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AN EVALUATION OF THE AUSTRALIAN ‘RECONCEPTUALISING EARLY MATHEMATICS LEARNING’ PROJECT: KEY FINDINGS AND IMPLICATIONS

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The Pattern and Structure Mathematics Awareness Project (PASMAP) has investigated the development of patterning and early algebraic reasoning among 4 to 8 year olds over a series of related studies. We assert that an awareness of mathematical pattern and structure (AMPS) enables mathematical thinking and simple forms of generalization from an early age. This paper provides an overview of key findings of the Reconceptualizing Early Mathematics Learning empirical evaluation study involving 316 Kindergarten students from 4 schools. The study found highly significant differences on PASA scores for PASMAP students. Analysis of structural development showed increased levels for the PASMAP students; those categorised as low ability developed improved structural responses over a short period of time.

In our PME 28, 29 and 30 research reports we describe a broad descriptive study of 103 first graders and 16 longitudinal case studies that found children’s perception and representation of structure generalised across a wide range of mathematical domains. Children’s strategies showing use of pattern and structure were determined from task-based interviews. A high positive correlation (0.944) was found between children’s performance on forty Pattern and Structure Assessment (PASA) tasks, and four stages of structural development: pre-structural, emergent, partial, and structural. Multiplicative structure, including unitising and partitioning, and ‘spatial structuring’, were found as critical to development of pattern and structure. They found not only that each student tended to show a single structural level in all their responses, but also that this level was strongly correlated with the total number of correct responses. They therefore argued that AMPS could be measured using the PASA interview, and that AMPS was indeed associated with mathematical understanding.

At PME 32 we introduced a new evaluation study, Reconceptualising Early Mathematics Learning describing the broad aims, design and instruments and pilot work. The purpose of this paper is to provide a summary of the key findings of the project: the implementation of a structural approach to early mathematics learning through the Pattern and Structure Mathematical Awareness Program (PASMAP) and the Pattern and Structure Assessment (PASA) interview. The theoretical bases for our study and background studies are highlighted in our recent volume (Mulligan, English,

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Mitchelmore & Crevensten, in press) and in our related PME37 symposium presentation, Reconceptualizing Early Mathematics Learning.

THEORETICAL PERSPECTIVE

Virtually all mathematics is based on pattern and structure. By mathematical *pattern*, we mean any predictable regularity involving number, space or measure. Examples are friezes, number sequences, units of measure and geometrical figures. By *structure*, we mean the way in which the various elements are organised and related. Thus, a frieze might be constructed by iterating a single "unit of repeat"; the structure of a number sequence may be expressed in an algebraic formula; and the structure of a geometrical figure is shown by its various properties. Structural thinking can emerge from, or underlie mathematical concepts, procedures and relationships. Mason, Stephens and Watson (2009) view structural thinking as more than simply recognising elements or properties of a relationship but having a deeper awareness of how those properties are used, explicated or connected.

Early childhood research on pattern and structure

There is an increasing body of research into young children's structural development of mathematics and early algebraic reasoning. Research in the area of number (Hunting, 2003; Mulligan & Vergnaud, 2006; Thomas, Mulligan & Goldin, 2002; van Nes & de Lange, 2007), patterning and reasoning (Clementset al., 2011; English, 2004; Papic, Mulligan & Mitchelmore, 2011), spatial measurement (Outhred & Mitchelmore, 2000), early algebra (Blanton & Kaput, 2005; Carraher, Schliemann, Brizuela, & Earnest, 2006; Warren & Cooper, 2008), and data modelling (English, 2012) have all shown how progress in student's mathematical understanding depends on a grasp of underlying structure.

A suite of studies by Mulligan and her colleagues (Mulligan, 2009) suggested that children who have developed an awareness of structure in one aspect of the early mathematics learning also tend to show a structural awareness in other aspects. Mulligan and Mitchelmore (2009) postulated the existence of a general construct called Awareness of Mathematical Pattern and Structure (AMPS).

The questions naturally arise, is it possible to improve students' AMPS by an appropriate intervention, and if so, does their general mathematical achievement also improve? Mulligan and colleagues developed a Pattern and Structure Mathematics Awareness Program (PASMAP) that focuses explicitly on raising primary school students' awareness of mathematical pattern and structure via a variety of well-connected pattern-eliciting experiences. Studies have included an extensive, whole-school professional development exercise across Kindergarten to Year 6; two year-long, single teacher studies in Years 1 and 2; and an intensive, 15-week individualised program with a small group of low-ability Kindergarten children (For details, see Mulligan, 2009). Many individual cases have been documented showing astonishing changes in children's structural awareness and development of mathematical concepts well beyond that expected for their age level. Some evidence has emerged that PASMAP also has an effect on their scores on independent mathematics assessments. More importantly the PASAMP aims to promote simple or 'emergent generalisation' in young children's mathematical thinking across a range of concepts.

The studies cited above lend strong support to the hypothesis that teaching young children about pattern and structure should lead to a general improvement in the quality of their mathematical understanding. However, none of the studies had a sufficiently large or representative sample, most lacked a comparison group and there was insufficient opportunity to track and describe in depth, the growth of structural development. The current study was therefore designed to evaluate the effects of PASMAP on student mathematical development in the first year of formal schooling.

METHOD

Participants: A purposive sample of four large primary schools, two in Sydney and two in Brisbane, representing 316 students from a diverse range of socio-economic and cultural contexts, participated in the evaluation throughout the 2009 school year. Two different mathematics programs were implemented: in each school, two Kindergarten teachers implemented the PASMAP and two implemented their standard program. The PASMAP framework was embedded into the standard Kindergarten mathematics curriculum, enabling schools to meet the required system-based learning outcomes for New South Wales and Queensland, respectively.

Procedure: Two different mathematics programs were implemented: In each school, two Kindergarten teachers implemented the PASMAP and two implemented their regular program. A researcher visited each teacher on a weekly basis and equivalent professional development was provided for all teachers. The PASMAP framework was embedded within but almost entirely replaced the regular Kindergarten mathematics curriculum. Features of PASMAP were introduced by the research team incrementally, at approximately the same pace for each teacher, over three school terms (May-December 2009). However, implementation time varied considerably between classes and schools, ranging from one 40-minute lesson per week to more than five 1-hour lessons per week.

Assessment Interviews and Classroom Data

All students were administered the I Can Do Maths (ICDM) standardized test of general mathematics achievement (Doig & de Lemos, 2000) at the beginning and end of the 2009 school year and again in mid-2010. From the pre-test data, two focus groups were selected in each class consisting of five students from the upper and lower quartiles, respectively. These students were interviewed in more detail using the PASA in February 2009, December 2009, and September 2010, the number of students varying from 190 to 170. An additional "extension" version of PASA was also administered in September 2010. The PASA items were parallel on all three occasions, but increased somewhat in complexity to take account of students' development.

Other evaluation data included video for a sample of PASMAP lessons for evidence of AMPS and students' articulation of emergent generalizations. Analysis focused on the high ability and low ability focus students. Students' explanations and drawn representations, and photos of their responses to tasks were collected during the implementation of PASMAP and were coded immediately after each lesson for level of structural development.

RESULTS

Quantitative outcome analysis

Analysis of the various PASA and ICDM scores showed the expected differences between ability levels and confirmed the equivalence of the two program groups. There was, however, a significant difference between the schools, with classes in the two Brisbane schools scoring lower than those in the two Sydney schools. No significant interactions were observed.

Table 1 Analysis of covariance of PASA scores at retention point

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1048.432[a]	17	61.672	10.380	.000
Intercept	53.229	1	53.229	8.959	.003
Covariate: PASA	158.346	1	158.346	26.650	.000
Covariate: ICDM	14.071	1	14.071	2.368	.126
School	117.125	3	39.042	6.571	.000
Ability	15.259	1	15.259	2.568	.111
Treatment	61.653	1	61.653	10.376	.002
School * Ability	11.643	3	3.881	.653	.582
School * Treatment	43.663	3	14.554	2.450	.066
Ability * Treatment	.217	1	.217	.037	.849
School * Ability * Treatment	13.589	3	4.530	.762	.517
Error	802.130	135	5.942		
Total	13412.000	153			
Corrected Total	1850.562	152			

R Squared = .567 (Adjusted R Squared = .512)

Total scores on the PASA and ICDM administered at the end of the intervention (December 2009) and at the retention point (September 2010) among the focus students were analysed using analysis of covariance (ANCOVA). In each case, the covariates were the initial PASA and ICDM scores and the factors were school (one of four), ability (high vs. low) and program (PASMAP vs. non-PASMAP).

Analysis of the ICDM scores indicated no significant interactions or main effects apart from a school effect. In other words, the PASMAP and regular students made very similar gains on ICDM over the period of the study, but Sydney students gained more. The analysis of the PASA scores also showed no significant interactions. However, there were two significant main effects at each point: a difference between schools, with the Sydney classes showing higher adjusted means than the Brisbane classes, and a difference between the program groups on each PASA assessment—modest at the end of the intervention ($p < 0.026$), highly significant at the retention point ($p < 0.002$), but only borderline ($p > 0.11$) for the extension section of the PASA. On each occasion, the PASMAP group scored higher than the regular group. Table 1 provides a summary the ANCOVA for the PASA at the retention point. We inferred that the PASMAP treatment was effective in promoting the conceptual understanding of early mathematics, as measured by the PASA but not in improving mathematical achievement as measured by ICDM.

Rasch scale analysis

The PASA total scores and the ICDM scores were used to construct a single Rasch scale that incorporated all items along a continuum. The main advantage of using Rasch analysis for constructing the PASA scale was that it could be used to link different versions of the PASA used in this study. The item map indicated that the PASA items and the students were reasonably well matched; in comparison, the ICDM items at the lower end of the scale did not sufficiently challenge the majority of students, although some more difficult ICDM items filled a gap between the PASA items. The scale's order of item difficulty on PASA items provided a measure of the students' overall level of AMPS. Thus a conceptual analysis of the item and its position on the scale reflected the complexity of the task in terms of pattern and structure as well as the reasoning required to complete it successfully. What we aimed to achieve with the scale was a picture of how the PASA measure of AMPS fitted with a standardized measure of general numeracy ability over time.

Structural outcomes analysis

To supplement the quantitative analysis, we provide some examples of the analysis of structural levels. Student responses on four PASA items requiring a drawn response at the three administrations were systematically coded for level of structural development (see Chapter 2). Coding showed an inter-rater reliability of 0.91. Figure 3.10 summarizes the results for the Sydney students. It can be seem that the PASMAP students were initially slightly more advanced than the regular program students, with about 5% more students in the partial structure and structural levels than the regular students. However, this difference grew in the subsequent administrations, reaching about 20% at the retention point.

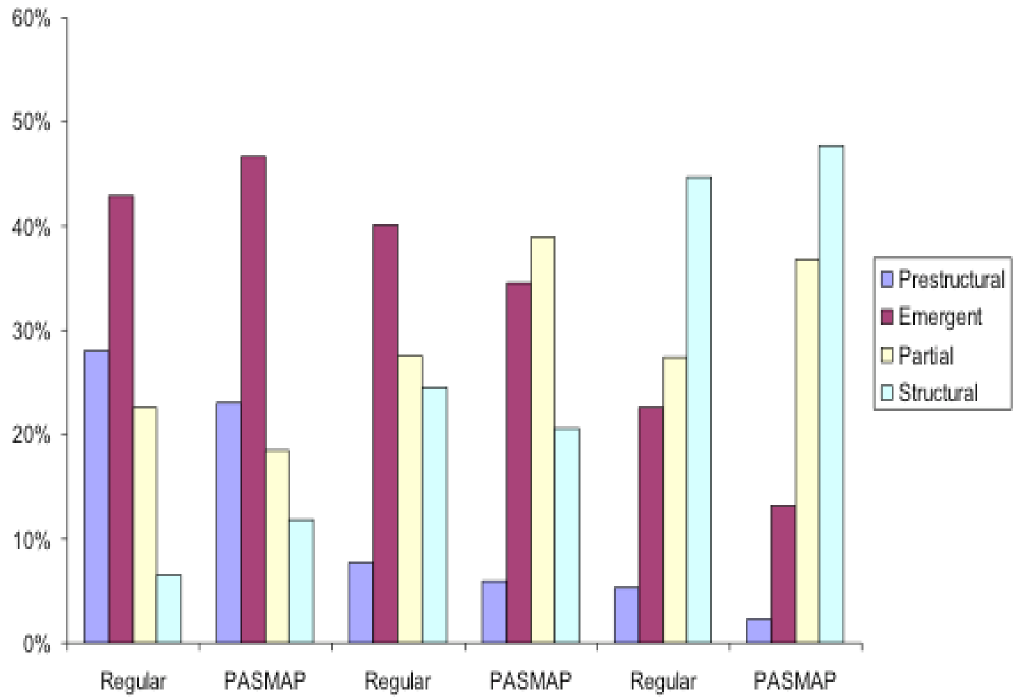

Figure 1. Structural development across selected PASA items at three interview points Feb 2009 (Pre-intervention), Dec 2009 (Post-intervention), Sept 2010 (Retention) in two Sydney schools.

DISCUSSION

The study produced a valid and reliable interview-based measure and a scale of AMPS that revealed new insights into students' mathematical capabilities at school entry. Clearly young students were able to solve a broad range of novel mathematical tasks, including repetitions and growing patterns, and multiplicative problems, not usually asked of students of this age.

PASMAP explicitly focused on the promotion of students' awareness of pattern and structure (AMPS). Particular gains were noted in the related areas of patterning, multiplicative thinking (skip counting and quotition), and rectangular structure (regular covering of circles and rectangles). As expected, a focus on pattern, structure, representation, and emergent generalisation advantaged the PASMAP students. However, students in the regular program were also able to elicit structural responses but had not been given opportunities to describe or explain their emergent generalised thinking that may have been developing. Thus, it was not possible to determine whether more advanced examples of structural development could be directly attributed to the program or innate developmental advances of more able students. One of the most promising findings was that the focus students categorised as low ability were able to develop structural responses over a relatively short period of time. Further analysis of the impact of PASMAP on structural development must consider individual teacher effect and school-based approaches to evaluate the program's scope and depth of achievement.

Our research has established that a large amount of children's mathematical thinking in early childhood can be described in terms of a growing awareness of pattern and structure. We have shown that children's levels of structural development can be reliably categorised, and that each child tends to be at the same level on different tasks typical of the early mathematics curriculum. This finding has led us to formulate the construct of Awareness of Mathematical Pattern and Structure (AMPS) that is prominent in children who achieve highly in mathematics in school and low in those who do not progress easily or develop learning difficulties. We regard the AMPS construct as a significant contribution to research into early childhood education. It provides a lens with which to examine children's thinking at a fundamental level and, in particular, to assess the deeper effects of early mathematics teaching.

FURTHER RESEARCH

We aim to explore further aspects of AMPS: the possibility that low AMPS in early childhood could predict poor performance in mathematics throughout schooling, particularly in relation to algebraic thinking. Extending the AMPS construct to the later years of schooling will involve studies of learning trajectories of students beyond the early years of schooling whose mathematical and scientific reasoning is enhanced by a structural approach. Our interest also lies in the application of the PASMAP approach to assisting those students with special needs; students with low levels of AMPS who may be prone to difficulties in learning mathematics and students with advanced AMPS who are able or gifted at mathematics (Mulligan, 2011).

A new phase of the research program is currently in progress, *Transforming Children's Mathematical and Scientific Development,* enabling the extension and application of this study utilising the same research team. This 3-year longitudinal study integrates the PASMAP pedagogical approach through novel experiences in data modelling and problem solving linked to the work of colleague Lyn English (English, 2012). An emphasis is placed on developmental features of how students structure data. The study tracks three cohorts of students initially employed in the *Reconceptualizing Early Mathematics Learning* project when in Kindergarten, through to Grades 2, 3 and 4. Two new cohorts of mathematically able students are being tracked from Kindergarten to Grade 2.

Other research applications include the *Patterns and Early Algebra* (PEAP) *Professional Development* (PD) *Program* (Papic, in press) that focuses on young children's patterning, early algebraic and mathematical thinking skills with the aim of closing the gap in numeracy achievement for Indigenous children in rural and regional early childhood settings in New South Wales state of Australia.

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