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DEVELOPMENT OF METHODOLOGICAL BASES FOR ASSESSING THE INTER-CALIBRATION INTERVALS OF MEASURING INSTRUMENTS

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The definition of the timing (periodicity) of the metrological verification of measurement devices as used in business or health care, while monitoring environmental conditions and proving health and safety at work, is an urgent issue in scientific research, upon which depends the quality of goods and services. Therefore, the article proposes a methodology for estimating intercalibration intervals, taking into account the concept of measurement uncertainty, which has been tested in tests of the inertia measure of electric motors.

Introduction. It is accepted that measurement instruments are technical devices which are characterised by standardised metrological characteristics [1 - 7]. The reliability of measurement devices is determined by their ability to withstand the metrological parameters of regulated limits. A result obtained beyond these set boundaries is classified as a metrological failure. The correspondence of metrological characteristics to their standardised values is established during the course of verification (metrological control) or metrological certification of measurement devices. The introduction of the concept of measurement uncertainty to international standards for the evaluation and definition of the characteristics of precision measurements [1] and the evaluation of quality electrotechnical items [2] requires the development of methods for the assessment (establishment) of the inter-verification interval of the measurement device, the procedure of which should be based on the theory of uncertainty of measurement [8, 9].

The development of evaluation methods for precision measurement and inter-verification interval measurement for measurement devices based on the concept of the uncertainty of measurement is needed in order to establish the timing of the next scheduled or unscheduled verification of the correspondence of metrological characteristics to their standardised values. The definition of the timing (periodicity) of the metrological verification of measurement devices as used in business or health care, while monitoring environmental conditions and proving health and safety at work, is an urgent issue in scientific research, upon which depends the quality of goods and services.

Given the above, *the purpose of this article* is to develop a method for evaluating the accuracy of performance measurement and the inter-verification interval for measurement devices based on the theory of the uncertainty of measurement, that will allow the timing to be established for scheduled verification of measurement devices, according to international standards that apply to the assessment of the quality of electrical products. In addition, the proposed evaluation method of measuring accuracy must be tested during the metrological certification of measurement device for the moment of inertia of electric motors.

Analysis of the status of research and publications. The existing academic resources quite satisfactorily consider separate theoretical approaches to evaluating and expressing uncertainty [8 - 12] and the theoretical approaches to determine intermediate verification (inter-verification) intervals of measuring instruments based on the limits of metrological characteristics of instability. The leeway in metrological characteristics is based on the theory of measurement error and the reliability of means without regard to the concept of uncertainty of measurement [12 – 14]. Thus, to date, there is no approach to determining the inter-verification range of measuring instruments based on the theory of uncertainty of measurement. Therefore there is a need to develop mathematical tools to determine and define the inter-verification range of measuring instruments based on international standards for evaluation characteristics of measurement accuracy - the theory of uncertainty of measurement.

We know that if it is possible to determine, at least to an approximate degree, the average number of metrological failures q in the general stream of rejections of measuring instruments, the estimated functional accuracy of measurements without measurement rejections $P_M(t)$ during the duration of operation t [9, 14] may be demonstrated by [4, 10]:

$$P_M(t) = 1 - q(t)[1 - P(t)], \quad (1)$$

where $P(t)$ is the probability of failure-free operation of the measuring device (technical reliability) for the time of operation t.

If the average number of metrological failures q (t) cannot be determined, then $P_M(t) = P(t)$ should be used.

We also know from the literature [6, 9, 10, 13, 14] that key indicators that can be used to calculate the characteristics of metrological reliability are: the probability of failure-free operation; the frequency of metrological failures; mean time to first failure in metrology; the parameter flow of metrological failures; and time to first metrological failure.

However, in the information on the means of measurement that are submitted for testing to provide type approval or metrological certification of the means of measurement, there is often no reliable information about the instability of the metrological characteristics of the measurement means required for justifying the assignment of an initial inter-verification interval for the measurement instrument. In these cases, it is possible to estimate such intervals by using fixed values for the reliability parameters as specified in the technical specifications and documentation for the measurement devices, or by using analogue information on the inter-verification intervals, followed by correction of operational values on the basis of data on the frequency of usage and the measurement conditions.

Theoretical approach to the definition of the inter-verification interval of measurement means based on the concept of the uncertainty of measurement. To determine the inter-verification interval of a measurement device based on the concept of measurement uncertainty, a theoretical approach is proposed below.

Experimental evaluation of measurement uncertainty in the lower and upper measurement range of a measurement device through digitized gradations, a series of measurements must be conducted at the lower limits of measurement of the measuring device (minimum values standardised by measurement instruments) within the measurement range of the measuring device, and the upper limits of measurement of the measurement device (the maximum value that is specified in the technical documentation). At the same time the input of the measuring device must be sampled in the set of values of measurement signals that correspond to the specified measurement range in the technical documentation. Hence, experimental research into the gradated limits of measurements may be performed using the methodology of sample measurements, sample signals, sample devices or comparison methods. Based on the experimental data, the standard uncertainty of type A for the lower limit of measurement for measuring mid-range and upper limit of measurement may be determined using the equation as follows [8, 15]:

$$u_A(\bar{x}_K) = \sqrt{\frac{\sum_{i=1}^{n}\left(x_{i,K} - \bar{x}_K\right)^2}{(n-1)n}}, \qquad (2)$$

where $x_{i,K}$ is the quantity of values obtained in the K-th group of observations according to the lower limit of measurement within the measurement range, and the upper limit of measurement; K is the number of groups monitored in the gradated increments of range measurements; $\bar{x}_K$ is the mean value of each group of observations under investigation; n is the number of measured values in the k-th group of observations [2, 15, 16].

From the results obtained by formula (2) of the experimental standard uncertainties of type A, the largest value is set by as a maximum of the standard uncertainty of type A $u_{Amax}(\bar{x})$, which is then used to determine the inter-verification interval of the measuring instruments.

The next stage in determining the inter-verification interval of the measurement device is the evaluation of the standard uncertainty of type B, which is determined by available information about the discarded remainder of repeatable effects that in theory, together with a justified degree of certainty, can be manifested in the process of measurement. In doing so, one should rely on information derived from prior measurements, acceptable working conditions for the means the measurement, the physical properties of the measured value, technical documentation data for the measurement device or means of reference data [17, 18]. After evaluating the theoretically possible components of standard uncertainties of type B, it is necessary to calculate the combined total standard uncertainty of type B u_{cB} based on the known forms of representation of combined uncertainty [17].

After calculating the total measurement uncertainty of type B, it is necessary to calculate the total uncertainty of the measurement result based on the maximum standard uncertainty of type A. The equation for calculating the total uncertainty of the measurement result in the absence of correlation is [8, 15, 19]:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)}, \quad (3)$$

where $\frac{\partial f}{\partial x_i} = c_i$ are the sensitivity coefficients for the equation of transformation of measurement; $u(x_i)$ is the standard uncertainty evaluated as type A where the maximum standard uncertainty of type A is $u_{Amax}(\bar{x})$) according to type B.

If we assume that the coefficient of sensitivity to uncertainty of type A is 1, and the sensitivity coefficient standard of uncertainty as estimated as type B is included in the calculation of the total uncertainty of type B, then equation (3) to estimate the total uncertainty of the measurement result may be written as:

$$u_c(y) = \sqrt{[u_{Amax}(\bar{x})]^2 + [u_{cB}]^2}. \quad (4)$$

If there is a correlation between the input variables, the equation for determining the total uncertainty of the measurement result will be:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} c_i^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_i c_j u(x_i) u(x_j) r(x_i, x_j)}, \quad (5)$$

$$r(x_i, x_j) = \frac{\sum_{l=1}^{N_{ij}} (x_{i_l} - \bar{x}_i)(x_{j_l} - \bar{x}_j)}{\sqrt{\sum_{l=1}^{N_{ij}} (x_{i_l} - \bar{x}_i)^2 \sum_{l=1}^{N_{ij}} (x_{j_l} - \bar{x}_j)^2}}$$

where the correlation coefficient.

After calculating the total uncertainty of the measurement result, it is necessary to define the expanded measurement of uncertainty, which is attributed to the measuring methods as a standardised value, and noted in the test report of the technical documentation for the measuring device. Expanded measurement uncertainty is obtained by multiplying the total uncertainty of the measurement result by the coverage coefficient [2, 8]:

$$U_N = k_P u_c(y), \quad (6)$$

where k_P is the coverage coefficient, which is determined by information on the confidence probability P, and the effective number of degrees of freedom ν_{eff}.

The value of the confidence probability P is usually stated in the specifications or technical manuals for a given means of measurement. If the technical documentation does not specify a confidence probability, it is determined either experimentally or determined a priori [8].

The effective number of degrees of freedom is calculated by the Welch–Satterthwaite equation:

$$\nu_{eff} = (n-1)\frac{u_c^4(y)}{[u_{Amax}(\bar{x})]^4}. \quad (7)$$

On the basis of the confidence probability P and the effective number of degrees of freedom ν_{eff} from the Student table, the coverage coefficient k_P is determined.

In the case that the effective number of degrees of freedom is greater than 30, $\nu_{eff} > 30$, the coverage coefficient is assumed to be $k_{0,9} = 1,64$ when probability P = 0.9; $k_{0,95} = 1,96$ when P = 0,95; $k_{0,99} = 2,58$ when P = 0,99 and $k_{0,9973} = 3$ when P = 0,9973.

After setting the standardised value of the expanded uncertainty of measurement under standard conditions, or the total measurement uncertainty value, it is necessary to define an operational longevity t for the use of the measurement instrument. This may be defined on the basis of information about the intensity of the exploitation of the measurement means (number of working hours of

operation per day), and also by the mean time to failure of the device or the stated value for operational duration to first metrological failure.

Having assigned certain standard values for the metrological characteristics this device should be used or tested under real applied conditions that differ from laboratory benchmarking, such conditions under which the measuring device will actually be used.

After lengthy trials of the measurement device, the total and summary expanded uncertainty is again calculated, based on the environmental conditions of actual use on-site. In this case, the calculations take into account the working conditions of operation, using real values of ambient temperature and other conditions of measurement. Thus, we may calculate operational uncertainty values using formulas (2) - (7). As a result of these calculations, operational values may be obtained for expanded uncertainty of measurement for U_E under operating conditions, which is then used to specify the inter-verification interval of measurement instruments.

Based on the values of uncertainty of type A, standardised and theoretically possible expanded measurement uncertainty and operational expanded uncertainty of the measurement under the assumption of symmetry of the distribution of uncertainty, the first assessment of the inter-verification interval of the measurement device T_1 may be calculated using values of uncertainty as follows:

$$T_1 = t\frac{ln\left(\frac{U_E}{k_{2P-1}u_{Amax}(\bar{x})}\right)}{ln\left(\frac{U_N}{k_P u_{Amax}(\bar{x})}\right)}, \qquad (8)$$

where k_{2P-1} is the coverage coefficient, corresponding confidence probability 2P-1, i.e. a probability value that corresponds to the probability of metrological serviceability of the measurement device at the time of the definition of the inter-verification interval of the measurement device; where t is the operational longevity of the measurement device [20].

The coverage coefficient ratio k_{2P-1} is determined from the Student table based information about the confidence probability 2P-1 and the effective number of degrees of freedom ν_{eff} .

The second evaluation of the inter-verification interval T_2 may be calculated by the formula:

$$T_2 = t\frac{U_E - k_{2P-1}u_{Amax}(\bar{x})}{U_N - k_P u_{Amax}(\bar{x})}. \qquad (9)$$

Based on calculated values of inter-verification intervals T_1 and T_2 , an inter-verification interval is determined for a measurement device, which is assumed to be equal to the minimum value between the values of T_1 and T_2, i.e. [9, 20]:

$$T = min[T_1, T_2]. \qquad (10)$$

Thus, the proposed method of determining the inter-verification interval for the means of measurement permits the establishment or specification of an inter-verification interval based on the concept of the uncertainty of measurement. This method meets international requirements for evaluating the accuracy of measurements, adhering to international unity for measurement definitions and can be used in the metrological certification of measuring instruments.

To verify the proposed theoretical statements used in calibration by a graduated method of determining inter-verification intervals of a measuring means, based on the concept of uncertainty, we consider the calculation of the inter-verification interval by taking the example of the metrological certification of a measurement device for measuring the moment of inertia of electric motors.

Results of testing the proposed evaluation method for the inter-verification interval during metrological certification of a device measuring the moment of inertia of electric motors. The principle of operation and mathematical model of the device measuring the moment of inertia is described in [21 - 23]. The equation of the conversion device of the moment of inertia of the electric motor is:

$$J = \frac{PT_0}{2\ln\left(\frac{M_k l \xi h^3}{er^4 C \Delta\delta}\right)} N_J, \quad (11)$$

where N_J is the number of pulses to the input of the timer-counter microcontroller that is part of the device to measure the moment of inertia over a time period of free oscillations of the rotor of the electric motor; M_k is the torque at slip S = 1, which is described in the Kloss equations [14, 15]; l is the length of the measuring lever; ξ is the modulus of elasticity of the membrane of the effort sensor; h is the thickness of the membrane of the effort sensor; e = 0,17; r is the radius of the membrane of the effort sensor; C is the coefficient of rigidity of the membrane of the effort sensor; $\Delta\delta$ is the standardised value of the absolute deviation of the measuring transducer; J is the moment of inertia as measured; P is the damping coefficient; T_0 is the duration of sample pulses, which fills the measuring period of the moment of inertia after the completion of the transition process (when electric motor power is cut and torque is diminishing as M_k approaches zero) [23].

The essence of the method of measurement is the production of torque from the time that an electric motor is connected to a power supply, which is described by the Kloss formulae [23]. This production of torque through the measuring lever that is on one side attached to the rotor of the electric motor, and on the other side through the effort sensor, which is an elastic element. As later the electric motor is disconnected from the power supply, the resulting torque M_k is reduced to zero during the time interval T_0. As the rotor of the electric motor undergoes free damped oscillations whose duration is due to the value of the moment of inertia of the rotor J, and the rigidity C of the effort sensor, then measuring the magnitude of the torque and the time interval from the moment of power-down to zero and knowing the value of C of the rigidity of the effort sensor, it becomes possible to determine the value of the moment of inertia of the electric motor (11).

So, for the metrological certification of the measurement device to determine the moment of inertia of electric motors, we need to set a fixed torque value M_k and then in the self-braking mode of operation of the electric motor perform measurements of the moment of inertia. The sample moment M_{exe} is suggested as being that created by using a sample set of weights. The equation to define the procedure for production of the sample torque is:

$$M_{exe} = gRm_{exe}, \quad (12)$$

where g is the rate of acceleration of gravity, which corresponds to 9,8066 m/s^2; R is the radius of the disk (9.9889 cm), secured to the shaft of an electric motor, which is part of the torque measurement transducer; m_{exe} is the mass of standard weights.

For calibration of a device to measure the moment of inertia, an asynchronous electric motor type AIR56A4 was used, having a nominal value of the moment of inertia of J_1=0,007Nm^2. To produce torque with slip S=1, it is necessary to create on the shaft of the electric motor a sample torque that corresponds to M_k=0,85Nm. To produce torque on a disk of radius R which is fixed to the shaft of the electric motor via string length l, the value of the sample mass should equal m_{exe}=867,726g.

From the results of previous measurements of the radius of the disk, it is known that the standard uncertainty of measurement of the radius of the disk is u_{Ar} = 1,04·10^{-3} mm, and with the technical specification data on the sample means of measuring mass with a maximum load of 1000 gm, it is known that the mass of the weights may be measured with an absolute deviation in measurement of Δ_m = 1 mg. The uncertainty of mass measurement if it is assumed that the absolute deviation is distributed evenly may be calculated by:

$$u_{Bm} = \frac{\Delta_m}{\sqrt{3}} = \frac{10^{-3} g}{1{,}73} = 0{,}58 \cdot 10^{-3} \text{ g}. \quad (13)$$

Thus, the total uncertainty in producing torque may be calculated by the formula:

$$u_c(M_{exe}) = \sqrt{\left(\frac{\partial M_{exe}}{\partial R}\right)^2 u_{Ar}^2 + \left(\frac{\partial M_{exe}}{\partial m}\right)^2 u_{Bm}^2}, \quad (14)$$

where $\frac{\partial M_{exe}}{\partial R} = gm_{exe} = 8{,}509\frac{m}{s^2}kg$ is the sensitivity coefficient dependent on the disk radius; $\frac{\partial M_{exe}}{\partial m_{exe}} = gR = 0{,}9796\frac{m^2}{s^2}$ is the sensitivity coefficient dependent on the mass of a sample weight.

Substituting the calculated sensitivity coefficients and uncertainty in the formula (14), we obtain the value of the total uncertainty of the production of the sample torque as $u_{c1}(M_{exe}) = 8.87\cdot10^{-6}$ Nm.

After setting the sample torque in the manner described above, a series of measurements of the moments of inertia is performed by using a device for the measurement of the moment inertia of electric motors. The results of measurements of moments of inertia are listed in the Table 1.

Table 1. Results of measurements of moments of inertia

№	Value of the moment of inertia J×10^{-3}, Nm^2	№	Value of the moment of inertia J×10^{-3}, Nm^2	№	Value of the moment of inertia J×10^{-3}, Nm^2
1	7,132	8	7,128	15	7,175
2	7,197	9	6,855	16	6,878
3	6,805	10	7,153	17	7,191
4	7,157	11	7,192	18	6,823
5	6,952	12	6,863	19	7,147
6	7,134	13	7,176	20	6,792
7	7,171	14	6,883	21	7,165

Based on the results of measurements of moments of inertia (Table 1), we may calculate the standard uncertainty of type A by the equation (2). Substituting the results of measurements in equation (2), we obtain a value for a standard experimental uncertainty of measurement by:

$$u_A(\bar{J}_1) = \sqrt{\frac{\sum_{i=1}^{n}(J_{i,1} - J_1)^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^{21}(J_{i,1} - 7{,}046\cdot10^{-3})^2}{21(21-1)}} = 34{,}16\cdot10^{-6}$$

Nm^2. (15)

For calibration of a device to measure the moment of inertia in a different range, an asynchronous electric motor type AIR80A2 was used [24], having a nominal value of the moment of inertia of J_2 = 0,015 Nm^2. To produce torque with slip S close to 1, the electric motor shaft must create an exemplary torque that corresponds to M_k = 5 Nm To produce torque on a disk of radius R which is fixed to the shaft of the electric motor via string length l, the value of the sample mass should equal m_{exe}=5104,273 g.

The total uncertainty in the production of a sample value of torque, which arises due to residual non-incorporated systematic effects related to the limited accuracy of measurement instruments of mass and the radius of the disk, is as according to formula (14): $\frac{\partial M_{exe}}{\partial R} = gm_{exe} = 50{,}056\frac{m}{s^2}kg$; $\frac{\partial M_{exe}}{\partial m_{exe}} = gR = 0{,}9796\frac{m^2}{s^2}$; $u_{c2}(M_{exe}) = 5{,}21\cdot10^{-5}$ Nm.

Having assigned sample values of the moments of inertia, a series of measurements was made that is listed in Table 2.

Substituting the experimental data as listed in Table 2, we may calculate the standard uncertainty of type A of the measurement of the moment of inertia of an electric motor (calibration uncertainty) by formula (2):

$$u_A(\bar{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}(J_{i,2} - \bar{J}_2)^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^{21}(J_{i,2} - 15{,}118\cdot10^{-3})^2}{21(21-1)}} = 28{,}36\cdot10^{-6}$$

Nm^2. (16)

Table 2. Results of measurements of moments of inertia

№	Value of the moment of inertia J×10^{-3}, Nm^2	№	Value of the moment of inertia J×10^{-3}, Nm^2	№	Value of the moment of inertia J×10^{-3}, Nm^2
1	15,121	8	15,315	15	15,205
2	15,017	9	15,122	16	15,012
3	14,89	10	15,112	17	15,323
4	15,116	11	15,015	18	15,303
5	15,102	12	15,313	19	14,97
6	15,087	13	15,018	20	15,114
7	15,211	14	14,91	21	15,196

For calibration of a device to measure the moment of inertia J_3 = 0,0042 Nm^2, an asynchronous electric motor type AIR56A2 was used [24]. For this type of electric motor to produce slip S close to 1, the electric motor must create an exemplary torque that corresponds to M_k = 0,64 Nm. To produce torque on a disk of radius R which is fixed to the shaft of the electric motor, the value of the sample mass should equal m_{exe}= 653,346 g.

The total uncertainty in the production of a sample value of torque, which arises due to residual non-incorporated systematic effects related to the limited accuracy of measurement instruments of mass and the radius of the disk, is as according to the formula (14): $\frac{\partial M_{exe}}{\partial R} = gm_{exe} = 6{,}407\frac{m}{s^2}kg$; $\frac{\partial M_{exe}}{\partial m_{exe}} = gR = 0{,}9796\frac{m^2}{s^2}$; $u_{c3}(M_{exe})$= 6,69·10^{-6} Nm.

The results of measuring the moment of inertia are presented in Table 3.

Table 3. Results of measurements of moments of inertia

№	Value of the moment of inertia J×10^{-3}, Nm^2	№	Value of the moment of inertia J×10^{-3}, Nm^2	№	Value of the moment of inertia J×10^{-3}, Nm^2
1	4,178	8	4,365	15	4,011
2	4,388	9	4,085	16	4,391
3	4,289	10	4,015	17	4,039
4	4,394	11	4,011	18	4,383
5	4,286	12	4,355	19	4,397
6	4,378	13	4,054	20	4,289
7	4,386	14	4,016	21	4,036

Substituting the experimental data from Table 3 in equation (2), we obtain the standard uncertainty of type A measuring moment of inertia of the electric motor (calibration uncertainty), that is:

$$u_A(\bar{J}_3) = \sqrt{\frac{\sum_{i=1}^{n}(J_{i,3} - \bar{J}_3)^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^{21}(J_{i,3} - 4{,}226\cdot 10^{-3})^2}{21(21-1)}} = 35{,}72\cdot 10^{-6}$$

Nm^2. (17)

The formula for determining the inter-verification interval of a measurement device incorporates the highest value, maximum, experimental standard uncertainty of type A. From the conducting of experimental research, it may be shown that standard uncertainty of type A may reach its maximum value when the smallest moment of inertia is measured with the value of $\bar{J}_3$ =0,0042 Nm^2. Therefore, to assess the inter-verification interval of a measurement device in subsequent calculations, we may use the maximum uncertainty of measurement of a moment of inertia of type A equal to $u_{Amax}(\bar{J}_3)$ = 35,72·10^{-6} Nm^2.

To calculate the total and expanded uncertainty of measurement, we should perform the evaluation of components with uncertainty of type B, which are manifested by non-incorporated residual systematic effects and limited properties of the constituent elements of a measurement means for the moment of inertia.

In as much as the constituent elements of the moment of inertia of the measurement device includes an effort sensor, then we may estimate the uncertainty of type B, which is due to the existence of a consolidated error γ =0,15 % when the maximum effort Q = 20 kg, assuming a uniform law of error distribution by the formula:

$$u_{B,s} = \frac{\gamma Q}{\sqrt{12}100\%} = 0{,}15\% \frac{20}{3{,}46 \cdot 100\%} = 8{,}67 \cdot 10^{-3} \text{ kg.} \quad (18)$$

Uncertainty of type B, which is caused by the presence of non-incorporated systematic effects associated with the presence of errors in the length of the measurement shaft which do not exceed Δ_l = ± 0,01·10^{-3} m, may be shown by:

$$u_{B,l} = \frac{\Delta_l}{\sqrt{12}} = \frac{\pm 0{,}01}{\sqrt{3}} = 5{,}77 \cdot 10^{-6} \text{ m.} \quad (19)$$

Let us place a value for the component of uncertainty due to the change in frequency of the quartz resonator during the formation period of sample pulses, when the frequency of the quartz resonator may be described by f_0 = 20 MGz, $T_0 = 1/f_0$ = 0,05·10^{-6} s which fills the measurement interval of the moment of inertia, given a temperature deviation in the ambient air temperature where t_v=25°C, from a normal temperature where t_n =20°C, through the temperature coefficient of frequency changes of the quartz resonator where k_t = ±1,5·10^{-6}/°C as specified in the technical documentation. This demonstrated by the equation:

$$u_{B,t} = T_0 \frac{|t_v - t_n|}{\sqrt{3}} k_t = 0{,}05 \cdot 10^{-6} \frac{(25-20)}{1{,}73} 1{,}5 \cdot 10^{-6} = 0{,}22 \cdot 10^{-12} \text{ s.} \quad (20)$$

The uncertainty caused by the presence of a certain response time by the analogue microcontroller comparator to the appearance of an input signal according to the specifications for the microcontroller does not exceed Δt = 0.5·10^{-6} s, as calculated by the formula:

$$u_{B,\Delta t} = \frac{\Delta t}{\sqrt{12}} = \frac{0{,}5 \cdot 10^{-6}}{3{,}46} = 0{,}14 \cdot 10^{-6} \text{ s} \quad . \quad (21)$$

The total standard uncertainty of type B with regard to the transformation equation (11) above and the estimated components of uncertainty of type B (14), (18) - (21), enables us to find the positive square root of the total variance of type B, which is described by the formula:

$$u_{Bc}^2 = \left(\frac{\partial J}{\partial M_k}\right)^2 u_{B,M_k}^2 + \left(\frac{\partial J}{\partial l}\right)^2 u_{B,l}^2 + \left(\frac{\partial J}{\partial \Delta\delta}\right)^2 u_{B,s}^2 + \left(\frac{\partial J}{\partial T_0}\right)^2 \left[u_{B,t}^2 + u_{B,\Delta t}^2\right] = 13{,}22 \cdot 10^{-4} \text{ Nm}^2, \quad (22)$$

where

$$\frac{\partial J}{\partial M_k} = -\frac{0{,}5 M_k^{-1} P T_0 N_{cp2}(M_{exe})}{ln\left(\left[\frac{M_k l \xi h^3}{er^4 C \Delta\delta}\right]^2\right)} = -39{,}47 \cdot 10^{-6} \text{ m}; \quad \frac{\partial J}{\partial l} = -\frac{0{,}5 l^{-1} P T_0 N_J}{ln\left(\left[\frac{M_k l \xi h^3}{er^4 C \Delta\delta}\right]^2\right)} = -2{,}64 \cdot 10^{-3} \text{ Nm}; \quad \frac{\partial J}{\partial T_0} = \frac{P N_J}{2 ln\left(\left[\frac{M_k l \xi h^3}{er^4 C \Delta\delta}\right]^2\right)} = 7{,}93 \cdot 10^3 \frac{Nm}{s}; \quad \frac{\partial J}{\partial \Delta\delta} = -\frac{0{,}5 \Delta\delta^{-1} P T_0 N_J}{ln\left(\left[\frac{M_k l \xi h^3}{er^4 C \Delta\delta}\right]^2\right)} = 7{,}94 \cdot 10^4 \frac{Nm^2}{kg}$$

the sensitivity coefficients are for the torque, for sensor error effects, the length of the measuring shaft, the period of sample pulses, respectively.

Substituting the calculated values of sensitivity coefficients and standard uncertainties of type B in equation (22), we obtain the value of the total standard uncertainty of type B, which is $u_{Bc} = 11{,}16 \cdot 10^{-4}$ Nm^2.

The total uncertainty of the measurement result for the moment of inertia with regard to the maximum experimental uncertainty of type A (17) and the total uncertainty of type B (22) may be calculated by the formula:

$$u_c = \sqrt{u_A^2(\bar{J}_3) + u_{Bc}^2} = 11{,}17 \cdot 10^{-4} \text{ Nm}^2. \quad (23)$$

To calculate the expanded uncertainty as defined in technical documentation for a device measuring the moment of inertia, we should calculate the effective number of degrees of freedom as follows:

$$\nu_{eff} = \frac{u_c^4}{\sum_{i=1}^{N} \frac{u_i^4}{\nu_i}} = (n-1)\frac{u_c^4}{u_A^4(\bar{J}_3)} = 20\frac{(11{,}17\cdot 10^{-4})^4}{(35{,}72\cdot 10^{-6})^4} = 19\cdot 10^6 \quad . \qquad (24)$$

Next, using the Student table, let us define the coefficient of coverage k_P using the values of the effective number of degrees of freedom (24) and confidence probability, which is assumed to be P = 0.95 based on information about the analogue probabilities [25]. This will be k_P = 1.96.

Knowing the coverage coefficient and the total uncertainty of the measurement result of the moment of inertia, we obtain an expanded uncertainty of measurement that is specified in the technical documentation for a measurement device for the moment of inertia of an electric motor, that is:

$$U_N = k_P u_c = 1{,}96\cdot 11{,}17\cdot 10^{-4} = 2{,}19\cdot 10^{-3} \text{ Nm}^2. \qquad (25)$$

Next, assuming that the intensity of operation of the measurement device is 7 hours per day, and setting the experimentally determined time to first failure, which for the measurement device that determines the moment of inertia for the electric motors is 3500 hours, we define a calendar service life of t, which equates to some 2 calendar years.

After prolonged use under real conditions using the measurement device, we may now calculate the components of uncertainty of type B.

These type B uncertainties result from:

- the presence of the consolidated errors of the effort sensor, as calculated by (18) and being $u_{B,s} = 8{,}67\cdot 10^{-3}$ kg;

- the presence of the non-consolidated systematic effects related to the limits of the ability to measure the measuring lever, as calculated by (19) and being equal to $u_{B,l} = 5{,}77\cdot 10^{-6}$ m;

- the presence of the response time of the analogue microcontroller comparator to the appearance of the input signal is calculated by formula (21) and is equal to the same value calculated by $u_{B,\Delta t} = 0{,}14\cdot 10^{-6}$ s;

- the deviation of the ambient temperature of the environment during testing when t_{v2} = 18 °C from the temperature under normal conditions when t_n = 20 °C, enables us to calculate through the temperature coefficient the change of frequency of the quartz resonator (k_t=±1,5·10^{-6}/°C) by the formula:

$$u_{B,t2} = T_0 \frac{|t_{v2} - t_n|}{\sqrt{3}} k_t = 0{,}05\cdot 10^{-6} \frac{|18-20|}{1{,}73} 1{,}5\cdot 10^{-6} = 8{,}67\cdot 10^{-14} \text{ s}. \qquad (26)$$

The value of combined measurement uncertainty of type B, u_{Bce}, may be calculated by formula (23) taking into account the calculated uncertainties of measurement of type B after the testing of the measurement device for the moment of inertia in actual use. As a result, we may obtain the value of the combined standard uncertainty of type B, which is $u_{Bce} = 11{,}16\cdot 10^{-4}$ Nm^2.

Because the value of the combined measurement uncertainty of type B remains unchanged, then the value of the combined uncertainty of the measurement resulting from real conditions remains as $u_{ce} = 11{,}17\cdot 10^{-4}$ Nm^2.

The effective number of degrees of freedom is determined by (24) and is also equal to the previous value $\nu_{eff} = 19\cdot 10^6$.

The coefficient of coverage k_{2P-1}, which corresponds to the confidence probability 2P-1, i.e. the probability value that corresponds to the probability of metrological serviceability of the measuring device under real conditions where the technical reliability of the measuring device is P = 0.95, and the probability of metrological serviceability is thus 2P–1=2*0,95-1=0.9, may be determined from the

Student table for the effective number of degrees of freedom where $\nu_{eff} > 30$ and the probability of metrological serviceability measuring device under actual conditions is $k_{2P-1} = 1{,}64$.

When the coverage coefficient is $k_{2P-1} = 1{,}64$, the value of the extended uncertainty U_E under real conditions of the operation of the measuring device is:

$$U_E = k_{2P-1} u_{ce} = 1{,}64 \cdot 11{,}17 \cdot 10^{-4} = 1{,}83 \cdot 10^{-3} \text{ Nm}^2. \quad (27)$$

Based on the values of the standardised expanded uncertainty that is theoretically possible, the expanded uncertainty under real conditions U_E (assuming a probability of metrological serviceability 2P-1 for the measurement device) and the maximum measurement uncertainty of type A $u_A(\bar{J}_3)$, let us calculate the initial assessment of the inter-verification interval T_1 of the measurement device for the moment of inertia of electric motors by formula (8), based on experimental period time to first failure t = 2 years. The value of the first inter-verification interval corresponds to:

$$T_1 = t \frac{\ln\left(\frac{U_f}{k_{2P-1} u_A(\bar{J}_3)}\right)}{\ln\left(\frac{U_f}{k_P u_{A\,max}(\bar{J}_3)}\right)} = 2 \frac{\ln\left(\frac{1{,}83 \cdot 10^{-3}}{1{,}64 \cdot 35{,}72 \cdot 10^{-6}}\right)}{\ln\left(\frac{2{,}19 \cdot 10^{-3}}{1{,}96 \cdot 35{,}72 \cdot 10^{-6}}\right)} \approx 1{,}99 \text{ years}. \quad (28)$$

The second value of the inter-verification interval for a measurement device for the moment of inertia of electric motors T_2, is calculated by formula (9), and is:

$$T_2 = t \frac{U_E - k_{2P-1} u_{A\,max}(\bar{J}_3)}{U_f - k_P u_{A\,max}(\bar{J}_3)} = 2 \frac{1{,}83 \cdot 10^{-3} - 1{,}64 \cdot 35{,}72 \cdot 10^{-6}}{2{,}19 \cdot 10^{-3} - 1{,}96 \cdot 35{,}72 \cdot 10^{-6}} = 1{,}67 \text{ years}. \quad (29)$$

Thus, the inter-verification interval for a measurement device for the moment of inertia of electric motors according to formula (10) may be assumed to be equal to the minimum values between T_1 &T_2:

$$T = min[T_1, T_2] = min[1{,}99, 1{,}67] = 1{,}67 \text{ years} = 20 \text{ months}. \quad (30)$$

The value of the inter-verification interval in months may best be chosen from a row of natural numbers: 0.25; 0.5; 1 and 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 15, 18; 21; 24; 30 and so on at 6-month intervals.

So, as a result of the metrological certification of the measurement device for the moment of inertia of electric motors, the accuracy of the measurements of the moment of inertia is based on the concept of uncertainty of measurement, the characteristics of which include the standard value of expanded uncertainty, which is $2{,}19 \cdot 10^{-3}$ Nm^2 with probability P = 0.95 and the inter-verification interval of the measuring device which equals 20 months.

Conclusion. The calibration method developed for inter-verification interval assessment of measurement devices allows standardised metrological characteristics of measurement devices to be set or refined and the timing of subsequent verifications may then be based on international requirements for the performance evaluation of the accuracy of measurement using the concept of uncertainty. This method allows for international consensus in measurements for the assessment of inter-verification intervals of measurement devices. Testing the calibration method of evaluating the accuracy of measurements and inter-verification intervals, made during the metrological certification of measuring devices for the moment of inertia of electric motors, demonstrates its validity and effectiveness.

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