ISSN 1330-3651 (Print), ISSN 1848-6339 (Online)
UDC/UDK 629.783.056.84:519.8

A POSITIONING METHOD OF BDS RECEIVER UNDER WEAK SIGNAL CIRCUMSTANCES BASED ON COMPRESSED FRACTIONAL-STEP

Peng Wu, Xiaofeng Ouyang, Wenxiang Liu, Feixue Wang

Original scientific paper
This article puts forward the compression fractional-step method which applies to the positioning of BDS receivers under weak signal circumstances. It then analyses the features of BDS constellation and improves the original algorithm based on various satellites' dynamic conditions of mixed constellation. With respect to the ambiguity of the decimal of milliseconds arising from the clock error of receivers in the process of calculation, this article advances a processing mode of extending search dimension and a fractional-step method of employing the implied elevation information of receivers which is then substituted into the equation as the observed quantity to reduce the calculation amount of ambiguity. It concludes that in the most adverse circumstances this method can reduce nearly half of the calculation amount of the whole second fuzzy search of signals' emission time. This article also brings forward the compression calculation method to compress the range between the approximate position and clock error based on the feature that the dynamic of non-MEO satellites among BDS satellites is small. Taking non-MEO satellites as calculation, satellites can reduce the accuracy requirements of the local clock of receivers. In theory, calculation through using pure GEO satellites can broaden the requirement from 187,5 s to 1500 s. In the light of simulation results, it turns out that when the elevation information is known and precise, the first step of fractional-step calculation is capable of meeting the requirement of positioning. Otherwise, the second step of calculation can obtain the accurate positioning and achieve the rapid positioning under weak signal circumstances.

Keywords: *BeiDou Navigation Satellite System (BDS), Global Navigation Satellite System (GNSS), signal transmission time recovery, weak signal*

Metoda pozicioniranja BDS prijemnika pri slabom signalu zasnovana na sažetom frakcijskom koraku

Izvorni znanstveni članak
U ovom se radu predlaže sažeta metoda frakcijskog koraka koja se odnosi na pozicioniranje BDS prijemnika u uvjetima slabog signala. Zatim se analiziraju karakteristike BDS konstelacije i poboljšava originalni algoritam zasnovan na dinamičkim uvjetima miješane konstelacije raznih satelita. U odnosu na neodređenost decimale tisućinke sekunde nastale zbog greške sata prijemnika pri izračunu, ovaj rad razvija način obrade proširenja dimenzija pretrage i korištenja metode frakcijskog koraka implicirane informacije o elevaciji prijemnika koja se zatim substituira u jednadžbu kao promatrana količina kako bi se smanjila izračunata količina neodređenosti. U zaključku se kaže da u najnepovoljnijim uvjetima ova metoda može smanjiti gotovo polovicu količine izračuna čitave druge fuzzy pretrage vremena emisije signala. U radu se također predlaže metoda skraćenja proračuna kako bi se smanjio raspon između približnog položaja i greške na satu zasnovana na karakateristici da je dinamika ne-MEO satelita među BDS satelitima mala. Uključujući ne-MEO satelite u računanje, sateliti mogu smanjiti potrebe za točnošću lokalnog sata prijemnika. Teorijski, računanje uporabom čistih GEO satelita može proširiti potrebu od 187,5 s na 1500 s. Imajući u vidu rezultate simulacije, proizlazi da kada je informacija o elevaciji poznata i točna, prvim se korakom računanja frakcijskog koraka može zadovoljiti potreba za pozicioniranjem. Inače, drugim se korakom računanja može dobiti točno pozicioniranje i postići rapidno pozicioniranje u uvjetima slabog signala.

Ključne riječi: *BeiDou navigacijski satelitski sistem (BDS), dobivanje vremena prijenosa signala, Globalni navigacijski satelitski sistem (GNSS), slabi signal*

1 Introduction

In the context of weak signals, it is very difficult for receivers to receive continuously available navigation message data and perform frame synchronization, thus affecting the obtainment of complete signal emission time and leading to the result that positioning and calculation is too slow to be accomplished [1 ÷ 3]. The position of satellites calculated by using the approximate position and time of receivers can make estimates for pseudo-range and acquire the estimated value of integer milliseconds of satellite signals' emission time all the way to recover the true value and obtain the precise position of receivers.

Acquiring the accurate local approximate position and local clock constitutes the key to the recovery algorithm of satellites' emission time. The literature ignores or conducts a separate research on the time carry of satellite integer milliseconds generated by the fractional part $\Delta\varphi^{\text{chip}}$ within the milliseconds of receivers' clock error. For example, literature [4] proposes that neglected or hypothesized search for $\Delta\varphi^{\text{chip}}$ through the initial value hypothesis method is unfavourable to practical application. Literature [5 ÷ 8] demonstrates that receivers can rapidly determine the integer milliseconds time of satellites by the pseudo-code phase measurements of five satellites under the condition that local time is known, the position is known or the position error of probability is within 150 km and the error of local time is within 187,5 s. And literature [9, 10], further studies the search range when the approximate position is uncertain and conducts a quantitative analysis on the calculation amount exceeding the limit of position error.

The local clock error of receivers can exert an influence on the calculation error of satellite positions, thereby resulting in the inaccuracy of pseudo-range estimated value in the solution equation. Therefore, this kind of error can also be considered as the equivalent approximate position error of receivers. In this article, in view of the equivalent approximate position error caused by the errors of local approximate position and local time and the influence of $\Delta\varphi^{\text{chip}}$, we label the sum of the above errors as that of the equivalent errors represented as ΔP:

$$\Delta P = \|\Delta u\| + |\Delta t_{\text{d}}||v_{\max}| + |\Delta\varphi^{\text{chip}}| \tag{1}$$

In the formula, $\|\Delta u\|$ stands for the approximate position error of receivers; $|v|_{\max}$ refers to the maximum speed of all observed satellites relative to ground receivers; and $|\Delta t_{\text{d}}|$ represents the time estimation error at boot time. When $\|\Delta u\|$ and $|\Delta t_{\text{d}}|$ are certain and $|v|_{\max}$

decreases, ΔP can be reduced. This method is conducive to expanding the scope of the algorithm and reducing the calculation amount of the ambiguity search on the satellite integer milliseconds during initial calculation. The difference is that the first two items have easier access to priori information thus facilitating the evaluation of their scope whereas $|\Delta\varphi^{\text{chip}}|$ has no or hardly has any access to priori information, which may result in the scope expansion of ΔP.

The GEO satellite characteristic of China's BDS has advantages for the large coverage of its signals and being visible for a long time. In addition, making use of the trait that the dynamics for GEO satellites is low or that $|v|_{\max}$ is smaller than average satellites serves to reduce ΔP. This article puts forward a solution method on the basis of BDS hybrid constellation - the compression fractional-step method. When there is auxiliary elevation information, we can directly utilize four GEO satellites for solution and obtain the position of receivers. Otherwise, we can exploit four CEO satellites to compress ΔP, which is followed by the solving of the precise position.

2 The Time recovery method of BDS receivers

2.1 The analysis of BDS constellation conditions

There exist three types of satellites in BDS constellation, the orbital data of which can be identified through Interface Control Document (ICD). But it is worth noting that the actual satellite orbit in motion does not completely comply with the definition of ICD mainly because satellites affected by various perturbing factors would deviate from the track. This entails rectification through orbital maneuvering especially when the orbital maneuvering of GEO satellites is more frequent [11]. Taking these factors into account, we collected the BDS message data in March 2013 and conducted a statistical analysis on its orbital information. Based on the above analysis of collected message data, the state of motion of various satellites is illustrated in Tab. 1.

Table 1 Statistic of satellite motion

Parameter name	GEO	IGSO	MEO
$s_{\min}$ / m	$4{,}215\times10^7$	$4{,}204\times10^7$	$2{,}783\times10^7$
$s_{\max}$ / m	$4{,}219\times10^7$	$4{,}227\times10^7$	$2{,}797\times10^7$
$v_{0\ \min}$ / m/s	0,54	$1{,}29\times10^3$	$2{,}61\times10^3$
$v_{0\ \max}$ / m/s	85,40	$2{,}89\times10^3$	$3{,}12\times10^3$
$\alpha_{\min}$ / °	23,68	89,58	89,77
$\alpha_{\max}$ / °	149,89	90,41	90,23

In the above table, s refers to the distance of the satellite orbit from the earth's centre; v_0 signifies the radial velocity of satellites; and α stands for the included angle between the velocity direction of satellites and the earth's surface. Determining the vector direction of satellites relative to receivers of the earth based on α helps to calculate the maximum speed of various satellites relative to stationary receivers on the surface.

Table 2 Max speed of satellites relative to the surface

Relative speed	GEO	IGSO	MEO
$|v_{\max}|$ / m/s	83	456	724

Literature [1] advises considering the worst case on MEO satellites in GPS and meanwhile taking into account the receiver dynamic in small range. So we set $|v|_{\max}$ at 800 m/s. Similarly, in BDS, we set the $|v|_{\max}$ of GEO, IGSO and MEO at 100 m/s, 500 m/s and 800 m/s respectively.

2.2 The pseudo-range residual equation under weak signals

Suppose c represents the light-speed and $d = 0{,}001\cdot c$, in the solving process of GNSS receivers, the pseudo-range residual equation P_i can be represented as follows:

$$\begin{aligned} P_i(u,t_{\text{d}}^{\text{r}},\varphi^{\text{chip}}) &= \rho_i - \left\| s_i(t_i^{\text{s}}) - u \right\| - ct_{\text{d}}^{\text{r}}\varepsilon_i = \\ &= D_i - ct_i^{\text{s,ms}} - \left\| s_i(t^{\text{r}} - t_{\text{d}}^{\text{r}} - \tau_i) - u \right\| - c(t_{\text{d}}^{\text{r,ms}} + t_{\text{d}}^{\text{r,chip}}) + \varepsilon_i = \\ &= D_i - c(t_i^{\text{s,ms}} + t_{\text{d}}^{\text{r,ms}}) - \left\| s_i(t^{r} - t_{d}^{r} - \tau_i) - u \right\| - ct_{\text{d}}^{\text{r,chip}} + \varepsilon_i = \\ &= D_i - ct_i^{\text{s,ms}} - \left\| s_i(t^{\text{r}} - t_{\text{d}}^{\text{r}} - \tau_i) - u \right\| - \varphi^{\text{chip}} + \varepsilon_i. \end{aligned} \tag{2}$$

In the above formula,

u − the position of receivers which can be obtained through auxiliary means, such as the mobile cellular network or WLAN

t_{d}^{r} − the local clock error of receivers with $t_{\text{d}}^{\text{r,ms}}$ and $t_{\text{d}}^{\text{r,chip}}$ corresponding to the millisecond section and the section below millisecond respectively

t_i^{s} − The emission time of signals with $t_i^{\text{s,ms}}$ and $t_i^{\text{s,chip}}$ corresponding to the millisecond section and the section below millisecond respectively. $t_i^{\text{s,chip}}$ can be obtained from the phase measurement information of receivers.

ρ_i − the pseudo-range of satellite i

$s_i(t_i^{\text{s}})$ − the position of satellite i at the time of t_i^{s}

τ_i − the signal propagation delay of satellite i

D_i − represented as $c(t_i^{\text{r}} - t_i^{\text{s,chip}})$

t^{r} − the local time of receivers that can be obtained directly through the punctual information of receivers

φ^{chip} − the Pseudo-range measurement error caused by the section within millisecond of the receiver clock error with the corresponding equations $\varphi^{\text{chip}} = ct_{\text{d}}^{\text{r,chip}}$ and $\Delta\varphi^{\text{chip}} \in \left[-d/2, d/2\right]$

ε_i − the observation noise.

In the corresponding Eq. (2), we can find that three unknowns $\left[u, t_{\text{d}}^{\text{r}}, \varphi^{\text{chip}}\right]$ determine the value of the residual P_i. In addition, as a result of weak signals, we are incapable of obtaining $t_i^{\text{s,ms}}$ from observation information. Meanwhile, we consider merging and then solving $t_{\text{d}}^{\text{r,ms}}$ and $t_{\text{d}}^{\text{r,ms}}$, after that we get a new unknown t_i^{ms}. Under the principle of LS that the residual P_i is 0 when the least square of the equation has a converged solution, we can identify the value range of t_i^{ms}, thereby recovering the emission time of satellite signals and figuring out the position of receivers and the clock error.

2.3 Calculation of the emission time of satellites

In the residual Eq. (3), given the influence of the initial values of satellite delay estimation τ_i and φ^{chip}, we unfold them at the approximate location and estimated clock error according to Taylor series while ignoring the more-than-second-order component. Then we can get:

$$\begin{aligned} &P_i\left(u, t_{\mathrm{d}}^{\mathrm{r}}, \tau_i, \varphi^{\text{chip}}\right) = \\ &P_i\left(u^{(0)}, t_{\mathrm{d}}^{\mathrm{r}(0)}, \tau_i^{(0)}, \varphi^{\text{chip}(0)}\right) + \\ &v_i\left(t_i^{\mathrm{s}(0)}\right)\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\left(\Delta t_{\mathrm{d}}^{\mathrm{r}} + \Delta\tau_i\right) + \\ &\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u - \Delta\varphi^{\text{chip}} + \varepsilon_i, \end{aligned} \tag{3}$$

where is in the Eq. (3):

$$\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right) = \frac{s_i\left(t_i^{\mathrm{s}(0)}\right) - u^{(0)}}{\left\| s_i\left(t_i^{\mathrm{s}(0)}\right) - u^{(0)} \right\|}$$

represents the satellite gaze vector of approximate location; $v_i(t_i^{\mathrm{s}(0)})$ stands for the maximum speed of satellites relative to receivers on the earth's surface at the time of $t_i^{\mathrm{s}(0)}$; and $\Delta\tau_i$ is the satellite position error caused by the estimated time delay of satellites.

According to literature [12], we can figure out that the satellite receiving time delay of GEO and IGSO is approximately 120 ÷ 140 ms and that of MEO is about 72 ÷ 91 ms within the coverage area of satellite signals. The estimated error of the emission time of satellite signals generated by substituting $\Delta\tau_i$ as the intermediate value does not exceed the range of 10 ms. And the estimated error's impact on the position error of the most dynamic satellite MEO does not exceed 8 m. Given that the error caused by $\Delta\tau_i$ is far less than the application condition that the approximate position error of ΔP does not surpass 150 km, we will just ignore the error in the equation.

To determine the solving conditions of the equation, we substitute the equation

$$\begin{aligned} &P_i\left(u^{(0)}, t_{\mathrm{d}}^{\mathrm{r}(0)}, \tau_i^{(0)}, \varphi^{\text{chip}(0)}\right) = \\ &D_i - ct_i^{\text{ms}} - \left\| s_i\left(t_i^{\mathrm{s}(0)}\right) - u^{(0)} \right\| - \varphi^{\text{chip}(0)} + \varepsilon_i \end{aligned}$$

Suppose the residual $P_i(u, t_{\mathrm{d}}^{\mathrm{r}}, \tau_i, \varphi^{\text{chip}})$ is 0, which means that the solution value has been obtained, then we have:

$$\begin{aligned} &D_i - ct_i^{\text{ms}} - \left\| s_i\left(t_i^{\mathrm{s}(0)}\right) - u^{(0)} \right\| - \varphi^{\text{chip}(0)} + \varepsilon_i = \\ &-v_i\left(t_i^{\mathrm{s}(0)}\right)\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\left(\Delta t_{\mathrm{d}}^{\mathrm{r}}\right) - \theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u + \Delta\varphi^{\text{chip}}. \end{aligned}$$

The emission time millisecond of satellites can be represented as:

$$\begin{aligned} N_i^{\text{ms}} = &\frac{1}{d}\left(D_i - \left\| s_i\left(t_i^{\mathrm{s}(0)}\right) - u^{(0)} \right\| - \varphi^{\text{chip}(0)} + \varepsilon_i + \right. \\ &\left. + v_i\left(t_i^{\mathrm{s}(0)}\right)\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\left(\Delta t_{\mathrm{d}}^{\mathrm{r}}\right) + \theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u - \Delta\varphi^{\text{chip}}\right). \end{aligned} \tag{4}$$

2.4 Determination of the search space dimension

In the Eq. (4), under the condition that the uncertain part

$$\frac{1}{d}\left| v_i\left(t_i^{\mathrm{s}(0)}\right)\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta t_{\mathrm{d}}^{\mathrm{r}} + \theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u - \Delta\varphi^{\text{chip}} \right| \le \frac{1}{2}$$

the emission time ambiguity of satellites N_i^{ms} is uniquely determined without multiple possibilities. But from the equation

$$\begin{aligned} &\left| v_i\left(t_i^{\mathrm{s}(0)}\right)\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta t_{\mathrm{d}}^{\mathrm{r}} + \theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u - \Delta\varphi^{\text{chip}} \right| \le \\ &v_i\left|\Delta t_{\mathrm{d}}^{\mathrm{r}}\right| + \left\|\Delta u\right\| + \left|\Delta\varphi^{\text{chip}}\right| \le \frac{d}{2} \end{aligned}$$

we can find that the expression on the left of the equal sign is a detailed presentation of the Eq. (1). So when calculating, we only need to take into consideration the carrying problem probably resulting from $\left(v_i\left|\Delta t_{\mathrm{d}}^{\mathrm{r}}\right| + \left\|\Delta u\right\|\right)$ and $\left|\Delta\varphi^{\text{chip}}\right|$. In the process of calculating, first and foremost we should identify two items that are easy to estimate.

The equation

$$\left| v_i\left(t_i^{\mathrm{s}(0)}\right)\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta t_{\mathrm{d}}^{\mathrm{r}} + \theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u \right| \le \frac{d}{2}, \tag{5}$$

means that $\left(v_i\left|\Delta t_{\mathrm{d}}^{\mathrm{r}}\right| + \left\|\Delta u\right\|\right)$ will not cause carry. So we can set the equation

$$\overline{N}_i^{\text{ms}} = \left[\frac{D_i - \left\| s_i\left(t_i^{\mathrm{s}(0)}\right) - u^{(0)} \right\|}{d} \right]$$

the physical meaning of which is the unique integer millisecond of satellite signals determined by the approximate location and local time information.

But in actual calculation, we have to take into account the item $\Delta\varphi^{\text{chip}}$ and the carry of the millisecond $\overline{N}_i^{\text{ms}}$ generated by $\Delta\varphi^{\text{chip}}$. The actual search $\overline{N}_i^{\text{ms}}$ should range within the space of $\left[\overline{N}_i^{\text{ms}}, \overline{N}_i^{\text{ms}} + 1\right]$ with ω, the dimension of possible values, being 2.

2.5 The compression fractional-step method

Regarding the pseudo-range residual equation

$$\begin{aligned} &D_i - ct_i^{\text{ms}} - \left\| s_i\left(t_i^{\mathrm{s}(0)}\right) - u^{(0)} \right\| - \varphi^{\text{chip}(0)} + \varepsilon_i = \\ &-\upsilon_i\left(t_i^{\mathrm{s}(0)}\right)\nu_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta t_{\mathrm{d}}^{\mathrm{r}} - \nu_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u + \Delta\varphi^{\text{chip}} \end{aligned}$$

t_i^{ms} namely $\overline{N}_i^{\mathrm{ms}}$ is an estimated known. At the same time, we ignore the unknown item $\Delta\varphi^{\mathrm{chip}}$ as the follow-up search. Then we can get:

$$\Delta\rho_i = -v_i\left(t_i^{\mathrm{s}(0)}\right)\theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta t_{\mathrm{d}}^{\mathrm{r}} - \theta_i^{(0)}\left(t_i^{\mathrm{s}(0)}\right)\Delta u. \qquad (6)$$

In the formula, $u^{(0)}$ represents the estimated approximate location of receivers. $t_i^{\mathrm{s}(0)}$ is obtained when t^{r} subtracts the estimated τ_i according to different intermediate values of various satellites' time delay.

We utilize receivers to conceal the elevation information and substitute the observation data of the fifth satellite for the ranging equation of elevation auxiliary which is represented as follows:

$$\frac{x_{\mathrm{u}}^{2}+y_{\mathrm{u}}^{2}}{(Ra+h)^2}+\frac{z_{\mathrm{u}}^{2}}{(Rb+h)^2}=1.$$

In the above formula, *Ra* stands for the short half axis of the Earth; *Rb* represents the long half axis of the Earth; and *h* refers to the estimated value of the ellipsoidal height.

Partial derivative on the pseudo-range residual Eq. (6) can be expressed in the following form:

$$\begin{bmatrix}\Delta\rho_1\\ \Delta\rho_2\\ \Delta\rho_3\\ \Delta\rho_4\\ m\Delta h\end{bmatrix}=\begin{bmatrix} l_1 & m_1 & n_1 & q_1 & 1\\ l_2 & m_2 & n_2 & q_2 & 1\\ l_3 & m_3 & n_3 & q_3 & 1\\ l_4 & m_4 & n_4 & q_4 & 1\\ \alpha & \beta & \gamma & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\\ \Delta t_{\mathrm{d}}^{\mathrm{r}}\\ \Delta\varphi^{\mathrm{chip}}\end{bmatrix}, \qquad (7)$$

where are in the Eq. (7):

$$\alpha=\frac{2\hat{x}_{\mathrm{u}}^{\mathrm{k}}}{(Ra+h)},\ \beta=\frac{2\hat{y}_{\mathrm{u}}^{\mathrm{k}}}{(Ra+h)},\ \gamma=\frac{2\hat{z}_{\mathrm{u}}^{\mathrm{k}}}{(Rb+h)},$$

$$m=2\left(\frac{\left(\hat{x}_{\mathrm{u}}^{\mathrm{k}}\right)^2+\left(\hat{y}_{\mathrm{u}}^{\mathrm{k}}\right)^2}{(Ra+h)^2}+\frac{(Ra+h)\left(\hat{z}_{\mathrm{u}}^{\mathrm{k}}\right)^2}{(Rb+h)^3}\right), \text{ and}$$

$$\begin{cases} l_i=-\dfrac{x_{\mathrm{s}}^{\mathrm{k}}+v_x^{\mathrm{k}}\hat{t}_{\mathrm{d}}^{\mathrm{r}}-\hat{x}_{\mathrm{u}}^{\mathrm{k}}}{\left\|s_i\left(t_i^{S(k)}+\hat{t}_d^{r}\right)-u^{(k)}\right\|}\\ m_i=-\dfrac{y_{\mathrm{s}}^{\mathrm{k}}+v_y^{\mathrm{k}}\hat{t}_{\mathrm{d}}^{\mathrm{r}}-\hat{y}_{\mathrm{u}}^{\mathrm{k}}}{\left\|s_i\left(t_i^{\mathrm{s(k)}}+\hat{t}_{\mathrm{d}}^{\mathrm{r}}\right)-u^{(\mathrm{k})}\right\|}\\ n_i=-\dfrac{z_{\mathrm{s}}^{\mathrm{k}}+v_z^{\mathrm{k}}\hat{t}_{\mathrm{d}}^{\mathrm{r}}-\hat{z}_{\mathrm{u}}^{\mathrm{k}}}{\left\|s_i\left(t_i^{\mathrm{s(k)}}+\hat{t}_{\mathrm{d}}^{\mathrm{r}}\right)-u^{(\mathrm{k})}\right\|}\\ q_i=\dfrac{\left(x_{\mathrm{s}}^{\mathrm{k}}+v_x^{\mathrm{k}}\hat{t}_{\mathrm{d}}^{\mathrm{r}}-\hat{x}_{\mathrm{u}}^{\mathrm{k}}\right)v_x^{\mathrm{k}}}{\left\|s_i\left(t_i^{\mathrm{s(k)}}+\hat{t}_{\mathrm{d}}^{\mathrm{r}}\right)-u^{(\mathrm{k})}\right\|}+\dfrac{\left(y_{\mathrm{s}}^{\mathrm{k}}+v_y^{\mathrm{k}}\hat{t}_{\mathrm{d}}^{\mathrm{r}}-\hat{y}_{\mathrm{u}}^{\mathrm{k}}\right)v_y^{\mathrm{k}}}{\left\|s_i\left(t_i^{\mathrm{s(k)}}+\hat{t}_{\mathrm{d}}^{\mathrm{r}}\right)-u^{(\mathrm{k})}\right\|}+\\ \quad+\dfrac{\left(z_{\mathrm{s}}^{\mathrm{k}}+v_x^{\mathrm{k}}\hat{t}_{\mathrm{d}}^{\mathrm{r}}-\hat{z}_{\mathrm{u}}^{\mathrm{k}}\right)v_z^{\mathrm{k}}}{\left\|s_i\left(t_i^{\mathrm{s(k)}}+\hat{t}_{\mathrm{d}}^{\mathrm{r}}\right)-u^{(\mathrm{k})}\right\|}. \end{cases}$$

After the integer millisecond time of satellite signal is figured out, the positioning is therefore achieved. If the elevation parameter is accurate, then positioning accuracy is limited by the elevation auxiliary accuracy; otherwise, the PDOP value should be optimized through adding non-GEO satellite for a further calculation.

When it comes to the compression fractional-step method, fractional-step actually refers to a method of calculation through two steps. To begin with, the implied elevation information should be used to determine the relative integer millisecond relationship of the signal transmission time of four satellites, then an integer millisecond estimate calculation of the fifth satellite signal transmission should be made according to the relatively accurate positioning result. Compared with the method of directly putting all these five satellites to calculate, this new method can effectively reduce the calculation of fuzzy search. Compression means that in the first step, relatively static satellite groups can be used to compress approximate position and clock bias information. These static satellites can include IGSO and GEO satellites which are more static than the MEO satellite, or just the GEO satellite which is even more static than IGSO.

We can see from the formula of Pseudo-range residuals that it will remain unchanged whatever types of satellites are. Therefore, this paper puts forward a step-by-step calculation of the compression fractional-step method which not only conforms to BDS application but also GPS.

3 Results and Discussion

Experimental data comes from the data with the B1 C collected by the self-developed BDS receiver in Changsha, China at 10 a.m. on April 16th, 2013. Satellites are marked as number 1, 2, 4, 5, 8, 11, among which No. 8 refers to the IGSO satellite and No. 11 represents the MEO satellite

The geometric dilution of precision (DOP) values of these experimental satellite constellations are as the following table illustrates:

Table 1 Comparison of simulated constellation DOP values

Elevation error / m	4 GEOs		4 GEOs and 1 IGSO		4 GEOs and 1 MEO	
	PDOP	TDOP	PDOP	TDOP	PDOP	TDOP
0	13,8	5,7	4,1	2,5	6,1	2,0
10	14,4	5,7				
10^2	41,7	6,8				
10^3	393,5	38,3				
10^4	3933	378,7				

Thanks to auxiliary elevation information, the equivalent DOP value of the GEO satellite constellation is related to the pseudo-range measurement accuracy and input accuracy. In this table, the distance measurement error is taken as 6 m.

3.1 Positional accuracy

The positional accuracy is counted after the adjustment of auxiliary elevation information accuracy and the analysis of the combination of three constellations, just as the Tab. 4 illustrated. The epoch interval among collected data is 1s and a total of 1000 epochs are used for counting.

Table 4 Positional accuracy of three constellations

Elevation error / m	Position error / m		
	4 GEOs	4 GEOs and 1 IGSO	4 GEOs and 1 MEO
0	15,8	38,0	41,23
10	22,64	38,0	40,05
10^2	107,5	37,3	40,97
10^3	1157	37,8	41,6
10^4	11981	39,9	42,1

Just as the table has indicated, when pure GEO constellations are used for positioning, the positional accuracy is mainly determined by the input elevation error and it fundamentally corresponds to the PDOP value. But when mixed constellations are used for positioning, elevation information has an impact not on the final positional accuracy but only on the equivalent error and the compression of ΔP. Therefore, in the second case, the positional accuracy is not influenced by the input elevation information accuracy.

3.2 Calculation analysis

When the initial value of $\Delta\varphi^{\text{chip}}$ is obtained without priori information, it can be taken as the extension of the search dimension ω. Under this circumstance, two problems should be paid attention to in the calculation analysis of the GPS algorithm: first, according to the equivalent position and the value range in the Eq. (5), when it is below $d/2$, the search range is $\left[\overline{N}_i^{\text{ms}}, \overline{N}_i^{\text{ms}}+1\right]$, and each initial integer millisecond estimate $\overline{N}_i^{\text{ms}}$ has the carrying potential. Thus the search space cannot be assumed as 1, and ω should exceed 2. In addition, there are many possible repeated search combinations when the value dimension of the integer millisecond of n satellites is ω. For instance, when $\overline{N}_i^{\text{ms}}$ and $\left(\overline{N}_i^{\text{ms}}+1\right)$ are adopted, the final disparity can be reflected in the receiver's local clock bias: 1ms. Just as the previous context has illustrated, such a millisecond difference will only exert a minor impact on the satellite position calculation and can therefore be ignored. Therefore, the second search combination can be regarded to result from an addition of the same numerical value like 1 or 2 to the first combination. It can thus be viewed as a redundant combination which needs no search.

Only a non-redundant combination is called for in the search process. First is to ensure that there are i out of n satellites which do not need a carry. Then, the combination possibilities are $(\omega-1)^{(n-i)}$ of the rest $(n-i)$ of satellites. Thus, when the search dimension of n satellites is ω, the maximum search frequency is

$$\sum_{i=1}^{n} C_n^i (\omega-1)^{(n-i)}.$$

Because at least 5 satellites are needed to calculate the time ambiguity degree through the traditional method, 31 searches will be carried out when the initial ambiguity degree is the most unfavourable. In comparison, this new method only calls for 4 satellites, and then a non-GEO satellite for a second calculation. In this perspective, two additional searches, at most, are involved. In a word, there are only (15+2) searches, which can effectively reduce calculation in the practical application. It must be pointed out that in the first integer millisecond search of satellite time, the new method involves at least iterate solutions even under the most ideal circumstances whereas the traditional method only involves one iteration.

Generally speaking, it is impossible to obtain auxiliary elevation information. To improve the geometrical distribution of the satellite, non-GEO satellites are always included in BDS positioning. When 4 GEO satellites are used for observation, two combinations should be taken into consideration: IGSO is included or only MEO is included in other satellites. Suppose that the approximate position error is 0, namely, the equivalent position and ΔP are only affected by $|\Delta t_{\text{d}}|$ and $\left|\Delta\varphi^{\text{chip}}\right|$, a comparison between the new method and the traditional one is as Tab. 5 and Tab. 6 demonstrate:

Table 5 The number of initial search range analysis when IGSO satellite exists

Time accuracy Δt / s	The traditional method		The method adopted in this paper	
	ω	Calculation/ times	ω	Calculation/ times
$0 \le \Delta t \le 300$	2	1,31	2	2,17
$300 < \Delta t \le 900$	3	1,211	2	2,17
$900 < \Delta t \le 1500$	4	1,781	2	2,17

When the method advocated in this paper is applied into GPS, there is an expectant optimal calculation from 1,31 to 2,17 under the first time accuracy condition in Tab. 6, namely when $\Delta t \le 187,5$. The only advantage the traditional method has over the new one in terms of calculation is that it only involves one search when the most ideal situation takes place, namely, when there is no carry of the integer millisecond estimate $\overline{N}_i^{\text{ms}}$ of all satellites.

Table 6 The number of initial search range analysis when only MEO satellite exists

Time accuracy Δt /s	The traditional method		The method adopted in this paper	
	ω	Calculation/ times	ω	Calculation/ times
$0 \le \Delta t \le 187,5$	2	1,31	2	2,17
$187,5 < \Delta t \le 562,5$	3	1,211	2	2,17
$562,5 < \Delta t \le 937,5$	4	1,781	2	2,17
$937,5 < \Delta t \le 1312,5$	5	1,2101	2	2,17

4 Conclusion

The solution put forward by this paper is dimension extension of the millisecond fraction distribution of the clock bias. On this basis, calculation can be reduced through the fractional-step method. In addition, concerning the BDS application conditions, the GEO satellite can be taken as the first step for calculation so as to lower the requirements of the local clock bias range. Theoretically, the requirement of GEO can be as much as 1500 s, which is a fundamental increase from 187,5 s of MEO.

There are also some limitations in this new method. For example, in the first step, user elevation information is used as the fifth measure indicator. Although the algorithm experiment has proved that the solution error can be controlled within the expectant limit even when the elevation is unknown, the second step calculation is also inevitable to guarantee the positional accuracy when the elevation is unknown. On the other hand, non-MEO satellites are required for the compression of the equivalent error and ΔP of the receiver with a view to improve the adaptation of the algorithm to the local clock bias. But such a requirement will limit receiver types and application regions, thus limiting the type to the BDS receiver and the region to areas around China.

To sum up, the compression fractional-step calculation advocated in this paper can facilitate the positioning of the BDS receiver under weak signal circumstances.

5 References

[1] Kaplan, E. D.; Hegarty, C. J. Understanding GPS: principles and applications, 2nd ed. // Norwood, MA, Artech House, 2006.

[2] Agarwal N, Basch J, Beckmann P. Algorithms for GPS operation indoors and downtown. // GPS Solutions, 6, (2002), pp. 149-160.

[3] Diggelen, F. V. Indoor GPS theory & implementation. // IEEE Position, Location & Navigation Symposium, 2002, pp. 40-247.

[4] SONG Cheng, WANG Fei-xue, ZHUANG Zhao-wen. Method for assisted-GPS positioning based on ambiguity resolution in weak signal environment. // Journal on Communications, 30, 9(2009).

[5] Sirola, N.; Syrjärinne, J. GPS Position Can be Computed without the Navigation Data. // ION GPS, 2002, pp. 2741-2744.

[6] Sirola, N. A Method for GPS Positioning without Current Navigation Data. // Tampere University of Technology, 2001.

[7] Sirola, N.; Syrjärinne, J. Solving GPS Time and Position without Navigation Data. // Proceedings of the ENC-GNSS, 2002.

[8] Sirola, N. Exhaustive Global grid search in computing receiver position from modular satellite range measurement. // Journal of Physics: Conference Series, 2006, pp. 73-82.

[9] Chen Mo-han, Ba Xiao-hui,Wang Yun, Chen Jie. A Fast Positioning Algorithm for Assisted-global Position System. // Science Technology and Engineering, 11, 10(2011).

[10] CAO Hui, YUAN Hong, Method for Time-of-transmission Recovery Based on Assisted-GPS Positioning. // Chinese J. Space Sci, 32, 3(2012), pp. 585-591.

[11] Zhenghang, Li; Weixing, Zhang; Xiaoying, Gong; Xiaochuan, Qu. Solution of orbit maneuver problem in autonomous orbit. // Geomatics and Information Science of Wuhan University, 36, 11(2011).

[12] BeiDou Navigation Satellite System Signal in Space Interface Control Document Open Service Signal B1I (Version 1.0). // China Satellite Navigation Office. December, 2012.

Authors' addresses

Peng Wu, PhD
Institution: College of Electronic Science and Engineering
Postal address: National University of Defense Technology, Changsha 410073, China
E-mail: wp4nnc@gmail.com

Xiaofeng Ouyang, Master
Institution: College of Electronic Science and Engineering
Postal address: National University of Defense Technology, Changsha 410073, China
E-mail: xfouyang@sina.com

Wenxiang Liu, Lecturer
Institution: College of Electronic Science and Engineering
Postal address: National University of Defense Technology, Changsha 410073, China
E-mail: liuwenxiang8888@163.com

Feixue Wang, Professor
Institution: College of Electronic Science and Engineering
Postal address: National University of Defense Technology, Changsha 410073, China
E-mail: wangfeixue365@sina.com