

# Simulating Counterfactual Representation\*

Andrew C. Eggers Benjamin E. Lauderdale January 2, 2016

#### ABSTRACT

We show how to use multilevel modeling and post-stratification to estimate legislative outcomes under counterfactual representation schemes that e.g. boost the representation of women or translate votes into seats differently. We apply this technique to two research questions: (1) Would the U.S. Congress be less polarized if state delegations were formed according to the principle of party proportional representation? (2) Would there have been stronger support for legalizing same-sex marriage in the U.K. House of Commons if Parliament more closely reflected the population in gender and age?

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### 1. INTRODUCTION

How would the voting outcomes of a legislature be different if it had more female members (Simon and Palmer 2010; Bump 2015; Sides 2015), more working-class members (Carnes 2012; Carnes and Lupu 2015), or more minority members (Cameron, Epstein, and O'Halloran 1996)? Answering these kinds of questions can help us understand existing variation in legislative voting and evaluate potential institutional reforms.

While counterfactuals about systems of representation can be illuminating, they can also be challenging to assess. A natural starting point is to estimate a behavioral model of legislative voting as a function of gender, class, race, or other features of interest, along with party and other predictors of voting decisions. We then want to predict the voting outcomes in a counterfactual legislature with a different distribution of legislator characteristics (e.g. more females). Complications arise, however, because the change being considered takes place at the aggregate level, while the model focuses on the individual level. For simple cases with linear models one can generate appropriate predictions (and confidence intervals) using algebra, but neither estimation nor inference is straightforward for models involving nonlinearities and interactions. It can also be challenging to incorporate the different kinds of uncertainty one can imagine in such a comparison, including uncertainty about the model and about the details of the counterfactual scenario being considered.

In this paper, we discuss a technique by which researchers can estimate and analyze aggregate counterfactual outcomes using legislative voting data. Our approach involves estimating a behavioral voting model and then using post-stratification to simulate representation under counterfactual distributions of legislator attributes. Post-stratification is most commonly used to address problems of *statistical* representation that arise in survey analysis with nonresponse (Särndal and Lundström 2005) and small-area estimation (Gelman and Little 1997; Park, Gelman, and Bafumi 2004). We suggest that this method is also useful in the counterfactual analysis of *political* representation.<sup>1</sup>

The key assumption behind our approach is that the relationship between legislator characteristics and legislator behavior would remain the same even as the distribution of characteristics changes. We call this the *behavioral stability assumption*. It is generally not testable and in many interesting counterfactual scenarios it is unlikely to hold. Even when we do not expect the behavioral stability assumption to hold, counterfactual exercises like the ones we carry out in this paper can be useful for highlighting the first-order effects of potential reforms, before any behavioral responses from voters, candidates, legislators and parties take place. Our approach could also be extended to incorporate the researcher's assumptions about how behavior *would* change as a result of particular reform efforts.

We apply the technique to two research questions. First, we consider how polarization in the U.S. Congress might change if state congressional delegations were chosen by proportional representation, such that e.g. 30% of the Massachusetts delegation would be Republican if 30% of voters in Massachusetts voted Republican. Our method shows that addressing party disproportionality at the state level would have yielded a small but noticeable reduction in polarization over the past twenty years, assuming that our behavioral model holds in the counterfactual scenario; this reduction in polarization occurs basically because relatively moderate Northeast Republicans and Southern Democrats were under-represented in this period. Polarization would have been *higher* un-

<sup>&</sup>lt;sup>1</sup>Multilevel regression with post-stratification for small area estimation involves estimating a general model using all the data and then simulating from the model given the known distribution of attributes for each small area. Our application is similar, except that in the second (post-stratification) step we use a counterfactual distribution of attributes.

der proportional representation in the 1960s, however. Second, we consider how the voting outcome on the 2013 same-sex marriage bill in the U.K. House of Commons might have been different if Parliament were more representative of the population in terms of age and gender. We show that addressing the overrepresentation of middle-aged men would probably have led to higher support for legalizing same-sex marriage (though with considerable uncertainty). Although the behavioral stability assumption is unlikely to strictly hold in either case, we view these as useful starting points for assessing the aggregate impacts of potential reforms.

The methods we discuss, if more broadly used, could improve the literature on legislative voting and representation in three main ways. First, in many cases researchers establish that legislator characteristics such as gender or class predict voting behavior. Although such studies tend to imply that aggregate changes in representation (due to e.g. changes in recruitment or campaign finance rules) would therefore have substantial impacts on aggregate legislative outcomes, these studies often do not provide explicit counterfactual analysis that illustrates the likely magnitude of these impacts. Our suggestions may encourage researchers to explicitly carry out these counterfactual analyses and thus help readers interpret their results. Second, researchers who do provide these counterfactuals often fail to grapple with the uncertainty in their estimates. We identify three types of uncertainty that one can consider incorporating into counterfactual analysis and provide a useful framework for incorporating them into estimation. Third, our approach enables researchers to explore counterfactuals that are not easily dealt with by examining single coefficients in a behavioral model; post-stratification is a more suitable approach when there are multiple features that change in the counterfactual and/or when those features enter into the behavioral model non-linearly or with interactions.

# 2. METHODS

Figure 1 provides a summary of the steps in our procedure for simulating counterfactual representation. The first step is to build a model of how the legislator's behavior of interest  $Y_i$  (voting, fundraising, etc.) varies as a function of the variables  $X_i$  whose distribution would be altered in the counterfactual scenario being considered. This model should take the form of a probability model  $p(Y_i|X_i,\Theta)$  describing the conditional distribution of the outcome as a function of the measured variables and estimated parameters  $\Theta$ . While a variety of modeling techniques could be applied here, multilevel models will be attractive for many of the same reasons they are attractive elsewhere (Gelman and Little 1997; Park, Gelman, and Bafumi 2004). The most important reason is that representation typically occurs within states, electoral districts, or other blocs, but often particular types relevant to the counterfactual (e.g. Massachusetts Republicans) are absent from the data. Multilevel models provide a framework for incorporating the necessary assumptions to fill in these gaps while also accounting for heterogeneity due to unobserved factors at the individual and state/region levels.

The second step is to generate a post-stratification scheme that reflects the counterfactual representation scenario. For legislative counterfactuals, this means specifying the count  $N_s$  corresponding to each type s (e.g. Republicans from New York; middle-aged female Conservatives) in the counterfactual legislature.<sup>2</sup> As in applications of multilevel regression and post-stratification (i.e. MRP) to small-area estimation, each type s specifies values of all attributes  $X_i$ in the behavioral model. One needs to specify the full joint distribution of all the attributes in the behavioral model so that outcome values can be simulated

 $<sup>^{2}</sup>$ These counts correspond to the weights typically used for post-stratification in the survey context. Using weights rather than counts would correspond to a hypothetical "infinite" legislature, where the granularity of individuals is ignored.

- 1. Estimate a behavioral model for legislators of the form  $p(Y|X, \Theta)$ , yielding posterior distribution  $p(\Theta|Y, X)$ .
- 2. Specify a post-stratification scheme either deterministically specifying a counterfactual set of legislator types X or a probability distribution p(X) from which each legislator's type  $X_i$  will be drawn.
- 3. For each  $t \in 1, 2, ..., T$  simulated legislatures of N legislators,
  - (a) Draw a realization of the behavioral model parameters from their posterior distribution  $\Theta \sim p(\Theta|Y, X)$
  - (b) If the post-stratification scheme is probabilistic, draw a realization of N legislator types  $X \sim p(X)$
  - (c) For each legislator i,
    - Draw a realization of the legislative behavior  $Y_i \sim p(Y_i|X_i, \Theta)$
  - (d) Characterize the aggregate behavior of all legislators in simulated legislature t
- 4. Characterize the distribution of the aggregate behavior of all legislators across T simulated legislatures.

Figure 1: Steps involved in estimating legislative behavior in a counterfactual legislature under the *behavioral stability assumption*, which is that the relationship between legislator characteristics and legislator behavior would remain the same even as the distribution of characteristics changes. These steps can be completed sequentially as above, or jointly via Bayesian simulation.

for each individual in the hypothetical legislature. In some applications, the researcher may want to treat this distribution as deterministic, i.e. specifying exactly how many of each type there will be in the counterfactual scenario; in others, the researcher may want to incorporate uncertainty about the distribution of types in the counterfactual scenario. Our first example below illustrates the first approach while the second illustrates the second.

The third step is to apply the behavioral model in the counterfactual legislature characterized by the post-stratification scheme and record the aggregate outcomes. The resulting predictions will accurately characterize outcomes in the counterfactual legislature only to the extent that the behavioral model  $p(Y|X, \Theta)$  estimated from observed behavior also applies under the counterfactual distribution of legislator types. This is the behavioral stability assumption.

The fourth and final step is to characterize the distribution of these outcomes over many simulations. The variation in aggregate outcomes across simulations will reflect three logically distinct sources of uncertainty that could be incorporated into the analysis. The first of these is model uncertainty, or uncertainty about parameters: how certain are we about how behavior varies as a function of representatives' attributes? This is taken into account in step 3a of Figure 1 either by simulating draws from the normal approximation to the MLE of the probability model (Gelman and King 1994; King, Tomz, and Wittenberg 2000) or by simulating draws from the Bayesian posterior of the probability model given some priors. The second source of uncertainty is post-stratification uncertainty, or uncertainty about the distribution of *types*: how certain are we about the prevalence of various relevant characteristics (e.g. party, gender) in the counterfactual legislature? This is taken into account in step 3b by drawing a new set of types from the probability distribution over  $N_s$  in each simulation. Third is finite-legislature uncertainty, or uncertainty about outcomes conditional on parameters and types: how uncertain are we about legislative outcomes (e.g. the results of a single roll call or the overall level of polarization) given that those outcomes are aggregated from individual realizations of a stochastic model for each of a finite number of hypothetical legislators? This is taken into account in step 3c by drawing outcomes for each legislator from the estimated  $p(Y_i|X_i, \Theta)$ rather than deterministically setting the outcome for each member of the counterfactual legislature to  $E(Y_i|X_i, \Theta)$ . Depending on the application, it may not be necessary to incorporate all of these types of uncertainty into the simulation, but the relevance of each to the desired counterfactual should be considered.

## 3. EMPIRICAL APPLICATIONS

# 3.1. Polarization in the U.S. Congress under State-Level Proportional Representation

As a first example of the kind of analysis we propose, we examine how polarization in the U.S. House of Representatives would differ if state congressional delegations were formed based on party proportional representation (PR) rather than on district-level plurality results. The basic approach is to model voting behavior as a function of party and state characteristics, and to use poststratification to estimate counterfactual outcomes in a legislature in which the number of Republicans and Democrats in each state delegation reflects statelevel voting patterns.

At the outset we note that the behavioral stability assumption introduced above is unlikely to hold in this counterfactual scenario. If congressional elections were based on state-wide PR, we might expect different candidates to be nominated and elected, different incentives for logrolling and strategic voting in the legislature, and different kinds of bills coming up for votes; each of these changes would violate the behavioral stability assumption. The counterfactual scenario we consider does not address any of these possible indirect effects of changing the current system of representation. Still, this and other similar counterfactual exercises clarify how individual-level behavioral variation relates to aggregate features of representation (e.g., are moderates under-represented, and to what extent?) while also giving insight into the first-order effects of possible reforms.<sup>3</sup>

We focus on legislators' DW-NOMINATE scores (Poole and Rosenthal 1997),

<sup>&</sup>lt;sup>3</sup>One can make an analogy to Duverger's classic distinction between mechanical and psychological effects: a change in the electoral system affects electoral outcomes through both channels, but studying each in isolation yields insights nonetheless (Duverger 1954).

which we model as a function of state-party means.<sup>4</sup> Because some states have single-party delegations, we cannot fit a fully-saturated regression model including all of the state-party interactions; even if we could fit such a model, it would perform poorly because most delegations are too small to provide much information about the state-specific means. Instead, we pool information across different states by modeling the state-party means hierarchically as a function of region, state-level presidential Democratic vote share  $P_j$  and congressional Democratic vote share  $C_j$  in the preceding election, for each party p. Thus, where j(i) is the state j of representative i, and r(j) is the region (Northeast, South, Midwest, West) of state j,

$$Y_i \sim N\left(\alpha_{j(i),p}, \sigma_p\right) \tag{1}$$

$$\alpha_{j,p} \sim N\left(\delta_{r(j),p} + \beta_{1,p} \mathbf{P}_j + \beta_{2,p} \mathbf{C}_j, \,\omega_p\right)$$
(2)

In words, the average DW-NOMINATE score for legislators from a given state and party is modeled as a linear function of the presidential and congressional voting outcomes in that state, with slope coefficients that vary by party and an intercept that varies by region and party. Clearly the behavioral model could be altered in various ways. We could, for example, allow nonlinearities in the relationship between electoral outcomes and voting behavior, or incorporate other predictors of congressional voting behavior such as measures of the relative locations of parties across state legislatures (Shor and McCarty 2011).

For the post-stratification stage of the simulation, we specify the distribution of seats across parties in the counterfactual PR legislature by applying the D'Hondt method<sup>5</sup> to party vote shares in the previous congressional election in

<sup>&</sup>lt;sup>4</sup>See Eggers and Lauderdale 2015, the replication archive for the analysis in this paper.

<sup>&</sup>lt;sup>5</sup>The D Hondt method of proportional representation is used to allocate seats in many PR systems including Spain, Denmark, the Netherlands, and Japan.

the state.<sup>6</sup> We chose to carry out the post-stratification step deterministically: we simply applied the formula to fixed electoral results to calculate the implied party composition as if the state-level party vote shares would not have changed under PR.<sup>7</sup> We then simulate 4,000 legislatures for each Congress from the 87th to the 112th, incorporating behavioral model uncertainty and finite legislature uncertainty through the Bayesian posterior simulation of the model.

In Figure 2 we show the trends in polarization, as measured by the distance between the average Republican and average Democrat. It has been well documented that polarization has risen over this period (McCarty, Poole, and Rosenthal 2006). Figure 2 shows that a large increase also would have taken place under the PR simulation over the same period, but with important differences. First, since about 1995 Congress would been less polarized in our PR counterfactual: in particular, the gap between the average Democrat and the average Republican would have been about 5% smaller. (Put differently, the increase in that gap between 1994 and 2010 would have been about 25% lower.) Second, between about 1975 and 1995 polarization would have been about the same in the actual and the counterfactual scenario. Third, before 1975 a hypothetical PR Congress would have been *more* polarized than the actual Congress was.

To explain these patterns, consider Figure 3. The left panel shows the average ideal point of Democrats and Republicans by region (South, Northeast, Midwest, and West) as estimated by our model. It shows that northern Repub-

 $<sup>^{6}</sup>$ We also ran the simulation using presidential vote shares, with similar results. A possible downside to using congressional results is that some races are uncontested and others are so lopsided that turnout and results may diverge from voting under a hypothetical PR system. The presidential and congressional results are most dissimilar in the 1960s, when Republican presidential candidates were popular in the South but southern congressional delegations remained solidly Democratic.

<sup>&</sup>lt;sup>7</sup>To incorporate post-stratification uncertainty, we would need a probabilistic model for the state-level vote shares under PR because D'Hondt is deterministic given vote shares; this model might then also include assumptions about how the state-level vote shares would change under PR.

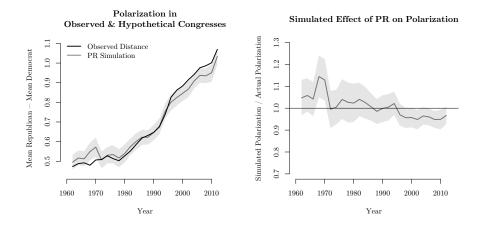


Figure 2: The left plot shows the distance between the average Republican and average Democrat as measured by DW-NOMINATE score, from the 87th to the 112th Congress, using the actual Congresses (black) and hypothetical Congresses elected by PR (blue). The 95% posterior bands around each line reflect uncertainty in the behavioral model and finite legislature uncertainty. The right plot shows the simulated effect of switching to PR on distance between the average Republican and average Democrat.

licans and (especially in the early period) southern Democrats have consistently been more moderate than their co-partisans in the other regions throughout the period we examine. The right panel shows the extent to which Republicans have been over-represented (and thus Democrats have been *under*-represented) in each region over time, compared to the PR counterfactual.<sup>8</sup> The most striking pattern relates to the South (dotted line): southern Republicans were consistently under-represented compared to PR (and thus southern Democrats were over-represented) until the early 1990s, at which point Republicans came to be over-represented (and thus Democrats were under-represented). Over the same period, the Republicans went from being over-represented in the Northeast to

<sup>&</sup>lt;sup>8</sup>These are shown relative to the mean over-representation in each election, which fluctuates substantially across elections because of the high responsiveness of the seats-votes curve. For the plot, we smooth the election-specific figures using a local linear regression with a normal kernel with a bandwidth (sd) of 4 years.

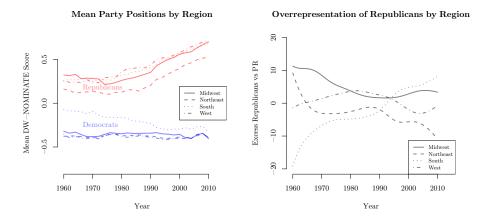


Figure 3: The left panel shows mean party position by region and party; northern Republicans and southern Democrats have consistently been more moderate than the rest of their party. The right panel shows the (smoothed) degree to which Republicans are over-represented in each region compared to a PR counterfactual, controlling for the overall level of Republican over- (or under-) representation in a given election. Democrats tended to be over-represented in the South before the 1990s and under-represented since then.

being under-represented. Combining this pattern with the one shown in the left panel of Figure 3, we can explain why (as we saw in Figure 2) polarization in our PR counterfactual is higher than the observed level of polarization in the 1960s but lower since about 1990: proportional representation makes outcomes more polarized in the 1960s primarily by curtailing the overrepresentation of moderate Democrats from southern states and moderate Republicans from northeastern states, while it makes outcomes less polarized since the 1990s by addressing the underrepresentation of moderate Democrats from southern states and moderate Republicans from northeastern states.

# 3.2. Voting on the U.K. Same-Sex Couples Act Under Improved Descriptive Representation by Age and Gender

As a second application of these methods, we consider a specific vote in the U.K. House of Commons.<sup>9</sup> On February 5, 2013, the House of Commons held its key vote on the passage of the Marriage (Same-Sex Couples) Bill, which proposed to legalize same-sex marriage in England and Wales. Each of the main party leaders offered a "free vote", meaning they did not formally instruct their MPs how to vote. As a result, and unusually for the House of Commons, voting revealed substantial internal disagreement within the parties: the bill was supported by 42% of Conservative MPs, 81% of Labour MPs, and 78% of Liberal Democrats (with the remainder either opposing or abstaining).

A cursory examination of the voting results indicates that female MPs were more likely to support the measure (74% support among female MPs compared to 57% among males) as were younger MPs (73% support among MPs under 40 compared to 59% among those 40); this suggests that support for the same-sex marriage bill might have been higher if the well-known under-representation of women and young people in the House of Commons were addressed. Our method is appropriate for assessing that conjecture.

As in the example of the previous section, we need two main elements to proceed: (1) a behavioral voting model that characterizes MPs' votes as a function of their characteristics, and (2) a counterfactual joint distribution of these characteristics that captures an alternative representation scheme. For our behavioral model, we fit a logistic regression model with party, sex, and age group (under 40, 40-59, 60 and over) without interactions.<sup>10</sup> We examine a counterfac-

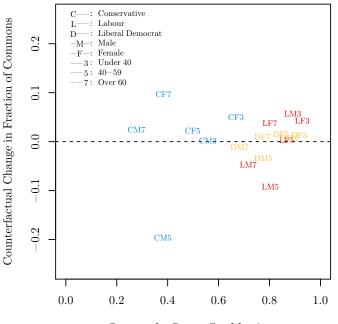
<sup>&</sup>lt;sup>9</sup>For data on MPs' age and gender, we used Hudson and Campbell 2015.

 $<sup>^{10}</sup>$ We also estimated a fully saturated interaction model, estimating the level of support for MPs in all of the 4 x 2 x 3 intersections of party, sex and age group. The results were nearly identical, as the logistic model fits the MP voting data remarkably well.

tual representation scheme in which the relative strengths of the different parties is held the same, but the joint distribution of age and gender within each party is changed to the distribution we observe for that party in the electorate; that is, we examine a situation in which Labour MPs look like 2010 Labour voters in terms of age and gender, while the overall share of Labour MPs is held the same, and likewise for the other parties.<sup>11</sup> We estimate the distributions of age and sex for supporters of each party using the 2010 BES post-election survey. In the counterfactual simulation, we do not fix the exact sex and age distribution for each party, but draw simulated MPs from that distribution, reflecting the fact that even if MPs were drawn from a pool that was representative of party supporters in terms of age and sex, absent a strict quota system there would not be an exact distributional match in any particular Parliament. Thus variation in outcomes across our 100,000 simulations of counterfactual parliaments voting on same-sex marriage reflects not just behavioral model uncertainty and finite legislature uncertainty (as in the previous example) but also post-stratification uncertainty.

The counterfactual analysis yields a mean posterior estimate of 65% support, with a 95% interval from 59% to 70%. In 88% of simulated votes the overall support is higher than the level of actual support for the bill (61%). We also carried out simulations in which we drew the distribution of age and gender by party from the *observed* joint distribution in the existing parliament. The 95% interval of the resulting distribution of voting outcomes runs from 56% to 66% support; in 81% of simulations the level of support was higher with the counterfactual distribution of age and gender by party than with the actual distribution. Thus taking into account uncertainty about model parameters, the distribution of types in the parliament, and the votes MPs cast, we have reason

 $<sup>^{11}</sup>$ Alternatively, we could have adjusted the party seat shares to be proportional to vote shares (as in our first example) or adjusted for gender and age without holding fixed the relative party strengths.



Support for Same-Sex Marriage

Figure 4: Age-gender categories that were relatively unsupportive of same-sex marriage tended to be over-represented in Parliament. The horizontal axis indicates each type of MP's predicted probability of supporting the same-sex marriage bill; the vertical axis indicates the difference between that type's share of a counterfactual Parliament that reflects the population in terms of age and sex (by party) and the actual Parliament.

to expect that there would be higher support for the same-sex marriage bill in Parliament if MPs more closely reflected the population in terms of gender and age, but the difference in expectation is not so large that we can be very confident of such a difference for any particular counterfactual Parliament.

Figure 4 helps explain the aggregate increase in support we see in our counterfactual simulations. For each type of MP (defined by party, gender, and age group), we show on the horizontal axis the estimated probability of supporting the bill and on the vertical axis the difference between the counterfactual share of that type and the actual share of that type (i.e. the change in weight between the two scenarios). By far the most over-represented group is Conservative males between 40 and 59 years of age (labeled "CM5"); the most underrepresented group is Conservative females over 60 ("CM7").<sup>12</sup> The reason we see the aggregate increase in support is that within each party the counterfactual scenario tends to involve down-weighting types that are relatively unsupportive of same-sex marriage (middle-aged men) and up-weighting types that are relatively supportive (women and MPs under 40).

Interestingly, this example serves as a reminder that improving descriptive representation need not improve the congruence between legislative behavior and public opinion. Recall that support among the public was around 55%, while support in the actual House of Commons was 61%; the expected level of support in a counterfactual House of Commons that was more descriptively representative with respect to age and gender is 65%. Thus addressing a lack of representativeness in terms of age and gender would tend to make the legislature less representative of public opinion on this particular issue. One possible explanation for this is that by making Parliament more descriptively representative dimensions (such as education and religious belief) where the unrepresentativeness of MPs tends to make them more favorable towards same-sex marriage.<sup>13</sup>

### 4. CONCLUSION

The statistical methods we have discussed in this paper offer a convenient way to communicate the implications of models of legislative behavior and to investigate

 $<sup>^{12}</sup>$ Recall that we hold party shares fixed in this counterfactual analysis, so the degree of over-representation across groups within a party must sum to zero.

<sup>&</sup>lt;sup>13</sup>Another explanation is one that arises in all assessments of congruence between public opinion and legislative voting: voting in Parliament is different from responding to a survey. It may be that our exercise has brought support in Parliament closer to what would have happened if a random sample of the population were actually asked to vote on the issue in Parliament.

the first-order impacts of possible institutional reforms, subject to the soundness of our behavioral model. These methods also allow us to convey our uncertainty about these implications.

These methods do not, of course, liberate the researcher from the usual difficulties of trying to say something about hypothetical scenarios. The approach we have highlighted tells us what would happen in a counterfactual legislature if the behavioral stability assumption holds: that is, if individual legislators' behavior remains the same even as the mix of legislators is altered. Of course, a huge literature on legislative politics tells us that agenda setting and legislative voting behavior would change in response to changes in the composition of the legislature. When the behavioral stability assumption seems unlikely to hold, researchers should adopt a more cautious interpretation of the results of counterfactual analysis: one might see the results as a measure of the first-order effects of a reform, or even as a way of interpreting the parameters of a behavioral model using illustrative hypothetical scenarios. Alternatively, if one has clear expectations about how a given reform would affect legislative behavior, one could relax the behavioral stability assumption and build these expectations into the model, extending the procedure we introduced above. While the methods we present do not solve any of the usual problems of counterfactual analysis, we hope that they do allow researchers to be clearer about the assumptions and implications of their counterfactual analyses.

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