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Aligned magnetic field of two-phase mixed convection flow in dusty Casson fluid over a stretching sheet with Newtonian heating

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Abstract. The effect of aligned magnetic field is numerically investigated for mixed convection flow of dusty Casson fluid over a stretching sheet. The governing equations of flow and heat transfer for the two-phase model (fluid and dust) with an appropriate thermal boundary condition which is Newtonian heating (NH) is presented. The similarity transformation is employed to transform the nonlinear governing equations for each phase into the ordinary differential equations which then solved numerically using Runge-Kutta Fehlberg (RKF45) method. Numerical solutions obtained for velocity and temperature distributions are illustrated through graph by varying several physical parameters. It is observed that the fluid velocity decreases with an increase in aligned magnetic field and particle-fluid interaction parameter.

1. Introduction

Engineering devices has many applications which embedded of mixed convection flow involving the heat transfer processes. This process also can be seen on atmospheric boundary layer flow, heat exchangers, solar collector, nuclear reactors and electronic equipment [1]. If the free convection and forced convection behave together then the mixed convection is arises in that flow. Such flow happened due to the difference temperature at the surface and ambient fluid which develop the buoyancy force. There have been several studies in the literature reporting the mixed convection flow for various physical aspects and geometry [2-7].

The above existing literatures are only restricted on the single flow (pure fluid). Actually, the foreign bodies or impurities in the form of shoot or ash may contain in a fluid. Thus, the development of mathematical model for predicting the accumulation of particles in fluid will help to determine its rheology for investigation the flow and heat transfer process of fluid. [8] studied the non-interacting particles in fluids over a semi-infinite flat plate. [9] and [10] discussed the effect of magnetic on the boundary layer flow of viscous fluid with dust particle, meanwhile [11] focuses on dusty nanofluid flow with the influence of buoyancy parameter. The non-Newtonian fluid of Casson model has been extensively used in food industries and is generally applied in manufacturing of chocolate [12]. The objective of the present paper is to determine whether the present of dust particles influences the behaviour of Casson fluid on the mixed convection flow over a vertical stretching sheet. Here, the

Casson fluid is interpreted as chocolate while the dust particle as copper. The mixture of chocolate and copper is numerically studied and hence it is expected to provide the understanding of two-phase boundary layer flow of non-Newtonian fluid contains dust particles.

2. Mathematical Model

Consider a steady, incompressible two dimensional mixed convection flow of dusty Casson fluid over a stretching sheet with linear velocity $u_w(x) = ax$ where an aligned magnetic field with an acute angle α_1 is applied to the flow as illustrated in figure 1. For simplification, the dust particles are assumed in spherical shape, uniform size and number density are taken as constant throughout the flow.

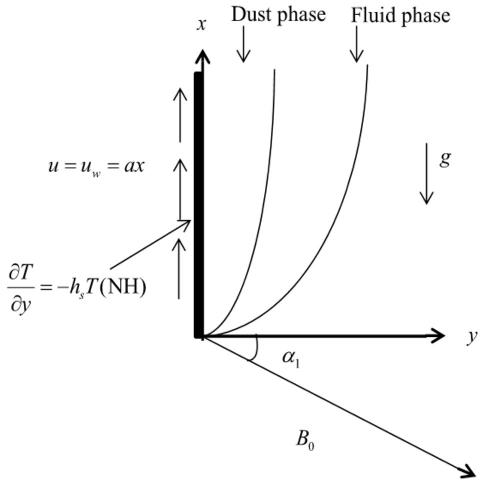

Figure 1. Flow configuration.

Under the boundary layer and Boussinesq approximations, the governing equations for two-phase flow as adopted from [13] and [14] can be written as:
Fluid phase:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\left(1+\frac{1}{A}\right)\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{\rho_p}{\tau_v}(u_p - u) - \sigma u B_0{}^2 \sin^2\alpha_1 + \rho g \beta^*(T - T_\infty), \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{\rho_p c_s}{\gamma_T}(T_p - T), \tag{3}$$

Dust phase:

$$\frac{\partial}{\partial x}(u_p) + \frac{\partial}{\partial y}(v_p) = 0 \tag{4}$$

$$\rho_p\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \frac{\rho_p}{\tau_v}(u - u_p), \tag{5}$$

$$\rho_p c_s\left(u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\right)=-\frac{\rho_p c_s}{\gamma_T}(T_p-T) \tag{6}$$

where (u,v) and (u_p,v_p) are the velocities components of the fluid and particle phase along x and y axes, respectively. μ is the coefficient of viscosity of the fluid, ρ and ρ_p are the density of fluid and dust phase, α_1 is the aligned angle, $\tau_v = 1/k$ is the relaxation time of particles phase, k is the Stoke's resistance (drag force), c_p and c_s are specific heat of fluid and dust particle, T and T_p are the temperature of fluid and particle phases, γ_T is the thermal relaxation time, B_0 is the magnetic-field strength, $A=\mu_B\sqrt{2\pi_c}/p_y$ is the Casson parameter, g is the gravity acceleration, and β^* is the thermal expansion coefficient.

The boundary conditions of NH are in the form

$$u=u_w(x)=ax,\quad v=0,\quad \frac{\partial T}{\partial y}=-h_s T \qquad \text{at } y=0$$

$$u\to 0,\quad u_p\to 0,\quad v_p\to v,\quad T\to T_\infty,\quad T_p\to T_\infty \quad \text{as } y\to\infty \tag{7}$$

where a is positive constant and h_s is heat transfer parameter.

The following similarity transformations were applied on equations (1)-(6)

$$u=axf'(\eta),\quad v=-(av)^{1/2}f(\eta),\quad \eta=\left(\frac{a}{v}\right)^{1/2}y,\quad \theta(\eta)=\frac{T-T_\infty}{T_\infty}$$

$$u_p=axF'(\eta),\quad v_p=(av)^{1/2}F(\eta),\quad \theta_p(\eta)=\frac{T_p-T_\infty}{T_\infty}, \tag{8}$$

where ψ is the stream function defined as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. Equations (1)-(6) are now transformed to ordinary differential equations as follow

$$\left(1+\frac{1}{A}\right)f'''(\eta)+f(\eta)f''(\eta)-\left(f'(\eta)\right)^2+\beta N\left(F'(\eta)-f'(\eta)\right)-M\sin^2\alpha_1 f'(\eta)+\lambda\theta(\eta)=0, \tag{9}$$

$$\theta''(\eta)+\Pr f(\eta)\theta'(\eta)+\frac{2}{3}\beta N\left(\theta_p(\eta)-\theta(\eta)\right)=0, \tag{10}$$

$$\left(F'(\eta)\right)^2-F(\eta)F''(\eta)+\beta\left(F'(\eta)-f'(\eta)\right)=0, \tag{11}$$

$$\theta_p'(\eta)F(\eta)+\frac{2}{3}\frac{\beta}{\Pr\gamma}\left(\theta(\eta)-\theta_p(\eta)\right)=0 \tag{12}$$

and the boundary conditions (7) are reduced to

$$f(0)=0,\ f'(0)=1,\ \theta'(0)=-b\left(1+\theta(0)\right) \quad \text{at } \eta=0$$

$$f'(\eta)\to 0, F(\eta)\to 0, F(\eta)\to f(\eta), \theta(\eta)\to 0, \theta_p(\eta)\to 0 \quad \text{as } \eta\to\infty \tag{13}$$

where a prime $(\ ')$ denotes differentiation with respect to η. $N=\rho_p/\rho$ is the mass concentration of particle phase, $M=\sigma B_0^2/\rho a$ is the magnetic field parameter, $\beta=1/a\tau_v$ is the fluid-particle interaction parameter, $\Pr=\mu c_p/k$ is the Prandtl number, $\gamma=c_s/c_p$ is the specific heat ratio of mixture, $b=-h_s\left(v/a\right)^{1/2}$ is the conjugate parameter for NH and $\lambda=Gr_x/\mathrm{Re}_x^{1/2}$ is the buoyancy parameter with $Gr_x=g\beta^* T_\infty x^3/v^2$ is the Grashof number and $\mathrm{Re}_x=u_w(x)x/v$ is the Reynolds number.

The exact solution of equation (9) which considering only Casson parameter is obtained by [15] and can be expressed as

$$f(\eta)=\left(1+\frac{1}{A}\right)^{1/2}\left(1-\exp\left(-\frac{\eta}{\left(1+1/A\right)^{1/2}}\right)\right) \tag{14}$$

The exact expression for temperature profile by using equation (14) is

$$\theta(\eta)=C_1\int_\eta^\infty e^{-\Pr\int_\eta^\infty f\,d\eta}d\eta, \qquad C_1=\frac{b(1-\theta(0))}{e^{-\Pr\int_\eta^\infty f\,d\eta}} \tag{15}$$

3. Numerical procedure

The numerical computation of equations (9)-(12) subjected to boundary conditions (13) are solved using Runge-Kutta Fehlberg (RKF45) method in Maple software because this scheme are stable, easy to implement and self-starting. The numerical results of flow and heat transfer for the two phase flow of dusty Casson fluid are determine in terms of velocity and temperature profiles with the influences of several physical parameters. The velocity and temperature profiles are asymptotically satisfied the boundary conditions for the maximum boundary layer thickness of $\eta_\infty=5$ and 8 as shown in figures (3)-(7).

4. Results and discussion

The mixture of chocolate with copper is discussed and their thermophysical properties are shown in table 1. The physical parameters of aligned angle α_1, magnetic field parameter M, particle-fluid interaction parameter β and buoyancy parameter λ are keeping to be fixed throughout the study in which the value of $\alpha_1=\pi/4$, $M=1$, $\beta=0.5$, $b=0.1$, $A=2$, $N=7.44$, $\Pr=16.7$, $\lambda=1$ and $\gamma=0.23$. The comparison of skin friction coefficient $f''(0)$ of the exact equation with the present numerical result are presented in table 2. It is observed that a very good agreement is attained and the present numerical results presented in this manuscript are acceptable and accurate.

Table 1. Thermo-physical properties of chocolate and copper.

	$\rho(Kgm^{-3})$	$C_p(JKg^{-1}K^{-1})$	$k(Wm^{-1}K^{-1})$	Source
Chocolate	1200	1670	0.26	[16]
Copper	8933	385	401	[11]

Table 2. Comparison of skin friction coefficient.

A	Exact equation (14)	Present
0.5	-0.577535	-0.577354
1.0	-0.707107	-0.707107
1.5	-0.774597	-0.774597
2.0	-0.186497	-0.816497
2.5	-0.845154	-0.845154
3.0	-0.866025	-0.866025

Figures 2 and 3 captured the velocity of the mixture of copper with Casson fluid $(\beta = 0.5)$ and pure Casson fluid $(\beta = 0)$ together with the effect of aligned angle α_1 and magnetic field parameter M respectively. The present of α_1 and M encourage the enhancement of opposite force or generally known as Lorentz force in the flow region which decelerates the fluid flow and it is noticed from these figures that the velocity is decrease with the increase in both parameters. Moreover, the velocity for Casson fluid with copper tend to achieve the asymptotic behaviour faster than single flow $(\beta = 0)$. It can be concluded the present of copper can also affected the velocity of Casson fluid together with the effect of α_1 and M, thus supress the fluid flow. The magnetic field is demolished if $\alpha_1 = 0$ and $M = 0$ where both parameters are influencing each other as denoted in fifth term of equation (9), so it can be seen from the figures that the curves at that respective value are overlapping.

The variation of particle-fluid interaction parameter β on the velocity and temperature for fluid and dust phases are presented in figures 4-5. The decreasing pattern of velocity and temperatures profiles is revealed in these figures as the β increases (decreases) for dust (fluid) phase. Theoretically, a large β reduces the relaxation time of particle phase and turns to develop the drag force on the fluid in contact with it. It is revealed from the figures that the dust phase will become equilibrium with the fluid phase for the bigger β. At this stage, the velocity and temperatures profiles for both phases become parallel.

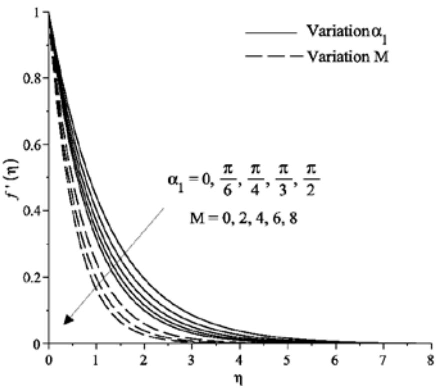

Figure 2. Variation of α_1 and M on velocity profile for pure Casson fluid.

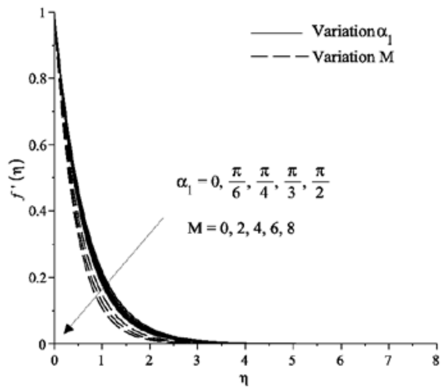

Figure 3. Variation of α_1 and M on velocity profile for dusty Casson fluid.

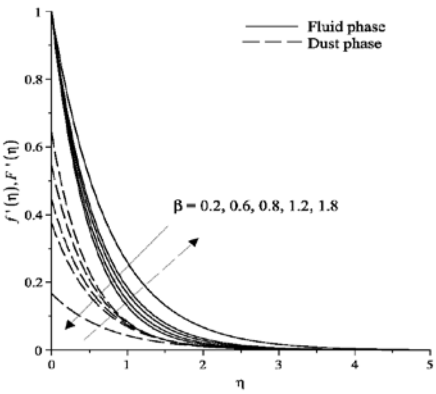

Figure 4. Variation of β on velocity profile.

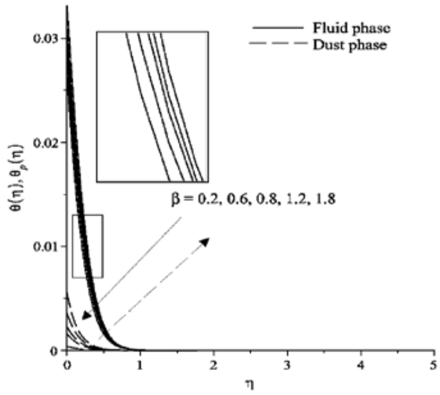

Figure 5. Variation of β on temperature profile.

5. Conclusions

The numerical solution of dusty Cassson fluid model over a heated stretching sheet subjected to NH is presented with the applied aligned magnetic field and buoyance force on the flow region. The governing equations for the fluid and dust phases are studied separately and the obtained numerical results of flow characteristic and heat transfer process for each phases are presented graphically. Based on the present findings, the flow of Casson fluid is affected by the present of dust particle. The understanding of the behaviour of two-phase flow can help to predict the flow processes of the contaminated fluid.

Acknowledgments

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