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CPT and Lorentz violation in the electroweak sector

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Abstract. Long ago, Carroll, Field and Jackiw introduced CPT-violation in the photon sector by adding a dimension-3 gauge-invariant term parametrized by a constant four-vector parameter k_{AF} to the usual (Maxwell) Lagrangian, deriving an ultra-tight bound from astrophysical data. Here, we will discuss recent work studying the extension of this term to the full electroweak gauge sector of the Standard Model. In the context of the Standard Model Extension, CPT and Lorentz violation arises from two gauge-invariant terms parametrized by the four vectors k_1 and k_2. First we will show how upon spontaneous breaking of the electroweak gauge symmetry these two terms yield Lorentz-violating terms for the photon and the W and Z bosons. As it turns out, the resulting modified dispersion relations for the W bosons yield spacelike momentum for one of its propagating modes at sufficiently large energy. This in turn allows for the possibility of Cherenkov-like W-boson emission by high-energy fermions such as protons, provoking their decay. Analysis of ultra-high-energy cosmic ray data allows for bounding the previously unbound parameter k_2, and, by combination with the ultra-tight bound on k_{AF}, the parameter k_1.

1. The CPT-odd gauge sector of the minimal Standard Model Extension

In the past two decades there has been a considerable interest in the possibility that Lorentz symmetry may not be exact in nature. The main theoretical motivation for this idea comes from a number of candidate theories for quantum gravity that have been shown to involve Lorentz-invariance violation as a possible effect. This may come about by spontaneous breaking of Lorentz symmetry in theories with Lorentz-invariant dynamics, such as in string field theory [1] or loop quantum gravity [2], or in theories that violate Lorentz invariance at a fundamental level, such as noncommutative geometry [3] (aspects of which are discussed by Orfeu Bertolami in this workshop) or Hořava-Lifshitz gravity [4, 5] (see the talks by Diego Blas and Kevin Grosvenor).

From the experimental point of view, the development of low-energy effective field theories with Lorentz-invariance violation, in particular the Standard-Model Extension (SME) [6], has been instrumental. If this framework, the Lagrangian of the matter sector contains all Lorentz-violating gauge-invariant effective operators that can be build from the conventional Standard-Model fields, coupled to vector and tensor coefficients that parametrize the Lorentz violation. The SME also contains all CPT-violating operators, since in any local interacting quantum field theory CPT violation implies Lorentz violation [7]. The SME can thus be used to provide a general quantification of the exactness of Lorentz and CPT symmetry in the form of observational constraints on the SME coefficients [8].

In 1991, well before the full SME was formulated, Carroll, Field and Jackiw introduced CPT-violation in the photon sector by adding a dimension-3 gauge-invariant term parametrized by a

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constant four-vector k_{AF} to the usual (Maxwell) Lagrangian [9]:

$$\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}k^{\mu}_{AF}A^{\nu}F^{\rho\sigma}. \tag{1}$$

They managed to extract an ultra-tight bound $|(k_{AF})^{\kappa}| \leq \mathcal{O}\big(10^{-43}\,\mathrm{GeV}\big)$ by analyzing astrophysical data. In the context of the $SU(2)\times U(1)$ electroweak sector of the Standard Model, CPT and Lorentz violation arises from two Chern-Simons-like SME terms parametrized by the four-vectors k_1 and k_2 [6]:

$$\epsilon^{\mu\nu\rho\sigma}k_{1\mu}B_{\nu}B_{\rho\sigma}+\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}k_{2\mu}\big[A^a_{\nu}A^a_{\rho\sigma}+\frac{2}{3}g_2\epsilon^{abc}A^a_{\nu}A^b_{\rho}A^c_{\sigma}\big], \tag{2}$$

where B_μ and A^a_μ (with $a=1,2,3$) represent the $U(1)_Y$ and $SU(2)_L$ gauge fields, $B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu$ and $A^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu$, while g_2 denotes the $SU(2)$ coupling constant. Both terms in expression (2) are gauge invariant up to a total derivative. There is a similar term corresponding to the strong-interaction sector, which we will ignore in this paper.

In the Standard Model, spontaneous symmetry breaking

$$SU(2)_L\times U(1)_Y\to U(1)_{EM}, \tag{3}$$

is provoked by assuming a Mexican-hat-type potential for the Higgs field $SU(2)$ doublet $\phi=\begin{pmatrix}\phi^+\\ \phi_0\end{pmatrix}$. In the unitary gauge, ϕ is taken to assume the vacuum expectation value $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$. The quadratic CPT- and Lorentz-violating Lagrangian of the electroweak gauge sector then becomes, in terms of the photon and the W and Z boson fields:

$$\begin{aligned}\mathcal{L}_{AWZ} &= \frac{1}{2}A_\mu D^{\mu\nu}A_\nu+\frac{1}{2}Z_\mu D^{\mu\nu}Z_\nu+W_\mu D^{\mu\nu}W^*_\nu+\frac{1}{2}m_Z^2Z_\mu Z^\mu+m_W^2W_\mu W^{*\,\mu}\\ &\quad+\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\left(k^\mu_{AF}A^\nu F^{\rho\sigma}+k^\mu_{ZZ}Z^\nu Z^{\rho\sigma}+2k^\mu_{WW}W^\nu W^{*\rho\sigma}+2k^\mu_{\mathrm{mix}}Z^\nu F^{\rho\sigma}\right),\end{aligned} \tag{4}$$

where $D^{\mu\nu}=\partial^2\eta^{\mu\nu}-(1-\xi^{-1})\partial^\mu\partial^\nu$, $m_W=c_w m_Z=g_2v/2$ as usual, and

$$k^\mu_{AF} = 2c_w^2k_1^\mu+s_w^2k_2^\mu\,, \tag{5}$$
$$k^\mu_{ZZ} = 2s_w^2k_1^\mu+c_w^2k_2^\mu\,, \tag{6}$$
$$k^\mu_{WW} = k_2^\mu\,, \tag{7}$$
$$k^\mu_{\mathrm{mix}} = c_ws_w(k_2^\mu-2k_1^\mu)\,, \tag{8}$$

with $c_w=\cos\theta_w$ and $s_w=\sin\theta_w$. The physical gauge boson fields are defined as usual

$$W_\mu=\frac{1}{\sqrt{2}}\left(A^1_\mu-iA^2_\mu\right), \qquad W^*_\mu=\frac{1}{\sqrt{2}}\left(A^1_\mu+iA^2_\mu\right), \tag{9}$$
$$Z_\mu=c_wA^3_\mu-s_wB_\mu\,, \qquad A_\mu=s_wA^3_\mu+c_wB_\mu\,. \tag{10}$$

2. Dispersion relations in the photon–Z-boson sector

The mixing term between the photon and the Z boson in (4) presents a considerable complication in interpreting the quadratic Lagrangian, as the asymptotic propagating modes no longer correspond exactly to the photon and Z-boson fields, but rather linear combinations thereof (and their derivatives).

As the existing bounds on k_{AF} are so stringent we will assume k_{AF} can be neglected with respect to k_{ZZ}, k_{WW} and k_{mix}. It is valid to do so as long as the bounds we consider on the latter parameters remain much larger than the bounds on k_{AF}. In that case, we see from (5) that $k_1^\mu = -\frac{1}{2}\tan^2(\theta_w)k_2^\mu$ such that $k_{\rm mix}^\mu = \frac{1}{2}\tan(2\theta_w)k_{ZZ}^\mu$. We now define the eight-component bi-four-vector $\mathcal{A}$, given by

$$\mathcal{A}^\mu = \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} . \tag{11}$$

Lagrangian (4), when restricted to the photon and Z boson fields, then becomes:

$$\mathcal{L}_{AZ} = \frac{1}{2}\mathcal{A}_\mu^T D^{\mu\nu}\mathcal{A}_\nu + \frac{1}{2}\mathcal{A}_\mu^T\mathcal{M}\mathcal{A}^\mu - \mathcal{A}_\mu^T\mathcal{X}k_{ZZ}^{\mu\nu}\mathcal{A}_\nu , \tag{12}$$

where $k_{ZZ}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}(k_{ZZ})_\rho\partial_\sigma$, and

$$\mathcal{M} = \begin{pmatrix} 0 & 0 \\ 0 & m_Z^2 \end{pmatrix} , \tag{13a}$$

$$\mathcal{X} = \begin{pmatrix} 0 & \frac{1}{2}\tan(2\theta_w) \\ \frac{1}{2}\tan(2\theta_w) & 1 \end{pmatrix} . \tag{13b}$$

Passing to momentum space, the equations of motion corresponding to Lagrangian $\mathcal{L}_{AZ}$ become

$$\Big[(p^2-\mathcal{M})\eta^\mu{}_\nu - (1-\xi^{-1})p^\mu p_\nu - 2i\mathcal{X}\epsilon^{\alpha\beta\mu}{}_\nu(k_{ZZ})_\alpha p_\beta\Big]\,\tau_\sigma^{(\lambda)\nu} \equiv \mathcal{S}^\mu{}_\nu\tau_\sigma^{(\lambda)\nu} = 0 \tag{14}$$

where $\tau_\sigma^{(\lambda)\nu}(\vec{p})$, with $\sigma = \pm 1$ and $\lambda \in \{0,3,+,-\}$, represent the eight eigenvectors of the equation-of-motion operator $\mathcal{S}^\mu{}_\nu$.

It can be shown [10] that the corresponding eigenvalues $\Omega_\sigma^\lambda(p)$ of $\mathcal{S}^\mu{}_\nu$ are given by

$$\Omega_{+1}^0(p) = \frac{1}{\xi}p^2 , \tag{15a}$$

$$\Omega_{-1}^0(p) = \frac{1}{\xi}(p^2 - \xi m_Z^2) , \tag{15b}$$

$$\Omega_{+1}^3(p) = p^2 , \tag{15c}$$

$$\Omega_{-1}^3(p) = p^2 - m_Z^2 , \tag{15d}$$

$$\Omega_{+1}^\pm(p) = p^2 - \frac{1}{2}m_Z^2 \pm \delta(p) + \frac{1}{2}\sqrt{(m_Z^2 \mp 2\delta(p))^2 + 4\tan^2(2\theta_w)\delta(p)^2} , \tag{15e}$$

$$\Omega_{-1}^\pm(p) = p^2 - \frac{1}{2}m_Z^2 \pm \delta(p) - \frac{1}{2}\sqrt{(m_Z^2 \mp 2\delta(p))^2 + 4\tan^2(2\theta_w)\delta(p)^2} . \tag{15f}$$

with $\delta(p) = \sqrt{(p\cdot k_{ZZ})^2 - p^2k_{ZZ}^2} = \frac{2}{\tan(2\theta_w)}\sqrt{(p\cdot k_{\rm mix})^2 - p^2k_{\rm mix}^2}$. The conditions $\Omega_\sigma^\lambda(p) = 0$ define the dispersion relations of the corresponding polarization modes $\tau_\sigma^{(\lambda)\nu}(\vec{p})$.

For "small" energies we can expand the square roots of the final two expressions and obtain

$$\Omega_{+1}^\pm(p) = p^2 + \tan^2(2\theta_w)\frac{\delta(p)^2}{m_Z^2} \pm 2\tan^2(2\theta_w)\frac{\delta(p)^3}{m_Z^4} + \cdots , \tag{16a}$$

$$\Omega_{-1}^\pm(p) = p^2 - m_Z^2 \pm 2\delta(p) - \tan^2(2\theta_w)\frac{\delta(p)^2}{m_Z^2} \mp 2\tan^2(2\theta_w)\frac{\delta(p)^3}{m_Z^4} + \cdots . \tag{16b}$$

From Eqs. (15) and (16) it is clear that the $\sigma = +1$ modes are massless and the $\sigma = -1$ modes are massive, at least in the limit of small Lorentz violation and low energies. We can therefore identify the former mode with the photon and the latter mode with the Z boson.

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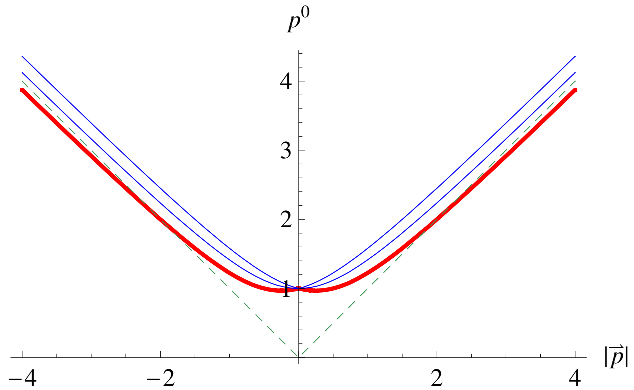

Figure 1. The dispersion relations (19) of the W boson for purely timelike k_2^μ, where we chose $k_2^0 = \frac{m_W}{4}$, in arbitrary units. Indicated in red is the + mode, which has spacelike four-momenta above an energy threshold.

It is not difficult to see from the Lorentz-violating dispersion relations (15e) and (15f) that

$$p^2 \begin{cases} \leq 0 & \text{for} \quad \sigma = +1 \\ > 0 & \text{for} \quad \sigma = -1 \end{cases} . \tag{17}$$

This shows that, if $k_{AF} = 0$ and $k_{ZZ} \neq 0$, the photon mode always has spacelike momenta, while the Z-boson momentum is always timelike. Note from (15c) that the longitudinal photon mode ($\lambda = 3$) is purely massless. It is unphysical and decouples, together with the gauge modes $\lambda = 0$, as usual upon applying, e.g., a BRST quantization procedure.

A more detailed analysis of the dispersion relations can be found in [10].

3. Cherenkov-like emission of W bosons

Let us now restrict Lagrangian (4) to the W boson field:

$$\mathcal{L}_W = W_\mu D^{\mu\nu} W_\nu^* + m_W^2 W_\mu W^{*\mu} + \epsilon_{\mu\nu\rho\sigma} k_2^\mu W^\nu W^{*\rho\sigma} . \tag{18}$$

It has been shown in [11] that (18) can be consistently quantized, as long as the components of k_2 are smaller than of the order of m_W, which we will of course assume to be the case.

As we will see, the resulting modified dispersion relations for the W bosons yield spacelike momentum for one of its propagating modes at sufficiently large energy. This makes it possible for Cherenkov-like W-boson emission to take place by fermions coupling to the W boson whose momentum exceeds a certain threshold value. In the case of composite fermions, such as protons, such a process will provoke their decay into lower-energy particles. Analysis of ultra-high-energy cosmic ray data allows for bounding the parameter k_2 and, by combination with the ultra-tight bound on k_{AF}, the parameter k_1.

The Lorentz-violating Lagrangian implies the dispersion relations

$$\Lambda_3(p) = p^2 - m_W^2 = 0, \qquad \Lambda_\pm(p) = p^2 - m_W^2 \pm 2\sqrt{(p \cdot k_2)^2 - p^2 k_2^2} = 0 , \tag{19}$$

(Here we suppressed the unphysical gauge mode $\lambda = 0$.) These dispersion relations are represented in Fig. 1 for the case of purely timelike k_2^μ. As it turns out, the $\lambda = 3$ and $\lambda = -$ gauge-boson polarization modes are timelike for any momentum. On the other hand, it follows from (19) that the gauge-boson momentum is spacelike for the $\lambda = +$ mode, if and only if

$$(p \cdot k_2)^2 > \frac{1}{4} m_W^4 . \tag{20}$$

Since spacelike momenta are a necessary condition for the desired Cherenkov-like processes, we will only consider W bosons in the + polarization mode.

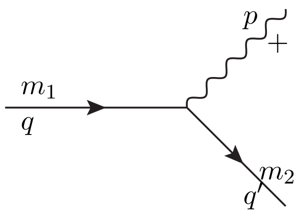

Figure 2. Relevant Feynman diagram for W-boson emission by an incoming fermion.

3.1. Emission by an elementary Dirac fermion

We will first consider the process indicated in Fig. 2 in which an elementary Dirac fermion with a mass m_1 decays to a W boson in the mode $\lambda = +$ and a Dirac fermion with a mass m_2. The Dirac fermions are taken to satisfy regular non-Lorentz-violating dispersion relations. We label the momenta of the particles as follows: the incoming fermion has momentum q, the emitted W boson has momentum p, and the out-going fermion has momentum $q' = q - p$.

Phase-space considerations show that a necessary condition for the decay to take place is

$$4(p \cdot k_2)^2 > (M^2 - (m_1 - m_2)^2)^2 + 4k_2^2(m_1 - m_2)^2. \tag{21}$$

Note that if $m_1 = m_2$ this condition reduces to the space-like condition (20).

The expression for the differential decay rate is

$$d\Gamma = \frac{1}{2q^0}\frac{d^3p}{(2\pi)^3}\frac{1}{\Lambda'_+(p)}\frac{d^3q'}{(2\pi)^3}\frac{1}{2q'^0}\left(\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2\right)(2\pi)^4\delta^4(q-p-q')\ . \tag{22}$$

The squared matrix element $|\mathcal{M}|^2$ is summed (averaged) over the final (initial) fermion spin. The unconventional factor

$$\Lambda'_+(p) = \frac{\partial\Lambda_+(p)}{\partial p^0}, \tag{23}$$

in the denominator defines a positive definite normalization in which the phase space and the matrix element are separately observer Lorentz invariant [11], i.e., invariant under simultaneous Lorentz transformations of the momenta and the Lorentz-violating four-vector. Note that this would not be the case if we were to choose a more conventional phase space normalization.

The matrix element corresponding to the process denoted in Fig. 2 is

$$i\mathcal{M} = \frac{ig_2}{2\sqrt{2}}\bar{u}(q')\gamma^\mu(1-\gamma^5)u(q)e_\mu^{(+)*}(p)\ , \tag{24}$$

where $u(q)$ and $u(q')$ are conventional Dirac spinors. (Analogous expressions can be written down for the case of anti-particles.) The four-vector $e_\mu^{(+)}(p)$ is the gauge-boson polarization vector that corresponds to the dispersion relation in Eq. (19). The explicit expression for the latter can be found in Ref. [11].

The decay rate can be evaluated to yield [12]

$$d\Gamma = -\frac{g_2^2}{64\pi^2 q^0}\int\frac{d^3p}{\Lambda'_+(p)}\theta(q^0-p^0)\delta((q-p)^2-m_2^2)p^2\left[(1\mp X)^2 - \frac{(m_1^2-m_2^2)^2}{p^4}\right]\ . \tag{25}$$

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where the upper (lower) sign holds for a decaying particle (antiparticle) and

$$X = \frac{p^2(p \cdot k_2 - 2q \cdot k_2) + (m_1^2 - m_2^2)(p \cdot k_2)}{p^2\sqrt{(p \cdot k_2)^2 - p^2 k_2^2}} \,. \tag{26}$$

It follows from formula (25) that for the decay to take place, the incoming fermion momentum has to exceed a threshold value:

$$|\vec{q}| > |\vec{q}|_{\mathrm{th}}, \qquad \text{where} \qquad |\vec{q}|_{\mathrm{th}} \approx \frac{M(M + 2m_2)}{2|\kappa|}, \tag{27}$$

where κ is a quantity of the order of the components of k_2: $\kappa \sim \pm|k_2^\mu|$ (see [12] for the exact expressions).

Another result that can be extracted from (25) is that the gauge bosons are emitted in a very narrow forward cone around the direction of the incoming fermion:

$$\cos\theta_{pq} = 1 + \mathcal{O}\left(\frac{\kappa^2}{M^2}\right), \tag{28}$$

where θ_{pq} is the angle between $\vec{p}$ and $\vec{q}$.

Integrating expression (25) over all W-boson momenta yields the total decay rate:

$$\Gamma = \frac{g^2|\kappa|}{64\pi} G(a)\theta(a-1). \tag{29}$$

where

$$\begin{aligned} G(a) = {} & \alpha(a)\left[\frac{1}{a}(-7+3y)\left(1 - \frac{m_2}{M}\right) - \frac{1}{a^2}(1-y)\left(1 - \frac{3m_2}{M}\right)\right] \\ & - 4\left(1 + \frac{1}{a}(1-y)\right)\left(1 - \frac{2m_2}{M}\right)\log\left(\frac{1 + a - m_2(1-a)/M - \alpha(a)}{1 + a - m_2(1-a)/M + \alpha(a)}\right) \\ & + \mathcal{O}\left(\frac{m_{1,2}^2}{M^2}, \frac{\kappa^2}{M^2}\right) . \end{aligned} \tag{30}$$

Here a is defined as the ratio of $|\vec{q}|$ to its threshold value, i.e., $a = |\vec{q}|/|\vec{q}|_{\mathrm{th}}$. The function $\alpha(a) = \sqrt{(a-1)^2 + 2m_2(a^2-1)/M}$ and $y = \pm\mathrm{sgn}(\kappa)$, where the upper (lower) sign applies to the particle (antiparticle).

A fermion that couples to a CPT-violating W boson will emit W bosons if it has an energy above threshold, each of which will take away an energy of at least the value $|\vec{q}|_{\mathrm{th}}$. From (29) it follows that the typical decay rate is of the order of 10^{-15} s if $\mathcal{O}(\kappa) = 10^{-7}$ GeV, corresponding to a bound we will find below. It also follows that it will approximately take a time of order $a \times 10^{-15}$ s for all fermions in a decay cascade to fall below threshold for such values of κ.

3.2. Cherenkov emission by a proton

In case the incoming fermion is a composite particle such as the proton, the emission of a W boson will provoke a break-up, since the typical momentum transfer lies in the range of the W-boson mass, which is well within the energy range of for example deep inelastic scattering. In this case, the proton-decay rate can be written as

$$\Gamma = \frac{1}{2q^0}\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{4\pi}{\Lambda'_+(p)} e_\mu^{(+)}(p) e_\nu^{(+)*}(p) W^{\mu\nu} \,, \tag{31}$$

where $W^{\mu\nu}$ is the hadronic part, given by

$$W^{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma} \langle q, \sigma | J^{\nu}(-p) \sum\!\!\!\!\!\!\int_X |X\rangle\langle X| J^{\mu}(p) | q, \sigma \rangle \ . \tag{32}$$

$|q, \sigma\rangle$ is a proton state with momentum q and spin σ, $J^{\mu}(p)$ is the hadronic current, while $\sum\!\!\!\!\!\!\int_X$ indicates a sum over all hadronic final states X along with the corresponding integrations over phase space. $W^{\mu\nu}$ can be evaluated in the parton model (see, e.g., Ref. [13] for a pedagogical introduction).

The final result for the proton decay rate is

$$\Gamma = \frac{g^2|\kappa|}{64\pi} \sum_q \int_0^1 dx \, (f_q(x) + \bar{f}_q(x)) \tilde{G}_q(ax) \theta(ax - 1) \ . \tag{33}$$

Here the functions $f_q(x)$ and $\bar{f}_q(x)$ are the parton distributions functions (PDFs) for the quarks and antiquarks of flavor q, respectively. They represent the chance of finding a quark with momentum fraction x inside the proton. The function $\tilde{G}_q(ax)$ in Eq. (33) is the function in Eq. (30) with the substitutions $m_2 \to x m_2$ and $y \to \tilde{y}_q = \mathrm{sgn}(\kappa) \frac{f_q(x) - \bar{f}_q(x)}{f_q(x) + \bar{f}_q(x)}$. The integral over x in Eq. (33) can be carried out numerically using fits for the PDFs. We refer to Ref. [12] for details.

The most important conclusion for us is that the threshold value for W emission by a proton to occur is still given by formula (27). The decay rate will depend on the details of the PDFs, but its order of magnitude value is the same as for the elementary fermion case (at least for values of a not very close to 1).

4. Limits from ultra-high-energy cosmic rays

We can use the fact that a proton with an energy above threshold will disintegrate to use astrophysical data to limit k_2^{μ}. More precisely, such a proton cannot reach Earth if its mean free path L is much smaller than the distance from its source to Earth. Since many ultra-high-energy cosmic ray particles (UHECR) with energies above $57\,\mathrm{EeV} \equiv |\vec{q}|_{\mathrm{obs}}$ have been observed, more or less from all directions [14], it follows that

$$|\kappa| \lesssim \frac{M^2}{|\vec{q}|_{obs}} \approx 1.1 \times 10^{-7}\,\mathrm{GeV} \equiv |\kappa|_0. \tag{34}$$

We see from (33) that the mean lifetime of protons (in the Earth's frame) t_p is proportional to $|\kappa|^{-1}$. A conservative (large) estimate gives a mean free path of

$$L \simeq c t_p \sim (\hbar c / |\kappa|_0) \times 10^{15} \sim 10^3\,\mathrm{km}\,. \tag{35}$$

It is clear that protons with an energy above this threshold will not be able to reach Earth from any viable UHECR source. This allows us to conclude that

$$|k_2^{\mu}| < 1.1 \times 10^{-7}\,\mathrm{GeV} \ . \tag{36}$$

Combining this with the ultra-tight bound on the components of k_{AF} and its expression (5) in terms of k_1 and k_2, we find the bound

$$|k_1^{\mu}| < 1.7 \times 10^{-8}\,\mathrm{GeV} \ . \tag{37}$$

on the components of k_1.

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