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To cite this article: C A Escobar and L F Urrutia 2018 *J. Phys.: Conf. Ser.* **952** 012013

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Vector models with spontaneous Lorentz-symmetry breaking

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Abstract. Even though models with spontaneous Lorentz-symmetry breaking also damage gauge invariance, an interesting possibility that emerges is to interpret the resultant massless Goldstone bosons as the gauge bosons of the related gauge theory. In this contribution we review the conditions under which gauge invariance is recovered from such models. To illustrate our general approach we consider the classical Abelian bumblebee and Nambu models. In the former case we prove its connection with electrodynamics by a procedure which takes proper care of the gauge-fixing conditions. In the case of the Abelian Nambu model its relation with electrodynamics is established in such a way that the generalization to the non-Abelian case is straightforward.

1. Introduction

An alternative way of looking at global active Lorentz-symmetry breaking (LSB) is to consider its spontaneous version, which yields Goldstone bosons (GBs) of tensorial character as opposed to the standard Higgs mechanism with scalar content. These GBs can be viewed as realizations of photons, gluons and gravitons, for example. One of the main questions posed by this approach is to determine the sector in the phase space where the model with broken Lorentz symmetry reduces to the gauge theory, to be called the mother gauge theory (MGT), the gauge bosons of which one intends to identify with the GBs of the Lorentz breaking.

The idea that gauge particles (photons and gravitons, for example) might arise as the GBs of a theory with spontaneous LSB has been widely studied and goes back a long way [1, 2, 3, 4]. Bumblebee models [5], which are vector theories exhibiting spontaneous LSB, have been useful in providing simple toy models to deal with this question [6, 7, 8, 9]. Also the study of Nambu models [10], where the LSB is imposed non-linearly in analogy with the description of pion interactions in the non-linear sigma model, brought new insights in this direction [11, 12, 13]. For additional references see for example Ref. [14].

We consider the simplest cases of the Abelian bumblebee and Nambu models which MGT is standard electrodynamics (ED). Nevertheless the method we use to determine the conditions for the equivalence between the Abelian Nambu model (ANM) and ED can be easily generalized to the non-Abelian case. The Abelian bumblebee model (ABM) and the ANM arise from the following Lagrangian density

$$\mathcal{L}_{\text{ABM}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - J_{\mu}B^{\mu} - \frac{\kappa}{2}V(\xi), \quad B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}, \quad \xi = B_{\mu}B^{\mu} - n_{\mu}n^{\mu}b^2, \quad (1)$$



and the difference between them depends on the rôle played by κ together with a specific choice of $V(\xi)$. In the former case κ is a constant with the usual choice $V(\xi) = \xi^n/n$, while in the latter situation κ is a Lagrange multiplier and $V(\xi) = \xi$. In Eq. (1), $b^2 > 0$, n^μ is a unit vector which indicates the direction of the symmetry breaking vacuum and the potential $V(\xi)$ has a minimum at $\xi = 0$, i.e. $V'(\xi)|_{\xi=0} = 0$ and $V''(\xi)|_{\xi=0} > 0$. The potential term breaks Lorentz symmetry spontaneously as well as gauge invariance. While bumblebee models contain massless and massive modes, which nature depends on the choice of the symmetry breaking vacuum, the constraint imposed on Nambu models via the Lagrange multiplier freezes the massive modes and restricts the dynamics to the Goldstone modes only.

The paper is organized as follows: in section 2 we review the conditions under which the ANM becomes equivalent to ED. Some comments regarding further generalizations of the method are included. Section 3 is devoted to a recap of the formulation of electrodynamics in terms of longitudinal and transverse variables. Here we pay careful attention to the use of a consistent gauge-fixing procedure at the classical level in electrodynamics, which clearly shows that the final Hamiltonian and Dirac brackets in terms of the transverse variables are fully independent of the gauge fixing. In this way we clarify some issues of gauge fixing arising in Ref. [15]. These results are subsequently employed in section 4 to obtain the phase space sector where the classical ABM is equivalent to ED. Our metric signature is $(+, -, -, -)$.

2. The Abelian Nambu model

In general, Nambu models can be derived from ABM models by taking κ to be a Lagrange multiplier and choosing $V(\xi) = \xi$. In practice, this means that the Lagrangian of the ANM is

$$\mathcal{L}_{\text{ANM}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - J_\mu B^\mu, \quad B_\mu B^\mu - n_\mu n^\mu b^2 = 0, \quad (2)$$

where the constraint must be solved and substituted in \mathcal{L}_{ANM} . Thus, the ANM has three degrees of freedom (d.o.f.), instead of the two required by gauge invariance in ED. We assume also that the ANM is coupled to a conserved current J_μ .

The problem now is how to recover gauge invariance from the ANM in a consistent manner. To this end, the main idea is to take advantage of the formal identity between the Lagrangians of the two theories, which will allow us to relate the corresponding canonical formulations through a canonical transformation, once we have identified the d.o.f. in the ANM which solve the constraint in Eq. (1). Let us consider the following independent coordinates Φ_3 and $\Phi_{\bar{k}}$, $\bar{k} = 1, 2$, together with the invertible velocity-independent transformation $B_i = B_i(\Phi_j)$

$$B_{\bar{k}} = \Phi_{\bar{k}}, \quad B_3 = \Phi_3 \left(1 - \frac{N}{4\Phi_3^2} \right), \quad N = (\Phi_{\bar{k}}\Phi_{\bar{k}} + n^2 b^2). \quad (3)$$

The additional choice

$$B_0 = \Phi_3 \left(1 + \frac{N}{4\Phi_3^2} \right), \quad (4)$$

guarantees the $B_\mu B^\mu = n^2 b^2$ is identically satisfied.

The above transformations (3) and (4) determine the velocities \dot{B}_μ as functions of $\dot{\Phi}_i$ and Φ_i and we can write $\mathcal{L}_{\text{ANM}} = \mathcal{L}_{\text{ANM}}(\Phi, \dot{\Phi})$. The calculation of the canonical momenta $\Pi = \partial\mathcal{L}_{\text{ANM}}/\partial\dot{\Phi}$ of the ANM indicates that we end up with a regular systems (no constraints) such that the velocities $\dot{\Phi}$ can be inverted in terms of the momenta Π . In other words, we can identify the three fields Φ_i as the d.o.f. of the ANM. The canonically-conjugated variables Φ and Π satisfy the standard Poisson algebra.

To make direct contact with ED, let us recall that \mathcal{L}_{ANM} in Eq. (2) can be rewritten as

$$\mathcal{L}_{\text{ANM}}(\Phi, \dot{\Phi}) = \frac{1}{2}(e_i e_i - b_i b_i) - J^\mu B_\mu, \quad (5)$$

in terms of the standard electric $e_i = \dot{B}_i - \partial_i B_0$ and magnetic field $b_i = \epsilon_{ijk} B_{jk}/2$, which now are functions of $(\dot{\Phi}, \Phi)$ and Φ , respectively. At this stage, e_i and b_i are only convenient labels to denote functions of $\dot{\Phi}$ and Φ .

Since (3) is a coordinate transformation we have

$$\frac{\partial \dot{B}_i}{\partial \dot{\Phi}_j} = \frac{\partial B_i}{\partial \Phi_j} \quad \Rightarrow \quad \Pi_i = \frac{\partial \mathcal{L}_{\text{ANM}}}{\partial \dot{\Phi}_i} = \frac{\partial B_j}{\partial \Phi_i} e_j, \quad e_i(\Phi, \Pi) = \frac{\partial \Phi_j}{\partial B_i} \Pi_j. \quad (6)$$

Using the transformation $(\Phi_i, \Pi_j) \rightarrow (B_i, e_j)$ we can write

$$\mathcal{H}_{\text{ANM}}(\Phi, \Pi) = \Pi_i \dot{\Phi}_i - \mathcal{L}_{\text{ANM}} = \frac{1}{2}(e_i e_i + b_i b_i) + J^i B_i - (\partial_i e_i - J^0) B_0, \quad (7)$$

where $B_0 = B_3 [(B_{\bar{i}} B_{\bar{i}} + n^2 M^2)/(B_3 B_3)]^{1/2}$ in terms of the original variables. Inserting the relation

$$\Pi_i \dot{\Phi}_i = \Pi_i \frac{\partial \Phi_i}{\partial B_j} \dot{B}_j = e_j \dot{B}_j. \quad (8)$$

in the Hamiltonian action of the ANM yields

$$S_{\text{ANM}} = \int d^4x \left(\Pi_i \dot{\Phi}_i - \mathcal{H}_{\text{ANM}}(\Phi, \Pi) \right) = \int d^4x \left(e_i^a \dot{B}_i^a - \mathcal{H}_{\text{ANM}}(B_j, e_k) \right), \quad (9)$$

which identifies e_i as the canonically-conjugated momenta to B_i .

The transformation $(\Phi_i^a, \Pi_j^b) \rightarrow (B_i^a, e_j^b)$ is generated only by the coordinate transformation (3), so that it is a canonical transformation. Thus, without any further calculation, we recover the canonical algebra of ED among the variables B_i and e_j from that of the ANM. In fact, using the relations (3) together with (6) we can explicitly verify this statement by using the canonical algebra of the ANM.

Now we undo the canonical transformation in (9) and look at the quantities B_i and e_j as independent canonical variables satisfying such a Poisson algebra. Also the Hamiltonian density (7) acquires now the same form as in ED. Nevertheless, in spite of this apparent identity, \mathcal{H}_{ANM} is still not the Hamiltonian of ED because we are missing the following two conditions: (1) $\Omega \equiv (\partial_i e_i - J^0)$ is not a constraint and (2) B_0 is not an arbitrary function. To recover these conditions we calculate the evolution of the Gauss function Ω . We can do the calculation in the dynamics of the ANM using $\mathcal{H}_{\text{ANM}}(\Phi, \Pi)$ together with the relations (3) and (6) or, in a simpler way, by considering directly $\mathcal{H}_{\text{ANM}}(B_i, e_j)$ plus the associated canonical algebra we have just obtained. The result is

$$\dot{\Omega} = -\partial_\mu J^\mu + F\Omega + \partial_i(G^i\Omega), \quad (10)$$

where the functions F, G^i depend only on the fields B_j . Since we have assumed current conservation, we observe that Eq. (10) implies that after demanding $\Omega = 0$ at some initial time, we obtain the Gauss-law constraint for all time. Then we can add this constraint to the Hamiltonian density in the form $\Theta\Omega$, with Θ being an arbitrary function, which in turn will make $(B_0 + \Theta)$ arbitrary. Since Ω is now a constraint, Eq. (10) assures us that no additional constraints arise.

Summarizing: the equivalence between the ANM and the ED, both coupled to a conserved external current, is obtained only after the Gauss law is imposed as an initial condition. The

above procedure can be directly generalized for a non-Abelian Nambu model when finding the conditions for its equivalence with the corresponding non-Abelian MGT [16].

We have also provided a proof of the quantum equivalence of the ANM and ED in the gauge $B_\mu B^\mu - n^2 b^2 = 0$, to all orders in perturbation theory, under the previously stated requirements. In this case the ANM is coupled to the conserved fermionic current $e\bar{\Psi}\gamma^\mu\Psi$ and the Gauss law is imposed upon the zeroth-order Lagrangian which reduces to ED in the axial gauge for $t \rightarrow \infty$ [17].

The study of Nambu models poses the following general problem in gauge theories: how do we recover gauge invariance after breaking it by the imposition of an additional constraint among the coordinates? This question was addressed in Ref. [18] for a Yang-Mills theory. We dealt with this problem in Ref. [14] by introducing what we call an extended Nambu model, defined by a Lagrangian density yielding first-class constraints only, plus an arbitrary relation among the coordinates restricted alone by very general conditions.

3. Electrodynamics in transverse and longitudinal variables

In preparation for the next section, let us briefly recall the formulation of ED in terms of these variables, which clearly shows that the resulting Hamiltonian and Dirac algebra are independent of the gauge fixing. To this end we decompose any vector field V^k in its transverse V_T^k and longitudinal $V_L^k = \partial_k V$ parts, such that $V^k = V_T^k + \partial_k V$, satisfying $\nabla \cdot \mathbf{V}^T = 0$ and $\nabla \times \mathbf{V}^L = 0$. In terms of the original vector we have

$$V_T^i = \left(\delta_{ik} - \frac{\partial_i \partial_k}{\nabla^2} \right) V^k, \quad V = \frac{1}{\nabla^2} \partial_k V^k, \quad \nabla^2 = \partial_k \partial_k. \quad (11)$$

For ED we rewrite the vector potential A^μ in terms of A_0, A_T^i, A and use analogous decomposition for the current J^μ . Using these variables, the standard Lagrangian density for ED becomes

$$\mathcal{L}_{ED} = \frac{1}{2} (\dot{A}_T^i)^2 - \frac{1}{2} \dot{A} \nabla^2 \dot{A} - (\nabla^2 A_0) \dot{A} - \frac{1}{2} A^0 \nabla^2 A^0 - \frac{1}{4} F_T^{ij} F_{Tij} - J^0 A^0 - J_T^i A_T^i - J \nabla^2 A. \quad (12)$$

The equations of motion are

$$-\ddot{A}_T^j + \partial_i F_T^{ij} + J_T^j = 0, \quad \nabla^2 (\dot{A} + A^0) + J^0 = 0, \quad \nabla^2 (\ddot{A} + \dot{A}_0 - J) = 0, \quad (13)$$

arising from the variations $\delta A_T^i, \delta A_0$ and δA respectively. Let us observe that current conservation $\partial_0 J^0 + \partial_i J^i = 0$ reads $\dot{J}^0 = -\nabla^2 J$ in such a way that the third equation in (13) is just the time derivative of the second equation.

The canonical momenta are

$$\Pi_0 = \frac{\partial L}{\partial \dot{A}^0} = 0, \quad \Pi_{iT} = \frac{\partial L}{\partial \dot{A}_T^i} = \dot{A}_T^i, \quad \Pi = \frac{\partial L}{\partial \dot{A}} = -\nabla^2 (\dot{A} + A_0). \quad (14)$$

yielding the non-zero, equal-time Poisson brackets

$$\begin{aligned} \{A^0(t, \mathbf{x}), \Pi_0(t, \mathbf{y})\} &= \delta^3(\mathbf{x} - \mathbf{y}), & \{A(t, \mathbf{x}), \Pi(t, \mathbf{y})\} &= \delta^3(\mathbf{x} - \mathbf{y}), \\ \{A_T^i(t, \mathbf{x}), \Pi_{Tj}(t, \mathbf{y})\} &= \left(\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \right) \delta^3(\mathbf{x} - \mathbf{y}). \end{aligned} \quad (15)$$

The extended Hamiltonian is

$$\mathcal{H}_E = \frac{1}{2} (\Pi_{iT})^2 + \frac{1}{4} F_{Tij} F_T^{ij} - J_T^i A_T^i - \frac{1}{2} \Pi \frac{1}{\nabla^2} \Pi + (J^0 - \Pi) A_0 + J \nabla^2 A + \alpha \Pi_0, \quad (16)$$

where we have explicitly introduced the primary constraint $\Pi_0 \approx 0$. The first-order action leading to the Hamilton equations reads

$$S = \int d^4x \left(\Pi_0 \dot{A}_0 + \Pi_{iT} \dot{A}_T^i + \Pi \dot{A} - \mathcal{H}_E \right), \quad (17)$$

Applying the standard Dirac procedure we are finally left with two first-class constraints

$$\phi_1 = \Pi_0 \approx 0, \quad \phi_2 = \Pi - J^0 \approx 0, \quad (18)$$

which lead to the final count of two independent d.o.f. in coordinate space. Let us observe that current conservation allows to rewrite the following two contributions in the Hamiltonian action as

$$\int d^4x (\Pi \dot{A} - J \nabla^2 A) = \int d^4x (\Pi \dot{A} + J^i \partial_i A) = \int d^4x (\Pi \dot{A} - \partial_i J^i A) = \int d^4x (\Pi - J^0) \dot{A} \approx 0. \quad (19)$$

In this way, after the complete gauge fixing, that is to say after the constraints (18) have been set strongly equal to zero, these two pieces will always cancel, irrespectively of the explicit form of the additional functions chosen for the gauge fixing. Also the contribution proportional to $(J^0 - \Pi)$ in the Hamiltonian density will be zero. Thus, after imposing (18) strongly, which means having fixed the gauge via two additional constraints, the Hamiltonian action always reduces to

$$S = \int d^3x \left(\Pi_{iT} \dot{A}_T^i - \frac{1}{2} (\Pi_{iT})^2 - \frac{1}{4} F_{Tij} F_T^{ij} + J_T^i A_T^i + \frac{1}{2} J^0 \frac{1}{\nabla^2} J^0 \right). \quad (20)$$

An important point to have in mind for a correct gauge fixing is to verify that the following conditions are satisfied. (a) The consistency of the additional constraints with the equations of motion, that is to say that the proposed choices of A_0 and A are compatible with the second equation in (13). (b) That the resulting system is in fact second class. (c) That the resulting Dirac brackets of the remaining transverse variables are unchanged with respect to their PB.

As a first general step, we can always use the constraint $\phi_1 = \Pi_0 \approx 0$ together with

$$\phi_3 = A_0 + \dot{A} + \frac{1}{\nabla^2} J^0, \quad (21)$$

which is precisely the second equation in (13), as a set of second class constraints to get rid of the variables Π_0 and A^0 . This choice maintains the Dirac brackets of the transverse degrees of freedom unchanged with respect to the original brackets in Eq. (15). Finally one has to set the constraint ϕ_2 strongly equal to zero. We illustrate this last gauge fixing by imposing a general coordinate constraint

$$\phi_4 = F(A^\mu) \approx 0. \quad (22)$$

In the variables we are using this means to set $F(A^0 = -\partial_t A - \frac{1}{\nabla^2} J^0, A_T^k, \partial_k A) \approx 0$, which defines a complicated non-linear equation for $A = A(A^k, J_0)$. Adopting a solution $Q(A^k, J_0)$ of this equation we replace ϕ_4 by

$$\bar{\phi}_4 = A - Q(A^k, J_0) \approx 0, \quad (23)$$

which is such that the set $\{\bar{\phi}_4, \phi_2\}$ is second class, thus fixing completely the remaining gauge symmetry. In practice, the explicit form of Q can be obtained only in very particular cases, but we assume its existence [19]. Again, the Dirac brackets of the transverse variables remain unchanged and the final Hamiltonian action is given by Eq. (20).

4. The Abelian bumblebee model

In this section we present a proof of the equivalence between the classical ABM and ED, under conditions to be specified in the following, which is alternative to that given in Ref. [15]. Let us recall that κ in Eq. (1) is now a constant instead of a Lagrange multiplier, which makes this theory different from the ANM discussed in the previous sections. To identify the phase space sector where the equivalence is achieved we follow closely the formulation of ED in terms of longitudinal and transverse variables discussed in section 3.

Since $\mathcal{L}_{ABM}(B^\mu) = \mathcal{L}_{ED}(B^\mu) - \frac{\kappa}{2}V(\xi)$, the equations of motion are

$$\begin{aligned} -\ddot{B}_T^j + \partial_i B_T^{ij} + J_T^j - \kappa V'(\xi) B_j &= 0, & \nabla^2(\dot{B} + B^0) + J^0 - \kappa V'(\xi) B^0 &= 0, \\ \nabla^2(\ddot{B} + \dot{B}_0 - J) - \kappa \partial_i (V'(\xi) B^i) &= 0. \end{aligned} \quad (24)$$

We always keep in mind that $B^i = B_T^i + \partial_i B$ but for notational simplicity we use the compact notation B^i (or B^μ), unless confusion arises.

Because the potential $V(\xi)$ does not include velocities, the canonically conjugated momenta of the ABM, together with the corresponding Poisson brackets are just given by the expressions (14) and (15). Also, the Hamiltonian density is $\mathcal{H}_{ABM} = \mathcal{H}_{ED} + \frac{\kappa}{2}V(\xi)$. The ABM has only two constraints

$$\Phi_1 = \Pi_0 \approx 0, \quad \Phi_2 = \Pi - J^0 - \kappa V'(\xi) B^0 \approx 0, \quad (25)$$

which are second class, implying that we have $\frac{1}{2}(2 \times 4 - 2) = 3$ d.o.f., instead of the two arising in ED. One possibility to cut this additional d.o.f. is to impose two additional second-class constraints.

In order to come closely to ED we propose to incorporate the additional constraints

$$\Phi_3 = B^0 + \dot{B} + \frac{1}{\nabla^2} J^0 \approx 0, \quad \Phi_4 = B_\mu B^\mu - n^2 b^2 = \xi \approx 0. \quad (26)$$

Nevertheless, in order to keep a sound theory we have to determine under which conditions such additional constraints are maintained for all time by the dynamics of the ABM, so that no further constraints are generated. We begin by calculating $\dot{\Phi}_3$ from Eq. (26), which involves $\dot{B}^0 + \ddot{B}$. Solving for this combination in the last equation in (24) yields

$$\dot{\Phi}_3 = \frac{1}{\nabla^2} \left(\partial_\mu J^\mu + \kappa \partial_i (V'(\xi) B^i) \right) = \frac{1}{\nabla^2} \left(\partial_\mu J^\mu + \kappa V'(\xi) \partial_i B^i + \kappa V''(\xi) B^i \partial_i (B_\mu B^\mu) \right). \quad (27)$$

From the above relation we conclude that after imposing current conservation and the condition $B_\mu B^\mu = n^2 b^2$ ($\xi = 0$, $\partial_i (B_\mu B^\mu) = 0$) at some initial time, $t = 0$ for example, the constraint $\Phi_3 \approx 0$ is conserved for all time.

The next step is to calculate $\dot{\Phi}_4 = 2(B_0 \dot{B}_0 - B_i \dot{B}_i)$, which we do by solving for \dot{B}^0 from the explicit expression for $\dot{\Phi}_2 \approx 0$. We have

$$\dot{\Phi}_2 = \dot{\Pi} - J^0 - \kappa \left(\dot{B}_0 V' + 2B_0 V'' (B_0 \dot{B}_0 - B_i \dot{B}_i) \right) = 0, \quad (28)$$

which leads to

$$\dot{B}_0 \approx \frac{1}{\kappa(V' + 2B_0^2 V'')} \left(\dot{\Pi} - J^0 + 2\kappa B_0 V'' B_i \dot{B}_i \right). \quad (29)$$

Recalling that $\Pi = -\nabla^2(\dot{B} + B_0)$, leading to $\dot{\Pi} = -\nabla^2(\ddot{B} + \dot{B}_0)$, and using again the full equation arising from the third expression in the relations (24) we obtain $\dot{\Pi} = -\partial_i J^i - \kappa \partial_i [V'(\xi) B^i]$ which we substitute in (29), yielding the final expression

$$\dot{B}_0 \approx \frac{1}{\kappa(V' + 2B_0^2 V'')} \left(-\partial_\mu J^\mu - \kappa \partial_i (V'(\xi) B^i) + 2\kappa B_0 V'' B_i \dot{B}_i \right). \quad (30)$$

The final step is to replace \dot{B}_0 in the expression for $\dot{\Phi}_4$ which leads to

$$\dot{\Phi}_4 \approx \frac{2B_0}{\kappa(V' + 2B_0^2 V'')} \left(-\partial_\mu J^\mu - \kappa \partial_i (V'(\xi) B^i) - \frac{\kappa V'(\xi)}{B_0} B_i \dot{B}_i \right). \quad (31)$$

Again, after imposing the initial conditions $\partial_\mu J^\mu|_{t=0} = 0$ and $\xi|_{t=0} = (B_\mu B^\mu - n^2 b^2)|_{t=0} = 0$, we obtain $\dot{\Phi}_4 \approx 0$ for all time.

To conclude, let us summarize the phase-space conditions under which the ABM turns into ED. As mentioned before, we set the initial conditions $\partial_\mu J^\mu|_{t=0} = 0$ and $\xi|_{t=0} = (B_\mu B^\mu - n^2 b^2)|_{t=0} = 0$. Then, at this time we recover the set of constraints (18), (21) and (22) given in the previous section, the latter with the choice $F(B^\mu) = B_\mu B^\mu - n^2 b^2$. Also, up to the constant $\kappa V(0)/2$, we have $\mathcal{H}_{ABM} = \mathcal{H}_{ED}$. That is to say, at $t = 0$ the above conditions turn the ABM into ED in the gauge $B_\mu B^\mu - n^2 b^2 = 0$. The constraints Φ_1 and Φ_2 in the ABM are automatically conserved by their own dynamics, and the above initial conditions guarantee also that $\dot{\Phi}_3 \approx 0$ and $\dot{\Phi}_4 \approx 0$ at $t = 0$. Then we reproduce the constraints (18), (21) and (22) together with \mathcal{H}_{ED} , now at $t = 0 + \delta t$, which means that the ABM is equivalent to ED at $t = 0 + \delta t$ in this sector of the phase space. We can iterate the previous analysis, since having Maxwell equations at $t = 0 + \delta t$, the identity $0 \equiv \partial_\mu \partial_\nu B^{\mu\nu} = \partial_\mu J^\mu$, leads to current conservation at this later time. In other words, the two previously defined initial conditions determine the sector of the phase space where the classical ABM becomes equivalent to ED for all time. A proof of the quantum equivalence between the two models to all orders in perturbation theory is given in Ref. [15], following similar steps to those in Ref. [17] for the ANM.

Acknowledgments

LFU thanks the Organizing Committee of the International Workshop on CPT and Lorentz Symmetry in Field Theory for a wonderful meeting in Faro. LFU has been partially supported by the project CONACyT # 237503. CAE and LFU acknowledge support from the project UNAM (Dirección General de Asuntos del Personal Académico) # IN104815. CAE is supported by the CONACyT Posdoctoral Grant No. 234745.

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