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Structural aspects of Lorentz-violating quantum field theory

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Abstract. In the last couple of decades the Standard Model Extension has emerged as a fruitful framework to analyze the empirical and theoretical extent of the validity of cornerstones of modern particle physics, namely, of Special Relativity and of the discrete symmetries C, P and T (or some combinations of these). The Standard Model Extension allows to contrast high-precision experimental tests with posited alterations representing minute Lorentz and/or CPT violations. To date no violation of these symmetry principles has been observed in experiments, mostly prompted by the Standard-Model Extension. From the latter, bounds on the extent of departures from Lorentz and CPT symmetries can be obtained with ever increasing accuracy. These analyses have been mostly focused on tree-level processes. In this presentation I would like to comment on structural aspects of perturbative Lorentz violating quantum field theory. I will show that some insight coming from radiative corrections demands a careful reassessment of perturbation theory. Specifically I will argue that both the standard renormalization procedure as well as the Lehmann-Symanzik-Zimmermann reduction formalism need to be adapted given that the asymptotic single-particle states can receive quantum corrections from Lorentz-violating operators that are not present in the original Lagrangian.

1. Introduction

The precision and sensitivity of the experiments probing potential violations of Lorentz and/or CPT symmetries have increased astonishingly [1]. This has allowed for stringent bounds on part of the parameter space of the Standard-Model Extension (SME) [2, 3] and in turn, it calls for better predictions from the model. Thus, one would like to go beyond tree-level to compute the ensuing phenomenology and contrast these predictions with experiments. Of course, this phenomenological motivation is not the only motivation for this work. We also consider it of utmost importance to consolidate the SME as a consistent theoretical framework to unravel the status of fundamental symmetry principles and therefore, without disregarding the importance of the former, we consider it crucial to contribute towards the understanding of structural aspects of quantum field theories with broken Lorentz and/or CPT symmetries. Other conceptual issues regarding perturbative quantum field theory analyses within the SME have been studied in [4, 5, 6, 7, 8, 9, 10].

In a conventional renormalizable quantum field theory, one of the implications of Lorentz symmetry is that after radiative corrections are taken into account, the Lagrangian terms that are quadratic in the fields can acquire a mass shift and field-strength factors only. Although these quantities are divergent, the latter implies that they can be dealt with systematically by the introduction of appropriate counterterms. This means that such contributions can be treated by the renormalization of quantities existing in the bare Lagrangian. In spite of the fact that even for asymptotic states self-interactions are always present, the whole construction is as if they can be disregarded altogether.

The external states spanning the asymptotic Hilbert space are crucial to properly determine the physics of free particles and equally important, to understand the meaning of the perturbative expansion upon which Feynman calculus is predicated. Perturbatively, this is seen by the fact that the propagators for the external legs, describing the quantum mechanical states of in- and out-states, are identical in structure to the momentum-space dispersion relations derived from the part of the Lagrangian that is quadratic in the fields. This provides the correct description of the propagation of free-particle states. A non-perturbative rigorous justification for this feature is given by the Lehmann–Symanzik–Zimmermann (LSZ) reduction formula [11, 12].

In the presence of Lorentz violation, specifically in the framework of the SME, previous analyses involving the properties of freely propagating particles have been performed under the tacit assumption that the latter holds. However, one may wonder whether or not this line of reasoning is still applicable. Thus the question regarding the determination of free-particle properties in the context of Lorentz-violation is structural for its description as a quantum field theory. The possibility of having finite radiative corrections and their implications in this respect will prove essential to the discussion. In this conference proceedings I wish to call the reader's attention to [13] where we shed some light on these issues.

For simplicity and on phenomenological grounds a subset of the minimal SME's electrodynamics sector will be considered. For this reason, it will suffice to work to first order in the SME coefficients. Despite the particularity of the model, it illustrates some subtleties that are worth considering.

2. The model

We start by considering the bare Lagrangian of single-flavour QED within the minimal SME.

$$\begin{aligned}\mathcal{L}_{\mathrm{SME}} &= \tfrac{1}{2} i\,\overline{\psi}_B\,\Gamma^\mu_B \overleftrightarrow{D}^B_\mu\,\psi_B - \overline{\psi}_B\,M_B\,\psi_B - \tfrac{1}{4}(F_B)^2 \\ &\quad - \tfrac{1}{4}(k^B_F)_{\mu\nu\rho\sigma}F^{\mu\nu}_B F^{\rho\sigma}_B + (k^B_{AF})^\mu A^\nu_B \tilde{F}^B_{\mu\nu}\,. \end{aligned} \tag{1}$$

The Lorentz-violating effects are contained in the generalized gamma matrices Γ^μ_B and the generalized mass matrix M_B, which are given by:

$$\begin{aligned}\Gamma^\mu_B &= \gamma^\mu + c^{\mu\nu}_B\gamma_\nu + d^{\mu\nu}_B\gamma_5\gamma_\nu + i f^\mu_B + \tfrac{1}{2}g^{\lambda\nu\mu}_B\sigma_{\lambda\nu} + e^\mu_B\,, \\ M_B &= m_B + a^\mu_B\gamma_\mu + b^\mu_B\gamma_5\gamma_\mu + \tfrac{1}{2}H^{\mu\nu}_B\sigma_{\mu\nu}\,. \end{aligned} \tag{2}$$

This flat-spacetime Lagrangian is multiplicatively renormalizable at one-loop order [6].

We will confine to the case where only $c^B_{\mu\nu}$ and $(k^B_F)_{\mu\nu\rho\sigma}$ are non-vanishing. Without loss of generality the $c^B_{\mu\nu}$ coefficient can be taken as symmetric since its antisymmetric part can be removed by a field redefinition [2, 3]. Further, we restrict the LV photon sector to that without birefringent effects. This amounts to taking $(k^B_F)^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\tilde{k}^{\nu\sigma}_B - \eta^{\nu\rho}\tilde{k}^{\mu\sigma}_B - \eta^{\mu\sigma}\tilde{k}^{\nu\rho}_B + \eta^{\nu\sigma}\tilde{k}^{\mu\rho}_B)$, where $\tilde{k}^{\mu\nu}_B = (k^B_F)^{\mu\alpha\nu}{}_\alpha$. This choice of parameter space is made to limit the discussion to a minimum, self-consistent, and nontrivial case which nonetheless has possible phenomenological consequences. Specifically, it can be shown that, at leading order in any fermion-photon system, the $c^{\mu\nu}$ and $\tilde{k}^{\mu\nu}$ coefficients are observationally indistinguishable and only the combination $2c^{\mu\nu} - \tilde{k}^{\mu\nu}$ is measurable [14, 15, 16]. Paralleling perturbative calculations in ordinary Lorentz

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symmetric case, we introduce a gauge parameter ξ and also implement a prescription for dealing with IR divergences by means of a soft-photon mass m_γ [17]. In the spirit of perturbation theory one usually chooses a zeroth-order system with known solutions, such that the remaining piece can be considered a small perturbation. Usually the zeroth-order system is chosen as the full quadratic part of the Lagrangian. In this case two reasonable choices can be made. In the first scheme, one defines as a basis the renormalized quadratic Lagrangian of the conventional Lorentz-symmetric case. All Lorentz-violating contributions to the Lagrangian are then taken as perturbations. In the second scheme, one defines as a basis the full renormalized quadratic Lagrangian, including the Lorentz-violating part, while the non-quadratic contributions to the Lagrangian are considered to be perturbations. Different advantages, drawbacks and interpretations of each scheme are described in [13]. It can be shown, however, that their predictions at one-loop level are equivalent, as it should be.

Having done this, what will be called the $c\tilde{k}$ model Lagrangian is:

$$\mathcal{L}_{c\tilde{k}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_{c.t.}\,, \tag{3}$$

where

$$\begin{aligned}
\mathcal{L}_0 &= \bar{\psi}\left[i\left(\gamma^\mu + c^{\mu\nu}\gamma_\nu\right)\partial_\mu - m\right]\psi - \frac{1}{4}\left[\eta^{\mu\nu} + \tilde{k}^{\mu\nu}\right]\left[\eta^{\alpha\beta} + \tilde{k}^{\alpha\beta}\right]F_{\mu\alpha}F_{\nu\beta} \\
&\quad -\frac{1}{2\xi}\left[\partial_\mu A^\mu + \tilde{k}^{\mu\nu}\partial_\mu A_\nu\right]^2\,, && (4)\\
\mathcal{L}_1 &= \bar{\psi}\left[-e\left(\gamma^\mu + c^{\mu\nu}\gamma_\nu\right)A_\mu\right]\psi\,. && (5)
\end{aligned}$$

The counterterm Lagrangian has been omitted for brevity, see [13] for details.

3. Fermion two-point function and self-energy

The quantity of concern for the determination of external fermion states is the on-shell limit of the two-point function. In this case it adopts the form: $\Gamma^{(2)}(p) = \Gamma^\mu p_\mu - m - \Sigma(p^\mu)$, where $\Gamma^\mu = \gamma^\mu + c^{\mu\nu}\gamma_\nu$. The one-loop determination of $\Sigma(p^\mu)$ will allow us to find the corresponding equation for the interacting spinor w up to that order. If in the conventional case this is $P_0 w_0 = 0$, with $P_0 = \not{p} - m$ and w_0 is the free spinor, then the desired one-loop expressions are:

$$\begin{aligned}
Z_R^{-1}\,\bar{P}(p)w &= 0, \qquad \bar{P}(p) \equiv P_0 + \delta P && (6)\\
(\Gamma^{(2)}(p))^{-1} &= \mathcal{Z}_R \bar{P}(p)^{-1} - R. && (7)
\end{aligned}$$

The first term in Eq. (7) gives the one-particle pole and the second term R encodes additional off-shell effects and possibly other multiparticle physics, though it is regular around the pole. The goal is to determine $\mathcal{Z}_R, \delta P$ and R perturbatively at one-loop and to first order in LV. Extracting the the one-particle propagator pole is nontrivial as $\Gamma^{(2)}(p)$ no longer depends on $\not{p}$ only. In fact, defining $c^p_\gamma \equiv c^{\mu\nu}p_\mu\gamma_\nu$ and similarly for $\tilde{k}^p_\gamma$ we observe that LV functions c^p_γ and $\tilde{k}^p_\gamma$ do not commute with $\not{p}$. However, a generalization of the Lorentz-symmetric case is given by

$$\Gamma^{(2)}(p) = \mathcal{Z}_R^{-1}\,\left((c,k)^p_p\right)\,\bar{P}(p) + \bar{P}(p)\;\Sigma_2(\not{p}, c^p_\gamma, \tilde{k}^p_\gamma, (c,k)^p_p)\,\bar{P}(p)\,, \tag{8}$$

in the sense that it indeed has the desired property given in Eq. (7) and also is in accordance with a general form suitable for extracting the pole from the full fermion propagator as in Ref. [18].

Separating the usual Lorentz-symmetric contributions, from the LV ones with tensor structure present in the $c\tilde{k}$ model Lagrangian, and from the LV contributions with structures not present in the original Lagrangian, the fermion self-energy can be written as:

$$\Sigma(p^\mu) \;=\; \Sigma_{\text{LI}}(\not{p}) + \Sigma_{\text{LV}}(p^2, c^p_\gamma, \tilde{k}^p_\gamma) + \delta\Sigma(p^\mu, c^{\mu\nu}, \tilde{k}^{\mu\nu}). \tag{9}$$

The first term on the RHS of Eq. (9) corresponds to the usual Lorentz-symmetric contribution to the fermion self-energy. On the contrary, the the last two terms contain Lorentz-violating contributions to the fermion self-energy. By considering the behaviour of electromagnetic interactions under C, P, and T [13] and by defining $c_p^p \equiv c^{\mu\nu} p_\mu p_\nu$ and similarly for $\tilde{k}_p^p$, we can write to linear order in LV:

$$\Sigma_{\rm LV}(p^2, c_\gamma^p, \tilde{k}_\gamma^p) \;=\; f_2^c(p^2)\, c_\gamma^p + f_2^{\tilde{k}}(p^2)\, \tilde{k}_\gamma^p, \tag{10}$$

$$\delta\Sigma(p^\mu, c_p^p, \tilde{k}_p^p) \;=\; f_3^c(p^2)\, \frac{c_p^p}{m} + f_4^c(p^2)\, \frac{\not{p} c_p^p}{m^2} + f_3^{\tilde{k}}(p^2)\, \frac{\tilde{k}_p^p}{m} + f_4^{\tilde{k}}(p^2)\, \frac{\not{p}\tilde{k}_p^p}{m^2}, \tag{11}$$

where the functions $f_i^c(p^2)$ and $f_i^{\tilde{k}}(p^2)$ are calculable to any order in the fine-structure constant α. The difference between these two LV contributions is that the terms in $\Sigma_{\rm LV}(p^2, c_\gamma^p, \tilde{k}_\gamma^p)$ are of a form that is already contained in the original (bare) Lagrangian, while those in from $\delta\Sigma(p^\mu, c_p^p, \tilde{k}_p^p)$ are not. To proceed we will choose the second perturbation scheme described above Eq. (3) to perform a one-loop computation. The corresponding Feynman rules are:

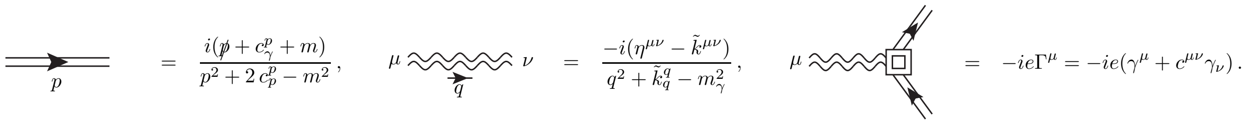

Figure 1. Feynman Rules in the $\xi = 1$ gauge.

4. Modified fermion kinetic operator and physically measurable quantities

From the one-loop expressions for the $f_i^c(p^2)$ and $f_i^{\tilde{k}}(p^2)$ functions obtained in [13] we determine the quantities we are looking for, i.e. $\mathcal{Z}_R$ and $\bar{P}$:

$$\mathcal{Z}_R^{-1} \;=\; 1 - \frac{\alpha}{\pi}\left[\ln\left(\frac{m}{m_\gamma}\right) - 1 + \frac{\gamma_E}{4} + \frac{1}{4}\ln\left(\frac{m^2}{4\pi\mu^2}\right)\right] - \frac{2\alpha}{3\pi m^2}(2\, c_p^p - \tilde{k}_p^p). \tag{12}$$

$$\bar{P}_1 \;=\; \not{p} + (c_{\rm ph})_\gamma^p - m_{\rm ph} + \frac{\alpha}{3\pi m}[2(c_{\rm ph})_p^p - (\tilde{k}_{\rm ph})_p^p]. \tag{13}$$

These are expressed in terms of the mass renormalization (which is as usual) and the corresponding expressions for the physically measurable LV coefficients, being:

$$m_{\rm ph} \;=\; m + \frac{\alpha}{\pi}\left[1 - \frac{3\gamma_E}{4} - \frac{3}{4}\ln\left(\frac{m^2}{4\pi\mu^2}\right)\right] m. \tag{14}$$

$$c_{\rm ph}^{\mu\nu} \;=\; c^{\mu\nu} - \frac{\alpha}{3\pi}\left[\frac{29}{12} - \gamma_E - \ln\left(\frac{m^2}{4\pi\mu^2}\right)\right](2c^{\mu\nu} - \tilde{k}^{\mu\nu}). \tag{15}$$

Strictly speaking we have not determined the physically measurable coefficient $\tilde{k}_{\rm ph}^{\mu\nu}$. As it differs from $\tilde{k}^{\mu\nu}$ by an order α expression we can substitute it in Eq. (13) consistently to first order in α.

The $\mathcal{O}(\alpha)$ LV radiative correction to the one-loop modified dispersion relation for asymptotic states is read off from the modified Dirac operator of Eq. (13). We observe that it is indeed proportional to $(2c_p^p - \tilde{k}_p^p)$ as required. Of course, we also find the usual shifts of conventional quantities as seen in Eqs. (12) and (14). This time, however, the fermion wave-function renormalization is no longer a constant but rather depends on the LV coefficients under

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consideration and on the external momentum. In addition to this we find asymptotic states in perturbative LV quantum field theory acquire higher-derivative μ-independent structures. These are the finite radiative corrections of tensor structures that were not present in the original Lagrangian referred to above and constitutes one of the key findings of this work. They stem from the $(c_{\rm ph})^p_p$ and $(\tilde{k}_{\rm ph})^p_p$ terms respectively. These features, by the way, are compatible with the general form of the one-particle fermion pole of the Källén-Lehmann representation derived in Ref. [18].

5. Asymptotic dispersion relation, the LSZ formula and S-matrix elements

The dispersion relation of asymptotic free-fermion states derived from the above is:

$$p^2 + 2(c_{\rm ph})^p_p - m^2_{\rm ph} + \frac{2\alpha}{3\pi}\left[2(c_{\rm ph})^p_p - (\tilde{k}_{\rm ph})^p_p\right] = 0. \tag{16}$$

In the Lorentz-symmetric case, scattering amplitudes are computed from on-shell external physical states. Presently, this is particularly delicate as the contributions of the finite radiative corrections whose structures are not present in the starting theory have no parallel in the usual derivation of S-matrix elements from the correlation functions. This leads to an adaptation of the LSZ reduction formula. This computation is too lengthy to reproduce here but we can outline its derivation and comment on its implications. Once in- and out- modified spinors are obtained a canonical quantization can be carried over and the Fourier expansion of the in- and out- fields are obtained from the correspondingly modified creation and annihilation operators. Next, the modified Feynman propagator can be obtained as the vacuum to vacuum time-ordered product of two in-field operators. As the wave-function renormalization is now dependent on LV quantities and also on the momentum of the external particle, this alters the usual "adiabatic" hypothesis, which relates the interacting-field operator at times much before/after the interaction takes place to the free-field operator. However, while considering all this, one can still express the fundamental scattering amplitude $\langle f|i\rangle$ in terms of vacuum to vacuum transition amplitudes of time-ordered products of interacting field operators and modified Dirac operators acting on the latter, together with wavefunction renormalization coefficients that account for self-interactions of the external states. In practical calculations, it is most useful to express the connected scattering amplitude in terms of truncated Green's functions as:

$$\begin{aligned}
\langle f|i\rangle_c &= (2\pi)^4\delta^4(\textstyle\sum p_i + \sum p'_i - \sum q_i - \sum q'_i)\cdots \\
&\times\ (-i)\mathcal{Z}_R^{\frac{1}{2}}((c,\tilde{k})^{q_1}_{q_1})\bar{u}_{in}(\vec{q}_1)\cdots i\mathcal{Z}_R^{\frac{1}{2}}((c,\tilde{k})^{p'_1}_{p'_1})\bar{v}_{in}(\vec{p}^{\,\prime}_1) \\
&\times\ G^{(2n)}_{\rm trunc}(-q'_1,\ldots,p_1,\ldots;-q_1,\ldots,p'_1,\ldots) \\
&\times\ u_{in}(\vec{p}_1)(-i)\mathcal{Z}_R^{\frac{1}{2}}((c,\tilde{k})^{p_1}_{p_1})\cdots v_{in}(\vec{q}^{\,\prime}_1)i\mathcal{Z}_R^{\frac{1}{2}}((c,\tilde{k})^{q'_1}_{q'_1})\cdots\,.
\end{aligned} \tag{17}$$

Eq. (17) embodies the Feynman rules for the scattering amplitude, incorporating:

- a momentum conserving delta-function;
- the amputated Green's function;
- a momentum-dependent wave-function-renormalization factor $\pm i\mathcal{Z}_R^{\frac{1}{2}}((c,\tilde{k})^p_p)$ for every external leg;
- a Dirac spinor for every external leg:
  - $u_{in}(\vec{p},\sigma)$ for an incoming fermion;
  - $\bar{u}_{in}(\vec{p},\sigma)$ for an outgoing fermion;
  - $v_{in}(\vec{p},\sigma)$ for an outgoing anti-fermion;
  - $\bar{v}_{in}(\vec{p},\sigma)$ for an incoming anti-fermion.

6. Discussion and conclusions

In this presentation we expect to have contributed to providing a detailed and thorough definition of perturbation theory in QFTs with a Lorentz-violating background. To do so we have focused on the on-shell limit of free-particle states, to first order in LV and the fine-structure constant α. A detailed computation of the fermion two-point function confirms our result is in agreement with other non-perturbative derivations [18]. Furthermore, the perturbative computations passed several cross-checks as the results were confirmed by two different perturbation schemes whose equivalence is not obvious from the beginning. Some results were also confirmed to be independent of the regularization method (by employing dimensional regularization and Pauli-Villars regularization. Also the relevant Ward-Takahashi identities were confirmed. We show a systematic reassessment of the adapted LSZ reduction formula that ultimately allows to express S-matrix elements as a series expansion in the fine-structure constant α with the corresponding set of Feynman rules to associate an amplitude to each term in the series. To illustrate our findings in [13] we analyze the non-trivial situation of infra-red divergences appearing in the scattering of a fermion off a stationary charge (Mott scattering). These issues endorse a laborious computation and allow us to summarize our conclusions.

(i) Asymptotic single-particle states receive concrete modifications due to radiative corrections. Contrary to the Lorentz symmetric case, some corrections appear as finite terms that were not in the original Lagrangian. Not all sub-sectors of the SME will necessarily suffer the same consequences, but at least it is the case for the model Lagrangian chosen, which is of considerable phenomenological relevance. However, one can expect that a more general LV theory or one with a wider parameter space might need to consider our formalism.

(ii) The wavefunction renormalization function depends on LV coefficients and on external momentum. The propagator pole is similarly altered, however, the pole structure of the fermion two-point function is expressible in a suitable manner, yielding appropriate interpretations for the mass-shell condition and renormalization conditions.

(iii) Non-trivial cancellation of IR divergences is also achieved in the fermion's dispersion relation and wavefunction renormalization function. This is illustrated for the Mott scattering cross section when the contribution of soft-photons from the external legs is considered.

(iv) All effects obtained up to the order considered are consistent with field redefinition that demands that only the combination $(2c^{\mu\nu} - \tilde{k}^{\mu\nu})$ is observable.

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