b) redundant as $\hat{m} = \min(\hat{m}_i) = 0$. Estimation of this model gave

$$\begin{aligned}\widehat{MN}_t = {} & \underset{(.854)}{-2.557} + \underset{(.09?)}{.554}MN_{t-1} + \underset{(.125)}{.269}Y_t + \underset{(.122)}{.347}Y_{t-1} - \underset{(.028)}{.093}RC_t - \underset{(.028)}{.061}RC_{t-1} - \underset{(.029)}{.093}RC_{t-2} \\ & - \underset{(.192)}{.215}P_t + \underset{(.200)}{.931}P_{t-1} - \underset{(.150)}{.361}P_{t-2} - \underset{(.008)}{.005}F_t + \underset{(.032)}{.044}V_t + \underset{(.182)}{.662}W_t - \underset{(.205)}{.587}W_{t-1} \\ & S = .00743298 \qquad \hat{\sigma} = .0133\end{aligned} \tag{4.13}$$

We see that both F_t and V_t, along with P_t, are insignificant and their deletion yields on re-estimation

$$\begin{aligned}\widehat{MN}_t = {} & \underset{(.771)}{-2.288} + \underset{(.084)}{.618}MN_{t-1} + \underset{(.110)}{.313}Y_t + \underset{(.113)}{.282}Y_{t-1} - \underset{(.022)}{.064}RC_t - \underset{(.026)}{.047}RC_{t-1} - \underset{(.026)}{.093}RC_{t-2} \\ & + \underset{(.171)}{.836}P_{t-1} - \underset{(.148)}{.388}P_{t-2} + \underset{(.146)}{.509}W_t - \underset{(.193)}{.566}W_{t-1}\end{aligned} \tag{4.14}$$

$$\hat{\lambda}_1 = 1.56 \quad \hat{\lambda}_2 = -.53 \quad \hat{\lambda}_3 = 1.17 \quad \hat{\lambda}_6 = -.15 \quad \hat{\mu}_1 = .62$$

$$S = .00791327 \quad \hat{\sigma} = .0133 \quad d_1 = 2.13 \quad h_1 = -.63 \quad Q(15) = 19.0$$

$$F_c(11,34) = 2.61, \; F(10,45) = 1.08 \quad F(3, 42) = .90$$

Although there is no evidence of mis-specification from the diagnostic statistics, and (4.14) is not rejected when tested against the maintained hypothesis by an F test, the wage rate elasticity is negative (although small) and once again the F_c statistic indicates structural instability. Accordingly, the dummy variable approach described by Gujarati (1970 a, b) was employed to allow the coefficients of (4.14) to vary between the first and second halves of the data period. Thus the dummy variable D_t, defined as

$$\begin{aligned} D_t &= 0 \text{ for } t = 1, 2, \ldots, 28 \\ &= 1 \text{ for } t = 29, 30, \ldots, 56 \end{aligned}$$

was introduced and estimation of the respecified version of (4.14), after deletion of insignificant coefficients, yielded

$$\boxed{\begin{aligned} MN_t = &- \underset{(.116)}{3.967D_t} + \underset{(.124)}{.330MN_{t-1}} + \underset{(.124)}{.208(D_tMN_{t-1})} + \underset{(.141)}{.344\,(D_tY_t)} \\ &+ \underset{(.106)}{.447Y_{t-1}} - \underset{(.052)}{.197RC_t} + \underset{(.055)}{.137(D_tRC_t)} - \underset{(.026)}{.068RC_{t-1}} - \underset{(.026)}{.115(D_tRC_{t-2})} \\ &+ \underset{(.162)}{.771P_{t-1}} - \underset{(.136)}{.396P_{t-2}} + \underset{(.129)}{.304W_t} - \underset{(.121)}{.245(D_tW_{t-1})} \qquad (4.15) \\ S &= .0067473 \qquad \hat{\sigma} = .0125 \end{aligned}}$$

From (4.15) we can derive the following functions for the two subperiods

<u>1964 I - 1970 IV</u>

$$\hat{MN}_t = .330MN_{t-1} + .447Y_{t-1} - .197RC_t - .068RC_{t-1} + .771P_{t-1} - .396P_{t-2} + .304W_t$$

$$\lambda_1 = .67 \quad \lambda_2 = -.40 \quad \lambda_3 = .56 \quad \lambda_6 = .45 \quad \mu_1 = .33 \qquad (4.16)$$

<u>1971 I - 1977 IV</u>

$$\hat{MN}_t = -3.967 + .538MN_{t-1} + .344Y_t - .447Y_{t-1} - .060RC_t - .068RC_{t-1} - .115RC_{t-2} + .771P_{t-1} - .396P_{t-2} + .304W_t - .245W_{t-1} \qquad (4.17)$$

$$\lambda_1 = 1.71 \quad \lambda_2 = -.53 \quad \lambda_3 = .82 \quad \lambda_6 = .13 \quad \mu_1 = .54$$

From (4.15) and the derived functions (4.16) and (4.17) the following conclusions may be drawn. The introduction of F_t and V_t into this demand for narrow money function is inappropriate, a result that is consistent with the theoretical development of F_t but not with that of V_t. The wage rate W_t is an important determinant of the demand for narrow money, but its true affect is only apparent when structural instability is accounted for. There does seem to be an important structural shift in the function, which may possibly be attributable to the introduction of CCC. Long run income and wage elasticities are considerably different for the two sub periods, in particular the income elasticity is well over twice as large in the later period than in the earlier period. Important dynamic affects are observed for the regressors in both sub periods with a somewhat more complicated lag pattern operating in the later period.

This finally accepted specification is radically different from the conventional model depicted in (4.10). Not only has the residual series been reduced to white noise, but more importantly a much more flexible function incorporating more general short run dynamics has been identified without loss of estimation precision (only one coefficient in (4.15) has a t ratio less than two) and with a thirty-five per cent decrease in the error standard deviation.

Finally, it is interesting to consider the estimation of the would-be "conventional" specification of the extended model, i.e. the AD(1, 0, 0, 0, 0, 0, 0) specification,

$$\begin{aligned}
\hat{MN}_t = \underset{(.088)}{.129} &+ \underset{(.082)}{.743}MN_{t-1} + \underset{(.109)}{.169}Y_t - \underset{(.039)}{.043}RC_t + \underset{(.166)}{.135}P_t - \underset{(.010)}{.006}F_t \\
&- \underset{(.041)}{.045}V_t + \underset{(.168)}{.112}W_t
\end{aligned}$$

$$\hat{\lambda}_1 = .66 \quad \hat{\lambda}_2 = -.17 \quad \hat{\lambda}_3 = .53 \quad \hat{\lambda}_4 = -.02 \quad \lambda_5 = -.18 \quad \lambda_6 = .44 \quad \mu_1 = .74 \tag{4.18}$$

$$S = .0179359 \quad \hat{\sigma} = .0193 \quad d_1 = 2.43 \quad h_1 = -2.04 \quad Q(15) = 17.0 \quad F(3,48) = .93$$

From these estimates, t tests show that all of the regressors are insignificant at conventional significance levels, with both the F test of the restriction to (4.10) and a comparison of error standard deviations confirming that the introduction of the additional regressors contributes nothing to the function. Even the presence of serial correlation, as shown by the diagnostic statistics, should not alter this conclusion for Granger and Newbold (1974) show that typically serial correlation inflates t ratios.

On first sight the decrease in significance of the $\underset{\sim}{X}_{1t}$ regressors on the introduction of the additional variables may plausibly be explained by multicollinearity, for a priori one might expect P_t and W_t and RC_t and F_t to be highly correlated. However, as pointed out by Davidson et al (1978), collinearity problems are often likely to occur in conjunction with omitted variable problems, and the addition of initially excluded regressors which are important in determining the regressand may well help to resolve what appears to be a collinearity problem between the originally included regressors. In hindsight, the problem of (4.18) is one of omitted variables for a more sophisticated specification incorporating additional variables modelling dynamic and structural affects has been identified. We can, therefore, offer here further empirical evidence supporting Davidson et al's (1978 p.677) view that

> It is not universally valid to assume that a group of badly determined estimates indicates the presence of collinearity (to be solved by reducing the dimensionality of the parameter space) rather than omitted variables bias (solved by increasing the dimensionality of the parameter space)."

4.5 SPECIFICATION SEARCH 2

In this section we investigate the specification searches undertaken for the vectors $\underset{\sim}{X}_2' = (MN, Y, RL, P)$ and $\underset{\sim}{X}_2^*$.

As it was found in chapter 3 that there was simultaneity between narrow money, MN, and the local authority interest rate, RL, OLS is therefore an inappropriate method of estimation for this particular combination of variables. The conventional AD(1, 0, 0, 0) specification was therefore estimated by instrumental variables (IV) with RL lagged one period, the public sector borrowing requirement and the logarithm of the vacancy rate being used as instruments for RL: the latter two variables being suggested by the models of interest rate determination developed and estimated by Demery and Duck (1978). Estimation gave

$$\hat{MN}_t = -.186 + .770MN_{t-1} + .235Y_t - .046RL_t + .203P_t$$
$$(.562)\quad(.056)\qquad(.063)\quad(.011)\quad(.044) \tag{4.19}$$
$$S = .0122524 \quad \hat{\sigma} = .0155 \quad \hat{\lambda}_1 = 1.02 \quad \hat{\lambda}_2 = -.22 \quad \hat{\lambda}_3 = .88 \quad \mu_1 = .77$$
$$d_1 = 2.66 \qquad Q(15) = 40.5$$

Unfortunately, d_1 and its associated statistics are invalid when calculated from the residuals of an IV regression (see Godfrey (1976, 1978a)) and, in any case, the Lagrange Multiplier tests can no longer be calculated as TR_n^2. In order to obtain valid diagnostic tests the AD(1, 0, 0, 0) specification was re-estimated by OLS, yielding

$$\hat{MN}_t = -.308 + .765MN_{t-1} + .252Y_t - .052RL_t + .206P_t$$
$$(.546)\quad(.056)\qquad(.060)\quad(.009)\quad(.044) \tag{4.20}$$
$$S = .0121586 \quad \hat{\sigma} = .0154 \quad \hat{\lambda}_1 = 1.07,\ \hat{\lambda}_2 = -.22,\ \hat{\lambda}_3 = .88,\ \mu_1 = .77$$
$$d_1 = 2.63 \quad h_1 = -2.61 \quad d_4 = 1.58 \quad h_4 = 1.37 \quad Q(15) = 38.9 \quad \tilde{Q}(15) = 47.0$$
$$\phi(1) = 6.76 \quad \phi(2) = 7.87 \quad \phi(4) = 8.48 \quad F_c(5, 46) = .72$$

The coefficient estimates and implied long run elasticities are almost identical with those of (4.19), thus suggesting that in this case it may not be crucial to incorporate the known simultaneity between the regressand and just one regressor. The residual diagnostic tests indicate the presence of second order serial correlation, confirmed by inspection of the residual correlogram, which shows $\hat{r}_1 = -.32$ and $\hat{r}_2 = .26$. The portmanteau statistics are rather large and may be accounted for by the presence of $\hat{r}_9 = .45$ and $\hat{r}_{14} = -.32$, in addition to the above correlations. Even though the F_c statistic does not indicate structural instability (such a test is, in any case, invalid in the presence of serial correlation), we must conclude that this conventional specification is seriously mis-specified and any interpretation of the long run elasticities would be unwarranted.

In view of the close similarity of (4.19) and (4.20) and to ease the computational burden the maintained hypothesis, the AD(4, 4, 4, 4) specification, was estimated by OLS. (In principle using the testing procedure in conjunction with IV estimation is admissable, see on this Mizon (1977a)). The resulting estimates are shown in Table 4.3.

The AD(1, 1, 0, 3) specification is obtained from stage (a) of the testing procedure irrespective of which conventional significance levels the individual t tests are performed at. Since stage (b) is necessarily redundant, OLS estimation of the above specification yields

$$\begin{aligned}\hat{MN}_t = {} & \underset{(.511)}{-.625} + \underset{(.063)}{.791MN_{t-1}} + \underset{(.113)}{.139Y_t} + \underset{(.119)}{.123Y_{t-1}} - \underset{(.008)}{.048RL_t} \\ & - \underset{(.132)}{.135P_t} + \underset{(.187)}{.719P_{t-1}} - \underset{(.193)}{.339P_{t-2}} - \underset{(.133)}{.072P_{t-3}}\end{aligned} \tag{4.21}$$

$S = .00900418 \qquad \hat{\sigma} = .0138$

TABLE 4.3

j	0	1	2	3	4	$\Sigma\theta_j$
MN_{t-j}	1	.699[4.66]	.176[1.07]	-.279[1.49]	.203[1.57]	.20
Y_{t-j}	.060[.50]	.363[2.41]	-.161[1.17]	-.176[1.45]	.163[1.43]	.25
RL_{t-j}	-.057[3.80]	.033[1.30]	-.041[1.61]	.024[.83]	-.028[1.42]	-.07
P_{t-j}	-.099[.74]	.771[4.06]	-.067[.33]	-.648[3.07]	.223[1.35]	.18

constant = -.642[1.25] S = .00480927 $\hat{\sigma}$ = .0116

TABLE 4.4

j	0	1	2	$\Sigma\theta_j$
MN_{t-j}	1	.590[3.67]	.182[1.21]	.23
Y_{t-j}	.011[.08]	.315[2.02]	-.087[.67]	.24
RL_{t-j}	-.075[3.60]	.042[1.44]	-.032[1.36]	-.07
P_{t-j}	-.179[.87]	.676[2.99]	-.098[.44]	.40
F_{t-j}	-.009[.94]	.004[.48]	-.006[.67]	-.01
V_{t-j}	.065[1.10]	-.015[.25]	-.022[.43]	.03
W_{t-j}	.422[2.21]	-.309[.96]	-.275[1.04]	-.16

constant = .404[.52] S = .00606266 $\hat{\sigma}$ = .0132

As a consequence of dropping Y_{t-1}, the least significant of the income regressors, and the two insignificant price regressors, P_t and P_{t-3}, the specification of the model is altered to that of AD(1, 0, 0, 2). Estimation of this specification yields

$$\hat{MN}_t = \underset{(.489)}{.476} + \underset{(.060)}{.803}MN_{t-1} + \underset{(.058)}{.233}Y_t - \underset{(.008)}{.049}RL_t + \underset{(.109)}{.595}P_{t-1} - \underset{(.122)}{.428}P_{t-2} \tag{4.22}$$

$$S = .00929659 \quad \hat{\sigma} = .0136 \quad d_1 = 2.61 \quad h_1 = -2.57 \quad Q(15) = 29.1$$

The diagnostic statistics indicate the presence of first order serial correlation (the only other significant residual correlation being $\hat{r}_9 = .46$) and, following Hendry (1974), the order of dynamics was increased to enable the AD(2, 1, 1, 3) specification to be considered. Estimation of this specification gives

$$\begin{aligned} \hat{MN}_t = & \underset{(.500)}{-1.070} + \underset{(.133)}{.510}MN_{t-1} + \underset{(.123)}{.359}MN_{t-2} + \underset{(.103)}{.104}Y_t + \underset{(.106)}{.126}Y_{t-1} \\ & \underset{(.015)}{-.067}RL_t + \underset{(.018)}{.006}RL_{t-1} + \underset{(.115)}{.499}P_{t-1} - \underset{(.200)}{.086}P_{t-2} - \underset{(.146)}{.297}P_{t-3} \end{aligned} \tag{4.23}$$

$$S = .00762736 \quad \hat{\sigma} = .0129 \quad F(4, 46) = 2.52$$

The F statistic confirms the rejection of (4.22) in favour of (4.23). A common autoregressive factor can now be extracted and estimation of the implied AD(1)(1, 0, 0, 2) specification yields an insignificant

$\chi^2(3)$ statistic, indicating the acceptance of the factorisation, and the following estimates

$$\hat{MN}_t = -.617 + .864MN_{t-1} + .189Y_t - .046RL_t + .589P_{t-1} - .470P_{t-2} - .378\hat{\varepsilon}_{t-1}$$
$$(.337)\quad(.043)\qquad(.042)\quad(.006)\quad(.092)\quad(.101)\quad(.124)$$

$$\hat{\lambda}_1 = 1.39 \qquad \hat{\lambda}_2 = -.34 \qquad \hat{\lambda}_3 = .88 \tag{4.24}$$

$$S = .00809493 \quad \hat{\sigma} = .0127 \quad Q(14) = 25.7, \ \chi^2(3) = 3.33, \ \chi^2(13) = 29.2$$

$$F_c(10,36) = .95$$

Although there is no evidence of structural instability (the F_c statistic being calculated from the unrestricted specification (4.23)), and all coefficients are precisely determined, implying an income elastic and an interest and price inelastic demand for narrow money function, further dynamic mis-specification is indicated from the residual correlogram. The significant residual correlations are $\hat{r}_3 = -.26$, $\hat{r}_7 = -.30$ and $\hat{r}_9 = .49$ and while the latter correlations may well be explained away as statistical artifacts, the former correlation, when coupled with the significant $\chi^2(13)$ statistic indicating rejection of (4.14) against the maintained hypothesis, leads to the conclusion that the $\underset{\sim}{X}_2$ vector of variables is inadequately capable of modelling the demand for narrow money.

It is again necessary, then, to consider the extended model incorporating the $\underset{\sim}{X}_2^*$ vector. Estimation of the maintained hypothesis AD(2, 2, 2, 2, 2, 2, 2) by OLS obtained the estimates shown in Table 4.4. Employment of the t tests of stage (a) with individual significance levels set at $\varepsilon_1 = \varepsilon_2 = .05$ leads to the acceptance of the

AD(1, 1, 0, 1, 0, 0, 0) specification, again rendering stage (b) of the procedure redundant. Estimation of this specification yields

$$\begin{aligned}\hat{MN}_t = \underset{(.580)}{.273} + \underset{(.062)}{.649}MN_{t-1} - \underset{(.122)}{.033}Y_t + \underset{(.113)}{.193}Y_{t-1} - \underset{(.011)}{.067}RL_t \\ - \underset{(.168)}{.406}P_t + \underset{(.143)}{.379}P_{t-1} - \underset{(.008)}{.014}F_t + \underset{(.031)}{.036}V_t + \underset{(.103)}{.296}W_t\end{aligned} \tag{4.25}$$

$$S = .00900225 \quad \hat{\sigma} = .0140 \quad d_1 = 2.05 \quad h_1 = -.21 \quad Q(15) = 28.2$$

Although there is no evidence of first order serial correlation, $\hat{r}_2 = .32$ is significant, along with $\hat{r}_9 = .31$ and $\hat{r}_{14} = -.30$. This mis-specification led to the inclusion of MN_{t-2} as an additional regressor and re-estimation after the deletion of the insignificant regressors Y_t, F_t and V_t gave

$$\begin{aligned}\hat{MN}_t = -\underset{(.498)}{.497} + \underset{(.106)}{.447}MN_{t-1} + \underset{(.104)}{.271}MN_{t-2} + \underset{(.068)}{.172}Y_{t-1} - \underset{(.010)}{.074}RL_t \\ - \underset{(.144)}{.293}P_t + \underset{(.137)}{.212}P_{t-1} + \underset{(.090)}{.300}W_t\end{aligned} \tag{4.26}$$

$$S = .00850891 \quad \hat{\sigma} = .0133 \quad Q(15) = 44.0$$

Although MN_{t-2} enters significantly and the respecification accounts for the previously observed second order serial correlation, the overall serial correlation problems, as indicated by the Q statistic, are further exacerbated. Inspection of the residual correlogram shows that the large correlations occur at high orders, although $\hat{r}_1 = .22$ is significant at the .10 level. This latter correlation led to further respecification using analogues of (4.23) and (4.24), i.e. including an additional lag on each regressor in (4.26) and then testing for the extraction of a common autoregressive factor of order one. This common factor was accepted, leading to the re-estimation

$$\begin{aligned}\hat{MN}_t = &-.391 + .278MN_{t-1} + .356MN_{t-2} + .243Y_{t-1} - .C79RL_t \\ &\quad(.526) \quad (.085) \qquad (.089) \qquad (.088) \qquad (.012) \\ &- .384P_t + .372P_{t-1} + .298W_t + .424\hat{\varepsilon}_{t-1} \\ &\quad(.128) \quad (.134) \qquad (.107) \quad (.121)\end{aligned} \tag{4.27}$$

$\lambda_1 = .66,\ \lambda_2 = -.22,\ \lambda_3 = -.0003,\ \lambda_6 = .81 \qquad \mu_1 = .74\ \mu_2 = -.47$

$S = .00760955\ \hat{\sigma} = .0126\quad Q(15) = 22.4\quad F_c(13,30) = .44\ \chi^2(3) = 6.57$

Although the Q statistic is insignificant at the .05 level, in view of its well known deficiencies the residual correlogram was checked. The only significant correlation was (again) $\hat{r}_9 = .35$ and as the F_c statistic (estimated from the unrestricted form of (4.27)) does not suggest structural instability the above specification was accepted. However, the long run price elasticity strongly implies that the price regressors should be combined as first differences, i.e. as

$\Delta P_t = P_t - P_{t-1}$, and this restriction, coupled with the deletion of the insignificant constant, was accepted by a χ^2 test, leading to

$$\hat{MN}_t = .265MN_{t-1} + .347MN_{t-2} + .219Y_{t-1} - .077RL_t - .378\Delta P_t$$
$$(.080) \qquad (.084) \qquad (.043) \qquad (.010) \qquad (.112)$$
$$+ .308W_t + .444\hat{\varepsilon}_{t-1} \tag{4.28}$$
$$(.038) \quad (.120)$$

$\hat{\lambda}_1 = .56$, $\hat{\lambda}_2 = -.20$, $\hat{\lambda}_3 = 0$, $\hat{\lambda}_6 = .79$, $\mu_1 = .67$, $\mu_2 = -.33$

$S = .00764988$ $\quad\hat{\sigma} = .0124$ $\quad\chi^2(2) = .30$

Of course, throughout this search the simultaneity between MN and RL has been ignored and we should now investigate the IV analogue of (4.27). For consistent estimates MN_{t-1}, MN_{t-2}, MN_{t-3}, Y_{t-1}, Y_{t-2}, RL_{t-1}, P_t, P_{t-1}, P_{t-2}, W_t and W_{t-1} must be included amongst the instruments (see Fair (1970)) and estimation using only these instruments yields

$$\hat{MN}_t = -.635 + .395MN_{t-1} + .314MN_{t-2} + .193Y_{t-1} - .078\hat{RL}_t$$
$$(.599) \quad (.122) \qquad (.121) \qquad (.078) \qquad (.014)$$
$$- .304P_t + .224P_{t-1} + .306W_t + .040\hat{\varepsilon}_{t-1} \tag{4.29}$$
$$(.161) \quad (.156) \qquad (.107) \quad (.134)$$

$S = .0106477$ $\quad\hat{\sigma} = .0149$

The estimate of the serial correlation coefficient is insignificantly different from zero and imposing this restriction along with the two mentioned above allows the estimation of the IV analogue of (4.28),

$$\hat{MN}_t = \underset{(.122)}{.443MN_{t-1}} + \underset{(.113)}{.241MN_{t-2}} + \underset{(.038)}{.174Y_{t-1}} - \underset{(.011)}{.069\hat{RL}_t}$$

$$- \underset{(.146)}{.245\Delta P_t} + \underset{(.037)}{.255W_t} \qquad (4.30)$$

$$\hat{\lambda}_1 = .55, \ \hat{\lambda}_2 = -.22, \ \hat{\lambda}_3 = 0, \ \hat{\lambda}_6 = .81, \ \mu_1 = .79, \ \mu_2 = .40$$

$$s^2 = .0110374 \qquad \hat{\sigma} = .0149 \qquad \chi^2(3) = 2.01$$

The χ^2 statistic accepts the restrictions and, although the coefficient estimates are less precisely estimated than the corresponding ones in (4.28), (a result that should be expected from IV estimation), the long run elasticities are almost identical. Again it seems that it is not crucial to take explicit account of the simultaneity between MN and RL.

Since (4.28) and (4.30) are virtually indistinguishable, we shall consider the inferences that may be drawn from the former model. This specification is not nested within the extended maintained hypothesis and this highlights the importance of using residual diagnostic tests in conjunction with the specification testing procedure, particularly when, as is the case here, the maintained hypothesis is not very general.

The wage rate, W_t, is shown to be an important determinant in the demand function whereas the other additional variables, F_t and V_t, are found to be insignificant. The demand function, as modelled by (4.28) is income and interest inelastic with a zero long run price elasticity, i.e. that it is changes in the price level that affect the level of narrow money balances. This is in almost total contrast to the elasticities implied by both the conventional specification and the specification chosen at the end of the initial search procedure. While the interest elasticity is consistently found to be less than unity, these latter models yield an income elasticity in excess of unity and a price elasticity only just less than unity.

Although further inferences regarding the relative performance of the two demand for narrow money functions are drawn in a later section, we have shown here the importance of dynamic specification testing for drawing appropriate conclusions as to both the short and long run interactions between money and its determinants.

4.6 SPECIFICATION SEARCH 3

In this section the specification searches undertaken for the vectors $\underset{\sim}{X}_3' = (MB, Y, RC, P)$ and $\underset{\sim}{X}_3^*$ are investigated.

There being no evidence of simultaneity between MB and the regressors, OLS estimation of the AD(1, 0, 0, 0) specification yields

$$\hat{MB}_t = \underset{(.632)}{-3.371} + \underset{(.041)}{1.064}MB_{t-1} + \underset{(.079)}{.279}Y_t - \underset{(.022)}{.073}RC_t - \underset{(.035)}{.072}P_t \qquad (4.31)$$

$S = .0118967 \quad \hat{\sigma} = .0153 \quad \mu_1 = 1.06$

$d_1 = 1.58 \quad h_1 = 1.65 \quad d_4 = 1.84 \quad h_4 = .31 \quad Q(15) = 26.2 \quad \tilde{Q}(15) = 32.6$

$\phi(1) = 2.51 \quad \phi(2) = 2.59 \quad \phi(4) = 6.99 \quad F_c(5,46) = 3.03$

The h_1 statistic is just significant at the .10 level with $\hat{r}_3 = .28$ being the only low order residual correlation significant at the .05 level, the others being $\hat{r}_9 = -.35$ and $\hat{r}_{14} = -.31$. However, even if one was prepared to ignore this limited evidence of residual serial correlation, the specification is unacceptable as it exhibits both structural instability, as evinced by the significant F_c statistic, and dynamic instability, a consequence of the root μ_1 of the lag polynomial on MB being greater than unity. For this reason no long run elasticities have been calculated, although all coefficients are precisely estimated. Both these forms of instability have been encountered in recent studies on the demand for broad money, notably Haache (1974), Artis and Lewis (1974, 1976) and Mills (1975, 1978).

Turning now to the estimates of the maintained hypothesis AD(4, 4, 4, 4) shown in Table 4.5, the use of stringent significance levels for the t tests in stage (a) of the testing procedure, e.g. $\varepsilon_i = 0.2$ for all i, leads again to the AD(1, 0, 0, 0) specification. Relaxing the significance levels for the later tests in the sequence, e.g. setting $\varepsilon_3 = \varepsilon_4 = .10$ (thus implying a significance level of .22 for the fourth test against the maintained), enables the AD(1, 2, 0, 1) specification to be considered. Estimation of this specification yields

$$\begin{aligned}\hat{MB}_t = &-3.972 + 1.046MB_{t-1} + .084Y_t + .121Y_{t-1} + .158Y_{t-2}\\ &\;(.656)\qquad(.041)\qquad\quad(.120)\quad(.150)\qquad(.114)\\ &-.075RC_t - .254P_t + .191P_{t-1}\\ &\;(.022)\qquad(.134)\quad(.127)\end{aligned} \tag{4.32}$$

$S = .0104091 \quad \hat{\sigma} = .0147 \quad \mu_1 = 1.05 \quad d_1 = 1.34 \quad h_1 = 2.59 \quad Q(15) = 40.0$

TABLE 4.5

j	0	1	2	3	4	$\Sigma\theta_j$
MB_{t-j}	1	1.207[7.24]	-.102[.37]	.041[.16]	-.223[1.37]	.08
Y_{t-j}	.218[1.79]	.083[.54]	.276[1.82]	-.162[1.13]	-.040[.34]	.38
RC_{t-j}	-.043[1.62]	.009[.30]	-.033[.97]	.015[.47]	-.031[1.18]	-.08
P_{t-j}	-.039[.23]	.449[1.96]	-.291[1.10]	-.030[.10]	-.017[.10]	.07

constant = -2.933[2.46]   S = .00549807 $\hat{\sigma}$ = .0124

TABLE 4.6

j	0	1	2	$\Sigma\theta_j$
MB_{t-j}	1	1.228[7.76]	-.253[1.49]	.03
Y_{t-j}	.206[1.75]	.249[1.85]	-.007[.06]	.45
RC_{t-j}	-.090[3.51]	-.003[.12]	-.076[3.05]	-.17
P_{t-j}	-.177[.95]	.409[2.03]	-.485[2.71]	-.25
F_{t-j}	.018[2.08]	-.004[.56]	.003[.34]	.02
V_{t-j}	-.004[.08]	.038[.73]	-.051[1.12]	-.02
W_{t-j}	.289[1.50]	-.111[.40]	.069[.34]	.25

constant = -5.358[4.77]   S = .00471167 $\hat{\sigma}$ = 0.116

Along with the continuing dynamic instability, the d_1 and h_1 statistics indicate the presence of first order serial correlation, although it may actually be of the third order, for $\hat{r}_3 = .37$. Using the approach to the presence of first order serial correlation considered in previous searches, the more general AD(2, 3, 1, 2) specification was estimated, giving

$$
\begin{aligned}
\hat{MB}_t = & -2.979 + 1.360MB_{t-1} - .346MB_{t-2} + .249Y_t + .072Y_{t-1} \\
& \quad (.831) \qquad (.126) \qquad\quad (.142) \qquad\quad (.166) \quad (.139) \\
& +.227Y_{t-2} - .259Y_{t-3} - .067RC_t - .002RC_{t-1} \\
& \quad (.142) \qquad (.112) \qquad (.024) \qquad (.026) \\
& -.179P_t + .499P_{t-1} - .343P_{t-2} \\
& \quad (.136) \quad (.205) \qquad (.134)
\end{aligned}
\tag{4.33}
$$

$S = .00735587 \qquad \hat{\sigma} = .0129 \qquad \mu_1 = 1.02 \qquad \mu_2 = .34$

$Q(15) = 16.6 \qquad F(4,44) = 4.57 \qquad F_c(12,32) = 1.20$

The F statistic rejects the restriction of (4.33) to (4.32) and thus the above respecification is justified. Although the Q and F_c statistics do not indicate the presence of serial correlation or structural instability, the specification is still bedevilled by dynamic instability. However, the extraction of a common first order autoregressive factor is more encouraging, for estimation of the implied AD(1)(1, 2, 0, 1) specification yields

$$\hat{MB}_t = -5.723 + .810MB_{t-1} + .203Y_t + .196Y_{t-1} + .412Y_{t-2}$$
$$(1.284) \quad (.071) \quad (.104) \quad (.113) \quad (.098)$$
$$-.039RC_t - .193P_t + .305P_{t-1} + .703\hat{\varepsilon}_{t-1} \qquad (4.34)$$
$$(.021) \quad (.121) \quad (.112) \quad (.095)$$
$$S = .00837292 \quad \hat{\sigma} = .0132 \quad \hat{\lambda}_1 = 4.27, \ \hat{\lambda}_2 = -.21, \ \hat{\lambda}_3 = -.59, \ \mu_1 = .81$$
$$Q(15) = 24.2 \qquad \chi^2(3) = 7.25 \qquad \chi^2(11) = 23.6$$

The common factor restriction is accepted by the $\chi^2(3)$ statistic and the model now displays dynamic stability. However, although the coefficients are precisely estimated the implied long run income elasticity is rather high while the price elasticity is incorrectly signed. Furthermore, the Q statistic is only just insignificant at the .05 level (recall Davies, Triggs and Newbold (1977)), with the residual correlations $\hat{r}_1 = -.22$ and $\hat{r}_2 = .22$ being significant at the .10 level. This evidence of serial correlation, plus the rejection of (4.34) when tested against the maintained hypothesis by the $\chi^2(11)$ statistic, are clearly worrying features of the above specification and point to the desirability of investigating the extended model incorporating the $\underset{\sim}{X}_3^*$ vector.

Estimates of the maintained hypothesis AD(2, 2, 2, 2, 2, 2, 2) are shown in Table 4.6 and stage (a) of the testing procedure leads to the acceptance of the AD(1, 0, 2, 2, 0, 0, 0) specification for all individual test significance levels between .01 and .05. Estimation of this specification yields

$$\hat{MB}_t = -5.690 + 1.046MB_{t-1} + .391Y_t$$
$$(.727) \quad (.043) \quad (.066) \qquad (4.35)$$
$$-.095RC_t - .026RC_{t-1} - .065RC_{t-2}$$
$$(.023) \quad (.023) \quad (.024)$$
$$-.235P_t + .309P_{t-1} - .413P_{t-2}$$
$$(.165) \quad (.179) \quad (.118)$$
$$+.016F_t - .023V_t + .275W_t$$
$$(.007) \quad (.033) \quad (.114)$$

$S = .00643134$ $\quad \hat{\sigma} = .0121$ $\quad \mu_1 = 1.05$

$d_1 = 1.47$ $\quad h_1 = 2.09$ $\quad Q(15) = 17.1$

Although the Q statistic does not indicate significant serial correlation the statistics designed specifically to test first order serial correlation are significant, thus highlighting the portmanteau statistic's lack of power against specific alternative hypotheses to the null hypothesis of white noise errors. The specification also exhibits dynamic instability but rather than respecifying the model by including additional lags on all variables only MB_{t-2} and Y_{t-1} were added as these regressors were reasonably significant in the maintained hypothesis. This extension defines an AD(2, 1, 2, 2, 0, 0, 0) specification, the estimation of which yields

$$\hat{MB}_t = -5.729 + 1.203MB_{t-1} - .222MB_{t-2} + .205Y_t + .262Y_{t-1}$$
$$(.814) \quad (.119) \quad (.131) \quad (.102) \quad (.096)$$
$$-.096RC_t - .012RC_{t-1} - .078RC_{t-2}$$
$$(.022) \quad (.022) \quad (.022)$$
$$-.235P_t + .368P_{t-1} - .428P_{t-2} \qquad (4.36)$$
$$(.162) \quad (.161) \quad (.107)$$
$$+.018F_t + .004V_t + .276W_t$$
$$(.006) \quad (.030) \quad (.109)$$

$S = .00491798$ $\quad \hat{\sigma} = .0108$ $\quad \mu_1 = .98$ $\quad \mu_2 = .23$

$Q(15) = 18.7$ $\quad F(2, 42) = 6.46$

The generalisation to (4.36) is justified on the evidence of the F statistic, with inspection of the residual correlogram revealing no significant residual correlations, in this case, therefore, confirming the inference from the Q statistic. Moreover, the roots of the lag polynomial on MB are less than unity, thus indicating dynamic stability. Deletion of the insignificant regressors RC_{t-1}, P_t and V_t yields on re-estimation

$$\hat{MB}_t = -5.342 + 1.288MB_{t-1} - .316MB_{t-2} + .272Y_t + .220Y_{t-1}$$
$$(.661) \quad (.098) \quad (.104) \quad (.087) \quad (.089)$$

$$-.081RC_t - .092RC_{t-2} + .283P_{t-1} - .446P_{t-2}$$
$$(.018) \quad (.019) \quad (.127) \quad (.102)$$

$$+.020F_t + .163W_t \tag{4.37}$$
$$(.006) \quad (.073)$$

$S = .00519364 \quad \hat{\sigma} = .0107 \quad \lambda_1 = 17.6, \lambda_2 = -6.2, \lambda_3 = -5.8,$

$\lambda_4 = .71, \lambda_6 = 5.8$

$\mu_1 = .96, \mu_2 = .33 \quad Q(15) = 20.6 \quad F(3,42) = .78 \quad F_c(11,34) = 1.91$

The three zero coefficient restrictions are jointly accepted by the F statistic and there is no evidence of residual correlation, with only $\hat{r}_9 = -.31$ being significant. The specification is also both structurally and dynamically stable. Unfortunately, the long run elasticities, apart from that of F_t are all rather implausible - a consequence of the near unit root on the MB polynomial ensuring that $\Sigma\theta_0$ is close to zero. Imposing this unit root restriction (i.e. $\Sigma\theta_0 = 0$) in addition to imposing identical coefficients on the Y and RC lag polynomials (i.e. restricting them to be zero order Almon polynomials) leads to the following re-estimation

$$
\begin{aligned}
\Delta\hat{MB}_t = & -5.368 + .287\Delta MB_{t-1} + .238(Y_t + Y_{t-1}) \\
& \;\;(.644) \quad\;\; (.095) \qquad\qquad (.033) \\
& -.089(RC_t + RC_{t-2}) + .278P_{t-1} - .439P_{t-2} \\
& \;\;(.012) \qquad\qquad\qquad\quad (.122) \qquad (.098) \\
& +.020F_t + .139W_t \qquad\qquad\qquad\qquad\qquad\qquad (4.38) \\
& \;\;(.006) \quad\; (.064)
\end{aligned}
$$

$S = .00529507 \qquad \hat{\sigma} = .0105 \qquad \mu_1 = 1 \qquad \mu_2 = .29$

$F(3,45) = .29 \qquad F(13,35) = .33$

On the basis of the two F statistics these restrictions are accepted and the specification is not rejected against the maintained hypothesis. In comparison with the originally estimated specification (4.31) the error standard deviation has been reduced by almost a third with an associated improvement in the precision of coefficient estimates. Both F and W are significant determinants of this broad money demand function, the importance of the former variable being particularly interesting as this was predicted from the theoretical development of the variable.

However, the successful extraction of a unit root from the MB lag polynomial does not allow any long run elasticities to be calculated (a consequence of the restriction $\Sigma\theta_0 = 0$). This implies infinitely long adjustment of MB to changes in the regressors and confirms the high absolute elasticities found in (4.37). The implications of such a model are that all long run information in the data and all a priori information from economic theory based on steady state arguments are lost. Davidson et al (1978) and Hendry (1979) have argued that it seems inappropriate to assume that short-run behaviour is independent of disequilibria in the levels of the variables and have proposed "error-correction" models of the form

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (x_{t-1} - y_{t-1}) + \varepsilon_t$$

as a modification of simple differenced models. However, inspection of the data determined specification (4.37) does not suggest that this particular demand for broad money function is capable of such re-specification, and either we must accept the economic implications of the final specification or embark on further empirical remodelling.

4.7 SPECIFICATION SEARCH 4

This section considers the final specification search undertaken for the vectors $\underset{\sim}{X}_4' = (MB, Y, RL, P)$ and $\underset{\sim}{X}_4^*$.

OLS estimation of the AD(1, 0, 0, 0) specification obtains

$$\hat{MB}_t = \underset{(.479)}{-2.516} + \underset{(.043)}{1.105}MB_{t-1} + \underset{(.075)}{.152}Y_t - \underset{(.011)}{.043}RL_t - \underset{(.039)}{.127}P_t$$

$$S = .0108887 \qquad \hat{\sigma} = .0146 \qquad \mu_1 = 1.11 \qquad (4.39)$$

$$d_1 = 1.56 \quad h_1 = 1.74 \quad d_4 = 2.18 \quad h_4 = -.32 \quad Q(15) = 19.0 \quad \tilde{Q}(15) = 23.0$$

$$\phi(1) = 2.45 \quad \phi(2) = 2.70 \quad \phi(4) = 4.08 \quad F_c(5,46) = 1.37$$

Equation (4.39) is very similar to the $\underset{\sim}{X}_3$ analogue (4.31) with the

h_1 statistic being significant at the .10 level and the root of the MB lag polynomial being greater than unity, thus indicating dynamic instability (although here there is no evidence of structural instability from the F_c statistic).

This conventional specification is again clearly inadequate and estimation of the maintained hypothesis AD(4, 4, 4, 4) obtains the results shown in Table 4.7. The use of stringent significance levels for the individual t tests of stage (a) of the testing procedure, say $\varepsilon_i = .02$ for all i, would select the AD(1, 0, 0, 0) specification but setting ε_3 and ε_4 at approximately .05 allows the AD(1, 2, 0, 0) specification to be considered, obviously rendering stage (b) redundant. Estimation of this specification yields

$$\begin{aligned}\hat{MB}_t = {} & \underset{(.051)}{-2.826} + \underset{(.045)}{1.082}MB_{t-1} + \underset{(.114)}{.061}Y_t - \underset{(.139)}{.009}Y_{t-1} + \underset{(.112)}{.158}Y_{t-2} \\ & - \underset{(.011)}{.042}RL_t - \underset{(.039)}{.113}P_t \end{aligned} \tag{4.40}$$

$S = .0102714 \qquad \hat{\sigma} = .0145 \qquad \mu_1 = 1.08$

$d_1 = 1.44 \quad h_1 = 2.17 \quad Q(15) = 25.0 \quad F(2,49) = 1.47$

The rather artificial nature of this generalised specification is emphasised by the F statistic, which accepts the restriction of (4.40) to (4.39). Serious first order serial correlation is now signalled, along with continuing dynamic instability, thus leading to the estimation of the further generalised AD(2, 3, 1, 1) specification, yielding

TABLE 4.7

j	0	1	2	3	4	$\Sigma\theta_j$
MB_{t-j}	1	1.048[6.28]	.157[.65]	-.110[.47]	-.114[.92]	.02
Y_{t-j}	.114[.92]	.087[.58]	.281[2.01]	-.183[1.29]	-.041[.33]	.26
RL_{t-j}	-.013[.73]	-.013[.51]	.002[.09]	-.012[.50]	-.007[.35]	-.04
P_{t-j}	-.125[.83]	.364[1.72]	-.137[.63]	-.286[1.35]	.183[1.23]	0

constant = -2.316[2.59] S = .00583463 $\hat{\sigma}$ = .0127

TABLE 4.8

j	0	1	2	$\Sigma\theta_j$
MB_{t-j}	1	1.095[6.41]	-.020[.10]	-.08
Y_{t-j}	-.029[.21]	.051[.32]	.104[.82]	.13
RL_{t-j}	-.043[2.01]	-.004[.13]	-.003[.15]	-.05
P_{t-j}	-.190[.91]	.022[.10]	-.125[.62]	-.29
F_{t-j}	.006[.61]	-.007[.83]	-.001[.05]	0
V_{t-j}	.010[.17]	.017[.28]	-.101[2.06]	-.07
W_{t-j}	-.049[.23]	.320[.99]	-.076[.30]	.20

constant = -2.929[3.78] S = .00602596 $\hat{\sigma}$ = .0131

$$\hat{MB}_t = -2.327 + 1.231MB_{t-1} - .178MB_{t-2} + .133Y_t - .015Y_{t-1} + .377Y_{t-2}$$
$$(.592) \quad (.142) \quad (.161) \quad (.113) \quad (.135) \quad (.128)$$
$$-.307Y_{t-3} - .013RL_t - .025RL_{t-1} - .131P_t + .060P_{t-1} \qquad (4.41)$$
$$(.110) \quad (.016) \quad (.017) \quad (.128) \quad (.117)$$
$$S = .00774751 \quad \hat{\sigma} = .0131 \quad \mu_1 = 1.06 \quad \mu_2 = .17$$

The extraction of a common autoregressive factor is not supported by the data, for estimation of the AD(1)(1, 2, 0, 0) specification yields a significant test statistic of $\chi^2(3) = 9.98$. Although the AD(2, 3, 1, 1) specification is therefore accepted a number of coefficients are insignificant in (4.41) and deletion of the regressors MB_{t-2}, Y_t, Y_{t-1}, RL_t and P_{t-1} leads, on re-estimation, to

$$\hat{MB}_t = -2.422 + 1.085MB_{t-1} + .431Y_{t-2} - .269Y_{t-3}$$
$$(.451) \quad (.034) \quad (.099) \quad (.103)$$
$$- .045RL_{t-1} - .101P_t \qquad (4.42)$$
$$(.009) \quad (.030)$$
$$S = .00847575 \quad \hat{\sigma} = .0130 \quad \mu_1 = 1.09 \quad F(5,45) = .85$$
$$d_1 = 1.91 \quad h_1 = .25 \quad Q(15) = 13.0 \quad F(14,36) = 1.16 \quad F_c(6,44) = 2.69$$

Although there is no evidence of serial correlation this specification is clearly unsatisfactory as it exhibits both dynamic and now structural instability and it is therefore necessary to consider the extended model incorporating the $\underset{\sim}{X}_4^*$ vector.

Estimates of the maintained hypothesis AD(2, 2, 2, 2, 2, 2, 2) are shown in Table 4.8, and setting the t test significance levels at, say, $\varepsilon_1 = \varepsilon_2 = .02$, enables stage (a) of the procedure to select the extended conventional specification AD(1, 0, 0, 0, 0, 0, 0), which on estimation yields

$$\begin{aligned}
\hat{MB}_t = & \underset{(.477)}{-2.876} + \underset{(.049)}{1.132}MB_{t-1} + \underset{(.080)}{.103}Y_t - \underset{(.011)}{.043}RL_t - \underset{(.110)}{.280}P_t \\
& + \underset{(.008)}{.016}F_t - \underset{(.033)}{.070}V_t + \underset{(.110)}{.123}W_t
\end{aligned} \tag{4.43}$$

$S = .00893884 \qquad \hat{\sigma} = .0136 \qquad \mu_1 = 1.13$

$d_1 = 1.62 \qquad h_1 = 1.53 \qquad Q(15) = 21.7$

However, setting $\varepsilon_1 = .05$ allows the AD(1, 0, 0, 0, 0, 0, 2) specification to be selected, the estimation of which obtains

$$\begin{aligned}
\hat{MB}_t = & \underset{(.441)}{-2.742} + \underset{(.045)}{1.124}MB_{t-1} + \underset{(.074)}{.097}Y_t - \underset{(.010)}{.046}RL_t - \underset{(.104)}{.236}P_t \\
& + \underset{(.007)}{.010}F_t + \underset{(.042)}{.006}V_t - \underset{(.043)}{.024}V_{t-1} - \underset{(.035)}{.082}V_{t-2}\ \underset{(.103)}{.109}W_t
\end{aligned} \tag{4.44}$$

$S = .00726162 \qquad \hat{\sigma} = .0126 \qquad \mu_1 = 1.12$

$d_1 = 1.86 \qquad h_1 = .56 \qquad F(2,46) = 5.31$

where the F statistic indicates rejection of (4.43) as a restriction of (4.44). Deletion of the insignificant V_t, V_{t-1} and W_t regressors and the replacement of the insignificant Y_t by Y_{t-1} yields on re-estimation

$$\hat{MB}_t = \underset{(.415)}{-2.927} + \underset{(.035)}{1.132MB_{t-1}} + \underset{(.061)}{.161Y_{t-1}} - \underset{(.009)}{.045RL_t}$$

$$\underset{(.034)}{-.126P_t} + \underset{(.006)}{.011F_t} - \underset{(.023)}{.104V_{t-2}} \qquad (4.45)$$

$S = .00719468 \qquad \hat{\sigma} = .0121 \qquad \mu_1 = 1.13$

$d_1 = 1.92 \quad h_1 = .31 \quad Q(15) = 18.9 \quad F_c(7,42) = 1.89, \; F(1,49) = 13.8$

Although there is no evidence of serial correlation (the only significant residual correlation being $\hat{r}_{14} = -.36$) or structural instability, the recurring problem of dynamic instability continues to present itself. Furthermore, the unit root restriction ($\theta_{01} = 1$) is strongly rejected by a conventional F test and hence no long run elasticities are calculated.

The specification (4.45) is therefore clearly unacceptable and leads us to the conclusion that the demand for broad money cannot be adequately modelled by either the $\underset{\sim}{X}_4$ or $\underset{\sim}{X}_4^*$ combination of variables, i.e. by a set of variables which includes the local authority rate as the representative short interest rate.

4.8 SUMMARY AND CONCLUSIONS

The specification searches have illustrated the importance of dynamic specification in the demand for money function, since not only have models been identified with high explanatory power but for all models important lag structures have been discovered which have fundamental effects on the long run relationships existing between money and its determinants.

With regard to the testing procedure employed to determine dynamic specification, a number of observations emerge from its empirical application. The choice of test significance levels is often found to be important in selecting specifications in stage (a) of the procedure, with the use of relatively large significance levels being frequently necessary to ensure selection of "sensible" specifications. Furthermore, it is also apparent that the careful use of residual diagnostic statistics is also essential to ensure arrival at appropriate final specifications, particularly when, as in the extended model (4.9), the maintained hypothesis under consideration is not able to be very general. In this context, the careful inspection of the residual correlogram and the use of statistics designed to test specific alternatives to the null hypothesis of white noise residuals (e.g. the Durbin-Watson and Lagrange Multiplier tests) are found to be more fruitful than inferences based on portmanteau statistics.

An important point highlighted by the specification searches is that the assumption of an autoregressive error process is not a universal solution to the problem of serial correlation, only one search arriving at a specification in which an autoregressive factor had been successfully extracted. Serial correlation in the residuals may arise from error autocorrelation, but it is at least as likely to arise from a multitude of potential mis-specifications, the most important here being omitted variables and mis-specified systematic dynamics. Note also that the presence of, say, first order error autocorrelation implicitly requires that each regressor appears as, at least, a first order lag polynomial, a requirement that may not be appropriate on either theoretical or empirical considerations.

Nevertheless, notwithstanding such complications the dynamic specification testing procedure has performed well, enabling dynamic specification to be investigated within a framework convenient for both estimation and testing and using sample sizes typical of those available for macroeconomic variables.

The specifications of the demand for money function selected in this chapter may be related to the findings of previous chapters. In comparing these multivariate descriptions of the money series' with the univariate models identified and estimated in chapter 2, the error standard deviation of the narrow money series is reduced by approximately 40 per cent with the inclusion of the additional variables, while their inclusion reduces the error standard deviation of the broad money series by up to 30 per cent depending on which combination of variables is used. The extension to a multivariate framework has also eliminated the nonstationarity found in the univariate narrow money models, but nonstationarity continues to be exhibited by the broad money series independently of the introduction of additional variables.

The endogeneity of the money series found in Chapter 3 is automatically confirmed by the identification of autoregressive distributed lag demand for money functions, and it is interesting to note that the single instance of simultaneity, between MN and RL, is corroborated by finding only contemporaneous correlation between these variables in the $\underset{\sim}{X}_2^*$ specification.

Turning now to the theoretical implications of the selected demand for money specifications, the two narrow money functions are sufficiently different to warrant close examination. Considering first their similarities; both functions are interest inelastic, although the long interest rate (consol yield) elasticity is over twice the absolute size of the short interest rate (local authority rate) elasticity. Neither F nor V appear significantly in the function, but the wage rate W is an important determinant, with both functions being wage inelastic.

The major difference between the functions is structural stability. The short interest rate specification, using the vector $\underset{\sim}{X}_2^*$ of variables, is structurally stable, yielding a function which is income inelastic but with a zero long run price elasticity, although price changes have an important negative effect. The long interest rate specification undergoes a structural shift and as a consequence displays an income elasticity which alters considerably between subperiods, being inelastic in the earlier period and highly elastic in the later period. In this specification the price level has a positive effect on narrow money, with an elasticity less than unity.

The divergent behaviour of the two specifications is somewhat difficult to explain simply as a consequence of the use of different interest rates and further modelling of the narrow money specifications would seem worthwhile before drawing any final conclusions.

For the broad money specifications problems of theoretical interpretation immediately arise. The own rate differential F, as theoretically predicted, is an important determinant of the demand for broad money and the wage rate W and the trade variable V both appear significantly, although not in the same specification. Both specifications are structurally stable, thus avoiding one of the problems traditionally found with this function, but no long run elasticities can be calculated as dynamic instability is a very serious problem. The long interest rate specification is modelled with changes in broad money, a consequence of a successful unit root restriction and this, in fact, implies infinite long run elasticities for all variables. The situation is even worse for the short interest rate specification, for this exhibits a root of the MB polynomial significantly greater than unity, thus implying an unstable demand function, with resultant explosive long run solutions.

Therefore, although the specification testing procedure has selected models with good statistical properties, the economic theory content of the demand for broad money functions is unacceptable and again suggests that further modelling would be worthwhile.

Thus, in conclusion, while the dynamic specification testing procedures developed and employed in this chapter have quite adequately produced good statistical specifications there are problems with the theoretical implications of the functions. While this may well be due to the inadequacy of the economic theory underlying the models further emprical modelling is desirable before we may make this unsatisfactory conclusion. These empirical extensions are developed in the next chapter.

CHAPTER FIVE :

FUNCTIONAL FORM IN THE DEMAND FOR MONEY

5.1 TRANSFORMATION OF VARIABLES AND FUNCTIONAL FORM

The empirical modelling of the interactions between the major macroeconomic variables developed in the previous chapters has used logarithmic transformations of the original data series as the actual variables appearing in the analysis.

In chapter two this transformation was justified on the statistical grounds that it helps to stabilise the variance and induce stationarity into time series while subsequently in chapter four, demand for money functions defined in terms of these logarithmically transformed variables were employed as they are particularly amenable to direct economic interpretation.

However, this logarithmic transformation, as indeed any particular transformation of variables, can be challenged from both statistical and economic theory standpoints. Taking logarithms may not necessarily be the most appropriate transformation for inducing desirable properties into a time series, a particularly interesting illustration of this being Chatfield and Prothero (1973) and the ensuing discussion. Certainly economic theory yields no a priori indication as to the correct functional form of the demand for money function and indeed the implied constant elasticities of the logarithmic model may be unduly restrictive. In particular, it may be argued that the doubling of the interest rate from 1% to 2% would be unlikely to have the same proportional affect on the demand for money as would a doubling from 10% to 20%.

A popular method of introducing flexibility into transforming individual time series and generalising functional form specification in regression models, which at the same time allows the appropriate transformation to be selected by the data itself, is the family of power transformations proposed by Box and Cox (1964). When transforming a series x_t, it may be defined as

$$
\begin{aligned}
x_t^{(\eta)} &= (x_t^{\eta} - 1)/\eta \qquad \text{for } \eta \neq 0 \\
x_t^{(0)} &= \ln x_t
\end{aligned}
\tag{5.1}
$$

In view of the rather unsatisfactory theoretical implications of the demand functions developed in the previous chapter, the transformation (5.1) was employed to generalise the functional form of the demand for money. This, in fact, is becoming an increasingly common procedure in studies of the demand for money function since the seminal paper by Zarembka (1968); see, e.g. previous studies of the U.K. demand for money by the present author, Mills (1975, 1978) and Mills and Wood (1977).

Thus, the analysis of functional form using the Box and Cox (1964) power transformation, extended to take into account serial correlation, is developed in section 5.2, and the resultant modelling of the alternative demand for money functions is presented in section 5.3, with conclusions being drawn in section 5.4.

5.2 FUNCTIONAL FORM ESTIMATION AND SERIAL CORRELATION

The method of using the power transformation (5.1) to determine functional form in a regression model is well known, and now appears frequently in textbooks (see, e.g. Kmenta (1971, ch.11.3)). Spitner

(1976) has considered the most general case in which different power transformations are used on each variable in the model.

Recently, Savin and White (1978) have extended the approach to jointly allow for the situation where the disturbances follow a first order autoregressive process. This extension is particularly useful in the present context and will be developed here.

Thus, consider the following model,

$$y^{(\eta_0)} = Z^{(\underset{\sim}{\eta})}\beta + u \tag{5.2}$$

where $y^{(\eta_0)} = [y_1^{(\eta_0)}, \ldots y_T^{(\eta_0)}]$ is a T x 1 vector of transformed observations on the dependent variable y, $Z^{(\underset{\sim}{\eta})} = [z_1^{(\eta_1)} \ldots z_k^{(\eta_k)}]$ is a T x k matrix of transformed observations on the k regressors, with $z_i^{(\eta_i)} = [z_{i1}^{(\eta_i)} \ldots z_{iT}^{(\eta_i)}]$, and $u = [u_1 .. u_T]$ is a T x 1 vector of errors. It is assumed that these errors follow a stationary AR(1) process

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad |\rho| < 1 \tag{5.3}$$

where the ε_t's are independent $N(0, \sigma^2)$ random variables.

For the model (5.2) in conjunction with (5.3) the loglikelihood function is

$$\begin{aligned} L(\eta_0, \underset{\sim}{\eta}, \rho, \beta, \sigma^2;\ y, Z) = {} & -\tfrac{T}{2}\ln(2\pi\sigma^2) + \tfrac{1}{2}\ln(1-\rho^2) \\ & - \frac{1}{2\sigma^2}(y^{(\eta_0)} - Z^{(\underset{\sim}{\eta})}\beta)'V^{-1}(y^{(\eta_0)} - Z^{(\underset{\sim}{\eta})}\beta) + \ln J \end{aligned} \tag{5.4}$$

where V^{-1} is the variance-covariance matrix of u,

$$V^{-1} = \begin{bmatrix} 1 & -\rho & 0 & \dots & 0 & 0 \\ -\rho & 1+\rho^2 & \rho & \dots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \dots & 0 & 0 \\ \vdots & & & & \vdots & \vdots \\ & & & & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix} \tag{5.5}$$

and J is the Jacobian of the transformation on the dependent variable,

$$J = \det \left[\frac{\partial y_t^{(\eta_0)}}{\partial y_t} \right] = \prod_{t=1}^{T} y_t^{\eta_0 - 1} \tag{5.6}$$

since

$$\left[\frac{\partial y_t^{(\eta_0)}}{\partial y_t} \right] = \begin{bmatrix} y^{\eta_0-1} & 0 & 0 & \dots & 0 \\ \rho y^{\eta_0-1} & y^{\eta_0-1} & 0 & \dots & 0 \\ \rho^2 y^{\eta_0-1} & \rho y^{\eta_0-1} & y^{\eta_0-1} & \dots & 0 \\ \vdots & \vdots & \vdots & & \\ \rho^{T-1} y^{\eta_0-1} & \rho^{T-2} y^{\eta_0-1} & \rho^{T-3} y^{\eta_0-1} & \dots & y^{\eta_0-1} \end{bmatrix} \tag{5.7}$$

Maximising (5.4) with respect to β and σ^2 given $\eta_0, \underset{\sim}{\eta}$ and ρ yields the estimators

$$\hat{\beta}(\eta_0, \underset{\sim}{\eta}, \rho) = (Z^{(\underset{\sim}{\eta})'} V^{-1} Z^{(\underset{\sim}{\eta})})^{-1} Z^{(\underset{\sim}{\eta})'} V^{-1} y^{(\eta_0)} \tag{5.8}$$

$$\hat{\sigma}^2(\eta_0, \underset{\sim}{\eta}, \rho) = T^{-1}(y^{(\eta_0)} - Z^{(\underset{\sim}{\eta})}\hat{\beta}(\eta_0, \underset{\sim}{\eta}, \rho))' V^{-1} (y^{(\eta_0)} \; Z^{(\underset{\sim}{\eta})}\hat{\beta}(\eta_0, \underset{\sim}{\eta}, \rho)) \tag{5.9}$$

Substituting (5.8) and (5.9) into (5.4) obtains the concentrated likelihood function,

$$L(\eta_0, \underset{\sim}{\eta}, \rho; y, Z) = -\frac{T}{2}(\ln(2\pi) + 1) - \frac{T}{2}\ln(\hat{\sigma}^2(\eta_0, \underset{\sim}{\eta}, \rho) + \frac{1}{2}\ln(1 - \rho^2)$$
$$+ (\eta_0 - 1) \sum_{t=1}^{T} \ln y_t \tag{5.10}$$

Maximum likelihood (ML) estimators, $\hat{\eta}_0$, $\hat{\eta}$ and $\hat{\rho}$, say, can be found by maximising (5.10) over η_0, $\underset{\sim}{\eta}$ and ρ, where the parameter space for ρ is restricted to the open interval (-1, 1). The ML estimators for β and σ^2 can now be obtained from (5.8) and (5.9) as $\hat{\beta} = \hat{\beta}(\hat{\eta}_0, \hat{\underset{\sim}{\eta}}, \hat{\rho})$ and $\hat{\sigma}^2 = \hat{\sigma}^2(\hat{\eta}_0, \hat{\underset{\sim}{\eta}}, \hat{\rho})$ respectively.

Of course, if ρ is set equal to zero a priori then the conventional Box and Cox analysis results, with $V^{-1} = I$ and $\ln(1 - \rho^2) = 0$.

In the model (5.2) the regression coefficients β_i are not of particular interest, apart from the precision with which they are estimated, since they apply to the transformed variables $z_i^{(\eta_i)}$ and not to the original variables z_i. Hence it is not meaningful to compare the estimated coefficients of models with different values of η_0 and $\underset{\sim}{\eta}$. However, estimated elasticities may be compared and, at any time t, these are given by

$$\lambda_{it} = \frac{\partial y_t}{\partial z_{it}} \frac{z_{it}^{\eta_i}}{y_t^{\eta_0}} \tag{5.11}$$

which, if there are no lagged values of $y^{(\eta_0)}$ contained in $Z^{(\eta)}$, becomes

$$\lambda_{it} = \beta_i \frac{z_{it}^{\eta_i}}{y_t^{\eta_0}} \tag{5.12}$$

or, if the first r variables in $Z^{(\eta)}$ are lagged values of $y^{(\eta_0)}$,

$$\lambda_{it} = \left[\frac{\beta_i}{1 - \sum_{j=1}^{r} \beta_j}\right] \cdot \frac{z_{it}^{\eta_i}}{y_t^{\eta_0}} \tag{5.13}$$

and, hence, except when $\eta_0 = \eta_i = 0$ (the logarithmic form), these elasticities are variable. It is often convenient to obtain average elasticities, calculated at the sample means of y and z_i, e.g.

$$\bar{\lambda}_i = \frac{\beta_i}{1 - \sum_{j=1}^{r} \beta_j} \cdot \frac{\bar{z}_i^{\eta_i}}{\bar{y}^{\eta_0}} \tag{5.14}$$

Inferences concerning the parameters of the model may be made on the basis of likelihood ratio (LR) tests. Such tests and associated confidence regions for the values of η_0, η and ρ may be constructed using the result that if Ω denotes the parameter space under the maintained hypothesis and ω denotes the space Ω restricted by the null hypothesis H_0, then a large sample LR test of H_0 is given by

$$2[L(\hat{\Omega}) - L(\hat{\omega})] \sim \chi^2(r)$$

where $L(\hat{\Omega})$ is the maximum of the likelihood function over Ω and $L(\hat{\omega})$ is the maximum of the likelihood function over ω, with r being the number of additional restrictions imposed by H_0.

Thus, a $100(1 - \alpha)\%$ confidence region for (η_0, η, ρ) is given by

$$2[L(\hat{\eta}_0, \hat{\underset{\sim}{\eta}}, \hat{\rho}) - L(\eta_0, \underset{\sim}{\eta}, \rho)] < \chi^2_\alpha(k + 2)$$

where $\chi^2_\alpha(k + 2)$ is the upper α significance point of χ^2 with k + 2 degrees of freedom. Correspondingly the critical region of the test of H_0: $\eta_0 = \eta_0^*, \underset{\sim}{\eta} = \underset{\sim}{\eta}^*, \rho = \rho^*$ against H_1: $\eta_0 \neq \eta_0^*, \underset{\sim}{\eta} \neq \underset{\sim}{\eta}^*, \rho \neq \rho^*$ is

$$2[L(\hat{\eta}_0, \hat{\eta}, \hat{\rho}) - L(\dot{\eta}, \underset{\sim}{\eta}, \rho)] > \chi^2_\alpha(k + 2)$$

Of the numerous possible conditional hypothesis tests the following would seem to be the most useful,

(i) $H_0\colon \eta_0 = \eta_0^*, \underset{\sim}{\eta} = \underset{\sim}{\eta}^*/\rho = \rho^*$ against $H_1\colon \eta_0 \neq \eta_0^*, \underset{\sim}{\eta} \neq \underset{\sim}{\eta}^*/\rho = \rho^*$

$$2[L(\hat{\eta}_0, \hat{\eta}, \rho^*) - L(\eta_0^*, \underset{\sim}{\eta}^*, \rho^*)] > \chi^2_\alpha(k + 1)$$

(ii) $H_0\colon \rho = \rho^*/\eta_0 = \eta_0^*, \underset{\sim}{\eta} = \underset{\sim}{\eta}^*$ against $H_1\colon \rho \neq \rho^*/\eta_0 = \eta_0^*, \underset{\sim}{\eta} = \underset{\sim}{\eta}^*$

$$2[L(\eta_0^*, \underset{\sim}{\eta}^*, \hat{\rho}) - L(\eta_0^*, \underset{\sim}{\eta}^*, \rho^*)] > \chi^2_\alpha(1).$$

and also

(iii) $H_0\colon \eta_0 = \eta_0^*, \underset{\sim}{\eta} = \underset{\sim}{\eta}^*$ against $H_1\colon \eta_0 \neq \eta^*, \underset{\sim}{\eta} \neq \underset{\sim}{\eta}^*$

$$2[L(\hat{\eta}_0, \hat{\underset{\sim}{\eta}}, \hat{\rho}) - L(\eta_0^*, \underset{\sim}{\eta}^*, \hat{\rho})] > \chi^2_\alpha(k + 1)$$

Thus having developed both an estimation and hypothesis testing framework for analysing functional form, this can now be employed on the alternative specifications of the demand for money function.

5.3 FUNCTIONAL FORM IN THE DEMAND FOR MONEY

The demand for money functions analysed in chapter four were generalised into the form of equation (5.2) in the following way. As it would be impractical to repeat the dynamic specification searches in conjunction with estimating functional form, the regressors entering into the finally chosen logarithmic model, without the imposition of any additional linear restrictions, were taken to be the variables comprising the z matrix for each specification. As the dependent variable y will be either MN or MB, the Z matrices will therefore comprise of lagged values of the particular money series, a constant, and contemporaneous and lagged values of Y, P, F, W, V and the appropriate interest rate RC or RL.

Two further simplifications were utilised. For three of the demand for money specifications, the finally accepted logarithmic models contained no autoregressive error, and this information was incorporated by imposing the restriction $\rho = 0$ a priori on the error process (5.3), so that for these specifications the analysis proceeds in the conventional manner of, e.g. Spitzer (1976). Secondly, previous modelling of the demand for money relationship by the author, Mills (1975, 1978), had found strong support for the restriction $\eta_i = \eta$ for all i, i.e. the same transformation can be used on each variable. In view of the simpler computational requirements this restriction was also imposed a priori, and hence models of the form

$$y_t^{(\eta)} = z_t^{(\eta)}\beta + u_t \tag{5.15}$$

were actually estimated. Such a restriction, although allowing easier comparison of, say, the logarithmic form (characterised by $\eta = 0$) and the linear form ($\eta = 1$), rules out the possibility of a different transformation on the interest rate, which may be theoretically desirable. Under this restriction the likelihood ratio test statistics for the most important null hypotheses become

(i) If $\rho = 0$ in the maintained hypothesis, then a $100(1 - \alpha)\%$ confidence interval for η is

$$2[L(\hat{\eta}, 0) - L(\eta, 0)] < \chi^2_\alpha(1).$$

(ii) A general $100(1 - \alpha)\%$ confidence region for η and ρ is

$$2[L(\hat{\eta}, \hat{\rho}) - L(\eta, \rho)] < \chi^2_\alpha(2)$$

(iii) $H_0: \eta = 0/\rho = 0$ against $H_1: \eta \neq 0/\rho = 0$

$$C(\eta) = 2[L(\hat{\eta}, 0) - L(0, 0)] > \chi^2_\alpha(1)$$

(iv) $H_0: \rho = 0/\eta = 0$ against $H_1: \rho \neq 0/\eta = 0$

$$C(\rho) = 2[L(0, \hat{\rho}) - L(0, 0)] > \chi^2_\alpha(1)$$

(v) $H_0: \eta = 0$ against $H_1: \eta \neq 0$

$$G(\eta) = 2[L(\hat{\eta}, \hat{\rho}) - L(0, \hat{\rho})] > \chi^2_\alpha(1)$$

(vi) $H_0: \rho = 0$ against $H_1: \rho \neq 0$

$$G(\rho) = 2[L(\hat{\eta}, \hat{\rho}) - L(\hat{\eta}, 0)] > \chi^2_\alpha(1)$$

(vii) $H_0: \eta = \rho = 0$ against $H_1: \eta \neq 0, \rho \neq 0$

$$J(\eta, \rho) = 2[L(\hat{\eta}, \hat{\rho}) - L(0, 0)] > \chi^2_\alpha(2)$$

Before discussing the individual models a final point should be noted. For the analysis the untransformed, "original", data series are required. However, as the logarithmically transformed series have been seasonally adjusted, apart from the interest rates, these adjusted series were antilogged to obtain the actual "untransformed" variables used in the subsequent analysis. In the discussion of the models the "untransformed" counterpart of the logarithmic variable X is denoted x.

Specification 1: $\underset{\sim}{X}^*_1$

For this specification the variables employed were taken as the untransformed counterparts of those appearing in equation (4.14). Thus the model (5.2) becomes

$$mn_t^{(\eta)} = Z_{1t}^{(\eta)}\beta + u_t \qquad (5.16)$$

where $Z_{1t}^{(\eta)} = [1,\ mn_{t-1},\ y_t,\ y_{t-1},\ rc_t,\ rc_{t-1},\ rc_{t-2},\ p_{t-1},\ p_{t-2},\ w_t,\ w_{t-1}]$

As no evidence of serial correlation was found in (4.14), ρ in (5.3) was set equal to zero a priori, u_t therefore being assumed to be distributed as $N(0, \sigma^2)$.

Maximum likelihood estimation of (5.16) obtained a transformation parameter estimate of $\hat{\eta} = -.74$. A plot of the loglikelihood function is shown in Figure 5.1, from which a 95% confidence interval for η is obtained as

$$-1.22 < \eta < -.08$$

This interval does not contain the logarithmic functional form, the corresponding LR test statistic being $C(\eta) = 25.8$, and thus equation (4.14) may be rejected in favour of the following equation containing the ML estimates of the β vector,

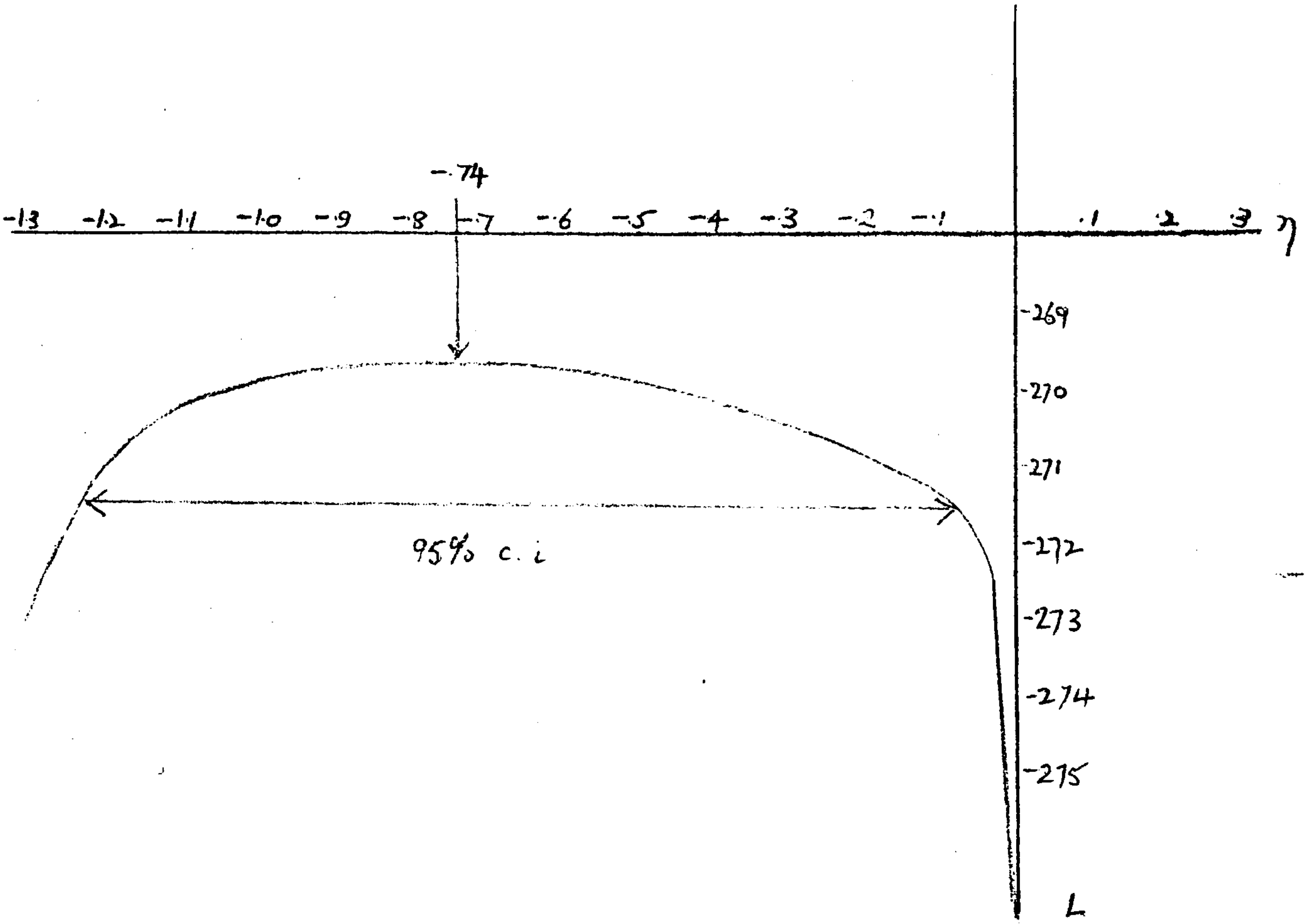
$$\begin{aligned}
mn_t^{(-.74)} = &\underset{(.170)}{-.191} + \underset{(.097)}{.508} mn_{t-1}^{(-.74)} + \underset{(.113)}{.325} y_t^{(-.74)} + \underset{(.115)}{.305} y_{t-1}^{(-.74)} \\
&\underset{(.043\times10^{-4})}{-.139\times10^{-4}} rc_t^{(-.74)} - \underset{(.052\times10^{-4})}{.126\times10^{-4}} rc_{t-1}^{(-.74)} - \underset{(.052\times10^{-4})}{.168\times10^{-4}} rc_{t-2}^{(-.74)} \\
&\underset{(.246\times10^{-3})}{+.866\times10^{-3}} p_{t-1}^{(-.74)} - \underset{(.186\times10^{-3})}{.406\times10^{-3}} p_{t-2}^{(-.74)} + \underset{(.005)}{.017} w_t^{(-.74)} - \underset{(.007)}{.014} w_{t-1}^{(-.74)}
\end{aligned}$$

$d_1 = 2.03,\quad h_1 = .16,\quad L = -269.69,\quad F_c(11,34) = 2.01$

(5.17)

FIGURE 5.1

LOGLIKELIHOOD PLOT FOR SPECIFICATION $\underset{\sim}{x}_1^*$



The h_1 statistic confirms the absence of first order autocorrelation and all coefficient estimates are significant. Of greater importance is the result of the test for structural stability, which yields an insignificant F_c statistic in contrast with the corresponding test on the logarithmic functional form. It may therefore be concluded that generalising the functional form not only rejects the logarithmic form but obtains a functional form in which the coefficients are structurally stable. Given the coefficient estimates of (5.17) the following restrictions are suggested; combining $y_t^{(-.74)}$ and $y_{t-1}^{(-.74)}$, combining $rc_t^{(-.74)}$, $rc_{t-1}^{(-.74)}$ and $rc_{t-2}^{(-.74)}$ and forming $\Delta w_t^{(-.74)} = w_t^{(-.74)} - w_{t-1}^{(-.74)}$. On imposing these restrictions, re-estimation of (5.17) led to

$$
\begin{aligned}
mn_t^{(-.74)} = {} & \underset{(.101)}{-.266} + \underset{(.067)}{.556}\, mn_{t-1}^{(-.74)} + \underset{(.051)}{.320}(y_t^{(-.74)} + y_{t-1}^{(-.74)}) \\
& \underset{(.019 \times 10^{-4})}{-.131 \times 10^{-4}}(rc_t^{(-.74)} + rc_{t-1}^{(-.74)} + rc_{t-2}^{(-.74)}) \\
& \underset{(.160\times10^{-3})}{+.931 \times 10^{-3}}\, p_{t-1}^{(-.74)} - \underset{(.165\times10^{-3})}{.409 \times 10^{-3}}\, p_{t-2}^{(-.74)} \qquad (5.18) \\
& \underset{(.004)}{+.017}\Delta w_t^{(-.74)}
\end{aligned}
$$

$L = -270.19, \quad \chi^2(4) = 1.00,$

$\bar{\lambda}_1 = 1.50, \ \bar{\lambda}_2 = -.50, \quad \bar{\lambda}_3 = .99, \quad \bar{\lambda}_6 = 0.$

The restrictions are accepted by a LR test, and from the calculated average elasticities the following features emerge. The average income elasticity is well in excess of unity, while the function is on average interest inelastic. The average price elasticity is almost exactly unity while, as a consequence of the imposed differencing restriction, there is a zero long run wage elasticity.

Thus generalising the functional form has led to a stable demand for narrow money function incorporating a long term interest rate with interesting and theoretically plausible long run properties.

Specification 2: $\underset{\sim}{X}^*_2$

For this specification evidence has been found of simultaneity between MN and RL. While simultaneity can be handled within the present framework (see Spitzer (1977)), the evidence from the logarithmic functional form search suggests that ignoring it in the present case does not significantly affect the coefficient estimates. Consequently such simultaneity was again ignored and the analysis of the preceding section performed.

The variables employed were taken as those appearing in equation (4.27) and thus the regressor matrix becomes

$$Z_{2t} = [1, mn_{t-1}, mn_{t-2}, y_{t-1}, rl_t, p_t, p_{t-1}, w_t]$$

and as a first order autoregressive error had been identified in the logarithmic form the general model of (5.2) accompanied by (5.3) was estimated. ML estimation obtained the estimates $\hat{\eta} = -.20$ and $\hat{\rho} = .42$, yielding the estimated functional form

$$\begin{aligned} mn_t^{(-.20)} = {} & \underset{(.377)}{.323} + \underset{(.087)}{.273}mn_{t-1}^{(-.20)} + \underset{(.091)}{.347}mn_{t-2}^{(-.20)} + \underset{(.092)}{.211}y_{t-1}^{(-.20)} - \underset{(.0011)}{.0072}rl_t^{(-.20)} \\ & - \underset{(.021)}{.062}p_t^{(-.20)} + \underset{(.022)}{.058}p_{t-1}^{(-.20)} + \underset{(.042)}{.124}w_t^{(-.20)} + .42\hat{u}_{t-1} \end{aligned}$$

$$L = -270.65 \qquad (5.19)$$

Inferences on η and ρ may be made on the basis of the loglikelihoods obtained from the various restricted models, with associated LR test statistics, shown in Table 5.1

TABLE 5.1

η	ρ	L
-.20	.42	-270.65
-.21	0	-273.53
0	.42	-271.00
0	0	-274.03

$C(\eta) = 1.00, \quad C(\rho) = 6.06$

$G(\eta) = .70, \quad G(\rho) = 5.76$

$J(\eta, \rho) = 6.76$

From these test statistics it is clear that the most parsimonious model not rejected against (5.19) is that characterised by $\eta = 0$, $\rho = .42$, i.e. the model (4.27) found by the logarithmic specification search. Indeed, the restricted model(4.28), which yields a loglikelihood of -271.16, cannot be rejected against (5.19), a LR test giving a value of 1.02, which is distributed as $\chi^2(3)$. Thus the analysis has shown that in this case a logarithmic functional form with an autoregressive error is the appropriate functional form.

Specification 3: $\underset{\sim}{X}_3^*$

For this specification the variables were taken to be the untransformed counterparts of those appearing in equation (4.37). Thus the regressor matrix becomes

$$Z_{3t} = [1, mb_{t-1}, mb_{t-2}, y_t, y_{t-1}, rc_t, rc_{t-2}, p_{t-1}, p_{t-2}, f_t, w_t]$$

and as no autoregressive error had been identified in the logarithmic functional form ρ was set equal to zero a priori. ML estimation obtained $\hat{\eta} = -.29$ with the accompanying estimated functional form

$$\begin{aligned} mb_t^{(-.29)} = & \underset{(.186)}{-1.257} \underset{(.103)}{+1.264} mb_{t-1}^{(-.29)} \underset{(.107)}{-.316} mb_{t-2}^{(-.29)} \underset{(.072)}{+.227} y_t^{(-.29)} \underset{(.074)}{+.187} y_{t-1}^{(-.29)} \\ & \underset{(.052\text{x}10^{-2})}{-.212\text{x}10^{-2}} rc_t^{(-.29)} \underset{(.055\text{x}10^{-2})}{-.247\text{x}10^{-2}} rc_{t-2}^{(-.29)} \\ & \underset{(.008)}{+.017} p_{t-1}^{(-.29)} \underset{(.006)}{-.025} p_{t-2}^{(-.29)} \qquad (5.20) \\ & \underset{(.093\text{x}10^{-3})}{+.315\text{x}10^{-3}} f_t^{(-.29)} \underset{(.016)}{+.035} w_t^{(-.29)} \\ & L = -297.43 \end{aligned}$$

A plot of the loglikelihood function is shown in Figure 5.2, where it is seen that a 95% confidence interval for η is

$$-.71 < \eta < .12$$

As $\eta = 0$ is contained within this interval, the actual LR test statistic being $C(\eta) = 2.08$, the logarithmic functional form (4.37) cannot be rejected when compared to (5.20). Furthermore, the restricted model

(4.38), which yields a loglikelihood of -299.01, also cannot be rejected against (5.20), the LR test statistic being 3.16 distributed as $\chi^2(4)$.

Thus it may be concluded that for this specification the logarithmic functional form is again appropriate.

Specification 4: $\underset{\sim}{X}^*_4$

For this specification the variables were taken to be those appearing in equation (4.45), the regressor matrix becoming

$$Z_{4t} = [1,\ mb_{t-1},\ y_{t-1},\ rl_t,\ p_t,\ f_t,\ v_{t-2}]$$

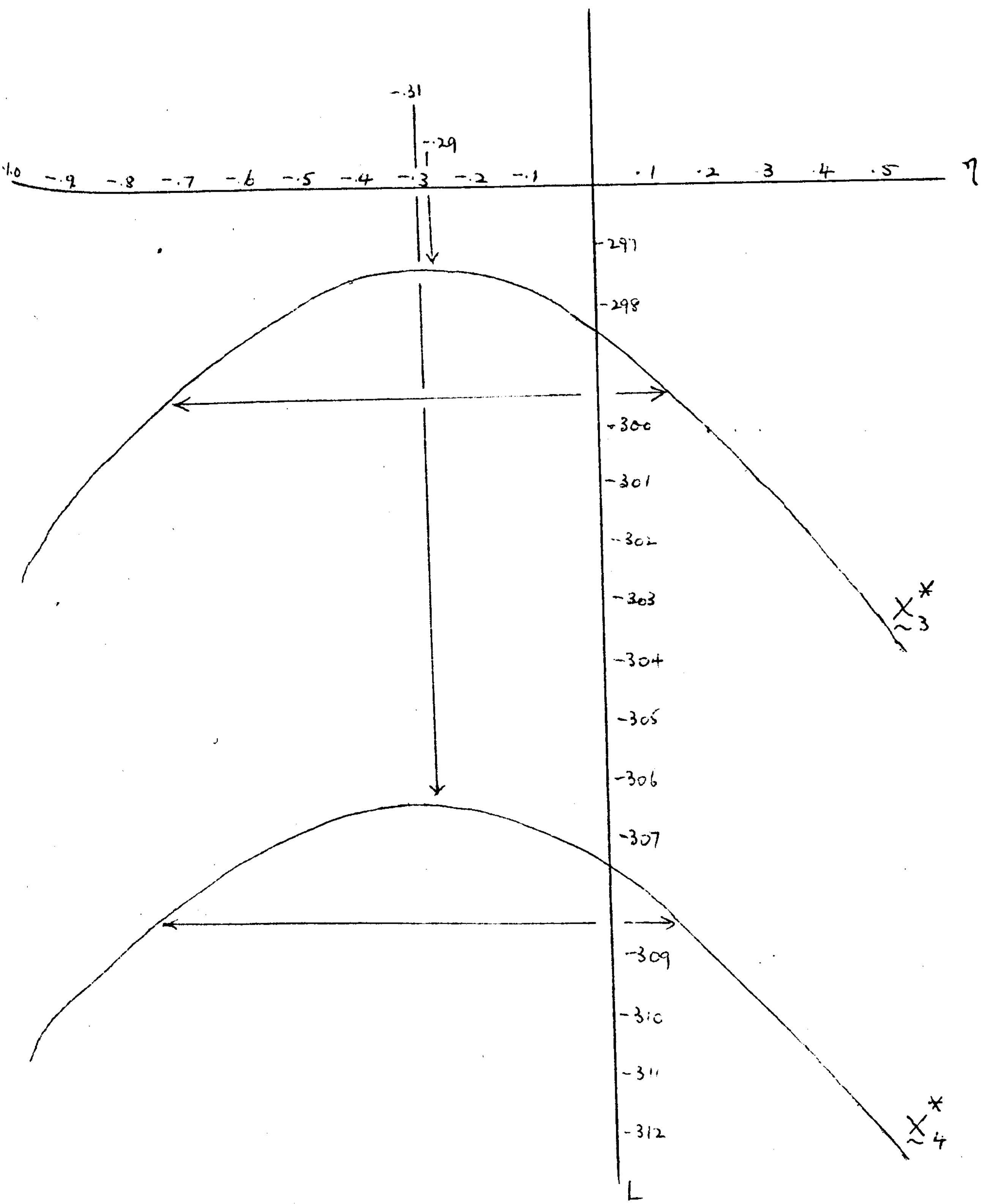
Again, as no autoregressive error had been previously identified ρ was set equal to zero a priori. ML estimation obtained $\hat{\eta} = -.31$ with the accompanying estimated functional form

$$\boxed{\begin{aligned} mb_t^{(-.31)} = {} & \underset{(.112)}{-.718} + \underset{(.036)}{1.112}mb_{t-1}^{(-.31)} + \underset{(.054)}{.121}y_{t-1}^{(-.31)} - \underset{(.178\text{x}10^{-3})}{.872\text{x}10^{-3}}\ rl_t^{(-.31)} \\ & \underset{(.168\text{x}10^{-2})}{-.533\text{x}10^{-2}}p_t^{(-.31)} + \underset{(.073\text{x}10^{-3})}{.128\text{x}10^{-3}}f_t^{(-.31)} - \underset{(.077\text{x}10^{-2})}{.336\text{x}10^{-2}}\ v_{t-2}^{(-.31)} \\ & L = -.306.52, \quad d_1 = 1.83, \quad h_1 = .66, \quad \mu_1 = 1.11 \end{aligned}} \tag{5.21}$$

The h_1 statistic confirms the absence of first order autocorrelation but generalising the functional form has not eradicated the problem of dynamic instability. This is not surprising as the 95% confidence interval for η obtained from the plot of the likelihood function shown in Figure 5.2 is

FIGURE 5.2

LOGLIKELIHOOD PLOTS FOR SPECIFICATIONS $\underset{\sim}{X}_3^*$ AND $\underset{\sim}{X}_4^*$



$$-.77 < \eta < .12$$

This interval, almost identical to that for the $\underset{\sim}{X}^*_3$ specification, contains $\eta = 0$ and, once again, the logarithmic functional form cannot be rejected by (5.21). The actual LR test statistic is $C(\eta) = 2.14$.

Again it may be concluded that the logarithmic form is the most appropriate functional form.

5.4 CONCLUSIONS

Generalising the functional form of the demand for money function has had somewhat mixed consequences. For three of the four specifications the logarithmic functional form has been found to be the most appropriate, thus confirming the original specifications of the previous chapter. Unfortunately, such a finding is rather disappointing in the case of the two broad money specifications, as the logarithmic functional forms do not possess particularly desirable thoeretical properties. Indeed, the major problem of dynamic instability does seem to be robust to functional form.

For the narrow money function for which the logarithmic functional form was rejected the generalisation has had desirable consequences, for the generalised function is now structurally stable. This particular question of why generalising the functional form has induced structural stability is considered further in the final chapter, where an attempt is made to determine the best functions with which to model the demand for narrow and broad money balances and to analyse the properties of these models in greater detail.

CHAPTER SIX :

FINAL COMPARISONS AND CONCLUSIONS

6.1 COMPARISON OF SELECTED DEMAND FOR MONEY FUNCTIONS

The analysis of the previous two chapters has enabled appropriate demand for money functions to be selected for the four alternative variable specifications. To assist in comparisons the estimated functions are set out in Table 6.1.

Although the two narrow money functions yield almost identical likelihoods, their parameter estimates imply very different long run elasticities. The constant elasticity function of the $\underset{\sim}{X}_2^*$ specification is income, interest and wage inelastic and has a zero long run price elasticity. However, the impact effect of price changes on the level of narrow money balances is negative and as approximately

$$\Delta P_t \simeq \ln \left(\frac{p_t - p_{t-1}}{p_{t-1}} \right)$$

this has the implication that the level of narrow money balances is negatively related to the rate of inflation, suggesting that aggregate transactions balances are reduced as the value of money falls.

In direct contrast, the variable elasticity function of the $\underset{\sim}{X}_1^*$ specification yields reasonably stable interest and price elasticities of around -0.5 and unity respectively and an income elasticity greater than unity and increasing over the sample period. These elasticities are plotted in Figure 6.1. For this function the long run wage elasticity is zero, but the positive impact effect of wage changes implies that the level of narrow money balances is positively related to "wage inflation".

Comparing the implications of the two functions one is immediately confronted with some curious differences. The finding of a short interest rate elasticity larger than the long rate elasticity does in fact confirm previous research and is consistent with the conventional explanation that this is because variations in short rates are in general greater than those in long rates (see Goodhart and Crockett (1970) and Table 1.1). However, the large difference in income elasticities is difficult to explain. Certainly the constant elasticity of below unity of the $\underset{\sim}{X}_2^*$ specification, implying economies of scale in money holding, is consistent with the majority of published research whereas the increasing income elasticity of the $\underset{\sim}{X}_1^*$ specification appears to be quite anomolous within advanced economies (see Laidler (1977, p.148-9)). The behaviour of prices and wages in the two functions is also curious, prices having an elasticity of almost unity and wages a zero elasticity when a long interest rate is employed, with these magnitudes being reversed on the inclusion of a short interest rate rather than the long rate. Also curious is the negative impact effect of inflation in this latter function.

The almost identical likelihoods yielded by the two functions suggests that discrimination between them may well be difficult, particularly as they are non nested. Peseran and Deaton (1978) have developed an elegant procedure for discriminating between such models but its computational complexity was felt to preclude its use here. However, from the behaviour of the long run elasticities it would appear that the construction of a composite model including both interest rates may allow for discrimination between the "simple" models and also resolve the interpretive difficulties with the elasticities. Indeed, it is quite conceivable

TABLE 6.1

Specification $\underset{\sim}{X}_1^*$: Equation (5.18)

$$\hat{mn}_t^{(-.74)} = -.266 + .556mn_{t-1}^{(-.74)} + .320(y_t^{(-.74)} + y_{t-1}^{(-.74)})$$
$$\quad (.101) \quad (.067) \qquad (.051)$$
$$-.131\text{x}10^{-4}(rc_t^{(-.74)} + rc_{t-1}^{(-.74)} + rc_{t-2}^{(-.74)})$$
$$(.019\text{x}10^{-4})$$
$$+.931\text{x}10^{-3}p_{t-1}^{(-.74)} - .409\text{x}10^{-3}p_{t-2}^{(-.74)}$$
$$(.160\text{x}10^{-3}) \qquad (.165\text{x}10^{-3})$$
$$+.017\Delta w_t^{(-.74)}$$
$$(.004)$$

$L = -270.19$, AIC = 556.38

$\lambda_1 = 1.50$, $\lambda_2 = -.50$, $\lambda_3 = .99$, $\lambda_6 = 0$, $\mu = .44$

Specification $\underset{\sim}{X}_2^*$: Equation (4.28)

$$\hat{MN}_t = .265MN_{t-1} + .347MN_{t-2} + .219Y_{t-1} - .077RL_t - .378\Delta P_t$$
$$(.080) \qquad (.084) \qquad (.043) \qquad (.010) \qquad (.112)$$
$$+.308W_t + .444\hat{u}_{t-1}$$
$$(.038) \qquad (.120)$$

$L = -271.16$, AIC = 556.32

$\lambda_1 = .56$, $\lambda_2 = -.20$, $\lambda_3 = 0$, $\lambda_6 = .79$, $\mu_1 = .67$, $\mu_2 = -.33$.

Specification $\underset{\sim}{X}_3^*$: Equation (4.38)

$$\Delta\hat{MB}_t = \underset{(.644)}{-5.368} + \underset{(.095)}{.287}\Delta MB_{t-1} + \underset{(.033)}{.238}(Y_t + Y_{t-1})$$

$$\underset{(.012)}{-.089}(RC_t + RC_{t-2}) + \underset{(.122)}{.278}P_{t-1} - \underset{(.098)}{.439}P_{t-2}$$

$$\underset{(.006)}{+.020}F_t + \underset{(.064)}{.139}W_t$$

$L = -299.01$, $AIC = 614.02$, $\mu_1 = 1$, $\mu_2 = .29$.

Specification $\underset{\sim}{X}_4^*$: Equation (4.45)

$$\hat{MB}_t = \underset{(.415)}{-2.927} + \underset{(.035)}{1.132}MB_{t-1} + \underset{(.061)}{.161}Y_{t-1} - \underset{(.009)}{.045}RL_t$$

$$\underset{(.034)}{-.126}P_t + \underset{(.006)}{.011}F_t - \underset{(.023)}{.104}V_{t-2}$$

$L = -307.59$, $AIC = 629.18$, $\mu_1 = 1.13$.

MN/RC ELASTICITIES

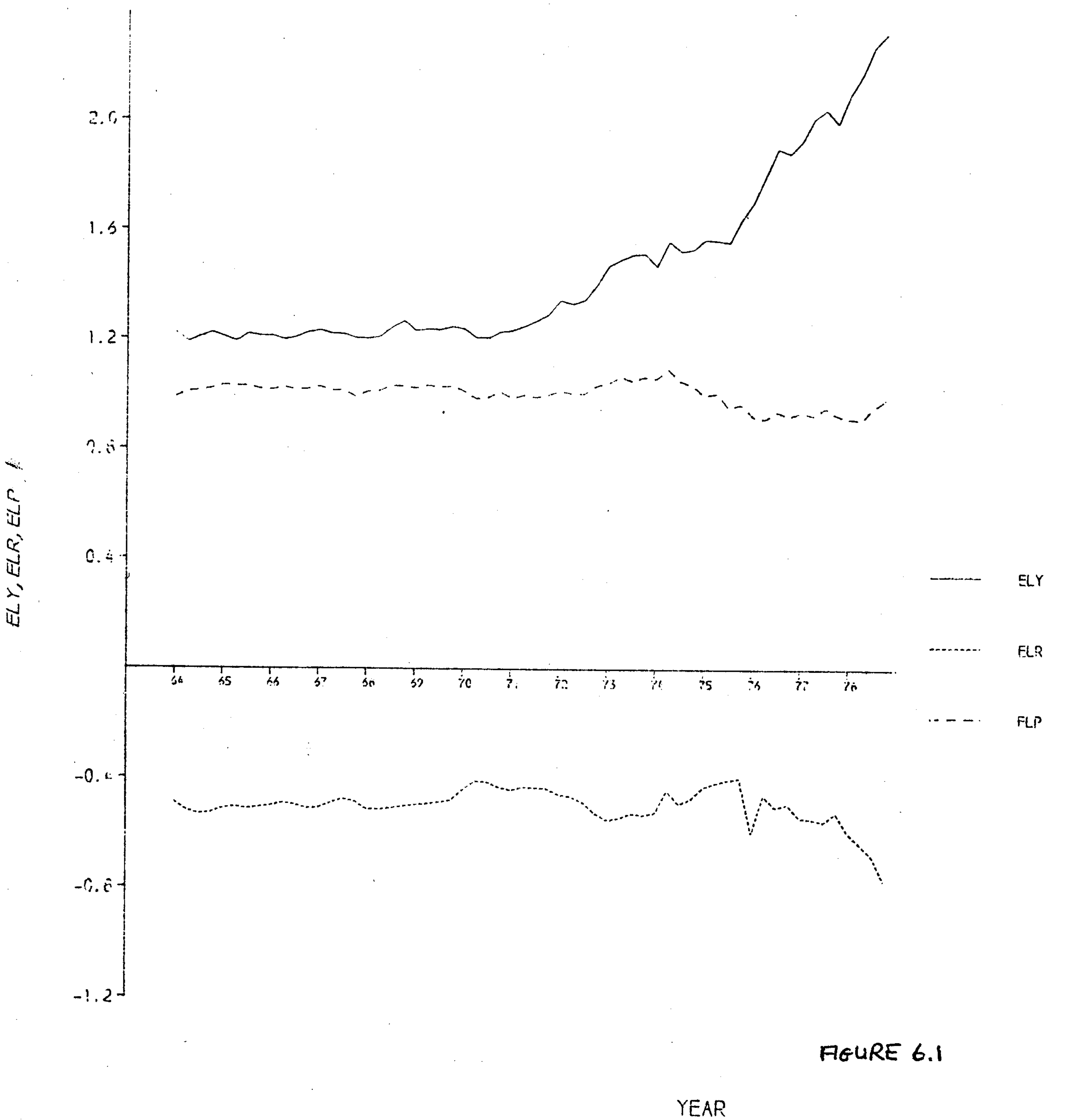


FIGURE 6.1

that both the selected functions may be rejected in favour of the composite model. Although the construction of such a model to discriminate between non nested models has been criticised in general terms by Peseran (1974) it is, in fact, feasible here and has the additional benefit of enabling the importance of the alternative interest rates to be determined jointly.

A composite model comprising both equations (4.28) and (5.18) as nested models is

$$\begin{aligned} mn_t^{(n)} = {} & \beta_0 + \beta_1 mn_{t-1}^{(n)} + \beta_2 mn_{t-2}^{(n)} + \beta_3 mn_{t-3}^{(n)} \\ & + \beta_4 y_t^{(n)} + \beta_5 y_{t-1}^{(n)} + \beta_6 y_{t-2}^{(n)} \\ & + \beta_7 p_t^{(n)} + \beta_8 p_{t-1}^{(n)} + \beta_9 p_{t-2}^{(n)} \\ & + \beta_{10} rc_t^{(n)} + \beta_{11} rc_{t-1}^{(n)} + \beta_{12} rc_{t-2}^{(n)} \\ & + \beta_{13} rl_t^{(n)} + \beta_{14} rl_{t-1}^{(n)} + \beta_{15} w_t^{(n)} + \beta_{16} w_{t-1}^{(n)} + u_t \end{aligned} \tag{6.1}$$

Equation (4.28), which may be written in general terms as

$$\begin{aligned} MN_t = {} & \alpha_1 MN_{t-1} + \alpha_2 MN_{t-2} + \alpha_3 Y_{t-1} + \alpha_4 RL_t \\ & + \alpha_5 (P_t - P_{t-1}) + \alpha_6 w_t + u_t \end{aligned} \tag{6.2}$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

may be obtained from equation (6.1) by placing the restrictions

$$\beta_0 = \beta_4 = \beta_{10} = \beta_{11} = \beta_{12} = \eta = 0$$

$$\beta_5\beta_{14}\beta_{15} = \beta_5\beta_{15}\beta_{16} = \beta_6\beta_{13}\beta_{15} = 0 \qquad (6.3)$$

$$\beta_7 + \beta_8 + \beta_9 = 0$$

$$\rho^3 - \beta_1\rho^2 - \beta_2\rho - \beta_3 = 0$$

where $|\rho| < 1$.

A test of the restriction to the nested model (6.2) can be constructed using the likelihood ratio approach outlined in Chapter five. Thus if Ω_1 denotes the parameter space for the maintained hypothesis (6.1) and ω_1 denotes the space Ω_1 restricted by the null hypothesis (6.3) then

$$2[L(\hat{\Omega}_1) - L(\hat{\omega}_1)] \sim \chi^2(11)$$

where $L(\hat{\Omega}_1)$ is the maximum loglikelihood of (6.1) and $L(\hat{\omega}_1)$ is the maximum loglikelihood of (6.2), there being 11 restrictions implicit in the set (6.3).

Similarly, equation (5.18) can be obtained by placing the restrictions

$$\beta_2 = \beta_3 = \beta_6 = \beta_7 = \beta_{13} = \beta_{14} = 0$$

$$\beta_{10} = \beta_{11} = \beta_{12}$$

$$\beta_4 - \beta_5 = 0 \qquad (6.4)$$

$$\beta_{13} + \beta_{16} = 0$$

Denoting as ω_2 the space Ω_1 restricted by the null hypothesis (6.4) then a test of the restriction to the model

$$mn_t^{(\eta)} = \gamma_0 + \gamma_1 mn_{t-1}^{(\eta)} + \gamma_2(y_t^{(\eta)}+y_{t-1}^{(\eta)}) + \gamma_3(rc_t^{(\eta)}+rc_{t-1}^{(\eta)}+rc_{t-2}^{(\eta)})$$

$$+ \gamma_4 p_{t-1}^{(\eta)} + \gamma_5 p_{t-2}^{(\eta)} + \gamma_6(w_t^{(\eta)} - w_{t-1}^{(\eta)}) + u_t \qquad (6.5)$$

is given by

$$2[L(\hat{\Omega}_1) - L(\hat{\omega}_2)] \sim \chi^2(10)$$

where $L(\hat{\omega}_2)$ is the maximum loglikelihood of (6.5) and there are 10 restrictions in the set (6.4).

Estimation of the composite model (6.1) by the methods of section 5.2 gave $L(\hat{\Omega}_1) = -260.68$ at the maximum likelihood estimate $\hat{\eta} = -.20$. As $L(\hat{\omega}_1) = -271.16$ and $L(\hat{\omega}_2) = -270.19$ we obtain the test statistics $\chi^2(11) = 20.96$ and $\chi^2(10) = 19.02$, both of which reject their respective null hypotheses at the .05 level of significance. As was intimated earlier, these tests imply rejection of both equations (6.2) and (6.5) in favour of a composite model containing both interest rates. However, a convenient simplification of (6.1) is obtained by considering the logarithmic functional form characterised by the null hypothesis ω_3, H_0: $\eta = 0$. Since $L(\hat{\omega}_3) = -260.91$ the associated likelihood ratio test yields the statistic $\chi^2(1) = .46$, thus accepting the restriction to the logarithmic functional form.

Estimation of this form of the composite model yielded

$$
\begin{aligned}
\hat{MN}_t = {} & -1.767 + .509MN_{t-1} + .235MN_{t-2} - .013MN_{t-3} \\
& \quad (.770) \quad (.142) \qquad\quad (.122) \qquad\quad (.112) \\
& +.145Y_t + .350Y_{t-1} - .131Y_{t-2} \\
& \quad (.115) \quad (.131) \qquad (.106) \\
& -.243P_t + .777P_{t-1} - .373P_{t-2} \\
& \quad (.171) \quad (.212) \qquad (.149) \\
& -.050RC_t - .005RC_{t-1} - .067RC_{t-2} \qquad\qquad (6.6) \\
& \quad (.016) \quad (.027) \qquad (.026) \\
& -.051RL_t + .017RL_{t-1} \\
& \quad (.017) \quad (.019) \\
& +.560W_t - .457W_{t-1} \\
& \quad (.176) \quad (.195)
\end{aligned}
$$

$S = .00532728 \qquad \sigma = .0117$

This equation is seen to be an autoregressive-distributed lag model in the vector $X^* = (MN, Y, P, RC, RL, W)$ of order AD(3, 2, 2, 2, 1, 1) and as such is amenable to the specification search procedure developed in chapter four. However, the insignificant coefficient of RL_{t-1} immediately precludes the extraction of an autoregressive factor, and deleting this variable, along with MN_{t-3}, Y_t, Y_{t-2}, P_t, RC_t and RC_{t-1}, and also combining P_{t-1} and P_{t-2} as $2P_{t-1} - P_{t-2} (= P_{t-1} + \Delta P_{t-1})$ and W_t and W_{t-1} as ΔW_t, yields on re-estimation

$$\hat{MN}_t = -1.013 + .465MN_{t-1} + .299MN_{t-2} + .310Y_{t-1}$$
$$(.486) \quad (.089) \quad (.090) \quad (.065)$$
$$+.242(P_{t-1} + \Delta P_{t-1}) - .065RC_{t-2}$$
$$(.040) \quad (.022)$$
$$-.053RL_t + .426\Delta W_t \tag{6.7}$$
$$(.008) \quad (.111)$$

$S = .00644923 \qquad \sigma = .0116 \qquad L = -266.26$

$d_1 = 1.94, \; h_1 = .30, \; Q(15) = 13.5, \; F_c(8,40) = 1.32, \; F(9,39) = .91$

$\lambda_Y = 1.31, \; \lambda_p = 1.03, \lambda_{RC} = -.28, \; \lambda_{RL} = -.23, \lambda_{\dot{W}} = 1.81, \; \mu_1 = .83,$
$\mu_2 = .36$

These nine restrictions are accepted and the model passes all diagnostic tests, with no significant residual correlations. Furthermore, the model is both dynamically and structurally stable and thus appears to be a satisfactory specification.

Both interest rates appear significantly, thus providing further evidence for Davidson et al's (1978) view that correct specification will resolve what often appears to be a problem of multicollinearity, the conventional argument for including only a single interest rate in the demand for money function. Both long run elasticities are of the same inelastic magnitude, with the long rate elasticity being now slightly larger (compare the conventional argument put forward earlier) but the short rate has a contemporaneous impact effect whereas the long rate's effect is lagged two periods, a reflection of the earlier finding of feedback between MN and RL.

The income elasticity remains in excess of unity, and while it has already been noted that this is unusual, recent research by Coghlan (1978) has found an elasticity of around unity when employing total final expenditure as the constraint variable in narrow money demand functions. The long run price elasticity is very close to unity, although the specification shows that in the short run both the price level and the change in the level (i.e. inflation) have a positive (and equal) effect on narrow money balances. The lagged reaction of narrow money to both real income and prices confirms its endogeneity with respect to these variables. The long run wage elasticity is constrained to be zero, but the coefficient of the ΔW_t variable implies that there is an elastic demand for narrow money balances with respect to wage inflation.

The model depicted by equation (6.7) thus appears to yield a satisfactory explanation of the demand for narrow money balances within the sample period 1963 I to 1977 IV, and is certainly superior to conventional functions specified with restricted dynamics, a single interest rate, and excluding the wage rate.

However, it is now widely recognised that within sample validation is not the only (nor indeed the best) test for the adequacy of an econometric model. A more stringent test is the predictive ability of the model outside of the sample period used for estimation. Unfortunately, the present study has been characterised by a total reliance on within-sample specification and estimation, a consequence of the requirement for both a sufficient number of observations to permit efficient estimation and a suitable number to model the possible structural shifts caused by institutional changes. This defect can be remedied to some extent by utilising the four observations from 1978 I to 1978 IV that are now available for post sample predictive purposes.

The predictive performance of equation (6.7) may be monitored by utilising the approach developed by Hendry (1979). This approach compares the within- and post-sample residual variances and uses the statistic defined as

$$z_{T_2} = \frac{\sum_{t=T+1}^{T+T_2} \hat{u}_t^2}{\hat{\sigma}^2}$$

where $T + 1$, $T + 2$, .. $T + T_2$ are the post-sample observations and the $\hat{u}_t$ are the prediction errors obtained from using the selected model. On the null hypothesis that the selected model (in this case equation (6.7)) is the "true" model then $z_{T_2} \sim \chi^2(T_2)$.

In the present case $T_2 = 4$ and applying the test to (6.7) yields the results shown in Table 6.2

TABLE 6.2

$\hat{u}_{T+1}$	$\hat{u}_{T+2}$	$\hat{u}_{T+3}$	$\hat{u}_{T+4}$	$\hat{\sigma}^2$
.0184	-.0598	-.0060	.0087	.000135

$z_4 = 19.8$ $\qquad$ $z_3 = 3.3$

The z_4 value strongly rejects the null hypothesis and obviously throws doubt on the validity of (6.7) as a model of the demand for narrow money balances. However, the large value of the statistic is almost totally due to the very large overprediction for 1978 II. If this prediction is removed, and the z_3 statistic calculated for the remaining three predictions, the value is well inside the region of acceptance of the null hypothesis. The large overprediction for 1978 II is a consequence of the unusually large increase in the wage rate, the rate for this quarter being almost 10 per cent higher than that for 1978 I, whereas the average quarterly increase for the years 1976 to 1978 was only 2.7 per cent. In view of this, and considering the accuracy of the subsequent predictions, the predictive performance of equation (6.7) is not too bad and hence, with some qualification, this model is considered to be an adequate explanation of the demand for narrow money balances.

Turning now to the broad money functions, we have already discussed the rather unsatisfactory theoretical implications of the two alternative specifications in chapter four. The approach employed above on the narrow money functions would, however, appear to be potentially useful in allowing the opportunity to develop a composite model containing both interest rates which may resolve these difficulties of theoretical interpretation for the demand for broad money.

Analogous to the development of the narrow money function, a composite model containing both equations (4.38) and (4.45) as nested models is

$$\begin{aligned} MB_t = \beta_0 &+ \beta_1 MB_{t-1} + \beta_2 MB_{t-2} + \beta_3 Y_t + \beta_4 Y_{t-1} \\ &+ \beta_5 P_t + \beta_6 P_{t-1} + \beta_7 P_{t-2} \\ &+ \beta_8 RC_t + \beta_9 RC_{t-1} + \beta_{10} RC_{t-2} \\ &+ \beta_{11} RL_t + \beta_{12} F_t + \beta_{13} W_t \\ &+ \beta_{14} V_t + \beta_{15} V_{t-1} + \beta_{16} V_{t-2} + u_t \end{aligned} \tag{6.8}$$

Equation (4.38) may be obtained from this model by imposing the nine restrictions

$$\begin{aligned} \beta_5 = \beta_9 = \beta_{11} = \beta_{14} = \beta_{15} = \beta_{16} &= 0 \\ \beta_8 - \beta_{10} &= 0 \\ \beta_3 - \beta_4 &= 0 \\ \beta_1 + \beta_2 &= 1 \end{aligned} \tag{6.9}$$

and hence a test of the restriction to (4.38) is given by

$$2[L(\hat{\Omega}_2) - L(\hat{\omega}_4)] \sim \chi^2(9)$$

where $L(\hat{\Omega}_2)$ is the maximum loglikelihood of (6.8) and $L(\hat{\omega}_4)$ the maximum loglikelihood of (6.8) under the set of restrictions ω_4 given by (6.9) i.e. the loglikelihood attached to (4.38). Similarly, equation (4.45) is obtained from (6.8) by imposing the restrictions

$$\beta_2 = \beta_3 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{13} = \beta_{14} = \beta_{15} = 0 \tag{6.10}$$

and thus a test of the restriction to (4.45) is given by

$$2[L(\hat{\Omega}_2) - L(\hat{\omega}_5)] \sim \chi^2(10)$$

where $L(\hat{\omega}_5)$ is the maximum loglikelihood of (6.8) under the restrictions ω_5 given by (6.10), i.e. the loglikelihood attached to (4.45). We should note that since the restrictions contained in the sets ω_4 and ω_5 are all linear, exact tests based on the F distribution may also be constructed.

Estimation of (6.8) yields $L(\hat{\Omega}_2) = -295.36$ and as $L(\hat{\omega}_4) = -299.01$ and $L(\hat{\omega}_5) = -307.59$ the test statistics $\chi^2(9) = 7.3$ and $\chi^2(10) = 24.5$ are obtained. From these statistics equation (4.45) may be rejected as a null hypothesis while equation (4.38) cannot be rejected, leading to the conclusion that this latter model, on the basis of parsimony, is the appropriate formulation of the demand for broad money function, i.e. that only the long interest rate is a determinant of broad money balances.

An alternative way of expressing (4.38) is as the following,

$$\begin{aligned}\Delta MB_t = -5.4 + .29\Delta MB_{t-1} + .12\overline{Y} - .045\overline{RC} \\ +.28\Delta P_{t-1} - .16P_{t-2} + .02F_t + .14W_t\end{aligned} \qquad (6.11)$$

where $\overline{Y} = (Y_t + Y_{t-1})/2$ and $\overline{RC} = (RC_t + RC_{t-2})/2$. This shows that the rate of change of broad money balances depends upon

(i) a constant,

(ii) the previous rate of change of broad money balances,

(iii) the recent "average" level of real income,

(iv) the recent "average" level of consol yields,

(v) the previous rate of inflation,

(vi) the price level two quarters previously,

(vii) the current differential between banks lending and borrowing rates,

and (viii) the current wage rate.

Equation (4.38) was also subjected to the predictive test introduced earlier for the four additional 1978 observations. The prediction errors and test statistic are shown as Table 6.3, from which we see that the null hypothesis that equation (4.38) is the true model is conclusively rejected, with the model considerably underpredicting for the last three quarters of 1978. We are therefore forced to conclude that although a model has been obtained which exhibits acceptable within sample statistical properties and sensible short run behaviour, it has no long run properties and performs particularly badly outside of the sample period.

TABLE 6.3

$\hat{u}_{T+1}$	$\hat{u}_{T+2}$	$\hat{u}_{T+3}$	$\hat{u}_{T+4}$	$\hat{\sigma}^2$
.0226	-.0485	-.0583	0.0256	.000110

$z_4 = 62.9$

Indeed, the ARIMA (2, 1, 0) model identified for the broad money series in chapter one (equation (1.5)), which yields a loglikelihood of -323.52, clearly inferior to (4.38), does appear to predict somewhat better. This is shown from the predictions of this model shown in Table 6.4. The z_4 statistic is not significant at the .025 level and the expected squared prediction error, defined as

$$\left(\sum_{t=1}^{T+T_2} \hat{u}_t^2\right)/T_2$$

is calculated to be .000063 compared to .000173 for equation (4.38). However, it must be noted that, under the assumptions that the predictions are unbiased and the prediction errors are uncorrelated, the two expected squared prediction errors are not significantly different. This conclusion is reached by using the fact that the two expected squared prediction errors will be equal if and only if the pair of random variables $\hat{u}^{(1)} \pm \hat{u}^{(2)}$ are uncorrelated, where $\hat{u}^{(1)}$ is the set of prediction errors obtained from equation (4.38) and $u^{(2)}$ the set obtained from (1.5). The correlation coefficient for these two variables is .55 which, since there are two degrees of freedom, is not significantly different from zero. (See Granger and Newbold

(1977, p.281) for details of this test).

TABLE 6.4

$\hat{u}_{T+1}$	$\hat{u}_{T+2}$	$\hat{u}_{T+3}$	$\hat{u}_{T+4}$	$\hat{\sigma}^2$
.0424	.0008	-.0269	-.0001	.000234

$$z_4 = 10.8$$

These results suggest that although we are able to build a reasonably coherent economic model for explaining the historical broad money series, there is some evidence that univariate ARIMA models will produce superior forecasts. This is rather disappointing in view of the time and cost spent on developing an econometric model, but may not be altogether surprising considering the conclusions that have been drawn from other comparisons of econometric models with time series models (see, e.g. Granger and Newbold (1977, ch.8).

6.2 CONCLUDING REMARKS

While results have been analysed and conclusions drawn in some detail throughout this thesis, it seems appropriate in this final section to bring together the major findings and to suggest areas of potentially useful future research.

The conclusions fall naturally into three categories. In chapter one univariate ARIMA time series models were built using the Box-Jenkin's (1970) methodology. Although there were some difficulties encountered in determining the appropriate degree of differencing required and in identifying a clearly superior model for some series, adequate within sample fits were obtained in each case. Only the model for the broad money series was employed for forecasting purposes, but this outperformed the econometric alternative on an expected squared prediction error criterion.

Chapters two and three were concerned with the paradox that exists between the standard IS/LM framework and direct estimation of the demand for money function. The empirical evidence confirmed that the exogeneity assumptions of the IS/LM model were invalid, and, in particular, that money was endogenously determined. Furthermore, there appeared to be only limited evidence of feedback from money to the other variables, this being restricted to narrow money and the short interest rate and broad money and the long interest rate. Most importantly, no feedback appeared to exist between money and either real income or prices, these latter variables being, therefore, classified as exogenous.

These results would seem to have important consequences for macroeconomic modelling of the U.K. economy. The IS/LM framework is clearly an inappropriate basis for empirical modelling and there is no support found for the monetarist contention that money directly influences income and prices. As a corollary, Artis and Lewis's (1976) models, which assume that money is exogenously determined, must also be rejected. In contradicting the previous findings on the causal relationship between money and income it is clear that a multivariate approach is more appropriate than the bivariate methods usually employed, as problems of "spurious causality" may be avoided.

However, it is possible that there does exist feedback from money to income and prices, but that the lags involved are too long to be picked up by the procedure used here. This would be consistent with a monetarist contention that the lag effect of money on these variables is long and variable. A further complication is the influence of alternative exchange rate regimes on the causal patterns, as discussed by Mills and Wood (1978). This last cavaet suggests a possible line of research - the use of monthly data over the period of floating exchange rates in operation since 1972. This would yield sufficient observations to allow a multivariate analysis to be performed, provided an appropriate real income proxy that is available monthly is used, perhaps an index of industrial production as employed by Goghlan (1978).

As a consequence of these results, the dynamic nature of the demand for money is apparent, and chapters four and five, along with the previous section of this chapter, have developed appropriate models for the broad and narrow money demand functions. The properties of these models have been comprehensively discussed in the previous section, but two of the general findings may be related to the broad consensus of the recent literature. The poor performance of the short interest rate as a determinant of broad money balances confirms previous findings, as does the general instability of the broad money function. Perhaps the most important features of these models, apart from the short run dynamic interactions, is the significant presence of the wage rate as a determinant of both money functions and the own rate differential as a determinant of the broad money function, both confirming a priori theoretical reasoning.

Ending on a more abstract, theoretical level, these last three chapters have developed empirical models of the demand for money through a series of specification searches. In the terminology of Leamer (1978, ch.1), the actual searches undertaken are a combination of specification searches, when the number of variables is excessively large, and post data model construction or "Sherlock Holmes inference" which occurs when the list of variables is incomplete. In the face of model uncertainty, search and decision processes of this type involving repeated significance test procedures are known to distort subsequent inferences and even invalidate classical statistical inference. (See the literature on the consequences of pre-test estimators, e.g. Judge and Bock (1978)). In particular, little is known about the sampling properties of the resulting estimators and certainly many forms of specification searches require the statistical evidence implied by the finally selected model to be appropriately discounted.

Nevertheless, as we have seen in this study, specification searches brought about by model uncertainty are an important and necessary feature of applied econometric research. One of the aims of this thesis, which it is hoped may be adopted in more applied studies, has been to describe the routes that have been taken in arriving at the finally chosen models, which, presumably could have been presented as if they had been initial choices. In this way, the reader is allowed to discount for himself the statistical significance of the finally selected models as he feels appropriate.

However, even in the light of these considerations, we feel that this thesis has presented a useful approach to econometric model building and obtained results which will aid in the understanding of the interactions existing between money, income and interest rates.

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