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Steady Aligned MHD Free Convection of Ferrofluids Flow over an Inclined Plate

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ABSTRACT

Numerical investigation is carried out for the MHD free convection laminar boundary layer flow that allows heat transfer of an electrically conducting Fe_3O_4–kerosene and Fe_3O_4–water-based ferrofluids. For this, an inclined plate is employed that has aligned magnetic effect as well as transverse magnetic field effect. Suitable similarity transformations are used to convert governing partial differential equations into coupled nonlinear ordinary differential equation. The Keller Box method, a well-known explicit finite difference scheme, is then employed to solve transformed equations numerically. For different values of physical parameters, a detailed parametric study is conducted. Means of graphs are extrapolated to determine the effects of all these parameters over temperature and the flow field. For various values of physical parameters, the numerical values are obtained and tabulated for skin friction coefficient and the rate of heat transfer as well. The results when compared with previously published studies were found to be in excellent agreement.

Keywords: *Ferrofluids, MHD, free convection, inclined plate, aligned and transverse magnetic field*

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Introduction

Nanofluids have wide application in various fields and have provided significant importance towards enhancement of heat transfer. Their applications include desalination, cavity problem, solar thermal collector, and so on [1–3]. Ferrofluids are magnetic nanofluids, which consist of a colloidal mixture of magnetic nanoparticles with size in the range of 10–20 nm and base liquid. Ferrofluids’ magnetic features are comparable to those of bulk magnetic materials and they allow retaining common Newtonian fluids’ flow characteristic. To provide ferrofluids with stability, surfactant is used to coat each tiny ferroparticle thoroughly. As indicated by several studies, outstanding heat transfer enhancement can be achieved by employing nanofluids as coolants when compared with ordinary fluids [4–5].

Magnethohydrodynamic (MHD) deals with the study of electrically conducting fluids that can move around in a magnetic field. An electric current is induced in the fluid when there is a change in the magnetic field that cuts the moving fluid [6]. Free convection flow that has magnetic field influence has caught the attention of many researchers with regards to their applications in modern material processing where magnetic fields can be employed to meticulously control and manipulate electrically conducting materials. Studies have been reviewed for laminar flow over an inclined plate. It should be noted that in this line, various investigations are conducted. An incompressible nanofluid’s steady mixed convection boundary layer flow was investigated by Ranaet al. [7] along an inclined plate that was embedded in a porous medium. In their study, it was observed that Nusselt number decreased with rise in thermophoresis number or Brownian motion number, whereas increasing the plate’s angle led to increase in Nusselt number. Also, the free convection flow from an isothermal plate that was inclined horizontally at a small angle was studied by Hossainet al. [8]. Recently, the numerical solution of MHD mixed convection nanofluid flow was analysed by Anjali Devi and Suriyakumar [9] over an inclined stretching plate. They considered the heat generation and suction effects. Thus, it was concluded that the non-dimensional velocity increases with increase in the inclination angle. However, the aim of the inclination angle effect was to decrease the temperature. Numerous authors (see [10-17]) have investigated the issues associated with the MHD boundary layer by considering different effects.

Many industrial applications, natural processes and chemical processing systems are faced with the free convection processes that involve combination mechanism of mass and heat transfer. An extensive research has been done to study free convective mass transfer flow as numerous engineering applications like cooling of nuclear reactors, rocket nozzles, high-speed aircraft and their atmospheric re-entry, high sinks in turbine blades, process equipment and chemical devices require an understanding of the effects of heat transfer along with mass transfer effects. S. Ostrach [18], who started the convection flow study, used an integral method to make a technical note on transient free convection flow's similarity solution past a semi-infinite vertical plate. Ingham et al. [19] studied the unsteady free convection flow on an isothermal surface near the attachment's three-dimensional stagnation point. Merkin and Mahmood [20] considered a vertical plate for examining free convection boundary layer with prescribed surface heat flux. Although many papers mention studies related to the flow and heat transfer of natural convection over embedded bodies using different media, only a few have considered nano- and microfluids.

However, to the extent of authors' knowledge, attempts are yet to be made to address the issues of MHD free convection heat transfer flow regarding ferrofluids over an inclined plate with transverse and aligned magnetic field. Hence, a study was conducted to understand free convection boundary layer flow over an inclined plate for two ferrofluids, namely Fe_3O_4-kerosene and Fe_3O_4-water. To solve the normalised boundary layer equations, an efficient Keller Box method is employed and an elaborate discussion is provided for the effects of material parameters on the heat transfer characteristics and flow field.

Mathematical Model

Consider the steady two-dimensional, incompressible, laminar, hydromagnetic free convection of ferrofluids flow over an inclined plate with aligned and transverse magnetic field. The plate is inclined at an angle of inclination γ measured in the clockwise direction and situated in an otherwise quiescent ambient fluid at temperature T_∞. The gravitational acceleration **g** is acting downward. The physical coordinates $\left(x, y\right)$ are chosen such that *x*-axis is chosen along the plate and the *y*-axis is measured normal to the surface of the plate (Figure 1). Water and kerosene are used as the base fluids with magnetite $\left(\mathrm{Fe_3O_4}\right)$ as a ferroparticle. The base fluids and ferroparticles are in thermal equilibrium and no slip occurs between them. The spherical shaped ferroparticles are considered. The viscous dissipation and radiation are neglected in the analysis.

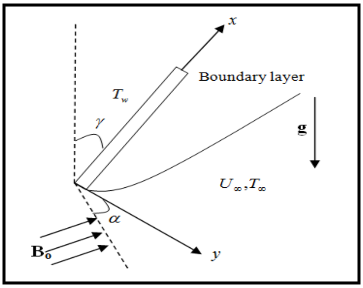


Figure 1: Geometry of the problem.

Under the above assumptions and following Tiwari and Das [21], the equations of MHD boundary layer flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}+\frac{(\rho\beta)_{nf}}{\rho_{nf}}g\cos\gamma\left(T-T_\infty\right)-\frac{\sigma B^2(x)}{\rho_{nf}}\sin^2\alpha\left(u-U_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions for the velocity and temperature of this problem are given by

$$\begin{aligned} &u(x,0)=0, \quad v(x,0)=0, \quad T(x,0)=T_w \\ &u(x,\infty)=U_\infty, \qquad\qquad T(x,\infty)=T_\infty \end{aligned} \tag{4}$$

where u and v are the x (along the plate) and the y (normal to the plate) component of velocities, respectively, T is the temperature of the ferrofluids, T_w is the ambient temperature of the ferrofluids, U_∞ is the constant free stream velocity and σ is the electrical conductivity. The transverse magnetic field assumed to be a function of the distance from the origin is defined as $B(x)=B_0x^{-\frac{1}{2}}$ with $B_0\neq 0$, where x is the coordinate along the plate and B_0 is the magnetic field strength. The effective properties of ferrofluids may be expressed in terms of the properties of base fluids, ferroparticles and the volume fraction of solid ferroparticles as follow [4]

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\qquad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},$$
$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s,\quad (\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s, \tag{5}$$
$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}},\qquad \frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}$$

where ρ_{nf} is the effective density, ϕ is the solid volume fraction, ρ_f and ρ_s are the densities of pure fluid and ferroparticles, respectively, μ_f is the dynamic viscosity of the base fluids, μ_{nf} is the effective dynamic viscosity, $(\rho C_p)_{nf}$ is the heat capacity of the ferrofluids, $(\rho C_p)_f$ is specific heat parameters of the base fluids, $(\rho C_p)_s$ is the specific heat parameters of ferroparticles, $(\rho\beta)_{nf}$ is the thermal expansion coefficient, α_{nf} is the thermal diffusivity of the ferrofluids, k_{nf} is the thermal conductivity of the ferrofluids, $(\rho C_p)_{nf}$ is the heat capacity of the ferrofluids and k_f and k_s are thermal conductivities of the ferrofluids and ferroparticles.

The continuity Eq.(1) is satisfied by introducing a stream function $\psi(x,y)$ such as

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x} \tag{9}$$

The following similarity variables are introduced

$$\eta=y\sqrt{\frac{U_\infty}{\nu_f x}}=\frac{y}{x}\sqrt{\mathrm{Re}_x},\ \psi=\nu_f\sqrt{\mathrm{Re}_x}\,f(\eta),\ \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{10}$$

where η is the similarity variable, $\mathrm{Re}_x=U_\infty x/\nu_f$ is the Reynolds number, $f(\eta)$ the non-dimensional stream function and $\theta(\eta)$ the non-dimensional temperature.

On the use of (5), (9) and (10), Eqs (2) and (3) reduce to the following nonlinear system of ordinary differential equations:

$$f''' + (1-\phi)^{2.5}\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)\frac{1}{2}ff'' + (1-\phi)^{2.5}M\left(1-f'\right)\sin^2\alpha + (1-\phi)^{2.5}\left(1-\phi+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\right)Gr_x\theta\cos\gamma = 0 \quad (11)$$

$$\left(\frac{k_{nf}}{k_f}\right)\theta'' + \frac{\Pr}{2}\left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)f\theta' = 0 \quad (12)$$

Subjected to the boundary conditions (4) which become

$$\begin{aligned} &f(0)=0, \quad f'(0)=0, \quad \theta'(0)=1 \\ &f'(\eta)=1, \quad \theta(\eta)=0, \quad \text{as } \eta\to\infty \end{aligned} \quad (13)$$

Where primes denote differentiation with respect to η , $M=\sigma B_0^2/\rho U_\infty$ is the magnetic parameter, $Gr_x = g\beta_f\left(T_w-T_\infty\right)x/U_\infty{}^2$ is the local Grashof number and $\Pr = \left(\mu C_p\right)_f/k_f$ is the Prantl number. In order to have a true similarity solution, the parameter Gr_x must be constant and independent of *x*. This condition will be satisfied if the thermal expansion coefficient β_f proportional to x^{-1} . Hence, assume [22], $\beta_f = ax^{-1}$ where *a* is a constant but have the appropriate dimension. Substituting $\beta_f = ax^{-1}$ into the parameter Gr_x will result $Gr = \dfrac{ag\left(T_w-T_\infty\right)}{U_\infty{}^2}$. The quantities of engineering interest are the skin-friction coefficient, C_f at the surface of the plate and local Nusselt number, Nu_x which are defined as:

$$C_f = \frac{\tau_w}{\rho_f U_\infty{}^2}, \quad Nu_x = \frac{xq_w}{k_f\left(T_w-T_\infty\right)} \quad (14)$$

where τ_w is the wall skin friction or shear stress at the plate and q_w is the heat flux from the plate, which given by:

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (15)$$

Substituiting (10) into (15) and using (14),

$$\frac{C_f}{(\mathrm{Re}_x)^{\frac{1}{2}}} = \frac{1}{(1-\phi)^{2.5}}f''(0), \quad \frac{Nu_x}{(\mathrm{Re}_x)^{-\frac{1}{2}}} = -\frac{k_{nf}}{k_f}\theta'(0) \quad (16)$$

Numerical Solution

Equations (11) and (12) subject to the boundary conditions (13) are solved numerically using Keller-box method as described in the books by Na [23] and Cebeci and Bradshaw [24]. The solution is obtained in the following four steps.

(i) Reduce (11) and (12) to first-order system.
(ii) Write the difference equations using central differences.
(iii) Linearize the resulting algebraic equations by Newton's method and write them in the matrix-vector form.
(iv) Solve the linear system by the block tridiagonal elimination technique.

Results and Discussion

To the system's pertinent parameters, physically realistic numerical values were assigned to gain meaningful insights from the flow structure relating to temperature, velocity, reduced Nusselt number and skin friction coefficient. For this, two different base fluids are considered, namely kerosene and water with magnetic ferroparticle, Fe_3O_4. Table 1 is referred to know the thermophysical properties of kerosene, water and Fe_3O_4.

The Prandtl number values for the base fluids, water and kerosene, are taken as 6.2 and 21 [25, 26], respectively. The solid ferroparticle's, ϕ, volume fraction effect is studied by considering the range $0 \le \phi \le 0.2$, where $\phi = 0$ signifies pure fluid water or kerosene. To validate the numerical method's accuracy, a direct comparison was made with the previously reported numerical results of Blasius [27] and Khan et al. [4] for Fe_3O_4-kerosene and Fe_3O_4-water, and aligned magnetic field parameter and in the absence of free convection parameter. Based on Table 2, the present results are observed to be in good agreement with those of the earlier findings.

The values of Prandtl number number for the base fluids, water and kerosene are taken to be 6.2 and 21 [25,26], respectively. The effect of the volume fraction of solid ferroparticle ϕ is studied in the range $0 \le \phi \le 0.2$, where $\phi = 0$ represents the pure fluid water or kerosene. The accuracy of the numerical method was validated by direct comparisons with the numerical results reported ealier by Blasius [27] and Khan et al. [4] for Fe_3O_4-water and Fe_3O_4-kerosene and in absence of free convection parameter and aligned magnecticfield parameter. From Table 2, the present results are in good agreement with the earlier findings.

Table 1 : Thermophysical properties of base fluids and ferroparticle [25,26].

Physical Properties	Water	Kerosene	Fe_3O_4
$\rho\left(kg/m^3\right)$	997.1	780	5200
$C_p\left(J/kgK\right)$	4179	2090	670
$k\left(W/mK\right)$	0.613	0.149	6
$\beta\times10^{-5}\left(K^{-1}\right)$	21	99	1.3

Table 2 : Comparison of the skin friction coefficient for different values of volume fraction of ferroparticles.

	Volume Fraction	Skin Friction $\left(Gr_x=0,\alpha=90^\circ,\gamma=0^\circ,M=0\right)$		
		Blasius [27]	**Khan *et* al.[4]**	**Present Study**
Pure Water	0	0.3321	0.33206	0.332059
Fe_3O_4-water	0.01	-	0.34324	0.343271
	0.05	-	-	0.389577
	0.1	-	0.45131	0.451635
	0.15	-	-	0.519813
	0.2	-	0.59517	0.595192
Pure Kerosene	0	-	-	0.332059
Fe_3O_4-kerosene	0.01	-	0.34557	0.345611
	0.05	-	-	0.400879
	0.1	-	0.47336	0.473745
	0.15	-	-	0.552809
	0.2	-	0.63950	0.640265

Numerical solutions are carried out for various values of physical parameters such as $\alpha=0,45^\circ,70^\circ,90^\circ$, $M=0,1,2,4$, $\gamma=0^\circ,45^\circ,60^\circ,90^\circ$, $Gr_x=0,0.1,2,3$ and $\phi=0,0.1,0.2$. Numerical computations of results are demonstrated through graphs over the flow field and temperature. Further, skin friction coefficient and the non-dimensional rate of heat transfer are found out and are presented by means of tables.

Figure 2 shows the effect of inclined angle of magnetic field on velocity and temperature profiles of both Fe_3O_4-water and Fe_3O_4-kerosene ferrofluids. It can be concluded that, an increase in the aligned angle enhances the velocity profiles and reduce the temperature profiles for both ferrofluids.

Further it is observed that increasing in α causes a decline in the momentum boundary layer and the thermal boundary layer for both ferrofluids. This is due to the fact that a raise in the value of aligned angle

($0° \leq \alpha \leq 90°$) to the plate strengthen the applied magnetic field. At $\alpha = 90°$, this aligned magnetic field acts like transverse magnetic field and ferroparticles are attracted by the magnetic field due to change in the positions of aligned magnetic field. It can be seen from Figure 3, the fluid velocity profiles increase monotonically with increase of magnetic field parameter while temperature decrease. Furthermore, momentum and thermal boundary layer decreases for both ferrofluids.

Figure 4 reveals there is a deceleration in the ferrofluids velocity as the inclination of plate γ increases. For $\gamma = 90°$, the plate is horizontal and for $\gamma = 0°$, the plate assumes a vertical position. The gravitational effect is minimum for $\gamma = 90°$ and maximum for $\gamma = 0°$. The momentum and thermal boundary layer increase with an increase of γ.

The positive values of local Grashof number, $Gr_x > 0$ is utilised in the computations. This corresponds to the cooling problem with respect to the application. The cooling problem is often encountered in engineering application. From Figure 5, the momentum boundary layer and thermal boundary layer thickness decreases with an increase in the value of local Grashof number (Gr_x) due to buoyancy effect.

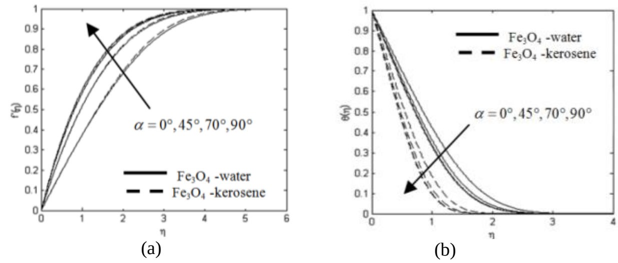


Figure 2: Effect of aligned magnetic field parameters on the velocity and temperature profiles for $M = 1, \gamma = 45°, Gr_x = 0.1$ and $\phi = 0.1$.

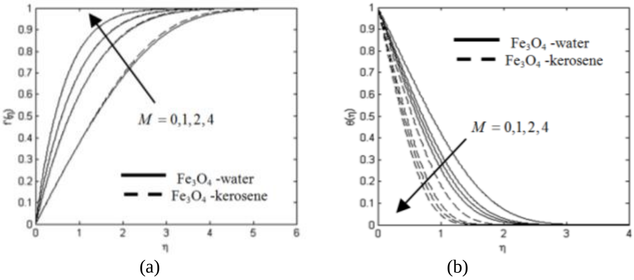
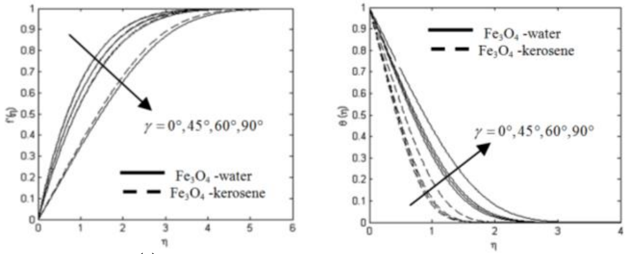
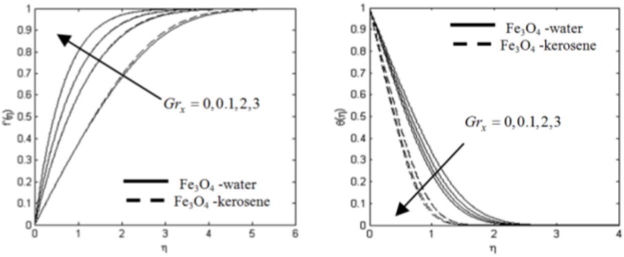


Figure 3: Effect of magnetic strength parameters on the velocity and temperature profiles for $\alpha = 90°, \gamma = 45°, Gr_x = 0.1$ and $\phi = 0.1$.



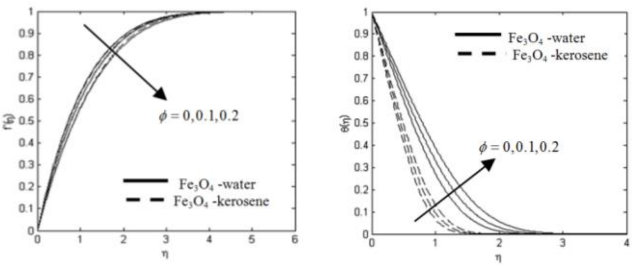
(a) (b)

Figure 4: Effect of inclined plate parameters on the velocity and temperature profiles for $M = 1, \alpha = 90°, Gr_x = 0.1$ and $\phi = 0.1$.



(a) (b)

Figure 5: Effect of local Grashof number on the velocity and temperature profiles for $M = 1, \alpha = 90°, \gamma = 45°$ and $\phi = 0.1$.



(a) (b)

Figure 6: Effect of volume fraction of ferroparticles on the velocity and temperature profiles for $M = 1, \alpha = 90°, \gamma = 45°$ and $Gr_x = 0.1$.

From Table 3, one can notice that the skin friction coefficient at the wall increase in magnitude with an increase in aligned magnetic field, magnetic strength, Grashrof number and volume fraction of ferroparticles for both ferrofluids. It is noticed that, the highest wall shear stress occurs when Grashof number increase. It is observed from Table 4, similar pattern occurs for Nusselt number and the highest rate of heat transfer occurs when volume fraction of ferroparticles increase. Heat transfer rate in Fe_3O_4- kerosene is more compare to Fe_3O_4- water based on Nusselt Number.

Table 3: Variation in skin friction for both ferrofluids at different parameters.

α	M	γ	Gr_x	ϕ	**Fe_3O_4-water**	**Fe_3O_4-kerosene**
0°	1	45°	0.1	0.01	0.402060	0.386466
45°					0.724095	0.713478
70°					0.896690	0.887523
90°					0.943150	0.934307
90°	0	45°	0.1	0.01	0.402060	0.386466
	1				0.943150	0.934307
	2				1.272749	1.265699
	4				1.754343	1.748982
90°	1	0°	0.1	0.01	1.105335	1.093859
		45°			0.943150	0.934307
		60°			0.810664	0.804031
		90°			0.343270	0.345653
90°	1	45°	0	0.01	0.907519	0.908326
			0.1		0.943150	0.934307
			2		1.562914	1.392773
			3		1.859068	1.614443
90°	1	45°	0.1	0	0.928462	0.919212
				0.05	1.005651	0.998455
				0.10	1.093266	1.088136
				0.15	1.193426	1.190262
				0.20	1.308866	1.307421

Table 4: Variation in Nusselt number for both ferrofluids at different parameters.

α	M	γ	Gr_x	ϕ	**Fe_3O_4-water**	**Fe_3O_4-kerosene**
0°	1	45°	0.1	0.01	0.651047	0.981540
45°					0.757053	1.167414
70°					0.799541	1.241693
90°					0.809822	1.259737
90°	0	45°	0.1	0.01	0.651047	0.981540
	1				0.809822	1.259737
	2				0.872270	1.370344
	4				0.940739	1.494548
90°	1	0°	0.1	0.01	0.842419	1.315714
		45°			0.809822	1.259737
		60°			0.779633	1.208362
		90°			0.630621	0.959292
90°	1	45°	0	0.01	0.802862	1.251962
			0.1		0.809822	1.259737
			2		0.915922	1.383890
			3		0.959155	1.436849

90°	1	45°	0.1	0	0.800628	1.235126
				0.05	0.846509	1.359926
				0.10	0.892185	1.489259
				0.15	0.937681	1.623448
				0.20	0.983003	1.762860

Conclusion

The problem of MHD free convection boundary layer flow of a ferrofluids through an inclined plate subjected to magnetic field has been analyzed for Fe_3O_4- water and Fe_3O_4- kerosene ferrofluids. The governing equations associated to the boundary layer condition were transformed to two nonlinear ordinary differential equations with help of similarity transformation equations. The solutions of the problem were numerically solved with the help of Keller Box method. The following results were investigated

(i) The velocity profiles of both ferrofluids increase with an increase in aligned magnetic field angle, magnetic strength and Grashof number. An increase in angle of inclined plate and volume fraction of ferroparticles results in decline of the velocity profiles for both ferrofluids.

(ii) An increase in angle of inclined plate and volume fraction of ferroparticles enhances the temperature profiles of both ferrofluids. The temperature profiles of both ferrofluids decrease with an increase in aligned magnetic field angle, magnetic strength and Grashof number.

(iii) Increasing of all parameters except for angle of inclined plate increase the skin friction and heat transfer rate of both ferrofluids.

(iv) In general, heat transfer rate in Fe_3O_4- kerosene ferrofluid is more compare with Fe_3O_4- water ferrofluid.

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