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Casimir effect in presence of spontaneous Lorentz symmetry breaking

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Abstract. The Casimir effect is one of the most remarkable consequences of the nonzero vacuum energy predicted by quantum field theory. In this contribution we study the Lorentz-violation effects of the minimal standard-model extension on the Casimir force between two parallel conducting plates in the vacuum. Using a perturbative method, we compute the relevant Green's function which satisfies given boundary conditions. The standard point-splitting technique allow us to express the vacuum expectation value of the stress-energy tensor in terms of this Green's function. Finally, we study the Casimir energy and the Casimir force paying particular attention to the quantum effects as approaching the plates.

1. Introduction

Interest in Lorentz violation has grown rapidly in the last decades since many candidate theories of quantum gravity [1, 2], such as string theory [3, 4] and loop quantum gravity [5, 6, 7], possess scenarios involving deviations from Lorentz symmetry. Nowadays, investigations concerning Lorentz violation are mostly conducted under the framework of the standard model extension (SME), initiated by Kostelecký and Colladay [8, 9]. The SME is an effective field theory that contains the standard model, general relativity, and all possible operators that break Lorentz invariance. The Lorentz-violating (LV) coefficients arise as vacuum expectation values of some basic fields belonging to a more fundamental theory, such as string theory [10, 11]. Some important features of the minimal SME comprise invariance under observer Lorentz transformations, energy-momentum conservation, gauge invariance, power-counting renormalizability [12], causality, stability and hermiticity [13].

The main goal of this contribution is to provide additional results regarding the local effects of the quantum vacuum in a particular sector of the electrodynamics limit of the SME, namely, the CPT-odd Maxwell-Chern-Simons term [14]. Concretely, we study the Casimir effect (CE) between two parallel conducting plates using a local approach based on the calculation of the vacuum expectation value of the stress-energy tensor via Green's functions satisfying the suitable boundary conditions.

In its simple manifestation, the CE is a quantum force of attraction between two parallel uncharged conducting plates [15]. More generally, it refers to the stress on bounding surfaces when a quantum field is confined to a finite volume of space. The boundaries can be material media, interfaces between two phases of the vacuum, or topologies of space. In any case, the modes of the quantum fields are restricted, giving rise to a macroscopically measurable force [16]. The CE has been previously considered within the SME framework in Refs. [17, 18, 19].



The authors in Ref. [18] used the zeta-function-regularization technique to compute the Casimir force between two parallel conducting plates within the (3+1)D Maxwell-Chern-Simons theory. The first attempt to tackle this problem was due to M. Frank and I. Turan [17]; however, as pointed out by O. G. Kharlanov and V. Ch. Zhukovsky [18], they used misinterpreted equations which led to an oversimplified treatment of the problem. More precisely, they considered that the photon dispersion relation corresponds to that for a massive photon; however, unlike the (2+1)D case, in (3+1)D the effect of the Maxwell-Chern-Simons term is a more complicated dispersion relation for the photon. Due to this wrong equation, Frank and Turan constructed also incorrectly the relevant Green's function (GF). One of the specific aims of this contribution is the construction of the correct Green's function within the (3+1)D Maxwell-Chern-Simons theory and the calculation of the Casimir-energy density and stress between two parallel conducting plates.

The outline of this contribution is as follows. Section 2 reviews some basics of the particular sector of the minimal SME to be considered in this work, namely, the (3+1)D Maxwell-Chern-Simons model. Using a perturbative method similar to that used for obtaining the Born series for the scattering amplitudes in quantum mechanics, in section 3 we compute the leading-order Green's function which satisfies given Dirichlet, Neumann or Robin boundary conditions, provided the smallness of the LV coefficients. In section 4 we use the standard point-splitting technique to express the vacuum expectation value of the stress-energy tensor in terms of the Green's function. The concrete calculation of the renormalized vacuum stress (and the Casimir force) between two parallel conducting plates is performed in section 5. We also discuss the local energy density, which is found to diverge as approaching the plates. We demonstrate that the divergent term does not contribute to the observable force. We summarize our results in section 6. Details about this investigation were recently published in Ref. [20].

2. Lorentz-violating electrodynamics

In the present contribution we are concerned with the CPT-odd sector of the SME. The relevant Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (k_{AF})_{\mu}A_{\nu}\tilde{F}^{\mu\nu} - j_{\mu}A^{\mu}. \quad (1)$$

Here, $j^{\mu} = (\rho, \mathbf{J})$ is the 4-current source that couples to the electromagnetic 4-potential A^{μ} , $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ is the electromagnetic field strength and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ its dual. From now on, we omit the subscript AF of the Lorentz- and CPT-violating $(k_{AF})^{\mu}$ coefficients and set $(k_{AF})^{\mu} \equiv k^{\mu} = (k^0, \mathbf{k})$. A nondynamical fixed k^{μ} determines a special direction in spacetime. For example, certain features of plane wave propagating along \mathbf{k} might differ from those of waves perpendicular to \mathbf{k} . Thus, particle Lorentz transformations are violated.

Varying the action $\mathcal{S} = \int \mathcal{L} d^4x$ with respect to A^{μ} yields the equations of motion:

$$\left(\square \eta^{\mu}_{\nu} - \partial^{\mu} \partial_{\nu} - 2k_{\beta} \epsilon^{\mu\beta\alpha} \partial_{\alpha} \right) A^{\nu} = j^{\mu}, \quad (2)$$

which extend the usual covariant Maxwell equations to incorporate Lorentz violation. Of course, the homogeneous Maxwell equations that express the field-potential relationship

$$\partial_{\mu} \tilde{F}^{\mu\nu} = 0, \quad (3)$$

are not modified due to the $U(1)$ gauge invariance of the action. The stress-energy tensor for this theory is, up to a total-derivative term, given by [9]

$$\Theta^{\mu\nu} = -F^{\mu\alpha} F^{\nu}_{\alpha} + \frac{1}{4} \eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} - k^{\nu} \tilde{F}^{\mu\alpha} A_{\alpha}. \quad (4)$$

Here $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ denotes the usual Minkowski flat space-time metric. Unlike the conventional case, $\Theta^{\mu\nu}$ cannot be symmetrized because its antisymmetric part is not longer a total derivative. By virtue of the equations of motion (2) and (3), the energy-momentum tensor obeys

$$\partial_\mu \Theta^{\mu\nu} = j_\mu F^{\mu\nu}. \quad (5)$$

Although the energy-momentum tensor is gauge dependent, it only changes by a total derivative under the gauge transformation $A^\mu \rightarrow A^\mu + \partial^\mu \Lambda$, i.e.

$$\tilde{F}^{\mu\alpha} A_\alpha \rightarrow \tilde{F}^{\mu\alpha} A_\alpha - \partial_\alpha (\tilde{F}^{\mu\alpha} \Lambda). \quad (6)$$

Consequently, the integrals over all space are gauge invariant. Note that the energy

$$\mathcal{E} = \int \Theta^{00} d^3 \mathbf{x} = \int \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2 - k^0 \mathbf{B} \cdot \mathbf{A}) d^3 \mathbf{x}, \quad (7)$$

is not positive definite due to the term $k^0 \mathbf{B} \cdot \mathbf{A}$, which may be negative. The appearance of this term in the energy density can introduce instability in the theory [21, 22], and it can be resolved by requiring that only spacelike components of k^μ are nonzero. However, this condition depends on the observer frame, so even an infinitesimal boost to another observer frame would reintroduce instability. Despite arising from a hitherto unobserved spontaneous breaking of the electromagnetic U(1) gauge symmetry, the photon mass can be introduced in this theory to eliminate the linear instability [23]. Although this idea might be physically acceptable, in this work we restrict ourselves to the minimal extension with a purely spacelike background $k^\mu = (0, \mathbf{k})$, which is fundamentally different from the theory with the purely timelike case $k^\mu = (k^0, \mathbf{0})$, as reviewed extensively in Refs. [24, 25].

3. Green's function method

To derive the GF for the previously discussed LV electrodynamics one can employ standard Fourier methods. As in conventional electrodynamics, the modified Maxwell operator appearing in parentheses in Eq. (2) is singular. To circumvent the non invertibility of the corresponding Minkowski matrix one can further work in the Lorentz gauge. The free-space GF (satisfying the standard boundary conditions at infinity) in momentum [26, 27] and coordinate [24, 25] representations can be obtained in a simple fashion. In the present work we are concerned with the effects of this Lorentz-violating electrodynamics on the Casimir force between two parallel conducting plates in the vacuum. To this end we employ a local approach consisting in the evaluation of the vacuum expectation value of the stress-energy tensor of the system, which can be expressed in terms of the appropriate Green's function. The presence of boundaries (e.g. the plates) makes the GF derived in Refs. [24, 25, 26, 27] not suitable for our purposes. Thus the aim of this section is the construction of the Green's functions which incorporates the presence of boundaries.

In the Lorentz gauge $\partial_\mu A^\mu = 0$, the field equations (2) take the form

$$\left(\square \eta^\mu{}_\nu - 2k_\beta \epsilon^{\mu\beta\alpha}{}_\nu \partial_\alpha \right) A^\nu = j^\mu, \quad (8)$$

where $\square = \partial_\mu \partial^\mu = \partial_t^2 - \nabla^2$ is the D'Alembert operator. To obtain the general solution of Eq. (8) for arbitrary external sources, we introduce the GF matrix $G^\mu{}_\nu(x, x')$ solving Eq. (8) for a pointlike source,

$$\left(\square \eta^\mu{}_\nu - 2k_\beta \epsilon^{\mu\beta\alpha}{}_\nu \partial_\alpha \right) G^\nu{}_\gamma(x, x') = \eta^\mu{}_\gamma \delta^4(x - x'), \quad (9)$$

in such a way that the general solution for the 4-potential in the Lorentz gauge is

$$A^\mu(x) = \int G^\mu{}_\nu(x, x') j^\nu(x') d^4x. \quad (10)$$

Since the timelike theory, without a photon-mass term, appears to be inconsistent (that is, the theory violates unitarity and causality, or both), in this work we specialize to the purely spacelike case $k^\mu \equiv (0, \mathbf{k}) \equiv (0, 0, 0, \kappa)$. Without loss of generality, we consider surfaces Σ_i which are orthogonal to \mathbf{k} in which Dirichlet, Neumann or Robin boundary conditions have been imposed. In this way, the GF we consider has translational invariance in the directions x and y , while this invariance is broken in the z -direction. Exploiting this symmetry we further introduce the Fourier transform in the direction parallel to the surfaces Σ_i , taking the coordinate dependence to be $\mathbf{R} = (x - x', y - y')$, and define

$$G^\mu{}_\nu(x, x') = \int \frac{d^2\mathbf{p}}{(2\pi)^2} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} g^\mu{}_\nu(\omega, \mathbf{p}, z, z'), \quad (11)$$

where $\mathbf{p} = (p_x, p_y)$ is the momentum parallel to Σ_i . Hereafter we will suppress the dependence of the reduced GF $g^\mu{}_\nu$ on ω and \mathbf{p} . The substitution of Eq. (11) into Eq. (9) yields the reduced GF equation

$$\left(\tilde{\square} \eta^\mu{}_\nu - 2i\kappa \epsilon^{3\mu\beta}{}_\nu p_\beta \right) g^\nu{}_\alpha(z, z') = \eta^\mu{}_\alpha \delta(z - z'), \quad (12)$$

where $p^\alpha = (\omega, \mathbf{p}, 0)$ and $\tilde{\square} = \mathbf{p}^2 - \omega^2 - \partial_z^2$. We now must solve the reduced GF equation for the various components. The solution to Eq. (12) is simple but not straightforward. Since the coefficient κ is assumed to be small, to solve it we employ a method similar to that used for obtaining the scattering amplitudes in quantum mechanics, in which the Schrödinger equation can be written as an integral equation, the Lippmann-Schwinger equation, which can be iterated to obtain the Born series. Indeed, the Lippmann-Schwinger equation for Green's operator is called the resolvent identity. In the problem at hand let us consider that the free (with $\kappa = 0$) reduced GF is known, being the solution of $\tilde{\square} \mathbf{g}(z, z') = \delta(z - z')$ in the region $\mathfrak{D} \subseteq \mathbb{R}$ and satisfying appropriate boundary conditions on the surfaces $\Sigma_i \subseteq \mathbb{R}^2$. Now Eq. (12) can be directly integrated using the free reduced GF. We thus establish the integral equation

$$g^\mu{}_\nu(z, z') = \eta^\mu{}_\nu \mathbf{g}(z, z') + 2i\kappa \epsilon^{3\mu\alpha}{}_\beta p_\alpha \int_{\mathfrak{D}} \mathbf{g}(z, z'') g^\beta{}_\nu(z'', z') dz''. \quad (13)$$

Suppose we take this expression for $g^\beta{}_\nu$, and plug it under the integral sign. Iterating this procedure, we obtain a formal series for $g^\mu{}_\nu$. At leading order in the LV coefficient κ , the reduced GF can be written as the sum of two terms, $g^\mu{}_\nu(z, z') = \eta^\mu{}_\nu \mathbf{g}(z, z') + \mathbf{g}^\mu{}_\nu(z, z')$, where the first term provides the propagation in the absence of Lorentz violation, while the second term, to second order in the LV coefficient κ , encoding the Lorentz symmetry breakdown is given by

$$\mathbf{g}^\mu{}_\nu(z, z') = 2i\kappa \epsilon^{\mu}{}_\nu{}^{\alpha 3} p_\alpha N(z, z') - 4\kappa^2 [p^\mu p_\nu - (\eta^\mu{}_\nu + n^\mu n_\nu) p^2] M(z, z'), \quad (14)$$

where $n^\mu = (0, 0, 0, 1)$ is the normal to the surfaces Σ_i . In deriving Eq. (14) we have used the identity $\epsilon^{3\mu\beta}{}_\alpha \epsilon^{3\alpha\gamma}{}_\nu p_\beta p_\gamma = p^\mu p_\nu - (\eta^\mu{}_\nu + n^\mu n_\nu) p^2$, with the definitions

$$N(z, z') = \int_{\mathfrak{D}} \mathbf{g}(z, z'') \mathbf{g}(z'', z') dz'', \quad M(z, z') = \int_{\mathfrak{D}} \mathbf{g}(z, z'') N(z'', z') dz''. \quad (15)$$

To calculate the $M(z, z')$ and $N(z, z')$ functions we require the suitable reduced Green's functions, which are given by

$$\mathbf{g}_0(z, z') = \frac{i}{2p} e^{ip(z_{>} - z_{<})}, \quad \mathbf{g}_{\parallel}(z, z') = \frac{\sin[pz_{<}] \sin[p(D - z_{>})]}{p \sin[pD]}, \quad (16)$$

in the free space \mathbf{g}_0 , which is defined on the real line, i.e. $\mathcal{D} = \mathbb{R}$, and \mathbf{g}_{\parallel} in presence of two parallel conducting plates separated by a distance D , which is defined on the domain $\mathcal{D}_{\parallel} = [0, D]$ and satisfies the boundary conditions $\mathbf{g}_{\parallel}(0, z') = \mathbf{g}_{\parallel}(D, z') = 0$, respectively. In the above expressions $z_{>}$ ($z_{<}$) is the biggest (smaller) of z and z' .

Notice that the full GF matrix G^{μ}_{ν} can be written as the sum of two terms, $G^{\mu}_{\nu}(x, x') = \eta^{\mu}_{\nu} \mathcal{G}(x, x') + \mathbf{G}^{\mu}_{\nu}(x, x')$, where \mathcal{G} and \mathbf{G}^{μ}_{ν} are the Fourier transformations of $\mathbf{g}(z, z')$ and \mathbf{g}^{μ}_{ν} , respectively, as defined in Eq. (11). It is worth mentioning that the second term satisfies the Lorentz gauge condition, i.e. $\partial_{\mu} \mathbf{G}^{\mu}_{\nu} = 0$. The proof follows from the reduced GF: $\partial_{\mu} \mathbf{G}^{\mu}_{\nu} \propto \int p_{\mu} \mathbf{g}^{\mu}_{\nu}$, which vanishes given that $\epsilon^{\mu}_{\nu} p_{\mu} p_{\alpha} = 0$ and $p_{\mu} n^{\mu} = 0$.

4. Vacuum stress-energy tensor

In section 2 we gave the stress-energy tensor (SET) for this theory and we showed that it can be written as the sum of two terms:

$$\Theta^{\mu\nu} = T^{\mu\nu} + \Xi^{\mu\nu}. \quad (17)$$

The first term,

$$T^{\mu\nu} = -F^{\mu\alpha} F^{\nu}_{\alpha} + \frac{1}{4} \eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}, \quad (18)$$

is the standard Maxwell stress-energy tensor, while the second,

$$\Xi^{\mu\nu} = -k^{\nu} \tilde{F}^{\mu\alpha} A_{\alpha}, \quad (19)$$

explicitly depends on the LV coefficients k^{μ} . Now we address the problem of the vacuum expectation value of the SET, to which we will refer simply as the vacuum stress (VS). This VS can be obtained from appropriate derivatives of the GF, in virtue of the formula

$$G^{\mu\nu}(x, x') = -i \langle 0 | \hat{T} A^{\mu}(x) A^{\nu}(x') | 0 \rangle. \quad (20)$$

Using the standard point splitting technique and taking the vacuum expectation value of the SET we can obtain

$$\langle \Theta^{\mu\nu} \rangle = \langle T^{\mu\nu} \rangle + \langle \Xi^{\mu\nu} \rangle, \quad (21)$$

where the first term,

$$\langle T^{\mu\nu} \rangle = i \lim_{x' \rightarrow x} \left[-\partial^{\mu} \partial'^{\nu} G^{\lambda}_{\lambda} + \partial^{\mu} \partial'_{\lambda} G^{\lambda\nu} + \partial^{\lambda} \partial'^{\nu} G^{\mu}_{\lambda} - \partial'^{\lambda} \partial_{\lambda} G^{\mu\nu} + \frac{1}{2} \eta^{\mu\nu} \left(\partial^{\alpha} \partial'_{\alpha} G^{\lambda}_{\lambda} - \partial^{\alpha} \partial'_{\beta} G^{\beta}_{\alpha} \right) \right], \quad (22)$$

is the VS of the standard Maxwell SET, and

$$\langle \Xi^{\mu\nu} \rangle = -2ik^{\nu} \epsilon^{\mu\alpha\beta\gamma} \lim_{x' \rightarrow x} \partial'_{\beta} G_{\gamma\alpha}, \quad (23)$$

encodes Lorentz-violating contributions.

5. Casimir effect

Now let us to consider the renormalized VS $\langle \Theta^{\mu\nu} \rangle_{\text{ren}}$, which is obtained as the difference between the VS in the presence of boundaries and that of the vacuum. For two parallel conducting plates separated by a distance D in the z direction, one can construct the renormalized expectation value of the stress-energy tensor, using conservation, tracelessness and symmetry arguments. We simplify our configuration by orienting the coordinate frame so that the plates are perpendicular to the background LV vector $k^\mu = (0, \mathbf{k}) = \kappa n^\mu$, however the general case proceeds along the same lines.

The explicit calculation of Eqs.(22)-(23), in the case of two parallel plates, allows us to derive $\langle \Theta^{\mu\nu} \rangle_{\text{ren}}$, which is given by

$$\langle \Theta^{\mu\nu} \rangle_{\text{ren}} = \langle t^{\mu\nu} \rangle_{\text{ren}} + (\eta^{\mu\nu} + 4n^\mu n^\nu) f_{\text{ren}}(\kappa, z) + n^\mu n^\nu g_{\text{ren}}(\kappa, z), \quad (24)$$

where $n^\mu = (0, 0, 0, 1)$ is the unit normal to the plates,

$$\langle t^{\mu\nu} \rangle_{\text{ren}} = -\frac{\pi^2}{720D^4} (\eta^{\mu\nu} + 4n^\mu n^\nu), \quad (25)$$

is the VS in the absence of Lorentz violation [28] and the functions $f(\kappa, z)$ and $g(\kappa, z)$, coming from the calculation of the GF function in Eq. (11), are defined as

$$f(\kappa, z) = \frac{4\kappa^2}{i} \int \frac{d^2\mathbf{p}}{(2\pi)^2} \int \frac{d\omega}{2\pi} \omega^2 \lim_{z' \rightarrow z} (p^2 + \partial_z \partial'_z) M(z, z'), \quad (26)$$

$$g(\kappa, z) = -8i\kappa^2 \int \frac{d^2\mathbf{p}}{(2\pi)^2} \int \frac{d\omega}{2\pi} p^2 \lim_{z' \rightarrow z} N(z, z'). \quad (27)$$

5.1. Global Casimir energy density

The Casimir energy is defined as the energy per unit area stored in the electromagnetic field between the plates, i.e.

$$\mathcal{E}_C = \int_0^D \langle \Theta^{00} \rangle_{\text{ren}} dz, \quad (28)$$

where $\langle \Theta^{00} \rangle_{\text{ren}}$ is the renormalized 00-component of the stress-energy tensor, which according to the procedure described above, it is obtained as the difference between the energy density in the presence of the plates and that of the free vacuum, i.e.

$$\langle \Theta^{00} \rangle_{\text{ren}} = \langle \Theta^{00} \rangle_{\parallel} - \langle \Theta^{00} \rangle_0. \quad (29)$$

Here, the labels \parallel and 0 mean that the expectation value is evaluated with the Green's function in the presence of the parallel plates and that of the vacuum, respectively. The final expression for the Casimir energy becomes

$$\mathcal{E}_C = -\frac{\pi^2}{720D^3} + \frac{\kappa^2}{32D}. \quad (30)$$

The Casimir stress is obtained by differentiating the Casimir energy with respect to D , i.e.

$$\mathcal{F}_C = -\frac{d\mathcal{E}_C}{dD} = -\frac{\pi^2}{240D^4} + \frac{\kappa^2}{32D^2}. \quad (31)$$

The first term is recognized as the usual Casimir force in Lorentz-symmetric electrodynamics, while the second represents the Lorentz-violating contribution. We observe that unlike the

$1/D^4$ dependence of the usual Casimir force, the LV part depends on $1/D^2$. As discussed in section 1, the authors in Ref. [18] analyzed the same Casimir configuration within the (3+1)D Chern-Simons extended electrodynamics. They obtained an attractive correction of the form $-\frac{25k_{AF}^2}{720D^2}$, while our result (31) is evidently repulsive. The origin of such disagreement is the Chern-Simons background tensor we work with; while the authors in Ref. [18] used a timelike background $k^\mu = (k_{AF}, \mathbf{0})$, we have chosen the spacelike vector $k^\mu = (0, \mathbf{k})$. Here we point out that the difference between our work and that of Ref. [18] is beyond the background tensor used. For example, they used standard techniques in Casimir physics (zeta function regularization and series summation via the residue theorem), while here we tackle the problem by means of Green's function techniques. The advantage of the local approach over the standard techniques is quite subtle: the former provides information about the local behavior of the quantum fields near a boundary, while the later only allows the calculation of the Casimir force. In 5.2 we discuss the local effects in (3+1)D Chern-Simons electrodynamics, which is the main aim of the present work.

5.2. Local effects

Heretofore, we have considered the global Casimir effect: the total energy of a field configuration or the force per unit area on a bounding surface [29, 30]. Local properties of the quantum vacuum induced by the presence of boundaries are of broad interest in quantum field theory [31] and they must be understood if one is to correctly interpret the inherent divergences in the theory.

The local energy density in Lorentz-symmetric electrodynamics has been discussed extensively in the literature [29, 30, 32]; however, the local effects in Lorentz-violating theories have not been considered. Here we aim to fill in this gap. We begin the analysis by considering an electromagnetic field confined between two parallel conducting plates at $z = 0$ and $z = D$, for which the energy density per unit volume between the plates is

$$\langle \Theta^{00} \rangle (z) = \langle t^{00} \rangle (z) + \langle \mathbf{T}^{00} \rangle (z). \quad (32)$$

A detailed analysis of the local effects due to the first term (in the absence of Lorentz violation) is presented in Ref. [29, 30]. Here we concentrate on the Lorentz-violating contribution,

$$\langle \mathbf{T}^{00} \rangle (z) = \frac{\kappa^2}{32D^2} [1 - 2 \csc^2(\pi Z)], \quad Z = \frac{z}{D}. \quad (33)$$

We observe that the z -independent term, $\frac{\kappa^2}{32D^2}$, corresponds to the global renormalized Lorentz-violating energy density obtained in 5.1. The second term encodes the local effects and it can be expressed as follows:

$$\mathcal{S}(Z) \equiv -\frac{\kappa^2}{16D^2} \csc^2(\pi Z). \quad (34)$$

We observe that it diverges quadratically as $z \rightarrow 0, D$. Its z integral over the region between the plates diverges linearly. This result reveals a close analogy with the one obtained from the Lorentz-symmetric part. In that case, the singular part diverges quartically as $z \rightarrow 0, D$. The less divergent Lorentz-violating contribution (34) can be understood as due to the dimension of Chern-Simons coupling κ . It is worth to mention that integrating this term over z ,

$$\int_0^D \mathcal{S}(Z) dz = -\frac{\kappa^2}{16\pi^2} \frac{2}{\Gamma(2)} \int_0^\infty d\rho, \quad (35)$$

we obtain a divergent constant term (D -independent), so it does not contribute to the observable force.

6. Conclusions

Let us summarize the main aims of the present contribution. We analyze the local effects of the quantum vacuum in a particular sector of the minimal SME, namely, the (3+1)D Maxwell-Chern-Simons term with a spacelike Lorentz-violating background field. Concretely, we use Green's function techniques to calculate the Casimir force between two parallel conducting plates separated by a distance D , focusing on the local properties of the quantum vacuum induced by the presence of the boundaries.

One of the contributions of the present work is the construction of the correct (indexed) Green's function for the theory we dealt with. We also analyze the behavior of the local energy density when approaching the plates.

The present work can be further generalized in a variety of ways. For example, the Green's function for different geometries can also be constructed using the same perturbative procedure. On the other hand, our analysis can also be applied to ponderable media. More precisely, we can consider a semi-infinite planar material medium with dielectric constant ε for which the reduced Green's function $\mathbf{g}_\varepsilon(z, z')$ is known. Now we can use it to evaluate the associated $N_\varepsilon(z, z')$ and $M_\varepsilon(z, z')$ functions, which are what we require to study the Lorentz-violating effects.

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