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# Demand Drives Growth all the Way

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## **Demand Drives Growth all the Way**

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**Abstract:** A demand-driven alternative to the conventional Solow-Swan growth model is analyzed. Its medium run is built around Marx-Goodwin cycles of demand and distribution. Long-run income and wealth distributions follow rules of accumulation stated by Pasinetti in combination with a technical progress function for labor productivity growth incorporating a Kaldor effect and induced innovation. An explicit steady state solution is presented along with analysis of dynamics. When wage income of capitalist households is introduced, the Samuelson-Modigliani steady state “dual” to Pasinetti’s cannot be stable. Numerical simulation loosely based on US data suggests that the long-run growth rate is around two percent per year and that the capitalist share of wealth may rise from about forty to seventy percent due to positive medium-term feedback of higher wealth inequality into its own growth.

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## Introduction

What sets the long run growth path of the economy? Following Robert Solow (1956) and Trevor Swan (1956) the conventional view is that growth is determined by factors contributing to aggregate supply -- capital deepening, population increase, and long-run growth of labor productivity. “Potential output” increases accordingly.<sup>1</sup>

The obvious alternative is to analyze growth from the side of demand. How do effective demand, endogenous productivity growth, and shifting income and wealth distributions influence and constrain the economy in the present and over time?

In addressing this question, the model presented here has seven salient characteristics.

First, as in almost all growth theory, profit income is assumed to flow directly to households (ignoring interest, dividends, capital gains, and all the other channels via which households receive payments from business). *Unlike* mainstream models, our specification initially maintains a class distinction between “capitalists” who receive profits and “workers” who get both labor and capital income. Capitalists save at a higher rate than workers. Wages received by capitalists are briefly discussed toward the end of the paper.

Second, growth models distinguish between “fast” and “slow” (or “state”) variables. The former vary in a “short” to “medium” run. They include capital utilization ( $u$  in what follows), the profit rate ( $r$ ), the investment/capital ratio ( $g$ ), the employment rate relative to population ( $\lambda$ ) etc. Time-derivatives of state variables such as the capital/population ratio ( $\kappa$ ), endogenous labor productivity ( $\xi$ ), and the share of wealth

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<sup>1</sup> Similar comments apply to Ramsey-type models which basically add a fancier saving function to Solow-Swan.

held by capitalists ( $Z$ ) are also determined in the short run. The state variables then cumulate over time. Contemporary mainstream models typically do not address dynamics of  $\xi$  and  $Z$ .

Third, in all time frames, mainstream models presuppose full employment of labor and capital and the existence of an aggregate production function with associated marginal productivity conditions which determine income distribution. In contrast, we assume that  $u$ ,  $r$ , and other fast variables are determined by interaction between functions  $u(r \dots)$  for effective demand and  $r(u \dots)$  for distribution. Both relationships have parameters included and also depend on  $\kappa$ ,  $\xi$ , and  $Z$ .

Fourth, as opposed to the Solow-Swan assumptions the specifications of  $u(r \dots)$  and  $r(u \dots)$  are based on observed business cycle behavior in rich economies. To reduce dimensionality, however, we suppress cyclical behavior in growth analysis and assume that *levels* of  $r$  and  $u$  are set by the joint solution of  $u(r \dots)$  and  $r(u \dots)$ .

Fifth, dynamics of aggregate capital  $K$  (measured at cost) are driven by real net investment. At prevailing output levels, capital is *not* a scarce factor of production subject to decreasing returns. Rather, its *level* sets the scale of the macro system. Its *growth* stimulates technical change.

Sixth, even though we do not assume full employment or decreasing returns to capital, dynamics of  $\kappa$  drive the state variables toward a steady state at which their growth rates would be equal at a level largely determined by population and productivity growth. We maintain the standard assumption that the growth rate of population ( $n$ ) is exogenous.

Seventh, away from the steady state, levels of fast variables are determined by the demand and distribution functions with their associated parameters. If the system were *at* a steady state (which will not be attained in finite time), equalized growth rates would override the effects of some demand-side parameters on levels of  $\kappa$ ,  $\xi$ , and  $Z$ . But demand does lead growth “all the way” toward the steady state.

Finally, we are dealing here with a fairly complicated system. Its behavior will to a large extent be described in terms of diagrams and signs of partial derivatives. More detailed analysis in terms of equation specifications and parameters will be provided in footnotes as we proceed.

Our specification draws freely on the works of several Keynesian economists, notably from the University of Cambridge. In the short run aggregate demand and distribution interact according to Richard Goodwin’s (and ultimately Karl Marx’s) model of cyclical growth in which a tighter labor market leads to a higher wage share and lower profit rate. Distribution influences demand via differential saving rates across classes and profitability figures in the determination of planned investment.

Over time, Nicholas Kaldor’s technological progress function along with induced innovation describe how productivity growth responds to the installation of new capital and shifts in the income distribution. Luigi Pasinetti pioneered the theory of wealth inequality. We adopt his approach by working with two distinct classes and tracing their wealth holdings over time.

There can be sustained growth satisfying Kaldor’s stylized facts (in many ways still the gold standard of growth theory) with the capitalists’ share of wealth settling

between zero and one. We present illustrative numerical simulations of how economic activity and wealth concentration may change over time en route toward a steady state.

### **Kaldor's and other stylized facts**

Sixty years ago, Nicholas Kaldor (1957) described six characteristics of economic growth. These “stylized facts” are still deemed the minimum requirement for any growth model. Kaldor pointed out that “over long periods”:

- i. labor productivity,  $\xi$ , grows at a steady exponential rate  $\hat{\xi} = (d\xi/dt)/\xi$  (with  $\xi = X/L$  where  $X$  is output and  $L$  employment);*
- ii. the ratio of capital to the population,  $\kappa$ , grows at a steady rate  $\hat{\kappa}$  (with  $\kappa = K/N$ );*
- iii. the profit share  $\pi$  is stable;*
- iv. the profit rate  $r$  is stable (with  $r = \pi X/K = \pi u$ );*
- v. the ratio of output to capital,  $u = X/K$ , is stable;*
- vi. the real wage,  $\omega$ , grows at the same rate as labor productivity.*

We can add

- vii. the employment ratio,  $\lambda = L/N$ , is stable in the long run;*
- viii. in national accounts data, undistributed business profits and corporate taxes are major sources of saving (both at a rate of 100%); distributed profits as well as capital gains on equity flow predominantly to high income households who have substantially higher saving rates than those further down in the size distribution whose incomes mostly come from wages (and fiscal transfers);*

*ix.* In the  $(u, \pi)$  plane for rich economies there is an observed clockwise business cycle around a stationary point with  $\pi$  leading  $u$  as the economy emerges from a trough or swings down from a peak.

### Model design

Stylized fact (*ix*) is the basis for a model of the medium run. The wage share  $\psi = 1 - \pi = \omega/\xi$  falls as the economy emerges from a slump -- the real wage stagnates while productivity grows. If investment demand responds to higher profits, capital utilization  $u$  and employment  $L = X/\xi$  rise. With  $\pi$  and  $u$  both increasing,  $r$  goes up as well. A tighter labor market ultimately bids  $\omega$  and  $\psi$  up. Profits are squeezed and firms implement labor-saving technical change. A downswing or “crisis” ensues.<sup>2</sup>

This cycle narrative appears in Marx’s *Capital* and *Theories of Surplus Value*, and was formalized by Goodwin (1967). For present purposes we suppress cyclicity and extend Goodwin’s relationships between income distribution and effective demand. Specifically,  $r$  responds negatively to  $\lambda$  (high-employment profit-squeeze) and  $u$  responds positively to  $r$  (profit-led demand).

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<sup>2</sup> The idea that the wage/profit distribution can influence effective demand traces back to the *General Theory* (John Maynard Keynes, 1936) and Josef Steindl (1952). Beginning with papers by Robert Rowthorn (1982) and Amitava Dutt (1984) the distribution vs demand linkage has been under active discussion. Amit Bhaduri and Stephen Marglin (1990) is an influential summary. Following Keynes’s (1939) repudiation of a counter-cyclical real wage, the mainstream version of dependence of distribution on the level of activity eventually emerged as a real wage Phillips curve. Econometric evidence about Marx-Goodwin cycles appears in Nelson Barbosa-Filho and Lance Taylor (2006), Peter Flaschel (2009), and David Kiefer and Codrina Rada (2015).

Observations (i), (ii), (v), and (vii) all apply to real variables at a steady state. Three level variables evolve over time – the total capital stock  $K$  (or alternatively the capital/population ratio  $\kappa$ ), the quantity of capital controlled by capitalists  $K_c$ , and labor productivity  $\xi$ . A steady state can be characterized by constant values of two ratio variables. One is  $Z = K_c/K$  or the capitalists' share of wealth. The other is  $\zeta = \kappa/\xi = \lambda/u$  or the ratio of capital “depth” to productivity which also equals the ratio of the employment rate to capital utilization. Constant  $Z$  and  $\zeta$  respectively imply that the pairs  $K_c$  and  $K$ , and  $\kappa$  and  $\xi$  change at the same exponential rate.

Growth of capital depth  $\hat{\kappa}$  is driven by the investment/capital ratio  $g = I/K$  with  $g$  responding positively to  $r$  and  $u$ . (In standard notation for any variable  $x$ ,  $\dot{x} = dx/dt$  and  $\hat{x} = \dot{x}/x$ .)

Productivity growth  $\hat{\xi}$  can be modeled following Kaldor's demand-side explanations. Over the years he introduced two versions of a “technical progress function.” In the first (Kaldor, 1957)  $\hat{\xi}$  is driven by  $\hat{\kappa}$ , with investment serving as a vehicle for more productive technology. The second (Kaldor, 1966) ties productivity growth to the output growth rate  $\hat{X}$  via economies of scale. To avoid too many logarithmic derivatives we follow the earlier variant. We also assume on Marxian lines that increasing tightness in the labor market will bid down the profit rate and induce innovation to speed productivity growth.

On these assumptions, we show below that the ratio  $\zeta$  converges to a steady state with  $\dot{\zeta} = 0$ . The long run investment rate  $g$  is affected by income distribution à la Pasinetti and is not equal to an exogenously determined “natural” level as in supply-



driven models. The employment rate and income distribution adjust to support the steady state so that observations (iii), (iv), (vi), and (vii) apply.

The stylized facts mentioned in (viii) are typically modeled in one of two ways. Following the traditional “Cambridge equation,” one approach is simply to assume that the saving rate from profit income exceeds the rate from wages. This version is relevant to determination of macro equilibrium and growth, but says nothing about accumulation of wealth.

Pasinetti’s (1962, 1974) distinction between two classes of households shifts the focus to wealth. In an initial specification, capitalists receive only profit income  $rK_c$  on their capital  $K_c$ ; workers get the rest of income  $(X - rK_c)$ . The classes’ saving rates from income are  $s_c$  and  $s_w$  respectively, with  $s_c > s_w$ . In an extension below, we allow capitalists to receive some wage income.

These assumptions underlie dynamics of capital concentration  $Z$ , with saving and investment setting the growth rates of  $K_c$  and  $K$ . Under appropriate assumptions discussed below  $\dot{Z}$  will converge to zero with  $Z > 0$ , setting up a joint steady state with  $\zeta$ . There is also a possibility that it will diverge toward a maximum possible level discussed below, in an “anti-dual” solution noted by William Darity (1981).

In the next section we specify the short- to medium-run equilibrium of the economy in terms of the level of aggregate demand and the functional distribution of income. These expressions and their dependence on  $Z$  and  $\zeta$  then allow us to spell out the details of the two-dimensional  $(Z, \zeta)$  long-run dynamical system.<sup>3</sup>

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<sup>3</sup> Amitava Dutt (1990) and Thomas Palley (2012) pointed out that variation in  $Z$  must play a role in long-run macroeconomic adjustment. This fact is not widely recognized,

### Short and medium term

The distributive side of temporary equilibrium can be set up in terms of either the profit share ( $\pi$ ) or profit rate ( $r = \pi u$ ). The latter gives more tractable short-run and steady state specifications so we opt for that.

A convenient formulation for gross investment is

$$(1) \quad g = I/K = g_0 + \alpha r + \beta u \quad .$$

Household saving per unit of capital is

$$(2) \quad \sigma = s_c r Z + s_w [(1 - \pi)u + r(1 - Z)] = (s_c - s_w)rZ + s_w u \quad .$$

Setting up macroeconomic balance just in terms of private investment and saving is traditional, but does not fit the data. Besides investment, exports and government purchases of goods and services are demand injections; imports and taxes are significant leakages.<sup>4</sup> Let  $\iota$  be a coefficient relating these injections to capital, with  $\nu$  scaling leakages to output. The macro balance condition becomes

$$(g + \iota) - (\sigma + \nu u) = 0 \quad .$$

To simplify algebra until we get to simulations below,, we hold  $\iota = \nu = 0$ . On this assumption, an expression for  $u$  becomes

$$(3) \quad u = [1/(s_w - \beta)]\{g_0 + [\alpha - (s_c - s_w)Z]r\} \quad .$$

An increase in  $r$  raises  $g$  by a factor  $\alpha$  and  $\sigma$  by a factor  $(s_c - s_w)Z$  so that demand will be profit-led if  $\alpha > (s_c - s_w)Z$  and the “Keynesian” stability condition for  $u$ ,  $s_w > \beta$ , applies.<sup>5</sup>

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but is highly relevant to contemporary debate. As far as we know, the significance of  $\zeta$  and its dependence on dynamics of  $\kappa$  and  $\xi$  have not been noted.

<sup>4</sup> For details, see Taylor (2017).

<sup>5</sup> If  $s$  is the overall saving rate, the standard Keynesian stability condition is  $s > \beta$ . Data suggest that  $s_w > \beta$  also applies.

On the distributive side, we formulate the Marx-Goodwin profit-squeeze distributive rule as a relationship between the profit rate and the employment ratio

$$(4) \quad r = \mu_0 - \mu_1 \lambda = \mu_0 - \mu_1 \zeta u$$

with  $\lambda = \zeta u$ , or employment is proportional to output, as the expression after the second equals sign shows. If  $\mu_0, \mu_1 > 0$  a higher level of  $u$  or  $\zeta$  increases  $\lambda$ , causing the rate of profit to fall:  $dr/du < 0$  and  $dr/d\zeta < 0$ .

Equations (3) and (4) specify equilibrium relationships between demand and distribution. In a Marx-Goodwin cycle model, they would be “nullclines” (loci along which  $\dot{u} = 0$  and  $\dot{r} = 0$ ) of a dynamical system in the  $(u, r)$  plane. The system would generate clockwise cycles around a stationary point. As discussed above, we suppress this cyclicity to concentrate on growth in the three-dimensional  $(\kappa, \xi, Z)$  system with the joint solutions to (3) and (4) setting levels of  $u$  and  $r$ .<sup>6</sup>

Figure 1 is a graphical representation showing how  $u$  and  $r$  respond in the short to medium run to shifts in  $Z$  and  $\zeta$ . Econometric results suggest that in high-income economies demand is weakly profit-led so the  $u(r)$  schedule is relatively steep in the  $(u, r)$  plane. The  $r(u)$  curve shows more responsiveness. The intercepts on the horizontal and vertical axes follow from (3) and (4) with  $r = 0$  and  $u = 0$  respectively.

### Figure 1

The point of intersection of the schedules, A, is the short to medium term equilibrium of the economy. Reduced form equations for  $u$  and  $r$  at point A are

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<sup>6</sup> Formal stability analysis of Marx-Goodwin cycles is readily available in the literature, e.g. Taylor (2004) and Flaschel (2009).

$$(5-u) \quad u = \frac{g_0 + (\alpha - \tilde{s}_c Z) \mu_0}{(s_w - \beta) + (\alpha - \tilde{s}_c Z) \zeta \mu_1}$$

and

$$(5-r) \quad r = \frac{(s_w - \beta) \mu_0 - g_0 \zeta \mu_1}{(s_w - \beta) + (\alpha - \tilde{s}_c Z) \zeta \mu_1}$$

with  $\tilde{s}_c = s_c - s_w$ .

These expressions can be used to assess responses to shifts in  $Z$  and  $\zeta$ . The formal algebra is messy but the intuition is clear from Figure 1. From (2) a higher value of  $Z$  shifts profit income from low-saving worker to high-saving capitalist households, lowering demand  $u$  for any given level of  $r$ : the  $u(r)$  schedule becomes steeper. The new equilibrium point is B. Using subscripts for partial derivatives, the outcome is  $u_Z < 0$ . Because of a weaker profit squeeze, the profit rate responds positively to  $Z$ ,  $r_Z > 0$ . With  $\pi = r/u$  we have  $\pi_Z > 0$ . The magnitude of  $\pi_Z$  is important in the analysis below of long-run stability of  $Z$ .

An increase in  $\zeta$  strengthens the profit squeeze for any given level of  $u$ , causing the  $r(u)$  schedule to have a steeper negative slope so that  $r$  falls. Due to the profit-led demand regime,  $u$  also falls:  $u_\zeta < 0$  and  $r_\zeta < 0$ . With a stable value of  $u$ , we get  $\pi_\zeta < 0$ . The new equilibrium point is C.<sup>7</sup>

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<sup>7</sup> An alternative medium-run model can be based on wage-led demand and a high employment wage-squeeze (decreasing returns to labor in a neoclassical specification or “forced saving” by workers in antique terminology). In a diagram like Figure 1, the slopes of  $u(r, Z)$  and  $r(u, \zeta)$  would be negative and positive respectively. An increase in  $Z$  would reduce both  $u$  and  $r$ , making  $\pi_Z > 0$  if the slope of  $r(u, \zeta)$  is relatively shallow (the elasticity of substitution is high in a neoclassical version). Wage-led/wage-squeeze appears to fit the data less well than profit-led/profit-squeeze. If  $\pi_Z < 0$ , long-run dynamics of  $Z$  will be stabilized (see discussion of equation (20) below).

## Dynamics of productivity and capital stock

Immediate interest lies with the growth rate of the capital stock. From (1)  $Z$  affects  $\hat{\kappa}$  ambiguously. Higher  $Z$  lowers  $u$  but raises  $r$  which decrease and increase investment respectively. If output is relatively insensitive to distribution then  $g_Z > 0$ .<sup>8</sup> The shifts in  $r$  and  $u$  just noted imply that  $g_\zeta < 0$ .

Dynamics of the capital-population ratio  $\kappa = K/N$  are the heart of all growth models. In growth rate form  $\kappa$  evolves over time according to

$$(6) \quad \hat{\kappa} = g - \delta - n$$

with  $\delta$  as the rate of depreciation and  $n$  as the exogenous population growth rate.

As discussed above, following Kaldor and Marx labor productivity growth can be assumed to respond to capital formation and distribution,<sup>9</sup>

$$(7) \quad \hat{\xi} = \gamma_0 + \gamma_1 \hat{\kappa} - \gamma_2 r$$

Putting (6) and (7) together gives the growth rate equation for  $\zeta$ :  $\dot{\zeta} = \hat{\kappa} - \hat{\xi}$  or

$$(8) \quad \dot{\zeta} = \zeta[(1 - \gamma_1)(g - \delta - n) - \gamma_0 + \gamma_2 r] .$$

The equation shows a trade off between  $g$  which boosts accumulation and  $r$  which retards productivity growth – increases in both variables raise  $\dot{\zeta}$ . At a steady state with  $\dot{\zeta} = 0$ , if one rises, the other must fall. With  $\zeta > 0$  the balancing condition is

$$(9) \quad g = [\delta + n + \gamma_0/(1 - \gamma_1)] - [\gamma_2/(1 - \gamma_1)]r = \bar{g} - [\gamma_2/(1 - \gamma_1)]r .$$

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<sup>8</sup> If the model is set up with  $\pi$  instead of  $r$  responding to  $\zeta$  then  $g_Z > 0$  unambiguously.

<sup>9</sup> The mainstream “induced innovation” literature beginning with John Hicks (1932) also points in the direction of a negative response of  $\hat{\xi}$  to  $r$ , consistent with microeconomic analysis of firm behavior.

Using boldface to signal variables at steady state we have

$$\bar{g} = \mathbf{g} + [\gamma_2/(1 - \gamma_1)]\mathbf{r}$$

in which  $\bar{g} = \delta + n + \gamma_0/(1 - \gamma_1)$  is the traditional long run investment/capital ratio, equal to the sum of rates of depreciation, population growth, and Kaldorian productivity growth  $\gamma_0/(1 - \gamma_1)$ .<sup>10</sup> Along a trajectory toward the  $\dot{\zeta} = 0$  point, a higher profit rate boosts  $\hat{\zeta}$  by cutting  $\hat{\xi}$ . At the steady state itself,  $\mathbf{r}$  and  $\mathbf{g}$  must adjust to the “natural rate”  $\bar{g}$ . With  $\hat{\kappa} = \hat{\xi}$ , both variables can grow indefinitely at the rate  $\bar{g} - [\gamma_2/(1 - \gamma_1)]\mathbf{r} < \bar{g}$ . Simple closed-form expressions for  $\mathbf{g}$  and  $\mathbf{r}$  are provided in (22) and (23) below.

Because  $g_\zeta < 0$  and  $r_\zeta < 0$ , (8) should be a stable differential equation with  $d\dot{\zeta}/d\zeta < 0$  at the steady state. Figure 2 plots  $\hat{\zeta} = \hat{\kappa} - \hat{\xi}$ . The slopes of the schedules show that an increase in  $\zeta$  cuts into investment but spurs productivity growth. A higher base rate  $\gamma_0$  of productivity growth shifts the  $\hat{\xi}(\zeta)$  locus upward, leading to a lower level of steady state  $\bar{\zeta}$ .

## Figure 2

### Wealth dynamics

Capitalist households receive income on their wealth holdings (ignoring any wage income at this stage) so their capital stock evolves according to

$$(10) \quad \hat{K}_c = s_c r - \delta$$

or

$$\dot{K}_c = (s_c r - \delta)K_c \quad .$$

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<sup>10</sup> We ignore the potential equilibrium at  $\zeta = 0$  which corresponds to the pre-capitalist state of zero employment and/or zero capital stock.

Because  $r_Z > 0$ , dynamics of  $K_c$  are unstable. As will be seen, the instability can be offset by the evolution of  $Z$  and  $\zeta$ . Meanwhile, along with (6) and (8), (10) describes our three-dimensional dynamical system.

Total capital stock grows at the rate of aggregate saving per unit of capital (2) minus depreciation,

$$(11-\sigma) \hat{K} = \sigma - \delta = (s_c - s_w)rZ + s_w u - \delta \quad .$$

Alternatively,

$$(11-g) \hat{K} = g - \delta = g_0 + \alpha r + \beta u - \delta \quad .$$

Stability of  $Z$  can be analyzed using either (11-g) or (11- $\sigma$ ) for  $\hat{K}$ . Begin with the latter.

With  $\hat{Z} = \hat{K}_c - \hat{K}$  we have the differential equation for  $Z$ ,

$$\dot{Z} = Z\{(s_c(1 - Z) + s_w Z)r - s_w u\}.$$

It is easier to conduct stability analysis in terms of  $\pi = r/u$  instead of  $r$  and  $u$  separately, so we rewrite this equation as

$$(12) \quad \dot{Z} = Z\{[s_c(1 - Z) + s_w Z]\pi - s_w\}u \quad .$$

From Figure 1, an increase in  $Z$  pushes up  $\pi$ . From (12),  $\dot{Z}$  can go up, destabilizing dynamics around a Pasinetti steady state with  $0 < Z < 1$  if the difference between  $s_c$  and  $s_w$  is small. After a quick look at characteristics of a Pasinetti solution, we take up details of convergence below.

### **Pasinetti steady state wealth**

For stock variables in a steady state, the ratios of their changes over time to their levels must all be equal. Combining the ratios for workers' and capitalists' capital gives the relationship

$$(13) \quad \mathbf{1} - \mathbf{Z} = [s_w/(s_c - s_w)] [(\mathbf{1} - \boldsymbol{\pi})/\boldsymbol{\pi}] \quad .$$

If  $\pi < 1$  and  $s_w > 0$ , there will be some saving from wages. The workers' share of capital,  $\mathbf{1} - \mathbf{Z}$ , has to be positive at steady state, setting an upper bound on  $\mathbf{Z}$ .

For capitalists' gross saving, from (10) the change-to-level ratio is  $s_c r K_c / K_c = s_c r$ . It must be equal to the economy-wide ratio of gross investment to capital, setting up Pasinetti's famous equation

$$(14) \quad s_c \mathbf{r} = \mathbf{g} \quad .$$

This formula implies that Thomas Piketty's (2014)  $r > g$  condition is a corollary of steady state accounting. It is not some new law of capitalism.

Finally, equating workers' change-to-level capital ratio to overall capital stock growth gives an expression alternative to (13),

$$\mathbf{1} - \mathbf{Z} = s_w(\mathbf{1} - \boldsymbol{\pi})\mathbf{u}/(\mathbf{g} - s_w \mathbf{r}) \quad .$$

If  $\mathbf{u}$  and  $\mathbf{r}$  are relatively stable, then this expression shows that the investment/capital ratio and concentration of wealth are positively related from the side of saving. Steady states in mainstream and the demand-driven model at hand bear a strong family resemblance.

### **Stability of the Pasinetti steady state**

Returning to dynamics of  $Z$  and holding  $\zeta$  constant, define

$$(15) \quad f(Z) = [s_c(1 - Z) + s_w Z]\pi - s_w = [s_c\pi - s_w] - (s_c - s_w)\pi Z \quad .$$

The derivative of  $f$  is

$$(16) \quad df/dZ = f_Z = -(s_c - s_w)\pi + [s_c(1 - Z) + s_w Z]\pi_Z$$

in which  $\pi_Z > 0$ . From (12) we have



$$(17) \quad \dot{Z} = Zfu$$

and

$$(18) \quad d\dot{Z}/dZ = Z[f_Z u + f u_Z] + fu \quad .$$

For a Pasinetti steady state we need  $f(\mathbf{Z}) = 0$  in (17), or

$$(19) \quad s_c \mathbf{r} = s_w [\mathbf{r} + (\mathbf{1} - \boldsymbol{\pi})\mathbf{u}/(\mathbf{1} - \mathbf{Z})] \quad .$$

which states that the growth rates of the capital stocks of both household classes have to be equal. Capitalist households receive only capital income, while worker households have additional income from wages (the second term in brackets). Rearranging (19) gives the explicit solution for  $\mathbf{Z}$  appearing in (13).

To check on stability of the Pasinetti solution, substitute (19) into (16) to get

$$(20) \quad f_Z = -(s_c - s_w)\pi + s_w(\pi_Z/\pi) \quad .$$

The Pasinetti steady state will be locally stable if the right-hand side is negative, requiring  $s_w$  to be well below  $s_c$  and  $\pi_Z$  small (or negative if the medium run is wage-led/wage-squeeze). If these conditions are not satisfied  $Z$  will diverge toward zero or the maximum level permitted by workers' saving (i.e. the Samuelson-Modigliani dual analyzed below or the Darity anti-dual solution). The potential divergence arises from positive feedback. An increase in  $Z$  raises  $\pi$  which from (16) can push up  $\dot{Z}$  – this is the destabilizing linkage via workers' saving noted above. On the other hand, higher  $\pi$  strengthens the stabilizing term  $-(s_c - s_w)\pi$ , which can hold  $Z$  below one as in the simulation below.

If we use (11-g) instead of (11-σ) to set  $\widehat{K}$ , working through similar analysis gives a stability condition as

$$(21) \quad (\alpha - s_c)r_Z > -\beta u_Z$$

so we need a small  $\beta$  (weak accelerator) and/or strongly profit-led demand with  $\alpha > s_c$  (recall that  $r_z > 0$  and  $u_z < 0$ ).

For either version, Figure 3 is a visualization of dynamics of  $Z$ . For stability,  $\hat{K}$  must respond more strongly than  $\hat{K}_c$  to an increase in  $Z$ .

### Figure 3

#### Explicit steady state solution

With productivity growth responding to  $r$  in (7), Pasinetti's formula (14) is a bridge between steady state solutions of  $\zeta$  and  $K_c$ . Substituting (14) into (9), letting

$A = \gamma_2/(1 - \gamma_1)$ , and solving gives values for  $r$  and  $g$ ,

$$(22) \quad r = \bar{g}/(s_c + A)$$

and

$$(23) \quad g = s_c \bar{g}/(s_c + A) \quad .$$

In practice  $\gamma_2$  and  $A$  will be small, but they create space for a long run investment rate  $g$  differing from  $\bar{g}$ . For the reasons discussed in connection with (9),  $g < \bar{g}$ .

Because  $\bar{g} = \delta + n + \gamma_0/(1 - \gamma_1)$ , a higher value of  $\gamma_0$ , the base rate of technical progress, leads to a higher long-term investment/capital ratio. The same is true of the Kaldor technical progress coefficient  $\gamma_1$  if  $\gamma_2$  is relatively small. A higher capitalist saving rate  $s_c$  reduces the profit rate but stimulates capital formation. Animal spirits (and workers' saving, etc.), on the other hand, do not affect  $r$  and  $g$  at a steady state.

Imposing a given investment/capital ratio  $g$  on equation (3) means that  $u$  and  $r$  would have to adjust if  $g_0$  were to increase. Such a response is a "theorem of accounting," valid if the system is really *at* a steady state, but is not relevant in other circumstances.

Finally, one can plug (22) and (23) into the investment function (1) and solve for  $u$ . The result turns out to be

$$u = [1/\beta(s_c + A)][(\alpha - s_c)\bar{g} - g_0(s_c + A)] .$$

The condition  $\alpha > s_c$  (with a strong inequality) discussed in connection with (21) is needed here to assure that  $u > 0$ .

### Digression on capitalist wage income

Before turning to long-run nullclines, it makes empirical sense to take a quick look at a specification in which capitalists receive wage income. The richest one percent of US households receive around seven percent of total labor compensation (largely through bonuses and stock options). How does this fact influence growth dynamics?

Equations (12) and (18) permit two well-known steady state solutions with  $\dot{Z} = 0$  to exist, one with  $Z = 0$  and the other with  $f(Z) = 0$ . The former is the “dual” steady state proposed by Paul Samuelson and Franco Modigliani (1966). A simple example arises when  $s_c = s_w = s$ . If saving rates are equal, workers are identical to capitalists, *except* for the fact that they also receive wages. Using this extra source of income, workers can outsave capitalists so that in the long run  $Z$  goes to zero. In more detail, from (15),  $f(0) = s(\pi - 1) < 0$ . At  $Z = 0$ , (18) becomes

$$d\dot{Z}/dZ = s(\pi - 1)u < 0$$

so the dual equilibrium is stable. For uniform saving rates, Pasinetti apparently reduces to Solow-Swan.

But in fact it is easy to show that even if saving rates are equal Solow-Swan breaks down if capitalists get wage income. Suppose that the capitalist class receives a

share  $1 - \theta$  of the wage bill  $(1 - \pi)X$ . Their saving is  $S_c = s_c[rK_c + (1 - \theta)(1 - \pi)X]$ .

Workers' saving is  $S_w = s_w[r(K - K_c) + \theta(1 - \pi)X]$ . Using these expressions, an

extended version of (12) is

$$\begin{aligned} \dot{Z} = Z\{ & [(s_c(1 - Z) + s_w Z)r - s_w u - [s_c(1 - \theta) + s_w \theta](1 - \pi)u]\} \\ & + s_c(1 - \theta)(1 - \pi)u. \end{aligned}$$

If  $Z = 0$  then  $\dot{Z} = s_c(1 - \theta)(1 - \pi)u > 0$  so the Samuelson-Modigliani steady state is unstable when  $\theta < 1$ . So long as capitalists receive some wage income, they can accumulate wealth at  $Z = 0$ . As noted in connection with (13), saving from workers' wages means that  $Z$  cannot reach a value of one. Similarly, saving from capitalists' wages can support a positive value of  $Z$  even if saving rates are equal.

### Accounting background for simulations

In annual data for the US economy, imports typically exceed exports so the rest of the world is a macroeconomic net lender. The sum of government current spending on goods and services, transfers to households, and net interest minus taxes is positive, making the sector a net borrower. The combined government and foreign sector is a net lender, meaning that it is accumulating wealth. Here is an explicit formulation.

In the notation introduced in connection with (1) and (2), in current data we have  $\nu u - \iota > 0$ . Let  $K_\phi$  be capital controlled by the foreign/government (FG) consolidated sector, and  $\Phi = K_\phi/K$ . Wealth accumulation is

$$\dot{K}_\phi = \nu X - \iota K$$

or

$$\hat{K}_\phi = (\nu u - \iota)/\Phi \quad .$$

There is no feedback from  $\Phi$  to  $Z$  and  $\zeta$ , so accounting consistency ensures that  $\Phi$  will converge to a steady state with  $\hat{K}_\phi = g$  if the other two state variables do so. Its steady state level will be

$$\Phi = (vu - \iota)/g \quad ,$$

or the ratio of FG net lending to overall gross investment. Levels of wealth held by the two classes and the FG sector must sum to  $K$ .<sup>11</sup> It is possible for the FG sector to be a net debtor in steady state, i.e.  $\Phi < 0$ . In that case, household wealth holdings would sum to  $K - \Phi$ , or capital plus consolidated foreign and government debt, as in standard national financial accounting.

### **Nullclines**

The next step is to assume that a Pasinetti steady state exists. We can examine slopes of its nullclines by using the derivatives of  $\dot{\zeta}$  in (8) and  $\dot{Z}$  in (12) with respect to  $\zeta$  and  $Z$ . Pasinetti's formula (14) is valid only at steady state, so we cannot employ it directly.

At a Pasinetti equilibrium we get  $\partial\dot{Z}/\partial\zeta < 0$  from (12) because  $\pi_\zeta < 0$ . Equation (20) already shows when  $\partial\dot{Z}/\partial Z < 0$  near a Pasinetti steady state. The nullcline for  $\zeta$  is a bit trickier. The discussion of (8) above suggests that  $\partial\dot{\zeta}/\partial\zeta < 0$ . In (1) if  $u$  is relatively insensitive to  $Z$  while  $r_Z > 0$ , then  $\partial\dot{\zeta}/\partial Z > 0$  via  $g_Z > 0$  along with  $r_Z > 0$ . We end up with a Jacobian with the sign pattern

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<sup>11</sup> In (13) the workers' share will now be  $\mathbf{1} - \mathbf{Z} - \Phi$ .

	$Z$	$\zeta$
$\dot{Z}$	-	-
$\dot{\zeta}$	+	-

The signs say that in the vicinity of a steady state the  $\dot{Z} = 0$  nullcline will have a negative slope, with the  $\dot{\zeta} = 0$  locus sloping upward. The Routh-Hurwitz conditions for local stability (trace  $< 0$ , determinant  $> 0$ ) are satisfied.

## Numbers

Figure 4 is a social accounting matrix (or SAM), very loosely based on US data, for an economy with a capital stock of 80 (trillion dollars). Output, defined as value-added plus FG leakages, is 20. To avoid a lot more algebra in the model, corporate sector accounts, capital wage income, and flows of fiscal and financial transfers (exceeding ten percent of GDP in the American economy) have been suppressed. The numbers in the matrix are *not* realistic in this sense.

### Figure 4

Even so, they suggest three observations.

The profit rate is 7.5%, with capitalists receiving an income of 2.4 corresponding to a share of 40% in total wealth or capital. Their income of 13% of GDP (total demand minus FG income) approximates the share of the top one percent of households in the USA. Workers would outnumber capitalists by a factor of almost one hundred, so the discrepancy in incomes per household is vast.

Using their profit income, capitalists provide 42.5% of total saving. Implied saving rates are  $s_c = 0.62$  and  $s_w = 0.117$ .

Initially, net saving of the FG sector is set to zero (fiscal and foreign deficits are equal), but this condition does not have to hold over time in the simulations.

On the basis of the SAM, Figure 5 shows nullclines for the model. There is a unique Pasinetti equilibrium, with  $Z = 0.69$  and  $\zeta = 2.42$ .

### Figure 5

### Simulation results

Based on these numbers for a stylized US economy, we simulate our model to gauge whether current trends of increasing wealth and income inequality may persist and to demonstrate that long term growth projections can be derived from models of the demand side. Figure 6 presents simulation results from the model.<sup>12</sup> Panels (a) and (b) show relatively slow convergence of GDP and the capital stock to approximately two percent growth, reflecting the intrinsic dynamics of the growth equation (6). The level of income per capita grows exponentially in panel (l).

### Figure 6

Panel (d) shows a sustained increase in  $Z$ . Panels (e) and (g) illustrate how the dynamics of  $Z$  incorporate positive feedback. The utilization rate  $u$  declines and the profit rate  $r$  goes up over time, in line with the description in Figure 1 of the effects of a higher level of  $Z$ . Because  $\pi = r/u$ , the profit share in panel (h) rises steadily. Together with high capitalist saving, the shift in the income distribution toward profits sets up increasing  $Z$  from (13). The shifts in saving in panels (i) and (j) mirror these trends. The higher profit rate spurs investment in panel (k).

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<sup>12</sup> We used *Mathematica* to generate the simulation runs. The notebook is available upon request.

In panel (c) the auxiliary variable  $\zeta$  is fairly stable in the range between 2.0 and 2.5 (with a steady state value of 2.42). With  $\zeta = \lambda/u$ , the decreasing utilization rate forces employment to drop off in panel (f). Finally, the evolution of FG net lending and wealth share in panels (m) and (n) reflects dynamics of  $u$  and  $g$ . The FG sector switches from being a net lender to a borrower, but plays a secondary role in the overall dynamics of the model.

Figure 7 shows the effects of a recession caused by an adverse demand shock in the year 2067 (five decades after the base year) to autonomous investment to produce an immediate six percent reduction in GDP, followed by gradual recovery. After the shock, variables revert toward the steady state with somewhat higher profits and investment (panels g, h, and k) than in the unshocked simulation. With lower employment in the recession (panel f)  $r$  and  $g$  jump above the unperturbed model's trajectories toward the steady state and then slowly decline. Capital utilization in panel (e) is lower and wealth concentration in panel (d) rises as saving by capitalists goes up in panel (i) and workers' saving in panel (j) drops off.

### Figure 7

In panel (c),  $\zeta = \kappa/\xi$  rises, in part due to faster growth of capital but also driven by a slower increase in productivity induced by the higher profit rate over time. Toward the end of the simulation, both productivity and income per capital fall by around one percent in comparison to the recession-free simulation. Deviations in trajectories toward the steady state are restrained, but visible. A *favorable* short-run demand shock would be beneficial for a long time. Demand does indeed drive growth all the way.



## Final Thoughts

Drawing heavily on multiple strands within Post-Keynesian economics, we construct a heterodox model of economic growth which ties short run variables describing aggregate demand and the distribution of income to long run variables including the stock of capital, the distribution of wealth, and productivities of labor and capital. Our alternative to the Solow-Swan model allows us to point to important similarities in steady state relationships as well as differences in the dynamics toward a steady state. In contrast to its neoclassical cousins, our demand-driven economy is prone to divergent instabilities. Even if conditions for stability are satisfied and variables such as the equilibrium rate of growth of capital are set by supply side parameters, demand and income distribution adjust and determine the equilibrium distribution of wealth.

Applying our model to a stylized data for the US economy, we find a rising concentration of wealth associated with a falling employment ratio and a more concentrated distribution of income. The reason is that via the paradox of thrift a higher *level* of  $Z$  cuts into effective demand. The resulting downward pressure on employment pushes up the profit rate and faster *growth* of wealth and income inequality. This narrative has a degree of verisimilitude in wealthy economies over recent decades. To retain analytic tractability, we assume no active policy in counteracting such developments and leave this important question for future research.

These results show that one does not need to rely solely on supply-side explanations for economic growth. Interactions between income distribution and effective demand, endogenous productivity change, and dynamics of wealth have their

own roles to play even in a model as simple as the one presented here. Important features of advanced capitalist societies, such as an elaborate financial sector with multiple assets and independent dynamics of their prices, active fiscal and monetary policy, and open economy complications (as outlined in Foley and Taylor, 2006), allow for more realistic interactions and deserve further exploration.

## Appendix

Parameters are scaled to match the US economy as represented in the SAM of Figure 5. In particular, parameter values are the following:

Saving, Investment, and FG parameters

$$s_c = 0.62, s_w = 0.117, g_0 = -0.015, \alpha = 0.6, \beta = 0.059, \nu = 0.1, \iota = 0.025$$

Distribution parameters

$$\mu_0 = 0.225, \mu_1 = 0.25$$

Parameters for capital, labor productivity, and population (assumed to follow logarithmic growth) dynamics

$$\delta = 0.025, \gamma_0 = 0.01, \gamma_1 = 0.5, \gamma_2 = 0.01, n_{2017} = 0.005, L_\infty = 500$$

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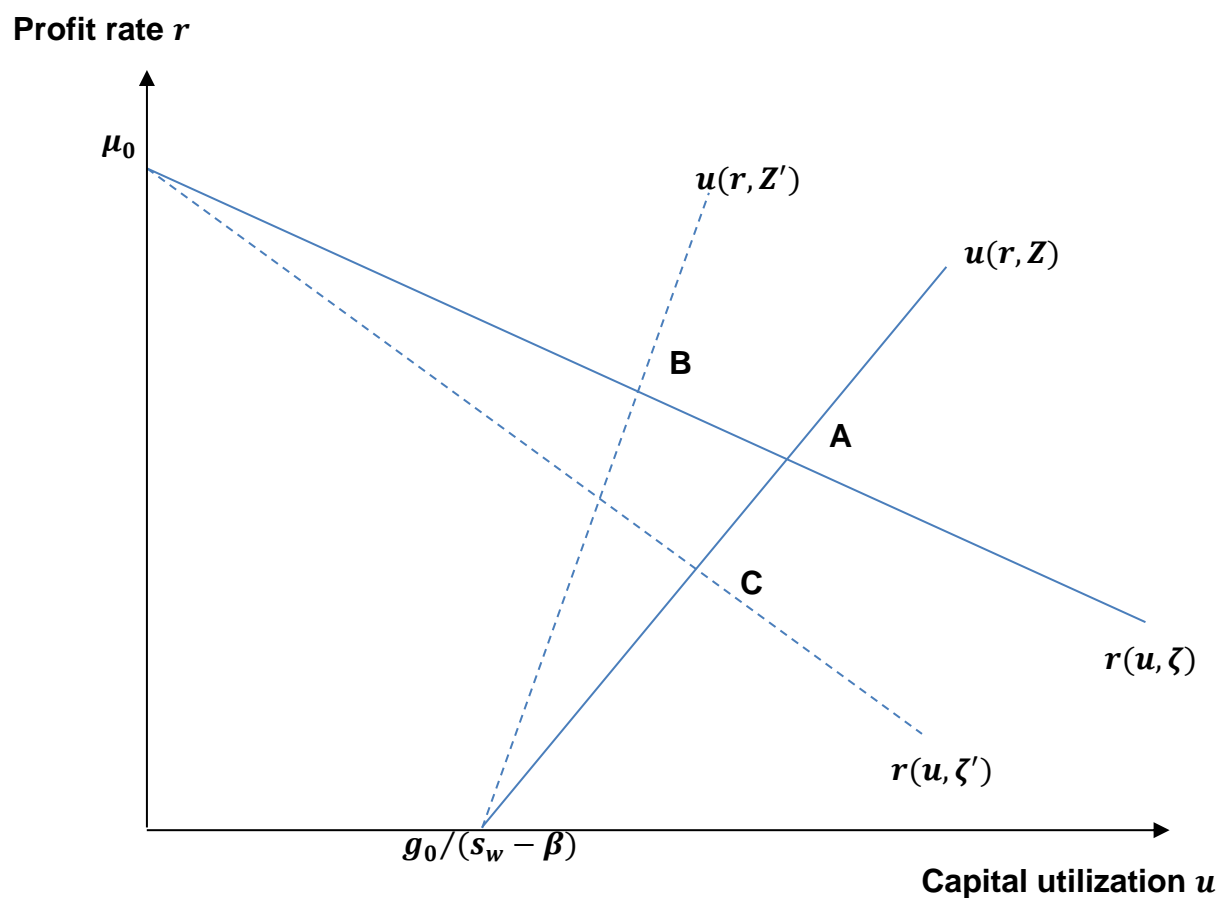
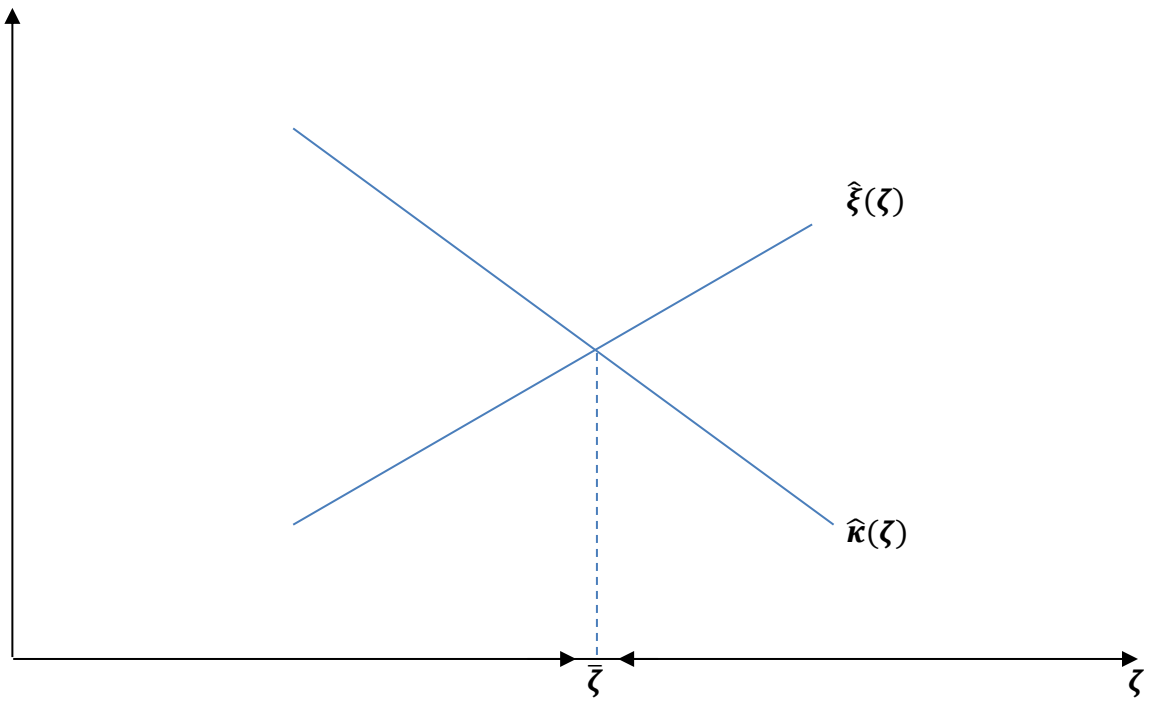


Figure 1: Short and medium run equilibrium as a function of  $Z$  and  $\zeta$ . An increase in  $Z$  lowers  $u$ , raises  $r$ , and shifts the equilibrium to Point B. Higher  $\zeta$  lowers  $r$  and  $u$  and shifts the equilibrium to point C.



**Figure 2: Dynamics of  $\zeta$ . There is a steady state at  $\bar{\zeta}$ .**

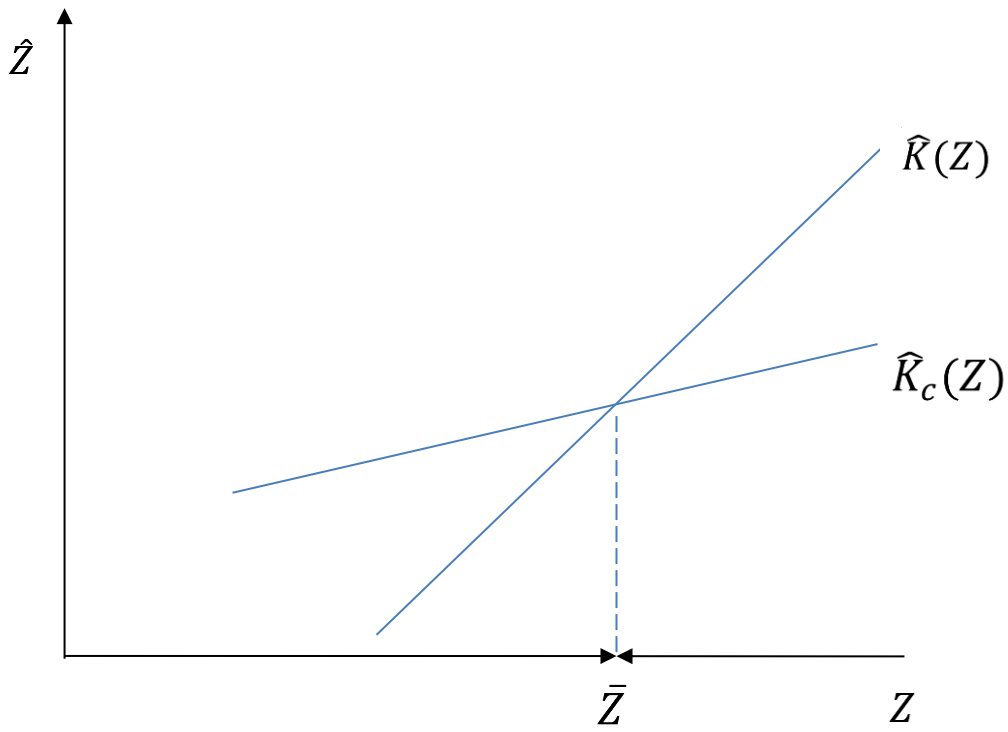
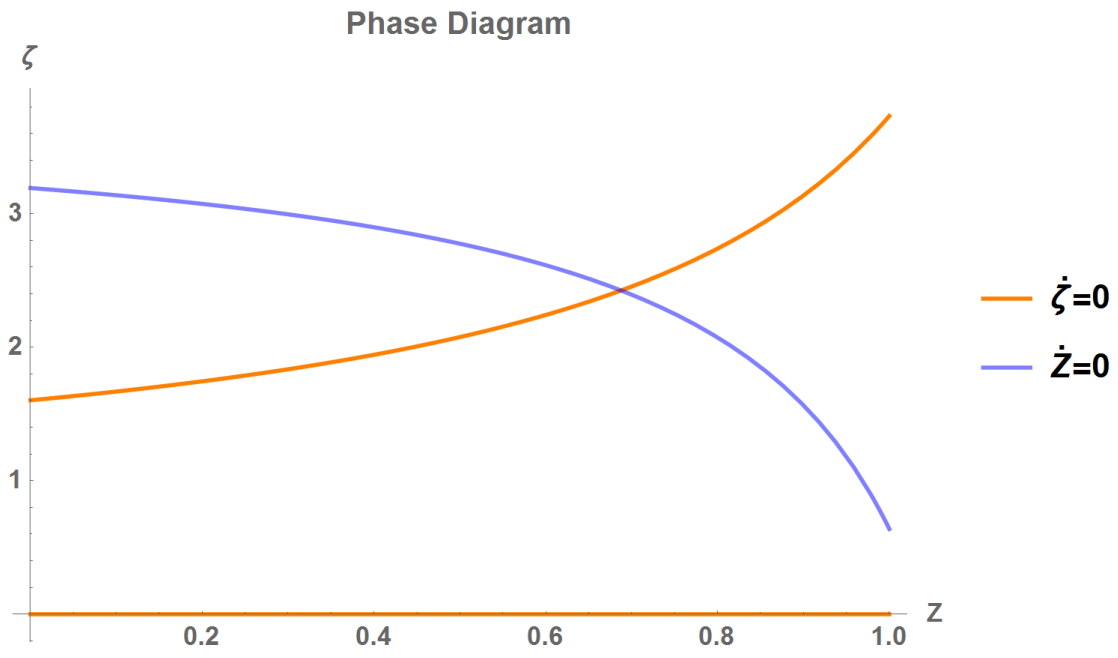


Figure 3: Dynamics of  $Z$  around a Pasinetti steady state at  $\bar{Z}$ .

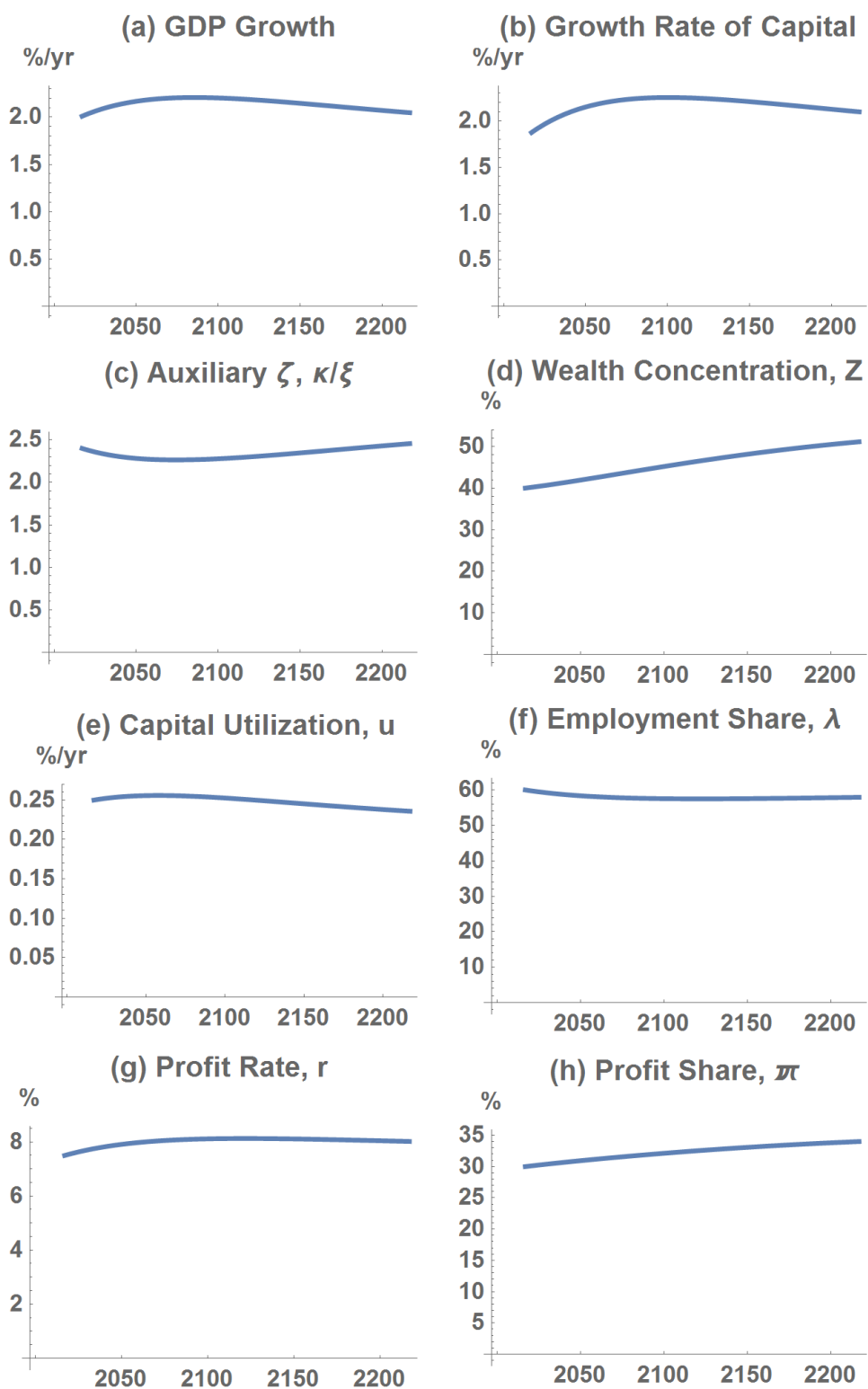


		Current spending		Demand injections		
		Worker uses of income	Capitalist uses of income	FG outlay	Investment	Row totals
		13.588	0.912	2	3.5	20
Worker wages	12					15.6
Worker profits	3.6					
Capitalist wages	0					2.4
Capitalist profits	2.4					
FG income	2					2
Macro balance		2.012	1.488	0	-3.5	0
Column totals	20	15.6	2.4	2	0	

**Figure 4: Social accounting matrix for simulations (initial capital stock = 80).**



**Figure 5: Phase diagram based on Figure 4 data for  $Z$  and  $\zeta$ .**



**Figure 6: Time-plots for simulation.**

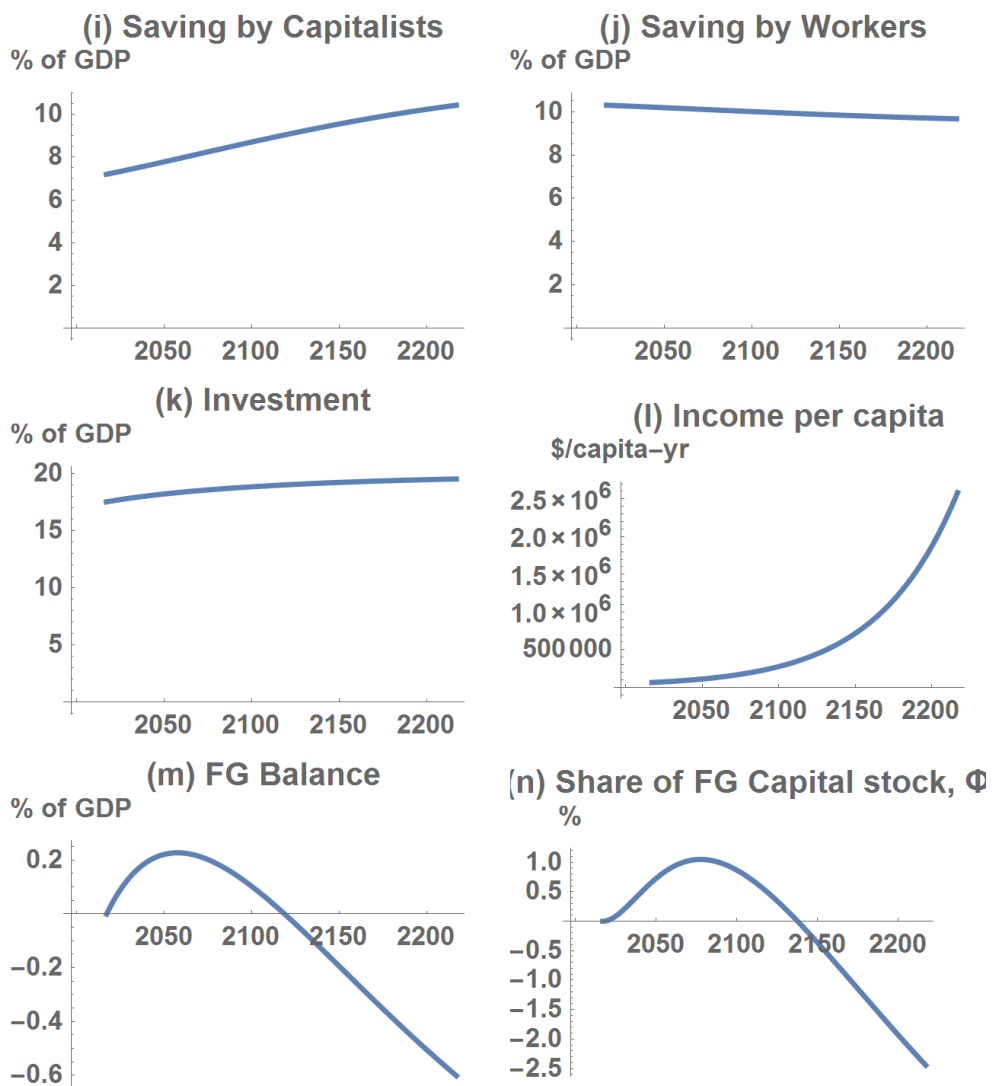


Figure 6 (cont'd): Time-plots for simulation.

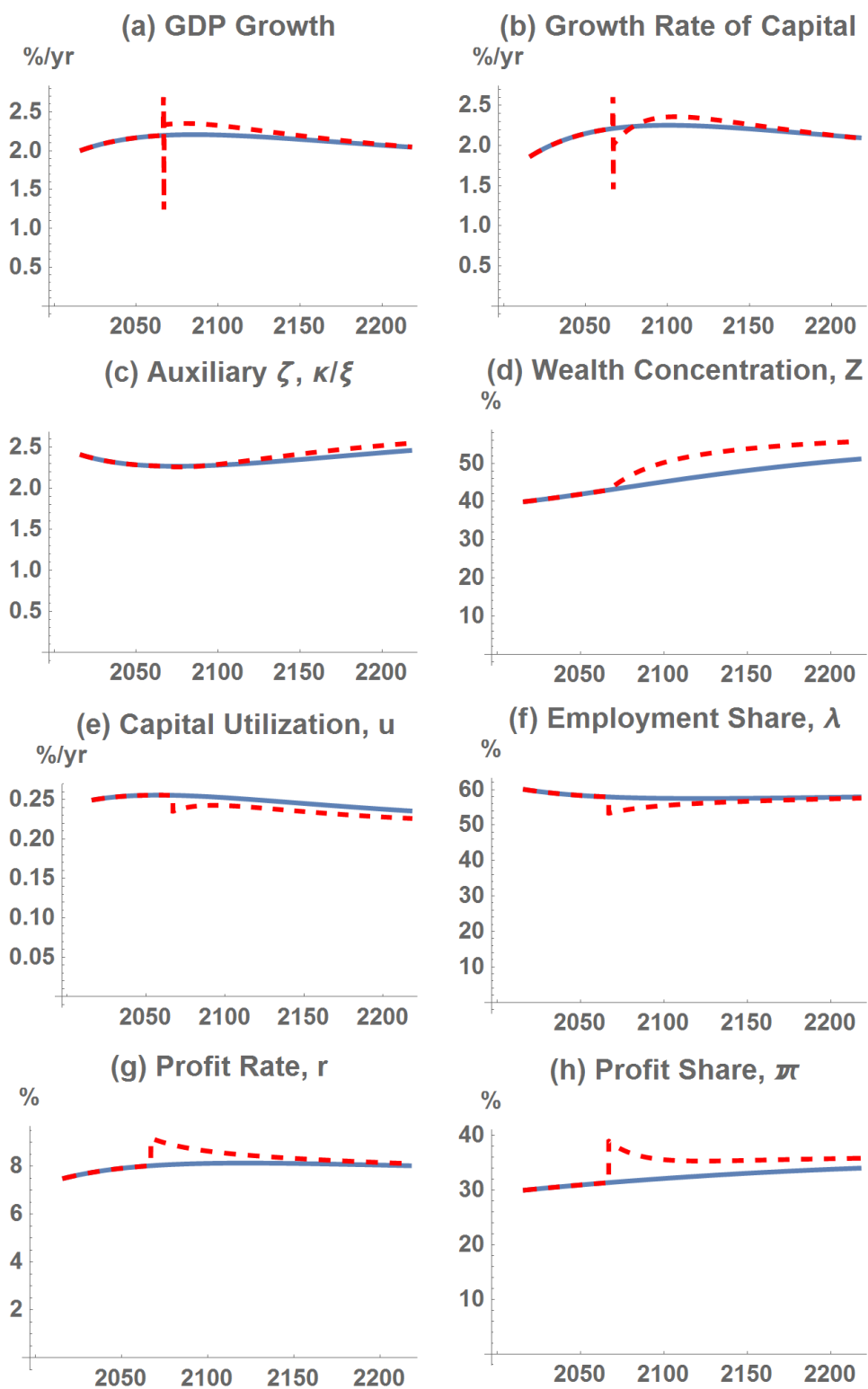
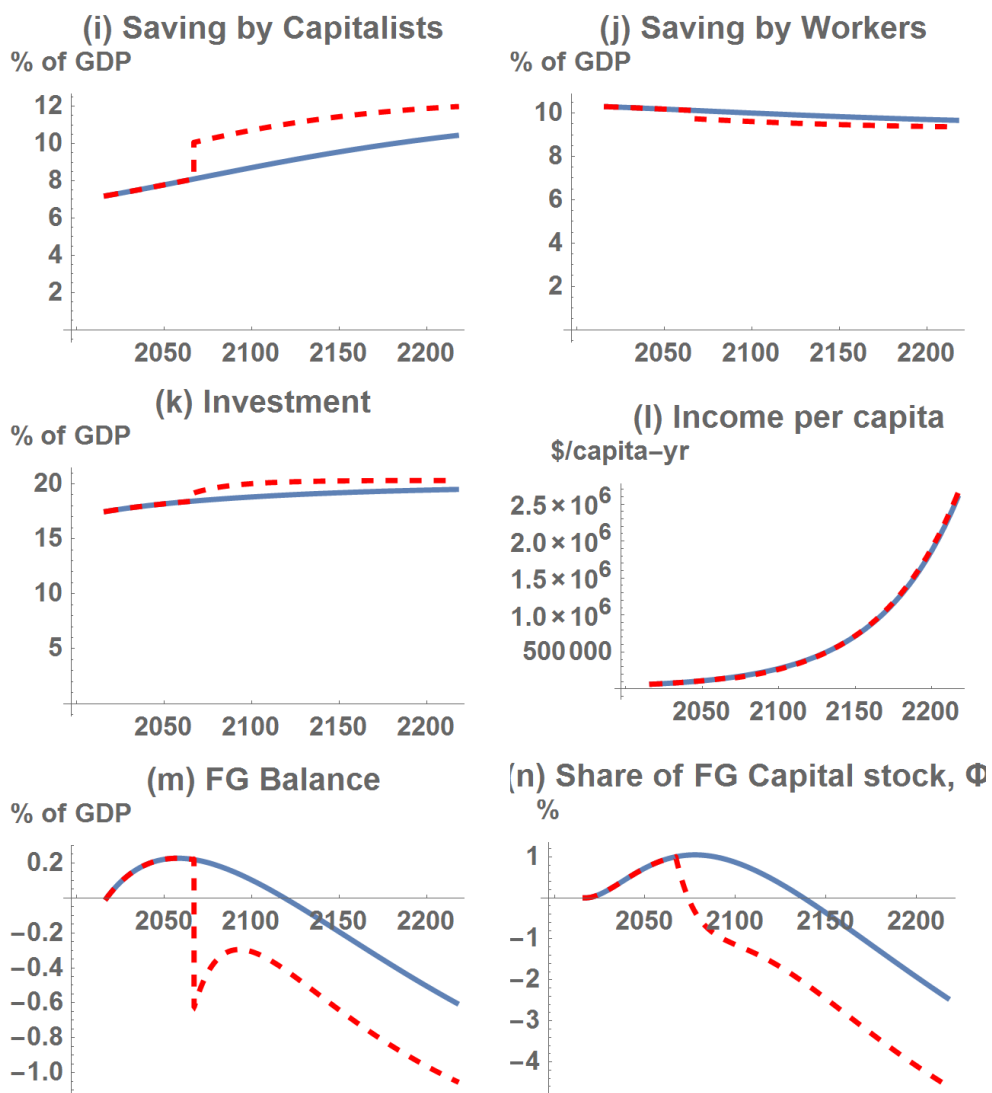


Figure 7: Time-plots for baseline (solid) and shocked (dashed) simulations



**Figure 7 (cont'd):** Time-plots for baseline (solid) and shocked (dashed) simulations.



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